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**Wealth Distribution,
Entrepreneurship and
Intertemporal Trade**

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Wealth Distribution, Entrepreneurship and Intertemporal Trade^{*}

Sanjay Banerji[†], Ngo Van Long[‡]

Résumé / Abstract

On étudie les échanges intertemporels entre deux pays ayant des distributions de richesses différentes. On montre que, à cause du risque moral et de l'imperfection du marché de capital, les gens très riches ou très pauvres choisissent de ne pas devenir entrepreneurs. À un taux d'intérêt donné, le pays dont la distribution de richesse est moins égalitaire a une offre d'entrepreneurs plus élevée, ce qui lui donne, en équilibre d'autarcie, un taux d'intérêt plus élevé. Par conséquent, quand le commerce international est permis entre deux pays, celui dont la distribution de richesse est moins égalitaire deviendra le pays créditeur. Les politiques redistributives peuvent donc influencer le profil du commerce intertemporel. Par exemple, si le gouvernement d'un pays adopte une politique d'aide aux entreprises qui font faillite, l'offre d'entrepreneurs de ce pays augmentera, ce qui, à son tour, haussera le taux d'intérêt en équilibre d'autarcie. Un pays créditeur deviendra un pays endetté quand une telle politique est introduite.

This paper examines the pattern of intertemporal trade between countries with different distribution of wealth. We also examine the consequences of redistribution policies in this framework. The driving force of our model are risk aversion, capital market imperfections, and costs associated with default. We show that under capital market imperfections due to moral hazard, the very rich and the very poor do not undertake any risk and become passive lenders. Only individuals whose wealth lies within a medium range choose to become entrepreneurs. At any given rate of interest, a country with a wealth distribution that is relatively less skewed to the left will have a greater supply of entrepreneurs, leading to a higher equilibrium interest rate under autarky. Hence, when the countries are opened to trade, those economies with highly skewed distribution (to the left) will become net lenders. Redistributive policies therefore will have impact on intertemporal trade. For example, if a government adopts a bail-out policy (a redistribution from successful entrepreneurs to unsuccessful ones) will increase the supply of entrepreneurs, driving up the autarkic interest

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rate. Consequently, a country which would be a lender if there were no bail-out policy is a net borrower under the bail-out policy.

Mots Clés : Échange intertemporel, distributions des richesses, risque moral, prudence, marché de capital imparfait, entrepreneurship

Keywords: Intertemporal trade, wealth distribution, moral hazard, prudence, imperfect capital market, entrepreneurship

JEL: O16, O15, F34

1 Introduction

This paper examines the pattern of intertemporal trade between countries with different distributions of wealth. We also examine the consequences of redistribution policies in this framework. The driving forces of our model are risk aversion, capital market imperfections, and costs associated with default. We show that under capital market imperfections due to moral hazard, the very rich and the very poor do not undertake any risk and become passive lenders. Only individuals whose wealth lies within a medium range choose to become entrepreneurs. At any given rate of interest, a country with a wealth distribution that is relatively less skewed to the left will have a greater supply of entrepreneurs, leading to a higher equilibrium interest rate under autarky. Hence, when the countries are opened to trade, those economies with highly skewed distribution (to the left) will become net lenders.

Redistributive policies therefore will have impacts on intertemporal trade. For example, if a government adopts a bail-out policy (a redistribution from successful entrepreneurs to unsuccessful ones), this will increase the supply of entrepreneurs, driving up the autarkic interest rate. Consequently, a country which would be a lender if there were no bail-out policy is a net borrower under the bail-out policy.

While the literature on risk aversion and entrepreneurship (see below for a brief survey) emphasizes the role of the coefficient of risk aversion in a loan market with perfect information, we show that it is the concept of prudence that plays an important role when risk-averse individuals borrow in a market under imperfect information.

An interesting feature of the paper is the endogenous determination of occupational choice. Individuals with identical utility functions but different wealth endowments self-select to be (or not to be) entrepreneur. In our model, the outcome of any investment project depends on the effort level of the entrepreneur in charge of the project. This effort level is not observable by lenders or financial intermediaries. Contracts are designed to give entrepreneurs incentives to exert effort. While these contracts mitigate against opportunism by entrepreneurs, they cannot replicate the outcome that would be obtained under symmetric information. An important implication of our results is that, in an extended version of the model, a well-designed redistribution of wealth may stimulate risk-taking activities and result in a higher growth rate.

Our paper builds on, and extends, earlier contributions to the literature

that connects income distribution to occupational choice in the context of capital market imperfections. It is useful to offer here a brief comparison of assumptions and results. Kihlstrom and Laffont (1979) consider a society with individuals having utility functions with different degrees of risk aversion. They show that more risk-averse individuals become workers while less risk-averse ones become entrepreneurs. In our model all individuals have the same utility function, but different endowments of wealth. It is the difference in initial wealths that causes individuals to choose different occupations. Kanbur (1979) also studies the relationship between risk taking and income distribution. He assumes that individuals have identical attitude to risk, but their production functions are ex-post different from each other because of independent productivity shocks which are observed before individuals decide on becoming entrepreneur or worker.

Galor and Ziera (1993) consider a model with irreversible investment involving fixed costs. They assume that agents borrow to finance their investment in skill acquisition, and bequeath some of their wealth. The lending rate is higher than the borrowing rate, and the cost of borrowing is higher for borrowers with low initial wealth. The amounts these borrowers bequeath are also smaller. The authors show that agents whose wealth lies below a threshold level find it optimal to choose unskilled jobs, with low wages. As a result, their descendents will also choose to be unskilled workers due to their low level of inherited wealth. A similar model is studied by Banerjee and Newman (1993) who assume that borrowers need to offer collaterals to lenders. Children of poor individuals do not inherit much, and therefore cannot offer sufficient collaterals to potential lenders. They are thus forced to choose not to be entrepreneurs. de Meza and Webb (1999) show that lack of information on the part of banks may lead to an over-provision of loans that encourages entry into entrepreneurship. If the associated incentive effects are strong, then there will be a positive relationship between wealth and entrepreneurial activities.

Aghion and Bolton (1997) formulate a model similar to ours, but they assume that all agents are risk-neutral. The probability of success of a project is dependent on the effort level chosen by the entrepreneur. In their model, individuals in the right-hand tail of the wealth distribution are entrepreneurs who do not borrow, while individuals in the middle section of the wealth distribution need to borrow to be entrepreneurs, because they do not have sufficient wealth to pay for the lumpy investment. This is in marked contrast to our result that very wealthy individuals may find it optimal not to be

entrepreneur. The difference is partly due to the fact that we assume (a) risk aversion (b) partial collateral, and (b) bankruptcy cost, while Aghion and Bolton assume risk neutrality and strictly limited liability (zero payments to banks in the event of project failure).

Newman (1995) assumes risk aversion, and find that, under moral hazard, the poorer individuals tend to be risk-takers and the wealthier individuals are risk-averters. This may be explained as follows. Optimal contract under moral hazard serve to resolve the tension between consumption- smoothing across states of nature, and efficient deployment of effort. We know that wealthier individuals tend to need less insurance at the margin. Optimal contracts prompt them to bear more risk at any given effort level. Individuals with greater wealth find it too costly (in terms of effort) to bear risks, as designed in the contracts.

All the models cited above share a common assumption: there is no transfer of resources from the entrepreneurs to the lenders in the event of project failure. This may be called the “strong limited liability” assumption. Our model allow a weaker version of limited liability: we assume that part of an entrepreneur’s private savings must be used to pay debts in the event of project failure. In addition, we assume the existence of a small real resource cost in the settling of a bankruptcy case. These twin assumptions play an important role in our model. We find that the very poor do not take risks, because the marginal utility of wealth in the event of bankruptcy is very high. We also find that the very rich do not become entrepreneurs, because the effort is very costly.

The papers cited above deal with closed economies, and most authors consider only a partial equilibrium setting. Grossman (1984) addressed the issue of the relationship between international trade and the formation of an entrepreneurial class. He showed that in a free trade equilibrium, there is a tendency to over-specialize in the non-risky sector and under-specialize in risk-taking activities. However, in his model, there was no risk sharing arrangements. The non-existence of financial market was exogenously imposed. As Dixit (1987) pointed out, once the incentive compatibility conditions are explicitly introduced, the market outcome is arguably a Pareto optimum. That is, once the sources of market imperfections are specified, risk-sharing arrangements will incorporate those features in incentive compatibility conditions, and it would be impossible to improve upon the market by means of public provision of insurance etc. Grossman and Maggi (2000) consider a model where the trade pattern reflects the distribution of talent across the

labor forces of the two countries. They show that a country with a more diverse work force exports the good for which individual success is more important than team production. Adverse selection tends to create a mismatch of talents which tend to be exacerbated under free trade. While Grossman and Maggi place emphasis on the interaction between distribution of talents, adverse selection in the labor market, and trade, our paper is concerned with interactions between wealth distributions, moral hazard, and imperfections in the credit market, in the context of a trading world.

The plan of the paper is as follows. In section 2, we develop a model of a closed economy, and state the assumptions and notation. In Section 3, we characterize equilibrium loan contracts under moral hazard, and show how the equilibrium riskless rate of interest and the occupational choice are determined. Section 4 considers a two-country world and shows how a country with a more skewed distribution of wealth tends to be the net lender. We also consider some extensions of the model, such as the implications of a bail-out policy. Some concluding remarks are offered in section 5.

2 The Autarkic Equilibrium

2.1 Assumptions and Notations

We begin by considering a model of a closed economy. There is a continuum of individuals in this economy. They differ in their initial wealth, denoted by w . The (cumulative) distribution of initial wealth is $F(w)$, and $f(w)$ denotes the corresponding density function. It is assumed that there is a closed interval $[w_L, w_H]$ over which $f(w)$ is strictly positive, and that $f(w) = 0$ for all w outside this interval. We assume $0 < w_L < w_H$. Then

$$\int_{w_L}^{w_H} f(w)dw = 1$$

and the per capita wealth is

$$\bar{w} = \int_{w_L}^{w_H} wf(w)dw.$$

There are only two periods. For simplicity, we assume that consumption takes place only in period two¹. In period one, each individual may choose

¹This assumption is also made by Gertler and Rogoff (1990), but in their model, unlike ours, entrepreneurship is not endogenous. Aghion and Bolton (1997) also assume that consumption takes place after the realization of the investment.

to be an entrepreneur (in which case he would carry out a risky business activity, using part of his own wealth, plus some additional borrowing, to finance his investment project), or he may choose not to be an entrepreneur (in which case he would keep all his wealth in the form of a deposit in a financial institution, which offers him the riskless gross rate of return $r > 1$). The endogenous determination of r will be explained in due course.

We assume that wealth, in its physical form, cannot be stored². It follows that all wealth must eventually be lent (via the financial institutions) to entrepreneurs, who use them as input in their investment projects. We adopt the standard assumption that financial institutions are risk neutral and perfectly competitive, so that their expected profits are zero.

Each entrepreneur can carry out only one investment project. Each project requires \bar{k} units of wealth. We assume that

$$\bar{k} > w_H \tag{1}$$

so that all entrepreneurs need external financing. A project can turn out to be a success or a failure. In the case of success, the pay-off of the project is $a > \bar{k}$. In the case of failure, the pay-off is zero. The probability of success is denoted by $\pi(e)$, where e is the effort level of the entrepreneur, which is not observable by the market. For simplicity, we assume that e can take only two possible values, 0 or 1. We write $p = \pi(1) > q = \pi(0)$. This indicates that the model exhibits the moral hazard property: the entrepreneur, who is a net borrower, may have an incentive to work at an effort level that is lower than what would be efficient in a world of perfect information. In what follows, unless otherwise stated, we assume that $p - q$ is sufficiently great, so that the equilibrium contracts have the following property: entrepreneurs are sufficiently rewarded for success that they have an incentive to set $e = 1$ even though e is not observable.

Since each entrepreneur needs \bar{k} units of capital as input, the assumption that $\bar{k} > w_H$ implies that in equilibrium, the endogenous number of entrepreneurs, N , is less than the number of individuals in this economy, M .

Each individual has the following utility function

$$U = U(y, e) = v(y) - \mu e$$

²An alternative assumption is that wealth can be stored but the rate of return on storage is lower than the equilibrium market rate of interest.

where y denotes his wealth in period 2, e is his effort level, $e \in \{0, 1\}$ and μ is a positive parameter. We set $\mu = 1$ by normalization³. The function $v(y)$ is increasing and strictly concave, with

$$v'(0) = \infty.$$

This property implies that each individual will invest some wealth in the riskless asset (i.e., lending to a financial institution), to avoid having zero wealth in period 2.

While the outcome of any given project is uncertain, we assume the probability of success of any given project is independent of those of other projects, and that the number of projects is large enough so that the law of large number applies. Thus, for the economy as a whole, if all entrepreneurs choose $e = 1$, aggregate output is paN where N is the measure of the set of individuals who, in equilibrium, choose to be entrepreneur.

Since the expected return on a project is pa and entrepreneurs have to incur effort cost, it is the case that the riskless rate of interest r must satisfy the condition that

$$r\bar{k} < pa + (1 - p)0 \tag{2}$$

2.2 A Benchmark Case: Observable Effort and No Bankruptcy Cost

If the effort level of each entrepreneur is observable, and if there is no bankruptcy cost, then we are in the first-best world. Since there is a continuum of individuals and a continuum of investment projects, in the aggregate there is no risk, and a perfect insurance market implies that all individuals are perfectly insured, given that their function $v(y)$ is strictly concave. We focus on the case where p/q is sufficiently great, so that optimal contracts specify that entrepreneurs must exercise full effort ($e = 1$).

Given the rate of interest r , competitive financial institutions offer contracts that specify, for any given w , the amount $\bar{k} - k(w)$ to be lent to the entrepreneur with wealth w , and the payments $R_s(w)$ and $R_f(w)$ that he must make in the events of success and failure respectively. Here $k(w)$ denotes the amount of equity that the entrepreneur puts in his project. The

³An entrepreneur can choose e to be 0 or 1. Non-entrepreneurs need not expend any effort, thus their e is 0.

equilibrium contract maximizes his expected utility, given that $e = 1$, subject to zero expected profit for the financial institutions:

$$\max pv[(w - k)r + a - R_s] + (1 - p)v[(w - k)r - R_f] - 1$$

subject to

$$pR_s + (1 - p)R_f = r(\bar{k} - k)$$

The solution yields constant utility across states of nature

$$(w - k)r + a - R_s = (w - k)r - R_f$$

At the optimum, $R_s + rk$ and $R_f + rk$ are determined by

$$R_s + rk = (1 - p)a + r\bar{k}$$

$$R_f + rk = R_s + rk - a$$

Thus, given r , the expected utility of an entrepreneur with initial wealth w who accepts a contract that stipulates $e = 1$, is

$$EV^e(w, 1; r) = v[wr + pa - r\bar{k}] - 1 \quad (3)$$

Similarly, a zero-expected-profit contract that maximizes the expected utility of an entrepreneur with initial wealth w , given that his effort is specified to be $e = 0$, yields the net utility

$$EV^e(w, 0; r) = v[wr + qa - r\bar{k}] - 0 \quad (4)$$

Define the function

$$\phi(w, r) \equiv EV^e(w, 1; r) - EV^e(w, 0; r) = v[wr + pa - r\bar{k}] - 1 - v[wr + qa - r\bar{k}] \quad (5)$$

This function measures, for an individual with wealth w , the relative attractiveness of being an entrepreneur with $e = 1$ (rather than being an entrepreneur with $e = 0$). It is decreasing in w for any given r : $\phi_w < 0$. We assume that, for all $r < pa/\bar{k}$, $\phi(w, r) > 0$ for $w = w_L$ and $\phi(w, r) < 0$ for w sufficiently great. Then there exists a “critical value” $w^c(r)$ such that individuals with wealth $w < w^c(r)$ will prefer being an entrepreneur with $e = 1$ to being one with $e = 0$. Since $\phi(w, r)$ is increasing in r for all $w < \bar{k}$,

if $w^c(r) < w_H$ (which is less than \bar{k} by assumption), an increase in r will increase w^c :

$$\frac{dw^c}{dr} = -\frac{\phi_r}{\phi_w} = -\frac{\{v'[wr + pa - r\bar{k}] - v'[wr + qa - r\bar{k}]\}(w - \bar{k})}{\phi_w} > 0$$

A pure lender, on the other hand, obtains the utility $v(wr)$. If $r > qa/\bar{k}$, [respectively, if $r < qa/\bar{k}$] the strategy of being a pure lender dominates [respectively, is dominated by] that of being an entrepreneur who exerts no effort. We assume that q is so small that in equilibrium, $r > qa/\bar{k}$. Let us define

$$\psi(w, r) = EV^e(w, 1; r) - 1 - v(wr)$$

This function measures, for an individual with wealth w , the relative attractiveness of being an entrepreneur with $e = 1$ (rather than being a pure lender.) Then, for all r in the interval $[qa/\bar{k}, pa/\bar{k}]$, $\psi(w, r)$ is decreasing in w : $\psi_w < 0$. There exists a value $w^b(r)$ such that $\psi(w^b(r), r) = 0$. Given the assumption that $r > qa/\bar{k}$, we can deduce that $w^b(r) < w^c(r)$, and

$$\frac{dw^b}{dr} = -\frac{\psi_r}{\psi_w} = -\frac{v'[wr + pa - r\bar{k}](w - \bar{k}) - v'(wr)w}{v'[wr + pa - r\bar{k}]r - v'[wr]r} < 0$$

Thus, in this economy, under perfect information (i.e., effort is observable), the demand for loans is

$$B(r) = \int_{w_L}^{w^b(r)} (\bar{k} - w)f(w)dw$$

and the supply of loans is

$$L(r) = \int_{w^b(r)}^{w_H} wf(w)dw$$

Since $B'(r) = (\bar{k} - w^b)f(w^b)\frac{dw^b}{dr} < 0$ and $L'(r) = -w^b f(w^b)\frac{dw^b}{dr} > 0$, the demand curve is downward sloping, and the supply curve is upward sloping. Their intersection determines the equilibrium interest rate, r^* . At r^* , demand equals supply, $B(r^*) = L(r^*)$, implying

$$\bar{k} \int_{w_L}^{w^b(r^*)} f(w)dw = \int_{w_L}^{w_H} wf(w)dw = \bar{w}$$

Let

$$w^* \equiv w^b(r^*)$$

and let $F(w)$ denote the cumulative distribution of wealth:

$$F(w) \equiv \int_{w_L}^w f(z)dz$$

then

$$F(w^*) = \frac{\bar{w}}{\bar{k}}$$

To summarize, under perfect information, the fraction of population who are entrepreneurs in equilibrium is \bar{w}/\bar{k} , and only individuals whose wealth is smaller than $w^* \equiv F^{-1}(\bar{w}/\bar{k})$ will choose to be entrepreneurs.

2.3 The Moral Hazard Case: Unobservable Effort

Now we turn to the case of unobservable effort levels. We continue to assume that each entrepreneur's wealth is *known* to the financial intermediaries. This, and the assumption that individuals have the same utility function and identical ability means that there is no adverse selection problem: no one can lie about his wealth or his utility function. The only problem is moral hazard: if how much an entrepreneur must pay back to the financial intermediaries is independent of his effort level, then he may have an incentive to exert no effort. Contracts must therefore be designed to provide sufficient incentive for entrepreneurs to choose $e = 1$ (This is of course based on the assumption that $p - q$ is sufficiently great to justify the choice $e = 1$.)

We now describe a contract for an entrepreneur with wealth level w . Recall that we assumed $w_H < \bar{k}$. The contract says that "if your wealth is w and you contribute an amount $k \leq w$ as your "equity" in your investment project (so that your borrowing from your financial institution is $\bar{k} - k \geq 0$) and you deposit the remaining $w - k$ at a financial institution, then you must pay back to your lender (the financial institution, or FI for short) an amount which depends on the outcome of your project. If the outcome is "success", your investment yields the gross return a , and you must pay an amount R to your FI; if your outcome is "failure" (the investment yields a gross return of zero), then, with your period two wealth $(w - k)r$, you must pay back to the FI an amount $\alpha(\bar{k} - k) > 0$ ". (In what follows, we assume $\alpha < r$.) The assumption that $\alpha > 0$ is meant to capture the fact that most real world credit contracts are at least partly secured by some sort of collateral. We do

not model the determination of α . It is exogenously specified, and is assumed to be very small.

It follows that if the entrepreneur exerts effort (i.e., $e = 1$), then his expected utility is

$$pv[(w - k)r + a - R] + (1 - p)v[(w - k)r - \alpha(\bar{k} - k)] - 1 \equiv G(w, k, R, 1)$$

and if he does not exert effort (i.e., $e = 0$), then his expected utility is

$$qv[(w - k)r + a - R] + (1 - q)v[(w - k)r - \alpha(\bar{k} - k)] \equiv G(w, k, R, 0)$$

We assume also that when a bankrupt entrepreneur pays the amount $\alpha(\bar{k} - k) > 0$ to his FI, the latter only gets a fraction β of it. In other words, the FI incurs a real cost $(1 - \beta)\alpha(\bar{k} - k)$ in collecting $\alpha(\bar{k} - k)$ from the failed entrepreneur. Bankruptcy costs are cost associated with the transfer of resources from the debtor to the creditor in the bad state of nature. These costs consist of legal fees, delay costs, etc. Several studies have shown that such costs can have an impact on the value of claims owned by creditors and are reflected in the pricing of loans.

Thus an FI that lends the amount $\bar{k} - k$ to an entrepreneur who does exert effort can expect to get

$$pR + (1 - p)\alpha\beta(\bar{k} - k)$$

On the other hand, the FI takes r as given, and must pay the amount $r(\bar{k} - k)$ to depositors. In equilibrium, we have the following zero expected profit condition (if $e = 1$):

$$pR + (1 - p)\alpha\beta(\bar{k} - k) = r(\bar{k} - k) \tag{6}$$

Competition among the FIs imply that, for given r , the FIs will offer to any entrepreneur with wealth w a contract that maximizes his expected utility, subject to the zero profit condition. We also assume that the contract provides enough incentive for the entrepreneur to exert effort ($e = 1$). More formally, FI j will offer to the entrepreneur with wealth w a pair of numbers (R, k) that maximizes $G(w, k, R, 1)$ subject to the *incentive compatibility constraint*

$$G(w, k, R, 1) \geq G(w, k, R, 0) \tag{7}$$

and the zero profit constraint (6).

3 Properties of the Equilibrium Contracts

We now turn to a fuller characterization of equilibrium contracts. We proceed as follows. First, we take the interest rate r as given, and show how the incentive compatibility constraint and the zero profit condition determine the contract for entrepreneurs at each wealth level. Then we show how the interest rate r is determined endogenously. In what follows, we normalize by setting $\mu = 1$.

3.1 Equilibrium loan contracts for entrepreneurs, given the interest rate on the safe asset

In equilibrium, profit will be zero, and the incentive compatibility constraint binds for each entrepreneur. It follows that, for given r , conditions (6) and (7) determine the equilibrium contract for entrepreneurs with wealth w . The following lemma characterizes the equilibrium contract, under the assumption that α is small:

LEMMA 1: (Equilibrium contract) In equilibrium, for given r , there exists a lower bound $\underline{w} > 0$ such that entrepreneurs with wealth $w \geq \underline{w}$ will be lent the amount $\bar{k} - k(w)$ and will be asked to pay the amount $R(w) \geq 0$ in the event of success, and $\alpha [\bar{k} - k(w)] \geq 0$ in the event of failure, where the pair $(R(w), k(w))$ satisfy the following conditions:

(i) the expected profit of the FI is zero

$$R = \frac{(\bar{k} - k)}{p} [r - (1 - p)\alpha\beta] \text{ for } k \leq \bar{k} \quad (8)$$

and (ii) the incentive compatibility constraint holds with equality

$$v [(w - k)r + a - R] - v [(w - k)r - \alpha(\bar{k} - k)] = \frac{1}{p - q} \quad (9)$$

Proof: See the Appendix.

REMARK 1: Equation (8) implies that, given the riskless rate of interest r , the repayment to the FI (in the event of success) per dollar borrowed is a constant, independent of the amount borrowed:

$$r_s \equiv \frac{R}{\bar{k} - k} = \frac{1}{p} [r - (1 - p)\alpha\beta].$$

(Note that r_s depends on r , and that the repayment received by the FI, in the event of failure, is $\alpha\beta$ per dollar borrowed.) Given α , which is exogenous in our model, financial institutions will be willing to lend only to individuals whose wealth exceeds a certain lower bound $\underline{w} > 0$.

REMARK 2: Equation (8) is represented in *Figure 1* by the downward sloping curve KK in the (k, R) space, with the vertical intercept $[r - (1 - p)\alpha\beta] \bar{k}/p$. If $p - q$ is sufficiently close to one, and α is sufficiently small, then the intercept of the second curve (defined by (9) and denoted as the curve VV) is below $[r - (1 - p)\alpha\beta] \bar{k}/p$ and hence the two curves will have an intersection $(k(w), R(w))$ with $k(w) > 0$ and $R(w) > 0$, if w exceeds a certain lower bound $\underline{w} > 0$.

PLEASE PLACE FIGURE 1 HERE

LEMMA 2: Entrepreneurs with greater w will borrow less (i.e. $\bar{k} - k(w)$ decreases with wealth) and hence *invest more (put more equity) in the risky project*, i.e. $k'(w) > 0$, given that contracts must satisfy (8) and (9).

Proof: Note that the curve KK is independent of w . For any given k , an increase in w will shift the curve VV down, resulting in an intersection to the right of the former intersection. Hence k^* increases, and R^* decreases. This means the individual borrows less when his wealth increases.

REMARK 3: One of the standard text-book results is that if the payoff per dollar invested in a risky asset in each state of nature is *independent* of the amount invested in the risky asset, then the amount an individual invests in the risky asset is an increasing function of his wealth if and only if his *absolute risk aversion is a decreasing function* of wealth. Our result in Lemma 2 is different, because the pay-off per dollar of equity in the event of success is

$$\delta_s = \frac{a - R}{k} = \frac{a}{k} - \frac{1}{pk} [r - (1 - p)\alpha\beta] (\bar{k} - k)$$

which is *dependent* on k (*decreasing* in k). (And similarly, the pay-off per dollar of equity in the event of failure increases in k .) Lemma 1 states that, with a concave utility function, but independently of whether *absolute risk aversion is a decreasing or increasing function*, entrepreneurs with greater wealth will contribute more equity in the project.

LEMMA 3: Given that contracts must satisfy (8) and (9), entrepreneurs with greater w will invest more in the riskless asset, (i.e., $w - k^*(w)$ increases with w), and at the same time putting more equity in the risky project, *if*

and only if the following condition holds

$$v'(s)(r - \alpha\beta) > [v'(u) - v'(s)]\beta] \alpha p \quad (10)$$

where u is the final wealth in the failure state,

$$u \equiv u(w) \equiv (w - k(w))r - \alpha(\bar{k} - k(w)) \quad (11)$$

and s is the final wealth in the success state,

$$s \equiv s(w) \equiv (w - k(w))r + a - R(w) \quad (12)$$

Proof: From (8) and (9), we get

$$\frac{dk}{dw} = \frac{[v'(u) - v'(s)]pr}{[v'(u) - v'(s)]pr + v'(s)(r - \alpha\beta) - [v'(u) - v'(s)]\beta] \alpha p} \quad (13)$$

which is less than 1 if and only if (10) holds.

REMARK 4: Condition (10) holds if α is small, which is assumed here.

3.2 An example: logarithmic utility

Let

$$v(y) = \ln y$$

then we obtain from (9)

$$\ln \left[\frac{(w - k)r + a - R}{(w - k)r - \alpha(\bar{k} - k)} \right] = \frac{1}{p - q}$$

hence

$$wr - kr + a - (\bar{k} - k)r_s = [wr - kr - \alpha(\bar{k} - k)] e^{1/(p-q)}$$

Thus

$$[r_s + (r - \alpha)e^{1/(p-q)} - r] k = [r_s - \alpha e^{1/(p-q)}] \bar{k} - a + rw [e^{1/(p-q)} - 1] \quad (14)$$

i.e.,

$$k(w) = \frac{[r_s - \alpha e^{1/(p-q)}] \bar{k} - a + rw [e^{1/(p-q)} - 1]}{[r_s + (r - \alpha)e^{1/(p-q)} - r]} \quad (15)$$

which is positive if w exceeds a certain lower bound $\underline{w} > 0$, and

$$\frac{dk}{dw} = \frac{[e^{1/(p-q)} - 1] r}{[e^{1/(p-q)} - 1] r + [r_s - \alpha e^{1/(p-q)}]}$$

which is positive and less than 1 provided α is small. This is consistent with Lemmas 2 and 3.

3.3 To be or not to be an entrepreneur

So far, we have characterized contracts that the FIs offer to potential entrepreneurs with wealth w , which would make him choose $e = 1$, *assuming* that he does want to be an entrepreneur. But depending on his wealth, an individual may find that the utility of being a lender, $v(rw)$, may exceed the expected utility of being an entrepreneur, i.e., it is possible that, for some w , we have

$$v(rw) > EU \equiv pv(s) + (1-p)v(u) - 1 \quad (16)$$

Clearly, if w is close to zero, condition (16) will be satisfied, because financial institutions lend a positive amount only to individuals with $w \geq \underline{w} > 0$. Thus the curve $v(rw)$ lies above the curve EU when w is small. At some $\hat{w} > 0$, the curve EU cuts the curve $v(rw)$ from below. The question is whether there exists some value $\tilde{w} > \hat{w}$ such that the curve $v(rw)$ again overtakes the curve EU . The answer turns out to depend on the magnitude of the *coefficient of prudence*, a concept introduced by Kimball (1990), which is in turn related to the convexity of the following function

$$\psi(y) \equiv \frac{1}{v'(y)}$$

which, in our model, measures the marginal rate of substitution between effort and income:

$$MRS_{ey} = -\frac{U_e}{U_y} = \frac{\mu}{v'(y)} = \frac{1}{v'(y)}$$

The function $\psi(r)$ is convex if and only if the coefficient of prudence is smaller than twice the coefficient of absolute risk aversion:

$$-\frac{2v''}{v'} \geq -\frac{v'''}{v''} \quad (17)$$

(the right-hand side of (17) is called the coefficient of prudence by Kimball, 1990).⁴

LEMMA 4:

(i) If $\psi(y)$ is concave, then

$$\frac{p}{v'(s)} + \frac{1-p}{v'(u)} \leq \frac{1}{v'(ps + (1-p)u)} \quad (18)$$

and if $\psi(y)$ is convex, then

$$\frac{p}{v'(s)} + \frac{1-p}{v'(u)} \geq \frac{1}{v'(ps + (1-p)u)} \quad (19)$$

(ii) if α is small, then

$$\frac{1}{v'(ps + (1-p)u)} \geq \frac{1}{v'(rw)} \quad (20)$$

Proof: Part (i) follows from Jensen's inequality. The inequality in part (ii) holds if and only if

$$rw < ps + (1-p)u \quad (21)$$

Now, if α is small, then $ps + (1-p)u = rw - rk(w) + pa - (\bar{k} - k(w))[r - (1-p)\alpha(\beta - 1)] \simeq rw + pa - r\bar{k} > rw$ because of (2).

LEMMA 5: If $\psi(y)$ is convex, then the curve EU may cut the curve $v(rw)$ from below at some value $\hat{w} > 0$, and then from above, at some value $\tilde{w} > \hat{w}$. (see *Figure 2*).

Proof: see the Appendix and Figure 2.

PLEASE PLACE FIGURE 2 HERE.

The following proposition follows immediately from the above lemma:

Proposition 1: if the coefficient of prudence is smaller than twice the coefficient of absolute risk aversion, then it is possible that, given r , there are

⁴Note that decreasing absolute risk aversion holds iff

$$-\frac{v'''}{v''} > -\frac{v''}{v'}$$

which is not inconsistent with (17). For further results on the role of v''' , see Hartwick (1999).

two values $\hat{w}(r) < \tilde{w}(r)$ such that only individuals whose wealth lies between them will choose to be entrepreneurs. Individuals with wealth exceeding $\tilde{w}(r)$, and those with wealth below $\hat{w}(r)$ will choose to invest in the safe asset (i.e., lend to financial intermediaries).

REMARK: Proposition 1 may be explained as follows: given that the marginal rate of substitution between effort and income is convex in y , individuals with very low wealth do not want to become entrepreneur because they are not willing to take risks, and individuals who are very wealthy do not want to become entrepreneur because they do not want to exert effort.

We now show how the two critical values $\hat{w}(r)$ and $\tilde{w}(r)$ change when r increases.

Proposition 2: $\hat{w}'(r) > 0$ and $\tilde{w}'(r) < 0$ provided that $\gamma(1-p)$ is sufficiently small, where $\gamma \equiv \{(r - \beta\alpha)/(r - \alpha)\} - 1 \geq 0$

Proof: see the Appendix.

3.4 Endogenous determination of the interest rate on the safe asset

So far, we have taken the interest rate r as given. Now we turn to its determination. This is given by the condition that the the interest rate must equate the aggregate lending by non-entrepreneurs (to the financial institutions) to the aggregate borrowing by entrepreneurs (from the financial institutions). The former is given by

$$L(r) = \int_{w_L}^{\hat{w}(r)} w f(w) dw + \int_{\tilde{w}(r)}^{w_H} w f(w) dw$$

and the latter is

$$B(r) = \int_{\hat{w}(r)}^{\tilde{w}(r)} (\bar{k} - w) f(w) dw$$

The excess demand (function) for fund is

$$D(r) = B(r) - L(r) = \bar{k} \int_{\hat{w}(r)}^{\tilde{w}(r)} f(w) dw - \int_{w_L}^{w_H} w f(w) dw = \bar{k} \int_{\hat{w}(r)}^{\tilde{w}(r)} f(w) dw - \bar{w}$$

Clearly, if r is very high, then $D(r) < 0$, and if $r = 0$, then $D(r) < 0$ (recall that $\bar{k} > \bar{w}$). By continuity, there exists a value r^* such that $D(r^*) = 0$.

Furthermore, such r^* is unique, because $D'(r) < 0$. Thus we obtain the following result:

Proposition 3: There exists a unique $r^* > 0$ such that the loan market clears. Individuals with wealth exceeding $\tilde{w}(r^*)$ lend all their wealth to the financial institutions, at the safe interest rate r^* , and so do individuals with wealth below $\hat{w}(r^*)$. Individuals whose wealth lies between these two critical values will be entrepreneurs, and they invest a fraction of their wealth in the safe asset, the remaining fraction being used as equity capital, which is always less than the total amount of capital invested in each risky project. The equilibrium fraction of the population who choose to become entrepreneurs is

$$\frac{N}{M} = \frac{\bar{w}}{k} = \int_{\hat{w}(r^*)}^{\tilde{w}(r^*)} f(w)dw. \quad (22)$$

4 Open Economies

4.1 International Borrowing Under the Moral Hazard Case

Now consider two countries with the same population size and the same level of per capita wealth, but different distributions of wealth. We will look at their autarkic equilibria, and examine the incentive for international borrowing. Let us assume there are two countries, A and B . The density functions of the two wealth distributions are respectively $f_A(w; \theta_A)$ and $f_B(w; \theta_B)$, where θ_A and θ_B are shift parameters. Now consider the autarkic equilibrium of country A , and let r_A be the autarkic interest rate. Consider the following thought experiment: Suppose the distribution $f_A(\cdot)$ undergoes a change: θ_A now increases to θ_A^* , which makes $f_A(w; \theta_A^*) < f_A(w; \theta_A)$ for all w in the interval $[\hat{w}(r_A), \tilde{w}(r_A)]$. Then clearly at the value r_A , the number of willing entrepreneurs will be less than N . To restore equilibrium, r_A must be lower.

To make the above argument more precise, let us write the equation that determines the equilibrium autarkic rate of interest as

$$F_A[\tilde{w}(r_A), \theta_A] - F_A[\hat{w}(r_A), \theta_A] = \frac{\bar{w}}{k}$$

Then

$$\frac{dr_A}{d\theta_A} = -\frac{\Omega}{\Gamma}$$

where

$$\Gamma \equiv \left\{ \frac{\partial}{\partial w} F_A [\tilde{w}(r_A), \theta_A] \right\} \frac{d\tilde{w}(r_A)}{dr} - \left\{ \frac{\partial}{\partial w} F_A [\hat{w}(r_A), \theta_A] \right\} \frac{d\hat{w}(r_A)}{dr} < 0$$

by Proposition 2, and

$$\begin{aligned} \Omega &\equiv \left\{ \frac{\partial}{\partial \theta_A} F_A [\tilde{w}(r_A), \theta_A] \right\} - \left\{ \frac{\partial}{\partial \theta_A} F_A [\hat{w}(r_A), \theta_A] \right\} \\ &= \frac{\partial}{\partial \theta_A} \int_{\hat{w}(r_A)}^{\tilde{w}(r_A)} f(w, \theta_A) dw = \int_{\hat{w}(r_A)}^{\tilde{w}(r_A)} \frac{\partial}{\partial \theta_A} f(w, \theta_A) dw < 0 \end{aligned}$$

It follows that $\frac{dr_A}{d\theta_A} < 0$.

Note that in the above argument, the condition $\frac{\partial}{\partial \theta_A} f(w, \theta_A) < 0$ over $[\hat{w}(r_A), \tilde{w}(r_A)]$ is sufficient, but not necessary, for Ω to be negative.

It follows from the above reasoning that if the density function $f_B(w; \theta_B)$ differs from $f_A(w; \theta_A)$ in that

$$f_B(w; \theta_B) < f_A(w; \theta_A) \text{ for all } w \in [\hat{w}(r_A), \tilde{w}(r_A)]$$

then country B will have a higher autarkic gross rate of return: $r_B > r_A$. Under these conditions, the opening of world financial markets will result in capital flow from A to B . We obtain the following proposition:

Proposition 4: Two countries with identical per capita wealth and identical individual utility function may engage in intertemporal trade if the distributions of wealth are different.

Corollary: A marginal redistribution of wealth away from the entrepreneurial class of a country can turn that country status from being a net lender to a net borrower.

4.2 Bailouts

In this section, we set up a framework for studying the implications of government policies that seek to help bankrupt entrepreneurs. Suppose the government taxes successful entrepreneurs, collecting from them amount T each, and pay the unsuccessful ones an amount η . Balanced budget requires that

$$pT \int_{\hat{w}(r)}^{\tilde{w}(r)} f(w) dw = \eta(1-p) \int_{\hat{w}(r)}^{\tilde{w}(r)} f(w) dt \quad (23)$$

where $[\hat{w}(r), \tilde{w}(r)]$ are solutions of the equation

$$v(rw) = pv[(w - k)r + (a - R) - T] + (1 - p)v[(w - k)r - \alpha(\bar{k} - k) + \eta] - 1$$

Does this bailout policy result in a lower or higher equilibrium interest rate? What is its implication on the country's net borrowing?

First, note that (23) implies that

$$\frac{dT}{d\eta} = \frac{1 - p}{p}$$

The contract between the financial intermediary (FI) and the entrepreneur with wealth w must now satisfy

$$(p - q)[v(s) - v(u)] = 1 \quad (24)$$

and

$$pR = (\bar{k} - k)[r - (1 - p)\alpha\beta] \quad (25)$$

where now

$$s \equiv (w - k)r + (a - R) - T$$

and

$$u \equiv (w - k)r - \alpha(\bar{k} - k) + \eta$$

From (24) and (25), we have

$$\frac{\partial k}{\partial T} = \frac{pv'(s)}{pv'(u)(r - \alpha) + (1 - p)v'(s)(r - \alpha\beta)}$$

and

$$\frac{\partial k}{\partial \eta} = \frac{pv'(u)}{pv'(u)(r - \alpha) + (1 - p)v'(s)(r - \alpha\beta)}$$

Hence

$$\begin{aligned} \frac{dk}{d\eta} &= \frac{\partial k}{\partial T} \frac{dT}{d\eta} + \frac{\partial k}{\partial \eta} \\ &= \frac{(1 - p)v'(s) + pv'(u)}{pv'(u)(r - \alpha) + (1 - p)v'(s)(r - \alpha\beta)} \end{aligned}$$

To find the response of the intersection points $\tilde{w}(r)$ and $\hat{w}(r)$ with respect to η , we use the equation of indifference, which is now

$$\frac{q}{p - q} + v[(w - k^*)r - \alpha(\bar{k} - k^*) + \eta] = v(wr) \quad (26)$$

where $k^* = k^*(w, T, \eta(T))$. From (26) we get

$$\frac{dw}{d\eta} = \left(\frac{1}{\Delta}\right) \frac{(1-p)v'(u)v'(s)\alpha(\beta-1)}{(1-p)v'(s)(r-\alpha\beta) + pv'(u)(r-\alpha)} \quad (27)$$

where

$$\Delta = v'(s) \left[r - (r - \alpha) \frac{\partial k}{\partial w} \right] - v'(NE)r \quad (28)$$

which is positive at $\hat{w}(r)$ and negative at $\tilde{w}(r)$. Hence, for a given r , $\hat{w}(r)$ increases and $\tilde{w}(r)$ decreases when there is an increase in η . This implies that the number of entrepreneurs increases, leading to an increase in the demand for fund. To restore equilibrium, the rate of interest r must increase.

Proposition 5: Bail-out policies of the type described above will lead to an increase in the equilibrium rate of interest. In a two-country world, this means that the country with such bail-out policies will become the debtor country.

REMARK: The role of the bankruptcy cost factor $\beta < 1$ is crucial here. If $\beta = 1$, then bailouts have no effect on the equilibrium interest rate.

5 Concluding remarks

We have set up a model to show that a country's wealth distribution influences the occupational choice of individuals: to be or not to be an entrepreneur. We have also showed that, under certain assumptions, only the middle-class individuals choose to be entrepreneurs. An implication is that countries that have the same level of per capita wealth and the same individual utility function may engage in mutual intertemporal trade, as long as they have different wealth distributions. The key elements in our model are (i) the tension between consumption smoothing across states of nature on the one hand, and contract design to overcome moral hazard on the other hand, (ii) the relationship between the coefficients of prudence and risk aversion, and (iii) bankruptcy cost and partial liability.

There are several directions of generalization, which we wish to pursue in our future work. An obvious extension is the process of capital accumulation. One would then be able to see how moral hazard and initial wealth distributions influence growth rates, and to determine conditions under which cycles may occur. Taxation policies, including transfers, may be studied in the context of moral hazard and endogenous choice of occupation.

APPENDIX

PROOF OF LEMMA 1

Consider the problem

$$\max_{R,k} G(R, k, w, 1) = pv((w - k)r + a - R) + (1 - p)v((w - k)r - \alpha(\bar{k} - k)) - 1$$

subject to

$$v((w - k)r + a - R) - v((w - k)r - \alpha(\bar{k} - k)) \geq \frac{1}{p - q} \quad (29)$$

and

$$pR - (\bar{k} - k)[r - (1 - p)\alpha\beta] \geq 0 \quad (30)$$

In the (k, R) space, the feasible set is the intersection of area above the line

$$R = \frac{(\bar{k} - k)}{p} [r - (1 - p)\alpha\beta] \quad (31)$$

with the area below the (positively sloped) curve defined by (29), see Figure 2. The absolute value of the slope of the line (31), dR/dp , is approximately r/p if α is small. Now it is easy to verify that the objective function $G(R, k, w, 1)$ is strictly concave and *decreasing* in (k, R) . This means that the iso-expected utility curves are concave in the (k, R) space, with negative slope given by

$$\frac{\partial R}{\partial k} = \frac{-G_k}{G_R}$$

If α is small, the absolute value of this slope is approximately $(r/p)[v'(u)/v'(s)] > (r/p)$. It follows that the maximum occurs at the point where both constraints hold with equality.

PROOF OF LEMMA 5:

Let $\phi(w) \equiv v(rw) - EU(w)$ be the difference between the utility of a pure lender with wealth w and the expected utility of an entrepreneur having the same wealth. We wish to find conditions which imply that $\phi(w)$ is a U -shaped curve, taking positive values for small w , negative values for intermediate values of w , and positive values again for large values of w .

By definition

$$\phi(w) = v(rw) - pv(s) - (1 - p)v(u) + 1 \quad (32)$$

Making use of (9) we can re-write $\phi(w)$ as

$$\phi(w) = v(rw) - [v(u) + q/(p - q)] \quad (33)$$

We want to show that $\phi'(w) < 0$ for small w and $\phi'(w) > 0$ for large w .
Now

$$\phi' = rv'(rw) - v'(u) [r(1 - k'(w)) + \alpha k'(w)]$$

where

$$\frac{\partial k}{\partial w} = \frac{pr [v'(u) - v'(s)]}{pv'(u)(r - \alpha) + (1 - p)v'(s)(r - \beta\alpha)} \quad (34)$$

$\phi'(w)$ is negative iff

$$H \equiv v'(u) \left[r - (r - \alpha) \frac{\partial k}{\partial w} \right] - rv'(rw) > 0 \quad (35)$$

which is equivalent to

$$r - (r - \alpha) \frac{\partial k}{\partial w} > \frac{rv'(rw)}{v'(u)}$$

i.e.,

$$\left[1 - \frac{v'(rw)}{v'(u)} \right] r > (r - \alpha) \frac{\partial k}{\partial w}$$

i.e.,

$$\left[\frac{v'(u) - v'(rw)}{v'(u)} \right] > (r - \alpha) \frac{p[v'(u) - v'(s)]}{pv'(u)(r - \alpha) + (1 - p)v'(s)(r - \beta\alpha)}$$

i.e.,

$$\left[\frac{v'(u) - v'(rw)}{v'(u)} \right] > \frac{p[v'(u) - v'(s)]}{pv'(u) + (1 - p)v'(s)\{(r - \beta\alpha)/(r - \alpha)\}} \quad (36)$$

Let $1 + \gamma = \{(r - \beta\alpha)/(r - \alpha)\} \geq 1$. Inequality (36) holds iff

$$\left[\frac{v'(u) - v'(rw)}{v'(u)} \right] > \frac{p[v'(u) - v'(s)]}{pv'(u) + (1 - p)v'(s)(1 + \gamma)} \quad (37)$$

Now (37) holds iff

$$\left[\frac{pv'(u) + (1-p)v'(s)(1+\gamma)}{v'(u)v'(s)} \right] [v'(u) - v'(rw)] > \frac{p[v'(u) - v'(s)]}{v'(s)} \quad (38)$$

Let

$$Z = \frac{pv'(u) + (1-p)v'(s)}{v'(u)v'(s)} \quad (39)$$

Then (38) holds iff

$$1 + \gamma(1-p) \left[1 - \frac{v'(rw)}{v'(u)} \right] > Zv'(rw) \quad (40)$$

iff

$$[Zv'(rw) - 1] - \gamma(1-p) \left[1 - \frac{v'(rw)}{v'(u(w))} \right] < 0 \quad (41)$$

Note that $v(rw) = v(u) + q/(p-q)$ implies that $rw > u(w)$ and hence $1 - \frac{v'(rw)}{v'(u(w))} > 0$. We want the inequality (41) to hold for small w and to be reversed for large w . Now, from Lemma 4, if α is small and $\psi(w)$ is convex, then

$$\frac{p}{v'(s)} + \frac{1-p}{v'(u)} > \frac{1}{v'(rw)}$$

and hence $Zv'(rw) > 1$, and $\phi(w)$ may be U -shaped if $\gamma(1-p) \left[1 - \frac{v'(rw)}{v'(u(w))} \right]$ is larger [respectively, smaller] than $[Zv'(rw) - 1]$ for small w , and smaller [respectively, larger] than it for large w .

It follows that if $\psi(w)$ is convex, then the EU curve may cut the $v(rw)$ curve from below at some \hat{w} , and then from above, at some $\tilde{w} > \hat{w}$.

PROOF OF PROPOSITION 2

From (8) and (9), we have

$$\frac{\partial k}{\partial w} = \frac{1}{\Delta} [v'(u) - v'(s)] pr > 0$$

and

$$\frac{\partial k}{\partial r} = \frac{1}{\Delta} [\{v'(u) - v'(s)\}(w-k)p + v'(s)(\bar{k}-k)]$$

where

$$\Delta = pv'(u)(r-\alpha) + (1-p)v'(s)(r-\alpha\beta) > 0$$

Differentiating the condition for indifference (equation (??)) with respect to r , we get

$$\frac{dw}{dr} = \frac{1}{H} \left[wv'(NE) - v'(u)(w - k) + (r - \alpha)v'(u)\frac{\partial k}{\partial r} \right] \quad (42)$$

where H , defined in (35) above, is positive at $\hat{w}(r)$ and negative at $\tilde{w}(r)$. The numerator in(42) must now be signed. It can be written as

$$wv'(NE) + v'(u) \left[\frac{p(w - k)\{v'(u) - v''(s)\} + v'(s)(\bar{k} - k)}{pv'(u) + (1 - p)v'(s)(1 + \gamma)} - (w - k) \right]$$

The term inside the square brackets can be written as

$$v'(s)(w - k) \left[\frac{\bar{k} - k}{w - k} - 1 - \gamma(1 - p) \right]$$

or

$$v'(s)(w - k) \left[\frac{\bar{k} - w}{w - k} - \gamma(1 - p) \right]$$

Now, since $w - k(w)$ is increasing in w , we have

$$\frac{\bar{k} - w}{w - k(w)} > \frac{\bar{k} - w}{w_H - k(w_H)} > \frac{\bar{k} - w_H}{w_H - k(w_H)}$$

which is positive if $\bar{k} > w_H$ and γ is sufficiently small.

References

- [1] Aghion, P. and P. Bolton, 1997, A Theory of Trickle-Down Growth and Development, *Review of Economic Studies* 64, pp151-172.
- [2] Banerjee, A. and A. Newman, 1993, Occupational Choice and the Process of Economic Development, *Journal of Political Economy* 101, pp. 274-98.
- [3] de Meza, D. and D. Webb, 1997, Wealth, Enterprise and Credit Policy, *Economic Journal* 109, pp 133-65.
- [4] Dixit, Avinash, (1987), Trade and Insurance with Moral Hazard, *Journal of International Economics* 23: 201-20.

- [5] Galor, O. and J. Ziera, 1993, Income Distribution and Macroeconomics, *Review of Economic Studies* 60, pp. 35-52.,
- [6] Gertler, Mark and Kenneth Rogoff, 1990, North-South Lending and Endogenous Domestic Capital Market Inefficiency, *Journal of Monetary Economics* 26, pp 245-266.
- [7] Grossman, Gene, (1984), International Trade, Foreign Investment, and the Formation of the Entrepreneurial Class, *American Economic Review* 74(4), 605-14.
- [8] Grossman, Gene, and Giovanni Maggi, (2000), Diversity and Trade, *American Economic Review* 90(5), 1255-75.
- [9] Hartwick, J.M. , 1999, Insuring and $u'''(y)$, *Economics Letters* 65, 205-212.
- [10] Kimball, M. S., 1990, Precautionary saving in the small and in the large, *Econometrica* 58, 53-73.
- [11] Kanbur, S. M. , 1979, Of Risk Taking and the Personal Distribution of Income, *Journal of Political Economy* 87(4), 769-797.
- [12] Kihlstrom R. E. and J.-J. Laffont, 1979, A General Equilibrium Entrepreneurial Theory of Firm Formation Based on Risk Aversion, *Journal of Political Economy* 87(4), 719-748.
- [13] Newman, A., 1995, Risk Bearing and Knightian Entrepreneurship, type-script.
- [14] Rogerson, W., 1985, Repeated Moral Hazard, *Econometrica* 62, pp 459-465.
- [15] Thiele, Henrik and Achim Wambach, 1999, Wealth Effects in the Principal Agent Model, *Journal of Economic Theory* 89, 247-60.

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