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Hypergame Analysis in E–Commerce: A Preliminary Report

Maxime Leclerc[†], *Brahim Chaib-draa*[‡]

Résumé / *Abstract*

Dans les jeux classiques, il est supposé que "tous les joueurs voient le même jeu", i.e., que les joueurs sont au courant des stratégies et des préférences des uns et des autres. Aux vu des applications réelles, cette supposition est très forte dans la mesure où les différences de perception affectant la prise de décision semblent plus relevées de la règle que de l'exception. Des tentatives ont été faites, par le passé, pour incorporer les distorsions aux niveaux des perceptions, mais la plupart de ces tentatives ont été essentiellement basées sur le "quantitatif" (comme les probabilités, les facteurs de risques, etc.) et par conséquent, trop subjectives en général. Une approche qui semble être attractive pour pallier à cela, consiste à voir les joueurs comme jouant "différents jeux" dans une sorte d'hyper-jeu. Dans ce papier, nous présentons une approche "hyper-jeu" comme outil d'analyse entre agents dans le cadre d'un environnement multi-agent. Nous donnons un aperçu (très succinct) de la formalisation d'un tel hyper-jeux et nous expliquerons ensuite, comment les agents pourraient intervenir via un agent-médiateur quand ils ont des perceptions différentes. Après cela, nous expliquerons comment les agents pourraient tirer avantage des perceptions différentes.

In usual game theory, it is normally assumed that “all the players see the same game”, i.e., they are aware of each other's strategies and preferences. This assumption is very strong for real life where differences in perception affecting the decision making process seem to be the rule rather the exception. Attempts have been made to incorporate misperceptions of various types, but most of these attempts are based on quantities (as probabilities, risk factors, etc.) which are too subjective in general. One approach that seems to be very attractive is to consider that the players are trying to play “different games” in a *hypergame*. In this paper, we present a hypergame approach as an analysis tool in the context of multiagent environments. Precisely, we first sketch a brief formal introduction to hypergames. Then we explain how agents can interact through communication or through a mediator when they have different views and particularly misperceptions on others' games. After that, we show how agents can take advantage of misperceptions. Finally, we conclude and present some future work.

Mots clés : Théorie des jeux, hyper-jeux, médiation.

Keywords: Game Theory, Hypergame, Mediation.

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1 Introduction

In classical game theory, it is normally assumed that "all the players see the same game", i.e., they are aware of each other's strategies and preferences. This assumption is very strong for real life where differences in perception affecting the decision made seem to be the rule rather than the exception. Attempts have been made to incorporate misperceptions of various types from at least as early as 1956 [Luce 1956]. Perhaps the most notable theoretical development is the work of Harsanyi [Harsanyi 1968] on game with incomplete information played by "Bayesian" players, i.e, players having a subjective probability distribution over all the alternative possibilities. Harsanyi shows that if these probability distributions are mutually "consistent" (i.e., that each can be considered as a conditional distribution derived from a common "basic" distribution) the situation can be modeled using a game in which a lottery determining the "type" of each player is first conducted by nature. Each player knows the probabilities governing this lottery, but is only partially informed of the actual outcome, and hence, of the "types" of players he faces. Because of its subjective probability distribution, doubts arise as to the applicability of this model to real-life applications.

Recently, uncertainty in game theory has been addressed under risk control by Wu and Soo [Wu 1999]. Precisely, Wu and his colleague have shown how the risk control can be carried out by a negotiation protocol using communication actions of asking guarantee and offering compensation via a trusted third party.

In this work, we have taken the same road but rather than introducing uncertainty into the model, we have considered that the players are trying to play "different games". This approach suggested by Bennett [Bennett 1993] permits all types of differences of perception while still allowing the model to remain reasonably simple. In its first and simplest form, this approach takes as a structure not a single game, but a set of perceptual games, each expressing a particular player's perspective of the situation in question. Such a set of games was termed a *hypergame*.

The rest of the paper consists of the following. Section 2 presents a brief formal introduction to hypergames. Section 3 describes how agents with different views can coordinate their actions through a mediator through communication. Section 4 examines some cases where agents can gain advantage from the misperceptions. Section 5 compares our approach to some related work. Finally, section 6 concludes and presents some future work.

2 A Brief formal introduction to Hypergames

A hypergame can be specified by the following elements:

Players: They are the parties (individual agents, groups, coalitions, etc.) that may affect the multiagent situation that we want to study using the hypergame.

Strategies: Each player may see a number of combinations of actions available to herself and to each of the other players. Notice that all players may not recognize the same actions as being available for each given player since they do not perceive the same actions as relevant.

Preferences: For each player, her various strategies define a set of perceived outcomes. Usually, she prefers some outcomes to others and has some beliefs about other players' preferences.

Definition 1 An *n-person hypergame* is a system consisting of the following:

1. a set P_n, on n players,
2. for each p, $q \in P_n$, a non-empty finite set S_p^q which reflects the set of strategies for player p as perceived by player q,
3. for each p, $q \in P_n$, an ordering relationship O_p^q, defined over the product space $S_1^q, \ldots, S_m^q$ and which represents the preference ordering of p's strategies as perceived by q.

Thus, S_p^q and O_p^q express q's perception of p's options and aims. The set $S_1^q, \ldots, S_m^q$ makes up q's strategy matrix and together with P_n and the ordering $O_1^q, \ldots, O_n^q$ reflect player q's game G^q within the hypergame G. Thus, an hypergame G can be considered as a set of n game, $G^1, \ldots, G^n$, one for each player.

We assume that each player i makes her strategy choice with full knowledge of her own game G^i. Obviously, a player may realize that others may perceive the situation differently: if so, she may have more or less an idea as to what games they are trying to play. Or she may see only her own game, which she assumes to represent her perception shared by all.

To give an initial illustration, here is an example of a 2-player hypergame for which q perceives an option available to player p, option (i.e., the option c) which is not available for herself.

Agent p's Game G^p

$S_p^p \setminus S_q^p$	α	β
a	1,4	2,3
b	4,1	3.2

Agent q's Game G^q

$S_p^q \setminus S_q^q$	α	β
a	1,4	2,3
b	4,1	3,2
c	3,2	5,0

Having defined our hypergame, the final step is of course to *analyze* it using general principles, and hence to draw some conclusions. One could hope to define a uniquely rational course of action for each agent-player. If used in a normative way, the hypergame approach would thus provide a very definite prescription for the decision-maker to follow; if used descriptively–*under an assumption that agents will act rationally*–it would give a prediction of the outcome to be expected.

In order to analyze a hypergame, we must introduce some set of decision rules for the players. Such rules are based on the notion of a "dominant" strategy as specified by classical authors of game theory (see for instance [Rasmussen 1989]).

Definition 2 A strategy is called *dominant* strategy for a player iff choosing it leads to an outcome at least as highly preferred by that player as those obtained using any other of her strategies, whatever the strategy choice of the other player(s).

Notice that according to this definition, it is theoretically possible for a player to have several such strategies. Starting from the dominance, we can introduce the following:

Definition 3 We say one allocation of payoffs *Pareto-dominates* another, or is *Pareto-superior* to another, if all players are at least as well off in the first as in the second, and at least one is better off.

Definition 4 We say an allocation is *Pareto-efficient* or *Pareto-optimal* if it is not dominated by any other allocation.

Now we can formulate some decision rules for a hypergame [Bennett 1980].

Rule 1: If a player has a *dominant* strategy, then this player chooses that strategy.

Rule 2: If a player perceives that another player has a dominant strategy, she chooses the most preferred outcome of those available when the other player uses her dominant strategy.

Rule 3: In a non-conflict-game, if a player perceives an outcome that is most preferred by *every* player, then he chooses that strategy enabling this outcome to be reached.

Notice that rule 1 says if an agent has a dominant strategy, then she should use it; whereas rule 2 says if an agent believes that another has a dominant strategy, he assumes that he will use it and act accordingly. In the case where neither player has a dominant strategy and players nevertheless have a preferred outcome, the decision-maker can facilitate the preferred outcome in the non-conflict game case. This is what rule 3 suggests.

In addition to the dominant strategy, each of the outcomes listed in the hypergame will be analyzed for *stability*, for each player separately and also across the players.

Definition 5 An outcome is *stable* for an individual player if it is not reasonable for her to change the outcome by switching her strategy.

One criterion for the stability is the Nash equilibrium which can be expressed by [Rasmussen 1989]:

> *An outcome of a game is a* **Nash equilibrium** *if no player has incentive to deviate from her strategy given that the other players do not deviate.*

This equilibrium does not refer to other's preferences and consequently we can assume it is also valid in a hypergame.

3 Coordination with a Mediator

Suppose that the two players p and q are two agents representing two companies, each desiring "not to be aggressive about the other (in the sense of market)" but suspicious of the other. We can give a hypergame model of this situation by assuming that each player has a choice between a cooperative (C) strategy and an aggressive one (A). Player p, we suppose, places the four possible outcomes in the following order of decreasing preference:

(C, C) Co-existence;

(A, C) Attack without q retaliating;

(A, A) Mutual aggression;

(C, A) Attack by q without reply.

However, these preferences are not correctly perceived by q. In fact, q believes[1] p to have the following preference order :

(A, C) Attack without q retaliating;

(C, C) Co-existence;

(A, A) Mutual aggression;

(C, A) Attack by q without reply.

On the other hand, q has the same preferences as p and these preferences are also not correctly perceived by p which perceived them as q perceived those of p. This situation can be represented by the following 2-person hypergame.

Agent p's Game G^p

$S_p^p \setminus S_q^p$	C	A
C	4,3	1,4
A	3,1	2,2

Agent q's Game G^q

$S_p^q \setminus S_q^q$	C	A
C	3,4	1,3
A	4,1	2,2

Considering the situation from p's point of view by looking at the game G^p. In this game, p does not have a dominant strategy and consequently, she can use rule

[1] Notice that with hypergame, we are taking into consideration "high-order" beliefs, that is players' perceptions of each other's perceptions of the situation.

2 since q has a dominant strategy which is A. In these conditions, p assumes that q will adopt this aggressive strategy and consequently he is faced with outcomes (C, A) and (A, A). According to rule 2, it chooses to be aggressive also, that is, it chooses (A, A) which seems to be for her a Nash equilibrium. q reasons similarly on G^q.

With classical game, we cannot see the players' differing perceptions and consequently we cannot understand exactly why players deviate from cooperation. In fact, if each player had not mistaken each other's preferences, both would converge on the cooperation option.

Now suppose that p and q want to verify their misperceptions by communicating or by consulting a mediator. It is clear here that mediation and communication are both important in the presence of suspicious perceptions. If players can communicate, they can tell each other what actions they will take. Sometimes this works, as the players have no motive to lie and they trust each other. If the players cannot communicate, or they have motive to lie, or they do not trust each other, a mediator may be able to help by suggesting a *Pareto-efficient* allocation. The players have no reason not to take this suggestion, and might use the mediator even if her services were costly [Rasmussen 1989].

Now suppose, that p and q communicate their actions. In the case where p trusts q and this latter does not, the matrices reflecting p's perception and q's perception are the following:

Agent p's Game G^p

$S_p^p \setminus S_q^p$	C	A
C	4,4	1,3
A	3,1	2,2

Agent q's Game G^q

$S_p^q \setminus S_q^q$	C	A
C	3,4	1,3
A	4,1	2,2

Looking at the situation from p's point of view, it will be seen that neither p nor q have a dominant strategy and as the game is considered by p as non-conflict game, this player applies rule 3 and chooses (C, C) which is Nash equilibrium which dominates (A, A). Looking now at the situation from q's point of view now, it will be seen that this player has not been convinced by p and consequently she maintains her misperception on p. His reasoning is: p has a dominant strategy A and she must act on the assumption that p will adopt this strategy (according to rule 2). In this situation, q is faced with two choices (C, A) and (A, A). As she is rational, she will opt for (A, A). From an external point of view, p and q have opted for (C, A), that is that p will cooperate and q attack. This is a very bad choice for p.

Thus, communication between agents is very risky in the case where agents are motivated by lie, or they do not trust each other. In this specific case, it is better to consider a mediator (by paying its services) which might suggest a Pareto-efficient allocation.

To achieve that, each agent communicate her "exact" preferences to the mediator since this latter is in charge to find the Pareto-efficient allocation for agents. As external observer, this mediator m sees the "exact" perceptions of p and q represented by the following matrix:

m's perception on p and q

$S_p^q \setminus S_q^q$	C	A
C	4,4	1,3
A	3,1	2,2

Now, p and q both trust m and their respective perceptions are the following:

Agent p's Game G^p

$S_p^p \setminus S_q^p$	C	A
C	4,4	1,3
A	3,1	2,2

Agent q's Game G^q

$S_p^q \setminus S_q^q$	C	A
C	4,4	1,3
A	3,1	2,2

Now each agent supposes she is in cooperative-game and applies rule 3 that leads her to the dominant strategy (C, C), a Pareto-efficient allocation which dominates (A, A).

We have assumed here that the "exact" perception was the perception of p. If conversely, the mediator has received from p and q as "exact" perception the perception of q , i.e., (A, C), (C, C), (A, A) and (C, A), then we obtain as final perceptions of p and q:

Agent p's Game G^p

$S_p^p \setminus S_q^p$	C	A
C	3,3	1,4
A	4,1	2,2

Agent q's Game G^q

$S_p^q \setminus S_q^q$	C	A
C	3,3	1,4
A	4,1	2,2

Game now turns out to be the famous "Prisoner's Dilemma" (PD) for which the dominant strategy equilibrium is (A, A). which is worse than the strategy (C, C). To force p and q to adopt both the strategy (C, C), we add a new rule.

Rule 4: If two players x and y agree to choose an outcome under the supervision of a mediator m, then as soon as one of them deviates from this outcome, m informs the other.

If our players p and q follow this rule, they adopt the dominant strategy “forced equilibrium” (C, C) (which Pareto-dominates (A,A)) since they know if one of them deviates from this “forced equilibrium”, the other knows it (informed by m) and both switch to (A, A). Our rule 5 reduces in fact the DP matrix to only *two* outcomes (C, C) and (A, A) and where the first one dominates the second one. In this case, choices of players p and q are facilitated.

Thus, the PD usually used to model many different situations, including oligopoly pricing, auction bidding, political bargaining, etc. does give a rationale for some behaviors. But without an hypergame representation, the essential element of the story –misunderstanding–is left out.

4 Gaining Advantage from Differences in Perception

Suppose a 2-player hypergame for which p perceives two options c and γ which are not available for q. In p’s point of view, option c is an option for p and γ is an option for q.

Agent p’s Game G^p

$S_p^p \setminus S_q^p$	α	β	γ
a	1,3	2,3	2,3
b	4,1	3.2	3,2
c	3,2	6,0	2,3

Agent q’s Game G^q

$S_p^q \setminus S_q^q$	α	β
a	1,3	2,3
b	4,1	3,2

From q’s point of view, it can be seen that q believes that p will play strategy b and she will play β in order to obtain the stable outcome (b, β). The player p is far from this point of view since she perceives two additional strategies that q does not see. From her point of view, q has a dominant strategy which is γ and as she assumes that q is rational, she believes that q will opt for that strategy. Knowing that, p will opt for b so that she gains the best payoff. We are faced with two points of views, according to p, the stable outcome is (b, γ), whereas according to q, the stable outcome is (b, β).

Suppose now that p is curious and wants to know if q has or not the same perceptions. In this case, she could ask a third party which knows p and q for instance and this third party informed her (this latter can do that, if p for instance share with her the advantage gained) that q has a limited view and she does not view options c and γ. Knowing that, p might let q opting for β with the intention to choose c in order to obtain a more preferable outcome (c, β) than (b, β).

Notice that this case is similar to the case where q sees two options that p does not perceive and which can be represented by the following matrices.

Agent p's Game G^p

$S_p^p \setminus S_q^p$	α	β
a	1,3	2,3
b	4,1	3,2

Agent q's Game G^q

$S_p^q \setminus S_q^q$	α	β	γ
a	1,3	2,3	3,2
b	4,1	3,2	0,6
c	3,2	2,3	3,2

Notice that the reasoning is similar for the following cases:

1. p (or q) perceives one option c (or γ) for her but which is not available for q (or p);

2. p (or q) perceives one option γ (or c) for the other agent but which is not available for herself;

3. etc.

Suppose now that the points of view p and q are the following:

Agent p's Game G^p

$S_p^p \setminus S_q^p$	α	β	γ
a	1,3	2,3	2,3
b	3,1	3,2	4,3

Agent q's Game G^q

$S_p^q \setminus S_q^q$	α	β
a	1,3	2,3
b	4,1	3,2

In this case, q's reasoning is the same as previously and she believes that stable outcome is (b, β). p believes that q has a dominant strategy which is γ and consequently, she will opt for the outcome b. However, as she is uncertain about what q perceives as outcomes, she communicates with her in order to tell her the different options that she perceives: α, β and γ. Once q is convinced, both agents perceive the same options and the same preferences and in this case, p and q opt for (b, γ).

5 Preliminary Prototype

We have developed a simple prototype based on hypergame as explained in this paper. This prototype helps users to analyze some multiagent situations in which agents are not well-informed of each other's preferences and strategies. We have currently underway a work for evaluating this tool in the context of e-commerce and particularly in situations where several bidders negotiate competitively with a manager and they do not see the same games.

6 Related Work

Wu and Soo [Wu 1999a] have proposed an approach where both players of a PD deposit with the trusted third party the defection free guarantee as it is shown for instance in the following example [Wu 1999a].

A special case of a PD with C: Cooperate and P: Pink

$p \setminus q$	C	P
C	$-1, -1$	$-10, 0$
P	$0, -10$	$-8, -8$

A dilemma-free game matrix.

$p \setminus q$	C	P
C	$-1, -1$	$-10, -1.1$
P	$-1.1, -10$	$-9.1, -9.1$

In this example, if p and q deposit with a trusted third party the defection–free guarantee, say 1.1, then the game matrix can be changed into the a dilemma–free one as shown above. In this game, both agents play C and thus get a payoff -1 and take back the guarantee from the trusted third party. Authors of this approach have argued that by doing so, both agents could escape from the prisoner's dilemma.

Recall that the following game

$p \setminus q$	C	P
C	a, a	b, c
P	c, b	d, d

is a Prisoner's Dilemma if $c > a > d > b$. In the case of Wu's approach when agents deposit with the trusted third party, the game is not a PD game anymore since the new game represented by the dilemma-free game does not respect the rule $c > a > d > b$. This means that players inform the third party that they have a new game (the dilemma–free) which is not a PD game and for which they have a stable outcome (C, C). It seems here that authors have changed totally the game switching from a PD game to a game which is not a PD anymore and for which the stable outcome (C, C) is obvious.

Wu and Soo [Wu 1999] have proposed recently to separate the risk preference from utility function as another dimension other than the expected payoff. To model the preference of taking risk, they defined three types of agents with different risk preferences. In this way, they redefined the concept of dominance so that the optimal strategy that agents select can take risk into account. Authors showed that different risk preference agents in the same uncertain game could lead to different outcomes that satisfy the agent's risk preference.

Some other researchers have investigated the game theoretical approach for a multiagent environment. Some of them have only underlined the assumption that agents model each other as rational agents [Axelrod 1984], [Brafman 1997], [Koller 1997], [Rosenschein 1994], [Sheory 1998], [Tennenholtz 1998]. This is in conformity with the classical game which assumes that "all the players see the same game", i.e., they are aware of each other's strategies and preferences. As we have explained previously, this assumption is very strong for real life where differences in perception affecting the decision made seem to be the rule rather the exception.

Similar to our approach is the Recursive Modeling Method (RMM) [Durfee 1995], [**?**]. RMM views a multiagent environment from the perspective of an agent that is individual trying to decide what action it should take right now. In addition to this, RMM uses a decision-theoretic paradigm of rationality where an agent attempts to maximize its expected utility given its beliefs. The basic building block of RMM's representation is a payoff matrix, which succinctly and completely expresses the agent's beliefs about its environment. In addition to this payoff matrix, the agent has other matrices representing what it knows about the other agents' decision-making situation, thus modeling them in terms of their own payoff matrices. It can be noticed that there are two differences between RMM and our approach. Firstly and conversely to RMM, representation of payoff matrices in our approach allows for all types of differences of perception while still allowing the model to remain reasonably simple, that is very close to the classical representation of game theory. Secondly, RMM uses probabilities and in this sense it is closely related to work in game theory of Harsanyi Aumann' and others [Aumann 1995], [Brand 1993], [Harsanyi 1968]. In some applications, it may be difficult to determine these probabilities.

We may noticed that by bringing perceptions into the picture, one has opened an infinite regress similar to common knowledge [Fagin 1995] and the recursive modeling method (RMM) [**?**]. Indeed, we have allowed that p's view may differ from q's view of p's view. Such reasoning can lead to p's view on q's view on p's view on q's view and so on so far. This means that with hypergame, we are taking into consideration "high-order" beliefs, that is p knowledge on q knowledge on p knowledge, etc. These nested beliefs can be finite if we want to be practical in realistic situations [Fagin 1995]. As suggested by Durfee [Durfee 1995], it is important in these situations "to know just enough" to coordinate (or to reason) well. To do this, we can be selective about the nested perceptions we use, or even using some of them by exploiting communication, mediation and negotiation. Another avenue might be to allow agents to learn strategies of their opponents and then correct their misperceptions. All these aspects are scheduled in our agenda for future work.

7 Conclusion and Future Work

In this paper, the assumption that the various parties are playing the same game is dropped from the outset, so that alternative games can be set up and their consequences explored. Precisely, we have shown that an analysis based on these alternative games (i.e, a hypergame) provides a means of modeling multiagent situations in which misperceptions occur. In this way, we are able to consider a wider range of assumptions that we are not able to see if we have considered the classical game theory. It can be seen that switching from classical game theory to a hypergame approach offers two advantages. Firstly, by allowing misperceptions, we may arrive at a model of a given multiagent situation that is more "realistic" than could be given before. Secondly, by relaxing the constraints of classical game theory, we have generated a wider set of alternative models which can be very useful for the analysis of a multiagent system-whether this analysis is descriptive or prescriptive.

There are many extensions to this work. Among these extensions, we see a lookahead-based exploration strategy for a model-based learning agent that enables exploration of the opponent's behavior during interaction in a multiagent system [Putro 2000, Carmel 1995, Goldman 1995]. By adopting such strategy, an agent might correct her misperceptions in our hypergame approach. Another extension consists in seeing how we can be selective about the nested perceptions that we use in the recursive hypergame in order to have a tractable approach.

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