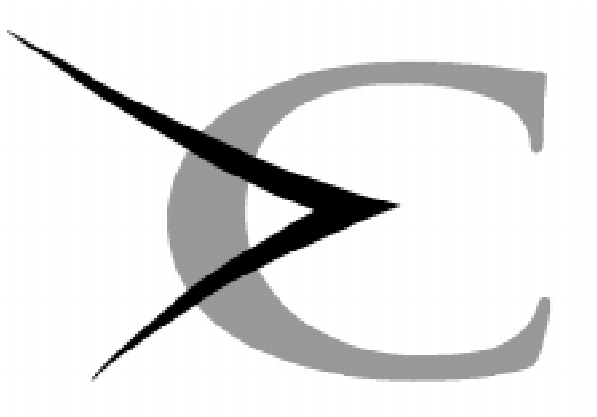

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Tests of Conditional Asset Pricing in the Brazilian Stock Market*

Marco Bonomo†, *René Garcia*‡

Résumé / Abstract

Dans cet article, nous testons une version du CAPM conditionnel par rapport au portefeuille de marché local, approximé par un indice boursier brésilien, au cours de la période 1976-1992. Nous testons également un modèle APT conditionnel en utilisant la différence entre les taux d'intérêt sur les dépôts de trente jours (Cdb) et le taux au jour le jour comme deuxième facteur en plus du portefeuille de marché pour capter l'important risque inflationniste présent durant cette période. Les modèles conditionnels CAPM et APT sont estimés par la méthode généralisée des moments (GMM) et testés sur un ensemble de portefeuilles construits selon la taille à partir d'un total de 25 titres échangés sur les marchés boursiers brésiliens. L'incorporation de ce deuxième facteur se révèle cruciale pour une juste valorisation des portefeuilles.

In this paper, we test a version of the conditional CAPM with respect to a local market portfolio, proxied by the Brazilian stock index during the period 1976-1992. We also test a conditional APT model by using the difference between the 30-day rate (Cdb) and the overnight rate as a second factor in addition to the market portfolio in order to capture the large inflation risk present during this period. The conditional CAPM and APT models are estimated by the Generalized Method of Moments (GMM) and tested on a set of size portfolios created from a total of 25 securities exchanged on the Brazilian markets. The inclusion of this second factor proves to be crucial for the appropriate pricing of the portfolios.

Mots Clés : CAPM conditionnel, APT conditionnel, efficacité des marchés, risque et rendements variables dans le temps

Keywords : Conditional CAPM, Conditional APT, Efficiency of Markets, Time-Varying Risk and Returns

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1 Introduction

Since the beginning of the nineties, the Brazilian stock market figures prominently among the star performers of the so-called emerging markets. A bright future seems also in store for this market if one considers the considerable inflows of capital that followed the successful recent "Real" plan. It seems worthwhile at this turning point in the Brazilian economy and in its financial markets to take a close look at how predictable risk and returns have been on the stock market in the last twenty years or so (between 1976 and 1992).

A first look at the unconditional moments of the returns series for the stock market index taken from the IFC Emerging Markets Data Base and reported in Table 1, shows an average return in US dollars of 21.15% and an average excess return in local currency of 28.82% . By industrialized country standards, these returns are high. However, as fundamental asset pricing models such as the CAPM or the APT tell us, high expected returns ought to be associated with high measures of risk with respect to a number of risk factors. One would therefore want to identify the set of fundamental sources of risk that affect the returns in this market. According to the CAPM, the expected return on a portfolio of assets is a function of the covariance of the portfolio return with the market portfolio return. Two different views can be taken however when selecting this market portfolio: one can consider that the Brazilian market is segmented and concentrate on local risk factors to explain local returns, or one can adopt the perspective of an international investor diversifying his portfolio worldwide. If enough investors diversify internationally their portfolios, the market will move towards integration, and expected returns in Brazil will be well described by the country's world risk exposure, the covariance of the Brazilian stock returns with the world market portfolio. This is the view taken by Harvey (1995) in a recent study on emerging markets. The author tests a dynamic factor asset pricing model in which the risk loadings are measured with respect to the world market return in excess of a risk-free asset return. Moreover, these risk loadings are allowed to vary through time. This feature is clearly essential in the context of emerging markets where the internal dynamics underlying the country's returns index along with unstable macroeconomic and political conditions can bring considerable variation in the factor loadings. The results for Brazil show that the beta with the world market return is not significantly different from zero and the unexpected part of the world risk premium is related to local market information such as the local dividend yield or a local interest rate. This suggests that the Brazilian market is either completely segmented from

or partially integrated with the world market.

Therefore, in this paper, we adopt the view according to which the Brazilian stock market is segmented and test a version of the conditional CAPM proposed by Bodurtha and Mark (1991) with respect to a local market portfolio, represented by the Brazilian stock index in the IFC database. The conditional CAPM is tested on a set of size portfolios created from a total of 25 securities exchanged on the Brazilian markets. In this CAPM model, as in Harvey (1995), the beta of a portfolio of assets is defined as the conditional covariance between the forecast error in the portfolio return and the forecast error of the market return divided by the conditional variance of the forecast error in the market return. In Harvey (1995), the returns are projected over a set of instruments in the information set of the investors. The distinctive feature of the model tested in this paper is that both components of the conditional beta are assumed to follow an ARCH process, a concept of conditional heteroskedasticity introduced by Engle (1982). This modelling choice can be rationalized in two ways. First, looking at the statistics in Table 1, one sees considerable autocorrelation in the squared market returns series, indicating the presence of ARCH effects. Second, the use of autoregressive processes might provide estimates that are more robust to structural change. Ghysels (1995) and Garcia and Ghysels (1996) show that models similar to Harvey (1991, 1995) or Ferson and Korajczyk (1995), where the returns are projected on a set of variables belonging to the information set such as a term spread, a risk spread, or a dividend yield, suffer from instability in the projection coefficients and therefore lead to systematic mispricing of the risk factors. By using the autoregressive structure, we hope to better forecast the returns and their variances and covariances. A shortcoming of our approach is that it assumes a fixed regime of segmentation throughout the period. The concept of time-varying integration proposed by Bekaert and Harvey (1995) addresses this shortcoming, as well as the problem of projection coefficient instability.

Since our period of estimation covers lapses of very high inflation (up to 30% a month), we also estimate an APT model using the excess return of a 30-day bond over the overnight rate as a second risk factor. The bond return should have a strong negative correlation with inflation surprises, and because of the high volatility of the monthly inflation rate during this period should capture an important risk factor[1]. This model offers the best estimates of the betas both with the market portfolio and

[1]We chose to use a 30-day bond because this is the longest maturity bond that was traded in the domestic market during the whole period.

with the 30-day bond. As predicted by the theory, the average market betas are increasing with the portfolio size. The average 30-day bond betas are negative for all portfolios. Since the excess return for those bonds are negatively correlated with inflation surprises, this indicates that the performance of those portfolios are positively affected by inflation innovations, with the big firm portfolio offering the best insurance against inflation. As a diagnostic test of this last and most complete model, we verify ex-post if the residuals are orthogonal to various variables in the information set of the agents. For example, we verify if the residuals are orthogonal to a January dummy to account for a possible January effect put forward in the US market studies, or to a dividend yield or lags of the risk-free asset.

The rest of the paper is organized as follows. Section 2 presents the conditional CAPM model, its econometric specification, and the estimation results. Section 3 mirrors section 2 for the APT model and ends with diagnostic tests of the model specification. Section 4 concludes.

2 The Conditional CAPM

2.1 The Model

The conditional CAPM can be stated as follows[2]:

$$E\left[r_i\left(t\right)|\Omega_t\right] = \beta_{it} E\left[r_M\left(t\right)|\Omega_t\right] \tag{1}$$

where $r_i\left(t\right)$ is the one-period return on portfolio i in excess of the risk-free asset return, $r_M\left(t\right)$ the excess return on the market portfolio, and β_t is given by the following expression:

$$\beta_{it} = \frac{Cov\left[r_i\left(t\right), r_M\left(t\right)|\Omega_t\right]}{Var\left[r_M\left(t\right)|\Omega_t\right]} \tag{2}$$

In this version of the CAPM, all moments are made conditional to the information available at time t represented by the information set Ω_t. Many asset pricing studies on the US stock markets (Ferson and Harvey (1991), for example) have shown that allowing the moments to vary with

[2]This equation can be deduced from the fundamental pricing equation: $E[r_i(t)r_M(t)|\Omega_t] = 1$, which is valid under the absence of arbitrage or as a first-order condition of an equilibrium model (see Duffie [1996], Chapter 1).

time is essential, since there is evidence that both the beta, the ratio of the covariance to the variance, and the price of risk $E\left[r_M\left(t\right)|\Omega_t\right]$ are time-varying. This is even more essential in emerging markets, where unstable macroeconomic and political conditions can translate into considerable variations in the factor loadings. To put model (1) into an estimable form, we decompose the returns into a forecastable part and an unforecastable part, namely:

$$r_i(t) = E\left[r_i\left(t\right)|\Omega_t\right] + u_i(t) \tag{3}$$

$$r_M\left(t\right) = E\left[r_M(t)\,|\Omega_t\right] + u_M(t) \tag{4}$$

where $u_i(t)$ and $u_m(t)$ are forecast errors orthogonal to the information in Ω_t. Equation (1) can therefore be rewritten as follows:

$$E\left[r_i\left(t\right)|\Omega_t\right] = \frac{E\left[u_i\left(t\right)u_M\left(t\right)|\Omega_t\right]}{E\left[u_M\left(t\right)^2|\Omega_t\right]} E\left[r_M\left(t\right)|\Omega_t\right] \tag{5}$$

To obtain a set of moment conditions suitable for GMM estimation, we need to specify parametric models for the expectations on the right hand side of (5). As mentioned in the introduction, following Bodurtha and Mark (1991), we choose to specify autoregressive processes for each of the expectations:

$$E\left[u_M\left(t\right)^2|\Omega_t\right] = \delta_{0_M} + \sum_{j=1}^{k_M 2} \delta_{j_M} u_M(t-j)^2 \tag{6}$$

$$E\left[u_i\left(t\right)u_M\left(t\right)|\Omega_t\right] = \delta_{0_i} + \sum_{j=1}^{k_i} \delta_{ji} u_i\left(t-j\right)u_M\left(t-j\right) \tag{7}$$

$$E\left[r_M\left(t\right)|\Omega_t\right] = \alpha_{0_M} + \sum_{j=1}^{k_M} \alpha_{j_M} r_M\left(t-j\right) \tag{8}$$

The number of lags k_{M^2}, k_i, k_M to be included in each of the equations above remains an empirical issue, given the constraint imposed by the number of available observations. The final form of the moment conditions that will be used for GMM estimation can therefore be written as follows:

$$
\begin{aligned}
& r_M(t) = \alpha_{0_M} + \textstyle\sum_{j=1}^{k_M} \alpha_{j_M} r_M(t-j) + u_M(t) \\
& u_M(t)^2 = \delta_{0_M} + \textstyle\sum_{j=1}^{k_{M^2}} \delta_{j_M} u_M(t-j)^2 + v_M(t) \\
& u_i(t)\, u_M(t) = \delta_{0_i} + \textstyle\sum_{j=1}^{k_i} \delta_{ji} u_i(t-j)\, u_M(t-j) + v_{iM}(t), i = 1, ..., N \\
& r_i(t) = \frac{\delta_{0i} + \sum_{j=1}^{k_i} \delta_{ji} u_i(t-j) u_M(t-j)}{\delta_{0_M} + \sum_{j=1}^{k_{M^2}} \delta_{j_M} u_M(t-j)^2} [\alpha_{0_M} + 1.25cm \textstyle\sum_{j=1}^{k_M} \alpha_{j_M} r_M(t-j)] \\
& \quad + u_i(t), i = 1, ..., N
\end{aligned}
\tag{9}
$$

where $v_M(t)$ and $v_{iM}(t)$ are the conditional forecast errors corresponding to the second-moment conditions.

2.2 Estimation method

The model of the last section is estimated by the Generalized Method of Moments (GMM), a method introduced by Hansen (1982). To implement the method, one needs to specify a set of instruments for each equation in system (9). The system has 2(N+1) equations, where N is the number of risky assets or portfolios. Suppose, for simplicity of exposition, that we have the same number of possibly different instruments for each equation, say q. Following Hansen (1982), we call $f_t(\beta)$ the vector formed by stacking the Kronecker products of each forecast error $\epsilon_{l,t}, l = 1, ..., 2(N+1)$ with the sets of q instruments, i.e. a vector of [2(N+1)⊗q] x1:

$$f_t(\beta) = [\epsilon_t \otimes Z_t] \tag{10}$$

where we have stacked in Z_t the sets of instruments $z_{1t}, z_{2t}, ..., z_{qt}$ and where β contains all the parameters of the model. As instruments, we choose the particular variables used in the projections to compute the forecast errors. For the forecast errors $u_M(t)$ and $u_i(t)$, one constant and

k_M lags of the market excess return have been used. For the other forecast errors corresponding to the asset covariances and market variance, we use respectively k_i and k_{M^2} lags of the dependent variable.

Since ϵ_t is a vector of forecast errors, the expectation of $f_t(\beta)$ evaluated at the true value of the parameters β_0 must be zero. The GMM estimator is given by:

$$b = \arg\min \left[\frac{1}{T}\sum_{t=1}^{T} f_t(\beta)\right]' \Sigma_T^{-1} \left[\frac{1}{T}\sum_{t=1}^{T} f_t(\beta)\right] \tag{11}$$

where $\Sigma_T^{-1} = \frac{1}{T}\sum_{t=1}^{T} f_t(b) f_t(b)'$. To carry out the estimation, we use a two-step procedure, with an identity matrix for Σ first and then a consistent estimate based on the Newey-West method (Newey and West, 1987). To test the model, we use the J-statistic, which is T times the value of the minimized value of the function. This statistic tests for the overidentifying restrictions imposed by the model. Under the null of a correctly specified model, this statistic is distributed asymptotically as a chi-square with 2(N+1)xq-($N(k_i+1)+2+k_M+k_{M^2}$) degrees of freedom.

To apply the GMM method, the projection variables used for the first and second conditional moments of the returns are lagged values of the dependent variable in each equation. Another common method is to use variables such as a lagged risk or term spread or a dividend yield variable. All variables are good candidates since they are in the information set but we believe that using autoregressions for both the first and second moments could provide estimates that are more robust to structural change. In Ghysels (1996) and Garcia and Ghysels (1996), it is shown that structural stability tests often reject models that pass the J-test where projections are made on other economic variables.

2.3 Data and Estimation Results

The IFC Emerging Markets Data Base of the World Bank provides data on stock prices and other financial variables for both the stock index and individual stocks in a series of developing and newly industrialized countries. For the sample of individual securities provided for Brazil, we selected a total of 25 common shares (see list of securities in Appendix 1) which were listed on the IBOVESPA stock exchange from 1976:1 to 1992:12. To test the model of section 2.1 we could theoretically use the returns on the individual securities, but for estimation purposes we have to limit the number of parameters. We follow common practice in grouping the securities into portfolios and testing the model on a small

set of portfolios. Given the limited number of available firms, we decided to form three size portfolios, where size refers to the capitalization value of the firms. We create these portfolios by first value-ranking the returns of the individual equities in each month and then by separating the returns into three size (value) quantiles. Within each quantile, we weight each return by the capitalization value of the firm relative to the total capitalization value of the firms in the quantile. The returns are computed in the local currency, the cruzeiro, in excess of the overnight rate.

Table 1 provides some sample statistics on both the returns and the excess return series for the Brazilian stock index. Although the mean of the raw local currency return series is quite high (159% in annualized returns) because of the high inflation that occurred during our sample period, the mean excess return is of the same magnitude as the mean return in US dollars. The squared series show a very strong autocorrelation, a usual feature in financial time series. All series depart also strongly from normality, as indicated mainly by the excess kurtosis statistic.

Table 2 reports some basic statistics for the three portfolios. Portfolios 1, 2, and 3 represent the small, medium, and large firms respectively. As can be seen on Table 2, the mean of the portfolios increases with size, while the variance decreases with size. This would not be compatible with an asset pricing model where risk would be measured by the variance of an asset or a portfolio, since a higher risk should lead to a higher return. It is however compatible with the CAPM model since the expected return would be lower for a small firm portfolio than for a large firm portfolio, because the small firm returns covary less with the market than the returns of the large firms. Therefore the small firms have an insurance value and investors require a lower return in equilibrium.

Figures 1 and 2 show the graphs of the market excess return series and the three portfolio excess returns series respectively. All graphs show the presence of strong autoregressive conditional heteroskedasticity. To start however, we estimate the simplest model, a constant beta CAPM, where all parameters in (9), except $\alpha_0, \delta_{0_M}, \delta_{0_i} (i = 1, 2, 3)$ are constrained to be zero. Estimation results are reported in table 3. Although the model cannot be rejected according to the overidentifying restriction criterion J, with a p-value close to 86%, one should be careful about this result. Since the number of moment conditions is large with respect to the number of observations, the conventional asymptotic inference might lose its validity (see Koenker and Machado (1996)[3]). The calculated

[3]They show that for the estimation of a linear model with general heteroskedastic-

betas are reported in Table 7. The higher mean return of portfolio 3 can be rationalized by its substantially higher β: 1.6, compared to values close to one for the two other portfolios.

Next, we introduce lagged terms in the mean, variance and covariance equations to estimate a conditional form of the CAPM. The specification chosen allows for ARCH effects in the market variance (a feature strongly present in the data) and in the portfolio covariances with the market. It is rich enough to test for restricted versions of interest, such as a constant beta model, a constant market price of risk or a constant conditional market variance. We have limited the autoregressions to a maximum of two lags in each of the equations. Overall, there are 14 parameters for 24 equations, which implies a $\chi^2(10)$ distribution for the J-test statistic. This specification is similar to the specification used in Bodurtha and Mark (1991). The parameter estimates for the Conditional CAPM with ARCH effects are shown in Table 4. Although all parameters estimates seem to be significantly different from zero, except in the conditional mean equation, the warning about the high number of moment conditions should be kept in mind. The mean betas (reported in row 2 in Table 7) seem too high, since they are all greater than 1.

To test for the restricted versions of the model mentioned above, we perform Wald tests. For the constant beta model, we test whether all parameters but the constants in each equation are equal to zero. The Wald statistic in this case will be distributed as a $\chi^2(10)$ variable. For the fixed market price of risk, we test whether the coefficients other than the constants in the market conditional mean and variance equations are zero. This is a $\chi^2(4)$ test. Finally, the test for a constant conditional market variance is a test for the equality to zero of δ_{1M} and δ_{2M}, which is a $\chi^2(2)$ test. All these restricted versions of the model are overwhelmingly rejected (at less than 0.01 in all cases). The high values of the mean betas and the time series behavior of the portfolio betas (shown in figure 3) suggest that an important risk dimension could be missing in the model. Indeed, the portfolio betas appear, if anything, to be more volatile during the first half of the sampling period, while the returns are much more volatile during the second half. Since Brazil was affected by a high and variable inflation especially during the second half of the period under study, we explore in the next section a conditional two-factor model, where the second factor aims at capturing the inflation risk.

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ity that $q^{5/2}/n \to 0$, where q is the number of moment conditions and n the number of observations, is a sufficient condition for the validity of conventional asymptotic inference about the GMM estimator. Indeed, using only 8 moment conditions (with only a constant as instrument), we obtain a p-value of 0.05 for the J statistic and the t-values drop to magnitudes of 3 and 4.

3 A Conditional Two-Factor Model

In this section we formulate and estimate an extension of the conditional CAPM estimated in section 2, where a second asset (a nominal bond called CD) is added as a second risk factor. Since the nominal return of this asset is fixed for a month, its real return is affected by the unforecastable component of inflation. It will therefore capture an important original risk factor in a high inflation economy, which would not be reflected fully in the market portfolio because its return is not fixed. This second factor contains also a real interest rate risk, associated with an unexpected change in monetary policy, but we believe that the variability of inflation is such that most of the nominal interest rate risk is due to inflation risk.

3.1 The Model

We assume the following conditional two-factor model for excess returns:

$$E\left[r_i\left(t\right)|\Omega_t\right] = \beta_{iM}(t)E\left[r_M\left(t\right)|\Omega_t\right] + \beta_{iF}(t)E[r_F(t) \mid \Omega_t] \tag{12}$$

where r_F is the excess return of the thirty-day CD over the overnight rate. We assume that the factors are conditionally uncorrelated and obtain the following expressions for the conditional betas:[4]

$$\begin{aligned} \beta_{iM}(t) &= \frac{Cov_t[r_i(t), r_M(t)|\Omega_t]}{Var_t[r_M(t)|\Omega_t]} \\ \beta_{iF}(t) &= \frac{Cov_t[r_i(t), r_F(t)|\Omega_t]}{Var_t[r_F(t)|\Omega_t]} \end{aligned}$$

While keeping the decomposition of the market portfolio return in (4), we also breakdown the return of the CD into a forecastable and an unforecastable term:

[4] This assumption simplifies the expressions and reduces the number of parameters to be estimated but also seems to be supported by the data. The unconditional correlation between the factors found in the data is low (-0.08), and the projections of the cross-products of the error terms u_{Mt} and u_{Ft} appear to be orthogonal to various variables in the information set.

$$r_F(t) = E[r_F(t) \mid \Omega_t] + u_F(t) \tag{13}$$

Then, the betas can be rewritten as follows:

$$\begin{aligned} \beta_{iM}(t) &= \frac{E[u_i(t)u_M(t) \mid \Omega_t]}{E[u_M(t)^2 \mid \Omega_t]} \\ \beta_{iF}(t) &= \frac{E[u_i(t)u_F(t) \mid \Omega_t]}{E[u_F(t)^2 \mid \Omega_t]} \end{aligned}$$

We maintain the autoregressive specification of section 2 for the conditional expectation of the market return, for the conditional variance of the forecast error and for the conditional covariance between the forecast errors in predicting the market portfolio return and each individual asset return. Similarly, we also specify an autoregressive process for the additional variables that appear as a consequence of the addition of a second factor:

$$\begin{aligned} E[r_F(t)|\Omega_t] &= \alpha_{0M} + \sum_{j=1}^{k_F} \alpha_{jM} r_F(t-j) \\ E[u_F(t)^2|\Omega_t] &= \delta_{0F} + \sum_{j=1}^{k_{F^2}} \delta_{jM} u_F(t-j)^2 \\ E[u_i(t)u_F(t)|\Omega_t] &= \delta_{iF} + \sum_{j=1}^{k_{iF}} \delta_{ji} u_i u_F(t-j) \end{aligned}$$

We finally obtain the following moment conditions for the GMM estimation:

$$
\begin{aligned}
& r_M\left(t\right) = \alpha_{0_M} + \textstyle\sum_{j=1}^{k_M} \alpha_{j_M} r_M\left(t-j\right) + u_M\left(t\right) \\
& u_M\left(t\right)^2 = \delta_{0_M} + \textstyle\sum_{j=1}^{k_M{}^2} \delta_{j_M} u_M(t-j)^2 + v_M(t) \\
& r_F(t) = \alpha_{0_F} + \textstyle\sum_{j=1}^{k_F} \alpha_{jF} r_F(t-j) + u_F(t) \\
& u_F(t)^2 = \delta_{0_F} + \textstyle\sum_{j=1}^{k_F{}^2} \delta_{j_F} u_F(t-j)^2 + v_F(t) \\
& u_i\left(t\right) u_M\left(t\right) = \delta_{0i_M} + \textstyle\sum_{j=1}^{k_{iM}} \delta_{ji_M} u_i\left(t-j\right) u_M\left(t-j\right) + v_{iM}(t),\ i = 1, ..., N \\
& u_i(t) u_F(t) = \delta_{0i_F} + \textstyle\sum_{j=1}^{k_{iF}} \delta_{ji_F} u_i(t-j) u_F(t-j) + v_{iF}(t)\,, i = 1, ..., N \\
& r_i\left(t\right) = \frac{\left(\delta_{0i_M} + \sum_{j=1}^{k_{iM}} \delta_{ji_M} u_i(t-j) u_M(t-j)\right)}{\left(\delta_{0_M} + \sum_{j=1}^{k_M{}^2} \delta_{j_M} u_M(t-j)^2\right)} \left[\alpha_{0_M} + \textstyle\sum_{j=1}^{k_M} \alpha_{j_M} r_M\left(t-j\right)\right] \\
& \quad + \frac{\left(\delta_{0i_F} + \sum_{j=1}^{k_{iF}} \delta_{ji_F} u_i(t-j) u_F(t-j)\right)}{\left(\delta_{0_F} + \sum_{j=1}^{k_M{}^2} \delta_{j_F} u_F(t-j)^2\right)} \left[\alpha_{0_F} + \textstyle\sum_{j=1}^{k_F} \alpha_{j_F} r_F(t-j)\right] \\
& \quad + u_i(t), i = 1, ..., N
\end{aligned}
\tag{14}
$$

3.2 Estimation Results and Comparison

As before, we first estimate a model with constant factor loadings where all parameters, except the ones with subscript zero, are constrained to be zero. Table 5 reports the results. The magnitude of the covariance parameters is small in absolute value, due to the small variance of the CD excess return, but large in relative terms: the CD betas of the second and third portfolios (reported in Table 7) are -5 and -30, respectively. Their negative values and the pattern followed by their magnitudes, which increases with the capitalization value, indicate that the high-value portfolios offer the best hedge against the inflation risk. It should be noticed that, by adding a second factor, all the market portfolio betas become slightly lower than one. As mentioned before in section 2.3, the p-value of the J-statistic is overinflated given the large number of moment conditions used in the estimation.[5] Table 6 reports the estimation results of the conditional two-factor model with ARCH effects. The results show that ARCH effects play an important role. First, it should be noticed (in Table 7) that the average market portfolio betas change in an important way: they are now all less than one and they increase with the size of the portfolio. The market beta for the large firm portfolio is almost twice as big as the one for the small firm portfolio. The time-varying betas are plotted in Figures 4 and 5. The betas of the three portfolios with respect to the market portfolio become much more volatile after 1987. This coincides with a period of higher

[5] Using only a constant as instrument, the p-value of the J-statistic falls to 0.16.

and more volatile inflation, suggesting that the ARCH effects are important mainly because of the volatility of inflation. This is in contrast with the variability of the betas produced by the CAPM model (see figure 3), where no clear pattern emerges. Because of the lack of reliability of the J-statistic with these many moment conditions, we perform in the next section various diagnostic tests on the residuals to assess the adequacy of the model.

3.3 Diagnostic Tests

We have already mentioned that the J-statistic could be misleading because of the large number of moment conditions used for estimation compared with the available number of observations. Yet, many more orthogonality conditions could be used for estimation that would be consistent with the implications of the asset pricing models we are testing. Newey (1985) proposed a test (called CS test) of orthogonality conditions not used in estimation but implied by the model. Intuitively, this test verifies whether the residuals in the various equations used for estimation are orthogonal to other variables in the information set that have not been used as instruments. It can therefore be seen as a diagnostic test of the specification maintained as the null hypothesis.

The CS statistic is computed as follows:

$$CS = T\left[L_T g_T\left(b_T\right)\right]' Q_T^{-1}\left[L_T g_T\left(b_T\right)\right], \tag{15}$$

where:

$$g_T(b_T) = [g_{1T}(b_T)' g_{2T}(b_T)']'$$

with: $g_{iT}(b_T) = \frac{1}{T}\sum_{t=1}^{T} f_{iT}(b_T), i = 1, 2.$

The f_{1T} and f_{2T} are respectively the orthogonality conditions used and not used in estimation. The vector $f_{2T}(.)$ of dimension s is formed by multiplying the residual by variables (say p) in the information set that have not been used in estimation. The rest of the variables defining the statistic CS are as follows:

$$\begin{aligned}
L_T &= \left[0_{s\times(p+s)} : I_s\right], \quad S_{ij,T} = \frac{1}{T}\sum_{t=1}^{T} f_{it}\left(b_T\right) f_{jt}\left(b_T\right)', (i = 1, 2; j = 1, 2), \\
B_T &= \left(H_{1,T}' S_{11,T}^{-} H_{1,T}\right) H_{2,T}', \qquad H_{i,T} = \frac{1}{T}\sum_{t=1}^{T} \frac{\partial}{\partial b} f_{iT}\left(b_T\right), (i = 1, 2), \\
Q_T &= S_{22,T} - S_{21,T} S_{11,T}^{-1} H_{1,T} B_T - B_T' H_{1,T}' S_{11,T}^{-1} S_{12} + H_{2,T} B_T,
\end{aligned} \tag{16}$$

and b_T is the minimizer of $g_{1T}(\beta)' S_{11,T}^{-1} g_{1T}(\beta)$. The results of the diagnostic tests are reported in Table 8.

There is always some arbitrariness in choosing the information variables that should be orthogonal to the residuals, but since we chose an autoregressive specification it seems natural to test if we put enough lags. Therefore, we test first if the residuals are orthogonal to six of their own lags.

All residuals related to the market portfolio conditions appear to be serially uncorrelated. Not too surprisingly, this is not the case however for the residuals corresponding to the inflation conditions. There is strong evidence of remaining serial correlation in these residuals. Given the high persistence (both in mean and variance) of the rates of inflation that Brazil experienced during this period, especially during the second part of the sample, long lags would be necessary to make these residuals uncorrelated.

Next, we test whether residuals from each equation are orthogonal to lagged returns: we use the corresponding portfolio excess returns for v_{iM}, v_{IF} and u_i, the market portfolio excess returns for u_M and v_M, and the CD excess returns for u_F and v_F. The null hypothesis of orthogonality cannot be rejected, except for the u_F residual. Again, this is an indication that more lags are necessary in the mean equation for the CD excess returns to account for persistent inflation.

The other way to build conditional asset pricing models has been to use variables that are deemed to help predict excess stock returns and returns volatility, as in Harvey (1995) for example. Based on data availability, we select three of these variables, a January dummy (to test if there is a January effect in Brazil), the risk-free rate (in our case the overnight rate), and the dividend yield. Given that we chose to test an autoregressive conditional asset pricing model, it is a good way to test if we omitted some important economic variables in our information set. Overall, the test results show little evidence that we left some important information aside. The main failure comes from residual in the CD variance equation, which confirms the results obtained with the other orthogonality tests.

To conclude, we can say that our diagnostic tests tend to support the specification chosen, apart from the equations modelling the mean, and the variance of the CD excess returns. The highly unstable behavior of the inflation rate during the second part of the sample, and its high persistence makes it difficult to come up with an effective parsimonious model. The introduction of the "inflation" factor is however essential

for the conditional asset pricing model[6]. We have seen that without it the betas with respect to the market portfolio are biased and therefore the portfolios are mispriced. The "inflation" factor reduces considerably this mispricing, but obviously not fully. A more careful modelling of the CD equations, which for example would take into account the stabilization plans introduced during the period, might improve somewhat the results. Our goal however was to show that the addition of this factor considerably improves the model and results in market portfolio betas that have a reasonable dynamic pattern and which average value conforms with the theory.

4 Conclusion

In this paper, we test various conditional asset pricing models for the Brazilian stock market. Our best specification involves a two-factor model, where the equilibrium returns are determined by their covariances with the market portfolio and with a factor capturing inflation risk. The time series obtained for the betas seem to characterize well the evolution of risk during the estimation period. To further assess the adequacy of the model, we performed various diagnostic tests to check the orthogonality of residuals with information not used in the estimation. Some misspecification of the conditions related to the inflation factor was detected both for the mean and the variance. Given the extremely volatile and persistent pattern of inflation during the second half of the sample, it is difficult to obtain a good parsimonious specification for this moment condition. The large number of parameters already estimated prevents us from going too far in the direction of a more complete specification. Even with these misspecifications, the introduction of the inflation factor is essential to reduce the mispricing of the portfolios that would result from its omission.

[6]Cati, Garcia, and Perron (1996) propose a time-series model for inflation accounting for various changes in regime brought about by the various stabilization plans introduced during the sampling period.

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Appendix 1

List of Securities Used to Form the Size Portfolios

Acesita-ON
Alpargatas-ON
Belgo-Mineira-ON
Brahma-ON
Brahma-OP
Brasil-ON (190.1)
Brasil-ON (190.2)
Brasmotor-PN
Docas-OP
Ford-OP
Klabin-ON
Light-OP
Lojas Am.-OP
Mannesmann-ON
Moinho Sant-ON
Moinho Sant-OP
Paranapanema-ON
Paul F. Luz-OP
Petrobras-ON
Pirelli-ON
Samitri-ON
Souza-Cruz -ON
Val R. Doce -ON
Vidr S. Marina -OP
Whit Martins -ON

Table 1
Sample moments for the Brazilian Stockmarket Return Series

	US$ Return Series		Local Currency Return Series		Local Currency Excess Return Series	
	x	x^2	x	x^2	x	x^2
Mean	21.15	26.51	159.46	83.52	28.82	50.14
Std. Dev.	60.26	18.98	79.15	51.06	70.49	37.63
Skewness	0.53	3.17	1.21	3.63	0.167	6.39
Exc. Kurt.	1.00	11.07	1.77	15.30	4.70	50.86
ρ_1	0.029	0.135	0.156	0.159	0.001	-0.015
ρ_2	-0.034	0.069	0.227	0.292	0.019	0.122
ρ_3	-0.035	0.0135	0.146	0.136	-0.062	0.061
ρ_4	-0.070	0.102	0.169	0.189	-0.019	0.182
ρ_5	-0.044	-0.025	0.107	0.072	-0.015	0.044
Box-Ljung statistic	7.82	22.70	60.00	61.95	15.03	46.99
P-value	0.65	0.012	3.6d-09	1.6d-09	0.13	9.5d-07

Table 2
Sample moments for the Three Size Portfolios
(Monthly Returns)

	Portfolio 1	Portfolio 2	Portfolio 3
Mean	2.36	3.04	6.86
Variance	11.53	5.49	5.29
Skewness	2.44	0.93	1.46
Exc. Kurt.	13.07	6.38	3.76

Table 3
CAPM with constant beta

Parameters of the Market Portfolio		
		α_{0M}
Mean		0.0309 (5.45)
		δ_{0M}
Variance		0.0308 (18.26)
Parameters of the Portfolio Covariances		
Portfolios		δ_{0i}
1		0.0298 (15.18)
2		0.0293 (16.78)
3		0.0503 (17.63)
Test of Orthogonality Conditions	J d.f. P-value	12.6455 19 0.8562

Notes: t-statistics are in parentheses. The system of equations (9), where all parameters, except α_{0M}, δ_{0M} and δ_{0i} (i=1,2,3), are constrained to be zero, is estimated by GMM with the following instruments: constant and 2 lags of market return (R_{Mt}) for market and portfolio returns; constant and 2 lags of portfolio covariances with market return ($u_{it}u_{Mt}$) for portfolio covariances; constant and 2 lags of market return variance (u_{Mt}^2) for market variance.

Table 4
Conditional CAPM with ARCH
Conditional Variances and Covariances

Parameters of the Market Portfolio			
	α_{0M}	α_{1M}	
Conditional Mean	0.0137 (1.40)	0.0206 (0.64)	—
	δ_{0M}	δ_{1M}	δ_{2M}
Conditional Variance	0.0033 (1.67)	0.4230 (15.63)	0.4760 (12.64)
Parameters of the Portfolio Conditional Covariances			
Portfolios	δ_{0i}	δ_{1i}	δ_{2i}
1	0.0087 (1.00)	0.3179 (3.91)	0.2656 (3.60)
2	0.0127 (2.38)	0.2080 (4.02)	0.2853 (6.79)
3	0.0485 (5.18)	-0.2300 (-4.08)	-0.2547 (-5.83)
Test of Orthogonality Conditions	J 10.0464	d.f. 10	p-value 0.4364

Notes: t-statistics are in parentheses. The system of equations (9), with $k_M = 1$, $k_M^2 = 2$, $k_i = 2$ (i = 1,2,3), is estimated by GMM with the following instruments: constant and 2 lags of market return (R_{Mt}) for market and portfolio returns; constant and 2 lags of portfolio covariances with market return ($u_{it}u_{Mt}$) for portfolio covariances; constant and 2 lags of market return variance (u_{Mt}^2) for market variance.

Table 5
Two-Factor Model with Constant Factor Loadings:
Market Portfolio and CD

Parameters of the first factor (Market Portfolio)			
		α_{0M}	
Mean		0.0214 (4.85)	
		δ_{0M}	
Variance		0.0409 (78.25)	
Parameters of the second factor (CD)			
		α_{0F}	
Mean		-0.0014 (-2.22)	
		δ_{0F}	
Variance		0.00002 (3.86)	
Parameters of the Portfolio Covariances			
Factors		Market Portfolio	CD
Portfolios		δ_{0iM}	δ_{0iF}
1		0.0386 (23.43)	1.38e-05 (0.20)
2		0.0337 (26.77)	-0.0001 (-1.82)
3		0.0382 (38.92)	-0.0006 (-3.47)
Test of Orthogonality Conditions	J d.f. P-value	13.2089 35 0.9997	

Notes for Table 5: t-statistics are in parentheses. The system of equations (14),

where all parameters, except α_{0M} , δ_{0M} α_{0F} , δ_{0F} and δ_{0iM} and δ_{0iF} , are constrained to be zero, is estimated by GMM with the following instruments: constant and 2 lags of market return (R_{Mt}) for market return; constant and 2 lags of CD return (R_{Ft}) for CD return; constant and 2 lags of portfolio covariances with market return ($u_{it}u_{Mt}$) for portfolio covariances with market return; constant and 2 lags of portfolio covariances with CD return ($u_{it}u_{Ft}$) for portfolio covariances with CD return; constant and 2 lags of market return variance (u_{Mt}^2) for market variance; constant and 2 lags of CD return variance (u_{iFt}^2) for CD variance; constant, 2 lags of market return (R_{Mt}), 2 lags of CD return (R_{Ft}) for portfolio returns.

Table 6
Two-Factor Model with ARCH
Conditional Variances and Covariances

Parameters of the first factor (Market Portfolio)						
	α_{0M}			α_{1M}		
Conditional Mean	0.0160 (1.52)			-0.0654 (-2.18)		
	δ_{0M}			δ_{1M}		
Conditional Variance	0.0320 (8.15)			0.1205 (4.97)		
Parameterms of the second factor (CDB)						
	α_{0F}			α_{1F}		
Conditional Mean	-0.0015 (-1.88)			0.0166 (1.80)		
	δ_{0F}			δ_{1F}		
Conditional Variance	4.1e-06 (1.22)			0.0005 (1.25)		
Parameters of the Portfolio Conditional Covariances						
Factors	Market Portfolio			CD		
Portfolios	δ_{0iM}	δ_{1iM}	δ_{2iM}	δ_{0iF}	δ_{1iF}	δ_{2iF}
1	0.0218 (2.25)	0.2630 (3.78)	-0.1975 (-4.57)	-1.0e-05 (-0.17)	0.0595 (2.18)	-0.0827 (-1.71)
2	0.0293 (7.51)	-0.0516 (-2.59)	0.0353 (1.66)	-1.7e-05 (-0.80)	0.0045 (1.61)	-0.0486 (2.43)
3	0.0304 (9.50)	0.1204 (7.21)	0.0019 (0.28)	-0.0001 (-1.63)	-0.0005 (-0.22)	-0.0046 (1.39)
Test of Orthogonality Conditions	J 11.0059		d.f 19		p-value 0.9236	

Notes for Table 6: t-statistics are in parentheses. The system of equations (14), with

k_M, k_M^2, k_F, and k_F^2 equal to 1, and k_{iM} and k_{iF} equal to 2, is estimated by GMM with the following instruments: constant and 2 lags of market return (R_{Mt}) for market return; constant and 2 lags of CD return (R_{Ft}) for CD return; constant and 2 lags of portfolio covariances with market return ($u_{it}u_{Mt}$) for portfolio covariances with market return; constant and 2 lags of portfolio covariances with CD return ($u_{it}u_{Ft}$) for portfolio covariances with CD return; constant and 2 lags of market return variance (u_{Mt}^2) for market variance; constant and 2 lags of CD return variance (u_{iFt}^2) for CD variance; constant, 2 lags of market return (R_{Mt}), 2 lags of CD return (R_{Ft}) for portfolio returns.

Table 7
Calculated Portfolio Betas for the Models

Models	Portfolio 1 (Low Capitalization)		Portfolio 2 (Medium Capitalization)		Portfolio 3 (High Capitalization)	
1- CAPM with Constant Beta	0.9675		0.9513		1.6331	
2- Conditional CAPM with ARCH Market (mean)	1.1017		1.2815		3.2328	
3- Factor Model with Constant Factor Loadings	Market Portfolio	CD	Market Portfolio	CD	Market Portfolio	CD
	0.94	0.69	0.82	-5.00	0.93	-30.00
4- Conditional Factor Model (mean)	Market Portfolio	CD	Market Portfolio	CD	Market Portfolio	CD
	0.58	-0.96	0.82	-1.17	0.95	-27.62

Table 8
Diagnostic Tests on Residuals

	6 Lags Residuals	6 Lags Re	January Dummy	Risk Free rate	Dividend Yield
v_{1M}	2.6530	9.8864	1.4334	3.4801	4.5456
v_{2M}	12.2732	8.1239	2.9404	9.0804	2.2916
v_{3M}	12.4749	10.2905	1.6851	9.9797	1.3459
v_{1F}	191.8595	6.8963	0.8751	12.3604	8.8475
v_{2F}	171.4355	7.9183	0.8240	14.1972	2.0839
v_{3F}	26.4639	5.9086	0.8084	5.6555	2.2142
u_M	2.0472	2.3051	1.3603	1.6926	2.6261
u_F	50.5790	465.5397	0.4684	10.7349	2.1296
u_1	0.1439	0.0283	3.85E-05	2.79E-16	0.0030
u_2	0.0004	7.55E-05	0.0076	1.03E-15	0.0033
u_3	1.23E-05	0.0001	0.0411	2.31E14	30.0435
v_M	4.9013	10.9817	1.8765	14.2590	1.7425
v_F	560.4755	5.0561	2.8866	47.1011	64.1722
df	6	6	1	3	3
$\chi^2_{5\%}$	12.5916	12.5916	3.8415	7.8147	7.8147
$\chi^2_{1\%}$	16.8119	16.8119	6.6439	11.3449	11.3449

Figure 1
Market Excess Returns in Local Currency

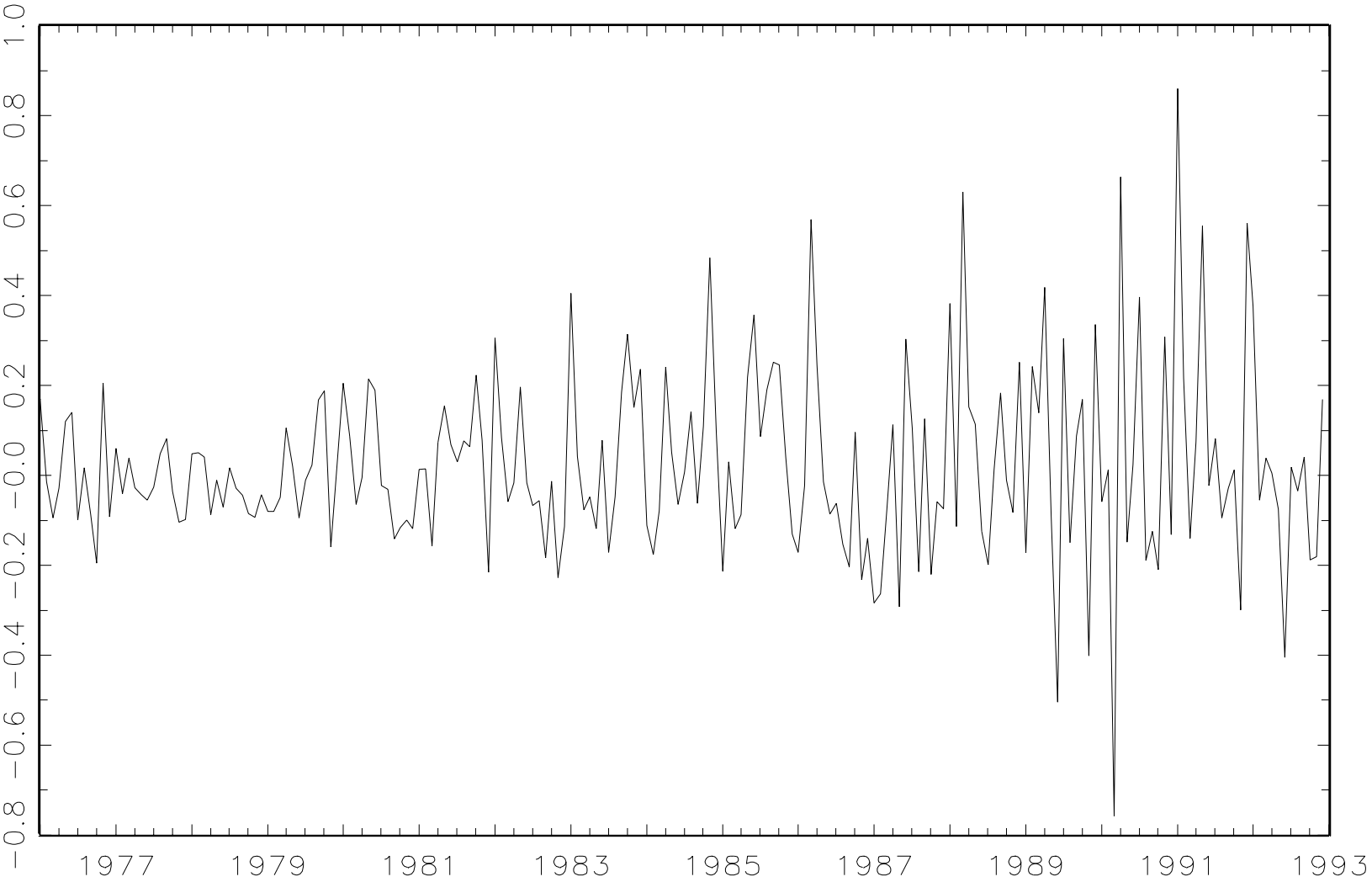


Figure 2

Size Portfolio Excess Returns in Local Currency

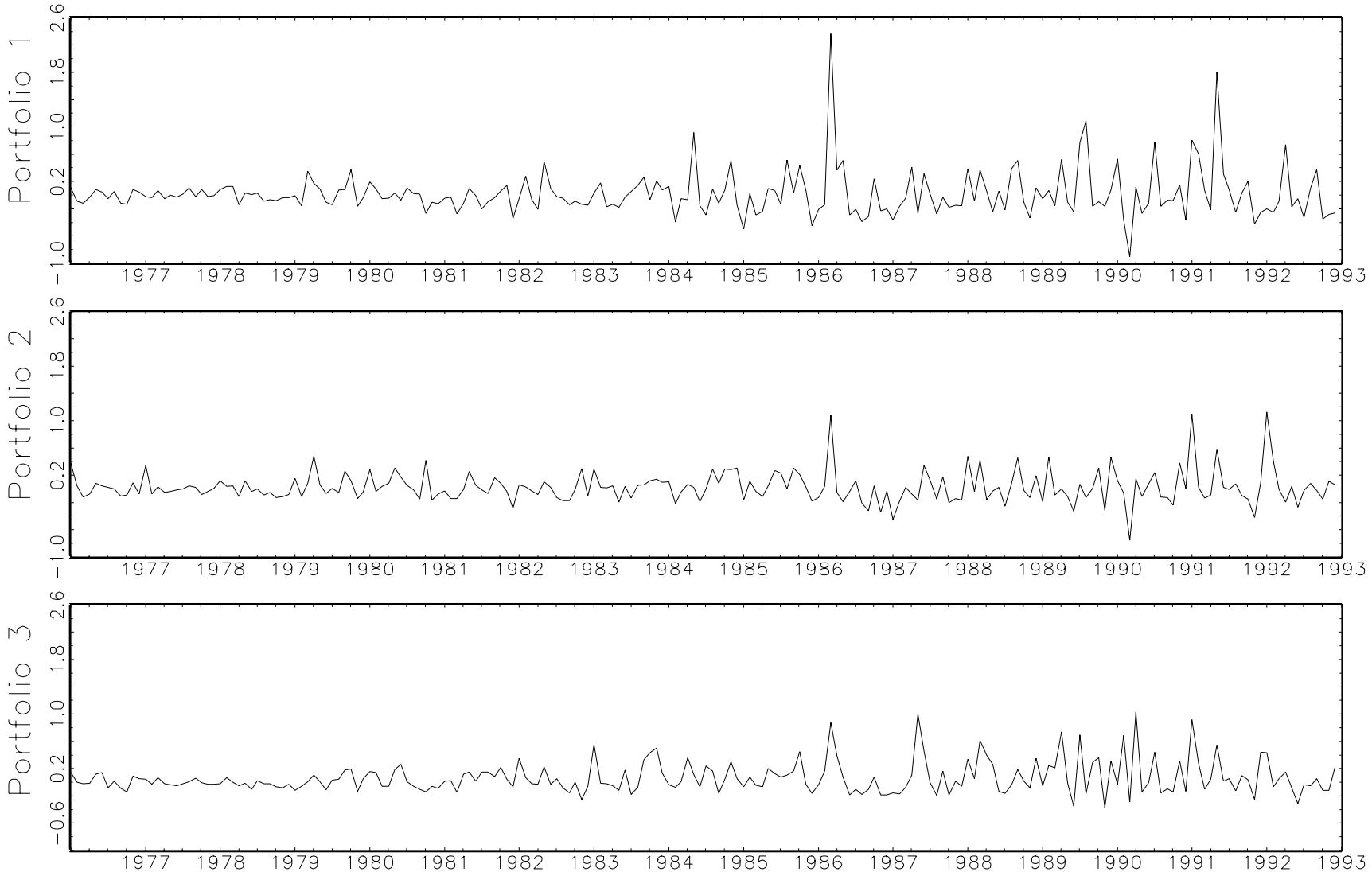


Figure 3
Conditional Market Betas for the CAPM Model

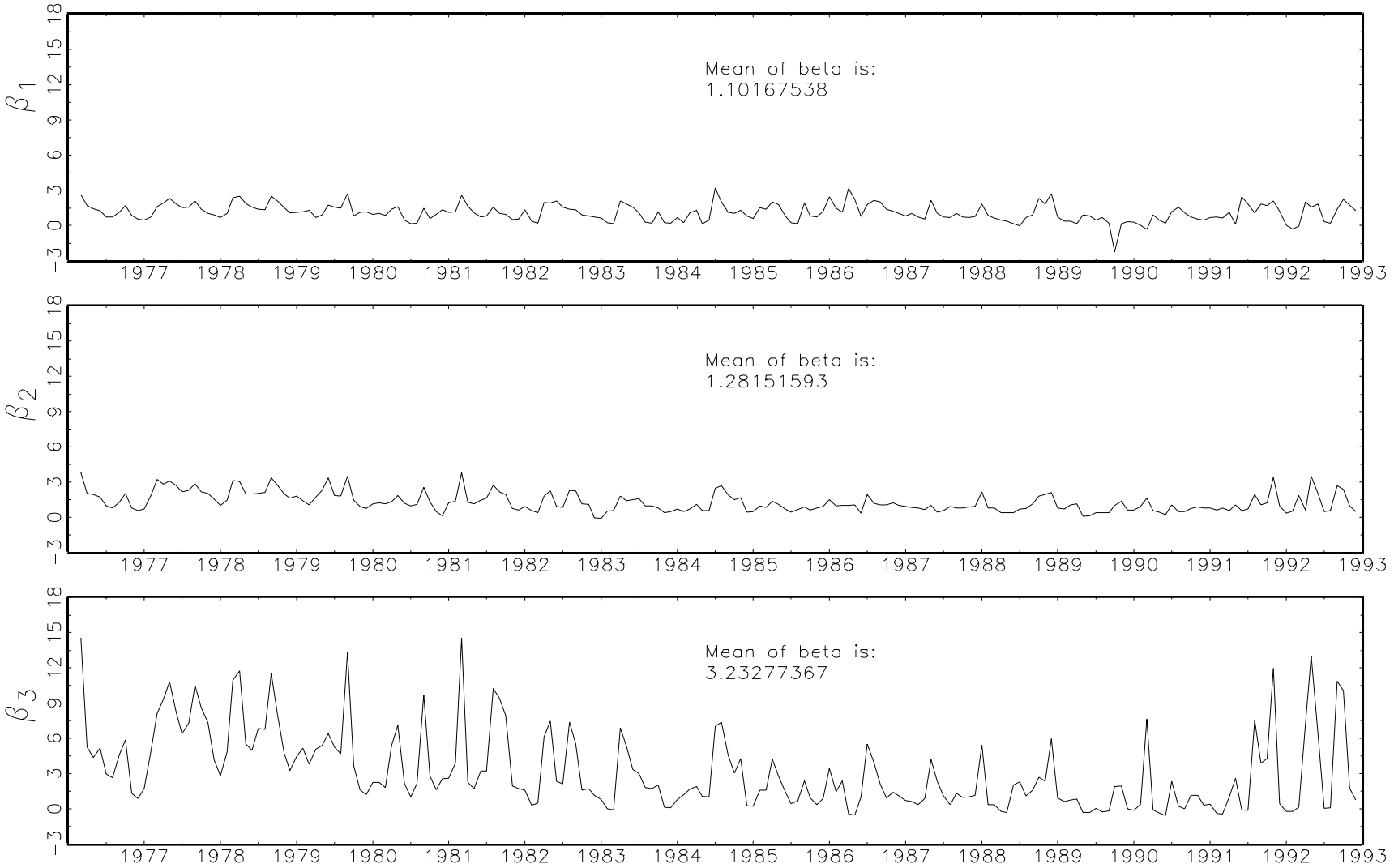


Figure 4
Conditional Market Betas for the Two–Factor Model

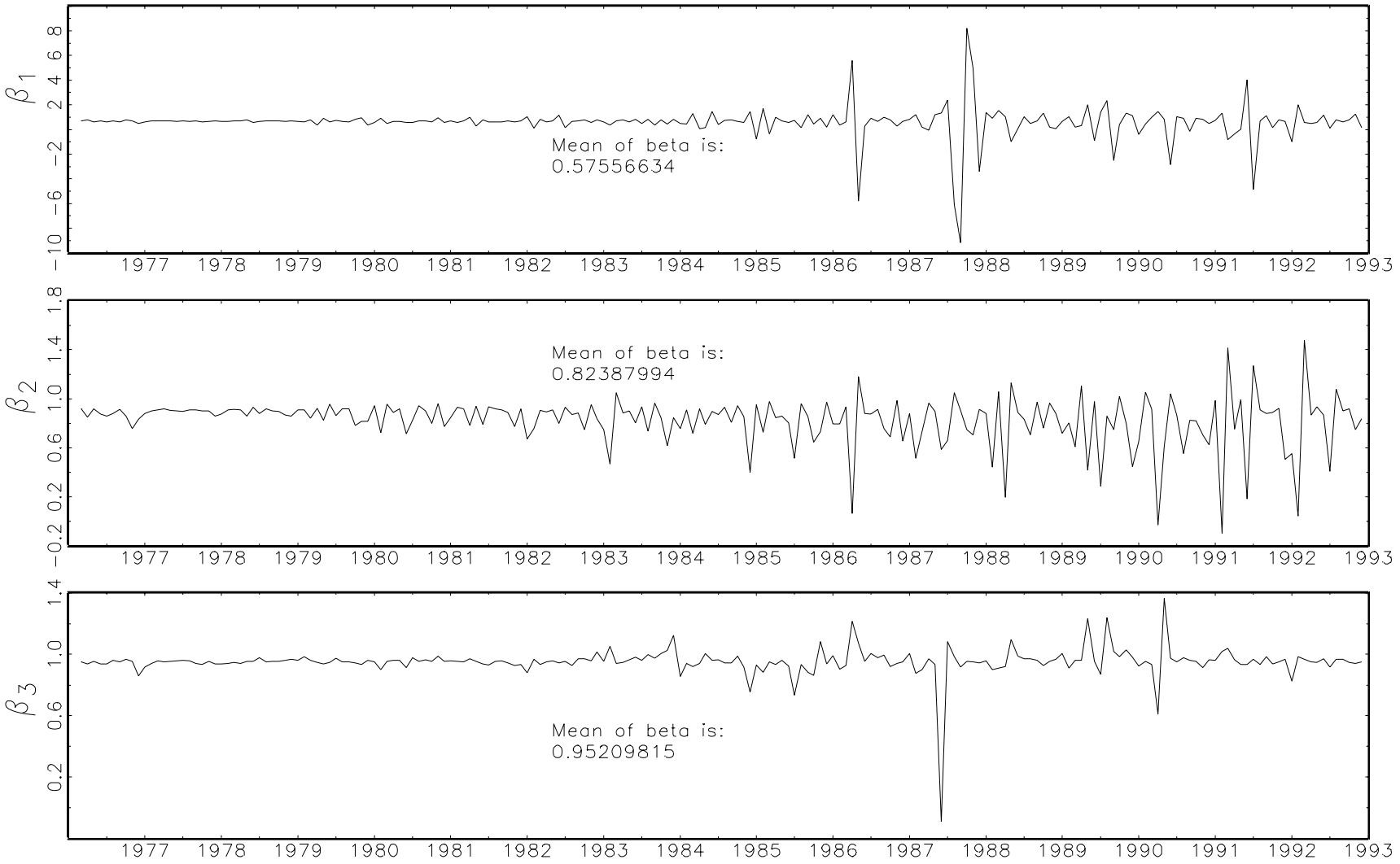
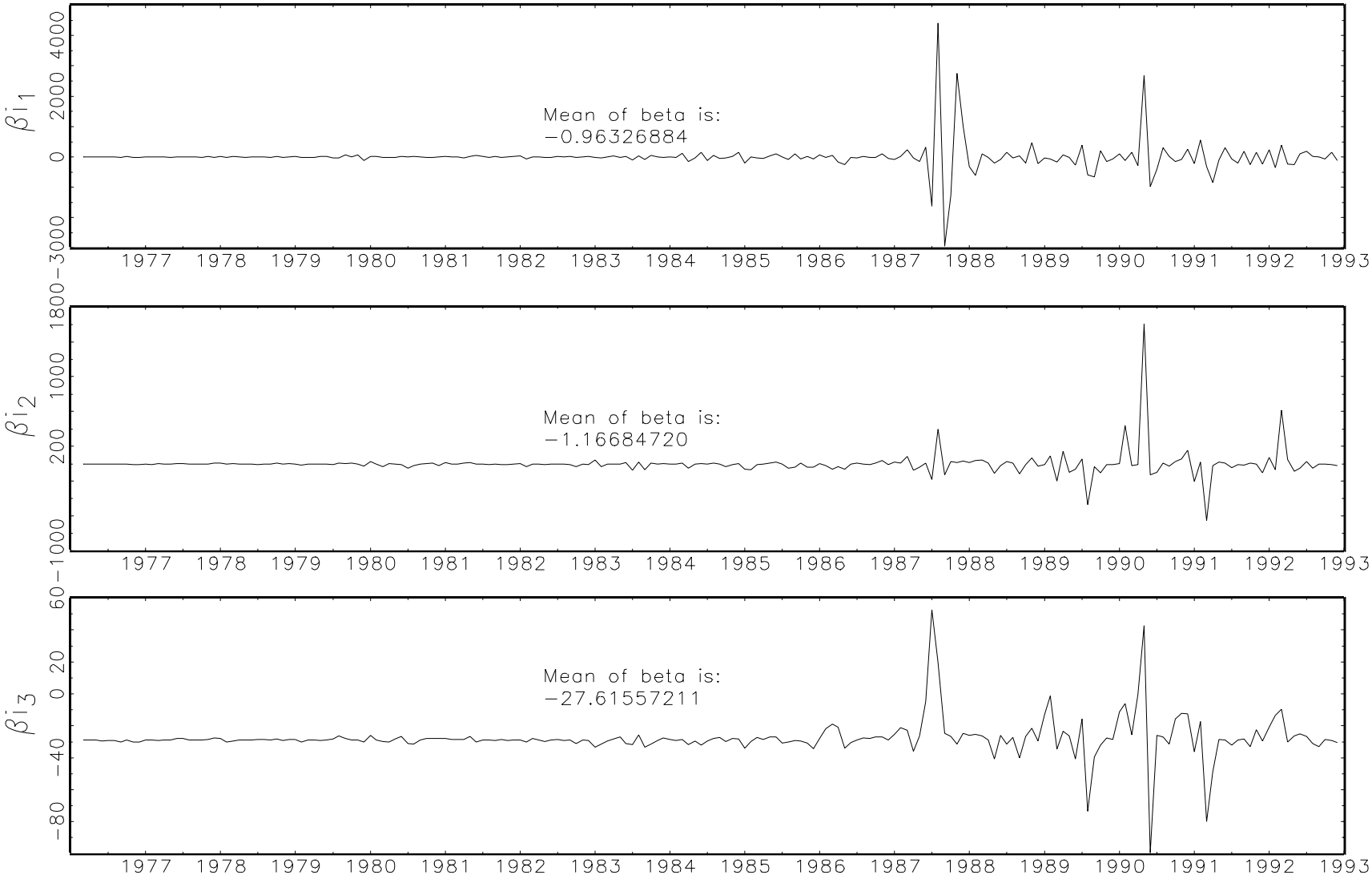


Figure 5
Conditional CD Betas for the Two-Factor Model



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