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Input Price Discrimination, Access Pricing, and Bypass*

Ngo Van Long†, *Antoine Soubeyran*‡

Résumé / Abstract

Nous examinons le problème des relations verticales. Quand un fournisseur discrimine, est-ce qu'il impose aux firmes à coût marginal plus bas un prix de l'input plus haut que celui qu'il impose aux firmes à coût marginal plus haut? Nous montrons que cela dépend de la capacité des firmes aval à partiellement produire l'input. Nous fournissons aussi une formule de charge d'accès dans le cas où les firmes aval sont des compétiteurs à la Cournot non identiques. Finalement, nous développons un modèle de discrimination par la qualité d'un input, et nous montrons que la firme amont peut trouver profitable de traiter différemment des firmes identiques.

This paper explores several aspects of the vertical relationship between an upstream firm and a number of downstream firms that are Cournot rivals relying on the inputs provided by the upstream firm. We address the following questions: (i) if the upstream firm can charge different prices to different downstream firms, will it charge higher prices to more efficient firms? (ii) if the upstream firm can provide different levels of quality of access to several ex ante identical downstream firms, will it provide a uniform quality of access? The answer to (i) depends on whether downstream firms can self-supply. As for (ii), we show that equals may be treated unequally.

Mots Clés : Prix d'un input, remises, intégration

Keywords: Input price, discount, self-supply

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1 Introduction

The vertical relationship between an "upstream" monopolist and a set of "downstream" firms that rely on the upstream monopolist for a vital intermediate input has received a great deal of attention in the literarure on industrial organization and regulation. The terms "upstream" and "downstream" should not be taken literally, as the following examples illustrate. In the petroleum industry, the upstream firm is the supplier of crude oil, and the downstream firms are oil refineries. However, in telecommunications, the downstream firms serve the market for long distance calls, while the upstream firm is the owner of the local network, without which the long-distance telephone companies cannot sell their products to the consumers. In the market for electricity, it is often the case that electricity transmission and distribution is controlled by one firm, but electricity generation is not. Downstream firms generate electricity and sell it to consumers, using an essential input which is the transmission network provided by the owner of the network, considered as an "upstream" firm. In some situations, an upstream firm can also be integrated with a downstream firm (e.g., the case of the owner of a local telephone network who also provides long distance services, in direct competition with several other "downstream" long-distance service firms).

Some of the major questions concerning the "upstream-downstream" relationship are: (i) does the upstream monopolist have an incentive to practice input price discrimination when downstream firms are not identical? (ii) does input price discrimination favour less efficient firms? (iii) how do the answers to the above questions change if (a) the downstream firms can also produce, perhaps at higher costs, the input themselves, or (b) the upstream firm is integrated with a downstream firm to supply the final good to consumers, in direct competition with other downstream firms? (iv) if the upstream firm can provide input at different *quality* levels to different downstream firms, will it give equal treatment to ex ante identical downstream firms? (v) in the case of input *quality* discrimination, if an upstream firm is integrated with a downstream firm, does the integrated firm have an incentive to reduce the quality of access to its rival downstream firms?

Partial answers to some of the above questions have been provided by DeGraba (1990) and Katz (1987). Assuming that downstream firms cannot produce the input, DeGraba shows[1] that, under linear demand, "when the

[1]DeGraba pointed out (p.1248) that this result was presented in Katz (1987) for

supplier is allowed to price-discriminate, he charges the firms with lower marginal cost a higher price than he charges the firm with the higher marginal cost"(p.1248). This theoretical result is called "discount reversal" because it predicts the exact opposite of the "quantity discount" phenomenon, i.e., the empirical observation that larger buyers tend to be charged less per unit than smaller ones. DeGraba explains that "the apparent contradiction stems from the fact that quantity discounts are used as a self-selection mechanism when the seller does not know the demand curves of the buyers"(p.1248). In DeGraba's model, because of the assumption of perfect information, such quantity discounts do not arise. DeGraba's explanation seems to suggest that under perfect information, one would not observe quantity discount.

However, Katz (1987) has shown that quantity discounts may arise even under perfect information, if the input supplied by the upstream firm can also be produced by the downstream firms, under a special form of increasing returns: constant marginal cost, and declining average cost, owing to a strictly positive fixed cost in the production of the intermediate input. Thus, according to Katz, a monopolist that sells an input would offer to a large buyer, such as a chain store, a better deal than the ones the monopolist offers to local stores, because the chain store can make the credible threat of producing the input itself as it has the potential advantage of economies of scale. Katz's model seems to suggest that increasing return is a crucial factor for quantity discount under perfect information.

In addition to the above "positive" issues, the "normative" issues of regulation have received a great deal of attention in the industrial organization literature. If there exists a regulator that seeks to maximize social welfare, what are the appropriate regulations on input prices (or access prices) and input quality? Recent works by Vickers (1995), Armstrong, Doyles, and Vickers (or ADV, 1996), Laffont, Rey, and Tirole (1996a, 1996b) have shed much light on these topics. Vickers (1995) considers the case where the downstream firms are *symmetric* Cournot rivals under *free* entry (implying zero profits), while ADV (1996) considers a downstream competitive fringe, that takes as given the price announced by a dominant integrated firm. ADV provides an ECPR (efficient component pricing rule) formula that relates the input price (or access price) to the direct cost and to the opportunity cost of providing access.

Cournot players. DeGraba's main interest is in how price discrimination affects downstream producers' long-run choice of technology.

Among the issues that we take up in our paper is the discount reversal result. We begin by showing that if downstream firms cannot self-supply the input, then discount reversal occurs even when the demand curve is not linear and marginal cost is not constant. Next, we consider the case where downstream firms can self-supply, and show that quantity discount can occur even under decreasing returns, in contrast to Katz's assumption of increasing returns.We also derive an access pricing formula for the case in which downstream firms are *asymmetric* Cournot rivals. Since we postulate that the objective is to maximize the profit of the upstream firm (or, in some cases, the vertically integrated firm) rather than to maximize social welfare, our access pricing formula is not directly comparable to those of ADV. However, broadly speaking, there is certain similarity in interpretation.

Another important issue that we address in this paper is input *quality* discrimination. As pointed out by Vickers (1995, p.14), input price is only one of several possible ways that an integrated firm could use to restrict access. Another dimension of restriction is the quality of access. Quality discrimination gives an integrated firm an alternative way of raising rivals' costs. A possible example is the interconnection of telecommunication networks. According to Vickers, "though the pricing terms on which British Telecom was to give access to its rival Mercury were set in 1985, there has been continuing dispute about the quality of that access in terms of delay, the quality of the lines of exchanges, etc., and the impact on Mercury's competitive position."(p. 14). Our paper complements Vickers' informal discussion on quality discrimination by providing a formal analysis of a model of input quality discrimination, where an integrated firm can provide access at different quality levels to several downstream rivals. We show that it can be optimal for the integrated firm to treat ex-ante identical rivals in non-identical ways. Our result, the optimality of the rule "unequal treatment of equals", indicates that models in which identical firms are *assumed* to be treated equally, can be misleading[2].

2 The Basic Model

In this section we consider the simplest case of vertical relationship: it is assumed that the downstream firms have no alternative sources of supply of

[2]For another instance of "unequal treatment of equals", see Long and Soubeyran (1997b).

the vital input, and the upstream firm does not participate in the final good market. Our model is similar to that of DeGraba (1990), but we replace his assumption of linear final demand by non-linear final demand, and we assume convex downstream cost instead of constant marginal costs. Furthermore, while DeGraba assumes that, in relation to input prices, downstream firms differ from each other in an additive way (i.e., firm i's per-unit cost of output is $t_i + c_i$, where t_i is the input price for firm i determined by the upstream monopolist, and c_i is an additional marginal cost of production which vary across firms), we assume that, in relation to input costs, downstream firms differ from each other in a multiplicative way (i.e., firm i's per-unit cost of output is $t_i D_i(q_i)/q_i$, where $D_i(q_i)$ is the input level necessary to produce output q_i.) We will show that "discount reversal" (i.e., the upstream firm charges a lower price to smaller downstream firms) occurs in this model, as it does in DeGraba's model.[3]

There are n downstream Cournot oligopolists producing a homogenous good, using an intermediate input produced by an upstream monopolist. The set of downstream firms is $N = \{1, 2, ..., n\}$. Let q_i denote the output of (downstream) firm i, and let $Q = \sum_{i \in N} q_i$. In order to produce the quantity q_i, the downstream firm i needs to use z_i units of the intermediate input: $z_i = D_i(q_i)$, where $D_i(0) = 0$, $D_i' > 0$ and $D_i'' \geq 0$. We refer to $D_i(.)$ as the *downstream input-requirement function* of firm i.The upstream supplier, denoted by S, charges firm i the input price t_i (per unit) and possibly a fixed fee T_i. Firm i also incurs a fixed cost $F_i \geq 0$ (exogenously given.) Let y_i be the amount of input that downstream firm i buys from firm S. In this section, since we assume that the downstream firms have no alternative sources of input supply, we have $y_i = z_i$.

We consider a two-stage game. In the first stage, the supplier S chooses firm-specific input prices $(t_1, ..., t_n)$ or firm-specific two-part tariffs (t_i, T_i), $i = 1, ..., n$. In the second stage, the downstream firms choose their outputs, and achieve a Cournot equilibrium.

The inverse demand function for the final good is $P = P(Q)$ with $P'(Q) < 0$. In addition, it is assumed that

$$QP''(Q)/[-P'(Q)] < n + 1 \tag{1}$$

[3]For the case of constant marginal costs and non-linear demand, see Long and Soubeyran (1997b), where the discount reversal is explained in terms of the "concentration motive theorem."

i.e., the demand curve is not too convex[4]. Given $t_1, ...t_n$, we have, at a Cournot equilibrium where all firms produce, the conditions

$$P'(\hat{Q})\hat{q}_i + P(\hat{Q}) = t_i D_i'(\hat{q}_i), \quad i \in N \tag{2}$$

where the hat denotes the Cournot equilibrium outputs. It is convenient to define the equilibrium marginal cost of firm i as

$$\theta_i \equiv t_i D_i'(\hat{q}_i) \tag{3}$$

Then (2) becomes

$$\hat{q}_i = \frac{P(\hat{Q}) - \theta_i}{[-P'(\hat{Q})]} \tag{4}$$

Firm i's profit function is:

$$\pi_i = P(Q)q_i - t_i D_i(q_i) - T_i - F_i$$

Using (2) to substitute for t_i,we can write the *equilibrium* profit of firm i as

$$\hat{\pi}_i = P(\hat{Q})\hat{q}_i - \left[\frac{P'(\hat{Q})\hat{q}_i + P(\hat{Q})}{D_i'(\hat{q}_i)}\right] D_i(\hat{q}_i) - T_i - F_i$$

or, more compactly,

$$\hat{\pi}_i = \left(1 - \frac{1}{\hat{\tau}_i}\right) P(\hat{Q})\hat{q}_i + \frac{1}{\hat{\tau}_i}[-P'(\hat{Q})]\hat{q}_i^2 - T_i - F_i \tag{5}$$

where $\hat{\tau}_i$ is the elasticity of the downstream input-requirement function of firm i, evaluated at the Cournot equilibrium:

$$\hat{\tau}_i \equiv \frac{\hat{q}_i D_i'(\hat{q}_i)}{D_i(\hat{q}_i)}$$

The profit function of the upstream firm is

$$\Pi_S = \sum_{i \in N} t_i y_i - C(y) + T$$

[4]For a complete set of assumptions that guarantees existence and uniqueness of a Cournot equilibrium, see Gaudet and Salant (1991). We adopt those assumptions for our model.

where $y = \sum_{i \in N} y_i$ and $C(y)$ is the upstream firm's cost of producing y, and where $T = \sum_{i=1}^{n} T_i$.Given $t_1, ...t_n$, the supplier's profit at the corresponding (downstream) Cournot equilibrium is

$$\hat{\Pi}_S = \sum_{i \in N} \left[\frac{P'(\hat{Q})\hat{q}_i + P(\hat{Q})}{D_i'(\hat{q}_i)} \right] D_i(\hat{q}_i) - C \left[\sum_{i \in N} D_i(\hat{q}_i) \right] + T$$

Equivalently,

$$\hat{\Pi}_S = \sum_{i \in N} \frac{1}{\hat{\tau}_i} \left[P'(\hat{Q})\hat{q}_i^2 + P(\hat{Q})\hat{q}_i \right] - C \left[\sum_{i \in N} D_i(\hat{q}_i) \right] + T \qquad (6)$$

In the first stage, the supplier, S, chooses the t_i's (and possibly the fixed charges T_i's) to maximize its profit. From (3) and (4), it is clear that the choice of the t_i's is *equivalent* to the manipulation of the marginal costs θ_i's of the downstream firms, which in turn is *equivalent* to choosing the equilibrium outputs $\hat{q}_i$'s. Of course the participation constraints $\hat{\pi}_i \geq 0$ must be satisfied.

In what follows, we focus on the benchmark case where S cannot use two-part tariffs nor other forms of non-linear pricing. Thus the T_i's are constrained to be zero. We further simplify the problem by assuming that the upstream cost is linear

$$C(y) = cy, \quad c > 0$$

and that the downtream firms' input requirement functions are convex and exhibit constant elasticity:

$$D_i(q_i) = \frac{d_i q_i^{\tau}}{\tau} \quad , \tau \geq 1, d_i > 0.$$

Then firm S's profit in the (downstream) Cournot equilibrium becomes

$$\hat{\Pi}_S = \frac{1}{\tau} P(\hat{Q})\hat{Q} - \frac{1}{\tau}[-P'(\hat{Q})]\hat{Q}^2 \hat{H} - \frac{c}{\tau} \sum_{i \in N} d_i \hat{q}_i^{\tau} \qquad (7)$$

where $\hat{H}$ is the Herfindahl index of concentration of the downstream industry:

$$\hat{H} = \sum_{i \in N} \left[\hat{q}_i / \hat{Q} \right]^2$$

As will be seen below, the discriminatory price structure chosen by S depends on the Herfindahl index of concentration and on the elasticity of the slope of the demand curve.

We now solve firm S's optimization problem. It is convenient to proceed in two steps. In the first step, we temporarily fixed the industry output $\hat{Q}$, and seek to characterize the monopolist's choice of the $\hat{q}_i$'s, conditional on $\sum_{i=1}^{n} \hat{q}_i = \hat{Q}$ (given). In the second step, we determine $\hat{Q}$.

The first step:

We re-write $\hat{\Pi}_S$ as

$$\hat{\Pi}_S = \frac{1}{\tau}\left[P(\hat{Q})\hat{Q} - \sum_{i=1}^{n} f_i(\hat{q}_i, \hat{Q})\right] \tag{8}$$

where

$$f_i(\hat{q}_i, \hat{Q}) \equiv [-P'(\hat{Q})]\hat{q}_i^2 + cd_i\hat{q}_i^\tau$$

For a given $\hat{Q}$, choose the Cournot equilibrium outputs, the $\hat{q}_i$'s, to maximize (8) subject to $\sum_{i=1}^{n} \hat{q}_i = \hat{Q}$ and the non-negativity of $\hat{q}_i$ and $\hat{\pi}_i$. (We will focus on the case where the solution is an interior solution, i.e., $\hat{q}_i > 0$ and $\hat{\pi}_i > 0$). The Lagrangian is

$$L = \frac{1}{\tau}\left[\{P(\hat{Q}) - \lambda\}\hat{Q} + \sum_{i=1}^{n}\{\lambda\hat{q}_i - f_i(\hat{q}_i, \hat{Q})\}\right]$$

and is strictly concave in the $\hat{q}_i$ for a given $\hat{Q}$. Then, at an interior solution,

$$\lambda - \frac{\partial f_i(\hat{q}_i, \hat{Q})}{\partial \hat{q}_i} = 0, \quad i \in N \tag{9}$$

Equation (9) implies

$$\lambda + 2P'(\hat{Q})\hat{q}_i^* = d_i c\tau(\hat{q}_i^*)^{\tau-1} > 0, \quad i \in N$$

It follows from this equation that $\hat{q}_i^* > \hat{q}_j^*$ if and only if $d_i < d_j$. Thus we have established the following result:

Proposition 2.1: The monopolist will adopt an input pricing scheme that ensures that low-cost firms (i.e., those with low d_i) produce more than high cost firms. Furthermore, marginal production costs, $d_i\tau(\hat{q}_i^*)^{\tau-1}$ are not equalized across firms.

The results that marginal production costs are not equalized across firms is due to the fact that the monopolist is constrained to use linear pricing for each downstream firm, leaving them with positive profits.[5]

Equation (9) can be inverted[6] to give

$$\hat{q}_i^* = \phi_i(\lambda, \hat{Q}) \tag{10}$$

and the optimal value of λ, denoted by $\lambda^*(\hat{Q})$, can thus be obtained from the condition

$$\sum_{i \in N} \hat{q}_i^* = \sum_{i \in N} \phi_i(\lambda, \hat{Q}) = \hat{Q} \tag{11}$$

(See the Appendix for two examples that illustrate this procedure). The optimal firm-specific input prices are

$$t_i^* = \frac{P'(\hat{Q})\hat{q}_i^* + P(\hat{Q})}{d_i(\hat{q}_i^*)^{\tau-1}} \tag{12}$$

which, together with (9), yields the formula for firm S's mark-up

$$t_i^* - c = \frac{(\tau-2)P'(\hat{Q})\hat{q}_i^* + (\tau P(\hat{Q}) - \lambda)}{\tau d_i(\hat{q}_i^*)^{\tau-1}} \tag{13}$$

The right-hand side of (13) is increasing in $\hat{q}_i^*$ for τ in the interval [1,2], and decreasing in d_i.This fact, together with Proposition 2.1 (which says that $\hat{q}_i^*$ is decreasing in d_i) yields the following result:

Proposition 2.2: For τ in the interval [1,2], the monopolist will practice "discount reversal", i.e., firms that are more efficient (those with a smaller d_i) must pay a higher price per unit of input supplied by the monopolist.

Another sufficient condition for "discount reversal" is $2P(\hat{Q}) - \lambda^*(\hat{Q}) > 0$ (given that $\tau \geq 1$). To see this, re-write (12) as

$$t_i^* = \frac{\tau\{P'(\hat{Q})\hat{q}_i^* + P(\hat{Q})\}}{2P'(\hat{Q})\hat{q}_i^* + \lambda^*(\hat{Q})}$$

[5] It is easy to verify that if the monopolist could use two-part pricing then T_i would be set so that $\hat{\pi}_i = 0$, in which case downtream marginal costs would be equalized.

[6] Because $\partial \hat{q}_i / \partial \lambda > 0$.

It follows from this equation that, for given $\hat{Q}$, t_i^* is increasing in $\hat{q}_i^*$ if $2P(\hat{Q}) - \lambda^*(\hat{Q}) > 0$.

Proposition 2.3: For $\tau \geq 1$, the monopolist will practice "discount reversal" if $2P(\hat{Q}) - \lambda^*(\hat{Q}) > 0$.

Remark: Proposition 2.3 requires the knowledge of $\lambda^*(\hat{Q})$. The examples in the Appendix show how $\lambda^*(\hat{Q})$ can be computed. Alternatively, as Proposition 2.4 below indicates, we can find sufficient conditions for $2P(\hat{Q}) - \lambda^*(\hat{Q}) > 0$ in terms of the curvature of the demand curve and the index of concentration of the downstream industry.

It remains to determine the monopolist's optimal $\hat{Q}$. This is done in the second step below.

The second step:

We now try to express the monopolist's profit as a function of $\hat{Q}$, having known how, for a given $\hat{Q}$, the $\hat{q}_i^*$ (and hence t_i^*) are optimally chosen. Following the duality approach used in Rockafellar (1970, Chapter 12), we define the "conjugate function" f^* of the original function $f_i(\hat{q}_i, \hat{Q})$ as follows:

$$f_i^*(\lambda, \hat{Q}) = \sup_{\hat{q}_i} \left[\lambda \hat{q}_i - f_i(\hat{q}_i, \hat{Q}) \right] \quad , \hat{q}_i \geq 0,$$

where $\hat{Q}$ is given. Then, the profit function of the monopolist, given the maximization performed in Step 1 above, is

$$\Pi_S^*(\hat{Q}) = L^*(\hat{Q}) = \frac{1}{\tau} \left[(P(\hat{Q}) - \lambda^*(\hat{Q}))\hat{Q} + \sum_{i \in N} f_i^*(\lambda^*(\hat{Q}), \hat{Q}) \right]$$

Assuming an interior solution, the optimal $\hat{Q}$ must satisfy the first order condition:

$$\tau \frac{d\Pi_S^*(\hat{Q})}{d\hat{Q}} = (P'(\hat{Q}) - \lambda^{*'}(\hat{Q}))\hat{Q} + (P(\hat{Q}) - \lambda^*(\hat{Q})) + \sum_{i \in N} \frac{\partial f_i^*}{\partial \lambda} \frac{d\lambda^*}{d\hat{Q}} + \sum_{i \in N} \frac{\partial f_i^*}{\partial \hat{Q}} = 0 \tag{14}$$

Using the envelope theorem, we have $\partial f_i^* / \partial \lambda = \hat{q}_i^*$, and (14) becomes

$$\tau \frac{d\Pi_S^*(\hat{Q})}{d\hat{Q}} = P(\hat{Q}) - \lambda^*(\hat{Q}) + P'(\hat{Q})\hat{Q}\left[1 + EH\right] = 0 \tag{15}$$

where H is the Herfindahl index of concentration $(1 \geq H \geq (1/n)^2)$ defined as

$$H = \sum_{i \in N} \frac{\hat{q}_i^{*\,2}}{\hat{Q}^2}$$

and E is the elasticity of the slope of the demand curve at $\hat{Q}$: $E = P''(\hat{Q})\hat{Q}/[-P'(\hat{Q})]$.

Remark: By definition, the Herfindahl index of concentration is at its maximum value ($H = 1$) if the industry output, Q, is produced by one firm, and H is at its minimum ($H = 1/n^2$) if all the n firms have identical market shares.

If $\Pi_S^*(\hat{Q})$ is strictly concave[7] in $\hat{Q}$ and the maximum is an interior one, then equation (15) uniquely determines the optimal $\hat{Q}^*$. At that optimum point,

$$2P(\hat{Q}^*) - \lambda^*(\hat{Q}^*) = P(\hat{Q}^*) - P'(\hat{Q}^*)\hat{Q}^*[1 + E^* H^*] \tag{16}$$

The right-hand side of this equation is positive if $E^* \geq 0$ (this inequality holds if the demand curve is linear or convex), or if $E^* < 0$ but $E^* H^* \geq -1$. Using this result and Proposition 2.3, we obtain:

Proposition 2.4: For "discount reversal" to occur, it is sufficient that the demand curve is linear or convex (implying $E^* \geq 0$), or that it is not too concave, i.e., $E^* H^* \geq -1$.

The optimal input price that the monopolist charges firm i is obtained from (12), (9), and (16):

$$t_i^* = \frac{c\tau[P'(\hat{Q}^*)\hat{q}_i^* + P(\hat{Q}^*)]}{2P'(\hat{Q}^*)\hat{q}_i^* + P(\hat{Q}^*) + P'(\hat{Q}^*)\hat{Q}^*[1 + E^* H^*]}$$

where the denominator is positive because it must be the same as the denominator of the right-hand side of (12), the left-hand side being t_i^* in both equations. This input price is dependent on the concentration index of the downstream industry, and on the elasticity of the slope of the demand curve.

3 Extension: Self-supply by Downstream Firms

In the preceding section, the downstream firms must rely entirely on the upstream monopolist for their input. We now relax that assumption and

[7] A set of sufficient conditions for this to hold is $\tau = 1$ and $P(Q)$ is linear.

consider the case where they can produce the input themselves. We wish to find out whether the dominant upstream firm still find it profitable to practice "discount reversal".

We continue to assume that, in order to produce q_i units of the final good, the downstream firm i needs $D_i(q_i)$ units of the intermediate input. It can satisfy this need by purchasing y_i units of the intermediate input from the upstream firm S, and producing x_i units of the intermediate input itself, such that $y_i + x_i = D_i(q_i)$. Let t_i be the firm-specific price charged by the upstream firm S. Let $U_i(x_i)$ be the cost to firm i of producing x_i. We assume that $U_i(x_i)$ is strictly convex, with $U_i(0) = 0$, $U_i'(0) = 0$ and $U_i'(\infty) = \infty$.The profit function of firm i is

$$\pi_i = Pq_i - t_i[D_i(q_i) - x_i] - U_i(x_i) - F_i - T_i$$

where T_i is the fixed fee imposed by the dominant upstream firm S if $y_i > 0$. It is important to note that since the downstream firm can produce the intermediate input with the cost $U_i(x_i)$ as specified above, the upstream firm can never charge a fee that would reduce firm i's profit to zero.

The timing of the game is as follows. In the first stage, the upstream firm S sets discriminatory input prices t_i, $i = 1, ..., n$, or two-part tariffs (t_i, T_i). In the second stage, the downstream firms simultaneously and non-cooperatively choose their output levels q_i. In the third stage, each downstream firm i makes its procurement decision: how much of the required input $D_i(q_i)$ is to be purchased from the upstream firm S, and how much is to be self-supplied.

As usual, the game is solved backwards. We consider first the choice made in the third stage. For given t_i, T_i and q_i, the firm i minimizes the cost of producing q_i. Let

$$C_i(t_i, q_i) = \min_{x_i}\{F_i + T_i + t_i[D_i(q_i) - x_i] + U_i(x_i)\} \tag{17}$$

subject to $D_i(q_i) \geq x_i \geq 0$. In what follows, we will restrict attention to the case where q_i is sufficiently large, and the cost $U_i(x_i)$ is sufficiently convex so that the constraint $D_i(q_i) \geq x_i$ is not binding. This means that we focus on equilibria where self-supply is only partial. Then problem (17) is equivalent to that of finding:

$$V_i^*(t_i) \equiv \max_{x_i}\{t_i x_i - U_i(x_i)\}$$

subject to $x_i \geq 0$. This problem yields $x_i = x_i(t_i)$, with

$$x_i'(t_i) = \frac{1}{U_i''(x_i)} > 0 \tag{18}$$

We can therefore express the cost function $C_i(t_i, q_i)$ as

$$C_i(t_i, q_i) = F_i + T_i + t_i D_i(q_i) - V_i^*(t_i)$$

We now turn to the second stage, when the firms choose their q_i, taking the t_i as given.. The necessary conditions for a Cournot equilibrium in this stage are the same as in the preceding section. Firm i's equilibrium profit is

$$\hat{\pi}_i = \left(1 - \frac{1}{\hat{\tau}_i}\right) P(\hat{Q})\hat{q}_i + \frac{1}{\hat{\tau}_i}[-P'(\hat{Q})]\hat{q}_i^2 - T_i - F_i + V_i^*(t_i)$$

and the profit of the upstream monopolist is

$$\Pi_S = \sum_{i \in N} t_i[D_i(\hat{q}_i) - x_i(t_i)] - C_S \left[\sum_{i \in N} \{D_i(\hat{q}_i) - x_i(t_i)\}\right] + \sum_{i \in N} T_i$$

where $T_i = 0$ if two-part tariff is not allowed.(Note that since firm i can make a positive profit without buying input from S, in the case where two-part tariffs are allowed, firm S cannot impose a value of T_i that would eliminate firm i 's profit.)

Consider the special case where the downstream cost functions $D_i(q_i)$ are linear: $D_i(q_i) = d_i q_i$. Then the Cournot equilibrium output q_i satisfies

$$\hat{q}_i = \frac{P(\hat{Q}) - \theta_i}{[-P'(\hat{Q})]} = \hat{q}_i(\hat{Q}, \theta_i) \tag{19}$$

where by definition θ_i is firm i's marginal cost: $\theta_i = d_i t_i$. Note that

$$\frac{\partial \hat{q}_i(\hat{Q}, \theta_i)}{\partial \theta_i} = \frac{1}{[-P'(\hat{Q})]} < 0 \tag{20}$$

Let $\theta_N = (1/n)\sum_{i \in N} \theta_i$. Summing (19) over all firms gives

$$\hat{Q} = \frac{nP(\hat{Q}) - n\theta_N}{[-P'(\hat{Q})]} \tag{21}$$

This equation shows that by choosing θ_N, the upstream monopolist can determine[8] the downstream industry output $\hat{Q}$. Therefore we write $\hat{Q} = \hat{Q}(\theta_N)$, and $\hat{q}_i = \hat{q}_i(\hat{Q}, \theta_i) = \hat{q}_i(\theta_N, \theta_i)$ (with a slight abuse of notation.) The quantity of input that firm i purchases from the upstream monopoly is

$$y_i = d_i \hat{q}_i - x_i(t_i) = d_i \hat{q}_i(\theta_N, \theta_i) - x_i(\theta_i / d_i) \equiv y_i(\theta_N, \theta_i)$$

with

$$\frac{\partial y_i(\theta_N, \theta_i)}{\partial \theta_i} = \frac{d_i}{P'(\hat{Q})} - \frac{x_i'(t_i)}{d_i} < 0$$

Now consider stage 1. (We do not consider two-part tariffs here). The upstream monopolist sets the t_i's (and hence θ_i and θ_N) to maximize its profit

$$\max_{\theta_i} \Pi_S = \sum_{i \in N} \frac{\theta_i}{d_i} y_i(\theta_N, \theta_i) - C_S \left[\sum_{i \in N} y_i(\theta_N, \theta_i) \right] \tag{22}$$

We wish to determine conditions under which the monopolist finds it profitable to practice discount reversal. To do this, it is convenient to solve the problem (22) in two steps. In the first step, θ_N is fixed, so that $\hat{Q} = \hat{Q}(\theta_N)$ is fixed, and the optimal θ_i's are determined subject to $\sum_{i \in N} \theta_i = n\theta_N$. In the second step we determine θ_N. The Lagrangian for the first step is

$$L = \sum_{i \in N} \frac{\theta_i}{d_i} y_i(\theta_N, \theta_i) - C_S \left[\sum_{i \in N} y_i(\theta_N, \theta_i) \right] + \lambda \left[\sum_{i \in N} \theta_i - n\theta_N \right]$$

Assuming an interior maximum, we get the first order conditions

$$\frac{y_i}{d_i} + \frac{\theta_i}{d_i} \frac{\partial y_i}{\partial \theta_i} - C_S' \frac{\partial y_i}{\partial \theta_i} + \lambda = 0 \tag{23}$$

This equation yields $\theta_i^* = \theta_i(\lambda, \theta_N)$. Therefore, substituting into the constraint, we get

$$\sum_{i \in N} \theta_i(\lambda, \theta_N) - n\theta_N = 0$$

which yields $\lambda = \lambda(\theta_N)$. Given θ_N, the monopolist's optimal input price t_i is

$$t_i^* = \frac{\theta_i^*}{d_i} = C_S' - \frac{1}{\frac{\partial y_i}{\partial \theta_i}} \left[\frac{y_i(\theta_N, \theta_i^*)}{d_i} + \lambda(\theta_N) \right] \tag{24}$$

[8] We assume that $nP(Q) + QP'(Q)$ is a decreasing function (see (1)) and that it is negative if Q is sufficiently large.

To proceed further, let us assume that $U_i(x_i) = (1/2)u_i x_i^2$. Then (24) gives

$$t_i^* = \frac{1}{2}\left[C_S' + [u_i d_i]\frac{P(\hat{Q}) + \lambda(\theta_N)[-P'(\hat{Q})]}{d_i^2 u_i + [-P'(\hat{Q})]}\right] \tag{25}$$

From (25), we obtains the following result:

Proposition 3.1: Under the assumption that $U_i(x_i) = (1/2)u_i x_i^2$.

(a) If the costs of downstream self-supply (the u_i's) are very high then the monopolist will practice discount reversal, i.e., a lower d_i implies a higher t_i :

$$sign\{t_i^* - t_j^*\} = -sign\{d_i - d_j\}$$

(b) If the costs of downstream self-supply are very low then the monopolist will not practice discount reversal.

(c) for any pair of downstream firms (i, j) with the same input-requirement functions (i.e., $d_i = d_j$), the firm with a lower cost of self-supply (i.e., a lower u) will be charged a lower input price. □

Proof: (a) and (b): from (25)

$$sign\frac{\partial t_i}{\partial d_i} = sign\{[-P'] - d_i^2 u_i\}$$

The right-hand side is negative if u_i is sufficiently great.

(c) from (25), $\partial t_i / \partial u_i > 0$.□

Proposition 3.1 is broadly in agreement with the result obtained by Katz (1987), who showed that if downstream firms can threaten to self-supply then the the upstream monopolist will give discounts to larger firms. However, Katz (1987) relied on the assumptions that self-supply involves a positive fixed cost and a constant marginal cost. On the contrary, we assume that self-supply involves no fixed cost, and the marginal cost of self-supply is increasing.

4 Vertically Integrated Input Supplier and Access Pricing

In the two preceding sections, the input supplier does not compete in the downstream market. We now consider the case where the input supplier is vertically integrated with a downstream firm and therefore treats other downstream firms as rivals. For instance, in telecommunications, the downstream sector serves the market for long-distance calls, and the upstream firm is the owner of the local telephone network, which may be vertically integrated with a long-distance service provider. Similarly, in the market for electricity, electricity transmission and distribution may be controlled by one firm, that also owns an electricity generation plant, in competition with other plants that rely on the transmission network provided by the integrated firm.

Using the model introduced in this section, we seek answers to the following questions: (i) does the vertically integrated firm has an incentive to practice discount reversal? (ii) could it be profitable for the integrated firm to treat identical downstream firms "unequally"? (iii) how strong is the incentive to raise rivals' cost? (iv) what form does the "Efficient Component Pricing Rule" (ECPR) take when the downstream firms are *non-identical* Cournot oligopolists?

Vickers (1995) addresses the question of access pricing under the assumptions that the downstream firms are *identical* Cournot rivals, and that downstream profits are zero due to *free entry.* Armstrong et al. (1996) assume that the downstream firms constitute a competitive fringe (i.e., they take the price of their output as given). We consider the case of *asymmetric* downstream firms that are Cournot rivals, and their number is fixed.

Let $N = \{1, 2, ..., n\}$ be the set of downstream firms. Partition this set into two subsets, denoted by $I = \{1, 2, .., n_I\}$ and $J = \{n_I + 1, ..., n_I + n_J\}$ where $n_I + n_J = n$. All members of I are integrated with the upstream firm S while all members of J are independent rivals. If I consists of more than one downstream firm, we assume that these downstream firms also compete with each others, i.e., the integrated firm behaves as if it has a multi-divisional structure that discourages collusion between the divisions. If the output of downstream firm h is q_h, its input need is $D_h(q_h)$. This need is satisfied partly by purchasing y_h from the upstream division of the integrated firm, and partly by self-supplying the quantity $x_h = D_h(q_h) - y_h$. The cost of self-supply is $U_h(x_h)$. The profits of the downstream firms are

$$\pi_h = Pq_h - t_h y_h - U_h(x_h), \quad h \in I \cup J \equiv N$$

where $y_h + x_h = D_h(q_h)$. For simplicity, we assume that

$$D_h(q_h) = d_h q_h, \quad h \in N$$

The profit of the upstream division (i.e., the input supplier S) of the integrated firm is

$$\Pi_S = \sum_{h \in N} (t_h - c) y_h$$

where $c > 0$ is the upstream firm's constant marginal cost. The total profit of the integrated firm is

$$\Pi_{IS} = \Pi_S + \sum_{h \in I} \pi_h = \sum_{j \in J} (t_j - c) y_j + \sum_{i \in I} (P - d_i c) q_i + \sum_{i \in I} [c x_i - U_i(x_i)]$$

The timing of the game is as followed. In the last stage, all the n downstream entities (the n_I divisions of the integrated firms and the n_J independent downstream firms) compete as Cournot rivals. Each entity h chooses its final output level q_h and its own production of intermediate input $x_h \leq d_h q_h$, while taking as given all the pairs (q_k, x_k) for $k \neq h$. They also take as pre-determined the input prices t_h dictated by the upstream entity S. Thus entity h seeks to maximize

$$\pi_h = P(Q_{-h} + q_h) q_h - t_h d_h q_h + [t_h x_h - U_h(x_h)]$$

subject to $d_h q_h \geq h_h \geq 0$.

The assumption that the downstream divisions of the integrated firms are Cournot rivals of each other may, or may not, be a good description of "real world" situations. Readers who feel uncomfortable with this assumption are requested to set $n_I = 1$.

Assuming an interior solution, we have $2n$ first-order conditions:

$$P'(Q) q_h + P(Q) = t_h d_h$$

$$t_h - U_h'(x_h) = 0$$

These conditions give $\hat{q}_h = \hat{q}_h(\theta_N, \theta_h)$ and $\hat{x}_h = \hat{x}_h(t_h)$, where $\theta_h = t_h d_h$ and $\theta_N = (1/n) \sum_{h \in N} \theta_h$.This stage gives the equilibrium profit of the downstream entities

$$\hat{\pi}_h = [-P'(\hat{Q})] \hat{q}_h^2 + V_h^*(t_h)$$

where $V_h^*(t_h) \equiv \max_{x_h} \{t_h x_h - u_h(x_h)\}$ s.t. $x_h \geq 0$.

We now turn to the first stage of the game, when the integrated firm chooses the t_h's to maximize its profit

$$\hat{\Pi}_{IS} = \sum_{j \in J} (t_j - c) \widehat{y}_j + \sum_{i \in I} (\hat{P} - d_i c) \hat{q}_i + \sum_{i \in I} [c \hat{x}_i - U_i(\hat{x}_i)]$$

where $\hat{P} = P(\hat{Q}(\theta_N))$, $\hat{q}_h = [\hat{P} - d_h t_h]/[-\hat{P}']$ for all $h \in N$, $\widehat{y}_j = d_j \hat{q}_j$ for all $j \in J$, and $\hat{x}_i = \hat{x}_i(t_i)$ for all $i \in I$. Recalling that $t_h = \theta_h / d_h$, we can formulate the optimization problem of the integrated firm as that of choosing the t_h' s to maximize $\hat{\Pi}_{IS}$. As in the preceding section, we solve this problem in two steps. In step 1, we fix θ_N (so that $\hat{P}$ is fixed, and optimize with respect to the t_h's subject to $\theta_N - (1/n)\sum_{h\in N} t_h d_h = 0$. The Lagrangian is

$$L = \hat{\Pi}_{IS} + \lambda \left[\sum_{h \in N} t_h d_h - n\theta_N \right]$$

Manipulations of the first-order conditions yield

$$t_i = c + \left(\frac{d_i}{[-\hat{P}']x_i'} \right) \left[\lambda[-\hat{P}'] - (\hat{P} - d_i c) \right], \quad i \in I \tag{26}$$

and, for all $j \in J$,

$$t_j = c + \left(\frac{d_j}{d_j^2 + [-\hat{P}']x_j'} \right) \left[\lambda[-\hat{P}'] + \hat{P} - d_j t_j - [-\hat{P}'](\hat{x}_j / d_j) \right] \tag{27}$$

These formulas are only implicit because the t_i (or t_j) appear on both sides of the equations. One may relate these formulas to the "efficient component pricing rule" (ECPR) derived by Armstrong, Doyles and Vickers (ADV, 1996):

Input price (or access price) = direct cost + opportunity cost of providing access.

However, we should note that ECPR was derived by ADV under the objective of maximizing welfare, not maximizing the profit of the integrated firm. Our component pricing rules (26) and (27) are for a monopolist. It contains the Lagrange multiplier λ which, as shown in the Appendix, can be solved for in terms of the given θ_N.

In order to proceed further, let us assume that

$$U_h(x_h) = \frac{u_h x_h^2}{2} \tag{28}$$

then we have

$$t_i = c + \left[\frac{d_i c + \lambda^*[-\hat{P}'] - \hat{P}}{[-\hat{P}']} \right] d_i u_i \quad , i \in I \tag{29}$$

$$t_j = \frac{1}{2} \left[c + \frac{\lambda^*[-\hat{P}'] + \hat{P}}{d_j^2 u_j + [-\hat{P}']} (d_j u_j) \right] \quad , j \in J \tag{30}$$

(Note that λ^* can be determined in terms of θ_N and other parameters; see the Appendix.) It follows that for any pair (i, i') such that $d_i = d_{i'}$, we have

$$\frac{t_i - c}{t_{i'} - c} = \frac{u_i}{u_{i'}} \quad , i, i' \in I$$

and for any pair (j, j'), we have

$$\frac{t_j - (c/2)}{t_{j'} - (c/2)} = \frac{\gamma_j}{\gamma_{j'}} \quad , j, j' \in J$$

where

$$\gamma_j = \frac{d_j u_j}{d_j^2 u_j + [-\hat{P}']}$$

(Note that $\partial \gamma_j / \partial u_j > 0$). Thus we have established the following results:

Proposition 4.1: If (28) holds, then

(i) the input prices for external downstream firms (that have the same d_j) are subject to discounts, i.e., firms with a lower u_j will be charged a lower t_j.

(ii) within the integrated firms, the transfer prices applied to downstream divisions are more favourable to those with lower costs of self-supply.□

Property (ii) implies that the less efficient divisions are "penalized", so that the integrated firm can achieve a greater profit by shifting market power to those divisions that have lower costs of self-supply. Property (i) was already derived in the preceding section, where the upstream firm is not vertically integrated.

Remark 4.1: From (30), we can ask the following question: for a given θ_N , (so that $\hat{Q}$ is fixed), and a given number n of downstream entities, how does t_j change if the set I of downstream divisions expands relative to the set J of independent downstream firms? To simplify, assume that the d_h's are the same for all $h \in N$. Compare the situation where I is the empty set (i.e., the upstream firm is not integrated with any downstream firm) with the situation where I consist of only one firm, which we denote as firm 1 without loss of generality. Let $\lambda_0^*(\theta_N)$ and $\lambda_1^*(\theta_N)$ denote the optimal value of the Lagrange multiplier in these two situations respectively. If u_1 is very small, then we can show (see the Appendix) that

$$\lambda_1^*(\theta_N) < \lambda_0^*(\theta_N) \tag{31}$$

This inequality implies that t_j decreases when the upstream firm is vertical integrated with firm 1. Thus, for a *given* θ_N, vertical integration does not result in a "raising rivals' cost" strategy. (This conclusion must be qualified: we would expect that vertical integration would lead to a change in the integrated firm's optimal choice of θ_N.)

Remark 4.2: From (30) and assuming the concavity[9] of $\hat{\Pi}_{SI}$ with respect to the t_j's, we conclude that, for any pair of identical downstream firms, the integrated firm charges them the same input price. Thus "equals are treated equally". As we will see in the following section, this property no longer holds in a model where the upstream firm S can choose quality levels that it offers to downstream firms.

Remark 4.3: The second step in the solving the optimization problem of the integrated firm consists of determining the optimal θ_N. This can be done using the approach taken in the preceding section.

5 Quality Discrimination and Access

So far, we have focussed on input price discrimination. As pointed out by Vickers (1995, p. 14), input price is only one of several possible ways that an integrated firm could use to restrict access. Another dimension of restriction is the quality of access. Quality discrimination gives firm S an alternative way of raising rivals' costs. According to Vickers, " a possible example is the interconnection of telecommunications networks. Though the pricing terms on which British Telecom was to give access to its rival Mercury were set in 1985, there has been continuing dispute about the quality of that access in terms of delays, the quality of lines and exchanges, etc., and the impact on Mercury's competitive position"(p.14).

[9]The function Π_{SI} is strictly concave in the t_j's if $U_h(x_h) = (u_h/\mu)x_h^{\mu}$ where $2 \geq \mu \geq 1$.(See the Appendix.)

In this section, we complement Vickers' informal discussion on quality of access, by developing a formal model. We will show that input quality discrimination exhibits a new feature not encountered in input price discrimination: under certain conditions, the upstream firm will find it profitable to offer identical downstream firms non-identical quality level[10].

There are n downstream firms. Firm i's output is q_i. Its unit production cost is $d_i = d_i(\mu_i)$ where μ_i is the quality level of the access supply by the upsream firm S. Firm S is not vertically integrated.We assume that

$$d_i(\mu_i) = d_{0i} - r_i(\mu_i)$$

where $d_{0i} > 0$ and $r_i(0) = 0, r_i' > 0$. Thus $r_i(\mu_i)$ is the reduction in unit cost when the quality of access exceeds the minimum level 0.

Quality is assumd to be objectively measurable, so that μ_i is the number of "units of quality" that firm i buys from firm S. Firm S announces to firm i that the price of each unit of quality is t_i. In addition, there is a fixed charge T_i. Firm i's total cost of producing q_i units of final output is

$$C_i = d_i(\mu_i)q_i + t_i\mu_i + T_i + F_i$$

Given any planned output q_i and given the pair (t_i, T_i) offered by the upstream firm, firm i chooses the quality level μ_i that it wants to purchase from S to minimize the per unit cost: Thus μ_i^* is given by

$$t_i/q_i = r'(\mu_i^*) \tag{32}$$

The cost function of firm i is then

$$C_i(q_i, t_i) = d_i(\mu_i^*(t_i/q_i))q_i + t_i\mu_i^*(t_i/q_i) + T_i + F_i$$

Define the marginal cost of output q_i as

$$\theta_i \equiv \frac{\partial C_i(q_i, t_i)}{\partial q_i} = d_i(\mu_i^*(t_i/q_i)) = d_{0_i} - r_i[\mu_i^*(t_i/q_i)] \tag{33}$$

(where we have made used of the property (32).) Note that since $r_i(.) \geq 0$,

$$\theta_i \geq d_{0i} \tag{34}$$

[10]This is another instance of a class of problems where "equals are treated unequally", see Long and Soubeyran (1997a).

From (33):

$$\mu_i^* = r_i^{-1}[d_{0i} - \theta_i] \tag{35}$$

Consider the Cournot equilibrium achieved by the downstream oligopolists, given the t_i's. The first order conditions yield

$$\hat{q}_i = \frac{\hat{P} - \theta_i}{[-\hat{P}']} \tag{36}$$

where $\hat{P} = P(\hat{Q}(\theta_N))$.

The profit of the upstream firm, in the case of linear tariffs (i.e. $T_i = 0$), is

$$\hat{\Pi}_S = \sum_{i \in N} t_i \mu_i^*(t_i/\hat{q}_i) - C_S \left[\sum_{i \in N} \mu_i^*(t_i/\hat{q}_i) \right] \tag{37}$$

Using (32), $t_i = \hat{q}_i r_i'(\mu_i)$, (35), and (36), we can express (37) as function of the θ_i's:

$$\hat{\Pi}_S = \sum_{i \in I} \frac{\hat{P} - \theta_i}{[-\hat{P}']} \left[r_i'(r_i^{-1}(d_{0i} - \theta_i)) \right] - C_S \left[\sum_{i \in N} r_i^{-1}(d_{0i} - \theta_i) \right] \tag{38}$$

Thus, the optimization problem of the upstream firm amounts to choosing the θ_i's (via the choice of the t_i's; see (33) and (36)) to maximize $\hat{\Pi}_S$, subject to (34), and $\hat{P} - \theta_i \geq 0$ (to ensure that the $\hat{q}_i$'s are non-negative). If the constraint (34) is binding for some i, this means that firm i is induced to purchase the lowest quality of access ($\mu_i = 0$) because t_i is too expensive.

Proposition 5.1 (Unequal treatment of equals): If the function $\hat{\Pi}_S$ in (38) is strictly convex in the θ_i's (for a given θ_N), then ex-ante identical firms will be given different quality levels.

Example 5.1:(Unequal treatment of equals)

Take the case where

$$C_S(\sum_{i \in N} \mu_i) = c \sum_{i \in N} \mu_i$$

and

$$r_i(\mu_i) = (\delta_i \mu_i)^\alpha, \quad 0 < \alpha < 1$$

then, from the definition of θ_i,

$$\mu_i = \frac{1}{\delta_i}\,(d_{0i} - \theta_i)^{1/\alpha}\,, \quad d_{0i} \geq \theta \geq 0$$

Thus

$$r_i'(\mu_i)\mu_i = \alpha r_i(\mu_i) = \alpha(d_{0i} - \theta_i)$$

and, for a given θ_N, the profit of the upstream firm is

$$\hat{\Pi}_S = A(\theta_N, \theta_1, ..., \theta_n) - B(\theta_1, ..., \theta_n)$$

where

$$A(\theta_N, \theta_1, ..., \theta_n) = \frac{\alpha}{[-\hat{P}']} \sum_{i \in N} (\hat{P} - \theta_i)(d_{0i} - \theta_i)$$

$$B(\theta_1, ..., \theta_n) = c \sum_{i \in N} \frac{1}{\delta_i}\,[d_{0i} - \theta_i]^{1/\alpha}$$

Notice that A and B are both convex functions. If $\alpha = 1/2$ and c is small, then $A - B$ is convex. Then, for a given θ_N, the optimum is at a corner; see Figures 1A and 1B for the case $n = 2$. Here, for a given θ_N, the vector (θ_1, θ_2) must lie on the line segment HK. The convexity of $A - B$ implies that the maximum for Π_S occurs at H or K.

Remark 5.1: Proposition 5.1 indicates that, from the point of view of firm S, optimal input quality discrimination is very different from input price discrimination. This is because the two types of inputs are quite different. In the former case, the input is the quality of access, which is independent of the output levels of the downstream firm. In the latter case, the input is produced materials that must be increased if the downstream firms expand their output levels.

6 Concluding Remarks

We have shown that, even with a general demand curve, "discount reversals" occur when an upstream firm practices input price discrimination for a vital input that downstream firms cannot produce themselves. However if downstream firms can, to some extent, self-supply the vital input, then discount reversals may no longer be profitable for the upstream firm. Moreover, if the upstream firm is integrated with one or several downstream firms, then in general it will give discount to larger downstream divisions. An integrated firm with several downstream divisions will not treat them the same way: the more cost-efficient divisions (with respect to self-supply) will be charged a lower price for the input sold by the upstream firm.

Quality of access provided by the upstream firm can vary accross downstream firms. We have shown that this can be the case even when all downstream firms are ex-ante identical. By treating ex-ante equal firms unequally, the upstream firm ensures that, for a given output level of the downstream industry, aggregate downtream production cost is minimized. Unlike raw materials, which tend to be proportional to final output level, quality of access is somewhat like capital equipment that shifts down the marginal cost curves. Thus it may be more efficient to concentrate this type of "investment" in one downstream firm, to exploit a sort of economy of scale.

We have also derived an access pricing rule from the point of view of an upstream firm that faces heterogenous downstream oligopolists. This rule resembles the "efficient component pricing rule" (ECPR) in the regulation literature.

In the models where self-supply is possible, we have assumed that a downtream firms cannot sell or buy the intermediate input from each other. This assumption was made to simplify the analysis. For a model which allows for the downstream market for the intermediate input, see Long and Soubeyran (1999).

Several tasks remain to de done. First, an ECPR should be derived from the point of view of a regulator. Second, asymmetric information should be introduced, because the regulator may not know the cost structure of the firms. Third, in the case of quality of access, we must find out whether there is a strong incentive for an integrated firm to raise rivals' costs. Other possible generalizations include (i) the case where downstream firms need several intermediate inputs, each being produced by a distinct upstream firm, and (ii) the case where each downstream firm produces, in addition to the final good, an intermediate input, which it exchanges for other intermediate inputs produced by other downstream firms.

APPENDIX A.1: Examples illustrating (10) and (11)

Example 2.1: with $\tau = 1$ (linear downstream costs), (10) gives

$$\hat{q}_i^* = \phi_i(\lambda, \hat{Q}) = \frac{\lambda - v b_i}{2[-P'(\hat{Q})]}$$

and (11) gives

$$\lambda^*(\hat{Q}) = \frac{2\hat{Q}[-P'(\hat{Q})]}{n} + \frac{v}{n} \sum_{i \in N} b_i > 0$$

Therefore

$$\hat{q}_i^*(\hat{Q}) = \frac{\hat{Q}}{n} - \frac{v}{2[-P'(\hat{Q})]} [b_i - b_N]$$

where $b_N \equiv (1/n) \sum_{i \in N} b_i$. Thus,

$$sign[\hat{q}_i^*(\hat{Q}) - \hat{q}_j^*(\hat{Q})] = -sign\,[b_i - b_j]$$

that is, the firm with lower cost will produce more, confirming Proposition 2.1.

Example 2.2: with $\tau = 2$ (quadratic downstream costs), (10) gives

$$\hat{q}_i^* = \phi_i(\lambda, \hat{Q}) = \frac{\lambda}{2\{[-P'(\hat{Q})] + v b_i\}} \equiv \lambda \psi_i(\hat{Q})$$

and (11) gives

$$\lambda^*(\hat{Q}) = \frac{\hat{Q}}{\sum_{i \in N} \psi_i(\hat{Q})}$$

Therefore

$$\hat{q}_i^*(\hat{Q}) = \frac{\psi_i(\hat{Q})\hat{Q}}{\sum_{i \in N} \psi_i(\hat{Q})}$$

Here, also, we obtain

$$sign\{\hat{q}_i^*(\hat{Q}) - \hat{q}_j^*(\hat{Q})\} = sign\{\psi_i(\hat{Q}) - \psi_j(\hat{Q})\} = -sign\,[b_i - b_j]$$

APPENDIX A.2: The determination of λ^* in Section 4.

Recall that $\sum_{h \in N} b_h t_h = n\theta_N$. Substituting (29) and (30) into the left-hand side, we obtain

$$\lambda^*(\theta_N) = \frac{A + B}{D} \tag{39}$$

where

$$A = n\theta_N - \sum_{i \in I} d_i c - \frac{1}{2} \sum_{j \in J} d_j c$$

$$B = \frac{1}{[-P'(\hat{Q})]} \sum_{i \in I} [\hat{P} - d_i c] d_i^2 u_i - \frac{\hat{P}}{2} \sum_{j \in J} \gamma_j$$

$$D = \frac{[-P'(\hat{Q})]}{2} + \sum_{i \in I} d_i^2 u_i$$

with

$$\gamma_j = \frac{d_j^2 u_j}{d_j^2 u_j + [-P'(\hat{Q})]}$$

APPENDIX A.3: Proof of (31)

Let $d_h = 1$ for all $h \in N$. Then $\lambda^*(\theta_N)$ in (39) becomes

$$\lambda^*(\theta_N) = \frac{A' + B'}{D'}$$

where

$$A' = n\theta_N - (c/2)(n + n_I)$$

$$B' = \frac{[\hat{P} - c]}{[-P'(\hat{Q})]} \sum_{i \in I} u_i - \frac{\hat{P}}{2} \sum_{j \in J} \gamma_j$$

$$D' = \sum_{i \in I} u_i + \frac{[-P'(\hat{Q})]}{2}$$

where

$$\gamma_j = \frac{u_j}{u_j + [-P'(\hat{Q})]}$$

When $n_I = 0$, we have $\lambda^*(\theta_N) = \lambda_0^*(\theta_N)$ where

$$\lambda_0^*(\theta_N) = \frac{E}{F}$$

with

$$E = n\theta_N - (nc/2) - \frac{\hat{P}}{2} \sum_{j \in N} \gamma_j$$

and

$$F = \frac{[-P'(\hat{Q})]}{2}$$

When $n_I = 1$, we have $\lambda^*(\theta_N) = \lambda_1^*(\theta_N)$ where

$$\lambda_1^*(\theta_N) = \frac{E+G}{F+u_1}$$

where

$$G = u_1 \left[\frac{[\hat{P}-c]}{[-P'(\hat{Q})]} + \frac{\hat{P}}{2\{u_1 + [-P'(\hat{Q})]\}} \right] - \frac{c}{2}$$

If u_1 is very small, then $\lambda_1^*(\theta_N) < \lambda_0^*(\theta_N)$. Thus, at an unchanged θ_N, the integration of an upstream firm with a downstream firm reduces the t_j's, $j \in J$. (However, θ_N would not be unchanged when the integration occurs.)

APPENDIX A.4: The concavity of Π_{IS}

$$\frac{\partial \Pi_{IS}}{\partial t_i} = -\left[\frac{d_i(\hat{P} - d_i c)}{[-\hat{P}']} \right] - (t_i - c)\hat{x}_i' + \lambda d_i$$

$$\frac{\partial \Pi_{IS}}{\partial t_j} = \hat{y}_j + (t_j - c)\frac{\partial \hat{y}_j}{\partial t_j} + \lambda d_j$$

$$\frac{\partial^2 \Pi_{IS}}{\partial t_i^2} = -\hat{x}_i' - (t_i - c)\hat{x}_i''$$

$$\frac{\partial^2 \Pi_{IS}}{\partial t_j^2} = 2\frac{\partial \hat{y}_j}{\partial t_j} + (t_j - c)\frac{\partial^2 \hat{y}_j}{\partial t_j^2}$$

where, from $t_h = U_i'(\hat{x}_i)$,

$$\hat{x}_i' = \frac{1}{U_i''(\hat{x}_i)} > 0$$

and

$$\hat{x}_i'' = -\frac{U_i'''(\hat{x}_i)}{[U_i''(\hat{x}_i)]^3} > 0$$

if $U_i'''(\hat{x}_i) < 0$.

We also have

$$\frac{\partial \hat{y}_j}{\partial t_j} = -\frac{d_j^2}{[-\hat{P}']} - \hat{x}_j' < 0$$

and

$$\frac{\partial^2 \hat{y}_j}{\partial t_j^2} = -\hat{x}_j''$$

It follows that

$$\frac{\partial^2 \Pi_{IS}}{\partial t_j^2} = -2 \left[\frac{d_j^2}{[-\hat{P}']} + \hat{x}_j' \right] - (t_j - u_j)\hat{x}_j''$$

Let us specify

$$U_h(x_h) = \left[\frac{u_h}{\mu} \right] x_h^{\mu} \quad , \mu \geq 1$$

then $U_h''' \leq 0$ if and only if $2 \geq \mu \geq 1$. In this case, $\hat{x}_j'' > 0$ and $\partial^2 \Pi_{IS}/\partial t_j^2 < 0$ if $t_j \geq c$.

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