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Harry J. Paarsch, Bruce Shearer

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Piece Rates, Fixed Wages, and Incentive Effects: Statistical Evidence from Payroll Records*

Harry J. Paarsch†, *Bruce Shearer*‡

Résumé / Abstract

Nous mesurons le gain de productivité réalisé quand les travailleurs sont payés à la pièce plutôt qu'à taux fixe; i.e., l'effet incitatif. Nos données proviennent des archives d'une compagnie qui s'occupe de la plantation d'arbres en Colombie-Britannique. Cette compagnie a payé ses travailleurs à la pièce et à taux fixe. Nos données contiennent des informations sur la productivité et sur les salaires quotidiens des travailleurs sur une période de presque six mois. De plus, nous observons les mêmes travailleurs sous les deux systèmes de paye, ce qui nous permet de contrôler les effets spécifiques aux individus dans les données. Nous développons et estimons un modèle simple du genre principal agent pour analyser le comportement des travailleurs sous les deux systèmes de paye. Le modèle implique un choix optimal de la part de la firme du système de paye en fonction des conditions de plantation. Nous utilisons le modèle pour déterminer les implications statistiques de la mesure de l'effet incitatif. Nous montrons que bien que les méthodes de régression simple ne sont pas capables d'identifier l'effet incitatif (à cause de l'endogénéité du système de paye), elles peuvent être utilisées pour calculer les bornes inférieures et supérieures de l'effet incitatif. Nous évaluons ces bornes à 5% et 32% de la productivité observée quand les travailleurs sont payés à la pièce. De plus, nous montrons que le modèle peut être estimé de façon structurelle en incluant directement dans l'estimation la décision optimale de la part de la firme au niveau de son choix du système de paye. Nos résultats structurels suggèrent que l'effet incitatif compte pour 9,1% de la productivité observée quand les travailleurs sont payés à la pièce.

We estimate the gain in productivity that is realized by paying workers piece rates rather than fixed wages; i.e., the incentive effect. Our data come from the payroll records of a British Columbia tree-planting firm that paid its workers both piece rates and fixed wages. These data contain information on

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* Correspondence Address: Bruce Shearer, CIRANO, 2020 University Street, 25th floor, Montréal, Qc, Canada H3A 2A5 Tel: (514) 985-4002 Fax: (514) 985-4027 e-mail: shearerb@cirano.umontreal.ca
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† University of Iowa

‡ Université Laval, CRÉFA and CIRANO

the daily productivity of workers over a period of nearly six months. Furthermore, we observe the same workers under both piece rates and fixed wages, allowing us to control for individual-specific effects in the data.We develop and estimate an agency model of worker behaviour under piece rates and fixed wages. The model implies optimal decision rules for the firm's choice of a compensation system as a function of planting conditions. We use the model to derive statistical implications for the incentive effect. We demonstrate that while simple regression methods cannot identify the incentive effect (due to the endogeneity of the payment system), they can provide upper and lower bounds to this effect. We estimate these bounds to be 5% and 32% of observed productivity on piece rate contracts. We also demonstrate that the model can be estimated structurally, wherein the firm's optimal choice of a compensation system is built directly into the estimation procedure. Structural results suggest that incentives accounted for 9.1% of observed productivity.

Mots Clés : Systèmes de compensation, effet incitatif, modèles principal-agent

Keywords : Compensation Systems, Incentive Effect, Principal-Agent Models

JEL : D2, J3, L2

1. Introduction

The role of compensation policy in influencing worker performance within the firm has been analysed extensively by economic theorists; see, for example, the work of Grossman and Hart (1982), Hart and Holmstrom (1985), and Holmstrom and Milgrom (1987). Recently, economists have begun to investigate the empirical implications of this research. Of particular interest has been the effects that compensation policies have on worker productivity, often called "incentive effects". Examples of this research include Lazear (1996) as well as the papers in Blinder (1990) and Ehrenberg (1990). One of the main obstacles to empirical work in this area is that compensation schemes cannot, in general, be considered exogenous. For example, Brown (1990) presents evidence that compensation policies are related to the observable characteristics of the firm. This suggests that the effects of compensation on worker productivity must be modelled simultaneously with the choice of a compensation system by the firm. In this paper, we present a first step toward this goal. We develop a model of the firm's choice of a compensation system that takes account of the worker's performance under the different options. We focus on the choice between paying a fixed-wage contract, under which a worker's wage is independent of his output, and a piece-rate contract, under which a worker's wage is an explicit function of his output. The model permits us to decompose the difference in observed productivity between piece-rate and fixed-wage contracts into an *incentive effect* (the difference in observed productivity due to changes in effort, holding conditions constant) and a *treatment effect* (the difference in observed productivity due to changing conditions beyond the worker's control). We estimate the size of the incentive effect using payroll data collected from a tree-planting firm located in the Province of British Columbia, Canada.

In many ways, the tree-planting industry is well-suited to measuring incentive effects. Worker output is easily observable and compensation systems vary across and (sometimes) within firms. We have collected data from a firm which wishes to remain anonymous and which paid its workers both fixed wages and piece rates. Our data contain information concerning the daily productivity and wages of tree planters, working under both fixed wages and piece rates, over a planting season in 1991, nearly six months.

A number of practical reasons also exist for studying the tree-planting industry in British Columbia. First, British Columbia produces around twenty-five percent of the softwood lumber in North America.[1] Any policies affecting timber supply within this province can have important implications for North American lumber markets. Next, the scope of reforestation in British Columbia is huge. At its peak, between 1981 and 1985, almost 2 billion seedlings were planted, an average of 400 million seedlings each year. While the pace of reforestation has slowed somewhat since then, approximately 200 million seedlings are still planted each year in British Columbia. Because the scope of planting is so large, small improvements in personnel policy can result in large savings. For example, an average tree costs about $0.50 to plant, so a ten percent saving can yield about $10 million per year. Third, conservationalists and environmentalists have actively supported improved methods of reforestation, but devising such methods requires information concerning how successful past and current methods have been as well as information concerning why

[1] For example, when statistics are reported for Canada, they are reported as "East of the Rockies" and British Columbia. Moreover, British Columbia is broken up into three regions – the coast, the southern interior, and the northern interior – each of which produces more timber than any province of Canada or state of the Union.

these efforts have or have not been successful. Finally, there has been a recent movement a foot to legislate that all workers be paid a fixed wage in the tree-planting industry. Firms in the industry have resisted this sort of change; we provide the first detailed evidence concerning why this change could make reforestation in British Columbia disasterously more expensive.

Controlling for treatment effects requires modelling the firm's decision to pay fixed wages or piece rates. Our model of contractual choice is based on interviews conducted with the managers of the firm which provided our data. In this manner, the paper resembles a case study. Yet, to the extent that workers are representative, our estimates on the size of the incentive effect have general applicability. Researchers working on models of contractual choice have developed a number of possible explanations for choosing one contract over another. These include risk aversion and transaction costs associated with determining the contract [Cheung (1969) and Ferrall and Shearer (1994)], quality-versus-quantity considerations [Stiglitz (1975) and Lazear (1986)], and financial constraints [Laffont and Matoussi (1995)]. We focus on the quantity-quality trade-off. Piece rates have the advantage of providing incentives for workers to work hard and consequently to plant a lot of trees. Yet, piece rates also reduce the incentive to the worker of producing quality output.

Workers are hired into the tree-planting industry to plant trees on land that has recently been logged. The planting process is well-suited for piece-rate pay since output is easily observable. In the extreme, one merely has to subtract the number of trees that the worker brings back at the end of the day from the number he was issued. Yet, planting conditions can vary a great deal from site to site, and these conditions determine the level of difficulty of planting trees. If conditions are very difficult, then workers may spend less time and effort ensuring that the trees are planted properly. Poorly planted trees will not survive, and this exposes the firm to possible fines by the government and the loss of future contracts. Thus, the quality of planting is a major concern of firms as well.

In our model, we incorporate asymmetric information between workers and the firm over planting conditions. The productivity of a worker is a function of his effort level and a random productivity shock. The productivity shock captures the planting conditions that are beyond the worker's control. Workers are assumed to observe the value of the productivity shock before they choose their effort level. These conditions, along with the method of pay, determine the number of trees the worker plants and whether the trees are planted well. In general, the value to the worker of planting trees well is more sensitive to conditions than is the value of planting trees poorly. A well-planted tree must be placed properly in the ground and appropriately distanced from the trees around it. If conditions are difficult, then planting properly will be very time consuming. For example, if the ground is very hard, then digging the holes to plant the trees will take more time and effort than if the ground is loose. Similarly, if the terrain is steep, then ensuring that trees are adequately spaced will require more time and effort than if the terrain is flat. This reduces the relative value of planting trees well. Planting poorly depends less on conditions since, in the limit, the worker can simply throw his trees away.

The key to the model is the different incentives embodied in the two payment schemes. Under fixed wages, the worker will always provide the minimum possible effort level. Since his pay is independent of his output, no benefit accrues to planting extra trees. Under piece rates, an incentive exists to plant extra trees. However, depending on planting conditions, the worker may prefer to plant these trees well or poorly. The worker cheats when conditions are poor because the effort cost associated with planting well is prohibitively high.

The firm can only observe the parameters of the distribution of production shocks (e.g., average conditions), and the total number of trees planted. We solve the model for the firm's optimal choice of compensation scheme as a function of these parameters. Evident from our discussions with firm managers is that piece rates are set such that they are inversely related to planting difficulty. We specify conditions under which the equilibrium piece rate will be negatively correlated with planting conditions and derive the implications for the observed data. We demonstrate that the firm is more likely to choose a fixed-wage compensation system when average planting conditions are poor. Under such conditions, paying the worker piece rates implies a high probability that the worker will cheat.

We derive the empirical implications of the model at several levels. First, we demonstrate that the difference between observed productivity under piece-rate contracts and fixed-wage contracts will be larger than the corresponding difference in observed total real wages. This result derives from the labour supply constraint of the worker; viz., despite low productivity under fixed-wage contracts, the worker must be paid enough money to keep him at the firm. Second, we show that the incentive effect is bounded from above by the observed difference in average productivity between the two types of contracts and bounded from below by the observed difference in real wages. Finally, we show that the model can be estimated structurally, permitting a more precise measurement of the incentive effect. We use the model to put structure on data collected from the payroll records of a tree-planting firm. We bound the incentive effect between 60 and 366 trees per day, or between 6% and 37% of observed output on piece-rate contracts. Our results from structural estimation suggest the incentive effect is equal to 88.53 trees per day, or 9.1% of observed output on piece-rate contracts.

Our results are consistent with those reported elsewhere. For example, Lazear (1996) attributes a 20% increase in individual worker productivity resulting from the introduction of a pay-for-performance system in an auto-glass company. In his review of early case-study evidence, Lawler (1971) reports that the introduction of individual incentive plans increased worker productivity between 10 and 20%. Our structural results lie at the lower end of this range suggesting the importance of controlling for endogeneity in the estimation of these models. Mitchell, Lewin and Lawler (1990) estimate the productivity gain due to profit sharing plans to be 8.4%. Jones and Kato (1995) estimate the productivity gain resulting from employee stock ownership plans (ESOP) to be 4 to 5%. The larger effects of individual incentive plans maybe due to an absence of the free-riding problem that plagues group-incentive schemes such as profit sharing and ESOPs.

The paper is organized as follows. In the next section, we discuss briefly the tree-planting industry in British Columbia. In section 3, we develop a model of contractual choice. In section 4, the empirical implications of the model are derived, while in Section 5, we discuss the data, and in section 6, we present the results. We conclude in section 7.

2. Tree Planting in British Columbia

While timber is a renewable resource, active reforestation can increase the speed at which forests regenerate and thus ensures a steady supply of timber to the North-American market. Extensive reforestation is undertaken by both the Ministry of Forests and the major timber-harvesting firms who hold Tree Farm Licenses.[2]

The mechanics of this reforestation are relatively straightforward. Prior to the harvest of any tract of coniferous timber, random samples of cones are taken from the trees of the tract, and seedlings are grown from the seeds contained in these cones. This ensures that the seedlings to be replanted are compatible with the local microclimates and soil as well as representative of the historical species composition.

Tree planting is a simple, yet physically exhausting, task. It involves digging a hole with a special shovel, placing a seedling in this hole, and then covering its roots with soil, ensuring that the tree is upright and that the roots are fully covered. The level of energy required to perform the task depends on the terrain on which the planting is done. In general, the terrain can vary a great deal from site to site. In some cases, after a tract has been harvested, the land is prepared for planting by burning whatever slash timber remains and by "screefing" the forest floor. Screefing involves removing the natural build-up of organic matter on the forest floor so that the soil is exposed. Screefing makes planting easier because seedlings must

[2] In British Columbia, nearly 90 percent of all timber is on government-owned (Crown) land. Basically, the Crown, through the Ministry of Forests, sells the right to harvest the timber on this land in two different ways. The most common way is through administratively set prices to thirty-four firms who hold Tree Farm Licenses. These licenses have been negotiated over the last three-quarter century, and require that the licensee adopt specific harvesting as well as reforestation plans. About 90 percent of all Crown timber is harvested by firms holding Tree Farm Licenses. The second, and less common way, to sell timber is at public auction through the Small Business Forest Enterprise Program. In this case, the Ministry of Forests assumes the responsibility of reforestation.

be planted directly in the soil. Sites that are relatively flat or that have been prepared are much easier to plant than sites that are very steep or have not been prepared. The typical minimum density of seedlings is about 1,800 stems per hectare, or an inter-tree spacing of about 2.4 meters, although this can vary substantially.[3] An average planter can plant about 900 trees per day, about half a hectare. An average harvested tract is around 250 hectares, so planting such a tract would take about 500 man days.

Typically, tree-planting firms are chosen to plant seedlings on harvested tracts through a process of competitive bidding. Depending on the land tenure arrangement, either a timber-harvesting firm or the Ministry of Forests will call for sealed-bid tenders concerning the cost per tree planted, with the lowest bidder being selected to perform the work. The price received by the firm per tree planted is called "the bid price." Bidding on contracts takes place in the late autumn of the year preceding the planting season which runs from early spring through to late summer. Before the bidding takes place, the managers of the tree-planting firms typically view the land to be planted and estimate the cost at which they can complete the contract. This estimated cost depends on the expected number of trees that a planter will be able to plant in a day which in turn depends on the general conditions of the area to be planted.

Tree-planting firms are typically quite small, usually having fewer than one hundred workers. These workers are generally paid using either piece-rate contracts or fixed-wage contracts. Under piece-rate contracts, workers are paid in proportion to their output; generally, no explicit base wage or production standard exists (although firms are governed by mini-

[3] One hectare is an area 100 metres square, or 10,000 square metres. Thus, one hectare is approximately 2.4711 acres.

mum wage laws). Output is typically measured in terms of the number of trees planted, although some area-based schemes are used as well.[4] Under fixed-wage contracts, worker pay is independent of output.

Tree-planting firms are mainly concerned with two aspects of worker output: quantity and quality. While quantity is important for the obvious reasons, the quality of planted seedlings determines the survival rate of those seedlings. The survival of seedlings depends on several aspects of the planting process. First, the planting spot must be acceptable. Unacceptable sites include areas of depression that are subject to flooding, areas next to major access roads, and areas under overhead obstacles that would interfere with growth. Second, there must be adequate spacing both between newly planted trees and between newly planted trees and existing trees. Third, the actual planting spot must be prepared correctly; i.e., the planting hole must be deep enough and wide enough to ensure that the root system will not be damaged during planting. Finally, the seedling must be placed vertically into the planting hole, so that the roots are not folded over, and the hole must be filled back in and firmly tamped down. These aspects of planting are monitored by the government or timber harvesting company. Poorly planted trees can cause tree-planting firms to be fined and can also hurt their chances of winning future contracts.

Firms can induce quality both by monitoring workers and by fining them for poorly planted trees. Planters work under the direction of a foreman who is responsible for monitoring the output of the workers. The foreman's pay is usually related to the performance of the workers under his direction, either in terms of a bonus or as residual claimant. Most firms employ one monitor for about every ten workers and this ratio is

[4] An area-based scheme is one under which workers are paid in proportion to the area of land they plant in a given day.

relatively constant across contracts. The schedule of fines is also relatively constant across contracts. This practice is seemingly inconsistent with the predictions of theoretical models of contracts which suggest that these elements of the contract should vary in response to changing planting conditions. One possible reason for keeping these elements constant is that there may be transaction costs associated with changing them every time conditions change.[5]

The majority of planting is done under piece rates. Areas to be planted are divided into plots. For each plot, the firm decides on a piece rate. This piece rate takes into account the expected number of trees that a planter will plant in a day and the expected wage of the worker. Thus, the piece rate will be negatively correlated with the planting conditions. Fixed wages tend to be paid when the conditions on the plot render piece rates infeasible. For example, if the conditions render planting very slow and difficult, then the value of the piece rate the firm must pay will be very high. This, coupled with the fact that the worker may prefer to cheat when planting is difficult, increases the value to the firm of a fixed-wage contract.

3. Theoretical Model

In our model, we consider the trade-off between quality and quantity in the tree-planting industry. A worker's productivity is a function of his effort level E and a productivity shock S. The productivity shock represents planting conditions that are beyond the worker's control such as the slope of the terrain, hardness of the ground, weather conditions, and the amount of ground cover. Throughout our analysis we take the bid

[5] For a related argument and evidence on the extent that transaction costs play in determining the form of observed contracts, see Ferrall and Shearer (1994).

price to be fixed; i.e., the productivity shock is drawn *after* the bid price is determined. This assumption captures the fact that planting conditions can change between the time that contracts are viewed for bidding and the time that planting takes place. Recall that the bidding takes place in the autumn of the year preceding planting. Changes in the terrain could occur, for example, due to severe weather blowing down existing trees or due to the extent of the underbrush changing. Bid prices can vary across contracts, something that will be accounted for in the empirical section of the paper.

We assume that S has a log-normal distribution; i.e., $\log S$ is distributed normally with mean μ and variance σ_S^2, so

$$f_S(s) = \frac{1}{s\sigma_S}\phi\left(\frac{\log s - \mu}{\sigma_S}\right)$$

where ϕ represents the standard normal probability density function. To capture the effect of monitoring, we allow for a minimum level of effort $\bar{e}$, greater than zero, that is enforceable by the firm. This effort level ensures that a certain number of trees are planted, and planted well. Beyond this minimum level of productivity, the worker can choose to plant a number of additional trees. Furthermore, he can choose to plant these additional trees well or poorly. To keep the model simple, we restrict the choice to be one or the other; i.e., the worker has discretion over the number of additional trees to plant, but either he plants them all well or he plants them all poorly. He cannot plant some of the extra trees well and some of them poorly. The relative benefits of planting well versus poorly will depend on the manner in which workers are paid and the conditions under which they are planting.

Let the productivity of a worker be given by the production function

$$
\begin{aligned}
&Y = Y_w + Y_p, \\
&Y_w = s(\bar{e} + E), \qquad (1) \\
&E \geq 0,\ Y_p \geq 0,\ EY_p = 0,
\end{aligned}
$$

where Y_w is the number of well planted trees, Y_p is the number of poorly planted trees, and E is worker effort devoted to planting additional trees well. Note that for s, a particular shock, $\bar{e}\,s$ trees will always be planted well. The condition that EY_p equals zero ensures that additional trees are either planted well or planted poorly. Note also that the number of poorly planted trees is independent of planting conditions. This reflects the fact that a worker can simply throw his trees away. For the same reason, no additional effort is required to plant trees poorly.

The worker's choice variables are E and Y_p; we denote the actual choices of these variables as e and y_p respectively. Furthermore, the worker is restricted to make one of the following three choices:

(i) $e = y_p = 0$ (provide only the minimum level of effort);

(ii) $e = 0, y_p > 0$ (plant additional trees poorly);

(iii) $e > 0, y_p = 0$ (plant additional trees well).

Trees that are planted poorly are detected with probability π by the government.[6] In the event that poorly planted trees are detected, the firm must pay a penalty γY_p^2 to the government. We also allow the firm to collect a penalty βY_p^2 from the worker. While the firm may have some

[6] We develop our model as a relationship between a tree-planting firm and the government. In practice, tree-planting firms are also hired by private timber-harvesting companies. Our analysis also applies to these relationships.

discretion over the choice of β, we make the simplifying assumption that the government chooses both β and γ. Given that the value of β is, in practice, largely independent of working conditions, this assumption does not appear restrictive. Note that γ may also be interpreted to include the cost of the loss of future reputation by the firm. Throughout the analysis, we assume that the firm cannot pass their penalties onto the workers completely; i.e., β is less than γ.

Workers are assumed to be risk neutral. This allows us to focus on the quality of output as the determinant of compensation system decisions, while abstracting from questions of risk aversion. In addition, several interviews with planters revealed that variation in their daily income was not a major consideration to them. The utility of a worker is defined over payment W, effort E, and poorly planted trees Y_p by

$$U(W, E, Y_p) = W - C(E, Y_p).$$

The cost function is

$$C(E, Y_p) = \begin{cases} \frac{\kappa}{2}(\bar{e} + E^2) & \text{if plant well;} \\ \frac{\kappa}{2}\bar{e} + \pi\beta Y_p^2 & \text{otherwise.} \end{cases}$$

Again, planting poorly involves no cost of effort beyond $\bar{e}$ since trees can be thrown away. They do, however, imply a fine of βY_p^2 with probability π. The worker chooses his effort level, E, and the number of trees to plant poorly, Y_p, to maximize his utility *after* S is revealed. The worker's alternative utility is denoted $\bar{u}$.

The firm can pay the worker a piece rate, or fixed wages; i.e.,

$$W = \begin{cases} \alpha Y & \text{if piece rates;} \\ \omega & \text{if fixed wages,} \end{cases}$$

where ω is independent of output.

The timing of the model is as follows:

0. the government chooses π, β, and γ;
1. the bid price P is determined at auction;
2. Nature chooses the pair (μ, σ_S^2), the parameters of the distribution of S;
3. the firm observes (μ, σ_S^2), and then chooses a payment system and its parameters; i.e., the fixed wage ω or the piece rate α;
4. the worker observes (μ, σ_S^2), and then accepts or rejects the contract;
5. Nature chooses s;
6. the worker observes s, chooses to plant well or poorly, and chooses an effort level producing output Y;
7. the firm observes Y and pays wages.

Note that the parameters of the distribution of S are determined after the bid price is fixed. This reflects the fact that planting conditions can change between the time of bidding and the time of planting. The parameters of the compensation system must be chosen based on the new conditions (μ, σ_S^2) since workers base their labour supply decisions on these conditions; i.e., the parameters of the compensation system must satisfy the worker's labour supply constraint. Furthermore, we assume that workers are mobile: to wit, the expected utility constraint is binding. This implies that the worker is indifferent *ex ante* between planting under piece rates and fixed wages. This assumption simplifies the analysis considerably and also seems consistent with observed practice in the tree-planting industry. Interviews with managers of tree-planting firms revealed that piece rates are set to ensure that workers earn a certain amount of money, on average, per day. This suggests that the expected utility constraint is a binding consideration in the setting of piece rates. We do not, however, impose a constraint on expected profits. This reflects the fact that our analysis is conditional on the bid price. Also, firms typically plant in a number of areas, each of which has a different contract associated with it. Profits need not be positive for each area planted, only on average across all of the contracts.

To solve the model, we work backwards. First, we solve for the worker's optimal action within each compensation system, and then we solve the firm's optimal choice of a compensation system, taking the reaction of the worker as given.

Worker's Problem

(a) *Fixed Wages.*

Under fixed wages, the worker will always supply the minimum level of effort $\bar{e}$. Since his wage is independent of output, there is no benefit to planting extra trees (either well or poorly), and both actions would entail a cost. Planting well entails an extra effort cost, and planting poorly entails a fine (with probability π). Both reduce utility. The wage is determined by the expected utility constraint. Letting $\hat{\omega}$ denote the wage that solves this constraint, we have

$$\hat{\omega} = \frac{\kappa}{2}\,\bar{e} + \bar{u}. \tag{2}$$

(b) *Piece Rates.*

Under piece rates, the worker has an incentive to plant trees in excess of $\bar{e}\,s$, since he is paid according to his output. However, the worker must decide whether to plant these trees well or to plant them poorly. This decision will depend on the indirect utility of each option.

Consider first the option of planting trees well.

$$W = \alpha(\bar{e} + E)s$$

$$U = \alpha(\bar{e} + E)s - \frac{\kappa}{2}(\bar{e} + E^2).$$

After observing s, the worker chooses effort E to equate his marginal benefit of effort to his marginal cost when

$$e = \frac{\alpha s}{\kappa},$$

giving indirect utility of

$$V_w = (\alpha s - \frac{\kappa}{2})\bar{e} + \frac{\alpha^2 s^2}{2\kappa}.$$

If the worker plants poorly, then

$$W = \alpha(\bar{e}s + Y_p) - \pi\beta Y_p^2$$

$$U = \alpha(\bar{e}s + Y_p) - \pi\beta Y_p^2 - \frac{\kappa}{2}\,\bar{e}.$$

The worker chooses Y_p to maximize his utility at

$$y_p = \frac{\alpha}{2\pi\beta},$$

giving indirect utility

$$V_p = (\alpha s - \frac{\kappa}{2})\bar{e} + \frac{\alpha^2}{4\pi\beta}.$$

Note that V_p is independent of conditions beyond the minimum enforceable effort level. This is because the worker can simply throw his trees away.

Conditional on s, the worker will plant well when

$$V_w > V_p$$

$$(\alpha s - \frac{\kappa}{2})\bar{e} + \frac{\alpha^2 s^2}{2\kappa} > (\alpha s - \frac{\kappa}{2})\bar{e} + \frac{\alpha^2}{4\pi\beta}$$

or

$$s > \sqrt{\frac{\kappa}{2\pi\beta}} = s^*.$$

The optimal decision rule of the worker is summarized in Figure 1. The worker plants well when s exceeds s^* (i.e., when conditions are good) and plants poorly when s is less than s^* (i.e., when conditions are bad). This result obtains because the value of planting poorly is independent of conditions. On the other hand, poor conditions reduce the value of planting trees well as the time and effort required to do so become more costly. This raises the relative value of planting poorly. Note that the value of s^* is independent of α since the piece rate is applied to the total number of trees planted independent of their quality.

In order for a worker to accept the contract, it must meet his expected utility constraint. Since the worker's decision to accept or to reject the contract is made before the value of s is revealed, this constraint will depend on the probability that the planter will plant well. Let q denote the probability the worker plants well; i.e.,

$$q = \Pr[S > s^*] = [1 - F_S(s^*)].$$

The expected utility constraint for piece-rate contracts is

$$\alpha\bar{e}\,\mathcal{E}[S] + \frac{q}{\kappa}\alpha^2\,\mathcal{E}[S^2|S > s^*] + \frac{(1-q)}{2\pi\beta}\alpha^2 - \frac{q}{2\kappa}\alpha^2\,\mathcal{E}[S^2|S > s^*] - \frac{(1-q)}{4\pi\beta}\alpha^2 - \frac{\kappa}{2}\,\bar{e} = \bar{u},$$

where $\mathcal{E}$ denotes the expectations operator. The first three terms are the expected gross wages. The fourth term is the expected effort cost of planting trees well. The fifth term is the expected fine due to poorly planted trees, and the sixth term is the cost of enforceable effort. Rearranging, gives

$$\alpha\bar{e}\,\mathcal{E}[S] + \frac{q}{2\kappa}\alpha^2\,\mathcal{E}[S^2|S > s^*] + \frac{(1-q)}{4\pi\beta}\alpha^2 = \frac{\kappa}{2}\,\bar{e} + \bar{u} = \hat{\omega}, \qquad (3)$$

where $\hat{\omega}$ is the fixed wage that would give the worker the same level of utility from (2).

We restrict ourselves to the economically relevant solutions of (3) for which α exceeds zero and denote this solution as $\hat{\alpha}(\mu, \sigma_S, \kappa, p, \beta)$ or $\hat{\alpha}$ for short.

Lemma 1

The value of $\hat{\alpha}$ that solves (3) is unique.

The proof of this lemma, like the proofs of all of our results, is contained in the Appendix.

Using the quadratic formula, it is straight forward to derive the following solution for $\hat{\alpha}$;

$$\hat{\alpha} = \frac{-\bar{e}\,\mathcal{E}[S] + \sqrt{\bar{e}^{\,2}\big(\,\mathcal{E}[S]\big)^2 + \left(\frac{2q}{\kappa}\,\mathcal{E}[S^2|S > s^*] + \frac{(1-q)}{\pi\beta}\right)\left(\frac{\kappa}{2}\,\bar{e} + \bar{u}\right)}}{\frac{q}{\kappa}\,\mathcal{E}[S^2|S > s^*] + \frac{(1-q)}{2\pi\beta}}. \tag{4}$$

We now derive conditions under which $d\hat{\alpha}/d\mu$ is less than zero.

Lemma 2

If the slope of the secant of the hazard function, evaluated between the points

$$\frac{\log s^* - \mu}{\sigma_S} - 2\sigma_S \quad \textit{and} \quad \frac{\log s^* - \mu}{\sigma_S}$$

is less than $1 + \Xi$, $\qquad \Xi > 0$, *then* $\frac{d\hat{\alpha}}{d\mu} < 0$.

In general, this condition states that the hazard function of the log-normal distribution must not be increasing too quickly. An increase in

μ will have opposing effects on expected income. First, it increases the worker's expected income from planting trees well. Second, it reduces the probability that he will plant poorly and so decreases the probability that he will receive $(\alpha/2\pi\beta)$. If the hazard function is increasing very quickly, then increasing μ decreases $\Pr[S|S > s^*]$ very quickly and the expected income from planting trees well receives less weight in the utility function.

For the rest of the paper, we assume that $d\hat{\alpha}/d\mu$ is less than zero in accordance with observed practice in the firm.

Note that expected wages under piece rates $\big(\mathcal{E}[W|p.r.]\big)$ are

$$\mathcal{E}[W|p.r.] = \hat{\alpha}\bar{e}\,\mathcal{E}[S] + q\frac{\hat{\alpha}^2}{\kappa}\,\mathcal{E}[S^2|S > s^*] + (1-q)\frac{\hat{\alpha}^2}{2\pi\beta}. \qquad (5)$$

Denoting the expected wage given fixed wages as $\mathcal{E}[W|f.w.]$ equal to $\hat{\omega}$, and using (3)

$$\mathcal{E}[W|p.r.] = \mathcal{E}[W|f.w.] + q\frac{\hat{\alpha}^2}{2\kappa}\,\mathcal{E}[S^2|S > s^*] + (1-q)\frac{\hat{\alpha}^2}{4\pi\beta}. \qquad (6)$$

Thus, expected wages under piece rates are higher than under fixed wages. The difference in expected wages compensates the worker for his extra costs under piece rates. There are two components to these costs. First, with probability q, the worker will supply a higher level of effort equal to $(\hat{\alpha}s/\kappa)$ towards planting trees well. The cost of this effort to the worker is $(\hat{\alpha}^2 s^2/2\kappa)$. Second, with probability $(1-q)$, the worker will plant $(\hat{\alpha}/2\pi\beta)$ trees poorly. These poorly planted trees will be detected with probability π implying an expected fine to the worker of $(\hat{\alpha}^2/4\pi\beta)$. The expected extra cost to the worker of planting under piece rates is

$$q\frac{\hat{\alpha}^2}{2\kappa}\,\mathcal{E}[S^2|S > s^*] + (1-q)\frac{\hat{\alpha}^2}{4\pi\beta}.$$

Firm's Problem

Taking the worker's reaction as given, the firm will choose to pay piece rates or fixed wages depending on the expected profits of each. Recall that p, the price per tree or bid price, received by the firm for planted trees is fixed.

(a) *Expected Profits under Fixed Wages.*

Given the worker always provides the minimum level of effort $\bar{e}$ under fixed wages, the expected profits per worker are

$$\mathcal{E}[Profits|f.w., p, \mu, \sigma_S] = p\,\mathcal{E}[\bar{e}S] - \hat{\omega}.$$

(b) *Expected Profits under Piece Rates.*

Expected profits (conditional on S being s) given the worker plants well are

$$(p-\hat{\alpha})(\bar{e}+e)s = (p-\hat{\alpha})(\bar{e}+\frac{\hat{\alpha}s}{\kappa})s.$$

If the worker plants poorly, then profits (conditional on S being s) are

$$(p-\hat{\alpha})\bar{e}s+(p-\hat{\alpha})y_p+\pi(\beta-\gamma)y_p^2 = (p-\hat{\alpha})\bar{e}s+(p-\hat{\alpha})\frac{\hat{\alpha}}{2\pi\beta}+\pi(\beta-\gamma)\frac{\hat{\alpha}^2}{4\pi^2\beta^2}.$$

Thus, expected profits from paying piece rates are

$$\mathcal{E}[Profits|p.r., p, \mu, \sigma_S] = q\left[(p-\hat{\alpha})\left(\bar{e}\,\mathcal{E}[S|S>s^*]+\frac{\hat{\alpha}}{\kappa}\,\mathcal{E}[S^2|S>s^*]\right)\right]$$
$$+(1-q)\left[(p-\hat{\alpha})\bar{e}\,\mathcal{E}[S|S<s^*]+(p-\hat{\alpha})\frac{\hat{\alpha}}{2\pi\beta}+\pi(\beta-\gamma)\frac{\hat{\alpha}^2}{4\pi^2\beta^2}\right].$$

(c) *Choice of Payment System.*

Having observed the parameters of the distribution of S, the firm will choose to pay piece rates if and only if the expected profits from doing so are larger than the expected profits from paying fixed wages; i.e.,

$$q\left[(p-\hat{\alpha})\left(\bar{e}\,\mathcal{E}[S|S>s^*]+\frac{\hat{\alpha}}{\kappa}\,\mathcal{E}[S^2|S>s^*]\right)\right]+(1-q)$$
$$\left[(p-\hat{\alpha})\bar{e}\,\mathcal{E}[S|S<s^*]+(p-\hat{\alpha})\frac{\hat{\alpha}}{2\pi\beta}+\pi(\beta-\gamma)\frac{\hat{\alpha}^2}{4\pi^2\beta^2}\right]>\bar{e}p\,\mathcal{E}[S]-\hat{\omega}.$$

But from (3),

$$\hat{\omega}=\hat{\alpha}\bar{e}\,\mathcal{E}[S]+\frac{q}{2\kappa}\hat{\alpha}^2\,\mathcal{E}[S^2|S>s^*]+\frac{(1-q)}{4\pi\beta}\hat{\alpha}^2.$$

Making the appropriate substitution and rearranging gives

$$\begin{aligned} qp\frac{\hat{\alpha}}{\kappa}\,\mathcal{E}[S^2|S>s^*]+(1-q)p\frac{\hat{\alpha}}{2\pi\beta} \;&>\; q\frac{\hat{\alpha}^2}{2\kappa}\,\mathcal{E}[S^2|S>s^*]+ \\ (1-q)\frac{\hat{\alpha}^2}{4\pi\beta}+(1-q)\pi(\gamma-\beta)\frac{\hat{\alpha}^2}{4\pi^2\beta^2}. \end{aligned} \tag{7}$$

The left hand side of (7) is the value of extra output the worker produces under piece rates. The first term is the price times expected number of well planted trees and the second term is the price times the expected number of poorly planted trees. The first two terms on the right hand side are the extra compensation that must be paid to the worker to compensate him for the extra effort he supplies under piece rates. The third term on the right hand side is the expected net fine that the firm must pay for poorly planted trees.

Lemma 3

A necessary condition for the firm to pay piece rates is $\hat{\alpha}\leq 2p$.

If the piece rate required to meet the expected utility constraint is too large relative to the bid price, then the firm will prefer to pay fixed wages. Note, there is no constraint that p exceed $\hat{\alpha}$ here since there is no zero profit constraint. To understand this result, note that the value of the extra output the worker produces under piece-rates is

$$p\frac{\hat{\alpha}}{\kappa}\int_{s^*}^{\infty} s^2 f_S(s)ds + (1-q)p\frac{\hat{\alpha}}{2\pi\beta}.$$

The extra wage that the firm must pay to compensate the worker for his additional effort is

$$\frac{\hat{\alpha}^2}{2\kappa}\int_{s^*}^{\infty} s^2 f_S(s)ds + (1-q)\frac{\hat{\alpha}^2}{4\pi\beta}.$$

Taking the difference between the two shows that the compensating wage differential is greater than the added productivity when $\hat{\alpha}$ exceeds $2p$. Under these circumstances, the firm will never want to pay piece rates.

Lemma 4.

If the worker plants extra trees poorly with probability one, then the firm will pay piece rates only if $\hat{\alpha} < \frac{2\beta}{\gamma}p$.

To interpret Lemma 4, note that rearranging (7) gives

$$\frac{q}{(1-q)} > \frac{\pi(\gamma-\beta)\frac{\hat{\alpha}}{4\pi^2\beta^2} - \left(p-\frac{\hat{\alpha}}{2}\right)\frac{1}{2\pi\beta}}{\left(p-\frac{\hat{\alpha}}{2}\right)\frac{1}{\kappa}\,\mathcal{E}[S^2|S>s^*]}. \tag{8}$$

The numerator on the right hand side of (8) is the net benefit to the firm of the worker cheating. If the firm earns a net gain when the worker cheats, then the firm will always prefer to pay piece rates since there is no added cost to doing so and worker productivity is higher. Rearranging this term yields that piece rates will always be preferred if

$$\pi(\gamma-\beta)\frac{\hat{\alpha}^2}{4\pi^2\beta^2} + \frac{\hat{\alpha}^2}{4\pi\beta} < p\frac{\alpha}{2\pi\beta}.$$

The left hand side is the extra cost to the firm of worker cheating consisting of the fine the firm must pay to the government plus the extra compensation the firm pays to the worker to compensate him for his fine. The right hand side is the extra benefit to the firm of the worker cheating. It is the value of the poorly planted trees that are not detected by the government. If, for example, the firm could pass all the fine onto the workers (i.e., γ equal to β), then the firm would pay piece rates whenever $2p$ exceeds $\hat{\alpha}$, as the firm would bear no cost in the event the worker cheated.

Lemmata 3 and 4 give the extreme cases of selecting a payment scheme. We now turn to the intermediate case of $\hat{\alpha}$ between $(2\beta p/\gamma)$ and $2p$. Note that this set of $\hat{\alpha}$s is not empty since γ exceeds β by assumption. Rearranging (8), piece rates will be paid when

$$(1-q) \leq \frac{\left(p - \frac{\hat{\alpha}}{2}\right)\frac{1}{\kappa}\int_{s^*}^{\infty} s^2 f_S(s)ds}{\frac{1}{2\pi\beta}\left(\pi(\gamma-\beta)\frac{\alpha}{2\pi\beta} - \left(p - \frac{\hat{\alpha}}{2}\right)\right)}.$$

The firm will pay fixed wages when the probability of cheating becomes too high. Recall that $\hat{\alpha}, q$, and $\int_{s^*}^{\infty} s^2 f_S(s)ds$ are functions of μ and p (we take $\sigma_S, \kappa, \beta, \pi$, and γ to be fixed).

Define

$$\psi(\mu|p) \equiv (1-q) - \frac{\left(p - \frac{\hat{\alpha}}{2}\right)\frac{1}{\kappa}\int_{s^*}^{\infty} s^2 f_S(s)ds}{\frac{1}{2\pi\beta}\left(\pi(\gamma-\beta)\frac{\alpha}{2\pi\beta} - \left(p - \frac{\hat{\alpha}}{2}\right)\right)}. \tag{9}$$

Note that the rule for the firm is now

$$\text{system of pay} = \begin{cases} \text{fixed wages} & \text{if } \psi(\mu|p) > 0 \\ \text{piece rates} & \text{if } \psi(\mu|p) \leq 0. \end{cases}$$

Denote $\mu^*(p)$ to be that value of μ that solves $\psi(\mu^*|p)$ equals zero given $\kappa, \pi, \gamma, \beta$, and σ_S. At a given bid price p, the firm is indifferent between paying piece rates and fixed wages along $\mu^*(p)$. We now consider the properties of $\psi(\mu|p)$.

Theorem 1

$\psi(\mu|p)$ is monotonically decreasing in μ if $\frac{d\hat{\alpha}}{d\mu} < 0$.

Theorem 1 demonstrates how general planting conditions affect the form of compensation system chosen by the firm for a given contract. Intuitively, two things occur when general planting conditions are poor, (i.e., the value of μ is low) both of which render piece rates less attractive to firms. First, since poor conditions reduce the expected payment the worker will receive under piece rates, the value of $\hat{\alpha}$ must rise relative to p to meet the expected utility constraint. Second, since low values of S are more likely when μ is low, the probability that the worker will cheat increases. The graph of $\psi(\mu|p)$ is presented in Figure 2. Note that there is a range of distributions, where $\mu^*(p)$ exceeds μ, for which the firm will pay fixed wages and a range of distributions, where μ exceeds $\mu^*(p)$, for which the firm will pay piece rates.

4. Implications of the Model

In order to discuss the empirical implications of the model, we must account for the fact that the bid price can differ across observed contracts. Interviews with the firm manager revealed that it is equally likely that changing conditions lead the firm to pay fixed wages on any contract. Therefore, we assume that the distribution of bid prices is independent of the payment system. We denote the density of bid prices as $f_P(p)$.

Assumption 1

$f_P(p|\mu > \mu^*(p)) = f_P(p)$ and $f_P(p|\mu < \mu^*(p)) = f_P(p)$.

Note that Assumption 1 does not rule out a correlation between μ and P since μ^* is a function of p as well.

Given Assumption 1, we can find the expected productivity of a worker given the firm paid piece rates. Recall that the expected productivity given piece rates and μ is

$$\mathcal{E}[Y|p.r.,\mu] = \bar{e}\,\mathcal{E}[S|\mu] + \frac{\hat{\alpha}}{\kappa}\,\Pr[S > s^*]\,\mathcal{E}[S^2|S > s^*,\mu] + \frac{\hat{\alpha}}{2\pi\beta}\,\Pr[S < s^*|\mu].$$

To find the expected productivity given piece rates, we simply integrate this term over all values of P and μ for which the firm paid piece rates.

$$\begin{aligned}\mathcal{E}[Y|p.r.] &= \int_P \left\{ \int_\mu \left[\int_Y y f_Y(y|p.r.,\mu,p)dy \right] g_\mu(\mu|p.r.,p)d\mu \right\} f_P(p)dp \\ &= \int_P \left\{ \int_{\mu^*(p)}^{\infty} \left[\bar{e}\,\mathcal{E}[S|\mu] + \frac{\hat{\alpha}}{\kappa}\,\Pr[S > s^*]\,\mathcal{E}[S^2|S > s^*,\mu] \right.\right. \\ &\left.\left. + \frac{\hat{\alpha}}{2\pi\beta}\,\Pr[S < s^*|\mu] \right] \frac{g_\mu(\mu|p)}{[1 - G_\mu(\mu^*(p))]} d\mu \right\} f_P(p)dp, \qquad (10)\end{aligned}$$

where $G_\mu(\mu(p))$ and $g_\mu(\mu|p)$ represent respectively the cumulative distribution and probability density functions of μ conditional on P being p.

To find the incentive effect on these contracts, we subtract the expected productivity the worker would have produced under *these same conditions* had he been paid fixed wages.

The expected productivity conditional on fixed wages and μ is

$$\mathcal{E}[Y|f.w.,\mu] = \bar{e}\,\mathcal{E}[S|\mu]$$

integrating this term over all P and all μ exceeding $\mu^*(p)$ gives the desired term

$$\int_P \left\{ \int_{\mu^*(p)}^{\infty} \mathcal{E}[Y|f.w.,\mu] \frac{g_\mu(\mu|p)}{[1 - G_\mu(\mu^*(p))]} d\mu \right\} f_P(p)dp$$

$$\int_P \left\{ \int_{\mu^*(p)}^{\infty} \bar{e}\,\mathcal{E}[S|\mu] \frac{g_\mu(\mu|p))}{[1 - G_\mu(\mu^*(p))]} d\mu \right\} f_P(p)dp. \tag{11}$$

Subtracting (11) from (10) gives the incentive effect for piece-rate contracts, denoted H; viz.,

$$H = \int_P \left\{ \int_{\mu^*(p)}^{\infty} h(\mu, p) d\mu \right\} f_P(p)dp,$$

where

$$h(\mu, p) = \left\{ \frac{\hat{\alpha}}{\kappa} \Pr[S > s^*|\mu]\,\mathcal{E}[S^2|S > s^*, \mu] + \frac{\hat{\alpha}}{2\pi\beta} \Pr[S < s^*|\mu] \right\} \frac{g_\mu(\mu|p)}{[1 - G_\mu(\mu^*(p))]}.$$

Theorem 1 suggests that a regression of productivity on the system of pay will not, in general, give an unbiased estimate of H. In order to measure such an effect, we would need to hold everything but the payment system constant, and then compare productivity under piece rates to productivity under fixed wages. Yet Theorem 1 implies that average conditions are not the same in observed piece rate contracts as they are in observed fixed wage contracts, and these average conditions affect worker productivity: observed productivity is lower under fixed wages than piece rates for two reasons. The first is the incentive effect; i.e., for any given set of conditions, workers have less incentive to work hard under fixed wages. The second is the treatment effect; i.e., it is poor conditions that lead the firm to pay fixed wages. The difference in observed productivity encompasses these two effects. A simple regression comparing productivity under the two systems will not be able to identify separately the effect of the payment system on output from the effect of the conditions on output. We are, however, able to bound the size of the incentive effect. Theorem 2 and its corollaries develop this argument.

Theorem 2

The difference in observed real wages between piece-rate and fixed-wage contracts will be smaller than the difference in observed productivity between these same contracts.

Comparing across payment schemes, productivity differences will be larger than the difference in real wages. The expected utility constraint implies that the fixed wage is independent of planting conditions. Even though productivity will be low under fixed wages (both because the conditions are poor and the lack of incentives provided to the worker) the firm must pay a wage that will satisfy the expected utility constraint in order to keep the worker at the firm. The firm can reduce the wage of the worker in response to the incentive effect; i.e., since the worker will supply less effort he no longer requires a compensating differential, yet the mobility of workers implies that the firm must absorb the loss of productivity due to the poor conditions.

Furthermore, we can bound the gain in productivity induced by paying workers piece rates rather than fixed wages.

Corollary 2.1

The productivity gain induced by paying workers piece rates on the set of contracts $\mu > \mu^*(p)$ is bounded from above by the difference in observed productivity between piece-rate and fixed-wage contracts.

Corollary 2.2

The productivity gain induced by paying workers piece rates on the set of contracts $\mu > \mu^*(p)$ is bounded from below by the difference in observed real wages between piece-rate and fixed-wage contracts.

The actual observed difference in productivity comprises two effects: the reduced effort of the workers due to the reduction in incentives, and the reduced productivity due to the poor conditions. Corollary 2.1 merely recognizes that the sum of these two effects must be at least as large as the incentive effect itself. Corollary 2.2 follows from the labour supply constraint of the worker and Lemma 3. The fact that fixed wages are independent of conditions implies that the fixed wage we observe is the same fixed wage as would be paid on contracts for which μ exceeds $\mu^*(p)$. From the labour supply constraint, the difference between fixed wages and the piece rate wages reflects the the compensating differential associated with the incentive effect. This compensating differential must be less than the incentive effect or the firm would never have paid piece rates on these contracts.

6. Data and Results

Our data come from the payroll records of a tree-planting firm located in British Columbia. In 1991, this firm paid its workers both piece rates and fixed wages. When workers planted under piece rates, the data contain daily information on the piece rate received by each worker and the number of trees the worker planted. When workers planted under fixed wages, the data contain daily information on the number of trees planted and the wage received. The planting season of this firm covers a period of nearly six months.

We include in our study only those workers who are observed planting under both piece rates and fixed wages. This gives us 983 observations on 17 workers of which 197 observations are under fixed wages and 786 are under piece rates. The data are summarized in Table 1. In the first part of the table, we summarize the data on the wages received, productivity (trees planted per day) and total wages under fixed-wage contracts (total wages are simply equal to the fixed wage in this case). In the second part of the table, we present data from the piece-rate contracts. In the first column, we summarize data concerning the piece rates received by workers, while in the second column we summarize the number of trees planted. In the third column, we summarize total wages, and in the fourth, we convert total wages to real values.

Our calculations of real wages were hampered by the fact that no data concerning the exact bid price are available. Yet interviews with firm managers revealed that the bid price is usually between 1.85 and 1.95 times the piece rate paid to workers. Therefore, we discount total wages received on piece-rate contracts by $(1.90 \times \alpha)$. Discounting wages from fixed-wage contracts is somewhat more problematic since we have no information on α for these contracts. Therefore, we use $(1.90 \times \bar{\alpha})$ as a discount factor, where $\bar{\alpha}$ is the average value of α observed in the data. Note, that since conditions on piece-rate contracts will, on average, tend to be better than those on fixed-wage contracts, $(1.90 \times \bar{\alpha})$ will tend to under-estimate the true price per tree received on these contracts. In turn, we will tend to over-estimate the real wage on fixed-wage contracts and consequently, we tend to under-estimate the difference in real wages. In this manner, our resulting estimate of the lower bound should be considered conservative.

In general, the planters were paid very well. The average daily wage was \$224.55 under fixed wages and \$242.52 under piece rates. Not everyone, however, enjoyed success on the job. For example, one individual earned only \$24 for a day of planting. Note that both the mean and standard deviation of trees planted is higher under piece rates (977.09, 419.10) than under fixed wages (614.43, 245.7) as is expected. Similarly, the total wage received is higher under piece rates than fixed wages reflecting the compensating differential paid to workers.

The large amount of variability in both trees planted and total wages suggests that individual heterogeneity may be important. To investigate this further, in Table 2 we present summary statistics concerning the number of trees planted per individual, ordered according to average (from lowest to highest). The first column shows the average number of trees planted per day, the second the standard deviation, the third and fourth give the maximum and minimum number of trees planted, and the fifth column shows the number of observations on each worker. Note that the average number of trees planted per day ranges from 442.5 to 1158, which underlines the importance of individual heterogeneity. A histogram of these average values is presented in Figure 3.

The Upper Bound.

Corollary 2.1 implies that the observed difference in average productivity between piece-rate contracts and fixed-wage contracts provides an upper bound to the incentive effect for piece-rate contracts. In order to calculate this difference in mean productivity, we regress daily productivity on an individual-specific constant and a dummy variable indicating fixed-wage contracts. The estimated equation is

$$Y_{i,t} = \rho_{0,i} + \rho_1 D_{i,t} + \epsilon_{Y,i,t},$$

where $Y_{i,t}$ is the number of trees planted on day t by individual i; the constant term $\rho_{0,i}$ is possibly individual specific; and $D_{i,t}$ is an indicator variable equal to one if individual i was planting under a fixed-wage contract on day t. Covariates are not generally available for the planters. However, given the short duration of the sample period, much of the variation in individual characteristics can be captured by the individual specific effect.

Results from regressing the number of trees planted on the payment system are presented in Table 3. Note that average productivity under fixed-wage contracts is approximately 362.67 trees below that of piece-rate contracts. In Table 4, we present results from the same regression, while controlling for individual-specific effects. To minimize the length of the table, we present simply the maximum, minimum, and average values of the individual-specific effects. A test of the hypothesis that these effects are zero has an F-statistic of 12.24; with 16 restrictions, the p-value is equal to zero. Controlling for individual-specific effects increases the difference in productivity between fixed-wage and piece-rate contracts to 366.2 trees per day.

The Lower Bound.

To estimate the lower bound, we regress daily real wages on a constant term and a dummy variable indicating fixed wage contracts. The estimating equation now becomes

$$W_{i,t} = \varphi_{0,i} + \varphi_1 D_{i,t} + \epsilon_{W,i,t},$$

where $W_{i,t}$ is the real wage received (expressed in terms of trees) on day t by individual i; the constant term $\varphi_{0,i}$ is possibly individual-specific; and $D_{i,t}$ is an indicator variable equal to one if individual i was planting under a fixed-wage contract on day t.

The results from regressing the daily real wage on the payment system are presented in Table 5. Note that the average real wage decreases by 59.71 trees under fixed wages. Again, including individual-specific effects increases this differential. These results are presented in Table

6. Here, the difference in average real wages is calculated to be 60.56 trees. An F-test of the restrictions that the individual-specific constants are insignificant is easily rejected, having a test statistic of 13.12 with 16 restrictions, so the p-value is zero.

7. Structural Estimation

In order to measure the incentive effect more precisely, we estimate the model structurally. In particular, we take the model developed in Section 3 as the data generating process and we estimate its parameters. We do not consider individual-specific effects since their inclusion had only marginal effects on the calculation of the upper and lower bounds to the incentive effect. We use the model to generate μ^* and s^* endogenously as functions of the structural parameters.

The parameters to be estimated are the following:

1. ν_μ, the mean of μ;
2. σ_μ^2, the variance of μ;
3. $\bar{e}$, the minimum effort level;
4. σ_S^2, the variance of S;
5. κ, the cost of effort;
6. $2\pi\beta$, the worker's expected fine for poorly planted trees;
7. γ, the firm's share of fines for poorly planted trees.

We denote the vector of these parameters as $\underline{\theta}$. Note s^*, the cut-off point for cheating on the distribution of S, is equal to $\sqrt{(}\kappa/2\pi\beta)$, so it is simply a ratio of these two parameters.

We make the following distributional assumptions:

Assumption A1: $\mu \sim N(\nu_\mu, \sigma_\mu^2)$;
Assumption A2: $S \sim \log N(\mu, \sigma_S^2)$.

Similarly, we make the following identification assumptions:

Assumption A3: $\nu_\mu = 0$;

Assumption A4: σ_S^2 is constant across contracts;

Assumption A5: Each contract is the same size;

Assumption A6: P is constant across contracts;

Assumption A7: $\gamma = \eta \times \beta, \quad \eta > 1$;

Assumption A8: $\bar{u} = 0$.

Assumption A5 implies that the number of man days worked on each contract is the same. Therefore, the ratio of the number of observations of fixed-wage planting to the total number of observations will identify the probability of paying fixed wages. As much as possible, firms do try to make sites of comparable size. Assumption A6 simplifies the estimation in that there is only one μ^* to solve for in the firm's problem. Assumption A7 implies that we only identify γ as a multiple of β. Finally, Assumption A8 specifies that the worker's alternative utility is equal to zero, an innocuous normalization.

We estimate the structural parameters by matching the population moments that are implied by the model to the sample moments of our data. Note that we lack data on the quality of output. This implies that the distribution of output under piece rates is a mixture of the distributions of output when the worker plants well and when he plants poorly.

The derivation of the population moments is provided in the Appendix. We denote sample equivalents of population moments with bars. Thus $\bar{y}_{fw}$ is the average productivity under fixed wages observed in the sample. Furthermore, we denote the difference between population moments and sample equivalents by m. We consider the following moment conditions:

$$
\begin{aligned}
m_1 &= \Phi\left(\frac{\mu^* - \nu_\mu}{\sigma_\mu}\right) - \frac{n_{f.w.}}{n_{f.w.} + n_{p.r.}}; \\
m_2 &= \mathcal{E}[Y|f.w.] - \bar{y}_{f.w.}; \\
m_3 &= \mathcal{E}[Y^2|f.w.] - \bar{y}^2_{f.w.}; \\
m_4 &= \mathcal{E}[Y|p.r.] - \bar{y}_{p.r.}; \\
m_5 &= \mathcal{E}[Y^2|p.r.] - \bar{y}^2_{p.r.}; \\
m_6 &= \mathcal{E}[W|f.w.] - \bar{w}_{f.w.}; \\
m_7 &= \mathcal{E}[W|p.r.] - \bar{w}_{p.r.}; \\
m_8 &= \mathcal{E}[\alpha|p.r.] - \bar{\alpha}_{p.r.}.
\end{aligned}
$$

We match the first two moments of productivity under both fixed wages and piece rates, the first moment of wages under both fixed wages and piece rates, the first moment of the piece rates paid to the workers and the proportion of observations that are paid fixed wages. We impose that the αs in the population moments satisfy the worker's expected utility constraint (4) and that μ^* solve $\psi(\mu)$ equals zero where $\psi(\mu)$ is defined in (9).

The parameters are estimated by minimizing the function

$$\mathbf{m}^\top \underline{\widehat{\Omega}}^{-1} \mathbf{m}$$

where $\mathbf{m}$ represents the vector of stacked moment conditions and $\underline{\widehat{\Omega}}$ is the estimated variance-covariance matrix of the moments. We calculated $\underline{\widehat{\Omega}}$ using the variance-covariance matrix of the sample moments and imposing the condition that the covariance between piece-rate moments and fixed-wage moments is zero.[7] The variance-covariance matrix of the parameter estimates, $\underline{\widehat{\theta}}$, is given by $\mathbf{G}^\top \underline{\widehat{\Omega}}^{-1} \mathbf{G}$, where $\mathbf{G}$ is a matrix with typical entry

$$G_{i,j} = \frac{\partial m_i}{\partial \theta_j}.$$

[7] See Greene (1993) pp. 372-374.

Results from this procedure are presented in Table 7.

The estimated incentive effect on piece-rate contracts, $\widehat{H}$, equals 88.53 trees. This number lies within the previously estimated upper and lower bounds and suggests that 24% of the observed difference in average productivity between piece rate and fixed wage contracts (366 trees) is due to incentives, the rest is due to differences in planting conditions. Furthermore, these results suggest that 9.1% of the observed productivity under piece rates (977 trees) is due to incentives.

8. Discussion and Conclusion

Empirical work in economics often suffers from a lack of experimental data. The comparison of worker productivity across payment systems is no different in this respect. The fact that firms choose their payment system suggests that the observed payment system may be endogenous and cause biased estimation results. We have modelled the tree-planting firm's choice of compensation system as a function of working conditions. An important result of our model is that firms are more likely to choose fixed-wage contracts when working conditions render planting difficult. This is because workers will tend to plant poorly when conditions are bad, exposing the firm to fines from the government.

Explicit modelling of the firm's choice process provides benefits at different levels. First, we are able to demonstrate why a simple comparison of average productivity between different compensation systems fails to identify the incentive effect. Second, the model allows us to bound the size of the incentive effect in terms of differences in observed variables; viz., average productivity and average real wages. We estimate the upper bound to the incentive effect to be 366 trees per day and the lower bound to be 60 trees per day. Given that average productivity under piece rates was 977 trees planted per day, this suggests that incentives are responsible for between 6% and 37% of this productivity. Third, the explicit modelling of behaviour enables us to estimate the model structurally and to measure more precisely the effect of incentives on productivity. Our results suggest that the incentive effect accounts for only 24% of the observed difference in average productivity between piece-rate and fixed-wage contracts (366 trees). This highlights the problem of endogeneity in the comparison of worker productivity across different compensation systems.

Finally, our results confirm the presence of an incentive effect; i.e., workers are more productive under piece rates than under fixed wages. It would be incorrect, however, to conclude that piece rates are better than fixed wages. The firm in our model chooses fixed wages or piece rates as a rational profit-maximizing response to planting conditions: under certain conditions, the gain in productivity is overridden by quality concerns, leading the firm to choose fixed wages. Therefore, our estimated incentive effect is an estimate of the output foregone when making such a choice.

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Appendix

Proof of Lemma 1.

Define

$$G(\alpha) \equiv \alpha \bar{e}\,\mathcal{E}[S] + \frac{q}{2\kappa}\alpha^2\,\mathcal{E}[S^2|S > s^*] + \frac{(1-q)}{4\pi\beta}\alpha^2 - \frac{\kappa}{2}\bar{e} - \bar{u}.$$

The proof follows from that fact that $G'(\alpha) > 0$ for $\alpha > 0$ and $G(0) < 0$.

Proof of Lemma 2.

Recall that the expected utility constraint satisfies

$$\hat{\alpha}\bar{e}\,\mathcal{E}[S] + \frac{\hat{\alpha}^2}{2\kappa}q\,\mathcal{E}[S^2|S > s^*] + \frac{(1-q)}{4\pi\beta}\hat{\alpha}^2 - \frac{\kappa}{2}\bar{e} = \bar{u}.$$

Totally differentiating this expression gives

$$\frac{d\hat{\alpha}}{d\mu} = -\frac{\hat{\alpha}\bar{e}\frac{\partial\,\mathcal{E}[S]}{\partial\mu} + \frac{\hat{\alpha}^2}{2\kappa}q\frac{\partial}{\partial\mu}\,\mathcal{E}[S^2|S > s^*] + \left[\frac{\hat{\alpha}^2}{2\kappa}\,\mathcal{E}[S^2|S > s^*] - \frac{\hat{\alpha}^2}{4\pi\beta}\right]\frac{\partial q}{\partial\mu}}{\bar{e}\,\mathcal{E}[S] + \frac{\hat{\alpha}}{\kappa}q\,\mathcal{E}[S^2|S > s^*] + \frac{(1-q)}{2\pi\beta}\hat{\alpha}}. \tag{A.1}$$

A sufficient condition for (A.1) to be negative is for the numerator,

$$\hat{\alpha}\bar{e}\frac{\partial\,\mathcal{E}[S]}{\partial\mu} + \frac{\hat{\alpha}^2}{2\kappa}q\frac{\partial}{\partial\mu}\,\mathcal{E}[S^2|S > s^*] + \left[\frac{\hat{\alpha}^2}{2\kappa}\,\mathcal{E}[S^2|S > s^*] - \frac{\hat{\alpha}^2}{4\pi\beta}\right]\frac{\partial q}{\partial\mu} > 0. \tag{A.2}$$

This expression has three parts. The first part is positive since $\mathcal{E}[S]$ equals $\exp\left(\mu + \sigma_S^2/2\right)$. The third part is also positive. To see this note that

$$q = 1 - F(s^*) = 1 - \Phi\left(\frac{\log s^* - \mu}{\sigma_S}\right) \Rightarrow \frac{\partial q}{\partial\mu} = \frac{1}{\sigma_S}\phi\left(\frac{\log s^* - \mu}{\sigma_S}\right) > 0.$$

As well,

$$\frac{\hat{\alpha}^2}{2\kappa}\,\mathcal{E}[S^2|S>s^*]-\frac{\hat{\alpha}^2}{4\pi\beta}\geq 0$$

since

$$\mathcal{E}[S^2|S>s^*]\geq\frac{\kappa}{2\pi\beta}=(s^*)^2.$$

To complete the proof we make the following definitions.

$$\Phi_1\equiv\Phi\left(\frac{\log(s^*)-\mu-2\sigma_S^2}{\sigma_S}\right),\qquad \phi_1\equiv\phi\left(\frac{\log(s^*)-\mu-2\sigma_S^2}{\sigma_S}\right)$$

$$\Phi_2\equiv\Phi\left(\frac{\log(s^*)-\mu}{\sigma_S}\right),\qquad \phi_2\equiv\phi\left(\frac{\log(s^*)-\mu}{\sigma_S}\right),$$

$$\xi\equiv\frac{2\kappa}{\hat{\alpha}^2}\left\{\hat{\alpha}\bar{e}\frac{\partial\,\mathcal{E}[S]}{\partial\mu}+\left[\frac{\hat{\alpha}^2}{2\kappa}\,\mathcal{E}[S^2|S>s^*]-\frac{\hat{\alpha}^2}{4\pi\beta}\right]\frac{\partial q}{\partial\mu}\right\}>0.$$

The numerator is positive if

$$\frac{\partial\,\mathcal{E}[S^2|S>s^*]}{\partial\mu}+\frac{\xi}{(1-\Phi_2)}>0$$

or

$$\left[\left(2(1-\Phi_1)+\frac{\phi_1}{\sigma_S}\right)(1-\Phi_2)-\frac{\phi_2}{\sigma_S}(1-\Phi_1)\right]$$
$$\frac{\exp\left(2\mu+2\sigma_S^2\right)}{(1-\Phi_2)^2}+\frac{\xi}{(1-\Phi_2)}>0.$$

Rearranging gives

$$2\sigma_S+\frac{\phi_1}{(1-\Phi_1)}-\frac{\phi_2}{(1-\Phi_2)}+\frac{\sigma_S}{(1-\Phi_1)}\,\xi\,\exp\left(-2(\mu+\sigma_S^2)\right)>0$$

or

$$\frac{\frac{\phi_2}{(1-\Phi_2)}-\frac{\phi_1}{(1-\Phi_1)}}{2\sigma_S}<1+\frac{1}{(1-\Phi_1)}\exp\left[-2(\mu+\sigma_S^2)\right]\frac{\xi}{2}$$

the left hand side of which is the slope of the secant of the hazard function evaluated at the points

$$\frac{\log(s^*) - \mu}{\sigma_S} \quad \text{and} \quad \frac{\log(s^*) - \mu}{\sigma_S} - 2\sigma_S.$$

Proof of Lemma 3.

Rearranging (7) gives

$$\frac{q}{\kappa}\left(p - \frac{\hat{\alpha}}{2}\right)\mathcal{E}[S^2|S > s^*] + \frac{(1-q)}{2\pi\beta}\left(p - \frac{\hat{\alpha}}{2}\right) > \frac{(1-q)\hat{\alpha}}{4\pi^2\beta^2}\pi(\gamma - \beta),$$

which can only be satisfied if $\hat{\alpha} < 2p$.

Proof of Lemma 4.

The proof follows directly from evaluating (7) with $q = 0$.

Proof of Theorem 1.

Throughout the proof, we make use of the following properties of the log-normal distribution:

$$\mathcal{E}[S] = \exp\left(\mu + \sigma_S^2/2\right) \tag{A.3}$$

$$\int_{s^*}^{\infty} s^2 f_S(s)ds = \exp\left(2\mu + 2\sigma_S^2\right)\left[1 - \Phi\left(\frac{\log(s^*) - \mu - 2\sigma_S^2}{\sigma_S}\right)\right] \tag{A.4}$$

$$F_S(s^*) = \Phi\left(\frac{\log(s^*) - \mu}{\sigma_S}\right) \tag{A.5}$$

where Φ denotes the cumulative standard normal density.

Differentiating ψ with respect to μ gives

$$\psi'(\mu) = -\frac{dq}{d\mu} - \left\{\left[\frac{1}{\kappa}\left(p - \frac{\hat{\alpha}}{2}\right)\frac{d}{d\mu}\int_{s^*}^{\infty} s^2 f_S(s)ds - \frac{1}{2\kappa}\int_{s^*}^{\infty} s^2 f_S(s)ds\frac{d\hat{\alpha}}{d\mu}\right] \times \right.$$

$$\left[\frac{1}{2\pi\beta}\left(\pi(\gamma-\beta)\frac{\hat{\alpha}}{2\pi\beta} - (p - \hat{\alpha}/2)\right)\right] - \left[\frac{1}{2\pi\beta}\left(\pi(\gamma-\beta)\frac{1}{2\pi\beta} + \frac{1}{2}\right)\frac{d\hat{\alpha}}{d\mu}\right] \times$$

$$\left.\left[\frac{1}{\kappa}\left(p - \frac{\hat{\alpha}}{2}\right)\int_{s^*}^{\infty} s^2 f_S(s)ds\right]\right\}\frac{1}{\left[\frac{1}{2\pi\beta}\left(\pi(\gamma-\beta)\frac{\hat{\alpha}}{2\pi\beta} - (p - \hat{\alpha}/2)\right)\right]^2}. \tag{A.6}$$

Recall

$$q = 1 - F_S(s^*) = 1 - \Phi\left(\frac{\log(s^*) - \mu}{\sigma_S}\right),$$

and

$$\frac{dq}{d\mu} = \frac{1}{\sigma_S}\phi\left(\frac{\log(s^*) - \mu}{\sigma_S}\right),$$

so the first term in (A.6) is negative. The sign of the second term depends on its numerator which can be broken into three parts as follows.

$$-\left[\frac{1}{\kappa}\left(p-\frac{\hat{\alpha}}{2}\right)\frac{d}{d\mu}\int_{s^*}^{\infty}s^2 f_S(s)ds\right]$$

$$\left[\frac{1}{2\pi\beta}\left(\pi(\gamma-\beta)\frac{\hat{\alpha}}{2\pi\beta}-(p-\hat{\alpha}/2)\right)\right] \qquad \text{(A.6.a)}$$

$$+\left[\frac{1}{2\kappa}\int_{s^*}^{\infty}s^2 f_S(s)ds\ \frac{d\hat{\alpha}}{d\mu}\right]$$

$$\left[\frac{1}{2\pi\beta}\left(\pi(\gamma-\beta)\frac{\hat{\alpha}}{2\pi\beta}-(p-\hat{\alpha}/2)\right)\right] \qquad \text{(A.6.b)}$$

$$+\frac{1}{2\pi\beta}\left(\pi(\gamma-\beta)\frac{1}{2\pi\beta}+\frac{1}{2}\right)\frac{d\hat{\alpha}}{d\mu}$$

$$\left[\frac{1}{\kappa}\left(p-\frac{\hat{\alpha}}{2}\right)\int_{s^*}^{\infty}s^2 f_S(s)ds\right] \qquad \text{(A.6.c)}$$

The term (A.6.a) is negative since

$$\int_{s^*}^{\infty}s^2 f_S(s)ds=\exp\left(2\mu+2\sigma_S^2\right)\left[1-\Phi\left(\frac{\log(s^*)-\mu-2\sigma_S^2}{\sigma_S}\right)\right],$$

and its derivative with respect to μ is positive. The terms (A.6.b) and (A.6.c) are negative if $\frac{d\hat{\alpha}}{d\mu}<0$. It follows immediately that a sufficient condition for $\psi'(\mu)<0$ is for

$$\frac{d\hat{\alpha}}{d\mu}<0.$$

Proof of Theorem 2

Equations (5) and (6) in the text calculate the expected wages under piece rates and fixed wages for a given value of μ. We rewrite (5) and (6) making the dependence on μ explicit.

$$\mathcal{E}[W|f.w.,\mu] = \hat{\omega} = \hat{\alpha}\bar{e}\,\mathcal{E}[S|\mu] + \frac{\hat{\alpha}^2}{2\kappa}\Pr[S > s^*|\mu]\,\mathcal{E}[S^2|S > s^*,\mu] +$$

$$\Pr[S < s^*|\mu]\frac{\hat{\alpha}^2}{4\pi\beta} \quad \text{(A.7)}$$

$$\mathcal{E}[W|p.r.,\mu] = \hat{\alpha}\bar{e}\,\mathcal{E}[S|\mu] + \frac{\hat{\alpha}^2}{\kappa}\Pr[S > s^*|\mu]\,\mathcal{E}[S^2|S > s^*,\mu] +$$

$$\Pr[S < s^*|\mu]\frac{\hat{\alpha}^2}{2\pi\beta} \quad \text{(A.8)}$$

To calculate the the expected real wages under piece rates and fixed wages, conditional on the firm's choice of payment system, we integrate over all values of p and μ that cause the firm to pay one system or the other. Recall that the firm chooses piece rates whenever $\mu > \mu^*(p)$. expected real wages given that the firm chose to pay piece rates are therefore

$$\mathcal{E}[W|p.r.] = \int_P \left\{ \int_{\mu^*(p)}^{\infty} \left\{ \hat{\alpha}\bar{e}\,\mathcal{E}[S|\mu] + \frac{\hat{\alpha}^2}{\kappa}\Pr[S > s^*|\mu]\,\mathcal{E}[S^2|S > s^*,\mu] + \right.\right.$$

$$\left.\left. \frac{\hat{\alpha}^2}{2\pi\beta}\Pr[S < s^*|\mu] \right\} \frac{g_\mu(\mu|p)}{[1 - G_\mu(\mu^*(p))]} d\mu \right\} \frac{1}{p} f_P(p)dp, \quad \text{(A.9)}$$

where $G_\mu(\mu(p))$ and $g_\mu(\mu|p)$ represent the distribution function and density of μ conditional on p. Similarly, expected wages given the firm chose fixed wages are $\mathcal{E}[W|f.w.]$ which equals $\hat{\omega}$ which is constant for all μ greater than $\mu^*(p)$ Using (A.7) we can write

$$\mathcal{E}[W|f.w.] = \int_P \left\{ \int_{\mu^*(p)}^{\infty} \left\{ \hat{\alpha}\bar{e}\,\mathcal{E}[S|\mu] + \frac{\hat{\alpha}^2}{2\kappa}\,\mathrm{Pr}[S > s^*|\mu]\,\mathcal{E}[S^2|S > s^*, \mu] + \right.\right.$$

$$\left.\left. \frac{\hat{\alpha}^2}{4\pi\beta}\,\mathrm{Pr}[S < s^*|\mu] \right\} \frac{g_\mu(\mu)}{[1 - G_\mu(\mu^*(p))]} d\mu \right\} \frac{1}{p} f_P(p) dp, \quad \text{(A.10)}$$

Subtracting (A.10) from (A.9) gives the difference in observed wages under the two payment schemes. Define

$$h(\mu, p) = \left\{ \frac{\hat{\alpha}}{\kappa}\,\mathrm{Pr}[S > s^*|\mu]\,\mathcal{E}[S^2|S > s^*, \mu] + \right.$$

$$\left. \frac{\hat{\alpha}}{2\pi\beta}\,\mathrm{Pr}[S < s^*|\mu] \right\} \frac{g_\mu(\mu|p)}{[1 - G_\mu(\mu^*(p))]},$$

then

$$\mathcal{E}[W|p.r.] - \mathcal{E}[W|f.w.] = \int_P \left\{ \int_{\mu^*(p)}^{\infty} \frac{\hat{\alpha}}{2p} h(\mu, p) d\mu \right\} f_P(p) dp. \quad \text{(A.11)}$$

Consider now a comparison of productivity under the two schemes. Under piece rates, expected productivity conditional on μ is

$$\mathcal{E}[Y|p.r., \mu] = \bar{e}\,\mathcal{E}[S|\mu] + \frac{\hat{\alpha}}{\kappa}\,\mathrm{Pr}[S > s^*]\,\mathcal{E}[S^2|S > s^*, \mu] +$$

$$\frac{\hat{\alpha}}{2\pi\beta}\,\mathrm{Pr}[S < s^*|\mu].$$

Expected productivity conditional on the the firm paying piece rates is

$$\mathcal{E}[Y|p.r.] = \int_P \left\{ \int_{\mu^*(p)}^{\infty} \mathcal{E}[Y|p.r., \mu] \frac{g_\mu(\mu|p)}{[1 - G_\mu(\mu^*(p))]} d\mu \right\} f_P(p) dp$$

$$= \int_P \Big\{ \int_{\mu^*(p)}^{\infty} \Big\{ \bar{e}\,\mathcal{E}[S|\mu] + \frac{\hat{\alpha}}{\kappa}\Pr[S > s^*]\,\mathcal{E}[S^2|S > s^*, \mu]$$

$$+ \frac{\hat{\alpha}}{2\pi\beta}\Pr[S < s^*|\mu]\Big\} \frac{g_\mu(\mu|p)}{[1 - G_\mu(\mu^*(p))]} d\mu \Big\} f_P(p)dp. \qquad \text{(A.12)}$$

Under fixed wages, expected productivity conditional on μ is

$$\mathcal{E}[Y|f.w., \mu] = \bar{e}\,\mathcal{E}[S|\mu]$$

and expected productivity conditional on the firm paying fixed wages is

$$\begin{aligned} \mathcal{E}[Y|f.w.] &= \int_P \left\{ \int_{-\infty}^{\mu^*(p)} \mathcal{E}[Y|f.w., \mu] \frac{g_\mu(\mu|p)}{[1 - G_\mu(\mu^*(p)]} d\mu \right\} f_P(p)dp \\ &= \int_P \left\{ \int_{-\infty}^{\mu^*(p)} \bar{e}\,\mathcal{E}[S|\mu] \frac{g_\mu(\mu|p))}{G_\mu(\mu^*(p))} d\mu \right\} f_P(p)dp. \qquad \text{(A.13)} \end{aligned}$$

Subtracting (A.13) from (A.12) gives the difference in observed productivity under the two payment schemes

$$\begin{aligned} \mathcal{E}[Y|p.r.] - \mathcal{E}[Y|f.w.] = &\int_P \Big\{ \int_{\mu^*(p)}^{\infty} h(\mu, p) d\mu \Big\} f_P(p)dp \\ &+ \bar{e}\Big[\int_P \Big\{ \int_{\mu^*(p)}^{\infty} \mathcal{E}[S|\mu] \frac{g_\mu(\mu|p)}{[1 - G_\mu(\mu^*(p))]} d\mu \\ &- \int_{-\infty}^{\mu^*(p)} \mathcal{E}[S|\mu] \frac{g_\mu(\mu|p)}{G_\mu(\mu^*(p))} d\mu \Big\} f_P(p)dp \Big] \qquad \text{(A.14)} \end{aligned}$$

Since the second term of (A.14) is positive, (A.14) will be larger than (A.11) if

$$\int_P \left\{ \int_{\mu^*(p)}^{\infty} h(\mu,p) d\mu \right\} f_P(p) dp \quad \geq$$
$$\int_P \left\{ \int_{\mu^*(p)}^{\infty} \frac{\hat{\alpha}}{2p} h(\mu,p) d\mu \right\} f_P(p) dp.$$

A sufficient condition for this to hold is for

$$\int_{\mu^*(p)}^{\infty} h(\mu,p) d\mu \geq \int_{\mu^*(p)}^{\infty} \frac{\hat{\alpha}}{2p} h(\mu,p) d\mu \quad \forall p,$$

and a sufficient condition for this to hold is for

$$h(\mu,p) \geq \frac{\hat{\alpha}}{2p} h(\mu,p) \quad \forall\ \mu > \mu^*(p),$$

or for

$$\hat{\alpha} \leq 2p \quad \forall\ \mu > \mu^*(p)$$

which must hold by Lemma 3.

Proof of Corollary 2.1

The expected productivity under piece rates for the contracts $\mu > \mu^*(p)$ is

$$\int_P \left\{ \int_{\mu^*(p)}^{\infty} \bar{e}\, \mathcal{E}[S|\mu] + \frac{\hat{\alpha}}{\kappa} \Pr[S > s^*]\, \mathcal{E}[S^2|S > s^*, \mu] + \right.$$
$$\left. \frac{\hat{\alpha}}{2\pi\beta} \Pr[S < s^*] \frac{g_\mu(\mu|p)}{[1 - G_\mu(\mu^*(p))]} d\mu \right\} f_P(p) dp.$$

The expected productivity under fixed wages for these same contracts is

$$\int_P \left\{ \int_{\mu^*(p)}^{\infty} \bar{e}\, \mathcal{E}[S|\mu] \frac{g_\mu(\mu|p)}{[1 - G_\mu(\mu^*(p))]} d\mu \right\} f_P(p) dp.$$

The difference in expected productivity is therefore

$$\int_P \left\{ \int_{\mu^*(p)}^{\infty} \frac{\hat{\alpha}}{\kappa} \Pr[S > s^*] \, \mathcal{E}[S^2 | S > s^*, \mu] + \right.$$
$$\left. \frac{\hat{\alpha}}{2\pi\beta} \Pr[S < s^*] \frac{g_\mu(\mu|p)}{[1 - G_\mu(\mu^*(p))]} d\mu \right\} f_P(p) dp$$
$$= \int_P \left\{ \int_{\mu^*(p)}^{\infty} h(\mu, p) d\mu \right\} f_P(p) dp.$$

From (A.14), the difference in *observed* average productivity under the two payment schemes is

$$\mathcal{E}[Y|p.r.] - \mathcal{E}[Y|f.w.] = \int_P \left\{ \int_{\mu^*(p)}^{\infty} h(\mu, p) d\mu \right\} f_P(p) dp$$

$$+ \bar{e} \left[\int_P \left\{ \int_{\mu^*(p)}^{\infty} \mathcal{E}[S|\mu] \frac{g_\mu(\mu|p)}{[1 - G_\mu(\mu^*(p))]} d\mu \right. \right.$$

$$\left. \left. - \int_{-\infty}^{\mu^*(p)} \mathcal{E}[S|\mu] \frac{g_\mu(\mu|p)}{G_\mu(\mu^*(p))} d\mu \right\} f_P(p) dp \right].$$

Proof of Corollary 2.2

From (A.11), the difference in observed wages is

$$\mathcal{E}[W|p.r.] - \mathcal{E}[W|f.w.] = \int_P \left\{ \int_{\mu^*(p)}^{\infty} \frac{\hat{\alpha}}{2p} h(\mu, p) d\mu \right\} f_P(p) dp \leq$$

$$\int_P \left\{ \int_{\mu^*(p)}^{\infty} h(\mu, p) d\mu \right\} f_P(p) dp$$

if

$$\frac{\hat{\alpha}}{2p} h(\mu, p) \leq h(\mu, p) \quad \forall \quad \mu > \mu^*(p)$$

which holds by Lemma 3.

Derivation of Moments

$$\mathcal{E}[Y^r|f.w.] = \int_{-\infty}^{\mu^*} \bar{e}^{\,r}\, \mathcal{E}[S^r|\mu] g_\mu(\mu|\mu < \mu^*) d\mu$$

$$= \bar{e}^{\,r} \exp\left(r\nu_\mu + \frac{r^2}{2}\sigma_\mu^2 + \frac{r^2}{2}\sigma_S^2\right) \frac{\Phi\left(\frac{\mu^* - \nu_\mu - r\sigma_\mu^2}{\sigma_\mu}\right)}{\Phi\left(\frac{\mu^* - \nu_\mu}{\sigma_\mu}\right)}\;;$$

$$\mathcal{E}[Y|p.r.] = \int_{\mu^*}^{\infty} \left\{ \bar{e}\,\mathcal{E}[S|\mu] + \frac{\hat{\alpha}}{\kappa} q\,\mathcal{E}[S^2|S > s^*, \mu] + (1-q)\frac{\hat{\alpha}}{2\pi\beta} \right\} \frac{g_\mu(\mu)}{[1 - G_\mu(\mu^*)]} d\mu\;;$$

$$\mathcal{E}[Y^2|p.r.] = \int_{\mu^*}^{\infty} \left\{ \bar{e}^{\,2} \exp(2\nu_\mu + 2\sigma_\mu^2 + 2\sigma_S^2) \frac{\left[1 - \Phi\left(\frac{\mu^* - \nu_\mu - 2\sigma_\mu^2}{\sigma_\mu}\right)\right]}{\left[1 - \Phi\left(\frac{\mu^* - \nu_\mu}{\sigma_\mu}\right)\right]} \right.$$

$$+ 2\bar{e}\frac{\hat{\alpha}}{\kappa} \exp(3\mu + 4.5\sigma_S^2) \left[1 - \Phi\left(\frac{\log(s^*) - \mu - 3\sigma_S^2}{\sigma_S}\right)\right]$$

$$+ \frac{\hat{\alpha}^2}{\kappa^2} \exp(4\mu + 8\sigma_S^2) \left[1 - \Phi\left(\frac{\log(s^*) - \mu - 4\sigma_S^2}{\sigma_S}\right)\right]$$

$$+ \frac{2\bar{e}\hat{\alpha}}{2\pi\beta} \exp(\mu + \sigma_S^2/2) \Phi\left(\frac{\log(s^*) - \mu - \sigma_S^2}{\sigma_S}\right)$$

$$\left. + \frac{\hat{\alpha}^2}{(2\pi\beta)^2} \Phi\left(\frac{\log(s^*) - \mu}{\sigma_S}\right) \right\} \frac{g_\mu(\mu)}{[1 - G_\mu(\mu^*)]} d\mu\;;$$

$$\mathcal{E}[W|p.r.] = \int_{\mu^*}^{\infty} \left\{ \hat{\alpha}\bar{e}\,\mathcal{E}[S|\mu] + \frac{\hat{\alpha}^2}{\kappa} q\,\mathcal{E}[S^2|S > s^*] + (1-q)\frac{\hat{\alpha}^2}{2\pi\beta} \right\} \frac{g_\mu(\mu)}{[1 - G_\mu(\mu^*)]} d\mu\,;$$

$$\mathcal{E}[W|f.w.] = \int_{\mu^*}^{\infty} \left\{ \hat{\alpha}\bar{e}\,\mathcal{E}[S|\mu] + \frac{\hat{\alpha}^2}{2\kappa} q\,\mathcal{E}[S^2|S > s^*] + (1-q)\frac{\hat{\alpha}^2}{4\pi\beta} \right\} \frac{g_\mu(\mu)}{[1 - G_\mu(\mu^*)]} d\mu\,;$$

$$\mathcal{E}[\hat{\alpha}|p.r.] = \int_{\mu^*}^{\infty} \hat{\alpha}(\mu) \frac{g_\mu(\mu)}{[1 - G_\mu(\mu^*)]} d\mu\,.$$

All integrals were simulated by drawing 20,000 random numbers from a normal distribution, truncated from below at μ^*.

Figure 1: Worker's Problem

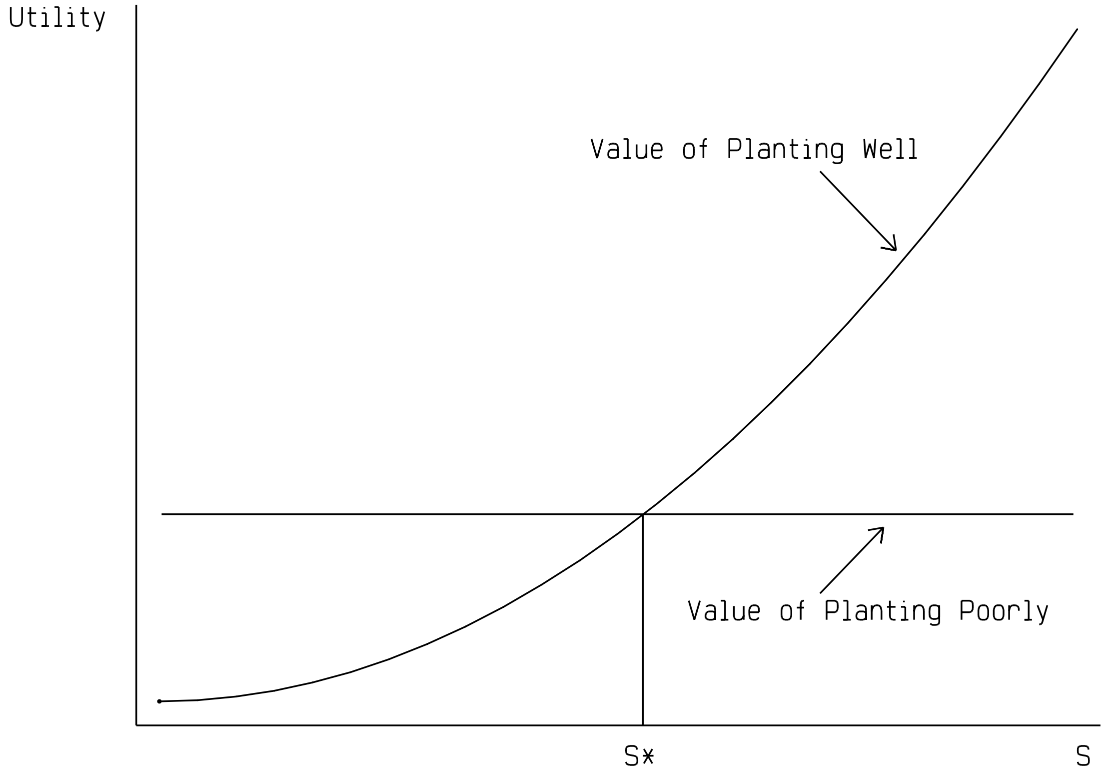

Figure 2: Firm's Problem

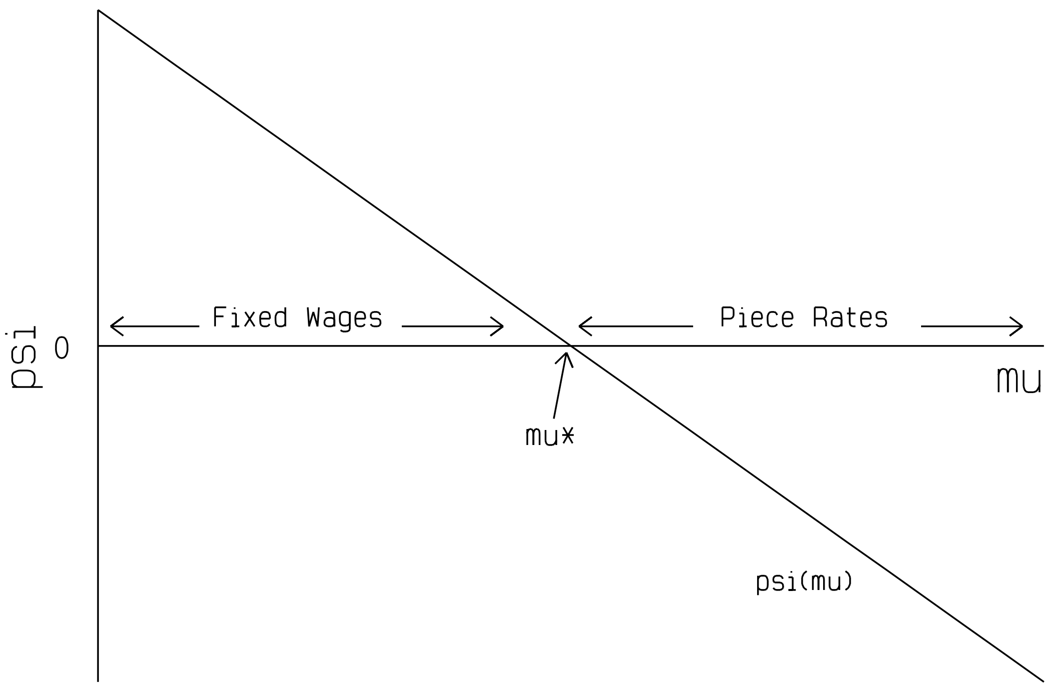

Figure 3: Average Trees Planted per Individual

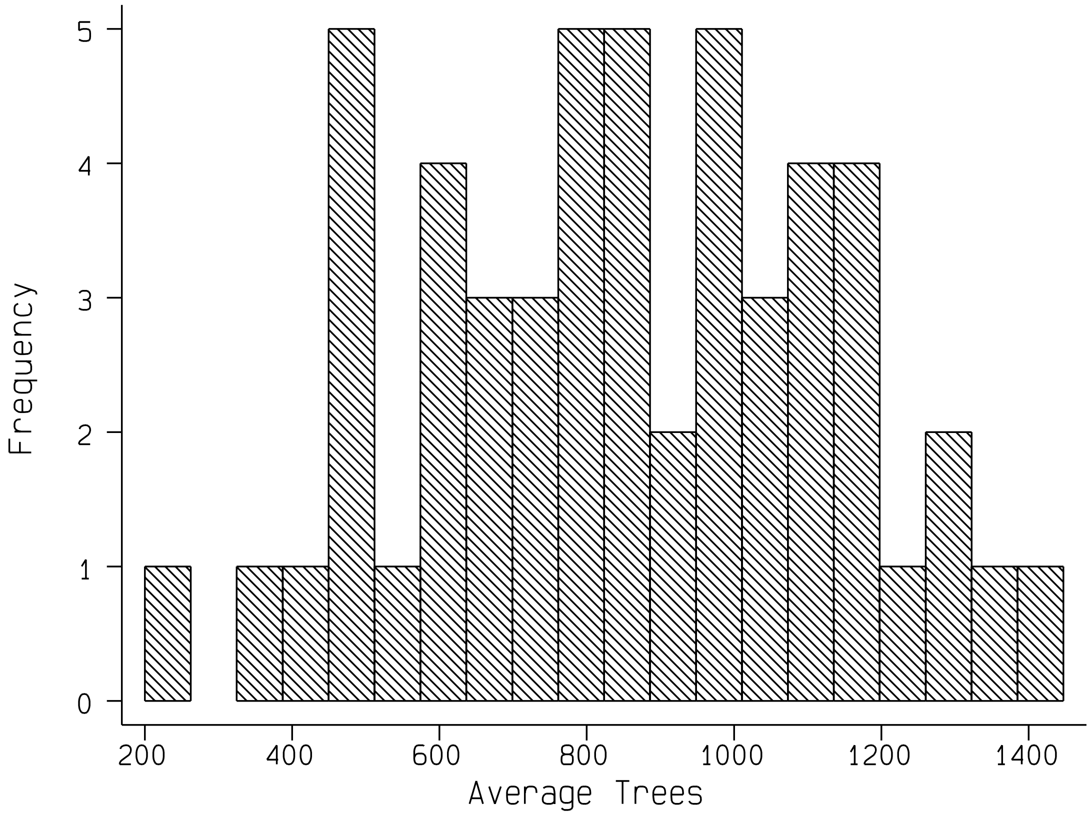

Table 1: Summary Statistics

Fixed Wages

	Wages	Trees	Total	Real Wage
Mean	224.55	614.43	224.55	454.55
Std.	45.09	245.7	45.09	91.29
Max	280	1575	280	566.80
Min	93	100	93	188.26
Obs	197	197	197	197

Piece Rates

	Rate	Trees	Total	Real Wage
Mean	0.27	977.09	242.52	514.26
Std. Dev	0.08	419.10	71.03	220.60
Max	0.45	2250	504	1184.21
Min	0.15	75	24	39.47
Obs	786	786	786	786

Table 2: Average Daily Productivity by Individual Planter

Individual	Observations	Average	Std Dev	Minimum	Maximum
1	24	442.50	169.56	120	840
2	38	662.89	398.48	100	1860
3	66	671.05	284.34	75	1260
4	51	739.63	330.74	90	1700
5	45	780.11	346.01	80	1440
6	53	816.42	313.11	205	1620
7	71	844.93	363.05	85	1800
8	70	857.30	378.90	185	2100
9	52	882.88	344.74	200	1720
10	70	884.57	350.54	200	1620
11	47	967.13	430.00	125	1820
12	85	971.32	348.46	120	1880
13	62	995.42	440.96	130	1940
14	47	1000.85	407.61	220	1640
15	74	1094.42	520.00	150	2200
16	63	1142.14	456.21	410	2250
17	65	1158.39	442.19	165	1900

Table 3: Regression of Daily Trees Planted on Payment System

Observations 983
F(1, 981) = 135.71
Prob > F = 0.00
R-square = 0.12

Variable	Coefficient	Std. Error	T-Statistic	P>|T|
Fixed Wage	-362.67	31.13	-11.65	0.00
Constant	977.09	13.94	70.11	0.00

Table 4: Regression of Daily Trees Planted on Payment System
Controlling for Individual Specific Effects

Observations 983
F(17, 965) = 20.97
Prob > F = 0.00
R-square = 0.27

Variable	Coefficient	Std Error	T-Statistic	P>|T|
Fixed Wage	-366.20	29.29	-12.50	0.00
Constant	711.08	58.39	12.18	0.00
Imax	553.13	74.01	7.47	0.00
Imin	-100.74	94.14	-1.07	0.29
Iave	257.82			

Notes: Imax is the maximum value of the individual effect coefficients
Imin is the minimum value of the individual effect coefficients
Iave is the average value of the individual effect coefficients

Table 5: Regression of Daily Real Wage on Payment System

Observations 983
F(1, 981) = 13.83
Prob > F = 0.0002
R-square = 0.0139

Variable	Coefficient	Std Error	T-Statistic	P>|T|
Fixed Wage	-59.71	16.06	-3.72	0.00
Constant	514.26	7.19	71.54	0.00

Table 6: Regression of Daily Real Wage on Payment System
Controlling for Individual Specific Effects

Observations 983
F(17, 965) = 13.32
Prob > F = 0.00
R-square = 0.19

Variable	Coefficient	Std Error	T-Statistic	P > |T|
Fixed Wage	-60.56	15.02	-4.03	0.00
Constant	390.90	29.94	13.06	0.00
Imax	261.99	37.62	6.97	0.00
Imin	-59.84	48.26	-1.24	0.22
Iave	118.54			

Notes: Imax is the maximum value of the individual effect coefficients
Imin is the minimum value of the individual effect coefficients
Iave is the average value of the individual effect coefficients

Table 7: Structural Estimates

Objective Function	19.15			

Parameter	Coefficient	Std Error	T-Statistic	P > |T|
sig2s	0.42	0.02	23.97	0.00
sigmu	0.22	0.02	9.50	0.00
ebar	712.36	17.20	41.42	0.00
k	0.66	0.01	55.44	0.00
2pbeta	3.02E-03	6.67E-04	4.52	6.92E-06
eta	3.12	0.09	36.39	0.00

Incentive Effect		88.53		

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