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Résumé / Abstract

Les méthodes simulées d'estimation sont de plus en plus utilisées pour l'estimation et l'évaluation de modèles structurels. Dans cette étude, nous introduisons un ensemble de tests de stabilité pour les modèles estimés à l'aide de la méthode des moments simulés (voir Duffie et Singleton (1993)). Ces tests sont basés sur les travaux récents, dans le cadre de la méthode des moments généralisés, de Andrews (1993) et Sowell (1996a, b). Nous obtenons la loi asymptotique de ces tests et nous montrons que cette loi ainsi que la puissance locale asymptotique ne dépendent pas du nombre de simulations. Une étude de Monte-Carlo révèle qu'en petit échantillon le nombre de simulations influence le niveau et la puissance des tests. Cependant, un nombre restreint de simulations semble suffisant pour obtenir des bonnes propriétés de petit échantillon.

Simulation-based estimation methods have become more widely used in recent years. We propose a set of tests for structural change in models estimates via Simulated Method of Moments (see Duffie and Singleton (1993)). These tests extend the recent work of Andrews (1993) and Sowell (1996a, b) which covered Generalized Method of Moments estimators not involving simulation. We derive the asymptotic distribution of various tests. We show that the number of simulations does not affect the asymptotic distribution nor the asymptotic local power of tests for structural change. A Monte Carlo investigation of the finite sample size and power reveals, however, that simulation uncertainty does affect the properties of tests. Nevertheless, even a relatively small number of simulations suffices to obtain tests with desirable small sample size and power properties.

Mots Clés : Méthode des moments simulés, tests de stabilité structurelle, tests optimaux

Keywords : Simulated method of moments, structural stability testing, optimal

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tests

JEL : C1, C12, C22

1 Introduction

The steady increase in computational speed of computers has enhanced the practical use of simulation-based estimators in econometrics. There is now a well established asymptotic distribution theory for a large variety of procedures, including the simulated method of moments estimator (henceforth SMM) which is the focus of our paper. Duffie and Singleton (1993) developped the asymptotic properties of the SMM estimation procedure in the context of dynamic econometric time series models. In such applications one often wants to test whether the parametric econometric model is invariant through time. Several tests for structural change with presumed breakpoint unknown already exist for the Generalized Method of Moments (henceforth GMM) estimation procedure which pre-dates SMM and does not involve simulations. Such tests were proposed by Andrews (1993), Andrews and Ploberger (1994), Sowell (1996a, b), Guay (1996), Hall and Sen (1996) and Ghysels, Guay and Hall (1997).

The purpose of our paper is to extend the tests proposed for the GMM estimator to cases involving estimation by simulation and rely on the SMM procedure. We introduce several Wald, LR-type, LM and Predictive tests for structural change with unknown breakpoint. The design of the tests is based on the optimality principles of local asymptotic power discussed by Andrews and Ploberger (1994), Sowell (1996a), Guay (1996) and Hall and Sen (1996). While the asymptotic distributions of the partial sample SMM estimators depend on a nuissance parameter, namely the number of simulations, we show that the asymptotic distributions of tests for structural change, which are functions of the partial sample SMM estimators, are nuissance parameter free. Hence, all the tests we propose in the paper have the same asymptotic distributions as their GMM counterpart with critical values tabulated in Andrews (1993), Andrews and Ploberger (1994), Sowell (1996a), Guay (1996) and Ghysels, Guay and Hall (1997). We also show that the number of simulations does not affect the asymptotic local power of tests for structural change. A Monte Carlo investigation of the finite sample size and power reveals, however, that simulation uncertainty does affect the properties of tests. Nevertheless, a relatively small number of simulations is sufficient to obtain tests with desirable small sample size and power properties.

In section 2 we fix notation and discuss the regularity conditions required to establish several large sample properties of SMM estimators which are used in the derivations of the asymptotic distribution of the tests. Section 3 is devoted to testing for structural change. The null hypothesis and the test statistics are formally defined and the main results of the paper, namely the asymptotic distributions of the tests, are presented. The next section 4 covers optimal tests. In section 5 we report the results of a Monte Carlo study of the finite sample properties of the tests. Section 6 concludes the paper.

2 Notation and regularity conditions

To establish the asymptotic distribution theory of tests for structural change we need to define first the class of data generating processes we can simulate, how to simulate them and how to define SMM estimators on (sub)samples of data. In a first subsection we present the class of data generating processes. The next subsection covers assumptions and definitions and a final subsections establishes asymptotic properties of partial sample SMM estimators. Before dealing with these issues we need to elaborate briefly on the specification of the parameter vector in our generic setup. We will consider parametric models indexed by parameters (β, δ) where $\beta \in B$, where $B \subset R^r$ and $\delta \in \Delta \subset R^s$. Following Andrews (1993) we make a distinction between pure structural change when no subvector δ appears and the entire parameter vector is subject to structural change under the alternative. Partial structural change corresponds to cases where only a subvector β is subject to structural change under the alternative. The generic null can be written as follows:

$$H_0 : \beta_t = \beta_0 \qquad \forall t = 1, \ldots, T \tag{2.1}$$

The majority of tests we will consider assume as alternative that at some point in the sample there is a single structural break, like for instance:

$$\beta_t = \begin{cases} \beta_1 & t = 1, ..., [\pi T] \\ \beta_2 & t = [\pi T] + 1, ..., T \end{cases}$$

where π determines the fraction of the sample before and after the assumed break point and [.] denotes the greatest integer function. Hence, we will consider a setup with a parameter vector which encompasses any kind of partial or pure structural change involving a single breakpoint. In particular, we consider a p dimensional parameter vector $\theta = (\beta_1', \beta_2', \delta')'$ where β_1 and $\beta_2 \in B \subset R^r$ and $\theta \in \Theta = B \times B \times \Delta \subset R^p$ where $p = 2r + s$. The parameters β_1 and β_2 apply to the samples before and after the presumed breakpoint. Therefore, we will formulate all our models in terms of θ. Special cases could be considered whenever restrictions are imposed in the general parametric formulation. One such restriction would be that $\theta = (\beta', \beta')'$, which would correspond to the null of a pure structural change hypothesis.

2.1 The Data Generating Process

Since we consider simulation-based inference we have a process of endogenous variables y_t which is generated by the following dynamic structural model:

$$g(y_t, y_{t-1}, x_t, \theta_0) = \varepsilon_t \tag{2.2}$$

where θ is a $p \times 1$ parameter vector, x_t is an observable exogenous process and ε_t is a process of disturbances with a *known* distribution. The processes y_t, x_t and ε_t can either be univariate or multivariate. For the moment we will not be very specific about the conditions we need to impose on y_t, x_t and ε_t. We assume however that we have a sample of observations $t = 1, \ldots, T$ for y_t and x_t. Furthermore, we also assume that the model in (2.2) has the following well defined reduced form:

$$y_t = H(y_{t-1}, x_t, \varepsilon_t, \theta_0). \tag{2.3}$$

Under (2.3), one can simulate values of y_t: (1) given initial values for y_0 and ε_0, (2) given value of the parameter vector θ, and (3) conditionnal on a path of the exogenous process x_t. Throughout the paper we will assume that the simulations of y_t^s are also conditional on the observed value of y_{t-1}.[1] Hence we will denote the simulated values as $y_t^s(x_t, \theta, y_0, y_{t-1}, \varepsilon_0)$. For simplicity, however, we will use the less cumbersome notation y_t^s as a shorthand for the entire expression. To make the presentation of the regularity conditions easier we will sometimes also pool the process $\{x_t\}_{t=-\infty}^{\infty}$ and the unobserved disturbances $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ into a single process $\{V_t\}_{t=-\infty}^{\infty}$. The SMM estimator involves moment conditions which are function of $\{y_t\}_{t=-l}^{+m}$, $\{x_t\}_{t=-l'}^{+m'}$ and of the simulated variables $\{y_t^s\}_{t=-l}^{+m}$. If we combine the processes $\{y_t\}_{t=-l}^{+m}$, $\{x_t\}_{t=-l'}^{+m'}$ into a single process z_t and the simulated counterpart into z_t^s [2] then the SMM estimator is based on the argument that:

[1]For a more elaborate discussion of different types of simulations in dynamic models see e.g. Gouriéroux and Monfort (1996) and Bilio and Monfort (1996).

[2]In fact, we consider that the observed and the simulated processes are given by two triangular arrays of random vectors z_{Tt} and z_{Tt}^s (see Assumption A.2 in the Appendix A and Section 2.2). For notational simplicity z_t and z_t^s denote z_{Tt} and z_{Tt}^s respectively.

$$E[m(z_t) - m(z_t^s, \theta_0)] = 0 \tag{2.4}$$

where m is a R^q-valued function of moment conditions and θ_0 is an element of the parameter space $\Theta \subset R^p$ where $q \geq p$. A sample equivalent can be written as follows:

$$f_T^S(\theta_0) = \frac{1}{T}\sum_{t=1}^{T}\left(m(z_t) - \frac{1}{S}\sum_{s=1}^{S} m(z_t^s, \theta_0)\right) \tag{2.5}$$

where S is the number of simulations. We can also replace the S simulations of size T samples by a single sample simulation of size TS and define:

$$f_T^S(\theta_0) = \frac{1}{T}\sum_{t=1}^{T} m(z_t) - \frac{1}{TS}\sum_{t=1}^{TS} m(z_t^s, \theta_0)$$

2.2 Assumptions and Definitions

We need to impose restrictions on the admissible class of functions and processes involved in estimation to guarantee well-behaved asymptotic properties of SMM estimators either involving the entire data sample or subsamples of observations. We first define the standard SMM estimator introduced by Duffie and Singleton (1993) using all the data.

Definition 2.1 *The full sample Simulated Method of Moments estimator* $\{\tilde{\theta}_T^S\}$ *is a sequence of random vectors such that:*

$$\tilde{\theta}_T^S = Argmin_\theta f_T^S(\theta)' \hat{W}_T f_T^S(\theta)$$

where $\hat{W}_T$ *is a random positive definite symmetric* $q \times q$ *matrix.*

The optimal weighting matrix W is defined to be the inverse of Ω^*, where $\Omega^* = (1 + \frac{1}{S})\Omega$ and

$$\Omega = \lim_{T\to\infty} Var\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T}[m(z_{Tt}) - Em(z_{Tt})]\right).$$

An estimator of Ω can also be obtained with simulated moments (see Duffie and Singleton (1993) and Gouriéroux and Monfort (1996) for a discussion).[3]

Several tests for structural change also involve partial sample SMM estimators similar to the partial sample GMM estimators defined by Andrews (1993). We consider two subsamples, the first is based on observations $t = 1, \ldots, [T\pi]$ while the second subsample covers $t = [T\pi] + 1, \ldots, T$ where $\pi \in \Pi \subset (0,1)$. The separation $[T\pi]$ represents a possible breakpoint which is governed by an unknown parameter π. We now formally define partial-sample SMM estimators for $\pi \in \Pi$ based on the first and the second subsamples.

Definition 2.2 *A partial-sample Simulated Method of Moments estimator $\{\hat{\theta}_T^S(\pi)\}$ is a sequence of random vectors such that:*

$$\hat{\theta}_T^S(\pi) = Argmin_\theta \bar{f}_T^S(\theta, \pi)' \hat{W}_T(\pi) \bar{f}_T^S(\theta, \pi)$$

for all $\pi \in \Pi$, where

$$\begin{aligned} \bar{f}_T^S(\theta, \pi) \quad &= \quad \frac{1}{T} \left[\begin{array}{c} \sum_{t=1}^{[T\pi]} \left(m(z_t) - \frac{1}{S} \sum_{s=1}^S m(z_t^s, \beta_1, \delta) \right) \\ 0 \end{array} \right] + \\ & \quad\quad \frac{1}{T} \left[\begin{array}{c} 0 \\ \sum_{t=[T\pi]+1}^{T} \left(m(z_t) - \frac{1}{S} \sum_{s=1}^S m(z_t^s, \beta_2, \delta) \right) \end{array} \right] \end{aligned}$$

where $\hat{W}_T(\pi)$ is a random positive definite symmetric $2q \times 2q$ matrix.

The partial-sample optimal weighting matrix is defined as the inverse of Ω^*, where $\Omega^* = (1 + \frac{1}{S})\Omega$ and

$$\begin{aligned} \Omega(\pi) \quad &= \quad \lim_{T \to \infty} Var \left(\frac{1}{\sqrt{T}} \left[\begin{array}{c} \sum_{t=1}^{[T\pi]} (m(z_t) - Em(z_t)) \\ 0 \end{array} \right] \right. \\ & \quad\quad \left. + \frac{1}{\sqrt{T}} \left[\begin{array}{c} 0 \\ \sum_{t=[T\pi]+1}^{T} (m(z_t) - Em(z_t)) \end{array} \right] \right). \end{aligned}$$

Before turning our attention to the regularity conditions we need to elaborate briefly on how we perform simulations with a structural break at $[T\pi]$. We have two choices. The first consists of simulating S draws of

[3] The optimal weighting can be estimated consistently using methods developed by Gallant (1987), Andrews and Monahan (1992), Newey and West (1994), among several others.

data generated for $t = 1, \dots, T\pi$ and $t = [T\pi] + 1, \dots, T$ conditional on particular values of β_1, β_2 and δ. Alternatively, instead of generating S data sets of size T with a breakpoint at π it is also possible to consider TS draws with a single breakpoint at $[TS\pi]$. In remainder of this section we discuss a set of regularity conditions we need to impose to obtain weak convergence of partial sample SMM estimators to a function of Brownian motions. To streamline the presentation we only summarize the assumptions and provide a detailed description of them in Appendix A.

Duffie and Singleton (1993) mention two reasons why the GMM regularity conditions cannot be used to show the convergence of SMM estimators. First, the initial conditions of the time series processes are in general not drawn from their stationary distribution which results in a local nonstationarity of the simulated process. Second, the first moment continuity assumption used by Hansen (1982) and Andrews (1987) is not sufficient to establish the uniform convergence of the sample criterion function to its population equivalent. Indeed, this continuity is not valid for data generated by simulations which depend on the unknown parameter vector. It will be assumed instead that observed and simulated series are near epoch dependent, a condition also used by Andrews (1993) in the context of tests for structural change which can accomodate local nonstationarity. This is covered by Assumption A.2. Yet, we also need to impose a number of regularity conditions which do not appear in Andrews (1993) or the more recent work on structural change tests for GMM estimators. In particular we need to impose a global Lipschitz condition on moment conditions and their total derivatives w.r.t. the parameter vector in order to obtain uniform convergence. The global Lipschitz condition was also used by Duffie and Singleton (1993), and results in modifications of Andrews (1993) to establish the asymptotic distribution of structural change tests. This condition appears in Assumption A.3 and A.5 and is a sufficient condition for stochastic equicontinuity of a triangular array of a random vector. Stochastic equicontinuity (strong or in probability) is a necessary and sufficient condition to go from pointwise to uniform convergence (strong or in probability).[4] Finally, we also impose standard identification assumptions and restrict the parameter space to be closed and bounded.

[4] See Andrews (1992).

2.3 Asymptotic Properties of Partial Sample SMM Estimators

We present two theorems in this section which establish the large sample properties of the partial sample SMM estimators under the null hypothesis. The first theorem establishes consistency whereas the second characterizes the asymptotic distribution.

Theorem 1 *Under Assumptions A.1 to A.3, for a fixed S, the partial sample SMM estimators $\hat{\theta}_T^S(\pi)$ satisfies $sup_{\pi\in\Pi}\|\hat{\theta}_T^S(\pi)-\theta_0\| \xrightarrow{p} 0$ for some θ_0 in the interior of Θ.*

Proof: See Appendix B.

For the case of one simulation of TS values, we define the following matrices:[5]

$$F = \lim_{T\to\infty}\frac{1}{TS}\sum_{t=1}^{TS} E\partial m(z_t^s,\theta_0)/\partial\theta' \in R^{q\times p},$$

$$F^\beta = \lim_{T\to\infty}\frac{1}{TS}\sum_{t=1}^{TS} E\partial m(z_t^s,\beta_0,\delta_0)/\partial\beta' \in R^{q\times r},$$

$$F^\delta = \lim_{T\to\infty}\frac{1}{TS}\sum_{t=1}^{TS} E\partial m(z_t^s,\beta_0,\delta_0)/\partial\delta' \in R^{q\times s},$$

$$F(\pi) = \left[\begin{array}{ccc} \pi F^\beta & 0 & \pi F^\delta \\ 0 & (1-\pi)F^\beta & (1-\pi)F^\delta \end{array}\right] \in R^{2q\times(2r+s)}.$$

We denote $\{B_1(\pi) : \pi \in [0,1]\}$ and $\{B_2(\pi) : \pi \in [0,1]\}$ as two q-dimensional vectors of mutually independent Brownian motion on $[0,1]$ and define

$$G(S,\pi) = \left(\begin{array}{c} \Omega^{1/2}\left[B_1(\pi) - \frac{1}{\sqrt{S}}B_2(\pi)\right] \\ \Omega^{1/2}\left[(B_1(1)-B_1(\pi)) - \frac{1}{\sqrt{S}}(B_2(1)-B_2(\pi))\right] \end{array}\right)$$

[5] In a similar manner, we can define the same matrices for the case of S simulations of T values.

Theorem 2 *Under Assumptions A.1 to A.5, for a fixed S, every sequence of partial sample SMM estimators* $\{\hat{\theta}_T^S(\cdot) : T \geq 1\}$ *satisfies*

$$\sqrt{T}(\hat{\theta}_T^S(\cdot) - \theta_0) \Rightarrow \left(F(\cdot)'W(\cdot)F(\cdot)\right)^{-1} F(\cdot)'W(\cdot)G(S,\cdot)$$

as a process indexed by $\pi \in \Pi$*, provided* Π *has a closure in* $(0,1)$.

Proof: See Appendix B.

It should be noted that the asymptotic distribution of the partial sample SMM estimators depends on a nuisance parameter, namely the number of simulations S. As $S \to \infty$, the asymptotic distribution coincides with the GMM case discussed in Andrews (1993). However, in practice, the number of simulations S is fixed and usually low. Fortunately, although the asymptotic distribution of the partial sample SMM estimators depends on S, we will see that the asymptotic distribution of the test statistics for structural change is independent of S, and hence is nuisance parameter free, regardless whether the breakpoint is known or unknown.

3 Tests for parameter constancy

In this section we introduce several tests for structural change and establish their asymptotic distribution. We present Wald, Lagrange multiplier and likelihood ratio-type tests for parameter constancy. Predictive tests will be discussed in the next section. The null hypothesis of our tests appeared in (2.1). In this section, we consider only the test statistics based on the optimal weighting matrix.[6] The first test statistic is the Wald statistic which is given by:

$$Wald_T^S(\pi) = T\left(\hat{\beta}_{1T}^S(\pi) - \hat{\beta}_{2T}^S(\pi)\right)' (\hat{V}_\Omega(\pi))^{-1} \left(\hat{\beta}_{1T}^S(\pi) - \hat{\beta}_{2T}^S(\pi)\right),$$

where $(\hat{V}_\Omega(\pi)) = \left(\hat{V}_1(\pi)/\pi + \hat{V}_2(\pi)/(1-\pi)\right)$ and $\hat{V}_j(\pi) = \left(\hat{F}_j^\beta(\pi)'\hat{\Omega}_j^{*-1}(\pi)\hat{F}_j^\beta(\pi)\right)^{-1}$ for $j = 1, 2$. When $j = 1$, the estimators of F^β and Ω^* are obtained with data from the first part of the sample $t = 1, \cdots, [T\pi]$ while for $j = 2$, the estimators are obtained with data from the remainder of the sample $t = T\pi + 1, \cdots, T$. The Lagrange Multiplier statistic does not involve estimators obtained from

[6]The result that the asymptotic distribution of the tests does not depend on S critically depends on the use of the optimal weighting matrix.

subsamples, rather it involves parameter estimates over the entire sample. More precisely, the $LM_T^S(\pi)$ statistic is based on the first order conditions for the partial sample SMM estimators evaluated at the full sample estimator:

$$LM_T^S(\pi) = c_T^S(\pi)' \left(\hat{V}_1(\pi)/\pi + \hat{V}_2(\pi)/(1-\pi)\right)^{-1} c_T^S(\pi)$$

where

$$c_T^S(\pi) = L \begin{bmatrix} \frac{1}{\pi}\hat{G}_1(\pi) & 0 \\ 0 & \frac{1}{(1-\pi)}\hat{G}_2(\pi) \end{bmatrix} \sqrt{T}\bar{f}_T^S(\tilde{\theta}_T^S, \pi)$$

$L = [I_p,\ -I_p]$, and $\hat{G}_j(\pi) = \left((\hat{F}_j^\beta(\pi))'\hat{\Omega}_j^{*-1}(\pi)\hat{F}_j^\beta(\pi)\right)^{-1}(\hat{F}_j^\beta(\pi))'\Omega_j^{*-1}(\pi)$. $\hat{F}_j^\beta(\pi)$ and $\hat{\Omega}_j^{*-1}(\pi)$ are respectively matrices evaluated at the restricted SMM estimator over the first and the second part of the sample. Andrews (1993) shows that the $LM_T^S(\pi)$ simplifies to:

$$\frac{T}{\pi(1-\pi)} f_{T\pi}^S(\tilde{\theta}_T^S)'\hat{\Omega}^{*-1}\hat{F}^\beta \left[(\hat{F}^\beta)'\hat{\Omega}^{*-1}\hat{F}^\beta\right]^{-1} (\hat{F}^\beta)'\hat{\Omega}^{*-1} f_{T\pi}^S(\tilde{\theta}_T^S)$$

where

$$f_{T\pi}^S(\tilde{\theta}_T^S) = \frac{1}{T}\sum_{t=1}^{[T\pi]} m(z_{Tt}) - \frac{1}{TS}\sum_{t=1}^{[TS\pi]} m(z_{Tt}^s, \tilde{\beta}, \tilde{\delta})$$

or

$$f_{T\pi}^S(\tilde{\theta}_T^S) = \frac{1}{T}\sum_{t=1}^{[T\pi]} \left(m(z_{Tt}) - \frac{1}{S}\sum_{s=1}^{S} m(z_{Tt}^s, \tilde{\beta}, \tilde{\delta})\right).$$

The LR-type statistic is defined as the difference between the objective function for the partial sample SMM evaluated at the full sample estimator and at the partial sample estimators:

$$\begin{aligned} LR_T^S(\pi) \quad = \quad & T\left(\bar{f}_T^S(\tilde{\theta}_T^S, \pi)'\hat{\Omega}_T^{*-1}(\pi)\bar{f}_T^S(\tilde{\theta}_T^S, \pi)\right. \\ & \left. - \ \bar{f}_T^S(\hat{\theta}_T^S(\pi), \pi)'\hat{\Omega}_T^{*-1}(\pi)\bar{f}_T^S(\hat{\theta}_T^S(\pi), \pi)\right). \end{aligned}$$

We state now the main theorem which establishes the asymptotic distribution of the Wald, LM and LR-type test statistics.

Theorem 3 *Under the null hypothesis H_0 in (2.1) and Assumptions A.1 to A.5, the following processes indexed by π for a given set Π whose closure lies in (0,1) satisfy:*

$$Wald_T^S(\pi) \Rightarrow Q_r(\pi), LM_T^S(\pi) \Rightarrow Q_r(\pi), LR_T^S(\pi) \Rightarrow Q_r(\pi),$$

with

$$Q_r(\pi) = \frac{BB_r(\pi)' BB_r(\pi)}{\pi(1-\pi)},$$

where $BB_r(\pi) = B_r(\pi) - \pi B_r(1)$ is a Brownian bridge and B_r is r-vector of independent Brownian motions.

Proof: See Appendix B.

The result in Theorem 3 tells us that the asymptotic distributions of the Wald, LR-type and LM statistics are the same as those obtained by Andrews (1993) for the GMM estimator. This implies that we can rely on the critical values computed by Andrews for instance for the socalled Supremum statistics which defined by the supremum over all breakpoints $\pi \in \Pi$ of $Wald_T^S(\pi)$, $LM_T^S(\pi)$ or $LR_T^S(\pi)$.

4 Optimal Tests

Several papers have explored socalled *optimal* tests since the original work of Andrews (1993) on testing for structural change with unknown breakpoint in the context of GMM estimators. See in particular, Andrews and Ploberger (1994), Sowell (1996a, b), Guay (1996) and Hall and Sen (1996). In this section we explore such optimal tests for SMM estimators. Following Sowell (1996b) we consider a sequence of local of alternatives based on the moment conditions:

Assumption 4.1 *Sequence of Local Alternatives:*

$$Ef_t(\theta_0) = h(\eta, s, \frac{t}{T})/\sqrt{T} \tag{4.1}$$

where $h(\eta, s, \pi)$, for $\pi \in [0,1]$, is a q-dimensional function that can be expressed as uniform limit of step functions, $\eta \in R^i$, $s \in R^j$ such that $0 < s_1 < s_2 < \ldots < s_j < 1$ and θ_0 is in the interior of Θ.

The sequence of local alternatives in (4.1) is expressed in terms of violations of moment conditions instead of parameters as in (2.1). This brings us to the subject of predictive tests for structural cange considered by Ghysels, Guay and Hall (1997) for the case of GMM estimators. They consider a single breakpoint, which amounts to the following null:

$$H_0 : E[m(z_t) - m(z_t^s, \theta_0)] = 0 \qquad \forall t = 1, \ldots, T$$

and alternative:

$$E[m(z_t) - m(z_t^s, \theta_0)] = \begin{cases} 0 & \forall t = 1, \ldots, [\pi T] \\ T^{-\frac{1}{2}}\mu_2 & \forall t = [\pi T] + 1, ..., T \end{cases}$$

with $\mu_2 \neq 0$. The predictive test is based on evaluating the sample moment conditions for the subsample $t = [\pi T] + 1, ..., T$ using $\hat{\theta}_{1T}^S(\pi)$, i.e. the parameter estimates from the first subsample. The test statistic is defined as:

$$Pred_T^S(\pi) = f_{T(1-\pi)}^S(\hat{\theta}_{1T}^S(\pi))' \hat{V}_{PR}^{-1}(\pi) f_{T(1-\pi)}^S(\hat{\theta}_{1T}^S(\pi))$$

where $\hat{V}_{PR}(\pi)$ is a covariance matrix defined in Ghysels, Guay and Hall (1997) and:

$$f_{T(1-\pi)}^S(\hat{\theta}_{1T}^S(\pi)) = \frac{1}{T} \sum_{t=[T\pi]+1}^{T} m(z_t) - \frac{1}{TS} \sum_{t=[TS\pi]+1}^{TS} m(z_t^s, \hat{\beta}_{1T}^S(\pi), \hat{\delta}_{1T}^S(\pi))$$

or its equivalent using S simulations of samples of size T. To proceed with the discussion we first extend Theorem 1 of Sowell (1996b) to the class of SMM estimators, namely:

Theorem 4 *Under Assumptions A.1 to A.5 and Assumption 4.1, then*

$$\begin{aligned} \sqrt{T} f_{T\pi}^S(\tilde{\theta}_T^S) \quad \Rightarrow \quad & \Omega^{1/2} B_1(\pi) - \frac{1}{\sqrt{S}} \Omega^{1/2} B_2(\pi) + H(\pi) - \\ & \pi F (F'WF)^{-1} F'W \left[\Omega^{1/2} B_1(1) - \frac{1}{\sqrt{S}} \Omega^{1/2} B_2(1) + H(1) \right]. \end{aligned}$$

where $H(\pi) = \int_0^\pi h(\eta, s, u) du$ *and* $B_1(\pi)$ *and* $B_2(\pi)$ *are two* q*-dimensional vectors of mutually independent Brownian motion and* $f_{T\pi}^S(\cdot)$ *is defined in Section 3.*

Proof: See Appendix B.

Sowell's asymptotic optimal tests are a generalization of the Neyman-Pearson approach to the case of two measures. The most powerful test is given by the Radon-Nikodym derivative of the probability measure implied by the local alternative with respect to the probability measure implied by the null hypothesis. These two probability measures are implied by the stochastic differential equation of the limiting stochastic processes derived in the next Corollary under the null and the alternative. In the remaining of the section, W_T will be an estimator of the inverse of the optimal weigthing matrix Ω^*. The next Corollary is the equivalent of Sowell's Corollary 1.

Corollary 4.1 *Under Assumptions A.1 to A.5 and the null hypothesis, there exists an orthonormal matrix C such that*

$$C\sqrt{T}W_T^{1/2} f_{T\pi}^S(\tilde{\theta}_T^S) \Rightarrow \begin{bmatrix} BB_p(\pi) \\ B_{q-p}(\pi) \end{bmatrix}.$$

and under the alternative in equation 4.1:

$$C\sqrt{T}W_T^{1/2} f_{T\pi}^S(\tilde{\theta}_T^S) \Rightarrow \begin{bmatrix} BB_p(\pi) + C_1\Omega^{*-1/2}\left(H(\pi) - \pi H(1)\right) \\ B_{q-p}(\pi) + C_2\Omega^{*-1/2}H(\pi) \end{bmatrix}.$$

where $BB_p(\pi)$ is a p-vector of standard Brownian bridge and $B_{q-p}(\pi)$ is a $(q-p)$-vector of standard Brownian motion and $C'\Lambda C[= \Omega^{*-1/2}F(F'\Omega^{*-1}F)^{-1}F'\Omega^{*-1/2}$ where

$$\Lambda = \begin{bmatrix} I_p & 0_{p\times(q-p)} \\ 0_{(q-p)\times p} & 0_{(q-p)\times(q-p)} \end{bmatrix}.$$

and C_1 is the matrix of the first p rows of C and C_2 the last $q-p$ rows of C.

Proof: See Appendix B.

The limiting stochastic processes in Corollary 4.1 are equivalent to the limiting stochastic processes for the GMM estimator. Corollary 4.1 shows that under the null hypothesis the limiting continuous stochastic processes are linear combinations of p Brownian bridges, one for each parameter estimated and spanning the space of *identifying restrictions*, and $q-p$ Brownian motions, spanning the space of *overidentifying restrictions*. The results in Theorem 4 and Corollary 4.1 imply that all the issues regarding the design of optimal tests raised in the context of GMM estimators readily apply to simulation-based SMM procedures. Following Hall (1997) we can consider the generic null and alternatives for the case of a single breakpoint:

$$H_0^I(\pi) = \begin{cases} P_F'\Omega^{*-1/2}E[m(z_t) - m(z_t^s,\theta_0)] = 0 & \forall t = 1,\ldots,[\pi T] \\ P_F\Omega^{*-1/2}E[m(z_t) - m(z_t^s,\theta_0)] = 0 & \forall t = [\pi T]+1,\ldots,T \end{cases}$$

which seperates the identifying restrictions across the two subsamples where $P_F = \Omega^{*-1/2}F(F'\Omega^{*-1}F)^{-1}F'\Omega^{*-1/2}$. Whereas the overidentifying restrictions are stable if they hold before and after the breakpoint. This is formally stated as $H_0^O(\pi) = H_0^{O1}(\pi) \cap H_0^{O2}(\pi)$ with:

$$H_0^{O1}(\pi) : (I_q - P_{F1}(\pi))\Omega_1^{*-1/2}(\pi)E[m(z_t) - m(z_t^s,\theta_0)] = 0 \qquad \forall t = 1,\ldots,[\pi T]$$
$$H_0^{O2}(\pi) : (I_q - P_{F2}(\pi))\Omega_2^{*-1/2}(\pi)E[m(z_t) - m(z_t^s,\theta_0)] = 0 \quad \forall t = [\pi T]+1,\ldots,T$$

where $P_{Fi}(\pi)$ and $\Omega_i^*(\pi)$ are the subsample equivalents of P_F and Ω^* respectively for $i = 1$,2. By projection decomposition appearing in Corollary 4.1 it is clear that instability must be reflected in a violation of at least one of the three hypotheses: $H_0^I(\pi), H_{01}^O(\pi)$ or $H_{02}^O(\pi)$. Various tests can be constructed with local power properties against any particular one of these three null hypotheses (and typically no power against the others). The Wald, LM and LR-type tests discussed in the previous section are based on the null $H_0^I(\pi)$, but have no power against violations of $H_{01}^O(\pi)$ or $H_{02}^O(\pi)$. Likewise, the predicitive tests have power against $H_0^I(\pi)$ and $H_{02}^O(\pi)$. Obviously, one can construct tests for stability of the identifying and overidentifying restrictions seperately (see Guay (1996), Hall and Sen (1996) and Sowell (1996b)). Moreover, following Andrews and Ploberger (1994) one can refine such tests for socalled *distant* and *close* alternatives. In addition one can further fine tune their setup with a priori information about breakpoints using a Bayesian interpretation to weighting densities defined on the set Π of possible breakpoints (see Andrews (1994) and Andrews and Ploberger (1994) for further discussion). So far we have not formally shown that the predictive test for SMM has the same distribution as that established for GMM nor any of the other tests suggested by Andrews and Ploberger (1994), Sowell (1996 a,b), Guay (1996), Hall and Sen (1996), or any others. However Corollary 4.1 combined with the continuous mapping theorem allows us from now on to establish the asymptotic distribution of any statistic for SMM based on the GMM results.

5 Finite Sample Properties

The results in sections 3 and 4 imply that the asymptotic distribution of simulation-based tests for structural change under the null as well as under a sequence of local alternatives is independent of S, the number of simulations. Theorem 3 covers the distribution under the null whereas the result in Corollary 4.1 shows that the local asymptotic power of the large class of Wald, LM, LR and Predictive type and optimal tests for structural change is independent of the number of simulations. Yet, one might expect that in finite samples both the size and power are affected by the number of simulations. We conduct a Monte Carlo study to appraise the extend to which the finite sample size and power depend on S. The setup we consider is the following:

$$y_t = \mu \mathbf{1}_{(t \geq \pi T)} + \varepsilon_t \tag{5.2}$$

where ε_t is i.i.d. $N(0, 1)$ and $\mathbf{1}_{(t \geq \pi T)}$ is one for $t \geq \pi T$ and zero otherwise.

Hence, we examine a shift in the mean with unknown breakpoint. Two sample sizes $T = 50$ and $T = 100$ are investigated with breaks at $\pi = .25, .50$ and $.75$ and values of $\mu = 0, 0.5$ and 1. Obviously, with $\mu = 0$ there is no break, which yields the finite sample size properties of the tests. The $SupLM$ statistic is taken as a representative case with $S = 1, 2, 5$ and 10. The Monte Carlo study is based on 1000 replications. The smallest of the two sample sizes, $T = 50$ is reported in Table 1, while $T = 100$ appears in Table 2. The top panel in both tables cover size with $\mu = 0$. A common pattern emerges from both Tables 1 and 2, namely with $S = 1$ and 2 there are clearly serious size distortions whereas with $S = 5$ and 10 they become less important and the difference between $S = 5$ and $S = 10$ are only minor. The power properties reveal a fairly similar pattern. This implies that chosing at least $S = 5$ appears adequate to avoid serious size properties and to have desirable power properties. We notice in fact that in this rather simple setup, even in modest sample sizes of $T = 50$ we have power of up to 77%. Another result of interest emerging, one not particularly surprising, is the symmetry of the results for $\pi = .25$ and $\pi = .75$. Finally, the maximal power is attained with $\pi = .50$ as would be expected.

6 Conclusion

In this paper we examined tests for structural change in the context of simulated method of moments estimators. We found that the asymptotic distribution for such tests coincide with their GMM counterpart regardless of the number of simulations and therefore the regardless of the simulation bias. Obviously, there are limitations to our results as well as unresolved challenges. Regarding the limitations we should mention the finite sample performance of the tests as noted in section 5. There is a fair amount of Monte Carlo evidence regarding GMM tests for structural change. In finite samples the simulation uncertainty is one more factor that may deteriorate the finite sample performance. Yet, in general the tests which we discussed have good size and power properties for structural change in the "middle" of the sample while the power properties deteriorate against structural change at the very beginning or the very end of a sample. Dufour, Ghysels and Hall (1994) proposed tests, with null and alternative similar to predictive tests, designed to handle such situations. Unfortunately, the SMM tests we discussed in this paper do not extend to situations considered by Dufour, Ghysels and Hall where one sample is large (the estimation sample) and a second sample is small (the prediction sample) even containing only one observation. These tests depend on the nuissance parameter S, though one could let $S \to \infty$, and obtain the same results as in Dufour, Ghysels and Hall. Since the second sample is small generating a large number of simulations would be practically feasible.

The next generation of estimators are simulation-based procedure involving two models, an auxiliary model and a model of interest. Such procedures, discussed by Gouriéroux, Monfort and Renault (1993) and Gallant and Tauchen (1996) add some nontrivial complications regarding testing since they involve parameters of two models. We leave testing for structural change in such settings for further research.

Appendices

A Detailed Description of Regularity Conditions

Assumption A.1 *The parameter space Θ is a closed and bounded subset of R^p.*

Assumption A.2 *The observed and simulated processes satisfy:*

- $\{z_{Tt} : t \leq T, T \geq 1\}$ *is a triangular array of Z-values random vectors that is L^0-near epoch dependent on a strong mixing base $\{V_{Tt} : t = ..., 0, 1, ...; T \geq 1\}$, where Z is a Borel subset of R^k.*[7]

- *For all $\theta \in \Theta$, $\{z^s_{Tt} : t \leq T, T \geq 1\}$ is a triangular array of Z-values random vectors that is L^0-near epoch dependent on a strong mixing base $\{V_{Tt} : t = ..., 0, 1, ...; T \geq 1\}$, where Z is a Borel subset of R^k.*

Assumption A.3 *The set of moment conditions satisfies:*

- *For some $r > 2$, $\{m(z_{Tt}) : t \leq T, T \geq 1\}$ is a triangular array of R^q-valued random vector that is L^2-near epoch dependent of size -1/2 on a strong mixing base $\{V_{Tt} : t = ..., 0, 1, ...; T \geq 1\}$ of size -r/(r-2), $\sup_{t \leq T, T \geq 1} E\|m(z_{Tt})\|^r < \infty$.*

- *For some $r > 2$, $\{m(z^s_{Tt}, \theta) : t \leq T, T \geq 1\}$ is a triangular array of R^q-valued random vector that is L^2-near epoch dependent of size -1/2 on a strong mixing base $\{V_{Tt} : t = ..., 0, 1, ...; T \geq 1\}$ of size -r/(r-2), $\sup_{t \leq T, T \geq 1} E\|m(z^s_{Tt}, \theta)\|^r < \infty$.*

- $\overline{\lim}_{T \to \infty} (1/T) \sum_{t=1}^{T} E \sup_{\theta \in \Theta} |m(z^s_{Tt}, \theta)|^{1+\varepsilon} < \infty$ *for some $\varepsilon > 0$.*

- *For all θ,$\theta' \in \Theta$ and $t \geq T$, there is a sequence $\{B_{Tt}\}$ not depending on θ with $\frac{1}{T} \sum_{t=1}^{T} EB_{Tt} = O_p(1)$ such that $\|m(z^s_{Tt}, \theta) - m(z^s_{Tt}, \theta')\| \leq B_{Tt} \|\theta - \theta'\|$*

- $\sup_{\pi \in \Pi} \|\hat{W}_T(\pi) - W(\pi)\| \overset{p}{\to} 0$ *for some $2q \times 2q$ matrices $W(\pi)$ for which $\sup_{\pi \in \Pi} \|W(\pi)\| < \infty$.*

[7] For a definition of L^p-near epoch dependence see Andrews (1993, p.830), Davidson (1994) or Gallant and White (1988).

- $\lim_{T\to\infty}(1/T)\sum_{t=1}^{[T\pi]}\Big((1/S)\sum_{s=1}^{S} Em(z^s_{Tt},\beta,\delta)\Big)$ *exists uniformly over* $(\beta,\delta,\pi)\in B\times\Delta\times\Pi$ *and equals* $\pi\lim_{T\to\infty}(1/TS)\sum_{t=1}^{TS} Em(z^s_{Tt},\theta)$ *and* $\lim_{T\to\infty}(1/T)\sum_{t=1}^{[T\pi]} Em(z_{Tt})$ *exists uniformly over* $\pi\in\Pi$ *and equals* $\pi\lim_{T\to\infty}(1/T)\sum_{t=1}^{T} Em(z_{Tt})$.

- $Var\Big(\frac{1}{\sqrt{T}}\sum_{t=1}^{T\pi}[m(z_{Tt})-Em(z_{Tt})]\Big)\to\pi\Omega$ *and* $Var\Big(\frac{1}{\sqrt{T}}\sum_{t=1}^{T\pi}[\Big(\frac{1}{\sqrt{S}}\sum_{s=1}^{S} m(z^s_{Tt},\theta)-Em(z^s_{Tt},\theta)\Big)]\Big)\to\pi\Omega$, $\forall\pi\in[0,1]$ *for some positive* $q\times q$ *matrix* Ω.

- $\tilde{m}(z_{Tt})=\tilde{m}(z^s_{Tt},\theta_0)$, *where* $\tilde{m}(z_{Tt})=lim_{T\to\infty}(1/T)\sum_{t=1}^{T} Em(z_{Tt})$ *and* $\tilde{m}(z^s_{Tt},\theta)=lim_{T\to\infty}\frac{1}{TS}\sum_{t=1}^{TS} Em(z^s_{Tt},\theta)$, *and for every neighborhood* $\Theta_0(\subset\Theta)$ *of* θ_0, $\inf_{\pi\in\Pi}\inf_{\theta\in\Theta/\Theta_0} f(\theta,\pi)'W(\pi)f(\theta,\pi)>0$, *where* $f(\theta,\pi)=(\pi(\tilde{m}(z_{Tt})-\tilde{m}(z^s_{Tt},\beta_1,\delta))',(1-\pi)(\tilde{m}(z_{Tt})-\tilde{m}(z^s_{Tt},\beta_2,\delta))')'$.

Assumption A.4 $F(\pi)'W(\pi)F(\pi)$ *is nonsingular* $\forall\pi\in\Pi$ *and has eigenvalues bounded away from zero.*

We define the total derivative as:

$$D_{\theta'}m(z^s_{Tt},\theta)=\frac{d}{d\theta'}m(z^s_{Tt},\theta) \tag{A.1}$$

Assumption A.5 *The total derivative satisfies:*

- *For some* $r>2$, $\{D_{\theta'}m(z^s_{Tt},\theta):t\le T,T\ge 1\}$ *is a triangular array of* R^q*-valued random vector that is* L^2*-near epoch dependent of size -1/2 on a strong mixing base* $\{V_{Tt}:t=...,0,1,...;T\ge 1\}$ *of size -r/(r-2) and* $\sup_{t\le T,T\ge 1}E\|D_{\theta'}m(z^s_{Tt},\theta)\|^r<\infty$.

- $m(z^s_{Tt},\theta)$ *is differentiable in* $(\beta,\delta)\in B_0\times\Delta_0$ $\forall z\in Z$, *where* B_0 *and* Δ_0 *are some neighborhood of* β_0 *and* δ_0 ,

- *For all* θ,$\theta'\in\Theta$ *and* $t\ge T$, *there is a sequence* $\{C_{Tt}\}$ *not depending on* θ *with* $\frac{1}{T}\sum_{t=1}^{T} EC_{Tt}=O_p(1)$ *such that* $\|D_{\theta'}m(z^s_{Tt},\theta)-D_{\theta'}m(z^s_{Tt},\theta')\|\le C_{Tt}\|\theta-\theta'\|$ *and* $\sup_{t\le T,T\ge 1}E\sup_{\theta\in\Theta_0}\|D_{\theta'}m(z^s_{Tt},\theta)\|^{1+\varepsilon}<\infty$ *for some* $\varepsilon>0$.

- $\lim_{T\to\infty}\frac{1}{TS}\sum_{t=1}^{[TS\pi]} ED_{\theta'}m(z^s_{Tt},\theta)$ *exists uniformly over* $\pi\in\Pi$ *equals* πF, $\forall\pi\in\Pi$.

B Proofs of Theorems

We need to use the following lemma for Theorem 1 and 2:

Lemma 1 *Suppose (a) Assumptions A.1 holds, (b) Assumptions A.2 holds, (c) for some* $r > 2$, $h(z_{Tt}^s, \theta)$ *is a triangular array of* R^q*-valued random vectors that is* L^2*-near epoch dependent of size -1/2 on a strong mixing base* $\{V_{Tt} : t = ..., 0, 1, ...; T \geq 1\}$ *of size -r/(r-2) and* $\sup_{t \leq T, T \geq 1} E\|h(z_{Tt}^s, \theta)\|^r < \infty$*, (d) the q-vector* $h(z_{Tt}^s, \theta)$ *follows a global Lipschitz condition in* θ *and*
(e) $\limsup_{T\to\infty} (1/T) \sum_{t=1}^{T} E \sup_{\theta\in\Theta} |h(z_{Tt}^s, \theta)|^{1+\varepsilon} < \infty$ *for some* $\varepsilon > 0$*. Then*

$$\sup_{\theta\in\Theta} \sup_{R\leq T} \left| \frac{1}{T} \sum_{t=1}^{R} [h(z_{Tt}^s, \theta) - Eh(z_{Tt}^s, \theta)] \right| \xrightarrow{p} 0$$

Proof of Lemma 1:

We define $G_{Tt}(\theta) = \sup_{R\leq T} |\frac{1}{T} \sum_{t=1}^{R} (h(z_{Tt}^s, \theta) - Eh(z_{Tt}^s, \theta))|$. By Theorem 21.9 of Davidson (1994),[8]

$$\sup_{\theta\in\Theta} G_{Tt}(\theta) \xrightarrow{p} 0$$

if and only if i) the pointwise convergence of $G_{Tt}(\theta)$ for $\theta \in \Theta_0$, where Θ_0 is a dense subset of Θ and ii) $\{G_{Tt}\}$ is stochastically equicontinuous. For i), by Assumption (a), Θ_0 is a dense subset of Θ. By Assumption (c), $h(z_{tT}^s, \cdot)$ is L^2-near epoch dependent, which implies that this process is L^2-approximable.[9] A L^2-approximable process is L^0-approximable. By Assumption (e), the approximators can be taken to be conditional means $\{Eh(z_{Tt}^s, \theta)|V_{Tt-r}, \cdots, V_{Tt+r} : t \leq T, T \geq 1, r \geq 1\}$. Thus $h(z_{Tt}^s, \cdot)$ is L^0-near epoch dependent. Using Lemma A2 of Andrews (1993) with X_{Tt} equal to an element of the q-dimensional vector $h(z_{Tt}^s, \theta) - Eh(z_{Tt}^s, \theta)$, we obtain i). For ii), by theorem 21.11 from Davidson (1994), global Lipschitz condition (Assumption (d)) is sufficient to obtain that $\{G_{Tt}\}$ is stochastically equicontinuous.

Proof of Theorem 1:

First, we need to show that

$$\sup_{\pi\in\Pi,\theta\in\Theta} |\bar{f}_T^S(\theta, \pi)' \hat{W}_T(\pi) \bar{f}_T^S(\theta, \pi) - f(\theta, \pi)' W(\pi) f(\theta, \pi)| \xrightarrow{p} 0$$

[8] The presentation of Davidson is drawn mainly from Andrews (1992). Newey (1991) provides also conditions for uniform convergence based on stochastic equicontinuity. However, Newey considers the more restrictive assumption that the parameter space is compact instead the weaker condition that the parameter is bounded.

[9] The concept of L^p-approximable process is due to Pötscher and Prucha (1991). See also Davidson (1994) for a presentation of L^p-approximable process.

Using Assumption A.3, the expression above holds if

$$\sup_{\pi\in\Pi}\sup_{\theta\in\Theta}|\bar{f}_T^S(\theta,\pi)-f(\theta,\pi)|\xrightarrow{p}0$$

Using $\sum_{[T\pi]+1}^{T}=\sum_{t=1}^{T}-\sum_{t=1}^{[T\pi]}$, the expression above holds if

$$\sup_{\theta\in\Theta}\sup_{[T\pi_1]\leq R\leq T}\left|\tfrac{1}{T}\sum_{t=1}^{R}\left[\left(m(z_{Tt})-\tfrac{1}{S}\sum_{s=1}^{S}m(z_{Tt}^s,\theta)\right)\right.\right. \tag{B.1}$$
$$\left.\left.-\ (Em(z_{Tt})-Em(z_{Tt}^s,\theta))\right]\right|\xrightarrow{p}0$$

where $\pi_1=\inf\{\pi:\pi\in\Pi\}>0$ and

$$\sup_{\theta\in\Theta}\sup_{\pi\in\Pi}\left|\tfrac{1}{T}\sum_{t=1}^{[T\pi]}\left[(Em(z_{Tt})-Em(z_{Tt}^s,\theta))\right.\right. \tag{B.2}$$
$$\left.\left.-\ (\tilde{m}(z_{Tt})-\tilde{m}(z_{Tt}^s,\theta))\right]\right|\xrightarrow{p}0.$$

For B.1, by the triangle inequality,

$$\sup_{R\leq T}\left|\frac{1}{T}\sum_{t=1}^{R}\left[\left(m(z_{Tt})-\frac{1}{S}\sum_{s=1}^{S}m(z_{Tt}^s,\theta)\right)-(Em(z_{Tt})-Em(z_{Tt}^s,\theta))\right]\right|$$
$$\leq\sup_{R\leq T}\left|\frac{1}{T}\sum_{t=1}^{R}(m(z_{Tt})-Em(z_{Tt}))\right|+\left|\frac{1}{T}\sum_{t=1}^{R}\left(\frac{1}{S}\sum_{s=1}^{S}m(z_{Tt}^s,\theta)-Em(z_{Tt}^s,\theta)\right)\right|.$$

By Assumption A.2, the first term on the right-hand side of the expression above converges to zero in probability. By using Lemma 1 for $m(z_{Tt}^s,\cdot)$ with Assumptions A.1 through A.3, we establish the uniform WLLN for $m(z_{Tt}^s,\cdot)$. Thus, the second term of the expression above also converges to zero in probability. Then, equation B.1 holds. Equation B.2 holds by Assumption A.3 and the triangle inequality.

We now apply Lemma A1 of Andrews (1993) with $Q_T(\theta,\pi)=\bar{f}_T^S(\theta,\pi)'\hat{W}_T(\pi)\bar{f}_T^S(\theta,\pi)$, $Q(\theta,\pi)=\bar{f}(\theta,\pi)'W(\pi)\bar{f}(\theta,\pi)$, we then obtain that $\sup_{\pi\in\Pi}\|\hat{\theta}(\pi)-\theta_0\|\xrightarrow{p}0$.

Proof of Theorem 2:

We have

$$\left(\frac{\partial}{\partial\theta'}\bar{f}_T^S(\hat{\theta}_T^S(\pi),\pi)\right)'\hat{W}_T(\pi)\sqrt{T}\bar{f}_T^S(\hat{\theta}_T^S(\pi),\pi)=o_p(1) \tag{B.3}$$

By Taylor's Theorem, in vector notation,

$$\sqrt{T}\bar{f}_T^S(\hat{\theta}_T^S(\pi),\pi)=\sqrt{T}\bar{f}_T^S(\theta_0,\pi) \tag{B.4}$$
$$+\left(\tfrac{\partial}{\partial\theta'}\bar{f}_T^S(\hat{\theta}_T^S(\pi),\pi)\right)\sqrt{T}(\hat{\theta}_T^S(\pi)-\theta_0).$$

where $\bar{\theta}^S_T(\pi)' = [\bar{\theta}^{S,(1)}_T(\pi)\ldots\bar{\theta}^{S,(p)}_T(\pi)]$ and $\bar{\theta}^{S,(k)}_T(\pi) = \lambda^{(k)}\theta^{(k)}_0 + (1-\lambda^{(k)})\hat{\theta}^{S,(k)}_T(\pi)$ for some $0 \leq \lambda^{(k)} \leq 1$ and $k = 1,\ldots,p$. Since $\hat{\theta}^S_T(\pi)$ is consistent for θ_0 (see Theorem 1 and by the consistency of the full sample estimator), $\bar{\theta}^S_T(\pi) \xrightarrow{p} \theta_0$.

We have to show that

$$\sup_{\pi\in\Pi}\left\|\frac{\partial}{\partial\theta'}\bar{f}^S_T(\bar{\theta}^S_T(\pi),\pi) - F(\pi)\right\| \xrightarrow{p} 0 \tag{B.5}$$

whenever $\bar{\theta}^S_T(\pi)$ satisfies $\sup_{\pi\in\Pi}\|\bar{\theta}^S_T(\pi)-\theta_0\| \xrightarrow{p} 0$. To establish this, we can write

$$\begin{aligned}\sup_{\pi\in\Pi}\left\|\frac{\partial}{\partial\theta'}\bar{f}^S_T(x_t,\bar{\theta}^S_T(\pi)) - F(\pi)\right\| \leq \\ \sup_{\pi\in\Pi}\left\|\frac{\partial}{\partial\theta'}\bar{f}^S_T(\bar{\theta}^S_T(\pi),\pi) - E\frac{\partial}{\partial\theta'}\bar{f}^S_T(\theta,\pi)\Big|_{\theta=\bar{\theta}^S_T(\pi)}\right\| \\ + \sup_{\pi\in\Pi}\left\|E\frac{\partial}{\partial\theta'}\bar{f}^S_T(\theta,\pi)\Big|_{\theta=\bar{\theta}^S_T(\pi)} - E\frac{\partial}{\partial\theta'}\bar{f}^S_T(\theta_0,\pi)\right\| \\ + \sup_{\pi\in\Pi}\left\|E\frac{\partial}{\partial\theta'}\bar{f}^S_T(\theta_0,\pi) - F(\pi)\right\|.\end{aligned} \tag{B.6}$$

For the first term of equation (B.6), we need to show a WLLN for $\partial\bar{f}^S_T(\theta,\pi)/\partial\theta'$. Using Lemma 1 for $D_{\theta'}m(z^s_{Tt},\theta)$ combined with Assumptions A.1, A.2, A.5, we obtain a uniform WLLN for $D_{\theta'}m(z^s_{Tt},\theta)$. Hence, the first term of the expression above converges to zero in probability.

For the second term of equation (B.6), we have

$$\begin{aligned}\sup_{\pi\in\Pi}\left\|E\frac{\partial}{\partial\theta'}\bar{f}^S_T(\theta,\pi)\Big|_{\theta=\bar{\theta}^S_T(\pi)} - E\frac{\partial}{\partial\theta'}\bar{f}^S_T(\theta_0,\pi)\right\| \leq \\ E\sup_{\pi\in\Pi}\left\|\frac{\partial}{\partial\theta'}\bar{f}^S_T(\theta,\pi)\Big|_{\theta=\bar{\theta}^S_T(\pi)} - \frac{\partial}{\partial\theta'}\bar{f}^S_T(\theta_0,\pi)\right\|\end{aligned}$$

By Taylor's Theorem, in vector notation,

$$\left\|\frac{\partial}{\partial\theta'}\bar{f}^S_T(\theta,\pi)\Big|_{\theta=\bar{\theta}^S_T(\pi)} - \frac{\partial}{\partial\theta'}\bar{f}^S_T(\theta_0,\pi)\right\| \leq \left\|\frac{\partial}{\partial\theta'}\bar{f}^S_T(\ddot{\theta}^S_T,\pi)\right\|\left\|\bar{\theta}^S_T(\pi)-\theta_0\right\|$$

where $\ddot{\theta}^S_T(\pi)' = [\ddot{\theta}^{S,(1)}_T(\pi)\ldots\ddot{\theta}^{S,(p)}_T(\pi)]$ and $\ddot{\theta}^{S,(k)}_T(\pi) = \lambda^{(k)}\theta^{(k)}_0 + (1-\lambda^{(k)})\bar{\theta}^{S,(k)}_T(\pi)$ for some $0 \leq \lambda^{(k)} \leq 1$ and $k = 1,\ldots,p$. Now, using the global Lipschitz condition with

$$C_{Tt} = \left\|D_{\theta'}m(z^s_{Tt},\ddot{\theta}^S_T(\pi))\right\|$$

by Assumption A.5 we have that $\frac{1}{T}\sum_{t=1}^{T} EC_{Tt} = O_p(1)$ for a neighborhood of θ_0. Since $\sup_{\pi\in\Pi}\|\bar{\theta}_T^S(\pi)-\theta_0\| \xrightarrow{p} 0$, we obtain that the second term converges to zero in probability. The third term of equation (B.6) converges to zero in probability by Assumption A.5. Now, we show that

$$\sqrt{T}\bar{f}_T^S(\theta_0,\cdot) \Rightarrow G(S,\cdot) \tag{B.7}$$

We define

$$f_{[T\pi]}^S(\theta,\pi) = \frac{1}{T}\sum_{t=1}^{[T\pi]}\left(m(z_t) - \frac{1}{S}\sum_{s=1}^{S} m(z_t^s,\theta)\right)$$

which is equal, under the null hypothesis, to

$$f_{[T\pi]}^S(\theta,\pi) = \frac{1}{T}\sum_{t=1}^{[T\pi]}[m(z_t) - Em(z_t)] - \frac{1}{TS}\sum_{t=1}^{[TS\pi]}[m(z_t^s,\theta) - Em(z_t^s,\theta)]$$

Both terms above have asymptotically independent increments and are mutually independent. Now, we define $v_{1,T}(\pi) = (1/\sqrt{T})\sum_{t=1}^{[T\pi]}[m(z_t) - Em(z_t)]$ and $v_{2,T}^S(\pi) = (1/\sqrt{TS})\sum_{t=1}^{[TS\pi]}[m(z_t^s,\theta) - Em(z_t^s,\theta)]$. By the fact that $\{v_{1,T}(\cdot): T\geq 1\}$ and $\{v_{2,T}^S(\cdot): T\geq 1\}$ have asymptotically independent increments and under Assumptions A.2 and A.3, we can apply Lemma A4 of Andrews (1993) to the sequences $v_{1,T}(\cdot)$ and $v_{2,T}^S(\cdot)$. Then $v_{1,T}(\cdot) \Rightarrow \Omega^{1/2}B_1(\cdot)$ and $v_{2,T}^S(\cdot) \Rightarrow \Omega^{1/2}B_2(\cdot)$. Since $\sqrt{T}f_{[T\pi]}^S(\theta_0,\pi) = ((v_{1,T}(\pi) - \frac{1}{\sqrt{S}}v_{2,T}^S(\pi))', ((v_{1,T}(1) - v_{1,T}(\pi)) - \frac{1}{\sqrt{S}}(v_{2,T}^S(1) - v_{2,T}^S(\pi)))')'$, moreover, by equations (B.3), (B.4), (B.5) and (B.7), Assumptions A.4 and A.5 and the continuous mapping theorem (see Pollard (1984)), we obtain the desired result.

Proof of Theorem 3:

The proof is a modification of Andrews' proof for the Wald test which takes into account the presence of simulated moments. We define $\Xi = [I_r,\ -I_r, 0_s] \in R^{r\times(2r+s)}$ and have:

$$\begin{aligned}\sqrt{T}\left(\hat{\beta}_{1T}^S(\pi) - \hat{\beta}_{2T}^S(\pi)\right) &= \Xi\sqrt{T}\left(\hat{\theta}_T^S(\pi) - \theta_0\right) \\ &\Rightarrow \Xi\left(F(\pi)'W(\pi)F(\pi)\right)^{-1} F(\pi)'W(\pi)G(S,\pi)\end{aligned}$$

For an optimal choice of the weighting matrix, we can write:

$$F(\pi)'W(\pi)F(\pi) = \begin{bmatrix} \pi(F^\beta)'\Omega^{*-1}F^\beta & 0 & \pi(F^\beta)'\Omega^{*-1}F^\delta \\ 0 & (1-\pi)(F^\beta)'\Omega^{*-1}F^\beta & (1-\pi)(F^\beta)'\Omega^{*-1}F^\delta \\ \pi(F^\delta)'\Omega^{*-1}F^\beta & (1-\pi)(F^\delta)'\Omega^{*-1}F^\beta & (F^\delta)'\Omega^{*-1}F^\delta \end{bmatrix}.$$

Therefore, by the lemma A5 of Andrews (1993):

$$\Xi \left(F(\pi)'W(\pi)F(\pi)\right)^{-1} F(\pi)'W(\pi)G(S,\pi) = [I_r : -I_r]$$

$$\times \quad \begin{bmatrix} \pi(F^{\beta})'\Omega^{*-1}F^{\beta} & 0 \\ 0 & (1-\pi)(F^{\beta})'\Omega^{*-1}F^{\beta} \end{bmatrix}^{-1}$$

$$\times \quad \begin{bmatrix} (F^{\beta})'\Omega^{*-1/2}\left((1+\frac{1}{s})^{-1/2}\right)\left[B_1(\pi) - \frac{1}{\sqrt{S}}B_2(\pi)\right] \\ (F^{\beta})'\Omega^{*-1/2}\left((1+\frac{1}{s})^{-1/2}\right)\left[(B_1(1)-B_1(\pi)) - \frac{1}{\sqrt{S}}(B_2(1)-B_2(\pi))\right] \end{bmatrix}$$

$$= \quad C\left(1+\frac{1}{S}\right)^{-1/2}\left(\frac{\left[B_1(\pi)-\frac{1}{\sqrt{S}}B_2(\pi)\right]}{\pi} - \frac{\left[(B_1(1)-B_1(\pi))-\frac{1}{\sqrt{S}}(B_2(1)-B_2(\pi))\right]}{(1-\pi)}\right)$$

where $C = \left((F^{\beta})'\Omega^{*-1}F^{\beta}\right)^{-1}(F^{\beta})'\Omega^{*-1/2}$ and $\left((1+\frac{1}{S})^{-1/2}\right)\left[B_1(\pi) - \frac{1}{\sqrt{S}}B_2(\pi)\right]$ is a q-dimensional vector of standard Brownian motions which will be denoted $B^*(\pi)$.

We have also

$$\frac{\hat{V}_1(\pi)}{\pi} + \frac{\hat{V}_2(\pi)}{(1-\pi)} \Rightarrow \frac{V}{\pi(1-\pi)} = \frac{CC'}{\pi(1-\pi)}.$$

where $V = \left((F^{\beta})'\Omega^{*-1}F^{\beta}\right)^{-1}$. The Wald statistic then converges to

$$\left(\frac{[B^*(\pi)-\pi B^*(1)]}{\pi(1-\pi)}\right)' C' \left(\frac{CC'}{\pi(1-\pi)}\right)^{-1} C \left(\frac{[B^*(\pi)-\pi B^*(1)]}{\pi(1-\pi)}\right).$$

We can decompose $C'(CC')^{-1}C$ as $C'(CC')^{-1/2}(CC')^{-1/2}C$. When we define the Brownian motions $C'(CC')^{-1/2}B^*(\pi)$, the asymptotic distribution result follows. In particular, the asymptotic distribution does not depend on the number of simulations S, and hence is nuisance parameter free. For the $LM_T(S,\pi)$ statistic and $LR_T(S,\pi)$, we can show that

$$LM_T^S(\pi) = Wald_T^S(\pi) + o_p(1)$$

and

$$LR_T^S(\pi) = Wald_T^S(\pi) + o_p(1)$$

using the same arguments as in Andrews for the GMM case. For brevity the proof is omitted.

Proof of Theorem 4:

We can show that

$$\sqrt{T} f_{T\pi}^{S}(\theta_0) \Rightarrow \Omega^{1/2} B_1(\pi) - \frac{1}{\sqrt{S}} \Omega^{1/2} B_2(\pi) + H(\pi) \qquad \text{(B.8)}$$

where $H(\pi) = \int_0^\pi h(r)dr$. Let us consider an expansion of the moment conditions for the full sample evaluated with the restricted estimator

$$f_T^S(\tilde{\theta}_T^S) = f_T^S(\theta_0) + \frac{\partial}{\partial \theta'} f_T^S(\bar{\theta}) \left(\tilde{\theta}_T^S - \theta_0 \right).$$

where $\bar{\theta}_T^{S\prime} = [\bar{\theta}_T^{S,(1)} \ldots \bar{\theta}_T^{S,(p)}]$ and $\bar{\theta}_T^{S,(k)} = \lambda^{(k)} \theta_0^{(k)} + (1 - \lambda^{(k)}) \hat{\theta}_T^{S,(k)}$ for some $0 \le \lambda^{(k)} \le 1$ and $k = 1, \ldots, p$. Multiplying both sides by: $\partial f_T^S(\tilde{\theta}^S)' W_T / \partial \theta'$ we obtain

$$\left(\tilde{\theta}_T^S - \theta_0 \right) = - \left[\frac{\partial}{\partial \theta'} f_T^S(\tilde{\theta}_T^S)' W_T \frac{\partial}{\partial \theta'} f_T^S(\bar{\theta}) \right]^{-1} \frac{\partial}{\partial \theta'} f_T^S(\tilde{\theta}_T^S)' W_T f_T^S(\theta_0) \text{(B.9)}$$

since $(\partial f_T^S(\tilde{\theta}_T^S)/\partial \theta')' W_T f_T^S(\tilde{\theta}_T^S) = o_p(1)$.

Now we expand the moment conditions for the first subsample evaluated at the restricted full sample estimator

$$f_{T\pi}^S(\tilde{\theta}_T^S) = f_{T\pi}^S(\theta_0) + \frac{\partial}{\partial \theta'} f_{T\pi}^S(\ddot{\theta}) \left(\tilde{\theta}_T^S - \theta_0 \right). \qquad \text{(B.10)}$$

where $\ddot{\theta}_T^{S\prime} = [\ddot{\theta}_T^{S,(1)} \ldots \ddot{\theta}_T^{S,(p)}]$ and $\ddot{\theta}_T^{S,(k)} = \lambda^{(k)} \theta_0^{(k)} + (1 - \lambda^{(k)}) \bar{\theta}_T^{S,(k)}$ for some $0 \le \lambda^{(k)} \le 1$ and $k = 1, \ldots, p$. We substitue (B.9) into (B.10)

$$\begin{aligned} f_{T\pi}^S(\tilde{\theta}_T^S) &= f_{T\pi}^S(\theta_0) - \frac{\partial}{\partial \theta'} f_{T\pi}^S(\ddot{\theta}) \left[\frac{\partial}{\partial \theta'} f_T^S(\tilde{\theta}_T^S)' W_T \frac{\partial}{\partial \theta'} f_T^S(\bar{\theta}) \right]^{-1} \\ &\quad \times \frac{\partial}{\partial \theta'} f_T^S(\tilde{\theta}_T^S)' W_T f_T^S(\theta_0) \end{aligned}$$

By (B.8) and Assumption (A.3) and eq(B.5)

$$\begin{aligned} \sqrt{T} f_{T\pi}^S(\tilde{\theta}_T^S) &\Rightarrow \Omega^{1/2} B_1(\pi) - \frac{1}{\sqrt{S}} \Omega^{1/2} B_2(\pi) + H(\pi) - \\ &\quad \pi F (F' W F)^{-1} F' W \left[\Omega^{1/2} B_1(1) - \frac{1}{\sqrt{S}} \Omega^{1/2} B_2(1) + H(1) \right]. \end{aligned}$$

Proof of Corollary 4:
The result follows from Theorem 4 and the fact that

$$\left(1+\frac{1}{S}\right)^{-1/2} C\left[B_1(\pi)-\frac{1}{\sqrt{S}}B_2(\pi)\right] \tag{B.11}$$

is a q-dimensional vector of standard Brownian motion.

References

[1] Andrews, D.W.K. (1992), " Generic Uniform Convergence," *Econometric Theory,* **8,** 241-257.

[2] Andrews, D.W.K. (1993), "Tests for Parameter Instability and Structural Change with Unknown Change Point," *Econometrica,* **61,** 821-856.

[3] Andrews, D.W.K. (1994), "The Large Sample Correspondence between Classical Hypothesis Tests and Bayesian Posterior Odds Tests," *Econometrica,* **62,** 1207-1232.

[4] Andrews, D.W.K. and J.C. Monahan (1992), "An Improved Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator," *Econometrica,* **60,** 953-966.

[5] Andrews, D.W.K. and W. Ploberger (1994), "Optimal Tests When a Nuisance Parameter Is Present Only under the Alternative," *Econometrica,* **62,** 1383-1414.

[6] Bilio, M and A. Monfort (1996), "Functional Indirect Inference", manuscript, CREST, Paris.

[7] Chow, G. C. (1960), "Tests of Equality between Sets of Coefficients in Two Linear Regressions," *Econometrica,* **28,** 591-605.

[8] Davidson, J. (1994), "Stochastic Limit Theory: An Introduction for Econometricians," Oxford University Press, Oxford.

[9] Duffie, D. and K. J. Singleton (1993), "Simulated Moments Estimation of Markov Models of Asset Prices," *Econometrica,* **61,** 929-952.

[10] Dufour, J.M., E. Ghysels and A. Hall (1994), "Generalized Predictive Tests and Structural Change Analysis in Econometrics," *International Economic Review,* **35,** 199-229.

[11] Gallant, A.R. (1987), "Nonlinear Statistical Models," John Wiley, New York.

[12] Gallant, A.R. and G.. Tauchen (1996), " Which Moments to Match?" *Econometric-Theory*, **12**, 657-681.

[13] Gallant, A.R. and H. White (1988), "A Unified Theory of Estimation and Inference for Nonlinear Dynamic Models," Basil Blackwell, New York.

[14] Ghysels, E., A. Guay and A. Hall (1997), "Predictive Test for Structural Change with Unknown Breakpoint," *Journal of Econometrics*, **82,** 209-233.

[15] Ghysels, E. and A. Hall (1990a), "A Test for Structural Stability of Euler Conditions Parameters Estimated Via the Generalized Method of Moments Estimator," *International Economic Review,* **31,** 355-364.

[16] Gouriéroux, C. and A. Monfort (1996), "Simulation-based econometric methods," Oxford University Press, Oxford.

[17] Gouriéroux, C., A. Monfort and E. Renault (1993), "Indirect Inference" *Journal of Applied Econometrics* **8**, S85-S118.

[18] Guay, A. (1996), "Structural Change Tests based on Moment Conditions," Discussion Paper UQAM.

[19] Hall, A. and A. Sen (1996), "Structural Stability Testing in models estimated by Generalized Method of Moments" *Journal of Business and Economic Statistics*, (forthcoming).

[20] Hansen, L.P. (1982), "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica,* **50,** 1029-1054.

[21] Newey, W.K. (1991), "Uniform Convergence in Probability and Stochastic Equicontinuity," *Econometrica,* **59,** 703-708.

[22] Newey, W.K. and K. West (1994), "Automatic Lag Selection in Covariance Matrix Estimation," *Review of Economic Studies,* **61,** 631-653.

[23] Pollard, D. (1984), "Convergence of Stochastic Processes," Springer-Verlag, New York.

[24] Potscher, B.M. and Prucha, I. R. (1991), " Basic Structure of the Asymptotic Theory in Dynamic Nonlinear Econometric Model, Part I: Consistency and Approximation Concepts," *Econometric Reviews,* **10,** 125-216.

[25] Sowell, F. (1996a), "Optimal Tests for Parameter Instability in the Generalized Method of Moments Framework," *Econometrica,* **64,** 1085-1107.

[26] Sowell, F. (1996b), "Tests for Violations of Moment Conditions," manuscript, Graduate School of Industrial Administration, Carnegie Mellon University.

Table 1

Size and power. $T = 50$

π	μ	S	1 %	5 %	10 %
	0	1	0.0	0.0	0.1
		2	0.4	2.8	4.8
		5	0.4	3.4	5.4
		10	1.0	3.4	5.8
.25	.5	1	0.0	0.0	0.1
		2	2.2	9.0	13.0
		5	3.3	12.1	17.6
		10	3.9	13.8	20.4
.5	.5	1	0.0	0.5	1.1
		2	2.7	12.9	19.0
		5	4.4	15.3	21.9
		10	4.8	17.5	26.4
.75	.5	1	0.0	0.2	0.6
		2	2.9	10.3	14.9
		5	2.9	11.9	17.4
		10	3.3	12.9	18.2
.25	1	1	0.0	6.0	13.2
		2	12.5	35.7	43.6
		5	21.6	50.0	60.3
		10	23.2	49.8	60.5
.5	1	1	0.2	11.6	21.4
		2	19.3	45.0	56.4
		5	33.2	61.8	71.0
		10	38.1	69.0	77.4
.75	1	1	0.1	5.4	11.2
		2	12.2	32.1	41.6
		5	17.5	45.4	56.0
		10	22.5	50.3	60.1

Table 2

Size and power. $T = 100$

π	μ	S	1 %	5 %	10 %
	0	1	0.0	0.0	0.1
		2	0.5	3.1	5.7
		5	0.7	3.5	5.3
		10	0.9	3.6	6.2
.25	.5	1	0.3	2.9	5.7
		2	9.3	24.8	30.9
		5	10.8	27.5	35.2
		10	10.5	28.7	37.5
.5	.5	1	1.0	5.8	10.2
		2	10.9	29.7	39.5
		5	15.0	35.1	45.0
		10	20.5	41.4	49.5
.75	.5	1	0.1	2.1	5.4
		2	7.3	21.1	29.8
		5	9.0	26.2	34.2
		10	12.7	30.1	36.7
.25	1	1	15.3	51.4	63.7
		2	48.6	77.1	85.2
		5	70.0	88.0	92.4
		10	73.3	89.1	93.1
.5	1	1	23.7	49.8	60.1
		2	67.2	84.6	90.0
		5	82.4	95.3	96.7
		10	86.9	94.8	97.2
.75	1	1	7.8	16.5	20.6
		2	50.9	77.5	84.0
		5	66.7	86.3	91.3
		10	73.2	90.4	93.5

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