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Elena Andreou and Eric Ghysels

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Detecting Mutiple Breaks in Financial Market Volatility Dynamics*

Elena Andreou† *Eric Ghysels*‡

Résumé / *Abstract*

Nous appliquons plusieurs nouveaux tests conçus pour déceler les ruptures structurelles dans la dynamique de variance et de covariance conditionnelles. Les tests s'appliquent à la fois aux processus de la classe ARCH et de type SV et tiennent compte des caractéristiques de mémoire longue. Nous les appliquons également aux estimateurs de volatilité engendrés par les données, en utilisant des données à haute fréquence et nous suggérons des applications multivariées. En plus de déterminer la présence des ruptures, les statistiques permettent d'identifier le nombre de ruptures ainsi que l'emplacement de ruptures multiples. Nous étudions la taille et la puissance des nouveaux tests pour divers modèles réalistes univariés et multivariés de variance conditionnelle et d'échantillonnage. L'article conclut avec une analyse empirique à partir de données provenant des marchés d'actions et de taux de change pour lesquels nous trouvons de multiples ruptures associées aux crises financières asiatiques et russes. Dans les échantillons sélectionnés avant et après les ruptures, nous trouvons des changements dans la dynamique et dans la mémoire longue de la volatilité.

We apply several recently proposed tests for structural breaks in conditional variance and covariance dynamics. The tests apply to both the class of ARCH and SV type processes and allow for long memory features. We also apply them to data-driven volatility estimators using high-frequency data and suggest multivariate applications. In addition to testing for the presence of breaks, the statistics allow to identify the number of breaks and the location of multiple breaks. We study the size and power of the new tests under various realistic univariate and multivariate conditional variance models and sampling schemes. The paper concludes with an empirical analysis using data from the stock and FX markets for which we find multiple breaks associated with the Asian and Russian financial crises. We find changes in the dynamics and long memory of volatility in the samples prior and post the breaks.

Mots clés : Ruptures structurelles, ARCH, mémoire longue, données à haute fréquence

Keywords : *Structural breaks, ARCH, long memory, high-frequency data*

JEL Classification : GI0, C15, C13

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† University of Cyprus
‡ University of North Carolina, Chapel Hill

Introduction

On this twentieth anniversary of Rob Engle's seminal paper on ARCH it is worth reflecting on some of the outstanding questions in the literature. It has long been conjectured that stock market volatility exhibit occasional breaks. Diebold (1986), Hendry (1986) and Lamoureux and Lastrapes (1990) were among the first to suggest that persistence in volatility may be overstated with the presence of structural breaks. More recently, Bos et al. (1999), Diebold and Inoue (2001), Granger and Hyung (1999), Mikosch and Starica (1999), Lobato and Savin (1998), among others, argued that the presence of breaks may explain the findings of long memory in volatility.

There is a substantial literature on testing for the presence of breaks in linearly dependent stochastic processes. A partial list of papers includes Antoch et al. (1997), Bai (1994, 1997), Bai and Perron (1998), Davis et al. (1995), Giraitis et al. (1996), Giraitis and Leipus (1990), Horváth and Kokoszka (1997), and Leipus (1994). There is a temptation to apply the tests for ARMA-type processes in the context of ARCH or stochastic volatility. For instance, one could view squared returns as an ARMA process and proceed with the application of tests suggested for testing breaks in the mean. Unfortunately, things are not so simple. The resemblance between ARMA models and GARCH is deceiving (see e.g. Francq and Zakoïan (2000b)). It took many years of research after the original work of Engle (1982) to clarify the asymptotics of GARCH(1,1) processes for instance (see Lee and Hansen (1984) and Lumsdaine (1996)) and the asymptotics of more general univariate and multivariate GARCH processes (see Ling and McAleer (1999 a,b)). In a recent paper Carrasco and Chen (2001) establish that GARCH processes are β-mixing, which precludes the application of many aforementioned tests for structural breaks requiring a much stronger mixing condition.[1] The class of discrete time stochastic volatility models also yields ARMA representations of squared returns (see e.g. Ghysels et al. (1996)) but care must again be taken of non-standard settings (see e.g. Francq and Zakoïan (2000a)). These issues become even more involved when allowing for the presence of long memory.

The purpose of this paper is to explore recent advances in the theory of change-point estimation for ARCH and SV models. A number of papers have

[1]Most tests proposed for linear processes impose ϕ-mixing or strong mixing conditions which are not satisfied by ARCH processes. For a general treatment of estimating so called weak GARCH models, see Francq and Zakoïan (2000b).

shown the consistency of various CUSUM type change-point estimators and tests for multiple breaks in the context of volatility models. The tests are not model-specific and apply to a large class of (strongly) dependent processes such as ARCH and SV type processes. The theoretical developments are described in a series of recent papers, see in particular Giraitis, Kokoszka and Leipus (1999, 2000), Horváth and Steinebach (2000), Kokoszka and Leipus (1998, 1999, 2000) and Laveille and Moulines (2000). So far only limited simulation and empirical evidence is reported about these tests. We enlarge the scope of applicability by suggesting several improvements that enhance the practical implementation of the proposed tests. We focus on the Kokoszka and Leipus and Laveille and Moulines tests and propose three types of extensions. First, the series used in the tests so far are either squared returns or absolute returns. We suggest to extend the application of change-point to more precise measures of volatility, including the high frequency data-driven processes studied by Andersen et al. (2001), Andreou and Ghysels (2000), Barndorff-Nielsen and Shephard (2000), Taylor and Xu (1997), among others. Second, we propose extending the tests to multivariate volatility settings. While there is no fully developed theory for multivariate processes we suggest to apply the existing tests to cross-products of returns and show this strategy is feasible and useful in practical applications. Finally, the finite sample performance of the new tests is examined via extensive Monte Carlo simulations.

We consider various financial series, including equity index returns for several financial markets in the Hong Kong, Japan, the U.K. and U.S. We also consider FX market series. The data series are similar to several prior studies, such as Andersen et al. (2001) and Granger and Hyung (1999), that found long memory properties in volatility. Our empirical analysis is particularly complementary to Granger and Hyung (1999) who use a statistic proposed by Inclán and Tiao (1994) and Bai (1997). The advantage of the Kokoszka and Leipus and Laveille and Moulines tests used here is their validity under a wide class of strongly dependent processes, including long memory, GARCH-type and nonlinear models. The Inclán and Tiao test applies in principle to independent series and is designed to find a break in the (unconditional) variance with unknown location. We show via Monte Carlo that the Inclán and Tiao test has nevertheless power and only minor size distortions when applied to dependent data, though it is not as powerful as the Kokoszka and Leipus (1998, 1999, 2000) and Laveille and Moulines (2000) tests.

The paper is organized as follows. In section 2 we describe the various tests. Section 3 presents the Monte Carlo design and results. Section 4 contains the empirical application and a final section concludes.

1 Test statistics for breaks in volatility dynamics

A classical statistical problem is to test the homogeneity of a process or the parameter constancy of models. There is a substantial literature on this question known as a change-point problem. The task is to test if a change or structural break has occurred somewhere in a sample and, if so, to estimate the time of its occurrence. The simplest form of departure from stationarity is a change in mean at some (unknown) point in the sample. This problem has received a great deal of attention, see for instance Csörgo and Horváth (1997) for a literature review. Financial returns series typically have constant mean, but exhibit noticeable and complex clustering patterns in volatility (see e.g. Bollerslev et al. (1994) for a survey of stylized facts). Such processes pose some non-trivial challenges as detecting a change in variance in an ARCH model can be rather difficult.[2] The section is divided in several subsections. The first covers CUSUM type tests for a single breakpoint. The second handles multiple break point tests. Finally, the final subsection covers the various series to which the tests are applied.

1.1 CUSUM type tests for a single break

In this paper we examine CUSUM type tests and estimators, which are simple and easy to implement. We focus only on a brief description of the theoretical aspects of the problem, starting with the following basic characterization of the CUSUM type tests and estimators. Suppose we have a sample of size N of a stochastic process $\{X_t\}$. We will think of X_t as a generic process satisfying certain regularity conditions that are sufficiently general to cover various return volatility related empirical series.

We will devote the entire final subsection to the description of the various processes to which the tests will apply. To facilitate the presentation, let us briefly mention some examples of X_t. Consider a return process, and let

[2] One could for instance think of extreme cases, where unconditional moments don't change, only the the conditional variance dynamics is perturbed.

$X_t = |r_t|^\delta$ for $\delta = 1, 2$. We will not provide all the details of the regularity conditions, yet it is clear that the progress made in recent years pertains to the types of temporal dependence allowed in the processes of interest, under the null of no breaks. The temporal dependence, in the context of heterogeneously distributed observations, is described by mixing conditions. Until recently, only weakly dependent or ϕ-mixing processes were covered by tests for structural breaks. Now, we can handle strongly depend, or strong mixing, i.e. α-mixing processes. To proceed further, it is worth recalling that measurable functions of mixing processes are mixing and of the same size (see White (1984, Theorem 3.49)). Hence, when the process r_t is α-mixing then $X_t = G(r_t, \ldots, r_{t-\tau})$, for finite τ is also α-mixing. While mixing conditions are not the only regularity conditions that need to be satisfied, it is clear that (1) those conditions play a key role in the recent advances and (2) once a primitive process, say returns at some sampling frequency, satisfies the assumed mixing conditions, then transformations such as $X_t = |r_t|^\delta$ for $\delta = 1, 2$, satisfy those conditions too. The choice of δ is of course important. For $\delta = 2$ we look at squared returns which relatw to volatility in an ARCH or SV context, since squared returns are the parent processes parametrically modelled in ARCH or SV-type models.[3] Without an explicit specification of an ARCH or SV model, the tests discussed in this section will examine whether there is evidence of structural breaks in the data generating process of squared returns (when $\delta = 2$). If we find a break, one must conclude that when fitting ARCH or SV-type processes, there will be breaks in their parametric structure. Alternatively, when $\delta = 1$, we examine absolute returns, which are another measure of volatility (see e.g. Ding et al. (1993)). Again without specifying a specific model, we will be looking at the presence of breaks in absolute returns for which various models have been proposed to mimic the long memory features of such processes. We can take this reasoning a step further and think of sampling returns intra-daily, denoted $r_{(i),t}$ for some intra-day frequency $i = 1, \ldots, m$, and form data-driven estimates of daily volatility by taking sums of squared intra-day returns. This is an example of $X_t = G(r_{(1),t}, \ldots, r_{(m),t})$. The high frequency process is α-mixing, and so is the daily sampled sum of intra-day squared returns, or various other empirical measures of quadratic variation. Using the notation of Andreou and Ghysels (2000) $X_t = (QVi)_t$ which are locally smoothed filters of the

[3] As noted before, Carrasco and Chen (2001) establish that GARCH and SV processes are, under suitable regularity condition, β-mixing.

quadratic variation using i days of high-frequency data. The case of $QV1$ corresponds to the filters studied by Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2000). With such high-frequency data-driven volatility measures we are again examining processes that relate to the ARCH and SV class of processes. Similar to squared returns, if we find a break in X_t representing high-frequency data-driven volatility, one must conclude that when fitting ARCH or SV-type processes, there will be breaks in their parametric structure (without specifying the actual parametric structure). The details of the various specifications for the X_t process will be discussed in the last subsection.

To test for breaks, Giraitis, Kokoszka and Leipus (1999, 2000) and Kokoszka and Leipus (1998, 2000) consider the following process:

$$U_N(k) = \left(1/\sqrt{N}\sum_{j=1}^{k} X_j - k/(N\sqrt{N})\sum_{j=1}^{N} X_j\right) \tag{1.1}$$

for $0 < k < N$. Kokoszka and Leipus (1998, 2000) consider $X_t = r_t^2$ in (1.1), i.e. squared returns, and study CUSUM type estimators $\hat{k}$ of an change point k^* where the parameters of an ARCH(∞) process change.[4] The CUSUM type estimators are defined as:

$$\hat{k} = \min\{k : |U_N(k)| = \max_{1\leq j\leq N} |U_N(j)|\} \tag{1.2}$$

The estimate $\hat{k}$ is the point at which there is maximal sample evidence for a break in the squared return process. To decide whether there is actually a break, one has also to derive the asymptotic distribution of $\sup_{0\leq k\leq N} U_N(k)$ or related processes such as $\int_0^1 U_N^2(t)dt$. Moreover, in the presence of a single break it has to be proven that $\hat{k}$ is a consistent estimator of k^*. More specifically, suppose that

$$r_t^2 = \begin{cases} r_{1,t}^2 & 1 \leq t \leq k^* \\ r_{2,t}^2 & k^* < t < N \end{cases}$$

then Kokoszka and Leipus (1998, 2000) show that $P\{|k^* - \hat{k}| > \varepsilon\} \leq C/(\delta\varepsilon^2\sqrt{n})$, for some positive constant C and δ depending on the ARCH

[4]Please recall that no ARCH or SV model are explicitly specified. Kokoszka and Leipus (1998) assume (i) a stationary ARCH(∞) process with short memory ie the coefficients decay exponentially fast, and (ii) the errors of the ARCH process are not assumed Gaussian but merely that they have a finite fourth moment.

parameters and they also show that $|k^* - \hat{k}| = O_p(1/n)$. Giraitis, Kokoszka and Leipus (1999, 2000), Horváth and Steinebach (2000) and Kokoszka and Leipus (1999, 2000) extend these results to include volatility processes with long memory (using $X_t = |r_t|$) , though they result in different convergence rates for $\hat{k}$. Finally, these authors also establish that under the null hypothesis of no break:

$$U_N(k) \rightarrow_{D[0,1]} \sigma B(k) \tag{1.3}$$

where $B(k)$ is a Brownian bridge and $\sigma^2 = \sum_{j=-\infty}^{\infty} Cov(X_j, X_0)$. Consequently, using an estimator $\hat{\sigma}$, one can establish that under the null:

$$\sup\{|U_N(k)|\}/\hat{\sigma} \rightarrow_{D[0,1]} \sup\{B(k) : k\epsilon[0,1]\} \tag{1.4}$$

which establishes a Kolmogorov-Smirnov type asymptotic distribution.[5]

The computation of the Kokoszka and Leipus (1998, 2000) test (henceforth K&L test) is relatively straightforward, with the exception of the computation of $\hat{\sigma}$ appearing in (1.4). The authors suggest to use a Heteroskedasticity and Autocorrelation Consistent (HAC) estimator applied to the X_j process. There are a multitude of such estimators, depending on the kernel function one uses. Examples of kernels which have been used by econometricians include: Hansen (1982) and White (1984) use the truncated kernel; the Newey and West (1987) estimator uses the Bartlett kernel; and the estimator of Gallant (1987) uses the Parzen kernel and that of Andrews (1991) uses the Quadratic Spectral (QS) kernel. We have experimented with a number of estimators in addition to the procedure of den Haan and Levin (1997) who propose a HAC estimator without any kernel estimation, which is called the Vector Autoregression Heteroskedasticity and Autocorrelation Consistent (VARHAC) estimator. This estimator has an advantage over any estimator which involves kernel estimation in that the circular problem associated with estimating the optimal bandwidth parame- ter can be avoided. For the VARHAC estimator involves fitting a parametric autoregressive model and a usual method to choose the order of AR such as the AIC is applied. The Monte Carlo evidence reported in den Haan and Levin (1997) indicates that the VARHAC estimator performs better than the nonprewhitened and prewhitened kernel estimators in many cases. Although we have not done a systematic study of various kernel HAC estimators versus the VARHAC

[5] Critical values can be found in most textbooks on nonparametric methods. The 90 %, 95 % and 99 % percentile (two-sided test) critical values are, respectively: 1.22, 1.36 and 1.63.

estimator, we found the latter to be the most reliable. All the results in the paper are therefore based on the VARHAC estimator for the variance $\hat{\sigma}$ appearing in (1.4).

The advantage of the K&L test is its validity under a wide class of processes, including long memory, GARCH-type and nonlinear models. In a study closely related to ours, Granger and Hyung (1999) use a different test, proposed by Inclán and Tiao (1994) in the context of linear models with breaks such as Chen and Tiao (1990) and Engle and Smith (1999).[6] The Inclán and Tiao test (henceforth I&T test) applies in principle to i.i.d. series and is designed to find a break in the (unconditional) variance with unknown location. The test statistic is defined as:

$$IT = \sqrt{N/2} \max_k |D_k| \tag{1.5}$$

where $D_k = \left[\left(\sum_{j=1}^{k} X_j / \sum_{j=1}^{N} X_j \right) - k/N \right]$. It is interesting to note that the asymptotic distribution of the statistic in (1.5) is the same as in (1.4), that is the supremum of Brownian bridge, and hence the same Kolmogorov-Smirnov type asymptotic distribution. In the Monte Carlo simulations we will examine how the the Inclán and Tiao test performs in non-i.i.d. settings and compare it to the Kokoszka and Leipus test, the latter applicable to long memory, ARCH, SV and other nonlinear processes.

1.2 Test statistics for multiple breaks

While testing for the presence of breaks in strongly dependent processes is a challenge, the literature has tackled an even greater challenge, namely testing for the number of breaks and, if there are multiple breaks, locating them. Bai and Perron (1996) have proposed a least squares estimation procedure to determine the number and location of breaks in the mean of linear processes with weakly dependent errors. The key result in Bai and Perron (1996), and prior work of Bai (1994), is the use of a Hájek-Rényi inequality to establish the asymptotic distribution of the test procedure. Recent work by Lavielle and Moulines (2000) has greatly increased the scope of testing for multiple breaks. They obtain similar inequality results for weakly as well as strongly dependent processes, including long memory processes, ARCH-type

[6] Note that Granget and Hyung also use the Bai (1997) test applied to absolute returns.

processes, etc.[7] The number of breaks is estimated via a penalized least-squares approach similar to Yao (1988). In particular, Laveille and Moulines show that an appropriately modified version of the Schwarz criterion yields a consistent estimator of the number of change-points.

To be more specific, consider the following generic model:

$$X_t = \mu_k^* + \varepsilon_t \quad t_{k-1}^* \leq t \leq t_k^* \quad 1 \leq k \leq r \tag{1.6}$$

where $t_0^* = 0$ and $t_{r+1}^* = T$, the sample size. The indices of the breakpoint and mean values μ_k^*, $k = 1, \ldots, r$ are unknown. It is worth recalling that X_t is a generic stand-in process. In practical applications, equation (1.6) applies to squared returns, absolute returns, high-frequency data-driven volatility estimates, etc. The Laveille and Moulines tests are based on the following least-squares computation:

$$Q_T(t) = \min_{\mu_k^*, k=1,\ldots,r} \sum_{k=1}^{r+1} \sum_{t=t_{k-1}+1}^{t_k} (X_k - \mu_k)^2 \tag{1.7}$$

Estimation of the number of break points involves the use of the Schwarz or Bayesian information criterion (following Yao (1988)) and hence a penalized criterion $Q_T(t) + \beta_T r$, where r is the number of break points and $\beta_T = 4\log(T)/T^{1-2d}$.[8]

If several change-points are suspected, the usual procedure is to divide the sample into two parts, before and after an estimated change point, and to test for the presence of a change point in each of the subsamples. A complementary procedure following this strategy was developed prior to Laveille and Moulines (2000). Inclán and Tiao (1994) propose an Iterated Cumulative Sum of Squares (ICSS) algorithm that covers testing for multiple breaks in the variances of independent processes. It has been applied by Aggarwal et al. (1999) and Granger and Hyung (1999). We combine the ICCS algorithm of Inclán and Tiao with the K&L test appearing in (1.1), to sequentially test for multiple breaks. We will use the sequential application of K&L tests in the empirical section to test for multiple breaks, in addition to using the L&M test.

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[7] The Laveille and Moulines tests apply to various classes of processes, including α-mixing processes. Since GARCH processes are β-mixing they are α-mixing and thus satisfy the Laveille and Moulines set of regularity conditions.

[8] This formula allows for the possibility of long memory, with d Hosking's long-range dependence parameter.

Given the sequential application of change-point tests in subsamples we may obviously end up with relatively small samples. It will therefore be important to appraise the power and size properties of change-point tests in small samples via Monte Carlo simulations.

1.3 Empirical processes

So far we discussed mostly X_t as a generic process. It was noted that besides squared return processes Kokoszka and Leipus (1999, 2000) suggest to apply their CUSUM type tests to absolute values of returns, which are covered by their regularity conditions. The interest in studying the absolute returns stems from Ding, Granger, and Engle (1993) and the subsequent work on long memory ARCH models, see Baillie, Bollerslev and Mikkelsen (1996), and long memory SV models, see Breidt, Crato and de Lima (1998), Comte and Renault (1998), among others. Hence, we examine CUSUM type tests with X_t equal to absolute returns. With squared returns we shall be examining whether there are breaks in ARCH and SV-type processes that are driven by squared returns. With absolute returns we investigate whether there are breaks in the type of processes often found in the literature on long memory in volatility. It should be stressed, however, that breaks in FIGARCH processes can be found through examining squared returns, whereas breaks in long memory volatility features in a more broadly defined sense, can be examined via absolute returns.

A departure from the limited number of applications found in the literature so far is to use actually estimates of conditional volatility through the use of high frequency data. The logic for considering such empirical processes is that squared returns can be viewed as noisy realizations of the underlying conditional volatility process (see Andersen and Bollerslev (1998) for a discussion). Hence, instead of considering the daily return process and square it, we can take advantage of high frequency intra-daily data to obtain daily estimates of volatility.[9] Using the notation $r_{(m),t}$ to represent high frequency data on day t sampled with frequency m we can study sums of squared returns $r^2_{(m),t}$ for different values of m, to produce the daily volatility measure: (i) $\hat{\sigma}_t^{QV1} = \sum_{j=1}^{m} r^2_{(m),t+1-j/m}$, $t = 1, ..., N$, where for

[9] We refrain here from discussion the diffusion details of this class of estimators as well as definitions of quadratic variation. For details we refer the reader to Andersen et al. (2001), Andreou and Ghysels (2000) and Barndorff-Nielsen and Shephard (2000).

the 5-minute sampling frequency the lag length is $m = 288$ for financial markets open 24 hours per day (e.g. FX markets) as in Andersen et al. (2001), Andreou and Ghysels (2000) and Barndorff-Nielsen and Shephard (2000) or (ii) One-day Historical Quadratic Variation (introduced in Andreou and Ghysels, 2000) defined as the sum of m rolling QV estimates: $\hat{\sigma}_t^{HQV1} = 1/m \sum_{j=1}^{m} QV1_{(m),t+1-j/m},\ t = 1, ..., N$. The intraday volatilities are denoted as QVi, $HQVi$ for window lengths $i = 1, 2, 3$. Clearly, the regularity conditions for squared daily returns, viewed as noisy estimates of quadratic variation, can be transplanted to more efficient filtering schemes like QVi (recall our discussion about mixing conditions and measurable functions).

The last empirical process we consider is one not currently covered explicitly by econometric theory and for which we can only conjecture that the regularity conditions are sufficient and justify the use of the existing asymptotic distributions. Namely, to study breaks in volatility co-movements we suggest to consider $X_t = r_{i,t} \times r_{j,t}$, that is the cross-product of returns for markets i and j. The advantage of using the cross-product is that we examine multivariate features through a univariate process, enabling us to apply the existing tests. Using Proposition 3.50 in White (1984), one would need to establish mixing conditions for the bivariate or multivariate GARCH process in order to derive mixing conditions for products like $X_t = r_t^i \times r_t^j$. At this points, no such conditions are available. Formally, we do not know the stochastic process properties of multivariate ARCH type models or discrete time SV type models and even for continuous time SV models very few results are known. Carrasco and Chen (2001) only establish mixing conditions for univariate ARCH type processes, and no results are available for multivariate processes. Our empirical analysis is therefore only speculative, in terms of its theoretical foundations. To provide some confidence in the reliability of the tests in a multivariate setting, we will conduct some Monte Carlo simulations that support the use of tests in multivariate settings. Since the data generating processes in the Monte Carlo simulations are representative, they provide some confidence to the usefulness of these tests beyond the current reach of econometric theory.

2 The Monte Carlo Design and Results

The aim of this section is to evaluate the performance of the Kokoszka and Leipus (1998, 2000) $\sup\{|U_N(k)|\}/\hat{\sigma}$ appearing in (1.4) as well as Inclán and

Tiao (1994) tests IT appearing in (1.5) (also refered to as K&L and I&T tests respectively) in detecting breaks in the volatility dynamics of financial asset returns. The observed absolute or squared returns transformations are the series monitored for breaks. The test recently proposed in Kokoszka and Leipus (1998, 2000) provides a framework for testing breaks in $(r_t)^2$ and $|r_t|$ where r_t follows an ARCH(∞) or a fractional ARIMA process.

The simulation design examines the size and power properties of the Kokoszka and Leipus (1998, 2000) test for GARCH type processes that can be considered as representative models of financial asset returns. The simulation results reported in Kokoszka and Leipus (2000) focus on the sampling distribution of the change-point estimator $\hat{k}$ for an ARCH(1) process. They find that its sampling distribution depends on the location of the change-point, the size of the variance change and its source. The extensive results presented in this section complement Kokoszka and Leipus (2000) in establishing the test's power for univariate and bivariate GARCH processes and a number of alternative change-point hypotheses often encountered in asset returns. The robust character of the test is also examined in the presence of outliers given the stylized fact of jumps or extreme observations observed in volatility and absolute returns which may lead to spurious nonlinearities or IGARCH effects (e.g. Lamoureux and Lastrapes, (1990), van Dijk et al. (1999 a,b), Franses et al. (2001)). Last but not least, the simulations provide positive results regarding the power of the tests for detecting breaks in the co-movements between financial series that can be considered as a useful direction for further theoretical analysis in the multivariate change-point problem.

The apparent similarity of the CUSUM-type statistics in K&L with that specified by Inclán and Tiao (1994) for independent sequences calls for an interesting comparison which brings about the connection between these two tests and their power in detecting change-points in GARCH processes as well as jumps in financial markets. The I&T test was recently applied to the squared returns of stock indices in emerging and Asian markets by Aggarwal et al. (1999) and Granger and Hyung (1999). In the former study the detection of breaks leads to incorporating dummies in the GARCH models whereas in the latter it leads to implications regarding the long-memory of the process that is assumed to follow the linear model with breaks developed by Chen and Tiao (1990) and Engle and Smith (1999).[10] For comparison

[10]Granger and Hyung (1999) also apply the Bai (1997) test to the absolute returns of

purposes both tests, namely K&L $\sup\{|U_N(k)|\}/\hat{\sigma}$ in (1.4) and IT in (1.5), are evaluated for absolute and squared returns whereas the latter test is also applied to the residuals of a GARCH process given that this test is originally designed for independent processes. In a first subsection the simulation design is discussed, followed by a subsection describing the results.

2.1 Simulation design

The simulated returns processes are generated from the following two types of DGPs: (i) a univariate Normal-GARCH process, and (ii) a multivariate GARCH process with constant correlation (M-GARCH-CC) (see e.g. Bollerslev (1990)). The choice of the M-GARCH-CC is due mainly for its simplicity and parsimony as well as the fact that it constitutes a multivariate design most closely related to the univariate GARCH models. More specifically, the two DGPs are:
(i) Univariate Normal-GARCH process:

$$\begin{aligned} & r_{i,t} = u_{i,t}\sqrt{h_{i,t}} \\ & h_{i,t} = \omega_i + \alpha_i u_{i,t-1}^2 + \beta_i h_{i,t-1}, \quad t = 1, ..., T \quad \text{and} \quad i = 0, 1. \end{aligned} \tag{2.1}$$

where $r_{i,t}$ is the returns process generated by the product of $u_{i,t}$ which is an $i.i.d.(0,1)$ series and the volatility process, $h_{i,t}^2$ that has a GARCH(1,1) specification, and
(ii) Multivariate GARCH process

$$\begin{aligned} r_{11,i,t} &= u_{1,i,t}\sqrt{h_{11,i,t}} + u_{2,i,t}h_{12,i,t} \\ r_{22,i,t} &= u_{2,i,t}\sqrt{h_{22,i,t}} + u_{1,i,t}h_{12,i,t}, \quad t = 1, ..., T \quad \text{and} \quad i = 0, 1. \end{aligned} \tag{2.2}$$

where $r_{11,i,t}$ and $r_{22,i,t}$ are the returns processes that are generated by $u_{1,i,t}$ and $u_{2,i,t}$ $i.i.d.(0,1)$ processes, GARCH conditional variances:

$$\begin{aligned} h_{11,i,t} &= \omega_{11,i} + \alpha_{11,i}u_{1,i,t-1}^2 + \beta_{11,i}h_{11,i,t-1} \\ h_{22,i,t} &= \omega_{22,i} + \alpha_{22,i}u_{2,i,t-1}^2 + \beta_{22}h_{22,i,t-1} \end{aligned} \tag{2.3}$$

and the conditional covariance:

$$h_{12,i,t} = \rho_{12,i}\sqrt{h_{11,i,t}h_{22,i,t}}. \tag{2.4}$$

the linear *STOPBREAK* model (Engle and Smith, 1999).

The process in (i) or (ii) without change points is denoted by $i = 0$ whereas a break in any of the parameters of the process is symbolized by $i = 1$. The models used in the simulation study are representative of financial markets data with the following set of parameters that capture a range of degrees of volatility persistence (measured by $\alpha_0 + \beta_0$). The vector parameters $(\omega_0, \alpha_0, \beta_0)$ in (2.1) describes the following Data Generating Processes: DGP1: $(0.4, 0.1, 0.5)$, DGP2: $(0.2, 0.1, 0.7)$ and DGP3 $(0.1, 0.7, 0.1)$. The three DGPs are characterized by low, average and high volatility persistence, respectively. In order to control the multivariate simulation experiment the volatility processes in the M-GARCH-CC model (2.3), are assumed to have the same parameterization. The sample sizes of $N = 500, 1000, 3000$ are chosen so as to examine not only the asymptotic behavior but also the small sample properties of the tests. The small sample features are particularly relevant for the sequential application of the tests in subsamples.

The models in (i) and (ii) without breaks ($i = 0$) denote the processes under the null hypothesis for which the simulation design provides evidence for the size of the K&L and I&T tests. The simulation results are discussed in the section that follows. Under the alternative hypothesis the returns process is assumed to exhibit breaks and four cases are considered to evaluate the power of the tests. The simulation study focuses on the simple single change-point hypothesis. In the context of (2.1) we study breaks in the conditional variance h_t which can also be thought as permanent regime shifts in volatility at change points πN ($\pi = .3, .5, .7$). Such breaks may have the following sources. H_1^A : a change in the volatility dynamics (or persistence), β_i. H_1^B : a change in the intercept, ω_i. H_1^C : a change in the tails of $u_{0,t}$ to $u_{1,t} \sim N(0, \sigma_u)$, ($\sigma_u = 1.1, 1.5$) at $t = \pi N + 1, ..., N$. H_1^D : outliers in the error, $u_{0,t}$ to $u_{1,t} \sim N(\mu_u, 1)$, with jump sizes $\mu_u = 4, 5$ and frequencies at given regular dates of a daily sample, $\Delta \cdot t_j$, (where $\Delta = 250, 500$ and $t_j = 1, 2, ..., \Delta/N$) and zero otherwise.[11]

The simulation investigation is organized as follows: Firstly, we consider the application of the K&L and I&T tests for a single series given by the univariate GARCH model in (i). The alternative hypotheses of a change point in (i) examine the power of the test in detecting breaks due to either changes in the parameters or the error of the GARCH process (as defined by $H_1^A, H_1^B, H_1^C, H_1^D$). Given the international linkages amongst financial mar-

[11] In our experiment the above simple jump process would facilitate the evaluation of the test's power in the presence of controled outliers.

kets it is interesting to examine whether breaks that occur in one market are transmitted and can be detected in another market. Hence we consider the framework of (ii) with a break that occurs in *one* of the GARCH processes. We test for a change-point in $r_{11,t}$ in equation (2.2) while the break is due to either $h_{22,t}$ in (2.3) or $h_{12,t}$ in (2.4). The power of the K&L test in the context of model (ii) is evaluated for the following alternative hypotheses: H_1^E : Breaks in the correlation coefficient ρ_{12} as well as $H_1^A, H_1^B, H_1^C, H_1^D$ that refer to the parameters or the error in $h_{22,t}$. Secondly, in the context of (ii) we extend the application of the tests in a bivariate framework for the cross-product between two processes. As discussed in section 1.3 we examine the cross-product of returns to appraise the performance of the tests in an area of great potential application, but with caution about the theoretical underpinnings that justify the asymptotic analysis.

2.2 Simulation Results

The simulation analysis commences with the examination of the asymptotic critical values of the K&L and I&T CUSUM-type tests for a GARCH process with different volatility persistence (given by DGP1 and DGP2) and for various sample sizes. The small samples ($N = 250, 500$) are particularly useful for the sequential application of the tests for multiple breaks whereas large samples are routinely encountered in applications of financial asset returns data. When the K&L test is applied to (r_t^2) and $|r_t|$ the critical values for the 90%, 95% and 99% percentiles are simulated using the VARHAC estimator for standardizing the statistic, over 10,000 replications. The results in Table 1 (top panel) show that the critical values for the 95% percentile are on average higher than the asymptotic critical value of 1.36 obtained by the supremum of the Brownian Bridge, i.e. the asymptotic distribution. Although the increase in the critical values is only marginal in the low persistent GARCH (DGP1) this increase is more evident for the more persistent GARCH (DGP2). These results also apply to the I&T test for the (r_t^2) process. In contrast, when the I&T test is applied to the errors (u_t) from the simulated GARCH, defined as $u_t = r_t/\sqrt{h_t}$ in model (2.1), the simulated critical values approximate the asymptotic level since this process is independent. Note, however, that there are still some minor fluctuations around the asymptotic critical values for various sample sizes, an indication that minor size distortions still may occur. The performance of the K&L test is further evaluated when the underlying process is a univariate Normal-

GARCH(1,1) by reporting the nominal sizes of the tests instead of the Monte Carlo simulation percentiles. In Table 2 we report results for tests that have an asymptotic size of 5 %. We find that there are only minor size distortions for GARCH models with low persistence (e.g. DGP1 where $a_0 + \beta_0 = 0.6$) and that these minor distortions remain as the sample size increases from $N = 500$ to 3000. Some mild distortions are found for DGP2 which yields a size around 10% whereas for the high-persistent process (DGP3) the test suffers more serious distortions up to 20%.

The power of the K&L test is evaluate by a number of alternative hypotheses as defined in the previous section. The results in Table 2 suggest that the tests have good power in detecting breaks under the following alternative hypotheses: Break in the constant (H_1^B) or dynamics (H_1^A) of volatility. The power of the K&L test is demonstrated for small changes (e.g. a 0.1 increase) in β_0 for all DGPs. Similar results apply to the alternative of a small change in the error term (H_1^C). The power of the tests increases with N in all DGPs. Note that the high nominal power for the persistent GARCH process (DGP3) needs to be weighted by the size distortions for near IGARCH processes mentioned above. In H_1^D we examine the K&L test when outliers or short-lived jumps are present in financial markets. These are generated in u_t and do not seem to have an adverse effect on the K&L test. The last panel shows that infrequent but large outliers are not mistakenly detected for permanent change points. Finally we evaluate the power of the K&L test for early change-points. The last three panels of Table 2 (for H_1^A, H_1^B, H_1^C) show that the K&L test can detect breaks that occur as early as $\pi = 0.3$ of the sample.

The size and power properties of the K&L test are compared with those of I&T. The latter is derived for independent series but has been applied to processes that exhibit dependence (Aggarawal et al. 1999, Granger and Hyung, 1999). Therefore we examine the properties of the test for $(r_t)^2$, $|r_t|$ as well as the errors of the GARCH process $(u_t)^2$ where $u_t = r_t/\sqrt{h_t}$ yields an independent series. Table 3 presents the nominal size and power simulation results of the I&T test under the same null hypothesis of a univariate Normal-GARCH(1,1) and all the alternative hypotheses discussed above (and also presented in Table 2). Let us first compare the performance of the I&T for $(r_t)^2$ and $|r_t|$. The I&T test for $(r_t)^2$ suffers from size distortions (above 10%) for all DGPs and sample sizes but appears to have good power in detecting even small changes in the GARCH coefficients or the error process (shown by the alternative hypotheses H_1^A, H_1^B, H_1^C) for large N. Nevertheless, its performance is adversely affected by outliers which appear to be consistently detected as change-points. If instead we adopt the $|r_t|$ transformation we note some interesting differences. The I&T test for $|r_t|$ appears seriously undersized and with relatively less power, when compare with $(r_t)^2$, in detecting breaks by any of the alternative hypotheses. However, it is interesting to note that for large N (e.g. 1000, 3000) and highly persistent GARCH processes (e.g. DGP3) the I&T test has good power properties and is not susceptible to outliers as opposed to $(r_t)^2$. Finally we examine the I&T test for $(u_t)^2$ which is by design an independent series. The results in Table 3 show that the size of the I&T statistic for $(u_t)^2$ is near the nominal 5% level. The I&T test for $(u_t)^2$ has power in detecting even small changes in the variance of error term (demonstrated by H_1^C) and is not seriously affected by outliers (H_1^D) for large samples, $N = 3000$. This evidence complements the results in I&T for i.i.d. series and small samples in that it shows that this test can be applied to the residuals of a GARCH for which it would have power to detect breaks only in the error term for very large samples. This statement is supported by the simulation results for the H_1^A and H_1^B for $(u_t)^2$ which show that it lacks power in detecting breaks in the conditional variance. The reason u_t lacks power is due to the standardization of the returns process $r_t/\sqrt{h_t}$ that offsets the corresponding changes in r_t and $\sqrt{h_t}$ and yields an *i.i.d.* error process, u_t.

The co-movements of asset returns in international stock markets are of particular importance in periods of financial crises during which shocks and structural changes can be transmitted between financial markets. The co-movements between returns are simulated based on a M-GARCH-CC as defined in equations (2.2)-(2.4). Th K&L test applies to a univariate ARCH(∞) process and we provide some preliminary simulations that examine the size and power of the test which we extend in the following two directions. First, we test for breaks in the returns of one asset, $r_{11,t}$, while change-points occur in the process of the second asset, $r_{22,t}$, or the correlation between them, ρ_{12}. This simulation experiment addresses the issue of the transmission of breaks. Second, we examine the change-point hypothesis in the cross-products of the returns ($r_{11,t} \times r_{22,t}$) process if breaks occur in $r_{22,t}$ or in the correlation between the two returns, ρ_{12}.

In Table 4 we evaluate the properties of the K&L test in detecting breaks in $r_{11,t}$ when it is generated by the bivariate GARCH process (2.2)-(2.4). Under the null hypothesis of homogeneity the average nominal size of the test is around 10% for the alternative DGPs (and different ρ_{12}'s). Under the alternative hypotheses of a single break in the parameters of $h_{22,t}$ the K&L test for $r_{11,t}$ yields some interesting results. The size and the location of the break in $h_{22,t}$ affects the power of the test. For instance, for a small break (e.g. 0.1 increase in $\beta_{22,0}, \omega_{22,0}$ and $\sigma_{u_{22,0}}$) the application of the K&L test in $r_{11,t}$ lacks power as opposed to the univariate GARCH model (2.1) case in Table 2. Increasing the size of break by 0.2 in $\beta_{22,0}$ and by 0.5 in $\sigma_{u_{22,0}}$ attributes better power to the K&L test for $r_{11,t}$ especially when N is large (e.g. $N = 3000$). These large sample results also apply to the power of the K&L test for $r_{11,t}$ in detecting breaks in the correlation coefficient, ρ_{12}. Summarizing, the simulation results of Table 4 show that if two financial assets follow a bivariate GARCH with constant correlation then a break in the correlation coefficient ρ_{12} of volatility co-movements can be detected in the returns process of either asset for large N. However, a change-point in the GARCH process of one asset ($h_{22,t}$) needs to be of a size equal to $1.5\sigma_{u_{22,0}}$ and $2\beta_{22,0}$ to be transmitted and detected as a change point in $r_{11,t}$.

As a final case in our simulation exercise we consider the change-point hypothesis in the cross-product of returns, $(r_{11,t} \times r_{22,t})$, and normalized returns, $\left(r_{11,t} \times r_{22,t}/\sqrt{h_{11,t}h_{22,t}}\right)$, in model (ii), shown in Table 5. The cross-product transformation reduces the dimensionality of multivariate models and allows the application of the K&L in a univariate framework. It is not surprising that when there is change-point in the correlation coefficient, ρ_{12}, (as given by H_1^D in Table 5), the K&L test has good power in the cross-products of both the returns and normalized returns, squared or absolute transformations. This property also holds when there is a relatively large (equal to 0.5) change in $\sigma_{u_{22,0}}$. The difference between the two transformations, returns and normalized returns, lies in the power of the K&L to detect change-points in $h_{22,t}$. Indeed, comparing the results for the H_1^A in Table 5 we observe that the normalized returns have less power because their cross-product reduces the process to the cross-covariance given by (2.4) and shown by:

$$\left(\frac{r_{11,t}}{\sqrt{h_{11,t}}} - u_{1,t}\right)\left(\frac{r_{22,t}}{\sqrt{h_{22,t}}} - u_{2,t}\right) = u_{1,t}u_{2,t}\left(\rho_{12}\right)^2\sqrt{h_{22,t}h_{11,t}} = u_{1,t}u_{2,t}\rho_{12}h_{12,t}$$

In contrast, the cross-product of returns involves all the conditional moments of the M-GARCH (2.2)-(2.4) as shown by:

$$r_{11,t} \times r_{22,t} = \left(u_{1,t}\sqrt{h_{11,t}} + u_{2,t}h_{12,t}\right) \times \left(u_{2,t}\sqrt{h_{22,t}} + u_{1,t}h_{12,t}\right)$$

which may explain the relatively higher power in detecting change points whose source is $h_{22,t}$.

Overall the simulation excercise focuses on single breaks in ARCH-type proccesses for which the K&L and I&T tests have good power properties. Extensions of the simulation experiment can include multiple breaks and long memory dependent series.

3 Empirical Results

There is a plethora of empirical evidence that the squared asset returns series are characterized by dynamic heteroskedasticity (e.g. Bollerslev et al., 1994) and the absolute returns process features long-range dependence (e.g. Granger and Ding, 1996). Empirical studies recognize that the existence of breaks or regime changes in financial markets affects volatility and long-range dependence in stock returns (e.g. Lamourex and Lastrapes, 1990, Mikosch and Starica, 1999, Granger and Hyung, 1999, Diebold and Ioune, 2001).

The empirical analysis aims to complement the simulation evidence in the following directions. We examine the change-point hypothesis in volatility dynamics and long-range dependence of international stock market indices and FX returns. The empirical performance of the tests, discussed in the previous sections, is evaluated by examining the relation of the change-points to economic events detected not only in the squared and absolute returns but also to a family of data-driven volatility filters. Moreover, we estimate the volatility and long-range dependence in subsample prior and post breaks in an attempt to verify changes in the dependence of the series. The empirical analysis also extends the simulation results to tests for multiple breaks using the Lavielle and Moulines test as well as applying the ICSS algorithm to the Kokoszka and Leipus test (henceforth K&L and M&L, respectively). The analysis also addresses change-points in the comovements between the major stock market indices in view of the global character of such markets and the importance of the transmission of financial crises.

The empirical analysis is performed using data from the stock and FX markets. The four international stock market returns indices, the Financial Times Stock Exchange 100 index (FTSE), the Hang-Seng Index (HSI), the Nikkei 500 index (NIKKEI) and the Standard and Poors 500 index (S&P500) are studied over the period 4/1/1989 - 19/10/2001 at daily frequency (sample size, $N = 3338$). The data source is Datastream. The choice of the sample is based on the recent experience of the Asian and Russian financial crises. We also study the Yen vis-à-vis the US dollar returns over the period 1/12/1986-30/11/1996 at 5-minute sampling frequency. The data source is Olsen and Associates. The original sample is 1,052,064 five-minute return observations (2653 days $\cdot$ 288 five-minute intervals per day). The returns for some days were removed from the sample to avoid having regular and predictable market closures which affect the characterization of the volatility dynamics. For the description of the data removed refer to Andersen et al. (2001). The final sample includes 705,024 five-minute returns reflecting $N = 2448$ trading days.

The empirical analysis commences with investigating the hypothesis of a single break in the four international stock market indices. The results in Table 6 provide evidence that neither the K&L nor the I&T tests support the null hypothesis of homogeneity in the absolute or squared returns of the stock market indices over the sample 1989-2001. These results hold for two alternative nonparametric estimators of $(r_t)^2$ and $|r_t|$ used for standardizing the $\max U_N(k)$ statistic defined in section (1.1): the VARHAC estimator and the NLS estimation of the ARMA(1,1) of squared and absolute returns (Francq and Zakoian, 2000b). The results hold whether we use the asymptotic critical value or the simulated critical values in Table 1. The tests detect a change-point in the volatility and long-range dependence of returns as approximated by the squared and absolute returns transformations. The overall picture of the four stock market returns series dates the change point in 1997 and in particular in the summer months of 1997 for the FTSE, HKI and NIKKEI. The same change-point dates are also supported by the Inclan and Tiao (I&T) test. Using the simulation evidence in Table 3 we note that for large sample sizes N the I&T test for $|r_t|$ is well-behaved in terms of size and power and is not distorted by outliers. It is interesting to note that the extention of the I&T statistic by Kim et al. (2000) (also reported in Table 6 as $B_N(C)$) does not detect any change-points. One possible explanation can be the poor power performance of the test in the presence of highly persistent GARCH processes as documented in Kim et al. and as is supported by the estimation of GARCH models for the four stock market indices.

The change-points detected in the three international stock market indices in Table 6 refer to the Asian crisis period. However the single change point hypothesis can mask the existence of multiple breaks which implies that in dating change-points it is advisable to follow a multiple breaks procedure. For this purpose Inclan and Tiao (1994) propose the ICSS algorithm which we apply to the Kokoszka and Leipus test. The multiple change-point hypothesis is also examined following the approach in Moulines and Lavielle (2000) also discussed in section 1.2. The results of these two tests are summarized in Table 7. In the M&L test we adopt two penalty function criteria, the first is the Bayesian Information Set (BIC) and the second is a modified BIC as proposed in Liu et al. (1996) (denoted by LWZ in Table 7) and we set the number of segments t_k equal to 3 and 5. The empirical findings show that irrespective of the choice of t_k the Moulines and Lavielle test consistently detects the same number of breaks, except when applied to the $|r_t|$ using the BIC which seems to estimate an increasing number of breaks as t_k increases. In the top panel of Table 7 we report the M&L test results. The overall results show that the combination of the BIC and $|r_t|$ tends to predict the largest number of breaks whereas the pair of LWZ and $(r_t)^2$ the smallest number of change-points. The Asian crisis period appears to be a common break in the above combinations (of processes and information criteria) and in all stock market indices that is revealed in different months of 1997. In July and August 1997 we detect the first change-points associated with the Asian crisis in the FTSE, HSI and NIKKEI followed by the October 1997 change-point in the S&P500 as well as the NIKKEI.[12] A second common break in the stock indices that is revealed in the L&M procedure is associated with the Russian crisis. In July 1998 we detect change-points in the FTSE and the S&P500 followed by the August 1998 break in the NIKKEI. The second panel of Table 7 reports tests for multiple breaks applying the ICSS algorithm to the Kokoszka and Leipus test. Comparing the results from the two tests we observe that the latter test detects a larger number of breaks especially when applied to the $|r_t|$ process. The two multiple change-point tests detect some common breaks in the same year mainly that of 1997.

Change-points are also detected in the nonlinear comovements between stock market indices. We adopt the cross-product of returns for pairs of the stock markets which can be considered as the cross-covariances of a general variance-covariance matrix in multivariate volatility models such as the M-GARCH. The simulation results in Table 5 in the context of a bivariate GARCH with costant correlation can be used for comparison. The results in Tables 8 and 9 show robust evidence that breaks have occurred in the

[12] A detailed chronology of the Asian financial crisis events in 1997 and 1998 produced by N. Roubini can be found at http://www.stern.nyu.edu/nroubini/asia/AsiaChronology1.hml.

comovements between the four international stock market indices in August and October 1997. These results are supported using both the K&L single change-point approach (in Table 8) as well as the M&L multiple breaks test (in Table 9) applied to the absolute and quadratic cross-product returns transformations of the four largest stock market indices.

As a final empirical application we test for change-points in the FX market applying the K&L test to the family of high-frequency volatility filters that estimate the Quadratic Variation (QV) of diffusion processes with stochastic volatility as briefly discussed in section 1.3. These high-frequency volatility estimates have been introduced by Merton (1980) and applied in Poterba and Summers (1986), French et al. (1987) and Hsieh (1991) interalia. More recently Andersen and Bollerslev (1998) reintroduced these filters using intraday data, similar to Hsieh (1991). Based on a continuous time diffusion process Andersen and Bollerslev (1998) estimate the one-day Quadratic Variation ($QV1$) which is also called integrated volatility and defined as the sum of the squared returns $r_{(m),t}$ for the intraday frequency m, to produce the daily volatility measure: $QV1$, discussed in section 1.3, using 5-minute sampling frequency the lag length is $m = 288$ for financial markets open 24 hours per day (e.g. FX markets). $QV1$ can be considered as an efficient estimate of the quadratic variation of a stock returns process. One reason for their efficiency being that they utilize the high-frequency intraday data information. The $QV1$ filters are generalized in Andreou and Ghysels (2001) using the results in Foster and Nelson (1996) to increase the window length $k = 2, 3$ days in QVk and to suggest rolling instead of block sampling schemes. The rolling estimation method yields the one-day Historical Quadratic Variation ($HQV1$) defined as the sum of m rolling QV estimates, as discussed in section 1.3, which is also extended to a k window length, $HQVk$. The rolling estimation method yields smooth volatility filters which answers one of the criticisms of the $QV1$ filter (see for instance Barnidoff-Nielsen and Shephard, 2001). The K&L and I&T tests are applied to these estimates of the quadratic variation and compared with the results for $(r_t)^2$. The results in Table 10 reveal the existence of a single change-point that is detected in all the QV type filters by the $U_{\max}/\hat{\sigma}_{VARHAC}$ and IT even at the 1% significance level as opposed to the mixed evidence of a change-point in $(r_t)^2$ and $|r_t|$. This change-point in the quadratic variation of the YN/US$ series is consistently estimated by the high-frequency volatility filters to be located on the 8/2/1993 and 9/2/1993 and associated with the highest increase of the YN vis-à-vis the US dollar since the 1970s and the possibility of Central

Bank interventions (as published in the Asian Wall Street Journal dated 23rd February, 1993).

The empirical analysis so far applied single and multiple breaks test procedures and identified the common dates estimated by the above tests as change-points. In an approach to verify that there was indeed a structural change in the asset returns processes we examine the volatility and long memory characteristics of the series in alternative subsamples - prior and after the breaks. The results in Table 11 report the estimated MLE parameters from a Normal GARCH(1,1) and the long memory parameter d using the semiparametric estimator in Robinson (1994). The varying estimated coefficients of volatility persistence, unconditional variance and long memory over the subsamples can be considered as further supportive empirical evidence that complements the change-point tests.

4 Conclusions

There is a substantial literature on testing for the presence of breaks in linearly dependent stochastic processes. The purpose of this paper is to explore recent advances in the theory of change-point estimation, using various new CUSUM type change-point estimators and tests for multiple breaks in the context of volatility models. The tests are not model-specific and apply to a large class of (strongly) dependent processes such as ARCH and SV type processes and were developed in a series of recent papers by in particular Giraitis, Kokoszka and Leipus (1999, 2000), Horváth and Steinebach (2000), Kokoszka and Leipus (1998, 1999, 2000) and Laveille and Moulines (2000). We focus on the Kokoszka and Leipus and Laveille and Moulines tests which monitor nonlinear transformations of returns processes (in square and absolute returns) without the need to specify any particular, restrictive functional form of the process. Moreover, the CUSUM type test of Kokoszka and Leipus (1998, 2000) and RSS minimization type test of Moulines and Lavielle (2000) are characterized by computational simplicity which is an additional advantage for the complex nonlinear structure of financial time series. So far only limited simulation and empirical evidence is reported about these tests. We enlarge the scope of applicability by suggesting several improvements that enhance the practical implementation of the proposed tests. The extensive simulation investigation regarding the performance of the recently proposed Kokoszka and Leipus test provides evidence that the test has good power properties in detecting even small changes in all the GARCH parameters and the error and appears robust to outliers, but suffers some size distortions in the persistent GARCH case.[13] The simulation experiment extends the performance of the test in testing for breaks in the nonlinear comovements of returns. The relatively good power results opens the route for theoretical extentions of these tests in multivariate settings. While there is no fully developed theory for multivariate processes we suggest to apply the existing tests to cross-products of returns and show this strategy is feasible and useful in practical applications. We also suggest the application of change-point to more precise measures of volatility, including the high frequency data-driven processes studied by Andersen et al. (2001), Andreou and Ghysels (2000), Barndorff-Nielsen and Shephard (2000), Taylor and Xu (1997), among others.

We consider various financial series, including equity index returns for several financial markets in the Hong Kong, Japan, the U.K. and U.S. The data series are similar to several prior studies, particularly Granger and Hyung (1999) who consider a longer but less recent sample. The applications of the Kokoszka and Leipus as well as the Lavielle and Moulines tests detect change-points in the volatility dynamics and long memory which are associated with the Asian and Russian financial crises. Evidence is provided that these events has also affected the nonlinear comovements between these stock market return indices. The empirical analysis is also performed using high frequency data from the FX markets. The above tests are applied to the Yen/US$ class of data driven volatility filters in an attempt to provide more efficient approximations of the quadratic variation of the process for which we also detect change-points.

[13] The IGARCH type of models violate the assumption of finite fourth moments required by the Kokoszka and Leipus tests.

References

[1] Aggarwal, R., Inclán, C. and R. Leal (1999), Volatility in Emerging Stock Markets", *Journal of Financial and Quantitative Analysis*, 34, 1-.

[2] Andersen, T. and T. Bollerslev (1998), "Answering the Skeptics: Yes, Standard Volatility Models Do Provide Accurate Forecasts", *International Economic Review*, 39, 885-905.

[3] Andersen, T., T. Bollerslev, F. X. Diebold and P. Labys (2001), "The Distribution of Exchange Rate Volatility", *Journal of American Statistical Association*, 96, 42-55.

[4] Andreou E. and E. Ghysels (2000), "Rolling sample volatility estimators: some new theoretical, simulation and empirical results" *Journal of Business and Economic Statistics*, forthcoming.

[5] Andrews, D. W. K. (1991), "Heteroskedasticity and Autocorrelation Consistent Covariance Ma-trix Estimation," *Econometrica*, 59, 817-858.

[6] Antoch, Jr., Huskova, M. and Z. Praskova (1997), "Effect of dependence on statistics for determination of change", *Journal of Statistical Planning and Inference*, 60, 291-310.

[7] Bai, J. (1994), "Least squares estimation of a shift in linear process", *Journal of Time Series Analysis*, 15, 453-472.

[8] Bai, J. (1997), "Estimating Multiple Breaks One at a Time", *Econometric Theory*, 13, 315-351.

[9] Bai, J. and Perron, P. (1998), "Estimating and Testing Linear Models with Multiple Structural Changes, *Econometrica*, 66, 47-78.

[10] Baillie, R., Bollerslev, T. and Mikkelsen, H. (1996), "Fractionally integrated generalized autoregressive conditional heteroskedasticity", *Journal of Econometrics* 74, 3-30.

[11] Barndorff-Nielsen, O. E. and N. Shephard (2000), "Econometric analysis of realised volatility and its use in estimating stochastic volatility models", Discussion Paper, Nuffield College, University of Oxford.

[12] Bollerslev, T. (1990), "Modelling the Coherence in Short Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model", *Review of Economics and Statistics*, 72, 498-505.

[13] Bollerslev, T., R. F. Engle and D. B. Nelson (1994), "ARCH Models", in Robert F. Engle and Dan McFadden (eds.), *Handbook of Econometrics*, Volume IV, 2959-3038. Amsterdam: North-Holland.

[14] Bos, C., Franses, Ph. H. and M. Ooms (1999), "Long memory and level shifts: re-analysing inflation rates", *Empirical Economics*, 24 , 427-450.

[15] Breidt, F.J., Crato, N. and P. de Lima (1988), "On the Detection and Estimation of Long Memory in Stochastic Volatility", *Journal of Econometrics*, 83, 325-348.

[16] Carrasco, M. and X. Chen (2001), "Mixing and Moment Properties of Various GARCH and Stochastic Volatility Models", *Econometric Theory* forthcoming.

[17] Chen, C. and G.C. Tiao (1990), "Random Level-Shift Time Series Models, ARIMA Approximations, and Level-Shift Detection", *Journal of Business and Economic Statistics*, 8, 83-97.

[18] Chu, C.-S. (1995), "Detecting parameter shift in GARCH models", *Econometric Reviews*, 14, 241-266.

[19] Comte, F. and E. Renault (1998), "Long Memory Continuous Time Stochastic Volatility Models", *Mathematical Finance*, 8, 291-323.

[20] Csörgo, M. and L. Horváth (1997), *Limit Theorems in Change-Point Analysis*, Wiley, New York.

[21] Den Haan, W.J., and A. Levin (1997), "A Practioner's Guide to Robust Covariance Matrix Estimation" in C.R. Rao and G.S. Maddala (eds.) *Handbook of Statistics - Vol. 15*, 291-341.

[22] Diebold, F.X. (1986), "Modeling the persistence of conditional variances: A comment", *Economtric Reviews*, 5, 51-56.

[23] Diebold, F.X. and Inoue, A. (2001), "Long Memory and Structural Change. Manuscript", *Journal of Econometrics* 105, 131-159.

[24] Ding, Z., Granger, C.W.J. and Engle, R.F. (1993) "A Long-memory Properties of Stock Market Returns and a New Model", *Journal of Empirical Finance*, 1, 83-106.

[25] Engle, R.F. (1982), "Autoregressive conditional heteroskedasticity with estimates of the variance of the United Kingdom inflation", *Econometrica*, 50, 987-1007.

[26] Engle R. F., T. Ito and W-L. Lin (1990), "Meteor Showers or Heat Waves? Heteroskedastic Intra-Daily Volatility in the Foreign Exchange Market", *Econometrica*, 58, 525-542.

[27] Engle R. F and A. Smith (1999), "Stochastic Permanent Breaks", *Review of Economics and Statistics*, 81, 553-574.

[28] Francq, C. and J.-M. Zakoïan (2000a), "Estimating Stochastic Volatility Models: A New Approach Based on ARMA Representations", CREST Discussion Paper No. 2000-47.

[29] Francq, C. and J.-M. Zakoïan (2000b), "Estimating Weak GARCH Representations", *Econometric Theory*, 16, 692-728.

[30] Franses, Ph. H., van der Leij, M. and R. Paap (2001), "Modeling Outliers and Temporary Level Shifts in Absolute Returns", Paper presented at International Conference on Modelling and Forecasting Financial Volatility,Perth, Australia, 7-9 September 2001.

[31] French, K.R., G.W. Schwert and R.F. Stambaugh (1987), "Expected Stock Returns and Volatility", *Journal of Financial Economics*, 19, 3-29.

[32] Gallant, A. R. (1987), *Nonlinear Statistical Models*, John Wiley and Sons, New York.

[33] Ghysels, E., Harvey, A.C. and Renault, E. (1996), "Stochastic volatility" in C.R. Rao and G.S. Maddala (eds.), *Handbook of Statistics - Vol. 14*, 119-191.

[34] Giraitis, L., Kokoszka, P. and Leipus, R. (1999), "Detection of long memory in ARCH models", Technical report, reprint

[35] Granger, C.W.J. (1998), "Real and spurious long-memory properties of stock-market data: comment", *Journal of Business and Economic Statistics*, 16, 268-269.

[36] Granger, C.W.J. and N. Hyung (1999), "Occasional Structural Breaks and Long Memory", Discussion Paper 99-14, Department of Economics, University of California, San Diego.

[37] Hansen, L. P. (1982), "Large Sample Properties of Generalized Method of Moments Estimators", *Econometrica*, 50, 1029-1054.

[38] Hendry, D. F. (1986), "An Excursion into Conditional Variance Land", *Economtric Reviews*, 5, 63-69.

[39] Horváth, L. (1997) "Detection of changes in linear sequences", *Annals Inst. Stat. Math.* 49, 271-283.

[40] Horváth, L. and P. Kokoszka (1997), "The effect of long-range dependence on change-point estimators", *Journal of Statistical Planning and Inference* 64, 57-81.

[41] Horváth, L., Kokoszka, P. and J. Steinebach (1999), "Testing for changes in multivariate dependent observations with applications to temperature changes", *Journal of Multivariate Analysis*, 68, 96-119.

[42] Horváth, L. and J. Steinebach (2000), Testing for changes in the mean and variance of a stochastic process under weak invariance, *Journal of Statistical Planning and Inference*, forthcoming.

[43] Horváth, L. and G. Teyssiere (2001), "Empirical process of the squared residuals of an ARCH sequence", *The Annals of Statistics*, 29, 445-469.

[44] Hsieh, D.A. (1991), "Chaos and Nonlinear Dynamics: Application to Financial Markets", *Journal of Finance*, 46, 1839-1877.

[45] Inclán, C. and G. Tiao (1994), "Use of cummulative sums of squares for retrospective detection of change of variance", *Journal of the American Statistical Association*, 89, 913-923.

[46] Kim, S., S. Cho and S. Lee (2000), "On the CUSUM test for parameter changes in GARCH(1,1) models", *Communications in Statistics*, 29, 445-462.

[47] Kokoszka, P. and R. Leipus (1998), "Change-point in the mean of dependent observations", *Statistics and Probability Letters*, 40, 385-393.

[48] Kokoszka, P. and R. Leipus (1999), "Testing for parameter changes in ARCH models", *Lithuanian Mathematical Journal* 39, 231-247.

[49] Kokoszka, P. and R. Leipus (2000), "Change-point estimation in ARCH models", *Bernoulli*, 6, 1-28.

[50] Kokoszka, P. and R. Leipus (2001), "Detection and estimation of changes in regime", in P. Doukhan, G. Oppenheim and M.S. Taqqu, eds. *Long-range Dependence: Theory and Applications*, Birkhauser, Boston, forthcoming.

[51] Lamoureux, C.G. and W.D. Lastrapes (1990), "Persistence in variance, structural change and the GARCH model", *Journal of Business and Economic Statistics*, 8, 225-234.

[52] Lavielle, M. and E. Moulines (2000), "Leat-squares Estimation of an Unknown Number of Shifts in Time Series", *Journal of Time Series Analysis*, 20, 33-60.

[53] Lee, S.W. and B.E. Hansen (1994), "Asymptotic Theory for the GARCH (1,1) Quasi-Maximum Likelihood Estimator", *Econometric Theory*, 10, 29-58.

[54] Ling, S. and M. McAleer (1999a), "Asymptotic Theory for a Vector ARMA-GARCH Model", *Econometric Theory*, forthcoming.

[55] Ling, S. and M. McAleer (1999b), "Stationarity and the Existence of Moments of a Family of GARCH Proccesses", *Journal of Econometrics*, forthcoming.

[56] Lobato, I.N. and N. Savin (1998), "Real and spurious long-memory properties of stock-market data", *Journal of Business and Economic Statistics*, 16, 261-268.

[57] Lumsdaine, R. (1996), "Consistency and Asymptotic Normality of the Quasi-Maximum Likelihood Estimator in IGARCH (1,1) and Covariance Stationary GARCH (1,1) Models", *Econometrica*, 64, 575-596.

[58] Lundbergh, S. and T. Terasvirta (1998), "Evaluating GARCH models", Technical Report No. 292, Working paper, Stockholm School of Economics.

[59] Merton, R. C. (1980), "On Estimating the Expected Return on the Market: An Exploratory Investigation", *Journal of Financial Economics* 8, 323-361.

[60] Mikosch, T. and C. Starica, C. (1999), "Change of structure in financial time series, long range dependence and the GARCH model", Technical Report, preprint available at `http,//www.cs.nl/ eke/iwi/preprints`.

[61] Newey, W. K., and K. D. West (1987), "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix", *Econometrica*, 55, 703-708.

[62] Poterba, J.M. and L. H. Summers, (1986), "The Persistence of Volatility and Stock Market Fluctuations", *American Economic Review*, 76, 1142-1151.

[63] Robinson, P. (1991), "Testing for strong serial correlation and dynamic conditional heteroskedasticity", *Journal of Econometrics*, 47, 67-84.

[64] Robinson, P. (1994), "Semiparametric analysis of long memory time series analysis", *Annals of Statistics*, 22, 515-539.

[65] Taylor, S. and X. Xu (1997), "The incremental volatility information in one million foreign exchange quotations", *Journal of Empirical Finance*, 4, 317-340.

[66] van Dijk, D., Franses, Ph. H. and A. Lucas (1999a), "Testing for Smooth Transition Nonlinearity in the Presence of Outliers", *Journal of Business and Economic Statistics*, 17, 217-235.

[67] van Dijk, D., Franses, Ph. H. and A. Lucas (1999b), "Testing for ARCH in the presence of additive outliers", *Journal of Applied Econometrics*, 14, 539-562.

[68] White, H. (1980) "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity", *Econometrica*, 48, 817-838.

[69] White, H. (1984) *Asymptotic Theory for Econometricians*, Academic Press.

[70] Yao, Y. (1988) "Estimating the number of change-points via Schwarz Criterion", *Statistics and Probablity Letters*, 6, 181-189.

Table 1: Empirical and Asymptotic Quantiles of the Kokoszka and Leipus (2000), $\max U_N(k)/\hat{\sigma}_{HAC}$, and the Inclán and Tiao (1994), *IT*, Statistics for a GARCH Process.

GARCH Model: $r_{0,t} = u_{0,t}\sqrt{h_{0,t}}$, $h_{0,t} = \omega_0 + \alpha_0 u_{0,t-1}^2 + \beta_0 h_{0,t-1}$												
	$(\omega_0 = 0.4, \alpha_0 = 0.1, \beta_0 = 0.5)$						$(\omega_0 = 0.2, \alpha_0 = 0.1, \beta_0 = 0.7)$					
	$\max U_N(k)/\hat{\sigma}_{HAC}$						$\max U_N(k)/\hat{\sigma}_{HAC}$					
Q_p	0.90		0.95		0.99		0.90		0.95		0.99	
X_t	$(r_t)^2$	$\|r_t\|$	$(r_t)^2$	$\|r_t\|$	$(r_t)^2$	$\|r_t\|$	$(r_t)^2$	$\|r_t\|$	$(r_t)^2$	$\|r_t\|$	$(r_t)^2$	$\|r_t\|$
N												
250	1.2538	1.2738	1.3996	1.4129	1.6518	1.6851	1.3508	1.3544	1.4921	1.5061	1.7586	1.7845
500	1.2675	1.2740	1.3986	1.4101	1.6914	1.6833	1.3653	1.3769	1.5164	1.5294	1.8307	1.8403
750	1.2536	1.2592	1.3921	1.4008	1.6737	1.6684	1.3851	1.3739	1.5341	1.5281	1.8151	1.8357
1000	1.2782	1.2866	1.4203	1.4329	1.7152	1.7199	1.3860	1.3904	1.5449	1.5373	1.8276	1.8510
1250	1.2865	1.2957	1.4309	1.4313	1.7126	1.7359	1.3836	1.3839	1.5366	1.5354	1.8314	1.8279
1500	1.2831	1.2840	1.4169	1.4249	1.6737	1.6888	1.3923	1.3626	1.5493	1.5411	1.8601	1.8558
2000	1.2758	1.2857	1.4268	1.4285	1.7180	1.7028	1.3938	1.3858	1.5476	1.5357	1.8500	1.8578
3000	1.2927	1.2931	1.4370	1.4343	1.7151	1.7325	1.4087	1.4000	1.5713	1.5538	1.8807	1.8680
3500	1.2907	1.2890	1.4258	1.4177	1.6719	1.6809	1.4105	1.4088	1.5706	1.5472	1.8606	1.8588
	IT						*IT*					
Q_p	0.90		0.95		0.99		0.90		0.95		0.99	
X_t	$(r_t)^2$	$(u_t)^2$	$(r_t)^2$	$(u_t)^2$	$(r_t)^2$	$(u_t)^2$	$(r_t)^2$	$(u_t)^2$	$(r_t)^2$	$(u_t)^2$	$(r_t)^2$	$(u_t)^2$
N												
250	1.3894	1.1802	1.5556	1.3169	1.9283	1.6110	1.5222	1.1791	1.7026	1.3151	2.0610	1.5669
500	1.4202	1.1954	1.5831	1.3268	1.8907	1.5791	1.5660	1.2003	1.7618	1.3317	2.0974	1.5947
750	1.4299	1.1938	1.5929	1.3259	1.9296	1.6043	1.5818	1.1952	1.7469	1.3216	2.1250	1.6089
1000	1.4345	1.1988	1.6119	1.3402	1.9410	1.6144	1.5930	1.2089	1.7714	1.3440	2.1682	1.6171
1250	1.4476	1.2088	1.6137	1.3452	1.9280	1.6010	1.5989	1.2080	1.7680	1.3319	2.1335	1.5868
1500	1.4434	1.2054	1.6210	1.3444	1.9662	1.6255	1.5967	1.2035	1.7797	1.3353	2.1426	1.6157
2000	1.4537	1.1997	1.6081	1.3333	1.9292	1.5989	1.6000	1.2031	1.7769	1.3305	2.1482	1.6069
3000	1.4597	1.2104	1.6275	1.3479	1.9733	1.6222	1.6222	1.2157	1.7938	1.3499	2.1717	1.6205
3500	1.4570	1.2078	1.6080	1.3370	1.9279	1.5977	1.6147	1.2072	1.7830	1.3373	2.1577	1.6225

Note: The Kokoszka and Leipus (1998, 2000) statistic is defined as $U_N(k) = \left(1/\sqrt{N}\sum_{j=1}^{k} X_j - k/(N\sqrt{N})\sum_{j=1}^{N} X_j\right)$. The $\max U_N(k)$ is standardized by the VARHAC estimator $\hat{\sigma}_{HAC}$ which is applied to X_t, either squared or absolute returns of the GARCH model. The standardized statistic converges to the supremum of a Brownian Bridge with asymptotic critical value 1.36. The Inclán and Tiao statistic $IT = \sqrt{N/2}\,\max|D_k|$ where $D_k = \left[\left(\sum_{j=1}^{k} X_j / \sum_{j=1}^{N} X_j\right) - k/N\right]$ is applied to the squared returns as well as the residuals $u_t = r_t/h_t^{0.5}$ from the GARCH process also converges to the sup of a Brownian Bridge. The Monte Carlo standard errors for $p = 0.90, 0.95, 0.99$ based on the Normal approximation of the binomial given by $3.92\sqrt{p(1-p)/N}$ are: 0.0118, 0.0085, 0.0037, respectively, estimated from 10,000 replicates of the GARCH model for N observations.

Table 2: Nominal Size and Power of the Kokoszka and Leipus (2000) test for a change-point in the volatility and long-range dependence based on a univariate GARCH process.

N	500		1000		3000	
X_t	$(r_t)^2$	$\|r_t\|$	$(r_t)^2$	$\|r_t\|$	$(r_t)^2$	$\|r_t\|$
H_0 : Univariate GARCH, $r_{0,t} = u_{0,t}\sqrt{h_{0,t}}$, $h_{0,t} = \omega_0 + \alpha_0 u_{0,t-1}^2 + \beta_0 h_{0,t-1}$						
$(\omega_0, \alpha_0, \beta_0)$						
DGP1: (0.4, 0.1, 0.5)	0.059	0.072	0.061	0.078	0.061	0.067
DGP2: (0.2, 0.1, 0.7)	0.083	0.098	0.097	0.112	0.116	0.116
DGP3: (0.1, 0.1, 0.8)	0.171	0.165	0.187	0.185	0.212	0.205
H_1^A : Break in the dynamics of volatility, β_0 (increase of 0.1) at $0.5N$.						
DGP1: $\beta_0 = 0.5$ to $\beta_1 = 0.6$	0.273	0.280	0.492	0.473	0.945	0.926
DGP2: $\beta_0 = 0.7$ to $\beta_1 = 0.8$	0.714	0.714	0.935	0.928	1.000	1.000
DGP3: $\beta_0 = 0.8$ to $\beta_1 = 0.9$	0.978	0.978	0.999	0.999	1.000	1.000
H_1^B : Break in the constant of volatility, ω_0 (increase of 0.1) at $0.5N$.						
DGP1: $\omega_0 = 0.4$ to $\omega_1 = 0.5$	0.210	0.204	0.353	0.353	0.809	0.787
DGP2: $\omega_0 = 0.2$ to $\omega_1 = 0.3$	0.470	0.455	0.743	0.723	0.985	0.984
DGP3: $\omega_0 = 0.1$ to $\omega_1 = 0.2$	0.718	0.702	0.913	0.915	1.000	1.000
H_1^C : Break in the error, $u_0 \sim N(0,1)$ (increase $\sigma_{u_1} = 1.1$) at $0.5N$						
DGP1: $u_1 \sim N(0, 1.1)$	0.287	0.277	0.548	0.520	0.921	0.917
DGP2: $u_1 \sim N(0, 1.1)$	0.368	0.356	0.624	0.613	0.960	0.948
DGP3: $u_1 \sim N(0, 1.1)$	0.449	0.437	0.710	0.700	0.982	0.975
H_1^D : Outliers in the error, $u_0 \sim N(0,1)$ ($\mu_{u_1} = 5$ every 250 observations).						
DGP1: $u_1 \sim N(5, 1)$	0.019	0.046	0.015	0.039	0.005	0.044
DGP2: $u_1 \sim N(5, 1)$	0.044	0.082	0.033	0.070	0.020	0.070
DGP3: $u_1 \sim N(5, 1)$	0.039	0.115	0.046	0.134	0.062	0.145

Table 2: Continued.
Properties of Kokoszka and Leipus (2000) test for an early change-point

N	500		1000		3000	
X_t	$(r_t)^2$	$\|r_t\|$	$(r_t)^2$	$\|r_t\|$	$(r_t)^2$	$\|r_t\|$
H_1^A : Break in the dynamics of volatility, β_0 (increase of 0.1) at $0.3N$.						
DGP1: $\beta_0 = 0.5$ to $\beta_1 = 0.6$	0.190	0.204	0.382	0.390	0.838	0.825
DGP2: $\beta_0 = 0.7$ to $\beta_1 = 0.8$	0.566	0.561	0.850	0.837	1.000	0.999
DGP3: $\beta_0 = 0.8$ to $\beta_1 = 0.9$	0.934	0.942	0.996	0.999	1.000	1.000
H_1^B : Break in the constant of volatility, ω_0 (increase of 0.1) at $0.3N$.						
DGP1: $\omega_0 = 0.4$ to $\omega_1 = 0.5$	0.148	0.153	0.254	0.262	0.674	0.634
DGP2: $\omega_0 = 0.2$ to $\omega_1 = 0.3$	0.336	0.327	0.568	0.557	0.959	0.951
DGP3: $\omega_0 = 0.1$ to $\omega_1 = 0.2$	0.552	0.573	0.851	0.844	0.999	0.999
H_1^C : Break in the error, $u_0 \sim N(0,1)$ (increase $\sigma_{u_1} = 1.1$) at $0.3N$						
DGP1: $u_1 \sim N(0, 1.1)$	0.195	0.199	0.329	0.333	0.833	0.804
DGP2: $u_1 \sim N(0, 1.1)$	0.290	0.304	0.492	0.482	0.885	0.870
DGP3: $u_1 \sim N(0, 1.1)$	0.376	0.386	0.548	0.548	0.932	0.923

Note: The Kokoszka and Leipus (1998, 2000) statistic is defined as $U_N(k) = \left(1/\sqrt{N}\sum_{j=1}^{k} X_j - k/(N\sqrt{N})\sum_{j=1}^{N} X_j\right)$. The $\max U_N(k)$ is standardized by the VARHAC estimator $\hat{\sigma}_{HAC}$ which is applied to X_t, either squared or absolute returns of the GARCH model. The normalized statistic converges to the sup of a Brownian Bridge with asymptotic critical value 1.36. The Normal GARCH (1,1) model is simulated (1,000 replications) where the superscirpts 1 and 0 in the variables and coefficients in the Table denote the cases with and without change-points, respectively.

Table 3: Nominal Size and Power of the Inclán and Tiao (1994) test for a change-point in the volatility and long-range dependence based on a univariate GARCH process.

N	500			1000			3000		
X_t	$(r_t)^2$	$\|r_t\|$	$(u_t)^2$	$(r_t)^2$	$\|r_t\|$	$(u_t)^2$	$(r_t)^2$	$\|r_t\|$	$(u_t)^2$
H_0 : Univariate GARCH, $r_{0,t} = u_{0,t}\sqrt{h_{0,t}}$, $h_{0,t} = \omega_0 + \alpha_0 u_{0,t-1}^2 + \beta_0 h_{0,t-1}$									
$(\omega_0, \alpha_0, \beta_0)$									
DGP1: (0.4, 0.1, 0.5)	0.133	0.000	0.038	0.131	0.000	0.041	0.165	0.000	0.042
DGP2: (0.2, 0.1, 0.7)	0.191	0.000	0.050	0.229	0.000	0.033	0.217	0.000	0.042
DGP3: (0.1, 0.1, 0.8)	0.286	0.000	0.038	0.337	0.005	0.041	0.353	0.003	0.051
H_1^A : Break in the dynamics of volatility, β_0 (increase of 0.1) at $0.5N$									
DGP1: $\beta_0 = 0.5, \beta_1 = 0.6$	0.404	0.001	0.047	0.633	0.023	0.047	0.966	0.294	0.058
DGP2: $\beta_0 = 0.7, \beta_1 = 0.8$	0.794	0.113	0.047	0.958	0.359	0.049	1.000	0.980	0.042
DGP3: $\beta_0 = 0.8, \beta_1 = 0.9$	0.989	0.621	0.047	1.000	0.969	0.046	1.000	1.000	0.047
H_1^B : Break in the constant of volatility, ω_0 (increase of 0.1) at $0.5N$									
DGP1: $\omega_0 = 0.4, \omega_1 = 0.5$	0.276	0.002	0.047	0.519	0.014	0.043	0.883	0.131	0.044
DGP2: $\omega_0 = 0.2, \omega_1 = 0.3$	0.576	0.023	0.037	0.806	0.108	0.036	0.998	0.672	0.053
DGP3: $\omega_0 = 0.1, \omega_1 = 0.2$	0.786	0.121	0.043	0.959	0.385	0.047	1.000	0.981	0.032
H_1^C : Break in the error, $u_0 \sim N(0,1)$ (increase $\sigma_{u_1} = 1.1$) at $0.5N$									
DGP1: $u_1 \sim N(0, 1.1)$	0.610	0.057	0.242	0.806	0.133	0.461	0.993	0.685	0.910
DGP2: $u_1 \sim N(0, 1.1)$	0.517	0.014	0.258	0.761	0.070	0.460	0.981	0.474	0.899
DGP3: $u_1 \sim N(0, 1.1)$	0.599	0.046	0.247	0.823	0.128	0.432	0.994	0.668	0.929
H_1^D : Outliers in the error, $u_0 \sim N(0,1)$ ($\mu_{u_1} = 5$ every 250 observations).									
DGP1: $u_1 \sim N(5, 1)$	0.357	0.000	0.231	0.271	0.000	0.112	0.219	0.000	0.079
DGP2: $u_1 \sim N(5, 1)$	0.442	0.001	0.223	0.401	0.003	0.135	0.360	0.000	0.077
DGP3: $u_1 \sim N(5, 1)$	0.505	0.001	0.257	0.500	0.002	0.136	0.481	0.002	0.079

Note: The Inclan and Tiao (1994) statistic $IT = \sqrt{N/2}\max|D_k|$ where $D_k = \left[\left(\sum_{j=1}^{k} r_j^2 / \sum_{j=1}^{N} r_j^2\right) - \frac{k}{N}\right]$ is specified for independent processes. It also converges to the sup of a Brownian Bridge with asymptotic critical value 1.36.

Table 4: Nominal Size and Power of the Kokoszka and Leipus (2000) test for a change-point in the volatility and long-range dependence based on a bivariate GARCH with constant correlation (ρ_{12}): Testing the change-point hypothesis in $r_{11,t}$ when the change-points occur in ρ_{12} and $r_{22,t}$.

N	500		1000		3000	
X_t	$(r_{11,t})^2$	$\lvert r_{11,t}\rvert$	$(r_{11,t})^2$	$\lvert r_{11,t}\rvert$	$(r_{11,t})^2$	$\lvert r_{11,t}\rvert$
H_0 : Bivariate constant correlation GARCH model where $(\omega_{ii,0}, \alpha_{ii,0}, \beta_{ii,0}, \rho_{ij,0})$:						
DGP1: (0.2, 0.1, 0.7, 0)	0.099	0.087	0.113	0.113	0.128	0.112
DGP2: (0.2, 0.1, 0.7, 0.3)	0.097	0.097	0.098	0.097	0.102	0.095
DGP3: (0.2, 0.1, 0.7, -0.3)	0.094	0.093	0.109	0.106	0.117	0.116
DGP4: (0.2, 0.1, 0.7, 0.5)	0.089	0.100	0.110	0.112	0.124	0.122
H_1^A : Break in the dynamics of $h_{22,t}$ volatility, $\beta_{22,0}$ (increase at $0.5N$)						
DGP2: $\beta_{22,0} = 0.7, \beta_{22,1} = 0.8$	0.113	0.110	0.109	0.113	0.177	0.162
DGP2: $\beta_{22,0} = 0.7, \beta_{22,1} = 0.9$	0.182	0.194	0.319	0.309	0.671	0.646
DGP4: $\beta_{22,0} = 0.7, \beta_{22,1} = 0.8$	0.098	0.100	0.107	0.111	0.168	0.180
DGP4: $\beta_{22,0} = 0.7, \beta_{22,1} = 0.9$	0.578	0.541	0.847	0.823	1.000	0.999
H_1^B : Break in the constant of $h_{22,t}$ volatility, $\omega_{22,0}$ (increase at $0.5N$)						
DGP2: $\omega_{22,0} = 0.2, \omega_{22,1} = 0.3$	0.087	0.082	0.110	0.111	0.134	0.137
DGP2: $\omega_{22,0} = 0.2, \omega_{22,1} = 0.4$	0.119	0.132	0.141	0.135	0.203	0.185
DGP4: $\omega_{22,0} = 0.2, \omega_{22,1} = 0.3$	0.112	0.123	0.143	0.141	0.239	0.230
DGP4: $\omega_{22,0} = 0.2, \omega_{22,1} = 0.4$	0.170	0.180	0.242	0.239	0.526	0.497
H_1^C : Break in the error, $u_{22,0} \sim N(0,1)$ (at $0.5N$)						
DGP2: $u_{22,1} \sim N(0,1.1)$	0.109	0.103	0.117	0.122	0.137	0.131
DGP2: $u_{22,1} \sim N(0,1.5)$	0.201	0.207	0.329	0.322	0.729	0.706
DGP4: $u_{22,1} \sim N(0,1.1)$	0.107	0.112	0.133	0.129	0.206	0.192
DGP4: $u_{22,1} \sim N(0,1.5)$	0.592	0.577	0.892	0.867	1.000	1.000
H_1^D : Break in the correlation coefficient, $\rho_{12,0}$						
DGP1: $\rho_{12,0} = 0,\ \rho_{12,1} = 0.3$	0.108	0.110	0.188	0.177	0.320	0.312
DGP1: $\rho_{12,0} = 0,\ \rho_{12,1} = 0.5$	0.341	0.333	0.541	0.516	0.921	0.908
DGP2: $\rho_{12,0} = 0.3,\ \rho_{12,1} = 0.5$	0.192	0.188	0.288	0.290	0.591	0.579
DGP4: $\rho_{12,0} = 0.5,\ \rho_{12,1} = 0.3$	0.182	0.191	0.282	0.265	0.578	0.559

Note: The simulated bivariate GARCH with constant correlation under the null of no breaks (denoted by the 0 superscript) is speficied as: $r_{11,t} = u_{1,0,t}\sqrt{h_{11,0,t}} + u_{2,0,t}\sqrt{h_{12,0,t}}$ and $r_{22,t} = u_{2,0,t}\sqrt{h_{22,0,t}} + u_{1,0,t}\sqrt{h_{12,0,t}}$ where $h_{11,0,t} = \omega_{11,0} + \alpha_{11,0}u_{1,0,t-1}^2 + \beta_{11,0}h_{11,0,t-1}$, $h_{22,0,t} = \omega_{22,0} + \alpha_{2,0}u_{2,0,t-1}^2 + \beta_{22,0}h_{22,0,t-1}$ and $h_{12,0,t} = \rho_{12,0}\sqrt{h_{11,0,t}h_{22,0,t}}$. The Kokoszka and Leipus test is summarized in the note of Table 1.

Table 5: Nominal Size and Power of the Kokoszka and Leipus (2000) test for a change-point in the volatility and long-range dependence based on a bivariate GARCH with constant correlation (ρ_{12}): Testing the change-point hypothesis in $r_{11,t} \times r_{22,t}$ when the change-points occur in ρ_{12} and $r_{22,t}$.

N	1000			
X_t	$(r_{11,t} \times r_{22,t})^2$	$\lvert r_{11,t} \times r_{22,t} \rvert$	$(u_{11,t} \times u_{22,t})^2$	$\lvert u_{11,t} \times u_{22,t} \rvert$
H_0 : Bivariate constant correlation GARCH model where $(\omega_{ii,0}, \alpha_{ii,0}, \beta_{ii,0}, \rho_{ij,0})$:				
DGP1: (0.2, 0.1, 0.7, 0)	0.079	0.099	0.031	0.038
DGP2: (0.2, 0.1, 0.7, 0.3)	0.059	0.094	0.040	0.039
DGP3: (0.4, 0.1, 0.5, 0.5)	0.033	0.061	0.037	0.043
DGP4: (0.2, 0.1, 0.7, 0.5)	0.061	0.090	0.042	0.055
H_1^A : Break in the dynamics of $h_{22,t}$ volatility, $\beta_{22,0}$ (increase at 0.5N)				
DGP2: $\beta_{22,0} = 0.7, \beta_{22,1} = 0.8$	0.477	0.641	0.069	0.087
DGP2: $\beta_{22,0} = 0.7, \beta_{22,1} = 0.9$	0.997	1.000	0.270	0.330
DGP4: $\beta_{22,0} = 0.7, \beta_{22,1} = 0.8$	0.999	1.000	0.092	0.132
DGP4: $\beta_{22,0} = 0.7, \beta_{22,1} = 0.9$	0.542	0.730	0.539	0.709
DGP3: $\beta_{22,0} = 0.5, \beta_{22,1} = 0.7$	0.770	0.914	0.099	0.157
H_1^B : Break in the constant of $h_{22,t}$ volatility, $\omega_{22,0}$ (increase at 0.5N)				
DGP2: $\omega_{22,0} = 0.2, \omega_{22,1} = 0.3$	0.276	0.425	0.055	0.072
DGP2: $\omega_{22,0} = 0.2, \omega_{22,1} = 0.4$	0.630	0.825	0.079	0.093
DGP4: $\omega_{22,0} = 0.2, \omega_{22,1} = 0.3$	0.277	0.417	0.065	0.099
DGP4: $\omega_{22,0} = 0.2, \omega_{22,1} = 0.4$	0.739	0.895	0.116	0.177
DGP3: $\omega_{22,0} = 0.4, \omega_{22,1} = 0.8$	0.865	0.965	0.139	0.205
H_1^C : Break in the error, $u_{22,0} \sim N(0,1)$ (at 0.5N)				
DGP2: $u_{22,1} \sim N(0,1.1)$	0.181	0.290	0.093	0.146
DGP2: $u_{22,1} \sim N(0,1.5)$	0.984	1.000	0.992	0.999
DGP4: $u_{22,1} \sim N(0,1.1)$	0.182	0.275	0.151	0.198
DGP4: $u_{22,1} \sim N(0,1.5)$	0.974	1.000	0.994	1.000
DGP3: $u_{22,1} \sim N(0,1.5)$	0.972	1.000	0.980	0.998
H_1^D : Break in the correlation coefficient, ρ_{12}				
DGP1: $\rho_{12,0} = 0,\ \rho_{12,1} = 0.3$	0.668	0.632	0.752	0.679
DGP1: $\rho_{12,0} = 0,\ \rho_{12,1} = 0.5$	0.989	0.997	0.996	1.000
DGP2: $\rho_{12,0} = 0.3,\ \rho_{12,1} = 0.5$	0.610	0.713	0.681	0.780
DGP4: $\rho_{12,0} = 0.5,\ \rho_{12,1} = 0.3$	0.607	0.710	0.683	0.775
DGP3: $\rho_{12,0} = 0,\ \rho_{12,1} = 0.5$	0.986	1.000	1.000	1.000

Note: The bivariate GARCH is defined in the note of Table 4. The Kokoszka and Leipus statistic is defined in the note of Table 1 and is applied to the absolute and squared transformations of the cross-product of returns and the residuals defined as $u_{ii,t} = r_{ii,t}/\sqrt{h_{ii,t}}$.

Table 6: Testing for a single change-point in the volatility and long-range dependence of daily Stock Market Indices (SMI) over the period 1989-2001

SMI Returns		*Change-point Statistics*					
		Change-point	*Kokoszka & Leipus Test*			*Inclán & Tiao Tests*	
		k	$\max U_N(k)$	$\frac{\max U_N(k)}{\hat{\sigma}_{HAC}}$	$\frac{\max U_N(k*)}{\hat{\sigma}_{ARMA}}$	IT	$B_N(C)$
FTSE	$\lvert r_t \rvert$	05/06/97	1.917	5.862*	6.665*	4.414*	0.599
	$(r_t)^2$	04/08/97	2.238	5.266*	3.511*	9.195*	1.249
HSI	$\lvert r_t \rvert$	18/08/97	3.460	4.619*	5.828*	4.954*	0.321
	$(r_t)^2$	18/08/97	7.104	2.181*	1.291	8.583*	0.556
NIKKEI	$\lvert r_t \rvert$	31/07/97	1.521	3.091*	3.806*	2.905*	0.449
	$(r_t)^2$	21/10/97	1.836	1.972*	1.305	4.427*	0.684
S&P500	$\lvert r_t \rvert$	04/02/97	2.395	6.882*	7.181*	5.837*	0.356
	$(r_t)^2$	26/03/97	2.718	4.888*	1.665*	11.103*	0.678

Notes: (1) The Stock Market Index (SMI) series refer to the Financial Times Stock Exchange index 100 (FTSE100), the Hang-Seng Index (HSI), the Nikkei 500 (NIKKEI), the Standards and Poors 500 index (S&P500). The daily sample over the period 4/1/1989 to 19/10/2001 yields $N = 3338$ observations. The series $r_t := \log p_t - \log p_{t-1}$ represents the returns on each index. The change-point tests are applied to the $(r_t)^2$ and $|r_t|$ transformations as well as $(u_t)^2$ where u_t is the residual from the GARCH.
(2) The Kokoszka and Leipus (1998, 2000) reported statistic is defined as $U_N(k) = \left(1/\sqrt{N}\sum_{j=1}^{k} X_j - k/(N\sqrt{N})\sum_{j=1}^{N} X_j\right)$. The $\max U_N(k)$ is standardized by the VARHAC estimator $\hat{\sigma}_{HAC}$ which is applied to X_t, either squared or absolute returns and ARMA estimators $\hat{\sigma}_{ARMA}$ of squared and absolute returns. The normalized statistic $\max U_N(k)/\widehat{\sigma}$ converges to the sup of a Brownian Bridge.
(3) The Inclán and Tiao (1994) statistic $D_k = \left(\sum_{j=1}^{k} r_j^2 / \sum_{j=1}^{N} r_j^2\right) - \frac{k}{N}$ specified for independent processes normalized as $IT = \sqrt{N/2}\max|D_k|$ also converges to the sup of a Brownian Bridge and is extended in Kim et al. (2000) for GARCH processes to be $B_N(C) = C\sqrt{N}\max|D_k|$ where $C^2$and κ are constants that are estimated by substituting the quasi-MLEs of the GARCH(1,1) $\hat{\omega}$, $\hat{\alpha}$, $\hat{\beta}$ and $N^{-1}\sum_{j=1}^{N} r_j^4$ to ω, α, β and $E(r_j^4)$.
(4) k refers to the location of the break and the * symbol attached to statistics denotes that the null hypothesis of no structural change is rejected using the asymptotic critical value of 1.36.

Table 7: Testing for multiple change-points in the volatility and long-range dependence of daily Stock Market Indices (SMI) over the period 1989-2001

SMI	Process	Selection Criterion		Number & Location of Breaks	
		Lavielle and Moulines Test			
FTSE	$\lvert r_t \rvert$	BIC	-2.616(2), -2.610(1)	2	16/11/92, 4/8/97
		LWZ	-2.599(1), -2.549(0)	1	4/8/97
	$(r_t)^2$	BIC	-2.123(1), -2.070(0)	1	13/7/98
		LWZ	-2.112(1), -2.069(0)	1	13/7/98
HSI	$\lvert r_t \rvert$	BIC	-1.121(3), -1.117(2)	3	3/7/92, 24/1/95, 15/8/97
		LWZ	-1.108(1), -1.074(0)	1	15/8/97
	$(r_t)^2$	BIC	2.005(1), 2.009(0)	1	15/8/97
		LWZ	2.010(0)	0	-
NIKKEI	$\lvert r_t \rvert$	BIC	-1.874(2), -1.867(1)	2	16/9/92, 31/7/97
		LWZ	-1.857(1), -1.851(0)	1	20/8/98
	$(r_t)^2$	BIC	-0.457(2), -0.452(1)	2	16/9/92, 14/10/97
		LWZ	-0.448(0)	0	-
S&P500	$\lvert r_t \rvert$	BIC	-2.525(3), -2.513(2)	3	21/8/91, 4/7/96, 22/7/98
		LWZ	-2.492(2), -2.491(1)	2	21/8/91, 4/2/97
	$(r_t)^2$	BIC	-1.602(1), -1.559(0)	1	15/10/97
		LWZ	-1.591(1), -1.559(0)	1	15/10/97

Notes: For brief data description refer to note 1, Table 6. The number of segments for multiple breaks is set equal to 3 and 5 and similar results are obtained. The selection criteria BIC and LWZ refer to the Bayesian or Schwarz Information Criterion and modified BIC proposed in Liu et al. (1996). The Lavielle and Moulines statistic is described in the second section of the paper and is compared with the results from the application of the ICSS algorithm to the Kokoszka and Leipus test for detecting multiple breaks.

Table 7: Continued. The ICSS type Algorithm for the Kokoszka and Leipus test in detecting multiple breaks.

		ICSS Algorithm for the Kokoszka and Leipus Test				
SMI	Process	Subsamples	k		$\frac{\max U_N(k)}{\hat{\sigma}_{HAC}}$	Number
		(observations)	Observ.	Date		of Breaks
FTSE	$\|r_t\|$	1-3338	2197	5/6/97	5.861*	5
		1-2197	1021	2/12/92	3.508*	
		2198-3338	2497	30/7/98	1.366*	
		1021-2197	1631	5/4/95	2.041*	
		2498-3338	2624	25/1/99	1.650*	
	$(r_t)^2$	1-3338	2239	4/8/97	5.265*	5
		1-2239	991	21/10/92	2.744*	
		992-2239	1556	21/12/94	1.734*	
		992-1556	1279	29/11/93	2.507*	
		1557-2239	2058	22/11/96	1.666*	
HSI	$\|r_t\|$	1-3338	2247	18/8/97	4.619*	6
		1-2247	1630	4/4/95	1.815*	
		2248-3338	2554	19/10/98	2.818*	
		1-1630	1239	4/10/93	2.416*	
		1240-1630	1404	23/5/94	2.540*	
		1405-1630	1534	21/11/94	1.953*	
	$(r_t)^2$	1-3338	2249	18/8/97	2.181*	2
		2250-3338	2554	22/10/98	2.072*	
NIKKEI	$\|r_t\|$	1-3338	2237	31/7/97	3.091*	5
		1-2237	966	16/9/92	2.476*	
		2238-3338	2792	16/9/99	2.161*	
		1-966	293	16/2/90	3.119*	
		2238-2792	2365	27/1/98	1.716*	
	$(r_t)^2$	1-3338	2295	21/10/97	1.972*	4
		1-2295	966	16/9/92	2.470*	
		2296-3338	2861	22/12/99	1.757*	
		1-966	295	20/2/90	2.162*	
S&P500	$\|r_t\|$	1-3338	2110	4/2/97	6.841*	4
		1-2110	776	30/12/91	4.143*	
		2111-3338	2495	28/7/98	1.863*	
		780-2110	1776	25/10/95	2.130*	
	$(r_t)^2$	1-3338	2146	26/3/97	4.888*	3
		1-2146	779	31/12/91	3.159*	
		780-2146	1814	18/12/95	2.343*	

Table 8: Testing for a single change-point in the comovements of daily stock market indices over the period 1989-2001

SMI Process		k	Kokoszka and Leipus Statistic $\max U_N(k)$	$\frac{\max U_N(k)}{\sigma_{HAC}}$	$\frac{\max U_N(k)}{\sigma_{ARMA}}$
HSI*FTSE	$\lvert r_{i,t} \times r_{j,t}\rvert$	14/08/97	2.7186	4.8168*	
	$(r_{i,t} \times r_{j,t})^2$	17/10/97	5.6168	2.3906*	
HSI*NIKKEI	$\lvert r_{i,t} \times r_{j,t}\rvert$	18/08/97	2.8441	3.6477*	
	$(r_{i,t} \times r_{j,t})^2$	28/8/97	9.6730	1.5906*	
HSI*S&P500	$\lvert r_{i,t} \times r_{j,t}\rvert$	14/08/97	2.7587	4.2474*	
	$(r_{i,t} \times r_{j,t})^2$	17/10/97	7.2665	1.4063*	
NIKKEI*FTSE	$\lvert r_{i,t} \times r_{j,t}\rvert$	04/08/97	1.3802	3.8403*	
	$(r_{i,t} \times r_{j,t})^2$	21/10/97	1.4674	1.4591*	
NIKKEI*S&P500	$\lvert r_{i,t} \times r_{j,t}\rvert$	15/10/97	1.4020	4.0103*	
	$(r_{i,t} \times r_{j,t})^2$	22/10/97	1.6199	2.0001*	
FTSE*S&P500	$\lvert r_{i,t} \times r_{j,t}\rvert$	5/8/97	1.7594	6.0627*	
	$(r_{i,t} \times r_{j,t})^2$	14/8/97	1.9434	3.6723*	

Notes: The data are briefly described in note 1, Table 6. The tests are applied on the cross-products of the returns between pairs of stock market returns indices ($r_{i,t} \times r_{j,t}$) and their transformations. Details of the test statistics can be found in note 2, Table 6.

Table 9: Testing for multiple change-points in the comovements of daily FX rates vis-a-vis the DM over the period 1989-2001

SMI	Process	Lavielle and Moulines Selection Criterion		Number & Location of Breaks	
HSI*FTSE	$\lvert r_{i,t} \times r_{j,t} \rvert$	BIC	-1.685(2), -1.684(1)	2	13/8/97, 14/7/99
		LWZ	-2.673(1), -1.635(0)	1	14/7/99
	$(r_{i,t} \times r_{j,t})^2$	BIC	1.343(1), -1.349(0)	1	16/10/97
		LWZ	1.349(0)	0	
HSI*NIKKEI	$\lvert r_{i,t} \times r_{j,t} \rvert$	BIC	-0.904(1), -0.882(0)	1	14/8/97
		LWZ	-0.893(1), -0.882(0)	1	14/8/97
	$(r_{i,t} \times r_{j,t})^2$	BIC	3.328(0)	0	
		LWZ	3.329(0)	0	
HSI*S&P500	$\lvert r_{i,t} \times r_{j,t} \rvert$	BIC	-1.556(2), -1.556(1)	2	13/8/97, 14/7/99
		LWZ	-1.545(1), -1.512(0)	1	13/8/97
	$(r_{i,t} \times r_{j,t})^2$	BIC	2.787(0)	0	
		LWZ	2.787(0)	0	
NIKKEI*FTSE	$\lvert r_{i,t} \times r_{j,t} \rvert$	BIC	-2.464(2), -2.462(1)	2	6/10/92, 23/9/97
		LWZ	-2.451(1), -2.436(0)	1	23/9/97
	$(r_{i,t} \times r_{j,t})^2$	BIC	-0.071(0)	0	
		LWZ	-0.071(0)	0	
NIKKEI*S&P500	$\lvert r_{i,t} \times r_{j,t} \rvert$	BIC	-2.638(2), -2.635(1)	2	24/8/92, 22/10/97
		LWZ	-2.624(1), -2.606(0)	1	22/10/97
	$(r_{i,t} \times r_{j,t})^2$	BIC	-0.649(1), -0.647(0)	1	22/10/97
		LWZ	-0.647(0)	0	
FTSE*S&P500	$\lvert r_{i,t} \times r_{j,t} \rvert$	BIC	-2.916(2), -2.915(1)	2	21/8/91, 5/8/97
		LWZ	-2.904(1), -2.844(0)	1	5/8/97
	$(r_{i,t} \times r_{j,t})^2$	BIC	-1.559(1), -1.540(0)	1	22/10/97
		LWZ	-1.549(1), -1.539(0)	1	22/10/97

Notes: As in notes 1 and 2 in Table 3.

Table 10: Testing for a single change-point in high-frequency volatility filters in the YN/US$ in the period 1986-1996

		Change-point Statistics				
Volatility		*Kokoszka & Leipus*			*Inclán & Tiao type tests*	
Filters	k	$\max U_N(k)$	$\frac{\max U_N(k)}{\hat{\sigma}_{HAC}}$	$\frac{\max U_N(k)}{\hat{\sigma}_{ARMA}}$	IT	$B_N(C)$
$\lvert r_t \rvert$	26/4/91	0.3538	1.493*	1.589*	1.996*	0.451
$(r_t)^2$	-	0.2676	1.120	1.273	1.151	0.260
$QV1$	9/2/93	0.3445	1.925*	3.845*	2.302*	-
$QV2$	9/2/93	0.3443	1.262	7.685*	2.301*	-
$QV3$	9/2/93	0.3442	1.021	11.212*	2.300*	-
$HQV1$	8/2/93	0.3428	1.804*	4.222*	2.291*	-
$HQV2$	8/2/93	0.3429	1.207	8.467*	2.292*	-
$HQV3$	9/2/93	0.3432	0.948	12.435*	2.294*	-

Notes: (1) The Yen vis-a-vis the US dollar returns over the period 1/12/1986-30/11/1996 at 5-minute sampling frequency. The data source is Olsen and Associates. The original sample is 1,052,064 five-minute return observations (2,653 days • 288 five-minute intervals per day). The returns for some days were removed from the sample to avoid having regular and predictable market closures which affect the characterization of the volatility dynamics. The final sample includes 705,024 five-minute returns reflecting N=2448 trading days.

(2) The one-day Quadratic Variation ($QV1$) is the sum of the squared returns $r_{(m),t}$ for the intraday frequency m, to produce the daily volatility measure: $QV1 = \sum_{j=1}^{m} r^2_{(m),t+1-j/m}$, $t = 1,...,n_{days}$, where for the 5-minute sampling frequency the lag length is $m = 288$ for financial markets open 24 hours per day. In $QV2$ and $QV3$ the window length is $k = 2, 3$ days, respectively. The rolling estimation method yields the one-day Historical Quadratic Variation ($HQV1$) defined as the sum of m rolling QV estimates: $HQV1 = 1/m \sum_{j=1}^{m} QV1_{(m),t+1-j/m}$, $t = 1,...,T_{days}$, which is also extended to a k window length, $HQVk$.

(3) The tests are described in notes (2) and (3) of Table 6.

Table 11: Estimating volatility dynamics and long range dependence in subsamples of stock market returns indices.

SMI	Process	Subsamples (observations)	k^* Date	Normal GARCH(1,1) Estimates ω	α	β	Long Memory d
FTSE	$\|r_t\|$	1-3338					0.398
		1-1009	16/11/92				0.450
		1010-2218	4/8/97				0.472
		2219-3338					**0.517**
	$(r_t)^2$	1-3338		0.003[4.409]	0.067[10.46]	0.919[111.8]	
		1-2484	13/7/98	**0.003**[3.669]	**0.058**[6.744]	**0.915**[64.18]	
		2485-3338		**0.017**[2.839]	**0.100**[4.410]	**0.845**[22.40]	
HSI	$\|r_t\|$	1-3338					0.388
		1-913	3/7/92				0.378
		914-1580	24/1/95				0.505
		1581-2269	15/8/97				**0.478**
		2270-3338					**0.518**
	$(r_t)^2$	1-3338		0.016[15.80]	0.124[19.29]	0.855[128.6]	
		1-2248	15/8/97	0.023[18.71]	**0.134**[17.04]	0.810[97.42]	
		2249-3338		0.029[4.387]	**0.092**[7.462]	0.878[63.48]	
NIKKEI	$\|r_t\|$	1-3338					0.407
		1-966	16/9/92				0.440
		967-2237	31/7/97				0.448
		2238-3338					0.470
	$(r_t)^2$	1-3338		0.005[6.979]	0.134[16.33]	0.859[105.4]	
		1-966	16/9/92	**0.002**[2.206]	**0.229**[11.11]	0.801[58.14]	
		967-2290	14/10/97	**0.007**[4.959]	**0.080**[7.805]	0.877[52.79]	
		2291-3338		**0.011**[3.125]	**0.100**[5.752]	0.875[41.36]	
S&P500	$\|r_t\|$	1-3338					0.374
		1-673	2/8/91				0.506
		674-2218	4/7/96				**0.427**
		2219-3338	22/7/98				0.457
	$(r_t)^2$	1-3338		0.0007[4.712]	0.039[12.979]	0.958[294.4]	
		1-2291	15/10/97	**0.0003**[3.256]	**0.018**[7.796]	**0.979**[409.1]	
		2292-3338		**0.013**[3.440]	**0.075**[5.801]	**0.883**[42.41]	

Notes: The Moulines and Lavielle (2000) multiple breaks detected in Table 7 for the absolute and squared returns processes are used to create various subsamples of each stock market return index. The estimated Normal GARCH(1,1) coefficients as well as the long memory parameter (Robinson, 1994) are reported for the total sample (N=1-3338) as the various subsamples determined by the estimated break points. The values in square brackets refer to t-values. Although not all subsamples have equal size some are approximately equal which allow for a better comparison of the estimated parameters. The bold parameters emphasize the change in the size of the volatility estimates in most subsamples (especially the parameters referring to the constant and ARCH effects of dynamic volatility).

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