2002s-59

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Série Scientifique
Scientific Series

Montréal
Juin 2002

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ISSN 1198-8177

Tests for Breaks in the Conditional Co-movements of Asset Returns*

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Résumé / *Abstract*

Nous proposons des procédures pour tester le changement structurel dans les co-mouvements conditionnels d'actifs financiers. L'approche est basée sur la forme réduite et la procédure à deux étapes. L'avantage est qu'on utilise des rendements normalisés par leurs volatilités, une transformation qui peut s'effectuer sans intervention explicite d'un modèle paramétrique. La deuxième étape consiste à tester le changement structurel dans les corrélations conditionnelles. Le papier contient une application empirique avec des taux de changes.

We propose procedures designed to uncover structural breaks in the co-movements of financial markets. A reduced form approach is introduced that can be considered as a two-stage method for reducing the dimensionality of multivariate heteroskedastic conditional volatility models through marginalization. The main advantage is that one can use returns normalized by volatility filters that are purely data-driven and construct general conditional covariance dynamic specifications. The main thrust of our procedure is to examine change-points in the co-movements of normalized returns. The tests allow for strong and weak dependent as well as leptokurtic processes. We document, using a ten year period of two representative high frequency FX series, that regression models with non-Gaussian errors describe adequately their co-movements. Change-points are detected in the conditional covariance of the DM/US$ and YN/US$ normalized returns over the decade 1986-1996.

Mots clés : Changement structurel, ARCH, corrélations conditionnelles, données de haute fréquence

Keywords : Change-point tests, multivariate GARCH models, conditional covariance, high-frequency financial data

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* The authors would like to thank the special issue editor, Ruey Tsay, two anonymous referees for valuable comments as well as M. Dacorogna for providing us the data. The first author is also grateful to the Marie Curie Individual Fellowship MCFI-2001-01645.

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1 Introduction

There are many circumstances where one may expect that the co-movements between financial assets undergo fundamental changes. For example, portfolio holders may worry about the impact of the deregulation of an industry on their optimal allocation of assets which depends on conditional covariances (in a mean-variance setting). The deregulation may cause fundamental shifts in the (conditional) correlations across the asset holdings. Likewise, hedging strategies involving foreign exchange may be adversely affected by central bank policy shifts. Emerging markets is another example where the potential of breaks in co-movements may occur. The world equity markets liberalization and integration may represent an example of structural changes in the relationship of these markets. Similarly, the recent evidence of the Asian and Russian financial crises, transmitted across markets, have serious effects for investors, corporations and countries. The global character of financial markets presents an additional reason for examining the transmission of breaks and their effects in the co-movements between financial as well as real assets. Most financial asset pricing theories and models assume that covariances between assets are stable (possibly time varying) whereas more recent empirical approaches recognize the presence of time heterogeneity such as regime changes (e.g. Bollen et al., 2000), institutional changes (e.g. Garcia and Ghysels, 1998, Bekaert et al., 2002) and extreme events (e.g. Hartmann et al., 2000). Pastor and Stambaugh (2001) have also recently shown that structural breaks could be one of the explanation of the equity premium puzzle.

We propose procedures designed to uncover structural changes in multivariate conditional covariance dynamics of asset returns. The procedures are based on testing for breaks in the conditional correlations involving normalized or risk adjusted returns which are defined as the returns standardized by the conditional variance process. Hence the conditional correlation is equivalent to the conditional covariance process of normalized returns that may exhibit a general form of dependence (e.g. $\phi-$ or $\alpha-$mixing) as well as heavy tails. We start from a multivariate dynamic heteroskedastic asset return process. Instead of trying to explore the co-movements via a parametric specification and test for structural change in the parameters, we adopt a reduced form approach which consists of testing for structural change in static or dynamic relationships involving marginalizations of the multivariate process. Our approach relates to a large class of multivariate ARCH-type models with constant or dynamic conditional correlation (see, for instance Bollerslev et al., 1994). Although there is some loss of information when we look at the individual normalized returns, these losses are offset by gains in reducing the overparameterized multivariate GARCH type models and by focusing on the conditional covariance specification. In addition this approach provides a simple and computationally efficient framework for testing and estimating the unknown (multiple) breaks in the co-movements of volatility in a context that allows general forms of dependence as well as heavy tails without having to explicitly estimate their form.

The choice of standardized returns as an object of interest is motivated by both finance and statistics arguments. From the finance point of view the standardized returns represent the fundamental measure of reward-to-risk consistent with conventional mean-variance analysis. The statistical arguments are a bit more involved. Our approach can be viewed as a two-stage method for reducing the dimensionality of multivariate heteroskedastic conditional volatility models to a framework involving returns normalized by purely data-driven volatility filters in the first stage and cross products of normalized returns in the second stage. Recently, Engle (2002), Engle and Sheppard (2001) and Tse and Tsui (2002) rely on a similar two-stage procedure to handle multivariate GARCH models. Their stages are both parametric whereas ours involve a first stage that is purely nonparametric. Our reduction approach does not aim in presenting an alternative specification or estimation of multivariate GARCH models. Instead, we adopt this two stage approach as a method to perform change-point tests in multivariate heteroskedastic models as well as to isolate the source of breaks that may occur in the conditional covariance. The two-stage procedure can be considered as a semiparametric approach since the second stage can allow for general types of dependence, data-driven spot and quadratic volatility measures as well as leptokurtic or asymmetric distributions. More specifically, let $r_{(m),t} := \log p_t - \log p_{t-m}$ be the discretely observed time series of continuously compounded returns with m measuring the time span between discrete observations. We compute $X_{(m),t} := r_{(m),t}/\hat{\sigma}_{(m),t}$ involving purely data-driven estimators $\hat{\sigma}_{(m),t}$. Foster and Nelson (1996) proposed several rolling sample type estimators. Their setup applies to ARCH as well as discrete and continuous time SV models (which are in our application marginalizations of multivariate processes). In addition to the Foster and Nelson rolling volatility filters we also consider high-frequency volatility filters, following the recent work of Andersen et al. (2001), Andreou and Ghysels (2002a), Barndorff-Nielsen and

Shephard (2002), among others. The data-driven measures of normalized returns provide the estimation of the first stage in multivariate heteroskedastic returns models. Moreover, keeping the first stage data-driven has the advantage that we do not specify, and therefore also not potentially misspecify, a parametric model for volatility. This may eliminate potential sources of misspecification and avoid erroneous inference on the presence of structural breaks. In addition, the data-driven normalized returns process, $X_{(m),t}$, is invariant to change-points in the univariate volatility dynamics. The second stage deals with the conditional covariance defined as the cross-product of normalized returns, say $Y_{12,(m),t} := X_{1,(m),t}X_{2,(m),t}$, for a pair of assets given by the vector $(1,2)'$. This process may exhibit constant, weak or strong dependence (as in multivariate constant or dynamic correlation GARCH and Factor models, respectively) as well as a general functional form driven by a heavy tailed distribution. The simulation and empirical results in the paper show that risk adjusted returns, using various volatility filters, are in most cases non-Gaussian with different types of temporal dependence structure.

The paper extends the application of recent change-point tests in Kokoszka and Leipus (1998, 2000) and Lavielle and Moulines (2000) to the conditional covariance of Multivariate GARCH (M-GARCH) models, using the above two stage procedure for detecting breaks in the co-movements of normalized returns. The simulation results show that these tests have good size and power properties in detecting large structural breaks.

The paper is organized as follows. In section 2 we discuss the general models and their reduced forms as well as the transformations of the data that form the basis of the testing procedure. Section 3 discusses the recent change-point tests, developed in a univariate context, and a method to apply them to the conditional covariance processes of multivariate heteroskedastic models. The fourth section presents a brief Monte Carlo experiment that examines the statistical properties of normalized returns and provides a justification for the testing strategies adopted. The size and power of the aforementioned tests are also investigated. In the empirical section we document using a ten year period of two representative high frequency FX series, YN/US$ and DM/US$, that the conditional covariance specified by regression models of daily risk-adjusted returns with non-Gaussian errors describe adequately their co-movements. The main thrust of our procedure is then to examine breaks in the co-movements of normalized returns using CUSUM and least-squares methods for detecting and dating the change-points. A final section concludes the paper.

2 Models and filters

It has long been recognized that there are gains from modeling the volatility co-movements. In practice one stumbles on the obvious constraint that any multivariate model is hopelessly overparameterized if one does not impose any type of restriction (see for instance, Engle (2001) for some of the open questions in multivariate volatility models). Bollerslev et al. (1994) provide an elaborate discussion of various multivariate ARCH type models and review the different restrictions which have been adopted to make multivariate volatility models empirically feasible. Ghysels et al. (1996) discuss various multivariate SV models, both in discrete and continuous time. In this section we describe the classes of multivariate heteroskedastic models that fall within the context of our statistical procedures for change-point testing the dynamic co-movements of asset returns. Broadly speaking there are two classes of multivariate volatility models, both being among the most widely applied parametric specifications. These are (1) multivariate factor models, see for instance Diebold and Nerlove (1989), Engle et al. (1990), Harvey et al. (1994), Ng et al. (1992) and many others and (2) the conditional correlation models, see for instance Bollerslev et al. (1988), Bollerslev (1990), Bolleslev et al., (1994) and more recently Engle (2002), Engle and Sheppard (2001) and Tse and Tsui (2002). Since the statistical procedures adopted here share many features with the latter we will devote the first subsection to the conditional correlation volatility specification. A second subsection is devoted to factor models and a final third subsection describes various volatility filters which are adopted for dynamic heteroskedastic series.

2.1 Conditional correlation models

The statistics developed in this paper apply to a two-step procedure that shares several features with the recent work on Dynamic Conditional Correlation (henceforth DCC) of Engle (2002), Engle and Sheppard (2001) and Tse and Tsui (2002). The appeal of DCC models is that they feature the flexibility and simplicity of univariate ARCH models but not the complexity of typical multivariate specifications. This decomposition also presents an advantage for change-point detection in multivariate heteroskedastic settings, discussed further in section 3. The statistical inference procedures proposed apply to several multivariate specifications given that the conditional covariance process satisfies some general regularity conditions. It will be convenient to start with a discrete time framework and to set notation we assume that an n-vector of returns R_t is observed. In the empirical applications n will be equal to 2, but our techniques extend to $n > 2$. Consider the ratio $X_{i,t} := r_{i,t}/\sigma_{i,t}$ where $r_{i,t}$ and $\sigma_{i,t}$ is the return and conditional volatility (standard deviation) of the i^{th} return process, respectively, using the *univariate* filtration of each series separately. Then the conditional correlation between pairs of assets, e.g. $(1,2)'$ is:

$$\rho_{12,t} = E_{t-1}\left(X_{1,t}X_{2,t}\right) := E_{t-1}\left(Y_{12,t}\right) \tag{2.1}$$

where we denote $Y_{12,t} := X_{1,t}X_{2,t}$. The original specification of Bollerslev (1990) assumed that $\rho_{12,t} := \rho_{12}$, yielding a CCC model, i.e. a Constant Conditional Correlation multivariate specification. It was noted that the CCC specification offered many computational advantages, but the assumption of constant ρ_{12} did not share much empirical support (see e.g. Engle (2002) Engle and Sheppard (2001) and Tse and Tsui (2002) for further discussion).

The procedures proposed in this paper also involve the $X_{1,t}$, $X_{2,t}$ and $Y_{12,t}$ processes. However, these processes are obtained in a much more general context not involving a parametric specification for the conditional standard deviation $\sigma_{i,t}$ for $i = 1,2$. Engle (2002), Engle and Sheppard (2001) and Tse and Tsui (2002) assume that $\sigma_{i,t}$ follows a GARCH(1,1) model. We adopt a purely data-driven specification for $\sigma_{i,t}$, and this has several advantages. First this approach covers processes more general than the GARCH specification some of which can account for asymmetries as well as jumps (given the results in Foster and Nelson (1996), Andersen et al. (2001) and Andreou and Ghysels (2002a)). The purely data-driven first stage also has the advantage that we do not potentially misspecify the parametric model for volatility. Moreover, this approach may eliminate potential sources of misspecification avoid erroneous inference on the presence of structural breaks. This is related to the second advantage of the method proposed in that it yields a semi-parametric setup for the second stage of the test procedure that also allows for general innovation distributions. The reason for focusing on the normalized returns process, $X_{i,t}$, rather than $r_{i,t}$ or $\sigma_{i,t}$, in multivariate GARCH type models is due to the fact that the former process is invariant to certain structural breaks in the univariate dynamics and therefore enables us to isolate breaks in the conditional covariance/co-movements from breaks in the univariate volatility dynamics. We elaborate further on these points later in sections 3 and 4.

In the remainder of this subsection we will discuss only the basic underpinnings of filtering $\sigma_{i,t}$. The notation will be simplified here by dropping the subscript i pertaining to a particular return series, i.e. instead of $r_{i,t}$ we will simply write r_t because we will adopt mainly a univariate framework. The computation of r_t/σ_t with data-driven σ_t is valid in a diffusion context as well as various discrete time processes such as various ARCH type models including GARCH, EGARCH, SV and other specifications. The setup is deliberately closely related to the work of Foster and Nelson (1996) on rolling sample volatility estimators. Consider the following discrete time dynamics:

$$r_{(m),t} = \mu_{(m),t} m^{-1} + M_{(m),t} - M_{(m),t-m} \equiv \mu_{(m),t} m^{-1} + \Delta_{(m)} M_{(m),t} \qquad (2.2)$$

which correspond to the so called Doob-Meyer decomposition of the m horizon returns into a predictable component $\mu_{(m),t}$ and a local martingale difference sequence. The decomposition is a natural starting point when returns are generated by a standard diffusion process with stochastic volatility. The decomposition in (2.2) is also the starting point for discrete time ARCH type processes. Conditional expectations and variances with respect to the (univariate) filtration $\{\mathcal{F}_{(m),t}\}$ will be denoted as $E_{(m),t}(\cdot)$ and $Var_{(m),t}(\cdot)$ respectively, whereas unconditional moments follow a similar notation, $E_{(m)}(\cdot)$ and $Var_{(m)}(\cdot)$. Consequently:

$$Var_{(m),t}(r_{(m),t}) \equiv E[(\Delta_{(m)} M_{(m),t} - \mu_{(m),t})^2 | \mathcal{F}_{(m),t}] = \sigma^2_{(m),t} m^{-1} \qquad (2.3)$$

where $\sigma^2_{(m),t}$ measures the conditional variance per unit of time. We will consider various data-driven estimators for $\sigma^2_{(m),t}$ which can generically be written as:

$$\hat{\sigma}^2_{(m),t} = \sum_{\tau=1}^{n_L} w_{(\tau-t)} (r_{(m),t+1-\tau} - \hat{\mu}_{(m),t})^2 \qquad (2.4)$$

where $w_{(\tau-t)}$ is a weighting scheme, n_L is the lag length of the rolling window and $\hat{\mu}_{(m),t}$ is a (rolling sample) estimate of the drift. The optimal window length and weights are discussed in Andreou and Ghysels (2002a) and applied in the empirical section.

2.2 Multivariate factor volatility models

Before discussing the specifics of the tests and filters, it is worth elaborating on the fact that the testing procedures are not only applicable in the context of DCC models despite the fact that our two-step procedure shares several features with such models. We noted earlier that the first step is nonparametric. This implies that we can cover a wider class of models than is allowed for in parametric CCC and DCC settings. In particular, we can handle different types of multivariate GARCH models, involving common factor and other specifications.

The commonality in volatility across markets has resulted in several specifications involving common factors. Diebold and Nerlove (1989), Engle (1987), Engle et al. (1990), Harvey et al. (1994), Ng et al. (1992) are some examples of alternative multivariate factor models of the ARCH or SV type. Many of the parametric model specifications are either observationally equivalent or share many common features. A nice illustration is the work of Sentana (1998) who provides a formal nesting of the factor GARCH model of Engle (1987) and the latent factor ARCH model of Diebold and Nerlove (1989).

It will be useful to start with a BEKK representation of multivariate ARCH models (see Engle and Kroner (1995)). Namely, the vector of returns is assumed to be described as:

$$\begin{aligned} R_t | \, (R_{t-p}, p = 1, \ldots) &\sim D(0, H_t) \\ vech\,(H_t) &= vech(\Phi) + \sum_{s=1}^{q} A_s vech(R_{t-s}R'_{t-s}) + \sum_{r=1}^{q} B_r vech(H_{t-r}) \end{aligned} \tag{2.5}$$

where A_s and B_r are square matrices of order $n(n+1)/2$ and Φ is a vector of the same order. It is worth noting that the matrix H_t comprises the conditional variance and covariance specifications $h_{ij,t}$ in the multivariate model which are by definition different from the univariate conditional volatility process denoted by $\sigma_{i,t}$. The general BEKK representation can be linked to several classes of models that will be of specific interest for our analysis. These are (1) the k-factor GARCH(p,q) and (2) the latent factor ARCH model and (3) the diagonal GARCH model.

We will not examine the full vector R_t, instead we examine the components separately, i.e. we look at the marginal processes $r_{i,t}$ for $i = 1, \ldots, n$. Nijman and Sentana (1996) and Meddahi and Renault (1996) provide formulas describing the volatility dynamics of linear combinations δR_t implied by (2.5). For example, in the case of H_t being a multivariate GARCH or common factor model, Nijman and Sentana (1996) describe the (weak) GARCH implied univariate process for any linear combination, including δ equal to for instance the vector picking the first return, namely $(1, 0, \ldots, 0)$. Meddahi and Renault (1996) provide similar results for stochastic volatility models. We will not rely on these mappings. Instead we will adopt a reduced form approach, allowing for the flexibility of leaving the parameters (2.5) unspecified. While we give up efficiency of filtering we gain by being able to rely on purely data-driven filters and simple statistical procedures. Moreover, the sequential analysis adopted for detecting and dating the change-points in section 3 is well-suited in this reduced form framework given the savings in the degrees of freedom and related estimation complexities involved in multivariate models. In the next subsection we digress further on the filters which yield estimates for $\sigma_{i,t}$.

2.3 Transformations for asset returns using data-driven volatility filters

The test statistics discussed in the next section are based on functions of normalized returns computed as $(r_{(m),t} - \hat{\mu}_{(m),t})/\hat{\sigma}_{(m),t}$, for some estimator of $\hat{\mu}_{(m),t}$ and $\hat{\sigma}_{(m),t}$, i.e. some sampling frequency m and weighting scheme $w_{(\tau-t)}$ in (2.4). The empirical setting that will be used involves very short spans of data with high frequency sampling. We can deal with the local drift either by estimating it as a local average sum of returns or, following the arguments in Merton (1980) among others, ignore any possible drift and set it to zero, i.e. $\hat{\mu}_{(m),t} \equiv 0$. For simplicity of our presentation, we will adopt the latter, i.e. set the drift to zero.

The setup in (2.2) and (2.3) is the same as Foster and Nelson (1996) who derive a continuous record asymptotic theory which assumes that a fixed span of data is sampled at ever finer intervals. The basic intuition driving the results is that normalized returns, $r_{(m),t}/\sigma_{(m),t}$, over short intervals appear like *approximately i.i.d.* with zero conditional mean and finite conditional variance and have regular tail behavior which make the application of Central Limit Theorems possible. Foster and Nelson impose several fairly mild regularity conditions such that the local behavior of the ratio $r_{(m),t}/\sigma_{(m),t}$ becomes approximately *i.i.d.* with fat tails (and eventually Gaussian for large m). In their setup local cuts of the data exhibit a relatively stable variance, which is why $\hat{\sigma}_{(m),t}$ catches up with the latent true $\sigma_{(m),t}$ with judicious choices of the weighting scheme and in particular the data window chosen to estimate the local volatility. The tests allow for some local dependence in the data and do *not* rely on Normality of the ratio $r_{(m),t}/\hat{\sigma}_{(m),t}$. The empirical evidence of the Normality of $r_{(m),t}/\hat{\sigma}_{(m),t}$ is mixed at the daily level at least. Zhou (1996) and Andersen et al. (2000) report near-normality for daily sampling frequencies. We find that different classes of volatility filters yield different distributional properties for the normalized returns process, $X_{(m),t}$.

A number of alternative volatility filters, $\hat{\sigma}_{i,(m),t}$, are considered below which differ in terms of the estimation method, sampling frequency and information set (further evaluated in Foster and Nelson, 1996, Andersen and Bollerslev, 1998, Andersen et al., 2001, and Andreou and Ghysels, 2002a). These data-driven variance filters belong to two classes of volatilities. First, the interday volatilities are: (i) The Exponentially Weighted Moving Average Volatility defined following the industry standard introduced by J.P. Morgan (see Riskmetrics Manual, 1995) as: $\hat{\sigma}_{RM,t} = \lambda\hat{\sigma}_{RM,t-1} + (1-\lambda)\, r_t^2$, $t = 1, ..., T_{days}$, where $\lambda = 0.94$ for daily data, r_t is the daily return and T_{days} is the number of trading days. (ii) One-sided Rolling daily window Volatility defined as: $\hat{\sigma}_{RV,t} = \sum_{j=1}^{n_L} w_j r_{t+1-j}^2$, $t = 1, ..., T_{days}$, where n_L is the lag length of the rolling window in days. When the weights w_j are equal to n_L^{-1} then one considers flat weights. In our simulations we will consider $n_L = 26$ and 52 days to conform with the optimality in Foster and Nelson and the common practice of taking (roughly) one month worth of data (see e.g. Schwert (1989) among others). These interday volatilities are denoted as $\hat{\sigma}_{i,t}$ where $i = RM$, $RV26$, $RV52$. The second class of intraday volatility filters is based on the quadratic variation of returns (see Andreou and Ghysels, 2002a for more details) and includes: (i) One-day Quadratic Variation of the process also called Integrated Volatility (e.g. Andersen and Bollerslev, 1998) is defined as the sum of squared log returns $r_{(m),t}$ for different values of m, to produce the daily volatility measure: $\hat{\sigma}_{QV1,t} = \sum_{j=1}^{m} r_{(m),t+1-j/m}^2$, $t = 1, ..., n_{days}$, where for the 5-minute sampling frequency the lag length is $m = 288$ for financial markets open 24 hours per day (e.g. FX markets). (ii) One-day Historical Quadratic Variation (introduced in Andreou and Ghysels, 2002a) defined as the sum of m rolling $QV1$ estimates: $\hat{\sigma}_{HQV1,t} = 1/m \sum_{j=1}^{m} \hat{\sigma}_{QV1,(m),t+1-j/m}$, $t = 1, ..., T_{days}$. The intraday volatilities are denoted as $\hat{\sigma}_{i,t}$ where $i = QVk, HQVk$, for window lengths $k = 1, 2, 3$, in the 5-minute sampling frequency case. For window lengths $k > 1$ the intraday volatility filters $(H)QVk$ are simple averages of $(H)QV1$ for k days. Table A1 (Appendix A) presents the window length asymptotic efficiency equivalence results.

3 Tests for structural breaks in co-movements

There is a substantial literature on testing for the presence of breaks in *i.i.d.* processes and more recent work in the context of linearly dependent stochastic processes (see for instance, Bai (1994, 1997), Bai and Perron (1998) interalia). Nevertheless, high frequency financial asset returns series are strongly dependent processes satisfying $\beta-$mixing. Chen and Carrasco (2001) provide a comprehensive analysis of such univariate processes and Bussama (2001), Chen and Hansen (2002) have shown that multivariate ARCH and diffusion processes are also β-mixing. This result precludes the application of many aforementioned tests for structural breaks that require a much stronger mixing condition. Following Kokoszka and Leipus (1998, 2000) and Lavielle and Moulines (2000) we explore recent advances in the theory of change-point estimation for strongly dependent processes. These papers have shown the consistency of CUSUM and least squares type change-point estimators, respectively, for detecting and dating change-points. The tests are not model-specific and apply to a large class of weakly and strongly dependent (e.g. ARCH and SV type) specifications. So far only limited simulation and empirical evidence is reported about these tests. Andreou and Ghysels (2002b) enlarged the scope of applicability by suggesting several improvements that enhance the practical implementation of the proposed tests. They also find via simulations that the VARHAC estimator proposed by den Haan and Levin (1997) yields good properties for the CUSUM-type estimator of Kokoszka and Leipus (2000).

The Lavielle and Moulines (2000) and Kokoszka and Leipus (2000) studies can handle univariate processes while here we investigate multivariate processes via the two-step setup. It is demonstrated that the two-stage approach adopted here for multivariate models can be considered as a simple reduced form and computationally efficient method for the detection of structural breaks tests in multivariate heteroskedastic settings. The procedures proposed apply to the empirical process $Y_{12,t} := X_{1,t}X_{2,t}$ for pairs of assets, appearing in (2.1), where $X_{i,t} := r_{i,t}/\sigma_{i,t}$, $i = 1, 2$, is obtained via the application of a data-driven filter described in the previous section. Note that in this section we treat the case of a generic filter without elaborating on the specifics of the filter. This process is invariant to change-points in the univariate GARCH parameters and feeds into the second step that tests for breaks in the conditional covariance $Y_{12,t}$. The β-mixing property of multivariate GARCH and diffusion processes (Bussamma, 2001, Chen and Hansen, 2002) implies that $Y_{12,t}$ is β-mixing too. This is valid for the M-GARCH with dynamic conditional correlation specifications. For instance, according to the M-GARCH-DCC (Engle, 2002) $Y_{12,t}$ has a GARCH specification which implies β-mixing. The exemption being the M-GARCH-CCC according to which $Y_{12,t}$ is assumed to be constant. Last but not least, we note that in dynamic correlation M-GARCH models the quadratic transformations such as $|Y_{12,t}|^d$ $d = 1, 2$ are also β-mixing since they are measurable functions of mixing processes, which are β-mixing and of the same size (see White (1984, Theorem 3.49 and Proposition 3.23)).

The analysis focuses on the bivariate case for ease of exposition. This two-step approach can be easily extended to the multivariate n number of assets in the M-GARCH framework for which $n(n-1)/2$ cross-covariances, $Y_{ij,t}$, would present the processes for testing the change-point hypothesis in pairs of assets. Netherless, it is worth noting that when n gets large this framework becomes useful if we impose some additional restrictions. For instance, in the M-GARCH-CCC model when n gets large we can test the null hypothesis of joint homogeneity in the correlation coefficients in the pairs of normalized returns, ρ_{ij}, versus the alternative that there is an unknown change-point in the any of these cross-correlations. A similar approach for $n-$dependent processes can be found in Horváth et al. (1999) which can be adapted to the conditional covariances of an M-GARCH-CCC model.

In the remainder of this section we discuss the specifics of the testing procedures.

3.1 CUSUM type tests

Without an explicit specification of a multivariate ARCH, the tests discussed in this section will examine whether there is evidence of structural breaks in the data generating process of $Y_{12,t}$. To test for breaks Kokoszka and Leipus (1998, 2000) consider the following process:

$$U_N(k) = \left(1/\sqrt{N}\sum_{j=1}^{k} Z_j - k/(N\sqrt{N})\sum_{j=1}^{N} Z_j\right) \tag{3.1}$$

for $0 < k < N$ where $Z_t = |Y_{12,t}|^d$ $d = 1, 2$ in (3.1) represents the absolute and squared normalized returns in an ARCH(∞) process. When the conditional covariance process exhibits an ARCH-type specification, like in most dynamic conditional correlation M-GARCH models, we need not specify the explicit functional form of $Y_{12,t}$. Kokoszka and Leipus (1998, 2000) assume that ARCH(∞) processes are (i) stationary with short memory i.e. the coefficients decay exponentially fast, and (ii) the errors are not assumed Gaussian but merely that they have a finite fourth moment. Horváth (1997) and Kokoszka and Leipus (1998) show that (3.1) holds if now the process $Z_t := Y_{12,t}$ is linearly dependent. The above moment conditions need also apply to M-GARCH processes. The CUSUM type estimators are defined as:

$$\hat{k} = \min\{k : |U_N(k)| = \max_{1 \leq j \leq N} |U_N(j)|\} \tag{3.2}$$

The estimate $\hat{k}$ is the point at which there is maximal sample evidence for a break in the Z_t process. To decide whether there is actually a break, one has also to derive the asymptotic distribution of $\sup_{0 \leq k \leq N} U_N(k)$ or related processes such as $\int_0^1 U_N^2(t)dt$. Moreover, in the presence of a single break $\hat{k}$ is a consistent estimator of k^*. Under the null hypothesis of no break:

$$U_N(k) \rightarrow_{D[0,1]} \sigma_Z B(k) \tag{3.3}$$

where $B(k)$ is a Brownian bridge and $\sigma_Z^2 = \sum_{j=-\infty}^{\infty} Cov(Z_j, Z_0)$. Consequently, using an estimator $\hat{\sigma}_Z$, one can establish that under the null:

$$\sup\{|U_N(k)|\}/\hat{\sigma}_Z \rightarrow_{D[0,1]} \sup\{B(k) : k\epsilon[0,1]\} \tag{3.4}$$

which establishes a Kolmogorov-Smirnov type asymptotic distribution. Further details about the computation of the statistics and its application to multiple breaks in a univariate GARCH context can be found in Andreou and Ghysels (2002b).

3.2 Least Squares type tests

Bai and Perron (1998) have proposed a least squares estimation procedure to determine the number and location of breaks in the mean of linear processes with weakly dependent errors. The key result in Bai (1994, 1997), Bai and Perron (1998) is the use of a Hájek-Rényi inequality to establish the asymptotic distribution of the test procedure. Recent work by Lavielle and Moulines (2000) has greatly increased the scope of testing for multiple breaks. They obtain similar inequality results for weakly as well as strongly dependent processes. The number of breaks is estimated via a penalized least-squares approach similar to Yao (1988). In particular, Lavielle and Moulines (2000) show that an appropriately modified version of the Schwarz criterion yields a consistent estimator of the number of change-points. In the present analysis we apply this test to the following generic model:

$$Y_{12,t} = \mu_k^* + \varepsilon_t \quad t_{k-1}^* \leq t \leq t_k^* \quad 1 \leq k \leq r \tag{3.5}$$

where $t_0^* = 0$ and $t_{r+1}^* = T$, the sample size. The indices of the breakpoint and mean values μ_k^*, $k = 1, \ldots, r$ are unknown. It is worth recalling that $Y_{12,t}$ is a generic stand-in process. In our application, equation (3.5) applies to the cross-products of normalized returns for examining the change-point hypothesis in the conditional covariance of M-GARCH-CCC and -DCC type models. For dynamic conditional correlation models (3.5) can be augmented to

$$Y_{12,t} = \theta_{12} + \eta_{12} Y_{12,t-1} + v_{12,t}. \tag{3.6}$$

When the M-GARCH conditional correlation is assumed constant or when dealing with a single observed factor model (e.g. the market CAPM) with constant correlation, another auxiliary equation that may yield power for testing the structural breaks hypothesis is the regression between normalized returns e.g. $X_{1,t} = \theta'_{12} + \eta'_{12} X_{2,t} + v_{12,t}$. Note that this regression is not strictly equivalent to (3.5) for the conditional covariance that is derived from the M-GARCH-CCC reduction approach. Nevertheless, it can be considered as another auxiliary regression that relates to the conditional co-movements between assets in factor models as well as most conditional mean asset pricing theories. A useful example of this approach can be considered in the context of the one factor model that is used to model the market CAPM model. Let $r_{M,t}$ and $r_{i,t}$ be the demeaned returns on the market (indexed by M) and on the individual firm stock i at time t :

$$r_{M,t} = \sigma_{M,t} u_{M,t} \tag{3.7}$$

$$r_{i,t} = \beta_{i,t} r_{M,t} + \sigma_{i,t} u_{i,t} \tag{3.8}$$

where $u_{M,t}$ and $u_{i,t}$ are uncorrelated $i.i.d.(0,1)$ processes, $\sigma_{M,t}, \sigma_{i,t}$ and $\beta_{i,t}$ are, respectively, the conditional variance of $r_{M,t}$, the firm specific variance of $r_{i,t}$, and the conditional beta of $r_{i,t}$ with respect to $r_{M,t}$. Beta is expressed in the following way:

$$\beta_{i,t} = \frac{E_{t-1}[r_{i,t} r_{M,t}]}{E_{t-1}[r_{M,t}^2]} := \frac{\sigma_{iM,t}}{\sigma_{M,t}^2} \tag{3.9}$$

In the market CAPM equation (3.8), we divide by the idiosyncratic risk, $\sigma_{i,t}$, and write explicitly beta to obtain:

$$\frac{r_{i,t}}{\sigma_{i,t}} = \frac{\sigma_{iM,t}}{\sigma_{M,t}\sigma_{i,t}} \frac{r_{M,t}}{\sigma_{M,t}} + \frac{\sigma_{i,t} z_{i,t}}{\sigma_{i,t}}$$

If we define the normalized returns by $X_{i,t}$ and $X_{M,t}$, then the following regression type model arises:

$$X_{i,t} = \frac{\sigma_{iM,t}}{\sigma_{M,t}\sigma_{i,t}} X_{M,t} + z_{i,t}$$

or

$$X_{i,t} = \rho_{iM,t} X_{M,t} + z_{i,t} \tag{3.10}$$

where $\rho_{iM,t}$ represents the conditional correlation between the returns of the two assets. Two interesting cases arise in the context of (3.10). If $\rho_{iM,t} = \rho_{iM}$ then constant conditional correlation implies the process (3.10) is $\phi-$mixing. If $\rho_{iM,t}$ is a dynamic conditional correlation then (3.10) is $\beta-$mixing. In both cases the Lavielle and Moulines test can be applied. Note that the above example is restricted to observable factors and can be extended to n risky assets to obtain n regressions of normalized returns with the risk adjusted market portfolio. The change-point could be performed to each equation (3.10) to assess the stability of the co-movements of risky stocks with the market portfolio.

The Lavielle and Moulines tests are based on the following least-squares computation:

$$Q_T(t) = \min_{\mu_k^*, k=1,\ldots,r} \sum_{k=1}^{r+1} \sum_{t=t_{k-1}+1}^{t_k} (Y_{12,k} - \mu_k)^2 \tag{3.11}$$

Estimation of the number of break points involves the use of the Schwarz or Bayesian information criterion (BIC) and hence a penalized criterion $Q_T(t) + \beta_T r$, where $\beta_T r$ is a penalty function to avoid over-segmentation with r being the number of changes and $\{\beta_T\}$ a decreasing sequence of positive real numbers. We examine the properties of this test using both the BIC and the information criterion proposed in Liu et al. (1997) (denoted as LWZ). It is shown under mild conditions that the change-point estimator is strongly consistent with T rate of convergence.

4 Monte Carlo Design and Results

In this section we discuss the Monte Carlo study which examines the properties of normalized returns in univariate and multivariate heteroskedastic parameterizations as well as the properties of the Kokoszka and Leipus (1998, 2000) and Lavielle and Moulines (2000) change-point tests applied in a multivariate heteroskedastic setting. The design and results complement the findings of Andreou and Ghysels (2002a,b) who propose extensions of the continuous record asymptotic analysis for rolling sample variance estimators and examine the aforementioned tests for testing breaks in the dynamics of univariate volatility models.

4.1 Simulation design

The simulated returns processes are generated from the following two types of DGPs: (i) a univariate GARCH process with Normal and Student's t errors, and (ii) a multivariate GARCH process with constant correlation (M-GARCH-CCC) (Bollerslev, 1990) as well as dynamic correlation such as the *vech* diagonal specification proposed in Bollerslev, Engle and Wooldridge (1988) (M-GARCH-VDC). The choice of the M-GARCH-CCC and M-GARCH-VDC models is mainly due to their simplicity and parsimony for simulation and parameterization purposes. Moreover, the former multivariate design is most closely related to the univariate GARCH for which the Kokoszka and Leipus (2000) test has been derived. More specifically, the DGPs examined are:
(i) Univariate GARCH process:

$$r_{q,t} = u_{q,t}\sqrt{\sigma_{q,t}}, \quad \sigma_{q,t} = \omega_q + a_q u_{q,t-1}^2 + \beta_q \sigma_{q,t-1}, \tag{4.1}$$

where $r_{q,t}$ is the returns process generated by the product of the error $u_{q,t}$ which is $i.i.d.(0,1)$ with Normal or Student's t distribution function and the volatility process, $\sigma_{q,t}$ that has a GARCH(1,1) specification. The process without change points is denoted by $q = 0$ whereas a break in any of the parameters of the process is symbolized by $q = 1$ to denote the null and the alternative hypotheses, respectively, outlined below.
(ii) Multivariate GARCH process for a pair of assets $(1, 2)$:

$$\begin{aligned} r_{1,q,t} &= u_{1,q,t}\sqrt{h_{11,q,t}} + u_{2,q,t}h_{12,q,t} \qquad (4.2)\\ r_{2,q,t} &= u_{2,q,t}\sqrt{h_{22,q,t}} + u_{1,q,t}h_{12,q,t}, \quad t = 1, ..., T \quad \text{and} \quad q = 0, 1. \end{aligned}$$

where $r_{1,q,t}$ and $r_{2,q,t}$ are the returns processes that are generated by $u_{1,q,t}$ and $u_{2,q,t}$ $i.i.d.(0,1)$ processes and M-GARCH conditional variances:

$$\begin{aligned} h_{11,q,t} &= \omega_{11,q} + a_{11,q}u_{1,q,t-1}^2 + \beta_{11,q}h_{11,q,t-1} \\ h_{22,q,t} &= \omega_{22,q} + a_{22,q}u_{2,q,t-1}^2 + \beta_{22,q}h_{22,q,t-1} \end{aligned} \tag{4.3}$$

The conditional covariance in the M-GARCH-CCC (Bollerslev, 1990) is given by:

$$h_{12,q,t} = \rho_{12,q}\sqrt{h_{11,q,t}h_{22,q,t}}. \tag{4.4}$$

Similarly the conditional covariance in the M-GARCH-VDC (Bollerslev et al., 1988) is given by:

$$h_{12,q,t} = \omega_{12,q} + a_{12,q}u_{1,q,t-1}u_{2,q,t-1} + \beta_{12,q}h_{12,q,t-1}. \tag{4.5}$$

The models used in the simulation study are representative of financial markets data with a set of parameters that capture a range of degrees of volatility persistence measured by $\delta = a + \beta$. The vector parameters (ω, a, β) in (4.1) describes the following Data Generating Processes: DGP1 has $(0.4, 0.1, 0.5)$ and DGP2 has $(0.1, 0.1, 0.7)$ and are characterized by low and high volatility persistence, respectively. In order to control the multivariate simulation experiment the volatility processes in the M-GARCH equations in (4.3) are assumed to have the same parameterization. The sample sizes of $N = 500$ and 1000 are chosen so as to examine not only the asymptotic behavior but also the small sample properties of the tests for realistic samples in financial time series. The small sample features are particularly relevant for the sequential application of the tests in subsamples for detecting multiple breaks or repeated application involving the combinations of pairs of normalized returns. For simplicity and conciseness the simulation design is restricted to the bivariate case whereas it can be extended to $n > 2$ assets and the tests are applied to the pair combinations just as in the bivariate model.

The models in (i) and (ii) without breaks $(q = 0)$ denote the processes under the null hypothesis for which the simulation design provides evidence for the size of the K&L test. The simulation results are discussed in the section that follows. Under the alternative hypothesis the returns process is assumed to exhibit breaks. Four cases are considered to evaluate the power of the tests. The simulation study focuses on the single change-point hypothesis and can be extended to the multiple breaks framework (see for instance, Andreou and Ghysels, 2002b). In the context of (4.1) we study breaks in the conditional variance $h_{q,t}$ which can also be thought as permanent regime shifts in volatility at change points πN ($\pi = .3, .5, .7$). Such breaks may have the following sources: H_1^A : A change in the volatility dynamics, β_q. H_1^B : A change in the intercept, ω_q. H_1^C : A change in the conditional correlation, given by $\rho_{12,q}$ in (4.4) or $H_1^D : \omega_{12,q}$ in (4.5), and $H_1^E : \beta_{12,q}$ in (4.5).

The simulation investigation is organized as follows. First we examine some of the probabilistic properties of the normalized returns series generated from univariate and multivariate GARCH models. Second we investigate the performance of the K&L and L&M tests using a univariate and multivariate normalized returns framework. The univariate normalized returns represents the first stage of estimation of M-GARCH models proposed in Engle (2002), Tse and Tsui (2002). We show that this stage is invariant to breaks that may have occurred in either H_1^A, H_1^B, H_1^C. In the second stage we test for breaks in the cross-product of normalized returns or the regression of normalized returns using the K&L and L&M tests. We find that the normalized returns transformation and its cross-product, $X_{i,t}$ and $Y_{12,t}$, are invariant to breaks that occur in the univariate GARCH parameters (H_1^A, H_1^B) tested by K&L and L&M whereas this procedure has power to detect large breaks that occur in the conditional covariance, $Y_{12,t}$, tested by H_1^C or H_1^D and H_1^E. The simulation as well as empirical analysis is performed using the GAUSS programming language.

4.2 The standardized returns processes

The statistical properties of daily returns standardized by the volatility filters outlined in section (2.3) are discussed in the context of univariate and bivariate dynamic heteroskedastic structures described in the previous subsection. For the intraday volatility filters and for the purpose of simulation and parameter selection we take the univariate representation of each GARCH process for alternative sampling frequencies following Drost and Werker (1996, Corollary 3.2) who derive the mappings between GARCH parameters corresponding to processes with $r_{(m),t}$ sampled with different values of m. Obviously the Drost and Werker formulae do not apply in multivariate settings, but they are used here for the marginal process, producing potentially an approximation error as the marginal processes are not exactly weak GARCH(1,1). Using the estimated GARCH parameters for daily data with $m = 1$, one can compute the corresponding parameters $\omega_{(m)}$, $\alpha_{(m)}$, $\beta_{(m)}$, for any other frequency m. The models used for the simulation study are representative of the FX financial markets, popular candidates of which are taken to be returns on DM/US$, YN/US$ exchange rates. We take the daily results of Andersen and Bollerslev (1998) and compute the implied GARCH(1,1) parameters $\omega_{(m)}$, $\alpha_{(m)}$ and $\beta_{(m)}$ for 1-minute and 5-minutes frequency, $m = 1440$ and 288, respectively, (reported in Appendix A, Table A2) using the software available from Drost and Nijman (1993).

The normalized returns transformation is the process of interest following the discussion in section 2. According to the univariate GARCH process, (4.1), the standardized returns process $X_{i,(m)} := r_{i,(m),t}/\sigma_{i,(m),t}$ is by definition $i.i.d.(0,1)$. The 'true' standardized returns of the univariate GARCH is given for the 1-minute sampling frequency and the corresponding parameters presented in Table A2 (Appendix A). The quadratic variation intraday estimators defined in section 2.3 are specified by aggregating the 'true' squared returns process for 5-, 30- and 60-minutes sampling frequency. The remaining volatility filters in section 2.3 are the spot volatilities which are specified here using daily frequencies. Evaluation of how well the returns standardized by the data-driven volatility filters approximate the true parametric structure is based on the contemporaneous Mean Square and Absolute Errors (MSE and MAE) using as benchmark the MSE (and MAE) of that derived for $X_{QV1,t}$. We also evaluate the statistical properties of these ratio transformations by testing their Normality assumption using the Jarque and Bera (1980) test as well as whether they adequately capture the nonlinear dynamics by testing for any remaining ARCH (Engle, 1982) in $X_{i,t}$. According to the bivariate GARCH process with constant or dynamic correlation, (4.4) or (4.5), respectively, the normalized returns is expected to be a dependent process. We note that one can generalize the above simulation design to a multivariate n-dimensional model as well as augment it for jumps.

The simulation results in Table 1 summarize the statistical properties of the daily returns standardized by the alternative volatility filters defined in section 2.3. The simulated processes refer to a bivariate GARCH-CCC with $\rho_{12} = 0$ and 0.5 (where the former case reduces to a univariate GARCH). The first panel of Table 1 presents the relative MSEs (MAEs) ratios vis-a-vis the MSE (MAE) of $X_{QV1,t}$ for assessing the relative efficiency of each data-driven standardized returns compared with the simulated 'true' return-to-volatility ratio. These results suggest that the lowest MSE (MAE) ratios are attributed to $X_{RV26,t}$, $X_{QV2,t}$ and $X_{HQV1,t}$ for the three different categories of data-driven volatility filters in section 2.3. These results are valid for both GARCH and M-GARCH-CCC processes, Normal and Student's t

errors and for both measures of efficiency. They are also consistent with the extensive simulation evidence in Andreou and Ghysels (2002a). The statistical properties of the returns-to-volatility simulated series are also assessed with respect to their distributional and temporal dependence dynamic properties. The Normality test results in the second panel of Table 1 show that under the Normal-GARCH, there is general simulation evidence in favor of the Normality hypothesis for all standardized returns series (at the 5% significance level) except for $X_{RM,t}$. However, under the more realistic assumption of a t-GARCH, arising from the heavy-tailed high-frequency data, we do not find supportive evidence of the Normality hypothesis in $X_{RM,t}$, $X_{RV52,t}$ and $X_{HQV3,t}$. It is worth emphasizing that the returns which exhibit heavy tails are standardized by the three different volatility filters all of which share some of the optimality features discussed in Foster and Nelson (1996) and Andreou and Ghysels (2002a), namely exponential weights, rolling estimation and optimal window length. Finally, in the last panel of Table 1 we present the simulation results from testing any remaining ARCH effects in normalized returns. We find evidence in favor of no remaining second-order dynamics in all risk-adjusted returns by interday and intraday volatility filters, under both Normal and Student's t univariate GARCH processes. The results present evidence that univariate returns process normalized by optimal volatility filters yield an approximately independent series with a distribution that has different tail behavior depending on the standardizing filter employed. If the process is generated by an M-GARCH process the normalized returns process is expected to exhibit second-order dependence and fat tails due to remaining heteroskedasticity. We find that $X_{i,t}$ are non-Gaussian and approximately independent processes for univariate GARCH models. Hence the cross product process $Y_{12,t} = X_{1,t}X_{2,t}$ is an independent process for univariate GARCH and M-GARCH-CCC processes whereas it exhibits nonlinear dependence for M-GARCH-VDC models. Generally the brief simulations support the assumptions required for the application of change-point tests discussed in section 3.

4.3 Simulation results of change-point tests

The theoretical and simulation evidence regarding the probabilistic properties of normalized returns satisfy the conditions of the Kokoszka and Leipus (2000) and Lavielle and Moulines (2000) tests discussed in section 3. Hence this section examines the properties of these tests for change-points in the conditional covariance of asset returns generated by M-GARCH processes with constant and dynamic conditional correlation. Note that the simulations discussion focuses on $N = 1000$ and $\pi = 0.5$ for conciseness purposes. Similar results are obtained for $\pi = 0.3, 0.7$ and $N = 500$, expect that the tests have less power in detecting early change points for the smaller sample.

In section 2 we discuss the reduced form approach adopted for M-GARCH models. The first stage involves the univariate specification and estimation of conditional variance dynamics which yields the normalized returns process for each asset, $X_{1,t}$ and $X_{2,t}$. The second stage involves the specification of the conditional covariance dynamics. For M-GARCH processes the conditional covariance is specified as the cross-product of pairs of normalized returns for assets 1 and 2 given by $Y_{12,t} = X_{1,t}X_{2,t}$. The equations for $Y_{12,t}$ which we use for change-point testing are given by:

$$Y_{12,t} = \rho_{12} + u_{12,t} \tag{4.6}$$

and

$$Y_{12,t} = \theta_{12} + \eta_{12} Y_{12,t-1} + v_{12,t} \tag{4.7}$$

Equations (4.6) and (4.7) represent the constant and dynamic conditional correlation of M-GARCH-CCC and M-GARCH-VDC models, respectively. The AR (4.7) of the conditional correlation as well its ARMA generalizations have been discussed in Engle (2002) and Tse and Tsui (2002) who propose that it can be used for the specification and estimation of a new class of M-GARCH models with dynamic conditional correlation. Similarly, (4.7) has been used in Tse (2000) as the auxiliary equation for testing the null hypothesis of constant conditional correlation ($\eta_{12} = 0$) implied by an M-GARCH-CCC.

The simulation results regarding the properties of change-point tests commence with the evaluation of the Kokoszka and Leipus (K&L) test for the M-GARCH-CCC. These results are reported in Table 2a. First we examine the change-point hypotheses in the first stage of specification and estimation which involves the univariate process of normalized returns, say $X_{1,t} = r_{1,t}/\widehat{\sigma}_{1,t}$ using two representative spot volatility filters to obtain $X_{RV26,t}$ and $X_{RM,t}$. Once more for conciseness purposes we do not report all the volatility filters for standardizing the returns process. For the change-point simulation analysis we focus on the $X_{RV26,t}$ and $X_{RM,t}$ series which are applicable in a broader sense given their daily sampling frequency as well as the relationship of the RiskMetricks with IGARCH models. Note that the empirical analysis considers all volatility filters discussed in section 2.3. The simulation results in Table 2a show that the K&L test has good size properties for $X_{RM,t}$ and $X_{RV26,t}$ and appears to be robust to any change-points in the univariate GARCH parameters or the correlation coefficient, as shown by any of the alternative hypotheses, H_1^A, H_1^B, H_1^C. This result is due to the multiplicative structure of GARCH type models according to which any change-point in the parameters of $h_{i,t}$ or $\rho_{12,t}$ is offset by the corresponding change in $r_{i,t}$ so that the ratio $r_{i,t}/\widehat{\sigma}_{i,t}$ is homogeneous. The second stage of the M-GARCH parameterization involves the cross-product of normalized returns. First we study the K&L test properties for $Y_{12,RV26,t} = X_{1,RV26,t} * X_{2,RV26,t}$ and $Y_{12,RM,t} = X_{1,RM,t} * X_{2,RM,t}$. The remaining results in Table 2a show that the K&L has good size properties for the conditional covariance involving $RV26$ as opposed to RM which yields an undersized K&L test in the $Y_{12,RM,t}$ case. The exemption being $\sigma_{Y_{12}}^{RM}$ which represents the Risk-Metrics series of the cross-product. This simulation evidence also relates to the Foster and Nelson (1996) arguments regarding the efficiency of exponential weights. Given the alternative hypotheses of change-points in the GARCH parameters, H_1^A and H_1^B, the cross-products of normalized returns, $Y_{12,RV26,t}, Y_{12,RM,t}, \sigma_{Y_{12}}^{RM}$, or their quadratic transformations yield no power. Summarizing, the K&L test simulation results show that the normalized returns series or the cross-product of such series are found to be robust to change-points in the univariate GARCH parameters. The last panel of Table 2 shows that the K&L test applied to $Y_{12,t}$ is able to detect large change-points in the correlation parameter ρ_{12} of the M-GARCH-CCC model. Note that the linear and quadratic (rather than absolute) transformation of $Y_{12,t}$ yield more power. In Table 2b we also examine the properties of the K&L test in the presence of the M-GARCH-VDC model and find it has less power in detecting change-points in the dynamics of conditional covariance (shown by H_1^D and H_1^E) as opposed to the M-GARCH-CCC (H_1^C) case. The Risk-Metrics of normalized returns, $\sigma_{Y_{12}}^{RM}$, turns out to be the transformation with the highest power especially in detecting large drops in the dynamics of the conditional covariance parameter. Recall that the test is undersized for the cross-product of normalized returns and is robust to breaks in the univariate GARCH, H_1^A and H_1^B.

The change-point hypothesis in the conditional covariance is also examined using the Lavielle and Moulines (L&M) test. Tables 3a and 3b report simulation evidence regarding the properties of the Lavielle and Moulines (L&M) in detecting relatively large breaks in the cross product of normalized returns using both M-GARCH-CCC and M-GARCH-VDC simulated processes. These Tables report the frequency distribution of the number of breaks. The highlighted numbers refer to the 'true' simulated number of breaks.[1] The results in Table 3a show that the L&M using both information criteria, the BIC and LWZ, have very good size properties but the former criterion yields better power except for small change-points. It is also interesting to note that the BIC criterion performs better for detecting downward rather than upward shifts in the ρ_{12} which is useful given the importance associated with downside risk in financial markets. The simulations also show that the tests are relatively more powerful in detecting breaks in the co-movements of M-GARCH-CCC rather in the -VDC processes. An explanation for the poor results in the latter process (shown in Table 3b) may be the static instead of the dynamic auxiliary regression for $Y_{12,t}$. Tables 4a and 4b also apply the L&M to the least squares regression of pairs of normalized returns:

$$X_{1,t} = \theta'_{12} + \eta'_{12} X_{2,t} + v_{12,t} \tag{4.8}$$

which is valid for the constant conditional correlation models. The results show that both information criteria, the BIC and LWZ, have very good size properties. In evaluating the power properties of the L&M test we could go directly to the last panel of Table 5a to examine H_1^C: a change in ρ_{12}. The highlighted results show that the BIC yields more power than the LWZ criterion for the L&M test which detects breaks in both directions and DGPs except when those are small in size (e.g. a 0.1 parameter change). The results regarding the remaining alternative hypotheses (H_1^A and H_1^B) show that the L&M test also detects breaks in the bivariate relationship of normalized returns when the source of these change-points rests in the univariate GARCH dynamics rather than in the co-movements. The size of such change-points plays an important role in that large changes (e.g. at least a 50% parameter increase) can be detected by the L&M test. The above results also hold if the simulated process is an M-GARCH-VDC shown in Table 5b, except that the size of the change-point needs to be even larger in either the conditional variance or covariance dynamics for the test to exhibit power. It is also interesting to note that in comparing the normalizing volatility filters we find that the regression involving $X_{RM,t}$ yields more power in detecting change-points in the conditional covariance of the M-GARCH-VDC whereas for the M-GARCH-CCC both $X_{RM,t}$ and $X_{RV26,t}$ yield similar power properties.

[1] The analysis refers to a single break but can be extended to multiple breaks as discussed in Andreou and Ghysels (2002b) univariate GARCH framework.

The brief simulation design for the properties of the K&L and L&M tests presents the following results: First, the K&L CUSUM and L&M least squares tests applied to $X_{1,t}, X_{2,t}$ and $Y_{12,t}$ have power in detecting large change-points only in the conditional covariance, $Y_{12,t}$, of M-GARCH-CCC and less so for VDC models. It has no power in detecting change-points in the univariate GARCH parameters. Hence, the simulation study recommends that if there is a change-point in the conditional covariance dynamics of financial asset returns, the cross-product of normalized returns yields not only a method for identification of the source of the break but also power in detecting the break. In contrast, the univariate normalized returns, $X_{i,t}$, are invariant to change-points. The returns process $|r_{i,t}|^d$, $d = 1, 2$ has power in detecting change-points in the univariate GARCH parameters but not in the conditional covariance process. Finally, the regression of pairs of normalized returns in the L&M test detects large breaks in *any* of the M-GARCH-CCC or -VDC parameters. Hence the K&L and L&M tests could be applied to the processes $r_{i,t}, X_{i,t}, Y_{ij,t}$ (and their transformations) in a complementary approach to identify the source of change-point.

5 Empirical Analysis

5.1 Co-movements of FX normalized returns

The empirical section of the paper investigates the bivariate relationship between the daily YN/US$ and DM/US$ risk adjusted returns over a decade and tests for structural breaks in their co-movements. The empirical results complement the Monte Carlo analysis by examining further the stochastic properties of risk-adjusted FX returns and investigate the presence of structural breaks. The discussion is organized as follows. First, we test the hypotheses of Normality and independence for all YN/US$ and DM/US$ standardized returns as well as the statistical adequacy of their regression representation. Second, we examine the stability of this bivariate relationship by testing for change-points using the Kokoszka and Leipus (2000), Horváth (1997) as well and Lavielle and Moulines (2000) tests which are valid for heavy tailed as well as weakly and strongly dependent processes. The timing and numbers of breaks are also estimated. The data source is Olsen and Associates. The original sample for a decade, from 1/12/1986 to 30/11/1996, is 1,052,064 five-minute return observations (2,653 days $\cdot$ 288 five-minute intervals per day). The returns for some days were removed from the sample to avoid having regular and predictable market closures which affect the characterization of the volatility dynamics. A description of the data removed is found in Andersen et al. (2001). The final sample includes 705,024 five-minute returns reflecting 2,448 trading days.

The statistical properties of daily returns normalized by a number of volatility filters are examined for the two FX series. First we focus on the temporal dependence and distributional properties of normalized returns. It is a well documented stylized fact that daily asset returns are characterized by a martingale difference with second-order temporal dynamics and a distribution that exhibits heavy-tails. Under the assumption that returns are described by parametric models such as univariate GARCH or SV it would be interesting to examine whether these purely data-driven volatility filters adequately capture the second-order dynamics of asset returns so as to yield standardized returns series that have no remaining nonlinear dynamics. This is examined by testing the hypothesis of ARCH effects in normalized returns. These empirical results are reported in Table 5 for the YN/US$ and DM/US$ which show two interesting features. First, for the 5-minute sampling frequency there are no remaining ARCH effects in any of the standardized returns series which implies that all volatility filters for both FX series appear equally efficient in capturing the non-linear dynamics. The second and most important finding is that this result does not extend to lower intraday sampling frequencies such as 30-minutes as shown by the remaining results in the same tables. Note that the same results applies to the 60-minute frequency filters which are not reported in the tables merely for conciseness purposes. The presence of ARCH effects in most of the lower frequency normalized returns suggests that the volatility filter and in particular its window length and estimation method are important in yielding a normalized returns process that captures all the nonlinear dynamics. The continuous record asymptotic analysis for the efficiency of rolling volatility filters in Foster and Nelson (1996) yields the optimal window length for different intraday sampling frequencies as discussed in Andreou and Ghysels (2002a) summarized in section 4.2 and Table A1. These theoretical asymptotic predictions of efficiency gain empirical support in Table 5 for the 30-minute sampling frequencies and both FX series. In particular, we find that the normalized returns based on rolling intraday volatility filters given by $X_{HQVi,t}$, $i = k, \ell$, where $k = 4, 8$ and $\ell = 6, 12$ days for the 30- and 60-minutes frequencies, respectively, capture the second-order dynamics exhibited by the FX returns at the 5% significance level. The spot volatility filters $X_{RM,t}$, $X_{RV26,t}$ and $X_{RV52,t}$ present mixed empirical evidence regarding the nonlinear temporal dependence at the 5% significance level. Yet at the 10% level the first two filters provide support for the null of no ARCH. Similar mixed results are obtained in Table 6 regarding the linear temporal dynamics for FX returns. Summarizing, the empirical results in Tables 5 and 6 show that the temporal dependence properties of normalized returns depend on the window length and estimation method of the volatility filter for intraday sampling frequencies. The normalized returns series $X_{HQVi,t}$, $i = k, \ell$, where $k = 4, 8$ and $\ell = 6, 12$ days, for 30- and 60-minutes, respectively, present empirical support for no remaining linear or nonlinear memory especially for the YN/US$ normalized returns. The nonlinear and linear dependence results for spot volatilities and $X_{QV1,k,\ell}$, especially for the DM/US$, provide evidence of weak and strong temporal dependence.

The distributional properties of normalized returns are assessed in Table 7 for the YN/US$ and DM/US$. Both the Jarque and Bera (1980) and Anderson and Darling (1954) test results provide no empirical support of the Normality hypothesis (at the 10% significance level) for any of the daily standardized returns series, mainly due to excess kurtosis in both the spot volatility (SV) normalized returns, $X_{SV,t}$, as well as the $X_{(H)QV,t}$ series. The exception to this result is $X_{QV1,t}$ which appears to support the Normality hypothesis only for the 5-minute sampling frequency. Nevertheless at the lower sampling frequencies $X_{QV1,t}$ is also non-Normal. At the 5-minute sampling frequency the sample skewness and kurtosis coefficients suggest that the empirical distributions for all standardized returns are leptokurtic except for $X_{QV1,t}$ which actually appears to be platykurtic with sample kurtosis coefficient below 3 for all intraday frequencies. Moreover, it is interesting to note that a longer window length beyond one day in QV filters as well as rolling instead of block sampling estimation methods yield excess kurtosis in the empirical distribution. A graphical representation of the non-Normal behavior of standardized returns is also presented in the Normal probability plots in Figures 1-4 for the 30-minute sampling frequency case. In Figure 1 we present the daily (unstandardized) YN/US$ returns based on the normalized returns using the Normal GARCH estimates, one of the popular parametric estimates of volatility. The plots (as well as the accompanying Anderson-Darling (AD) statistics) show that both series empirically invalidate the Normality assumption. Similar behavior is presented by the spot volatility normalized returns in Figure 2. The plots in Figure 3 show the returns normalized by the quadratic variation filters yield less heavy tails than the corresponding unstandardized returns. Nevertheless, in all cases except $X_{QV1,t}$ the AD test results suggest lack of fit of the empirical distribution due to heavy tails which increase with the volatility window length. It is also interesting to compare Figures 3 and 4 and observe that $X_{QV,t}$ and $X_{HQV,t}$, respectively, produce different tail behavior compared to the benchmark of unstandardized returns. In particular, $X_{HQV,t}$ exhibit more peakness than the $X_{QV,t}$ or the returns process. This result is also verified by the nonparametric kernel estimates (which are not presented mainly for conciseness). Using the kernel estimates for two benchmark cases, the simulated Normal *i.i.d.* process and the normalized returns by the GARCH estimates, the remaining kernels for standardized returns by data-driven volatility filters suggest the following stylized facts. The spot volatility filters produce standardized returns very similar to those of the GARCH filter. In contrast, the quadratic volatility filters yield normalized returns that are neither close to the Normal nor the $X_{GARCH,t}$ benchmarks. The $X_{QV,t}$ filters have less heavy tails and appear closer to the Normal except that they are relatively platykurtic. In contrast, $X_{HQV,t}$ yield an empirical distribution which has similar tail behavior to $X_{GARCH,t}$, but it is nonetheless more peaked, with a higher concentration of observations in the middle. Last but not least, we note that the rolling volatility method for both spot and integrated volatilities yield normalized returns that exhibit some skewness effects as opposed to the block sample estimation of the QV filters. It is worth noting that the daily and most intraday volatility filters result in non-Normality due to both excess kurtosis and in most cases asymmetry. This result may be due to an underlying non-Normal distribution and/or the presence of jumps and breaks in the risk adjusted returns process. Indeed the simulation results in Table 1 are generally in favor of the Normality hypothesis for a Normal GARCH process.

Summarizing, the univariate empirical analysis of the standardized returns presents the following four results. First, the efficiency of volatility filters plays an important role in terms of capturing all the second-order dynamics exhibited by returns. This efficiency depends on the sampling frequency, window length and estimation method. The combination of rolling estimation and optimal window produces nearly independent standardized FX returns series. Second, temporal aggregation of intraday returns requires a longer lag of volatility so as to capture the dependence in normalized returns and the empirical findings support the continuous record asymptotics of the efficiency of volatility filters. Third, the empirical tail behavior implied by $X_{QV,t}$ and $X_{HQV,t}$ differ and the latter are found to be relatively more leptokurtic. Moreover, as the window length increases for both QV and SV filters, the distribution of the respective standardized returns becomes more leptokurtic.

The above results suggest that the ratio transformation of daily returns-to-volatility based on data-driven volatility filters can yield a process with a relatively simple statistical structure. This transformation relates to empirical measures of investment performance such as the historical Sharpe ratio. Moreover, as discussed in section 2 it can be considered as a useful nonparametric two stage approach which reduces the dimensionality of a multivariate structure for asset returns and volatility. Similarly, Engle (2002), Tse and Tsui (2002) adopt this approach for specification and estimation of M-GARCH models. In this setup and given the empirical univariate results above, we examine the multivariate relationship of normalized returns in a regression context. First we examine the dynamic structure of risk adjusted returns using Granger causality tests and the existence of a linear regression relationship for YN/US$ and DM/US$ normalized returns. Table 6 also presents these results for the bidirectional causality between the YN/US$ and DM/US$ risk adjusted returns. It is shown that there is no significant empirical evidence of a lead-lag relationship between the co-movements of the two FX series. This result appears robust to the different specifications of volatility and sampling frequencies, the choice of lag length in the VAR(p) representation for studying the causality relationship as well as when that is augmented by the contemporaneous regressor. In contrast to the inexistence of a dynamic relationship between risk adjusted returns there is significant correlation between the YN/US$ and DM/US$ standardized returns. This is examined using two methods. The first method applies the Tse (2000) test (which has good properties in the presence of non-normality) for which the two FX standardized returns provide empirical evidence that supports that null hypothesis of constant conditional correlation. The second method examines the relationship of the two normalized FX returns using the simple linear regression OLS results in Table 8 for the 5- and 30-minute frequencies. In all cases the estimated regression coefficient is highly significant and ranges from 0.6 to 0.75 as representing the contemporaneous covariance structure of standardized returns in the DM and YN vis-a-vis the US$. The statistical adequacy of this regression relationship is examined and the reported residual misspecifications tests show that the Normality assumption is empirically invalidated (which is expected given the univariate analysis above). The additional misspecification results show that the residuals in all $X_{SV,t}$ and $X_{(H)QV,t}$ regressions support the independence hypothesis except $X_{QV1,t}$ in the 30-minute sampling frequency. Summarizing, the empirical results show a static regression representation with a non-Normal conditional distribution for the two FX risk adjusted returns. This analysis opens the route for regression type techniques in detecting change-points and suggests that the empirical conditional covariance process does not exhibit significant dynamics.

5.2 Empirical evidence for breaks in FX co-movements

The above empirical regularities of the DM/US$ and YN/US$ normalized returns suggest that we can apply the least squares methods of Bai and Perron (1998) and Lavielle and Moulines (2000) as well as the CUSUM test of Horváth (1997) and Kokoszka and Leipus (1998, 2000) discussed in section 3, in order to examine the presence of structural breaks.

The K&L change-point test results for the conditional covariance between the DM/US$ and YN/US$ are reported in Table 10. The results show that the univariate normalized returns (using any volatility filter transformation) appear to be time-homogeneous processes. However, for the cross-product of the two FX normalized returns the K&L test shows that there is strong evidence of a change-point in their co-movements. The breaks are detected in all specifications of normalized returns and they occur at the same point in time, namely at 23/3/1995 at which the sequential statistic first exceeds the 5% control limit. This event is related to a period of high uncertainty and a series of bilateral interventions by the Bank of Japan and the Fed (see for instance the Asian Wall Street Journal). It is worth mentioning the parametric CUSUMSQ test (Brown et al., 1975) presents empirical evidence for the instability in the linear regression of the two FX risk adjusted returns. However, we emphasize that these results are based on the statistical adequacy of the Normal, linear regression model. The presence of heavy tailed distributions in normalized returns (or generally deviations from Normality) requires more efficient statistical inference methods for testing the existence of breaks. Similarly, although the parametric CUSUM is robust to deviations from Normality this result does not extend to the CUSUM of squares (Ploberger and Kramer, 1986). Note that an application of the parametric CUSUM does not detect any change-points.

These results are complemented by testing for multiple breaks using the L&M regression method and the two information criteria, BIC and LWZ. Applying the L&M to the cross-product, $Y_{12,t}$, in equation (4.6) supports the null hypothesis of time homogeneity in the co-movements of the two FX series. Given the empirical results in the previous section which support a static regression framework for the two FX normalized returns, we apply the L&M test in the context of equation (4.8). The number and timing of breaks detected (reported in Table 12) not only vary depending on the information criterion but also on the specification of normalized returns. The general result is that the tests choose between zero, one and two change-points and the break dates are relatively more consistent for $X_{(H)QV,t}$ using both criteria. This is also related to the empirical results comparing the different normalizations. The two change-points detected are associated with the events of the US stock market crash in October 1987 and the period before the repeated bilateral FX market interventions in March 1995. From the simulation results we learn that the BIC criterion is relatively more powerful and this is complemented by the empirical evidence which in most cases detects two change-points. Concluding we find that the co-movements in YN/US$ and DM/US$ risk adjusted returns for the most efficient class of filters present evidence for change-points using the recent CUSUM and least-squares methods in K&L and L&M, respectively. Both approaches yield consistent results about the change-points in the co-movements whereas the latter procedure complements the former by detecting an additional break in the sample.

6 Conclusions

We propose reduced form procedures designed to uncover breaks in the co-movements of financial markets via testing for change-points in linear relationships involving returns normalized by conditional volatility. There are several advantages to using normalized returns. Among the advantages we noted that (1) the covariance of normalized returns capture conditional correlations, (2) they reduce the complexity of multivariate volatility models along the same lines as Engle (2002), Engle and Sheppard (2002) and Tse and Tsui (2002), (3) they allow to separate breaks in univariate processes and breaks in co-movements, (4) they enable us to adopt two-stage procedure consisting of a purely data-driven nonparametric first stage and a semiparametric second stage. Though our procedures shares some features with the two-stage estimation procedure of DCC models, we take a reduced form view that suffices for the change-point test purpose. Since the parametric structure of the volatility co-movements are largely left unspecified we cover a larger class of multivariate specifications, including factor ARCH models. Another main advantage of employing the two-step procedure is that the statistical inference methods allow for departures from normality and therefore are robust to heavy tailed distributions. It should also be noted that the returns-to-volatility process and related measures are used often to appraise portfolio performance. Such measures include the Treynor ratio which is the square of the Sharpe ratio (Treynor and Black, 1973). Our two-stage procedure also applies to various alternative functional forms of normalized returns. Hence, we can examine structural breaks in Treynor-Black and other measures, and again not require normality assumptions to do so (similar to the Jobson and Korkie (1980,1981) approach for the Normal case).

We document, using a ten year period from 1986 to 1996 of YN/US$ and DM/US$ series, that regression models with non-Gaussian errors describe adequately their co-movements. We find that the co-movements in YN/US$ and DM/US$ risk adjusted returns for the most efficient class of filters present evidence for change-points using both the Kokoszka and Leipus (2000) and Lavielle and Moulines (2000) tests. These structural breaks are associated with the 1987 stock market crisis as well as the 1995 bilateral FX interventions of the Bank of Japan and the Fed.

In the paper we restrict the simulation and empirical investigations in bivariate models. Extensions to the multidimensional vector of n assets are routes for further research. The methods proposed can be adapted to examine the $n-$homogeneity of the conditional correlation of the cross-section of assets when n is large in the context of M-GARCH-CCC models in a similar way to Horváth et al. (1999) for the mean of n-dependent observations. Further research in a system of conditional covariance equations for testing change-points is a useful extension of the present analysis.

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A Data sources and filters

Table A1: Asymptotically Equivalent One-Sided Equal Weighting Schemes for Volatility Filters

		Equivalence to:					
		26-day filter		30-day filter		52-day filter	
Frequency, m	m per day	Lags	Days	Lags	Days	Lags	Days
Hourly	5.7	62	10.9	71	12.6	124	21.8
Half-hourly	11.3	87	7.7	101	8.9	175	15.4
Fifteen-minute	22.7	124	5.5	143	6.3	247	10.9
Five-minute	68	218	3.2	245	3.6	428	6.3
One-minute	340	476	2.4	544	1.6	952	2.8

Note: The entries to the table report numerical calculations for window length using the asymptotic equivalence of the one-sided rolling volatility based on the Foster and Nelson (1996) equation, $C^F_{(m),t} \approx \frac{\theta}{n_R+n_L} + \sqrt{\theta\Lambda\rho}\frac{n_R-n_L}{n_R+n_L} + \Lambda\frac{n_R^3+n_L^3}{3(n_R+n_L)^2}$, where the superscript F refers to the flat weighting scheme, n_L and n_R are the window length parameters for lags and leads, respectively and Λ, θ and ρ are higher-order moments (see Foster and Nelson, 1996 for details) which depend on the sampling frequency m and the characteristics of the underlying diffusion process, but are assumed constant as in Andreou and Ghysels (2000a). This representation allows us to make relatively simple comparisons of asymptotically equivalent sampling schemes involving sampling at different frequencies m. For instance, when n_R, n_L, θ, Λ and ρ are fixed, then a one-sided window of length n_L with daily data yields the same asymptotic efficiency as a one-sided window of length $n_L m^{-1/2}$ with intra-daily sampled data at frequency $1/m$. We set $n_L = 26, 30{,}52$.

Table A2: GARCH(1,1) Models in Simulation Design

	Daily, $m = 1$		5min frequency, $m = 288$		1min frequency, $m = 1440$	
	DM/US\$	YN/US\$	DM/US\$	YN/US\$	DM/US\$	YN/US\$
$\phi_{(m)}$	0.022	0.026	0.000078	0.000093	0.0000155	0.0000185
$\beta_{(m)}$	0.068	0.104	0.005869	0.009267	0.0026574	0.0041994
$\gamma_{(m)}$	0.898	0.844	0.994011	0.990547	0.9973186	0.9957635
$\kappa(m)$	3	3	2.494283	2.655314	2.6412370	2.4754561
$v(m)$	0.647	0.250	0.647044	0.480798	0.645833	0.498652
$\kappa(m).(v(m))^2$	1.256	0.750	1.613911	1.276670	1.101663	0.615532

Note: The GARCH model parameters are, $\phi_{(m)}, \alpha_{(m)}$ and $\beta_{(m)}$, as defined in section 3.1. The kurtosis parameter is $\kappa(m)$. The unconditional variance is $v(m) = \phi_{(m)}/(1 - \alpha_{(m)} - \beta_{(m)})$. The daily parameters for the DM/US \$ and YN/US \$ were obtained from Andersen and Bollerslev (1998) and cover the period 01/10/87 - 30/09/92.

In the Monte Carlo design for the evaluation of the high frequency volatility filters we consider the following sample sizes, T, for the 24 hour traded FX markets:

T_{years}	T_{days}	$T_{5\,\text{min}}$	$T_{1\,\text{min}}$
5	1,250	360,000	1,800,000
10	2,500	720,000	3,600,000

We assume that 1 year has 250 trading days. Each experiment is performed with 500 replications. Note that for the one-sided rolling estimates we create sufficient data before the effective sample (equivalent to one year). Samples of T=1250 and 2500 trading days (5 and 10 years) are studied.

Table 1: **Monte Carlo Simulations of MSEs and MAEs Ratios**, **Normality and Second-order Dependence Test Results for Daily FX** $X_i = r_i/\widehat{\sigma}_i$ **of the YN/US$ calculated at the 5-minute frequency**

	MSE and MAE Ratios				Jarque Bera Normality Test				ARCH Test	
	N-GARCH		t-GARCH		N-GARCH		t-GARCH		N-GARCH	t-GARCH
					$\rho_{12} = 0$	$\rho_{12} = 0.5$	$\rho_{12} = 0$	$\rho_{12} = 0.5$	$\rho_{12} = 0$	$\rho_{12} = 0$
X_i	MSE	MAE	MSE	MAE	JB	JB	JB	JB	ARCH(5)	ARCH(5)
					p-value	p-value	p-value	p-value	p-value	p-value
X_{RM}	0.777	0.994	0.685	0.983	9.481	9.622	16.107	6.389	1.586	1.585
					(0.009)	(0.008)	(0.000)	(0.041)	(0.173)	(0.255)
X_{RV26}	0.714	0.832	0.602	0.849	3.634	3.481	4.514	2.913	1.586	1.586
					(0.163)	(0.175)	(0.105)	(0.233)	(0.173)	(0.255)
X_{RV52}	0.799	0.866	0.663	0.880	2.154	2.228	23.305	9.279	0.983	0.942
					(0.341)	(0.328)	(0.000)	(0.009)	(0.515)	(0.533)
X_{QV1}	-	-	-	-	1.874	1.954	2.011	2.162	0.993	0.996
					(0.392)	(0.376)	(0.366)	(0.339)	(0.509)	(0.519)
X_{QV2}	0.698	0.843	0.976	0.999	1.925	1.969	2.584	2.278	0.997	1.002
					(0.382)	(0.374)	(0.275)	(0.320)	(0.504)	(0.499)
X_{QV3}	0.730	0.860	0.999	0.998	1.948	1.975	3.805	2.564	0.998	1.014
					(0.378)	(0.373)	(0.149)	(0.277)	(0.504)	(0.487)
X_{HQV1}	0.779	0.893	0.674	0.856	1.971	1.999	3.923	2.803	0.997	1.046
					(0.373)	(0.368)	(0.141)	(0.246)	(0.504)	(0.479)
X_{HQV2}	0.890	0.949	0.989	0.997	1.994	1.993	9.121	5.072	0.998	1.030
					(0.368)	(0.369)	(0.010)	(0.079)	(0.503)	(0.482)
X_{HQV3}	0.838	0.938	0.999	0.999	2.005	2.009	14.100	9.624	0.997	1.041
					(0.366)	(0.366)	(0.001)	(0.008)	(0.503)	(0.475)

Note: The simulation design is described in section 3. We consider Normal and Student's t (with 3 degrees of freedom) M-GARCH-CCC processes. The volatility filters are defined in section 2.3. The relative Mean Square and Absolute Errors (MSE and MAE) ratios are obtained vis-a-vis the MSE and MAE of X_{QV1} for comparison purposes. The standardized returns are tested for Normality using the Jarque-Bera (JB) test. We also examine any remaining second-order temporal dependence in standardized returns using the ARCH test. The number in parentheses refer to the lag length. Similar results were obtained for alternative lag lengths. The total sample size is 2500 observations which is adjusted for the subsample of 2250 due to the standardized returns by rolling volatilities.

Table 2a: **Nominal Size and Power of the Kokoszka and Leipus (2000) test for a change-point in the normalized returns and their co-movements from a bivariate GARCH-CCC**.

Samples $N = 1000$ & Change-point timing, $\pi = 0.5$									
Processes	$X_{1,RV26,t}$	$X_{1,RM,t}$	$X_{1,RV26,t} * X_{2,RV26,t}$			$X_{1,RM,t} * X_{2,RM,t}$			
			$U_{\max}/\widehat{\sigma}_{VARHAC}$						
Transformations	$X_{1,t}$	$X_{1,t}$	$Y_{12,t}$	$(Y_{12,t})^2$	$\|Y_{12,t}\|$	$Y_{12,t}$	$(Y_{12,t})^2$	$\|Y_{12,t}\|$	$\sigma^{RM}_{Y_{12}}$
H_0 : $(\omega_{11,0}, \alpha_{11,0}, \beta_{11,0})$ and $(\omega_{11,1}, \alpha_{11,1}, \beta_{11,1})$									
DGP1: (0.4, 0.1, 0.5)	0.037	0.057	0.014	0.075	0.053	0.000	0.000	0.000	0.052
DGP2: (0.1, 0.1, 0.8)	0.027	0.070	0.010	0.058	0.052	0.000	0.000	0.000	0.038
H_1^A : Break in the dynamics of volatility, $(\beta_{11,0}, \beta_{11,1})$ and $(\beta_{22,0}, \beta_{22,1})$									
DGP1: (0.5, 0.8)	0.060	0.030	0.011	0.065	0.074	0.061	0.004	0.004	0.023
DGP1: (0.5, 0.1)	0.050	0.063	0.010	0.068	0.036	0.146	0.078	0.019	0.057
DGP1: (0.5, 0.4)	0.033	0.057	0.015	0.079	0.044	0.001	0.005	0.006	0.063
DGP2: (0.8, 0.4)	0.023	0.050	0.013	0.039	0.022	0.141	0.013	0.002	0.123
DGP2: (0.8, 0.5)	0.053	0.060	0.007	0.043	0.023	0.028	0.000	0.000	0.077
DGP2: (0.8, 0.3)	0.040	0.040	0.012	0.041	0.018	0.309	0.003	0.000	0.127
H_1^B : Break in the constant of volatility, $(\omega_{11,0}, \omega_{11,1})$ and $(\omega_{22,0}, \omega_{22,1})$									
DGP1: (0.4, 0.8)	0.040	0.053	0.038	0.045	0.031	0.007	0.007	0.002	0.020
DGP1: (0.4, 0.2)	0.068	0.040	0.061	0.093	0.071	0.087	0.062	0.009	0.077
DGP1: (0.4, 0.1)	0.040	0.057	0.097	0.142	0.188	0.197	0.003	0.092	0.117
DGP2: (0.1, 0.2)	0.033	0.043	0.020	0.060	0.044	0.003	0.014	0.000	0.025
DGP2: (0.1, 0.05)	0.040	0.013	0.020	0.089	0.089	0.014	0.042	0.005	0.047
DGP2: (0.1, 0.4)	0.053	0.030	0.008	0.055	0.027	0.072	0.003	0.005	0.023
H_1^C : Break in the correlation coefficient, $(\rho_{12,0}, \rho_{12,1})$									
DGP1: (0.5,0.6)	0.053	0.043	0.018	0.077	0.012	0.001	0.007	0.000	0.045
DGP1: (0.5,0.9)	0.023	0.020	**0.301**	0.068	0.037	0.065	0.004	0.006	**0.286**
DGP1: (0.5,0.4)	0.070	0.050	0.025	0.092	0.070	0.042	0.054	0.009	0.063
DGP1: (0.5,0.1)	0.033	0.047	**0.915**	**0.620**	**0.150**	**1.000**	**0.626**	**0.178**	**0.403**
DGP2: (0.5,0.6)	0.060	0.050	0.015	0.054	0.052	0.000	0.014	0.003	0.033
DGP2: (0.5,0.9)	0.060	0.020	**0.304**	0.061	0.025	0.073	0.006	0.006	**0.190**
DGP2: (0.5,0.4)	0.060	0.083	0.028	0.077	0.090	0.072	0.073	0.023	0.034
DGP2: (0.5,0.1)	0.057	0.037	**0.934**	**0.615**	**0.195**	**1.000**	**0.739**	**0.250**	**0.354**

Note: (1) The Kokoszka and Leipus (2000) test statistic is $U_k = \left(\frac{1}{\sqrt{T}}\sum_{j=1}^{k} Z_j - \frac{k}{T}\frac{1}{\sqrt{T}}\sum_{j=k+1}^{T} Z_j\right)$. The max $U_T(k)$ is standardized by the VARHAC estimator, $\widehat{\sigma}_{VARHAC}$, which is applied to the $X_t = r_t/\widehat{\sigma}_{i,t}$ i=1,2 and $Y_{12,t} = X_{1,t}X_{2,t}$ transformation from the bovariate GARCH with assets (a,b). The normalized statistic $U_{\max}/\widehat{\sigma}_{VARHAC}$ converges to the sup of a Brownian Bridge with asymptotic critical value 1.36 at the 5% significance level. (2) The simulated bivariate GARCH with constant conditional correlation under the null hypothesis of no breaks is speficied as: $r_{1,t} = u_{1,t}\sqrt{h_{11,t}} + u_{2,t}\sqrt{h_{12,t}}$ and $r_{2,t} = u_{2,t}\sqrt{h_{22,t}} + u_{1,t}\sqrt{h_{12,t}}$ where $h_{11,t} = \omega_{11} + \alpha_{11}u^2_{1,t-1} + \beta_{11}h_{11,t-1}$, $h_{22,t} = \omega_{22} + \alpha_{22}u^2_{2,t-1} + \beta_{22}h_{22,t-1}$ and $h_{12,t} = \rho_{12} * \sqrt{h_{11,t} * h_{22,t}}$. The model is simulated (1,000 replications) where the superscirpts 1 and 0 in the variables and coefficients denote the cases with and without change-points, respectively. Under the alternative hypotheses H_1^A, H_1^B the change in parameters refer to both GARCH processes.

Table 2b: **Nominal Size and Power of the Kokoszka and Leipus (2000) test for a change-point in the normalized returns and their co-movements from a bivariate GARCH with time varying correlation**.

Samples $N = 1000$ & Change-point timing, $\pi = 0.5$									
Processes	$X_{1,RV26,t}$	$X_{1,RM,t}$	$X_{1,RV26,t} * X_{2,RV26,t}$			$X_{1,RM,t} * X_{2,RM,t}$			
			$U_{\max}/\widehat{\sigma}_{VARHAC}$						
Returns Transformation	$X_{1,t}$	$X_{1,t}$	$Y_{12,t}$	$(Y_{12,t})^2$	$\lvert Y_{12,t}\rvert$	$Y_{12,t}$	$(Y_{12,t})^2$	$\lvert Y_{12,t}\rvert$	$\sigma^{RM}_{Y_{12}}$
H_0 : $(\omega_{11,0}, \alpha_{11,0}, \beta_{11,0})$ and $(\omega_{11,1}, \alpha_{11,1}, \beta_{11,1})$									
DGP1: Low persistence	0.067	0.040	0.004	0.056	0.004	0.003	0.038	0.002	0.006
DGP2: High persistence	0.037	0.060	0.001	0.002	0.002	0.006	0.049	0.007	0.006
H_1^A : Break in the dynamics of volatility, $(\beta_{11,0}, \beta_{11,1})$ and $(\beta_{22,0}, \beta_{22,1})$									
DGP1: (0.5, 0.8)	0.027	0.043	0.000	0.001	0.000	0.245	0.062	0.008	0.130
DGP1: (0.5, 0.1)	0.043	0.040	0.186	0.003	0.007	0.002	0.249	0.302	0.012
DGP1: (0.5, 0.4)	0.023	0.037	0.002	0.002	0.002	0.004	0.102	0.082	0.001
DGP2: (0.8, 0.4)	0.040	0.040	0.002	0.001	0.001	0.075	0.106	0.052	0.075
DGP2: (0.8, 0.5)	0.040	0.057	0.001	0.002	0.001	0.028	0.090	0.024	0.023
DGP2: (0.8, 0.3)	0.060	0.033	0.001	0.005	0.002	0.171	0.131	0.091	0.177
H_1^B : Break in the constant of volatility, $(\omega_{11,0}, \omega_{11,1})$ and $(\omega_{22,0}, \omega_{22,1})$									
DGP1: (0.4, 0.8)	0.017	0.043	0.000	0.001	0.000	0.014	0.067	0.016	0.001
DGP1: (0.4, 0.2)	0.037	0.033	0.002	0.000	0.000	0.009	0.050	0.004	0.003
DGP1: (0.4, 0.1)	0.037	0.037	0.001	0.001	0.000	0.055	0.095	0.044	0.000
DGP2: (0.1, 0.2)	0.037	0.003	0.001	0.005	0.001	0.017	0.051	0.008	0.013
DGP2: (0.1, 0.05)	0.037	0.023	0.073	0.009	0.005	0.005	0.053	0.005	0.012
DGP2: (0.1, 0.4)	0.053	0.050	0.008	0.018	0.012	0.239	0.152	0.120	0.242
H_1^E : Break in the covariance dynamics, $(\beta_{12,0}, \beta_{12,1})$									
DGP1: (0.5,0.8)	0.037	0.047	0.000	0.000	0.000	0.063	0.072	0.036	0.005
DGP1: (0.5,0.1)	0.063	0.057	0.000	0.003	0.000	0.015	0.060	0.009	0.014
DGP1: (0.5,0.4)	0.040	0.057	0.000	0.000	0.000	0.003	0.060	0.008	0.006
DGP2: (0.8,0.1)	0.037	0.047	**0.204**	0.030	0.060	**0.412**	0.014	0.003	**0.774**
DGP2: (0.8,0.4)	0.053	0.040	0.013	0.003	0.003	0.081	0.024	0.004	**0.318**
DGP2: (0.8,0.5)	0.050	0.047	0.003	0.004	0.002	0.012	0.007	0.001	**0.155**
DGP2: (0.8,0.3)	0.030	0.043	0.013	0.012	0.008	**0.286**	0.044	0.021	**0.518**

Note: The notes in Table 2a apply.

Table 3a: **Frequency Distribution of the number of change-points obtained with the Lavielle and Moulines (2000) test applied to the $Y_{12,t}$ when there is a *single break* in a M-GARCH-CCC**

Samples, $T = 1000$ and change point, $\pi = 0.5$ and *Segments*, $t_k = 5$												
Normalized returns regression $Y_{12,\hat{\sigma}_i,t} = \mu_{12} + v_{12,t}$												
Volatility Filter, $\hat{\sigma}_i$ i	*RV*26						*RM*					
Lavielle & Moulines	BIC			LWZ			BIC			LWZ		
Number of Breaks	0	1	≥ 2	0	1	≥ 2	0	1	≥ 2	0	1	≥ 2
H_0 : $(\omega_{11,0}, \alpha_{11,0}, \beta_{11,0})$ and $(\omega_{22,0}, \alpha_{22,0}, \beta_{22,0})$												
DGP1: (0.4, 0.1, 0.5)	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**0.98**	0.02	0.00	**1.00**	0.00	0.00
DGP2: (0.1, 0.1, 0.8)	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00
H_1^A : Break in the dynamics of volatility with $(\beta_{11,0}, \beta_{11,1})$ and $(\beta_{22,0}, \beta_{22,1})$												
DGP1: $(0.5, 0.6)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP1: $(0.5, 0.8)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP1: $(0.5, 0.1)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP1: $(0.5, 0.7)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP1: $(0.5, 0.2)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP2: $(0.8, 0.7)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP2: $(0.8, 0.4)$	0.30	**0.70**	0.00	1.00	**0.00**	0.00	0.30	**0.70**	0.00	1.00	**0.00**	0.00
DGP2: $(0.8, 0.5)$	0.84	**0.16**	0.00	1.00	**0.00**	0.00	0.90	**0.10**	0.00	1.00	**0.00**	0.00
H_1^B : Break in the constant of volatility with parameters $(\omega_{11,0}, \omega_{11,1})$ and $(\omega_{22,0}, \omega_{22,1})$												
DGP1: $(0.4, 0.1)$	0.80	**0.20**	0.00	1.00	**0.00**	0.00	0.72	**0.28**	0.00	1.00	**0.00**	0.00
DGP1: $(0.4, 0.2)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP1: $(0.4, 0.3)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP1: $(0.4, 0.8)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP2: $(0.1, 0.3)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP2: $(0.1, 0.5)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	0.92	**0.08**	0.00	1.00	**0.00**	0.00
DGP2: $(0.1, 0.2)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
H_1^C : Break in the correlation coefficient $(\rho_{12,0}, \rho_{12,1})$												
DGP1: $(0.5, 0.1)$	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00
DGP1: $(0.5, 0.3)$	0.63	**0.37**	0.00	1.00	**0.00**	0.00	0.60	**0.40**	0.00	1.00	**0.00**	0.00
DGP1: $(0.5, 0.2)$	0.00	**1.00**	0.00	0.80	**0.20**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00
DGP1: $(0.5, 0.4)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP1: $(0.5, 0.8)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP2: $(0.5, 0.1)$	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00
DGP2: $(0.5, 0.3)$	0.67	**0.33**	0.00	1.00	**0.00**	0.00	0.47	**0.53**	0.00	1.00	**0.00**	0.00
DGP2: $(0.5, 0.2)$	0.00	**1.00**	0.00	0.80	**0.20**	0.00	0.00	**1.00**	0.00	0.00	**0.20**	0.00
DGP2: $(0.5, 0.4)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP2: $(0.5, 0.8)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00

Notes: The Lavielle and Moulines (2000) test is described in section 3. The Bayesian Information Criterion (BIC) and its modification by Liu et al. (1997) denoted as LWZ are used. The simulations focus on DGP1, DGP2, $T = 1000$ for 500 trials. For comparison purposes the alternative hypotheses of change points are similar to the K&L simulations (Table 2) and extended to larger breaks. Reported is the frequency distribution of the breaks detected. The highlighted numbers refer to the true number of change-points in the simulated process.

Table 3b: **Frequency Distribution of the number of change-points obtained with the Lavielle and Moulines (2000) test applied to the $Y_{12,t}$ when there is a *single break* in a M-GARCH with dynamic conditinal correlation**

Samples, $T = 1000$ and change point, $\pi = 0.5$												
Normalized returns regression $Y_{12,\hat{\sigma}_i,t} = \mu_{12} + v_{12,t}$												
Volatility Filter, $\hat{\sigma}_i$, i		*RV*26						*RM*				
Lavielle & Moulines		BIC			LWZ			BIC			LWZ	
Segments, $t_k = 5$												
Number of Breaks	0	1	≥ 2	0	1	≥ 2	0	1	≥ 2	0	1	≥ 2
H_0 : $(\omega_{11,0}, \alpha_{11,0}, \beta_{11,0})$ and $(\omega_{22,0}, \alpha_{22,0}, \beta_{22,0})$												
DGP1: (0.4, 0.1, 0.5)	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00
DGP2: (0.1, 0.1, 0.8)	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00
H_1^A : Break in the dynamics of volatility with parameters $(\beta_{11,0}, \beta_{11,1})$ and $(\beta_{22,0}, \beta_{22,1})$												
DGP1: (0.5, 0.6)	0.40	**0.22**	0.36	0.92	**0.08**	0.00	0.38	**0.28**	0.32	0.92	**0.08**	0.00
DGP1: (0.5, 0.8)	0.16	**0.26**	0.58	0.94	**0.06**	0.00	0.40	**0.20**	0.34	0.92	**0.06**	0.02
DGP1: (0.5, 0.1)	0.38	**0.24**	0.36	0.94	**0.02**	0.04	0.38	**0.22**	0.40	0.94	**0.06**	0.00
DGP1: (0.5, 0.2)	0.28	**0.18**	0.50	0.88	**0.12**	0.00	0.36	**0.26**	0.36	0.94	**0.04**	0.02
DGP2: (0.8, 0.7)	0.42	**0.22**	0.34	0.88	**0.10**	0.02	0.36	**0.26**	0.34	0.94	**0.06**	0.00
DGP2: (0.8, 0.5)	0.42	**0.14**	0.52	0.92	**0.04**	0.04	0.44	**0.20**	0.34	0.98	**0.02**	0.00
DGP2: (0.8, 0.4)	0.22	**0.20**	0.58	0.96	**0.02**	0.02	0.40	**0.20**	0.38	0.98	**0.02**	0.02
H_1^B : Break in the constant of volatility with parameters $(\omega_{11,0}, \omega_{11,1})$ and $(\omega_{22,0}, \omega_{22,1})$												
DGP1: (0.4, 0.5)	0.24	**0.22**	0.52	0.92	**0.06**	0.02	0.36	**0.24**	0.36	0.98	**0.02**	0.00
DGP1: (0.4, 0.8)	0.40	**0.16**	0.38	0.96	**0.04**	0.00	0.40	**0.20**	0.38	0.98	**0.02**	0.00
DGP2: (0.1, 0.3)	0.32	**0.20**	0.40	0.92	**0.06**	0.02	0.28	**0.30**	0.42	0.90	**0.08**	0.02
DGP2: (0.1, 0.5)	0.34	**0.16**	0.38	0.94	**0.04**	0.02	0.42	**0.10**	0.48	0.96	**0.04**	0.00
DGP2: (0.1, 0.2)	0.32	**0.20**	0.42	0.94	**0.06**	0.00	0.50	**0.18**	0.30	0.96	**0.04**	0.00
H_1^D : Break in the constant of the conditional covariance coefficient $(\omega_{12,0}, \omega_{12,1})$												
DGP1: (0.4, 0.1)	0.38	**0.12**	0.42	0.92	**0.08**	0.00	0.24	**0.36**	0.40	0.94	**0.06**	0.00
DGP1: (0.4, 0.2)	0.34	**0.16**	0.44	0.96	**0.02**	0.02	0.26	**0.30**	0.33	0.98	**0.02**	0.00
DGP1: (0.4, 0.3)	0.30	**0.38**	0.28	0.94	**0.04**	0.02	0.36	**0.18**	0.42	0.92	**0.06**	0.02
DGP1: (0.4, 0.8)	0.36	**0.26**	0.36	0.96	**0.04**	0.00	0.44	**0.24**	0.30	0.96	**0.04**	0.00
DGP2: (0.1, 0.3)	0.36	**0.10**	0.50	0.88	**0.10**	0.02	0.42	**0.22**	0.34	0.90	**0.10**	0.00
DGP2: (0.1, 0.5)	0.32	**0.28**	0.34	0.88	**0.08**	0.04	0.40	**0.28**	0.32	0.98	**0.02**	0.00
DGP2: (0.1, 0.2)	0.34	**0.32**	0.30	0.98	**0.02**	0.00	0.44	**0.14**	0.36	0.96	**0.04**	0.00
H_1^E : Break in the dynamics of the conditional covariance coefficient $(b_{12,0}, b_{12,1})$												
DGP1: (0.5, 0.8)	0.38	**0.22**	0.39	0.94	**0.04**	0.02	0.38	**0.20**	0.36	0.98	**0.02**	0.00
DGP1: (0.5, 0.2)	0.34	**0.16**	0.44	0.96	**0.02**	0.02	0.42	**0.14**	0.40	0.96	**0.04**	0.00
DGP1: (0.5, 0.1)	0.30	**0.38**	0.28	0.94	**0.04**	0.02	0.48	**0.12**	0.32	0.98	**0.02**	0.00
DGP1: (0.5, 0.4)	0.18	**0.32**	0.42	0.92	**0.08**	0.00	0.32	**0.32**	0.34	0.92	**0.08**	0.00
DGP2: (0.8, 0.4)	0.26	**0.26**	0.46	0.94	**0.04**	0.02	0.44	**0.28**	0.28	0.96	**0.04**	0.00
DGP2: (0.8, 0.5)	0.38	**0.20**	0.34	0.96	**0.04**	0.00	0.32	**0.26**	0.38	0.92	**0.06**	0.02
DGP2: (0.8, 0.7)	0.40	**0.16**	0.42	0.96	**0.02**	0.02	0.32	**0.20**	0.46	1.00	**0.00**	0.00

Notes: The notes in Table 3a apply.

Table 4a: **Frequency Distribution of the number of change-points obtained with the Lavielle and Moulines (2000) test applied to the regression of normalized returns when there is a *single break* in a M-GARCH-CCC process**

Samples, $T = 1000$ and change point, $\pi = 0.5$ and *Segments*, $t_k = 5$

Normalized returns regression $X_{1,\hat{\sigma}_i,t} = \theta_{12} + \eta_{12} X_{2,\hat{\sigma}_i,t} + v_{12,t}$

Volatility Filter, $\hat{\sigma}_i$ *i*	*RV*26						*RM*					
Lavielle & Moulines	BIC			LWZ			BIC			LWZ		
Number of Breaks	0	1	≥ 2	0	1	≥ 2	0	1	≥ 2	0	1	≥ 2
H_0 : $(\omega_{11,0}, \alpha_{11,0}, \beta_{11,0})$ and $(\omega_{22,0}, \alpha_{22,0}, \beta_{22,0})$												
DGP1: (0.4, 0.1, 0.5)	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**0.98**	0.02	0.00	**1.00**	0.00	0.00
DGP2: (0.1, 0.1, 0.8)	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00
H_1^A : Break in the dynamics of volatility with parameters $(\beta_{11,0}, \beta_{11,1})$ and $(\beta_{22,0}, \beta_{22,1})$												
DGP1: (0.5, 0.6)	0.96	**0.04**	0.00	1.00	**0.00**	0.00	0.98	**0.02**	0.00	1.00	**0.00**	0.00
DGP1: (0.5, 0.8)	0.00	**1.00**	0.00	0.44	**0.56**	0.00	0.00	**0.98**	0.02	0.38	**0.62**	0.00
DGP1: (0.5, 0.1)	0.70	**0.30**	0.00	1.00	**0.00**	0.00	0.48	**0.52**	0.00	1.00	**0.00**	0.00
DGP1: (0.5, 0.7)	1.00	**0.00**	0.00	1.00	**0.00**	0.00	0.46	**0.54**	0.00	0.98	**0.02**	0.00
DGP1: (0.5, 0.2)	1.00	**0.00**	0.00	1.00	**0.00**	0.00	0.82	**0.18**	0.00	1.00	**0.00**	0.00
DGP2: (0.8, 0.7)	0.02	**0.96**	0.02	0.98	**0.02**	0.00	0.88	**0.12**	0.00	1.00	**0.00**	0.00
DGP2: (0.8, 0.4)	0.00	**1.00**	0.00	0.78	**0.22**	0.00	0.46	**0.54**	0.00	0.98	**0.02**	0.00
DGP2: (0.8, 0.5)	0.02	**0.96**	0.02	0.98	**0.02**	0.00	0.04	**0.94**	0.02	0.94	**0.06**	0.00
H_1^B : Break in the constant of volatility with parameters $(\omega_{11,0}, \omega_{11,1})$ and $(\omega_{22,0}, \omega_{22,1})$												
DGP1: (0.4, 0.1)	0.04	**0.96**	0.00	0.92	**0.08**	0.00	0.06	**0.94**	0.00	0.94	**0.06**	0.00
DGP1: (0.4, 0.2)	0.76	**0.24**	0.00	1.00	**0.00**	0.00	0.62	**0.38**	0.00	1.00	**0.00**	0.00
DGP1: (0.4, 0.3)	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP1: (0.4, 0.8)	0.10	**0.90**	0.00	1.00	**0.00**	0.00	0.20	**0.80**	0.00	1.00	**0.00**	0.00
DGP2: (0.1, 0.3)	0.10	**0.90**	0.00	1.00	**0.00**	0.00	0.96	**0.04**	0.00	1.00	**0.00**	0.00
DGP2: (0.1, 0.5)	0.00	**0.98**	0.02	0.00	**1.00**	0.00	0.00	**0.98**	0.02	0.00	**1.00**	0.00
DGP2: (0.1, 0.2)	0.74	**0.26**	0.00	1.00	**0.00**	0.00	0.92	**0.08**	0.00	1.00	**0.00**	0.00
H_1^C : Break in the correlation coefficient $(\rho_{12,0}, \rho_{12,1})$												
DGP1: (0.5, 0.1)	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**0.98**	0.02	0.00	**1.00**	0.00
DGP1: (0.5, 0.3)	0.00	**1.00**	0.00	0.88	**0.12**	0.00	0.00	**1.00**	0.00	0.78	**0.22**	0.00
DGP1: (0.5, 0.4)	0.90	**0.10**	0.00	1.00	**0.00**	0.00	0.92	**0.08**	0.00	1.00	**0.00**	0.00
DGP1: (0.5, 0.8)	0.00	**1.00**	0.00	0.26	**0.74**	0.00	0.00	**0.95**	0.05	0.35	**0.65**	0.00
DGP2: (0.5, 0.1)	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.02	**0.98**	0.00	0.80	**0.20**	0.00
DGP2: (0.5, 0.3)	0.02	**0.98**	0.00	0.92	**0.08**	0.00	0.00	**0.98**	0.02	0.88	**0.12**	0.00
DGP2: (0.5, 0.4)	0.90	**0.10**	0.00	1.00	**0.00**	0.00	0.90	**0.10**	0.00	1.00	**0.00**	0.00
DGP2: (0.5, 0.8)	0.00	**1.00**	0.00	0.30	**0.70**	0.00	0.88	**0.12**	0.04	1.00	**0.00**	0.00

Notes: The notes in Table 3a apply.

Table 4b: **Frequency Distribution of the number of change-points obtained with the Lavielle and Moulines (2000) test applied to the regression of normalized returns when there is a *single break* in a M-GARCH with dynamic conditional covariance**.

Samples, $T = 1000$ and change point, $\pi = 0.5$												
Normalized returns regression $X_{1,\hat{\sigma}_i,t} = \theta_{12} + \eta_{12} X_{2,\hat{\sigma}_i,t} + v_{12,t}$												
Volatility Filter, $\hat{\sigma}_i$, i	*RV*26						*RM*					
Lavielle & Moulines	BIC			LWZ			BIC			LWZ		
Segments, $t_k = 5$												
Number of Breaks	0	1	≥ 2	0	1	≥ 2	0	1	≥ 2	0	1	≥ 2
H_0 : $(\omega_{11,0}, \alpha_{11,0}, \beta_{11,0})$ and $(\omega_{22,0}, \alpha_{22,0}, \beta_{22,0})$												
DGP1: (0.4, 0.1, 0.5)	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00
DGP2: (0.1, 0.1, 0.8)	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00
H_1^A : Break in the dynamics of volatility with parameters $(\beta_{11,0}, \beta_{11,1})$ and $(\beta_{22,0}, \beta_{22,1})$												
DGP1: (0.5, 0.6)	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP1: (0.5, 0.8)	0.00	**1.00**	0.00	1.00	**0.00**	0.00	0.00	**1.00**	0.00	0.95	**0.05**	0.00
DGP1: (0.5, 0.1)	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP1: (0.5, 0.7)	0.96	**0.04**	0.00	1.00	**0.00**	0.00	0.98	**0.02**	0.00	1.00	**0.00**	0.00
DGP1: (0.5, 0.2)	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP2: (0.8, 0.7)	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP2: (0.8, 0.5)	0.54	**0.46**	0.00	1.00	**0.00**	0.00	0.59	**0.41**	0.00	1.00	**0.00**	0.00
DGP2: (0.8, 0.4)	0.00	**1.00**	0.00	1.00	**0.00**	0.00	0.00	**1.00**	0.00	1.00	**0.00**	0.00
H_1^B : Break in the constant of volatility with parameters $(\omega_{11,0}, \omega_{11,1})$ and $(\omega_{22,0}, \omega_{22,1})$												
DGP1: (0.4, 0.5)	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP1: (0.4, 0.8)	0.80	**0.20**	0.00	1.00	**0.00**	0.00	0.44	**0.56**	0.00	1.00	**0.00**	0.00
DGP2: (0.1, 0.3)	0.14	**0.86**	0.00	1.00	**0.00**	0.00	0.01	**0.99**	0.00	1.00	**0.00**	0.00
DGP2: (0.1, 0.5)	0.00	**1.00**	0.00	0.32	**0.68**	0.00	0.00	**1.00**	0.00	0.26	**0.74**	0.00
DGP2: (0.1, 0.2)	0.96	**0.04**	0.00	1.00	**0.00**	0.00	0.98	**0.02**	0.00	1.00	**0.00**	0.00
H_1^D : Break in the constant of the conditional covariance coefficient $(\omega_{12,0}, \omega_{12,1})$												
DGP1: (0.4, 0.1)	0.00	**1.00**	0.00	1.00	**0.00**	0.00	0.00	**1.00**	0.00	0.26	**0.74**	0.00
DGP1: (0.4, 0.2)	1.00	**0.00**	0.00	1.00	**0.00**	0.00	0.40	**0.60**	0.00	1.00	**0.00**	0.00
DGP1: (0.4, 0.3)	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP1: (0.4, 0.8)	0.80	**0.20**	0.00	1.00	**0.00**	0.00	0.00	**1.00**	0.00	0.22	**0.78**	0.00
DGP2: (0.1, 0.3)	0.10	**0.90**	0.00	1.00	**0.00**	0.00	0.96	**0.04**	0.00	1.00	**0.00**	0.00
DGP2: (0.1, 0.5)	0.00	**1.00**	0.00	0.30	**0.70**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00
DGP2: (0.1, 0.2)	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
H_1^E : Break in the dynamics of the conditional covariance coefficient $(b_{12,0}, b_{12,1})$												
DGP1: (0.5, 0.8)	0.00	**1.00**	0.00	1.00	**0.00**	0.00	0.00	**1.00**	0.00	0.94	**0.06**	0.00
DGP1: (0.5, 0.2)	1.00	**0.00**	0.00	1.00	**0.00**	0.00	0.92	**0.08**	0.00	1.00	**0.00**	0.00
DGP1: (0.5, 0.1)	1.00	**0.00**	0.00	1.00	**0.00**	0.00	0.42	**0.58**	0.00	1.00	**0.00**	0.00
DGP1: (0.5, 0.4)	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP2: (0.8, 0.4)	0.00	**1.00**	0.00	0.20	**0.80**	0.00	0.00	**1.00**	0.00	0.01	**0.99**	0.00
DGP2: (0.8, 0.5)	0.00	**1.00**	0.00	0.92	**0.08**	0.00	0.00	**1.00**	0.00	0.66	**0.34**	0.00
DGP2: (0.8, 0.7)	0.98	**0.02**	0.00	1.00	**0.00**	0.00	0.90	**0.10**	0.00	1.00	**0.00**	0.00

Note: The notes in Table 4a apply.

Table 5: **Nonlinear Dependence Test Results for Daily YN/US$ and DM/US$ Standardized Returns based on various intraday sampling frequencies**

	YN/US$				DM/US$			
	5min. frequency		30min. frequency		5min. frequency		30min. frequency	
X_i	ARCH(1) p-value	ARCH(5) p-value	ARCH(1) p-value	ARCH(5) p-value	ARCH(1) p-value	ARCH(5) p-value	ARCH(1) p-value	ARCH(5) p-value
X_{RM}	0.361	0.257	3.072	0.868	0.039	0.199	3.972	2.860
	(0.548)	(0.936)	(0.079)	(0.501)	(0.843)	(0.963)	(0.049)	(0.014)
X_{RV26}	0.387	1.278	5.736	1.938	1.601	0.843	4.375	2.126
	(0.534)	(0.270)	(0.017)	(0.085)	(0.206)	(0.519)	(0.037)	(0.059)
X_{RV52}	0.026	0.257	13.326	4.229	1.120	2.491	10.772	2.974
	(0.872)	(0.936)	(0.000)	(0.001)	(0.289)	(0.029)	(0.001)	(0.011)
X_{QV1}	2.314	0.921	4.099	1.553	6.517	2.535	9.001	3.330
	(0.128)	(0.466)	(0.043)	(0.170)	(0.011)	(0.027)	(0.003)	(0.005)
X_{QVk}	2.254	0.900	5.266	2.169	5.271	2.392	9.284	4.078
	(0.133)	(0.480)	(0.022)	(0.055)	(0.022)	(0.036)	(0.002)	(0.001)
$X_{QV\ell}$	-0.011	0.741	0.105	0.929	1.143	2.453	5.738	2.421
	(0.553)	(0.593)	(0.745)	(0.461)	(0.285)	(0.032)	(0.017)	(0.034)
X_{HQV1}	4.801	1.604	8.037	3.074	7.173	2.654	13.274	4.446
	(0.029)	(0.155)	(0.005)	(0.009)	(0.008)	(0.021)	(0.000)	(0.000)
X_{HQVk}	0.836	1.197	0.035	1.705	2.074	2.338	1.193	3.099
	(0.361)	(0.308)	(0.851)	(0.130)	(0.149)	(0.039)	(0.275)	(0.009)
$X_{HQV\ell}$	0.006	1.008	0.542	1.006	0.855	2.494	0.074	1.067
	(0.936)	(0.412)	(0.462)	(0.412)	(0.355)	(0.029)	(0.786)	(0.377)

Note: The volatility filters are defined in section 2.3. The data set refers to the 5-minute YN/US$ from 1/12/86 to 30/11/96 which yields a daily sample size of T=2446 days and is adjusted for a subsample of 2346, excluding the first 100 observations as a result of the rolling volatility estimators. The window lengths k=2,4,6 and l=3,8,12 days for the 5-, 30- and 60-minutes frequency, respectively. The ARCH test for alternative lag lengths and respective p-values in parentheses are reported.

Table 6: **Linear Dependence and Granger Causality Test Results for Daily YN/US$ and DM/US$ Standardized Returns based on various intraday sampling frequencies**

	Linear Temporal Dependence Tests												Granger Causality Test Results between				
	YN/US$						DM/US$						$YN(\widehat{\sigma})$ and $DM(\widehat{\sigma})$ Normalized Returns				
	5min. frequency			30min. frequency			5min. frequency			30min. frequency							
X_i	LM(1)	LM(5)	LM(20)	LM(1)	LM(5)	LM(20)	LM(1)	LM(5)	LM(20)	LM(1)	LM(5)	LM(20)	Direction of Causality	5-minute		30-minute	
	p-value	p-value	p-value	p-value	p-value	p-value	p-value	p-value	p-value	p-value	p-value	p-value		F-test	p-value	F-test	p-value
X_{RM}	0.361	0.674	1.283	7.347	3.048	2.417	0.326	1.127	0.792	5.093	1.604	1.805	YN(RM_1), DM(RM)	0.315	(0.575)	0.038	(0.846)
	(0.548)	(0.644)	(0.179)	(0.007)	(0.009)	(0.000)	(0.568)	(0.344)	(0.726)	(0.024)	(0.156)	(0.016)		2.807	(0.094)	4.659	(0.031)
X_{RV26}	0.114	0.813	1.253	8.244	3.307	2.397	0.682	1.197	0.847	5.745	1.687	1.755	YN(RV26_1), DM(RV26)	0.099	(0.753)	0.003	(0.959)
	(0.735)	(0.540)	(0.201)	(0.004)	(0.006)	(0.001)	(0.409)	(0.308)	(0.657)	(0.017)	(0.134)	(0.020)		2.694	(0.101)	4.135	(0.042)
X_{RV52}	0.376	0.473	1.365	9.779	3.557	2.386	0.183	1.255	0.759	4.913	1.341	1.832	YN(RV52_1), DM(RV52)	0.436	(0.509)	0.050	(0.822)
	(0.539)	(0.797)	(0.129)	(0.002)	(0.003)	(0.000)	(0.669)	(0.281)	(0.765)	(0.027)	(0.244)	(0.014)		3.434	(0.064)	4.034	(0.045)
X_{QV1}	2.007	1.298	1.949	2.353	1.511	2.068	0.098	1.311	0.883	0.278	1.556	1.112	YN(QV1_1), DM(QV1)	0.678	(0.400)	0.255	(0.614)
	(0.157)	(0.262)	(0.021)	(0.125)	(0.183)	(0.004)	(0.754)	(0.257)	(0.610)	(0.598)	(0.169)	(0.328)		3.278	(0.070)	3.669	(0.056)
X_{QVk}	0.716	0.824	1.585	0.807	0.891	1.622	0.003	1.088	0.876	0.010	1.217	0.918	YN(QVk_1), DM(QVk)	0.927	(0.336)	0.688	(0.407)
	(0.398)	(0.532)	(0.048)	(0.369)	(0.486)	(0.039)	(0.955)	(0.365)	(0.619)	(0.919)	(0.299)	(0.564)		3.159	(0.079)	2.766	(0.096)
$X_{QV\ell}$	0.559	1.191	1.482	0.154	1.048	0.074	0.239	1.242	0.907	0.005	1.194	0.917	YN(QVℓ_1), DM(QVℓ)	0.492	(0.482)	0.163	(0.686)
	(0.454)	(0.311)	(0.077)	(0.695)	(0.387)	(0.785)	(0.625)	(0.287)	(0.579)	(0.944)	(0.308)	(0.563)		2.743	(0.098)	1.799	(0.179)
X_{HQV1}	0.782	0.773	1.455	0.699	0.629	1.536	0.010	0.951	0.829	0.029	0.974	0.839	YN(HQV1_1), DM(HQV1)	1.203	(0.273)	0.974	(0.324)
	(0.377)	(0.569)	(0.087)	(0.403)	(0.678)	(0.060)	(0.919)	(0.447)	(0.680)	(0.864)	(0.432)	(0.665)		2.975	(0.085)	2.467	(0.117)
X_{HQVk}	0.522	0.674	1.455	0.393	0.624	1.420	0.167	1.071	0.891	0.849	1.209	0.922	YN(HQVk_1), DM(HQVk)	0.849	(0.357)	0.789	(0.374)
	(0.470)	(0.643)	(0.087)	(0.531)	(0.682)	(0.102)	(0.683)	(0.375)	(0.599)	(0.357)	(0.302)	(0.559)		3.202	(0.074)	2.452	(0.117)
$X_{HQV\ell}$	0.452	0.596	1.427	0.075	0.628	1.208	0.568	1.054	1.227	0.622	1.271	0.928	YN(HQVℓ_1), DM(HQVℓ)	0.734	(0.392)	0.259	(0.611)
	(0.502)	(0.703)	(0.099)	(0.784)	(0.679)	(0.237)	(0.451)	(0.384)	(0.221)	(0.430)	(0.274)	(0.550)		3.063	(0.080)	2.253	(0.134)

Note: The volatility filters are defined in section 2.3. The data set refers to the 5-minute YN/US$ from 1/12/86-30/11/96, T=2446 days and is adjusted for a subsample of 2346, excluding the first 100 observations as a result of the rolling volatility estimators. The window lengths k=2,4,6 and l=3,8,12 days for the 5-, 30- and 60-minutes frequency, respectively. The sample linear dependence hypothesis is examined using Lagrange Multiplier (LM) tests for alternative lag lengths along with their respective p-values. The normalized returns YN(.) and DM(.) denote the YN/US$ and DM/US$ risk adjusted returns, respectively. The direction of noncausality runs from the lagged variable to the contemporaneous one. The reverse causality for each case is given by the second line of each pair of normalized returns.

Table 7: Normality Test Results for Daily YN/US$ Standardized Returns based on various intraday sampling frequencies

	YN/US$						DM/US$					
	5min. frequency			30min. frequency			5min. frequency			30min. frequency		
X_i	Sk.	AD	BJ	Sk.	AD	BJ	Sk.	AD	BJ	Sk	AD	BJ
	Kr.	p-value	p-value	Kr.	p-value	p-value	Kr.	p-value	p-value	Kr.	p-value	p-value
X_{RM}	-0.215	4.305	51.511	-0.174	9.062	167.08	-0.012	1.890	8.210	0.142	7.589	170.59
	3.585	(0.000)	(0.000)	4.260	(0.000)	(0.000)	3.289	(0.000)	(0.017)	4.290	(0.000)	(0.000)
X_{RV26}	-0.251	7.566	148.74	-0.226	15.403	446.74	-0.019	4.233	55.713	0.256	12.430	451.08
	4.127	(0.000)	(0.000)	5.089	(0.000)	(0.000)	3.754	(0.000)	(0.000)	5.086	(0.000)	(0.000)
X_{RV52}	-0.309	11.196	327.21	-0.380	25.022	1471.9	-0.030	6.788	132.50	0.277	19.598	1359.3
	4.722	(0.000)	(0.000)	6.805	(0.000)	(0.000)	3.989	(0.000)	(0.000)	6.688	(0.000)	(0.000)
X_{QV1}	-0.030	0.558	1.064	-0.055	1.029	10.407	-0.011	0.418	6.605	-0.012	1.214	17.845
	2.915	(0.149)	(0.588)	2.693	(0.010)	(0.000)	2.741	(0.328)	(0.037)	2.573	(0.004)	(0.000)
X_{QVk}	-0.091	2.720	35.943	-0.093	1.384	12.914	-0.005	0.880	2.479	-0.004	0.491	0.256
	3.579	(0.000)	(0.000)	3.312	(0.001)	(0.000)	3.159	(0.024)	(0.289)	3.051	(0.219)	(0.880)
$X_{QV\ell}$	-0.113	5.598	105.5	-0.192	7.459	151.9	-0.021	1.945	3.292	0.009	3.215	3.699
	3.992	(0.000)	(0.000)	4.193	(0.000)	(0.000)	3.359	(0.000)	(0.001)	3.194	(0.000)	(0.157)
X_{HQV1}	-0.138	5.248	120.04	-0.134	3.355	59.811	-0.110	2.942	132.12	-0.092	2.676	82.549
	4.073	(0.000)	(0.000)	3.736	(0.000)	(0.000)	4.142	(0.000)	(0.000)	3.902	(0.000)	(0.000)
X_{HQVk}	-0.191	8.683	245.61	-0.149	9.976	314.04	-0.082	4.649	151.52	-0.059	5.664	132.39
	4.539	(0.000)	(0.000)	4.769	(0.000)	(0.000)	4.235	(0.000)	(0.000)	4.159	(0.000)	(0.000)
$X_{HQV\ell}$	-0.202	10.719	327.82	-0.179	11.298	380.72	-0.054	5.555	154.06	-0.099	6.671	280.06
	4.787	(0.000)	(0.000)	4.943	(0.000)	(0.000)	4.251	(0.000)	(0.000)	4.683	(0.000)	(0.000)

Note: The volatility filters are defined in section 2.3. The data set refers to the 5-minute YN/US$ from 1/12/86 to 30/11/96 which yields a daily sample size of T=2446 days and is adjusted for a subsample of 2346, excluding the first 100 observations as a result of the rolling volatility estimators. The window lengths k=2,4,6 and l=3,8,12 days for the 5-, 30- and 60-minutes frequency, respectively. The sample Skewness and Kurtosis (Sk and Kr., respectively) are reported. The test statistics reported refer to the Anderson-Darling (AD), Bera-Jarque (BJ) along with their respective p-values.

Table 8: **Linear Regression Results of Daily YM/US$ on DM/US$ Standardized Returns based on Intra-day Sampling Frequencies**

	5-minute sampling frequency								30-minute sampling frequency							
	OLS results		Residual Misspecification results						OLS results		Residual Misspecification results					
X_i	const.	beta	BJ	Sk.	ARCH(1)	ARCH(5)	LM(1)	LM(5)	const.	beta	BJ	Sk.	ARCH(1)	ARCH(5)	LM(1)	LM(5)
	p-value	p-value	p-value	Kr.	p-value	p-value	p-value	p-value	p-value	p-value	p-value	Kr.	p-value	p-value	p-value	p-value
X_{RM}	-0.017	0.603	601.95	-0.566	2.468	1.115	1.220	0.702	-0.032	0.746	52786	-2.068	0.010	0.073	1.749	2.569
	(0.276)	(0.000)	(0.000)	5.209	(0.116)	(0.350)	(0.269)	(0.622)	(0.011)	(0.000)	(0.000)	25.862	(0.919)	(0.996)	(0.186)	(0.025)
X_{RV26}	-0.021	0.604	884.21	-0.597	1.847	1.091	1.298	0.917	-0.032	0.743	84087	2.463	0.022	0.044	0.854	2.345
	(0.208)	(0.000)	(0.000)	5.760	(0.174)	(0.363)	(0.225)	(0.469)	(0.024)	(0.000)	(0.000)	31.907	(0.883)	(0.999)	(0.355)	(0.039)
X_{RV52}	-0.023	0.603	1542.4	-0.766	4.217	1.987	1.619	0.729	-0.038	0.722	175997	-3.336	1.229	0.039	1.229	2.051
	(0.172)	(0.000)	(0.000)	6.664	(0.040)	(0.078)	(0.203)	(0.601)	(0.009)	(0.000)	(0.000)	44.895	(0.268)	(0.999)	(0.268)	(0.069)
X_{QV1}	0.004	0.605	54.153	-0.223	1.508	3.238	0.394	0.440	0.007	0.600	31.273	-0.193	0.786	3.492	0.180	0.459
	(0.759)	(0.000)	(0.000)	3.595	(0.219)	(0.006)	(0.530)	(0.821)	(0.659)	(0.000)	(0.000)	3.414	(0.375)	(0.004)	(0.671)	(0.807)
X_{QV2}	-0.004	0.607	284.72	-0.400	0.507	1.524	1.603	1.524	0.0006	0.607	183.84	-0.329	0.475	1.789	1.281	0.523
	(0.784)	(0.000)	(0.000)	4.507	(0.476)	(0.179)	(0.206)	(0.179)	(0.971)	(0.000)	(0.000)	4.204	(0.491)	(0.112)	(0.258)	(0.759)
X_{QV3}	-0.003	0.609	283.44	-0.397	0.513	1.538	1.540	0.588	-0.016	0.618	609.2	-0.485	1.535	0.350	1.028	0.499
	(0.821)	(0.000)	(0.000)	4.505	(0.474)	(0.175)	(0.215)	(0.709)	(0.016)	(0.016)	(0.000)	5.244	(0.215)	(0.882)	(0.311)	(0.777)
X_{HQV1}	-0.0002	0.607	442.81	-0.422	0.069	0.959	2.335	0.599	0.0002	0.605	201.57	-0.325	0.031	1.208	1.465	0.352
	(0.861)	(0.000)	(0.000)	4.902	(0.793)	(0.442)	(0.127)	(0.701)	(0.938)	(0.000)	(0.000)	4.282	(0.861)	(0.303)	(0.226)	(0.881)
X_{HQV2}	-0.0003	0.603	1117.7	-0.614	0.174	0.675	2.418	0.607	-0.0006	0.632	803.44	-0.514	1.223	0.716	1.679	0.485
	(0.611)	(0.000)	(0.000)	6.152	(0.676)	(0.643)	(0.120)	(0.694)	(0.648)	(0.000)	(0.000)	5.681	(0.269)	(0.612)	(0.195)	(0.788)
X_{HQV3}	-0.0003	0.602	1435.1	-0.662	0.420	0.679	2.274	0.598	-0.0007	0.609	1187.5	-0.572	0.777	0.196	1.618	0.474
	(0.530)	(0.000)	(0.000)	6.597	(0.517)	(0.639)	(0.132)	(0.702)	(0.407)	(0.000)	(0.000)	6.297	(0.574)	(0.964)	(0.204)	(0.796)

Note: The notes in Tables 6,7 and 8 apply.

Table 9: Kokoszka and Leipus (2000) Change-point Test Results of Daily YM/US$ on DM/US$ Standardized Returns based on 30 minute Intra-day Sampling Frequency

	Univariate Normalized Returns						Comovements of Normalized Returns and Break Dates		
	$X_{YN/US\$,\hat{\sigma}_i}$			$X_{DM/US\$,\hat{\sigma}_i}$			$Y_{YN/US\$DM/US\$,\hat{\sigma}_i}$		k*
$\hat{\sigma}_i$	$U_{\max}$	$U_{\max}/\hat{\sigma}_{VARHAC}$	k*	$U_{\max}$	$U_{\max}/\hat{\sigma}_{VARHAC}$	k*	$U_{\max}$	$U_{\max}/\hat{\sigma}_{VARHAC}$	
RM	0.692	0.706	-	0.828	0.839	-	6.674	5.215*	Mar.95
$RV26$	0.802	0.810	-	0.786	0.788	-	1.921	1.413*	Oct.87
$RV52$	0.822	0.806	-	0.870	0.856	-	1.777	1.178	-
$QV1$	1.066	1.106	-	0.933	0.937	-	3.836	3.503*	Oct.87
$QV4$	1.138	1.133	-	0.957	0.929	-	3.934	2.980*	Oct.87
$QV8$	1.188	1.184	-	0.951	0.914	-	3.316	2.245*	Oct.87
$HQV1$	0.165	1.086	-	0.139	0.879	-	8.265	2.453*	Oct.87
$HQV4$	0.088	1.128	-	0.079	1.003	-	1.845	1.984*	Oct.87
$HQV8$	0.063	1.149	-	0.052	0.945	-	0.855	1.818*	Oct.87

Note: The Kokoszka and Leipus (1998, 2000) test is discussed in section 3. The test statistic is $U_k = \left(\frac{1}{\sqrt{T}}\sum_{j=1}^{k} X_j^2 - \frac{k}{T}\frac{1}{\sqrt{T}}\sum_{j=k+1}^{T} X_j^2\right)$. The $\max U_T(k)$ is standardized by the VARHAC estimator, $\hat{\sigma}_{VARHAC}$, which is applied to the X_t transformation from the multivariate GARCH model. The normalized statistic $U_{\max}/\hat{\sigma}_{VARHAC}$ converges to the sup of a Brownian Bridge with asymptotic critical value 1.36 at the 5% significance level. k* refers to the timing of the break. The test is applied to the univariate normalized returns series as well as the cross product of the YN/US$ and DM/US$ risk adjusted returns.

Table 10: **Lavielle and Moulines (2000) Multiple Breaks Test Results of Daily YM/US$ on DM/US$ Standardized Returns based on 30 minute Intra-day Sampling Frequency**

	Univariate Normalized Returns						Comovements of Normalized Returns and Break Dates			
	$X_{YN/US\$,\hat{\sigma}_i}$			$X_{DM/US\$,\hat{\sigma}_i}$			$X_{YN/US\$,i} = \theta + \eta X_{DM/US\$,\hat{\sigma}_i} + u_t$			k*
$\hat{\sigma}_i$	SIC(k)	LWZ(k)	k*	SIC(k)	LWZ(k)	k*	SIC(k)	LWZ(k)		
RM	-0.042 (0)	-0.041 (0)	-	-0.028 (0)	-0.027 (0)	-	-0.298 (1)	-0.285 (1)	Oct.87, Mar.95	Mar.95
							-0.301 (2)	-0.184 (0)		
$RV26$	-0.014 (0)	-0.013 (0)	-	-0.004 (0)	-0.004 (0)	-	-0.497 (1)	0.495 (0)	Oct.87	-
							-0.496 (0)			
$RV52$	0.037 (0)	0.037 (0)	-	0.032 (0)	0.033 (0)	-	-0.438 (2)	-0.435 (0)	Oct.87, Mar.95	-
							-0.437 (1)			
$QV1$	-0.067 (0)	-0.066 (0)	-	-0.004 (0)	-0.004 (0)	-	-0.529 (2)	-0.515 (1)	Oct.87, Mar.95	Oct.87
							-0.528 (1)	-0.512 (0)		
$QV4$	0.015 (0)	0.015 (0)	-	0.066 (0)	0.066 (0)	-	-0.469 (2)	-0.454 (1)	Oct.87, Mar.95	Oct.87
							-0.467 (1)	-0.452 (0)		
$QV8$	0.060 (0)	0.060 (0)	-	0.078 (0)	0.079 (0)	-	-0.438 (2)	-0.426 (0)	Oct.87, Mar.95	-
							-0.435 (1)			
$HQV1$	-3.684 (0)	-3.684 (0)	-	-3.766 (0)	-3.765 (0)	-	-4.309 (1)	-4.295 (1)	Oct.87	Oct.87
							-4.286 (0)	-4.285 (0)		
$HQV4$	-5.085 (0)	-5.085 (0)	-	-5.110 (0)	-5.109 (0)	-	-5.629 (2)	-5.614 (1)	Oct.87, Mar.95	Oct.87, Mar.95
							-5.628 (1)	-5.611 (0)		
$HQV8$	-5.803 (0)	-5.802 (0)	-	-5.827 (0)	-5.827 (0)	-	-6.337 (2)	-6.323 (2)	Oct.87, Mar.95	Oct.87, Mar.95
							-6.336 (1)	-6.321 (1)		

Note: The Lavielle and Moulines (2000) test is discussed in section 3. The Bayesian Information Criterion (BIC) and its modification by Liu et al. (1997) denoted as LWZ are used in the above regression for the comovements between the normalized returns of the DM/US$ and YN/US$. k* refers to the timing of the break. The application of the test to the univariate series does not detect any change-points.

Figure 1:

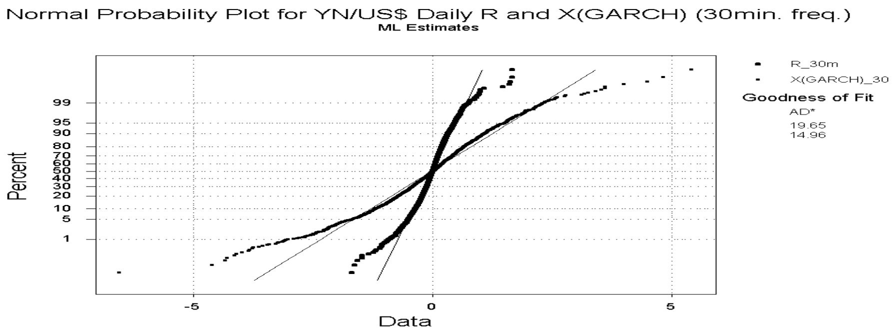

Figure 2:

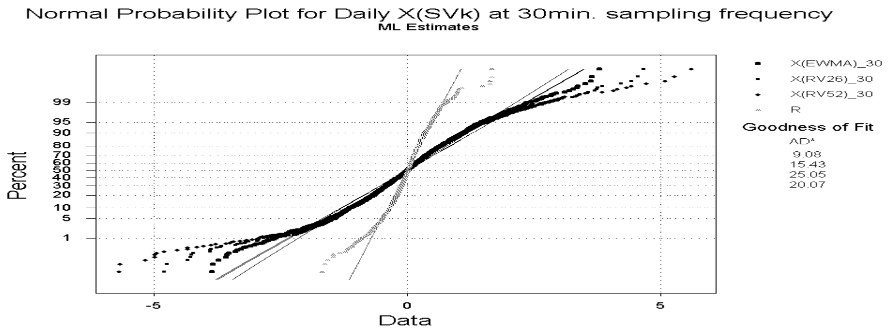

Figure 3:

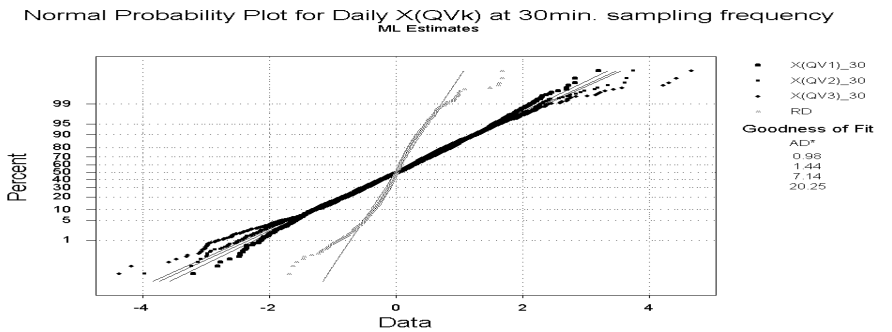

Figure 4: