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Trading Patterns, Time Deformation and Stochastic Volatility in Foreign Exchange Markets*

Eric Ghysels[†], Christian Gouriéroux[‡], Joanna Jasiak[⁺]

Résumé / Abstract

La globalisation des échanges sur le marché mondial des taux de change est une des sources principales des effets saisonniers — journaliers et hebdomadaires — dans la volatilité des prix. Une façon de modéliser ces phénomènes consiste à utiliser la spécification d'un processus subordonné pour formaliser la relation entre la volatilité et l'intensité des échanges. Cet article, fondé sur les idées de Clark (1973), Dacorogna et al. (1993) et Ghysels et Jasiak (1994), présente un modèle de volatilité stochastique avec la déformation du temps pour les séries des taux de change. La déformation du temps est déterminée par la dynamique du flux des cotations à travers la journée, les fourchettes de prix passées ainsi que les rendements antérieurs. Dans la partie empirique, nous appliquons ce modèle aux données de haute fréquence de Olsen and Associates. La méthode d'estimation que nous avons employée est le Quasi-Maximum de Vraisemblance proposé par Harvey et Stock, adapté par Ghysels et Jasiak aux processus déformés du temps.

Globalization of trading in foreign exchange markets is a principal source of the daily and weekly seasonability in market volatility. One way to model such phenomena is to adopt a framework where market volatility is tied to the intensity of (world) trading through a subordinated stochastic process representation. In this paper we combine elements from Clark (1973), Dacorogna et al. (1993) and Ghysels and Jasiak (1994), and present a stochastic volatility model for foreign exchange markets with time deformation. The time deformation is based on daily patterns of arrivals of quotes and bid-ask spreads as well as returns. For empirical estimation we use the QMLE algorithm of Harvey et al. (1994), adopted by Ghysels and Jasiak for time deformed processes, and applied to the Olsen and Associates high frequency data set.

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1. Introduction

The interbank FX market for foreign exchange transactions is one of the prime examples of the recent trends in globalization of trading in international financial markets. The Bid/Ask prices quoted by various firms and banks are recorded over the 24 hours per day and displayed worldwide by news services, such as Reuters or Telerate. Due to the overlapping periods of activity of market makers located over the three continents — America, Europe and Asia, a sequential pattern in intra-day trading is observed. Works by Wasserfallen and Zimmerman (1985), Wasserfallen (1989), Feinstone (1987), Ito and Roley (1987), Müller et al. (1990), Goodhart and Figliouli (1991), Bollerslev and Domowitz (1993) and Dacorogna et al. (1993) are examples of a growing interest in research in this area. Most of them document and examine both daily and weekly seasonalities in the volatility of foreign exchange rates.

The seasonal phenomena in the volatility of foreign exchange markets can be modelled in a variety of ways. One possibility to accommodate seasonality is to modify the traditional ARCH or GARCH type models.[1] Another strategy is to seasonally adjust the data, a practice quite common for economic time series but which is not without its longstanding controversies.[2] Alternatively, the market volatility can be tied to the intensity of trading via a subordinate stochastic process representation, as suggested by Clark (1973). This approach has been adopted in some recent works by researchers from Olsen and Associates [see, for example, Dacorogna et al. (1992, 1993), Müller et al. (1992)]. Instead of modelling asset price behavior in calendar time, price movements can be represented as being driven by an information arrival process which itself evolves randomly yet with certain predictable patterns through time. Formally, daily returns, $x(\Delta t) = \log(p(t)/p(t-1))$, are hence redefined as $\log p(t)/p(t-1) = x(\Delta g(t))$ where $g(t)$ is a positive, increasing stochastic process, sometimes called directing process. This setup can be referred to as time deformation since the relevant time scale is no longer calendar time t but *operational* time $g(t)$. Let us point out some advantages of this approach. As emphasized by Mandelbrot and Taylor (1967), it easily accommodates leptokurtic distributions for asset returns.

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[1] ARCH models with seasonality are discussed in Bollerslev and Ghysels (1994).

[2] See Ghysels (1994) and Miron (1994) for further discussion as well as Andersen and Bollerslev (1994) for applications.

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Clark (1973) has shown that within this framework, comovements between trading volume and asset returns can easily be modelled. Finally, time deformation yields a random variance similar to a *stochastic volatility* model. These ideas have been refined and extended in several ways for foreign exchange markets. Dacorogna et al. (1993) proposed time scales related to a measure of worldwide activity, based on an empirical scaling law of returns relating the mean absolute change of the logarithmic middle price to calendar time. It is intuitively based on the notion that as the world market time "slows down", depending on the number of markets active and on their local intra-day pattern, price volatility decreases and vice versa. Since this time deformation concept is based on *average* market activity at any point in time, it accommodates the repetitive seasonal pattern. Dacorogna et al. (1993) do not fully exploit, however, the framework of subordinated processes suggested by Clark as they forego the information in the *current* market activity. In this paper we adopt the generic framework of Ghysels and Jasiak (1994) and propose a stochastic volatility model with time deformation which blends features of an average *and* a conditional market activity.

The empirical work is based on the data provided by Olsen and Associates. The series consist of DEM/USD, JPY/USD and JPY/DEM exchange rates and contain all quotes that appeared on the interbank Reuters network over the entire year from October 1, 1992 through September 29, 1993. Although this data bank contains a bid and ask price for each quote along with the time to the nearest even second, several researchers (see, for example, Dacorogna et al. (1993), Müller et al. (1993), Moody and Wu (1994)) consider a single price series constructed as a logarithmic average of asks and bids. In section 2 we examine the data and discuss the advantages and shortcomings of this approach. The stochastic volatility model in its generic form is presented in section 3. In section 4, we discuss observable stochastic processes which approximate the market activity and appear in our specification of operational time. In section 5 we report the empirical estimates of the stochastic volatility model with time deformation based on intra-day market activity. Section 6 concludes the paper.

2. Market Dynamics and the Distributional Properties of Asks and Bids

In this section we provide a statistical analysis of asks and bids and study the behavior of their geometric average. We examine the descriptive statistics, the autocorrelation patterns and also investigate marginal and joint empirical densities. Since the outcomes of time scale adjustments are for us of primary concern, the data are both analyzed on a real time (tick-by-tick) basis, and over a fixed 20 minute sampling interval.

The high frequency data consist of interbank FX price quotes for three exchange rates: the Deutschmark/US Dollar (DEM/USD), the Japanese Yen/US Dollar (JPY/USD) and the Yen/Deutschmark (JPY/DEM) rate. The numbers of observations in the three samples are, respectively, 1,472,266, 570,839 and 159,004. Although the ask and bid sequences are reported simultaneously for every transaction, a vast majority of researchers study a single price series constructed as a logarithmic average of asks and bids. Following the notation adopted by Dacorogna et al. (1993), the returns on the foreign exchange market are thus defined as:

$$\Delta x(t) = x(t) - x(t-1)$$
$$= \tfrac{1}{2}[(\ell og\,\text{ask}(t) + \ell og\,\text{bid}(t)) - (\ell og\,\text{ask}(t-1) + \ell og\,\text{bid}(t-1))],$$

or,

$$\Delta x(t) = \tfrac{1}{2}[(\ell og\,\text{ask}(t) - \ell og\,\text{ask}(t-1)) + (\ell og\,\text{bid}(t) - \ell og\,\text{bid}(t-1))]$$
$$= \tfrac{1}{2}[\Delta \ell og\,\text{ask}(t) + \Delta \ell og\,\text{bid}(t)].$$

Usually it is assumed that the dynamics of the $x(t)$ series reflect the general pattern of market activity. One could argue, however, that the logarithmic middle price averages out outcomes of distinct trading strategies of buyers and sellers. Indeed, the real time data reveal several differences between asks and bids. Table 2.1 presents the summary statistics of $\Delta \ell og\,\text{ask}(t)$ and $\Delta \ell og\,\text{bid}(t)$, the two components of $\Delta x(t)$, as well as of $\Delta x(t)$ compared across markets in real time. Table 2.2 contains the same statistical summary over a fixed 20 minute interval of time scale. We report the mean, variance, standard deviation, skewness coefficient, excess kurtosis (i.e., the

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empirical kurtosis -3), the minimum and maximum values as well as the range. A 95% confidence interval of the mean and variance estimators are also provided.

In real time, we find in general that $\Delta \ell og \; \text{ask}(t)$ has a higher mean and a larger variance than $\Delta \ell og \; \text{bid}(t)$. However, the first two moments of asks and bids differ only marginally as compared to the discrepancies reported in moments of order 3 or 4. In fact, the most relevant differences arise in terms of asymmetry and tail properties. On the JPY/USD and JPY/DEM markets, the ask series are skewed to the right, while the bids are skewed to the left. The quotes on the DEM/USD exchange rates are both skewed to the right and show little differences in absolute values of the skewness coefficients. On the contrary, on the JPY/DEM market, we report a 434 times higher absolute value of the skewness coefficient of asks compared to bids. More excess kurtosis is found in the ask series as well. The difference is either slight, as it is the case of the most active and hence, most regularly behaved DEM/USD market, moderate in the JPY/USD quotes, where excess kurtosis in asks is almost 3.5 times higher than in bids or extreme on the JPY/DEM market where the ask coefficient is almost 1523 times larger than the excess kurtosis of the bid series.

Two observations can be made regarding the third and fourth moment statistics reported in tables 2.1 and 2.2. First, the differences in skewness and kurtosis for bids and asks in the tick–by–tick data indicate that there are far more extreme changes in the ask quotations than there are in the bids. As noted before, these differences are particularly important for the JPY/DEM and JPY/USD markets. A second observation is with respect to the comparison of the kurtosis statistics obtained from real time and twenty minute sampling. In Ghysels, Gouriéroux and Jasiak (1995) it is shown that for a time deformed process $X(\Delta g(t))$ there is an increase in kurtosis due to time deformation when the mechanism generating $\Delta g(t)$ is independent of X. This would yield larger excess kurtosis for the twenty minute sampled series in comparison with the tick–by–tick series. The results in tables 2.1 and 2.2 show that this is the case for the DEM/USD series and for the bid series of the JPY/USD market. All other series do not have this feature.

The logarithmic middle price seems to follow the asymmetric pattern of the bid quotes, both in terms of the sign and the magnitude of skewness coefficients. The thickness of tails in the $\Delta x(t)$ series appears also to be determined rather by bids

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than by asks at least on those markets where the largest bid-ask discrepancies in terms of excess kurtosis were reported, i.e., JPY/USD and JPY/DEM.

The descriptive statistics resulting from data sampled over 20 minute intervals, presented in Table 2.2, provide us some insights on the time scale adjustment effects. The results in Tables 2.1 and 2.2 indicate that the sampling scheme has an immediate and very strong impact on the distributional properties of the data. We report largely different values of the first four moments of quotes on the same exchange rates sampled on the adjusted time scale.[3] Besides, data show much less variety across the markets in a sense that the basic statistics defining the distinct character of the three data sets become much less dissimilar. It seems that on the aggregated time scale, some of the properties identifying the individual series are getting attenuated. Accordingly, we do not observe either the bid-ask discrepancies, at least to the extent reported in the real time. For this reason, statistics on both quote sequences and their logarithmic average appear more coherent as well.

To visualize the differences between the $\Delta \ell og \, \text{ask}(t)$, $\Delta \ell og \, \text{bid}(t)$ and $\Delta x(t)$ series in terms of their distributional properties, we present plots of the corresponding empirical univariate densities. (See figure 2.1, appendix 1.) For clarity of exposition, we cover only one market, namely JPY/DEM featuring extreme bid-ask discrepancies in real time.

Figures 2.2–2.3 display the bivariate distributions of $[\Delta \ell og \, \text{ask}(t)$, $\Delta \ell og \, \text{bid}(t)$], the univariate distributions of the two series, as well as the contour plots of quotes recorded both in real time and on the adjusted time scale. A typical shape of the bivariate density can be described as a sudden, very pronounced peak surrounded by some smaller ones within a large domain of infrequently quoted values. In all data sets, the empirical densities are stretched out along one axis of the ellipse, indicating a strong positive correlation between $\Delta \ell og \, \text{ask}(t)$ and $\Delta \ell og \, \text{bid}(t)$. The shapes shown on the contour plots confirm a higher variance of data sampled at 20 minute intervals and suggest more correlation between both quote sequences on the 20 minute grid.

The issue that remains to be investigated is whether the distributional properties revealed by quotes recorded over one year are shared by samples over shorter time

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[3]This phenomenon has been documented for series aggregated from daily to weekly or to monthly sampling frequencies [see Drost and Nijman (1993)].

horizons, like one month or one day. A closely-related problem is the stability of the empirical densities through calendar time to uncover the presence of seasonal patterns.

We selected 6 monthly subsamples consisting of quotes recorded in October and December 1992 as well as in January, March, May and July 1993. We analyzed both the tick-by-tick data and quotes sampled at 20 minute intervals. For convenience, we report again the results for one market only, i.e., the JPY/USD. (See figures 2.4–2.5, appendix 1.)

The variety of shapes of the bivariate empirical densities of monthly subsamples throughout the year reveals the complexity of seasonal phenomena in exchange rates. In the monthly tick-by-tick data sets, asks and bids show less discrepancies than in the entire sample. The JPY/USD market is particular in a sense that $\Delta \ell og$ ask(t) still take values over a larger range than $\Delta \ell og$ bid(t) , has a higher variance and longer tails. (See figure 2.4, appendix 1.) Especially ask quotes on the JPY/USD rates remain symmetric in October, while bids display a strong skewness. Interestingly, the October asymmetry is common to all bid sequences and exhibited also by asks on the JPY/DEM exchange rates.

The density of the average JPY/USD price, $\Delta x(t)$, seems, in general, to take on values over the bid's range. However, it does not reflect the bid's asymmetry. On the remaining markets where the ask and bid densities are more similar in terms of range and variance, skewness in the $\Delta x(t)$ sequence seems to be determined by the skewness of bids.

Quotes on the JPY/USD rates, sampled at 20 minute intervals, do not reveal the "October skewness". (See figure 2.5, appendix 1.) Instead, an asymmetry in the asks' density is observed in January, while bids display asymmetric behavior either in January, May, July and December. The bids and asks prices recorded on the remaining two markets have similar asymmetric distributions in almost all monthly subsamples. Apparently, on the adjusted time scale, every month has its own particular rhythm and pattern of trading, as reflected by a characteristic tail behavior.

Since asks and bids on DEM/USD and JPY/DEM rates exhibit on the 20 minute grid similar distributional properties, the general tendency of these markets is ex-

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pected to be well-approximated by $\Delta x(t)$. In case of the JPY/USD quotes, their logarithmic average mimics the tail behavior of bids rather than asks.

The empirical densities corresponding to a given day are shown in figures 2.6–2.7, appendix 1. Our data consists of quotes recorded over 4 days of the week of October 5 through 12, 1992. Daily patterns are examined on the most active DEM/USD market. The empirical densities of daily subsamples differ again in terms of shape. In the tick-by-tick records, Wednesday's and Sunday's distributions are more stretched out and are more symmetric than the Monday's and Friday's ones. On Monday, both asks and bids are characterized by long right tails, while on Friday the asymmetry is exhibited by bids only. The middle price $\Delta x(t)$ reflects again the distributional properties of bids.

The quotes sampled over the 20 minute grid show a variety of daily densities although due to a small number of observations, Sunday's data can be disregarded. As we have observed in the monthly data, the adjustments of the time scale imparts asymmetries and thus seasonality on days (months) where they are not reported in the tick-by-tick data. For example, thick and uneven Wednesday's tail reappear on all markets.

Many of the seasonal phenomena may also be recovered within the autocorrelation patterns of the series. As this issue will be discussed in section 4, we concentrate on serial dependencies on the real and the adjusted time scales up to lag 100.

The autocorrelations in bids and asks on both time scales are persistent and do not reveal any new facts. Although we do not report the cross-correlation functions, some insights on the price dynamics are worth being presented. As we have inferred from the empirical densities, asks and bids sampled at 20 minute intervals are more correlated than asks and bids in the real time. In fact, covariances of data on the adjusted time scale are almost equal to one on all markets and vary between 0.7–0.8 across markets in the tick-by-tick samples. In terms of the lagged dependence, the first tick is of primary importance for the ask and bid price adjustments. The cross-correlation at lag 1 is negative and varies on the markets between –0.2 and –0.3. The cross-correlations drop dramatically within the next tick indicating still a significant, although a very low, positive dependence (less than 0.03) at lag 2. At higher lags, the dependence between ask and bid series remains extremely low and occasionally takes

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on significant values. On the 20 minute scale, the real time cross-correlations sum up to one significant lag observed on all markets of a negative value close to -0.1.

To investigate the persistence in $(\Delta x(t))^2$, modelled within the SV framework, we computed the autocorrelation functions of $(\Delta \ell og \, ask(t))^2$, $(\Delta \ell og \, bid(t))^2$ and $(\Delta x(t))^2$. On all markets, squared values of returns, asks and bids in real time show similar, persisting patterns of serial dependence. (See figure 2.8, appendix 1.) The same behavior is revealed by data sampled at 20 minute intervals on the DEM/USD and JPY/USD markets. The squares of returns $(\Delta x(t))^2$ on the JPY/DEM exchange rates are exceptional, as they do not follow the autocorrelation pattern of squared values of ask and bids (see figure 2.9, appendix 1).

The time scale adjustments have shown, so far, either to alleviate some extremes in the distributional structure of the tick-by-tick data or to impart some phenomena related to the seasonality unobserved in real time. Two interpretations seem to be plausible: by sampling at fixed time intervals, we either extract the necessary information out of the noisy tick-by-tick records and reveal the essential properties of the data, or we forego important information and hence obtain an oversimplified image of the true underlying processes. The evidence we have presented, indicates that, apart from some exceptions, the behaviors of asks and bids, the two components of the logarithmic middle price, are much more coherent on the 20 minute time scale. Hence, the middle price increments $\Delta x(t)$ approximate better the general tendency of the quotes. By choosing the 20 minute grid to model volatility in the $\Delta x(t)$ series, we need to make the necessary adjustments in the traditional SV model to accommodate several aspects of seasonality. In the next section we explain how this can be achieved by modelling the stochastic volatility within a time deformation framework.

Table 2.1: Summary Statistics (real time (tick-by-tick) data)

DEM/USD

	mean	var	std. dev.	skewness	excess kurtosis	min	max	range
$\Delta \ell og\,\mathrm{ask}(t)$	0.992E-07 (-0.345E-06, 0.543E-06)	0.754E-07 (0.752E-07, 0.755E-07)	0.00027	0.04352	5.4334	-0.00603	0.00972	0.01576
$\Delta \ell og\,\mathrm{bid}(t)$	0.900E-07 (-0.337E-06, 0.517E-06)	0.697E-07 (0.696E-07, 0.699E-07)	0.00026	0.05181	4.5265	-0.00660	0.00923	0.01584
$\Delta x(t)$	0.992E-07 (-0.302E-06, 0.501E-06)	0.617E-07 (0.615E-07, 0.618E-07)	0.00024	0.05438	6.0310	-0.00616	0.00947	0.01565

JPY/USD

	mean	var	std. dev.	skewness	excess kurtosis	min	max	range
$\Delta \ell og\,\mathrm{ask}(t)$	-0.218E-06 (-0.119E-05, 0.760E-06)	0.141E-06 (0.140E-06, 0.141E-06)	0.00037	0.02470	21.9921	-0.00944	0.00939	0.01885
$\Delta \ell og\,\mathrm{bid}(t)$	-0.233E-06 (-0.117E-05, 0.705E-06)	0.130E-06 (0.129E-06, 0.130E-06)	0.00036	-0.00158	6.5750	-0.00907	0.00916	0.01824
$\Delta x(t)$	-0.217E-06 (-0.109E-06, 0.655E-06)	0.112E-06 (0.112E-06, 0.113E-06)	0.00033	-0.00174	10.7756	-0.00921	0.00925	0.01847

JPY/DEM

	mean	var	std. dev.	skewness	excess kurtosis	min	max	range
$\Delta \ell og\,\mathrm{ask}(t)$	-0.260E-03 (-0.344E-03, -0.175E-03)	0.293E-03 (0.291E-03, 0.296E-03)	0.01714	167.02110	31359.7910	-0.03059	3.21939	3.24999
$\Delta \ell og\,\mathrm{bid}(t)$	-0.166E-05 (-0.338E-05, 0.481E-07)	0.121E-06 (0.120E-06, 0.122E-06)	0.00034	-0.38500	20.5896	-0.00975	0.00995	0.01971
$\Delta x(t)$	-0.169E-05 (-0.337E-05, -0.236E-07)	0.115E-06 (0.114E-06, 0.116E-06)	0.00033	-0.39260	18.8470	-0.00968	0.00662	0.01631

Table 2.2: Summary Statistics (20 minute sampling interval)

DEM/USD

	mean	var	std. dev.	skewness	excess kurtosis	min	max	range
$\Delta log\,\mathrm{ask}(t)$	0.760E-05 (-0.547E-05, 0.208E-04)	0.846E-06 (0.829E-06, 0.864E-06)	0.00092	0.2974	9.0182	-0.00841	0.01344	0.02186
$\Delta log\,\mathrm{bid}(t)$	0.757E-05 (-0.564E-05, 0.207E-04)	0.840E-06 (0.824E-06, 0.858E-06)	0.00091	0.2048	9.1872	-0.00842	0.01345	0.02187
$\Delta x(t)$	0.770E-05 (-0.533E-05, 0.207E-04)	0.830E-06 (0.813E-06, 0.847E-06)	0.00091	0.2496	9.3027	-0.00842	0.01344	0.02186

JPY/USD

	mean	var	std. dev.	skewness	excess kurtosis	min	max	range
$\Delta log\,\mathrm{ask}(t)$	-0.663E-05 (-0.200E-04, 0.681E-05)	0.873E-06 (0.856E-06, 0.891E-06)	0.00093	0.17402	12.6636	-0.00923	0.01097	0.02021
$\Delta log\,\mathrm{bid}(t)$	-0.663E-05 (-0.198E-04, 0.656E-05)	0.841E-06 (0.824E-06, 0.858E-06)	0.00091	0.01485	10.2305	-0.00924	0.01000	0.01925
$\Delta x(t)$	-0.664E-05 (-0.197E-04, 0.611E-05)	0.824E-06 (0.807E-06, 0.841E-06)	0.00090	0.09053	10.9400	-0.00924	0.01049	0.01973

JPY/DEM

	mean	var	std. dev.	skewness	excess kurtosis	min	max	range
$\Delta log\,\mathrm{ask}(t)$	-0.161E-04 (-0.313E-04, -0.100E-05)	0.100E-05 (0.979E-06, 0.102E-05)	0.00100	-0.06358	7.3136	-0.00988	0.01143	0.02132
$\Delta log\,\mathrm{bid}(t)$	-0.162E-04 (-0.311E-04, -0.126E-05)	0.972E-06 (0.951E-06, 0.993E-06)	0.00098	-0.22131	7.1505	-0.01003	0.01028	0.02032
$\Delta x(t)$	-0.161E-04 (-0.311E-04, -0.123E-05)	0.963E-06 (0.949E-06, 0.990E-06)	0.00098	-0.13480	7.1586	-0.00996	0.01086	0.02082

3. Stochastic Volatility and Time Deformation

In this section we provide a brief summary of the stochastic volatility model with time deformation presented in Ghysels and Jasiak (1994) and Ghysels, Gouriéroux and Jasiak (1995). Following the work by Hull and White (1987), Johnson and Shanno (1987), Scott (1987), Wiggins (1987), Chesney and Scott (1989), Stein and Stein (1991) and Heston (1993), we call a stochastic volatility model the following set of equations:

$$dy(t) = \mu y(t)\, dt + \sigma(t)\, y(t)\, dW_1(t) \,, \qquad (3.1a)$$

$$d \log \sigma(t) = a(b - \log \sigma(t))\, dt + c dW_2(t) \,, \qquad (3.1b)$$

where $W_1(t)$ and $W_2(t)$ are two independent, standard Wiener processes. Ghysels and Jasiak (1994) suggested to adopt the framework of equations (3.1a) and (3.1b) and define the volatility process as a subordinated stochastic process evolving in a time dimension driven by market activity. This approach has been motivated by the works of Mandelbrot and Taylor (1967) as well as Clark (1973). The complex and quite frequently irregular behavior of asset prices becomes simpler and hence easier to model once we assume that the volatility is tied to some observed or unobserved variables, like the information arrival, which determine the dynamics of tradings.[4] Hence, we assume that there exist an operational time scale of the volatility process, with $s = g(t)$, a mapping between operational and calendar time t, such that:[5]

$$dy(t) = \mu y(t)\, dt + \sigma(g(t))\, y(t)\, dw_1(t) \,, \qquad (3.2a)$$

$$d \log \sigma(s) = a(b - \log \sigma(s))\, ds + c dw_2(s) \,. \qquad (3.2b)$$

Following Stock (1988), we use the notation $g(t)$ for the directing process to indicate some *generic* time deformation, which may include trading volume besides many other series that help to determine the pace of the market. Before discussing what might determine $g(t)$, we would like to make some observations regarding equations (3.1a) and (3.1b). Indeed, it should first be noted that the equations collapse to the usual stochastic volatility model if $g(t) = t$. Obviously, there are several possible

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[4]The microfoundations for time deformation and the process of price adjustments can be found most explicitly in Easley and O'Hara (1992).

[5]The mapping $s = g(t)$ must satisfy certain regularity conditions which will be discussed later.

specifications of $\sigma(g(t))$. Moreover, one could correctly argue that defining volatility as a subordinated process amounts to suggesting a more complex law of motion in comparison to the Ornstein-Uhlenbeck (henceforth, O-U) specification appearing in (3.1b). This interpretation is valid, yet it should be noted that, through $g(t)$, one can associate many series other than the security price $y(t)$ to explain volatility; hence, one implicitly deals with a multivariate framework. Moreover, as we have pointed out, the time deformation setup enables us to handle rather complex structure through the subordinated representation.

To enhance our understanding of the mechanism of the process, we first consider the system (3.2) in its continuous and discrete time versions. To simplify the presentation, let us set $b = 0$ and discuss a continuous time AR(1). An investor's information can be described by considering the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and the nondecreasing family $F = \{\mathcal{F}_t\}_{t=0}^{+\infty}$ of sub-σ-algebras in calendar time. Furthermore, we let Z_t be a m-dimensional vector process adapted to the filtration F, i.e., Z_t is $\mathcal{F}_t$-measurable. The increments of the time deformation mapping g will be assumed to be $\mathcal{F}_{t-1}$ measurable via the logistic transformation:

$$\frac{dg(\tau; Z_{t-1})}{d\tau} \equiv g(\tau; Z_{t-1}) \equiv \exp(c'Z_{t-1}) / \left\{ \frac{1}{T} \sum_{t=1}^{T} \exp(c'Z_{t-1}) \right\} , \qquad (3.3)$$

for $t - 1 \leq \tau < t$.[6] Equation (3.3), setting the speed of changes in operational time as a measurable function of calendar time process Z_{t-1}, is completed by the additional identification assumptions:

$$0 < g(\tau; Z_{t-1}) < \infty , \qquad (3.4)$$

$$g(0) = 0 , \qquad (3.5)$$

$$\frac{1}{T} \sum_{t=1}^{T} \Delta g(t) = 1. \qquad (3.6)$$

These three conditions guarantee that the operational time clock progresses in the same direction as calendar time without stops or jumps.[7] Given that g is constant between consecutive calendar time observation via (3.3), its discrete time analogue $\Delta g(t) \equiv g(t) - g(t-1)$ takes the same logistic form appearing in (3.3). At this point,

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[6]The fact that the denominator in (3.3) contains a sample average may suggest that $\sigma(g(t))$ is not measurable with respect to the filtration $\mathcal{F}_t$ in calendar time. However, the denominator in (3.3) is there for reasons of numerical stability of the algorithms. Since it is only a scaling factor, its presence is of no conceptual importance.

[7]See Stock (1988) for a detailed discussion of the identification assumptions.

we will not present the components of the Z_{t-1} vector. As we will discuss this issue in the next section, let us just indicate that, in principle, Z_{t-1} consists of any processes related to the information arrival. Ghysels and Jasiak (1994) show that the solution in calendar time can be expressed as:

$$\Delta \log y_t - a_1 \Delta \log y_{t-1} - \lambda = e^{h_t} \varepsilon_t \,, \tag{3.7}$$

$$h_t = [(1 - \exp(a\Delta g(t)))]b + \exp(a\Delta g(t))h_{t-1} + v_t \,, \tag{3.8}$$

$$v_t \sim N(0, -\Sigma(1 - \exp(2a\Delta g(t)))/2a) \,. \tag{3.9}$$

Equations (3.7) and (3.8) constitute the basic set of equations for the discrete time representation of the SV model with a subordinated volatility process which evolves at a pace set by $\Delta g(t)$. A linear state-space representation of the system (3.7)–(3.8) can be estimated by maximizing the conditional maximum likelihood function within the Kalman filter framework. Following Harvey, Ruiz and Shephard (1994), we rewrite equation (3.7) as:

$$\log[\Delta \log y_t - a_1 \Delta \log y_{t-1} - \lambda]^2 = h_t + \log \varepsilon_t^2 \,, \tag{3.10}$$

where: $E \log \varepsilon_t^2 = -1.27$ and $Var \log \varepsilon_t^2 = \pi^2/2$. Defining $\varsigma_t = \log \varepsilon_t^2$, we obtain:

$$\log[\Delta \log y_t - a_1 \Delta \log y_{t-1} - \lambda]^2 = -1.27 + h_t + \varsigma_t \,. \tag{3.11}$$

Apart from the parameter λ, whose treatment is discussed for instance by Gouriéroux, Monfort and Renault (1993), the coefficients of this state-space model are time-varying and, hence, similar to the specification proposed by Stock (1988), except for the properties of the ς_t process which is no longer Gaussian. Consequently, the estimation procedure based on the Kalman filter will result here in a quasi-maximum likelihood estimates, as pointed out by Harvey, Ruiz and Shephard (1994). The details of the QMLE algorithm for time deformed SV models are discussed in Ghysels and Jasiak (1994); while Ghysels, Gouriéroux and Jasiak (1995) present a detailed account of subordinated process theory and their estimation.

4. Directing Processes for Market Activity

The model structure described in the previous section is a generic one where, apart from some regularity conditions and the logistic form, the specification of $\Delta g(t)$ was left open. Clark (1973), Tauchen and Pitts (1983) and Ghysels and Jasiak (1994) studied stock returns and used a time deformation model with trading volume as proxy for market activity. It is well known that for foreign exchange markets trading volume is difficult to obtain. Hence, we need to consider other series. The *Olsen and Associates* data base provides several possibilities to model market activity. The purpose of this section is to discuss the different approaches one could consider.

Our strategy will consist of distinguishing "regular" or average market activity, and deviations from the expected level of activity. For example, when European financial markets open and start active trading in say the DEM/USD currency exchange, each market participant has a certain expectation of the number of quotes arriving during the first five minutes, the next five minutes, and so on. Some mornings, trading is more brisk or even sometimes frenzy-like. On other mornings, the market activity is down relative to its usual rhythm. Every part of the trading day has a certain reference norm of activity against which one portrays the latest quote arrivals. What is true for quote arrivals holds also for other market indicators like bid-ask spreads, returns, absolute value of returns, etc. The model specification strategy which we will adopt is to incorporate into $\Delta g(t)$ measures of "regular" or average market activity and series representing deviations from average trading patterns. To continue with the quote arrival example, we can formulate $\Delta g(t)$ as:

$$\Delta g(t) = \exp(c' Z_{t-1}) \equiv \exp(\Theta_{qa} nqa_{t-1} + \Theta_{qd}(nqa_{t-1} - nq_{t-1})) , \qquad (4.1)$$

where the scaling constant appearing in (3.3) has been omitted from (4.1). Hence, from (4.1) we have that: $Z_{t-1} = (nqa_{t-1}, (nq_{t-1} - nq_{t-1}))$ where nqa_{t-1} is the mean number of quotes arriving over the interval $t-1$, while nq_{t-1} is the actual number of quotes which arrived in $t-1$.

To clarify this, let us consider the plots appearing in figure 4.1, appendix 2. The figure consist of six plots, the left side displaying graphs with results from data sampled at 5 minute intervals and the right panel containing the 20 minute sampling frequency equivalent. We study the three markets of the Olsen data set, namely DEM/USD,

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JPY/DEM and JPY/USD. Each plot covers a span of a week, omitting the weekends, and displays the average number of quotes, computed over the entire sample, for each 5 (left) or 20 (right) minute time intervals of the week. The plots display the repetitive intra-day cycle which is so typical for high frequency exchange rate data. The 5 minute plots are, of course, more jagged than the 20 minute ones, but each shows clearly the patterns of quote arrivals repeating each 24 hour cycle. The graphs displayed in figure 4.1 represent the nqa_{t-1} series used to model $\Delta g(t)$. The number of quote arrivals is one candidate series to measure market activity, besides other series which we shall discuss shortly.

Before turning to these other series, it is worth drawing attention to a special case of time deformation. Suppose for the moment that $\Theta_{qa} = 0$ in (4.1). Then, $\Delta g(t)$ is purely a function of the repetitive daily pattern of $\{nqa_t\}$ which amounts to volatility being a periodic autoregressive process:

$$h_t = \gamma_t + \alpha_t h_{t-1} + W_t , \qquad (4.2)$$

where γ_t and α_t are changing every 5 or 20 minutes, depending on the sampling frequency, with a 24 hour repetitive cycle, i.e., $\gamma_t = \gamma_s$, $\alpha_t = \alpha_s$ with $s = t + 24$ hours.[8] A periodic model like (4.2) resembles the class of periodic ARCH processes proposed by Bollerslev and Ghysels (1994) in analogy with periodic ARMA models for the mean which have been extensively studied. Of course, the parameter variation in (4.2) is determined by $\gamma_t = (1 - \exp(a\Delta g(t)))$ and $\alpha_t = (a\Delta g(t))$.

We noted that quote arrivals are not the only measure of market activity, and indeed several other series in the Olsen data file could be considered. Figure 4.2 of appendix 2 displays the intra-daily pattern of bid-asks spreads. The figure has the averages computed on a weekly basis of the average bid-ask spreads during 5 or 20 minute intervals. We notice in figure 4.2 a reasonably regular 24 hour pattern for bid-ask spreads but by far not as pronounced and regular as the quote arrivals displayed in figure 4.1 of appendix 2. Following the example in (4.1), we can formulate a directing process as follows, using the same principle:

$$\Delta g(t) \equiv \exp(\Theta_{sa} spa_{t-1} + \Theta_{sd}(spa_{t-1} - sp_{t-1})) , \qquad (4.3)$$

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[8]Since the averages nqa_t were computed on a weekly basis, there might be some slight differences from one day to the next one over an entire week. Yet, judging on the basis of figure 4.1, those differences appear minor.

where spa_{t-1} is the sample average computed on a weekly basis of the mean spread over the interval $t-1$ while sp_{t-1} is the mean spread actually realized.

Last, but certainly not least, we can use absolute return. The weekly averages are displayed in figure 4.3 of appendix 2. The absolute return series has been used by Müller et al. (1990) to model an activity scale. These authors have observed that absolute returns exhibited clear structures reflecting market activity through the repetitive cycle of business hours. Indeed, we recover such a pattern in absolute returns, although it appears again to be not as regular as in the case of quote arrivals. If we were to use only absolute returns, we could construct a directing process:

$$\Delta g(t) \equiv \exp(\Theta_{ra} ara_{t-1} + \Theta_{rd}(ara_{t-1} - ar_{t-1})) , \qquad (4.4)$$

where ara_{t-1} is the sample average of absolute returns while ar_{t-1} are the actual realization for time interval $t-1$.

Whatever measure suits best to formulate the directing process is ultimately an empirical question of model specification and diagnostics. In (4.1) through (4.4) we considered each of the series separately in their expected value format and deviations from the mean. However, one could easily combine the series and create a generic directing process:

$$\Delta g(t) \equiv \exp(c'Z_{t-1}) \equiv \exp(\Theta_{qa} nqa_{t-1} + \Theta_{sa} spa_{t-1} + \Theta_{ra} ara_{t-1} \qquad (4.5)$$

$$+ \Theta_{qa}(nqa_{t-1} - nq_{t-1}) + \Theta_{sd}(spa_{t-1} - sp_{t-1}) + \Theta_{rd}(ara_{t-1} - ar_{t-1})) .$$

A priori one should expect that the formulation (4.5) has a lot of redundancy, particularly with respect to the nqa, spa and ara time series. Presumably, the best representation is to pick one of the averages as representative to measure market activity as a combination of the selected average processes and add the series measuring deviations from regular market activity. The latter could be represented either by one, two or all three surprise variables in (4.5). This is precisely the modelling strategy which we will adopt in the next section.

Before presenting the estimation results, we need to discuss the time series properties of the $(nqa_t - nq_t)$, $(spa_t - sp_t)$ and $(ara_t - ar_t)$ series, i.e., the series measuring deviations from regular market activity. To do this, we examine the autocorrelation

function of each series. They are plotted in figures 4.4 through 4.6 of appendix 2, and they are complementary to the plots of the weekly averages. The first of the three figures covers the ACF of the $\{nqa_t - nq_t\}$ series. We notice a very strong and repetitive pattern in all three markets. This means that average quote arrivals, as displayed in figure 4.1 of appendix 2 are not the only source of periodic patterns appearing in equation (4.1), since the deviations from market average are also strongly autocorrelated with seasonal patterns. When we turn our attention to figure 4.5 of appendix 2, which covers the bid-ask spread series $\{spa_t - sp_t\}$, we observe less seasonal autocorrelations, at least on a daily basis, but still within a weekly lag. Since weekends were deleted prior to computing the autocorrelation functions, one recovers a positive autocorrelation at around 360 lags. This weekly pattern is present on both the DEM/USD and JPY/USD markets. On the JPY/DEM market, we find a daily seasonal pattern, however, quite similar to that in figure 4.4. Finally, we turn our attention to the absolute return market deviation series $\{ara_t - ar_t\}$ in figure 4.6. Unlike the two previous series, it exhibits no particular regular patterns in the corresponding ACF. Instead, we find a slowly decaying pattern starting from a first order autocorrelation which is much higher than the previous ones, namely .25 instead of around .05 as those appearing in figures 4.4 and 4.5. Even after 800 lags, we still have an autocorrelation above .05.

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5. Empirical Results

There are three currency exchange markets available in the Olsen and Associates data set, and hence, we will devote a subsection to each market. We begin with the most active DEM/USD market which is covered in section 5.1 followed by JPY/DEM and finally, JPY/USD markets which are covered in section 5.2. Before discussing the actual results, a few observations are in order regarding estimation. It was noted in the previous section that the details of the QMLE algorithm are omitted here as they appear in Ghysels and Jasiak (1994). The numerical optimization of the quasi-likelihood function was accomplished via simulated annealing. The algorithm, which is described in Goffe et al. (1994), appeared to be the best equipped to deal with the multiplicity of local maxima which tricked most other conventional algorithms we tried. Also, for reasons of numerical stability, we rescaled the quote and spread series by 1.e–03 while the absolute return series was rescaled by 1.e–01.

5.1 The DEM/USD Foreign Exchange Market

In section 4 we noted that our modelling strategy of the mapping between calendar time and operational time would consist of picking one of the three series measuring anticipated market activity and combine it with the set of series reflecting deviations from averages. Table 5.1 reports the estimation results obtained from the 20 minute sampling interval for three model specifications, each involving different measures of average market activity, as appearing in equations (4.1) through (4.5), completed with several combinations of the deviations from average market activity. To avoid reporting too many empirical results, we present models with nqa and ara variables of average market activity and omit those with spa which yield quite similar results. The three surprise terms appear either simultaneously or separately in models summarized in table 5.1. Besides the point estimates, we also report standard errors which were computed using a heteroskedasticity consistent QMLE covariance matrix estimator. One should recall that the QMLE procedure is asymptotically inefficient, yet the standard errors in Table 5.1 reveal that all series entering $\Delta g(t)$, no matter what specification is used, appear significant. Hence, the standard errors do not give us

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much guidance on what model specification to chose. Before elaborating further on model choice, let us first discuss the interpretation of the estimates. One can note immediately that all the coefficients Θ_{ij} have negative signs. Obviously, each coefficient measures a partial effect. However, from microstructure models we know, for instance, that as the time interval between quotes decreases, one expects spreads to increase (see, for instance, Easley and O'Hara (1992) for further discussion). Hence, each of the series reflects movements that are obviously not unrelated. The mapping between calendar time and operational time we investigate is, of course, one based on statistical fit. Let us distinguish first the coefficients related to average market activity from those related to deviations from the normal pace. The first example covers average quotes. Negative coefficients Θ_{qa} and a imply that when the average number of quotes is high, market volatility becomes more persistent and less erratic. Obviously, high quote arrivals do not necessarily reflect a high information content, but often it means that many markets are active simultaneously. Comparing figures 4.1 and 4.2, we note that high average quote arrivals appear to be associated with higher bid-ask spreads, at least for the DEM/USD market discussed here. Likewise, comparing figures 4.1 and 4.2, we make the same observation for absolute returns, at least again on the DEM/USD market.

The coefficients related to deviations from normal market activity are also negative. Since deviations are measured as average minus actual realizations, it is clear that, with negative a coefficient, above normal market activity increases volatility and vice versa. Also, operational time increases (decreases) when market activity is above (below) average. It must also be noted that each specification of $\Delta g(t)$ involves lagged values of the deviations from market activity. This is, of course, done in order to guarantee that $\Delta g(t)$ is based on variables that are measurable with regard to $t - 1$ information. From the autocorrelation functions in figures 4.4 through 4.6, we also know, however, that the first order autocorrelations for each of the market activity deviation processes are positive.

With all entries being significant for the 20 minute DEM/USD specifications, we must rely on other criteria to discriminate among models. In the remainder of this section, we will focus on the models appearing in the first column of tables 5.1 through 5.3. These models contain all three measures of deviations from average market

activity combined with each of the three measures of average market activity. We first turn our attention to the plots of squared returns paired with the sample paths of market time as obtained from the estimated $\Delta g(t)$ processes. They are displayed in figure 5.1 of appendix 3. The three $\Delta g(t)$ processes appear quite similar, although upon closer examination, it is clear that the time deformation involving average quotes looks quite distinct from the other two specifications.

Since we have computed the $\Delta g(t)$ process, we may also proceed as in Müller et al. (1993) and analyze returns not in calendar time, but rather in operational time. It is a useful tool, as Müller et al. (1993) suggest, to study "deseasonalized" returns. It should be noted though that while the Olsen and Associates activity scale is purely based on average (repetitive) patterns, our uses direct dynamic effects. We compute the autocorrelation function in operational time estimated from our models, by using an approximation, namely, by defining the $[\Delta \log y_t - a_1, \ \Delta \log y_{t-1} - \lambda]^2 / \Delta g(t)$ process as being the normalized returns, relative to market time. Obviously, when $\Delta g(t) \equiv 1$, we recover the calendar time process. Otherwise, we recover a squared return process adjusted for serial dependence and drift which is normalized by operational time changes. This normalized process is used to compute an autocorrelation function. For comparison, we plot first the squared returns ACF in calendar time followed by the ACF computed from the Olsen and Associates time scale.[9] (See figures 5.2–5.3, appendix 3.) We observe that all operational time autocorrelation functions, namely, the Olsen and Associates and our specifications look very different. Those involving average spreads which were not reported in table 5.1 show significant autocorrelations at weekly, biweekly, etc. lags. In sharp contrast, the ACF's involving operational time scales with average quotes and particularly with absolute returns, indicate that the normalized squared returns are almost white noise series which do not show any long memory properties. Judging on the basis of these ACF, it appears that the model involving absolute returns is probably the most appealing to estimate from the DEM/USD data.

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[9]We are grateful to Michael Dacorogna for providing us with the ACF. It should be noted that the sample used in Müller et al. (1993) and the one used here is not exactly the same. We ignore this aspect here.

5.2 The JPY/USD and JPY/DEM Foreign Exchange Markets

We now turn our attention to tables 5.2 and 5.3, each covering empirical results from one market. Again, we present model specifications involving two different measures of average market activity, *nqa* and *ara*, combined with three measures of deviations introduced in the previous section.

There are several differences between the parameter estimates based on the DEM/USD sample and those reported on the JPY/USD and JPY/DEM markets. In tables 5.2 and 5.3 we notice immediately positive as well as negative signs of Θ_{ij} and we also see that some coefficients became insignificant. Exceptionally, the parameter estimates of the JPY/USD model involving the *nqa* variable (table 5.2, top panel) have similar signs as the coefficients of the analogue specification estimated from the DEM/USD data. Consequently, both models yield a similar interpretation of the volatility behavior. The operational time slows down when the number of expected quotes increases and it accelerates while the current number of quotes, the current level of spread or returns exceeds the expected values. Thus, changes in the volatility appear to be driven by the extent in which the actual market activity deviates from the average level. The results presented in the bottom panel of table 5.2 indicate that the surprise terms have the same effect in the specification involving the *ara* variable. A high level of expected returns, contrary to the average quote arrival, speeds up the operational time and the volatility adjustments.

The results based on the JPY/DEM sample are difficult to interpret. In section 2 we have pointed out several distinct distributional properties of JPY/DEM quotes. Some particular seasonal patterns of this series have also been discussed in section 4. It appears that the only variable accelerating the operational time and, hence, changes in the volatility process, is the instantaneous excess return. The coefficients on the remaining variables are positive throughout both specifications indicating an opposite effect. It may also be worth recalling from section 2 that the excess kurtosis statistics for the tick–by–tick data in comparison to the equally sampled data were not conform with a time deformation framework $X(\Delta g(t))$ were $\Delta g(t)$ is independent of X, on the JPY/DEM market. This fact may also help to explain the poor performance. In conclusion, the JPY/DEM model yields results which are not plausible, and it

seems appropriate to estimate volatility on this particular market within a different framework.

Table 5.1

QML Estimates of Stochastic Volatility Models with Time Deformation

20 Minute Sampling Intervals — DEM/USD Market

Model: $\log[\Delta \log y_t - a_1 \Delta \log y_{t-1} - \lambda]^2 = -1.27 + h_t + \varsigma_t; \; h_t$
$$= [(1 - \exp(a \Delta g(t)))]b + \exp(a \Delta g(t))h_{t-1} + v_t$$

$$\Delta g(t) \approx \exp[\Theta_{qa} nqa_{t-1} + \Theta_{qd}(nqa_{t-1} - nq_{t-1}) + \Theta_{sd}(spa_{t-1} - sp_{t-1}) + \Theta_{rd}(ara_{t-1} - ar_{t-1})]$$

$$v_t \sim N(0, -\Sigma(1 - \exp(2a\Delta g(t)))/2a)$$

	(1)		(2)		(3)		(4)	
	Est.	St. Er.	Est.	St. Er.	Est.	St. Er.	Est.	St. Er.
Θ_{qa}	−0.0106	0.0011	−0.0107	0.0011	−0.0114	0.0032	−0.0148	0.0016
Θ_{qd}	−0.0197	0.0029	−0.0196	0.0029	0.0320	0.0057	—	—
Θ_{sd}	−1.5040	0.0054	−1.4805	0.0053	—	—	−1.6044	0.0262
Θ_{rd}	−4.5236	0.0049	—	—	—	—	−4.7899	0.0050
a	−0.4206	0.0050	−0.3800	0.0053	−0.1357	0.0305	−0.6455	0.0226
Σ	1.1714	0.0049	1.0581	0.0049	0.3758	0.1050	1.7842	0.0114
b	−14.9361	0.0050	−14.9369	0.0050	−14.9702	0.0773	−14.8807	0.0375

$$\Delta g(t) \approx \exp[\Theta_{ra} ara_{t-1} + \Theta_{qd}(nqa_{t-1} - nq_{t-1}) + \Theta_{sd}(spa_{t-1} - sp_{t-1}) + \Theta_{rd}(ara_{t-1} - ar_{t-1})]$$

	(1)		(2)		(3)		(4)	
	Est.	St. Er.	Est.	St. Er.	Est.	St. Er.	Est.	St. Er.
Θ_{ra}	−2.2965	0.0050	−2.3450	0.0050	−2.2099	0.6004	−1.8602	0.0049
Θ_{qd}	−0.0270	0.0028	−0.0270	0.0028	−0.0268	0.0047	—	—
Θ_{sd}	−1.7471	0.0161	−1.7464	0.0157	—	—	−1.8360	0.0142
Θ_{rd}	−3.0309	0.0049	—	—	—	—	−2.6463	0.0049
a	−0.7601	0.0269	−0.7418	0.0245	−0.1257	0.0189	−0.8759	0.0276
Σ	2.0766	0.0115	2.0260	0.0106	0.3382	0.0629	2.2712	0.0120
b	−14.8290	0.0060	−14.8193	0.0119	−14.8360	0.0614	−14.6385	0.0106

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Table 5.2

QML Estimates of Stochastic Volatility Models with Time Deformation

20 Minute Sampling Intervals — JPY/USD Market

Model:
$$\log[\Delta \log y_t - a_1 \Delta \log y_{t-1} - \lambda]^2 = -1.27 + h_t + \varsigma_t; \quad h_t$$
$$= [(1 - \exp(a \Delta g(t)))]b + \exp(a \Delta g(t))h_{t-1} + v_t$$

$$\Delta g(t) \approx \exp[\Theta_{qa} nqa_{t-1} + \Theta_{qd}(nqa_{t-1} - nq_{t-1}) + \Theta_{sd}(spa_{t-1} - sp_{t-1}) + \Theta_{rd}(ara_{t-1} - ar_{t-1})]$$

$$v_t \sim N(0, -\Sigma(1 - \exp(2a\Delta g(t)))/2a)$$

	(1)		(2)		(3)		(4)	
	Est.	St. Er.	Est.	St. Er.	Est.	St. Er.	Est.	St. Er.
Θ_{qa}	−0.0153	0.0043	−0.0167	0.0066	−0.0070	0.0024	−0.0070	0.0042
Θ_{qd}	−0.0239	0.0036	−0.0243	0.0083	—	—	—	—
Θ_{sd}	−0.2002	0.0049	—	—	0.2970	0.1401	—	—
Θ_{rd}	−0.8204	0.0049	—	—	—	—	−0.5879	0.0049
a	−0.2189	0.0053	−0.2152	0.0411	−0.1943	0.0069	−0.1983	0.0399
Σ	0.6899	0.0111	0.6777	0.1756	0.5801	0.0239	0.5951	0.0079
b	−14.9240	0.0050	−14.9306	0.0596	−14.7899	0.0212	−14.7950	0.0050

$$\Delta g(t) \approx \exp[\Theta_{ra} ara_{t-1} + \Theta_{qd}(nqa_{t-1} - nq_{t-1}) + \Theta_{sd}(spa_{t-1} - sp_{t-1}) + \Theta_{rd}(ara_{t-1} - ar_{t-1})]$$

	(1)		(2)		(3)		(4)	
	Est.	St. Er.	Est.	St. Er.	Est.	St. Er.	Est.	St. Er.
Θ_{ra}	6.2526	0.0049	6.2884	2.6309	6.0877	3.8865	5.2324	0.0049
Θ_{qd}	−0.0178	0.0041	−0.0181	0.0054	—	—	—	—
Θ_{sd}	−0.2864	0.0050	—	—	−0.3251	0.1398	—	—
Θ_{rd}	−0.7404	0.0049	—	—	—	—	−0.5565	0.0049
a	−0.2063	0.0161	−0.2063	0.0202	−0.1921	0.0068	−0.1943	0.0154
Σ	0.6357	0.0643	0.6354	0.0804	0.5702	0.0234	0.5777	0.0596
b	−14.8784	0.0339	−14.8860	0.0388	−14.7786	0.0213	−14.7838	0.0024

Table 5.3

QML Estimates of Stochastic Volatility Models with Time Deformation

20 Minute Sampling Intervals — JPY/DEM Market

Model:
$$\log[\Delta \log y_t - a_1 \Delta \log y_{t-1} - \lambda]^2 = -1.27 + h_t + \varsigma_t; \; h_t$$
$$= [(1 - \exp(a\Delta g(t)))]b + \exp(a\Delta g(t))h_{t-1} + v_t$$

$$\Delta g(t) \approx \exp[\Theta_{qa} nqa_{t-1} + \Theta_{qd}(nqa_{t-1} - nq_{t-1}) + \Theta_{sd}(spa_{t-1} - sp_{t-1}) + \Theta_{rd}(ara_{t-1} - ar_{t-1})]$$

$$v_t \sim N(0, -\Sigma(1 - \exp(2a\Delta g(t)))/2a)$$

	(1)		(2)		(3)		(4)	
	Est.	St. Er.	Est.	St. Er.	Est.	St. Er.	Est.	St. Er.
Θ_{qa}	0.0273	0.0074	0.0022	0.0132	0.0186	0.0134	−0.0050	0.0131
Θ_{qd}	−0.0204	0.0097	0.0201	0.0111	—	—	—	—
Θ_{sd}	0.4924	0.0054	—	—	0.4822	0.0800	—	—
Θ_{rd}	−0.2240	0.0052	—	—	—	—	0.1387	0.5881
a	−0.1773	0.0098	−0.1743	0.0088	−0.1781	0.0091	−0.1767	0.0089
Σ	0.3195	0.0272	0.3163	0.0212	0.3232	0.0215	0.3241	0.0216
b	−14.3923	0.0022	−14.3674	0.0202	−14.4061	0.0199	−14.3817	0.0196

$$\Delta g(t) \approx \exp[\Theta_{ra} ara_{t-1} + \Theta_{qd}(nqa_{t-1} - nq_{t-1}) + \Theta_{sd}(spa_{t-1} - sp_{t-1}) + \Theta_{rd}(ara_{t-1} - ar_{t-1})]$$

	(1)		(2)		(3)		(4)	
	Est.	St. Er.	Est.	St. Er.	Est.	St. Er.	Est.	St. Er.
Θ_{ra}	6.6248	5.4780	7.7326	5.6690	6.4390	3.0797	7.6165	0.0052
Θ_{qd}	−0.0214	0.0116	0.0195	0.0111	—	—	—	—
Θ_{sd}	−0.4577	0.0787	—	—	−0.1573	0.0978	—	—
Θ_{rd}	−0.2292	0.6664	—	—	—	—	−0.1189	0.0054
a	−0.1805	0.0093	−0.1748	0.0088	−0.1812	0.0171	−0.1750	0.0147
Σ	0.3276	0.0220	0.3178	0.0212	0.3309	0.0417	0.3199	0.0356
b	−14.4100	0.0198	−14.3696	0.0196	−14.4160	0.0252	−14.3790	0.0133

6. Conclusions

In this paper we discussed the dynamics of three exchange markets: DEM/USD, JPY/USD and JPY/DEM, and we proposed a stochastic volatility model for exchange rates sampled at high frequencies.

We first examined the complexity of market dynamics emphasizing the seasonal patterns in return, bid, and ask series. The analysis has been based both on unequally spaced data as well as on series sampled at fixed 20 minute intervals. We have pointed out that the choice of the time scale is crucial for the accuracy and the informational content of the results. In the tick-by-tick records, we observed some interesting shifts in the *entire* distributions from one month to the other and even throughout the week. The equally spaced data exhibit similar radical changes in the behavior of the empirical distributions through time. The complexity of the seasonals in high frequency records requires, thus, a more sophisticated framework than simple mean shift models of standard adjustment techniques developed in the (macro) time series analysis, which are transplanted to volatility models (i.e. adding mean shifts to GARCH type models, etc.) Finally, we presented evidence that the usual geometric average of bids and asks is an appropriate measure of returns on the 20 minute time scale but is an unreliable indicator of mean price changes in the tick-by-tick records.

Next, we investigated a new approach to deal with the seasonal effects in high frequency data and proposed a time deformation framework of stochastic volatility. It is worth emphasizing that it is the first attempt to fit this type of model to high frequency exchange rate series. We examined two specifications of the relationship between the volatility of quotes and the expected values of some relevant variables approximating the market activity as well as the instantaneous deviations from their average behavior. In general, the models successfully explained the market dynamics at least in two out of three data sets.

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Appendix 1

Figure 2.1
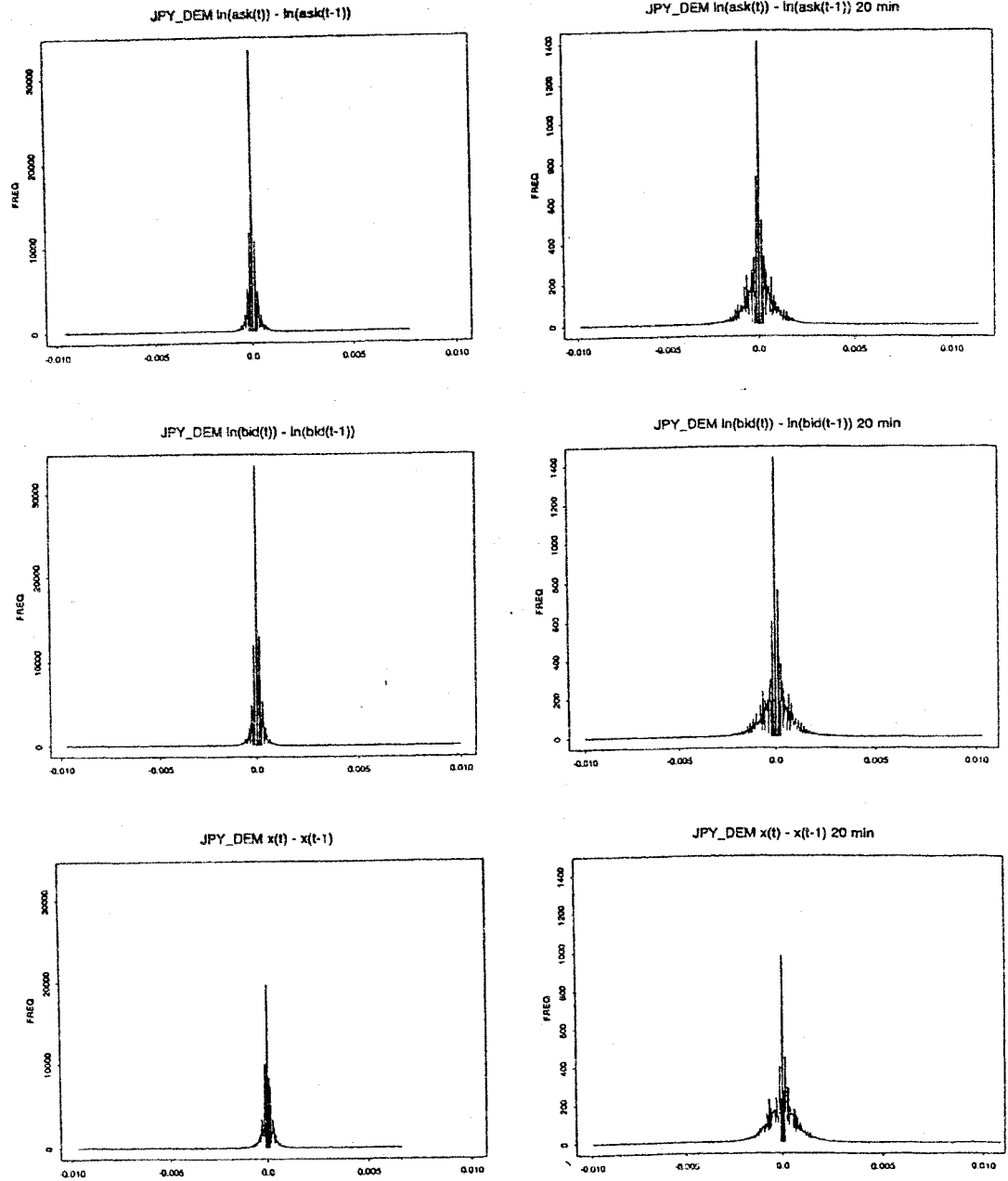
Figure 2.2
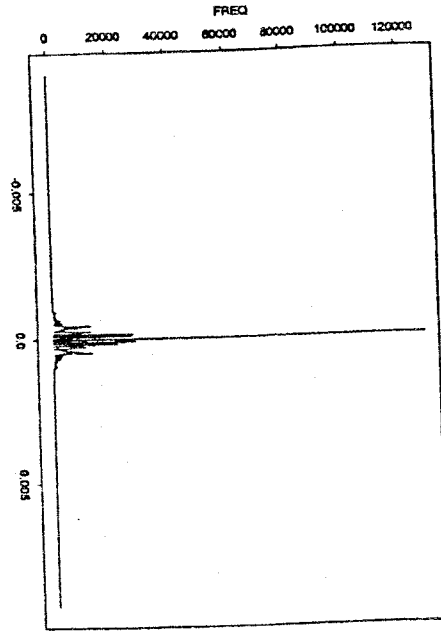
Figure 2.3

JPY_USD ln(bid(t)) - ln(bid(t-1)) 20min
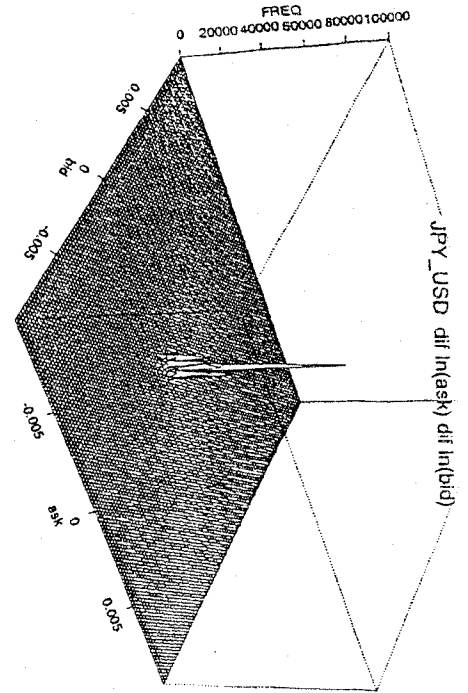
JPY_USD dif ln(ask) dif ln(bid) 20min
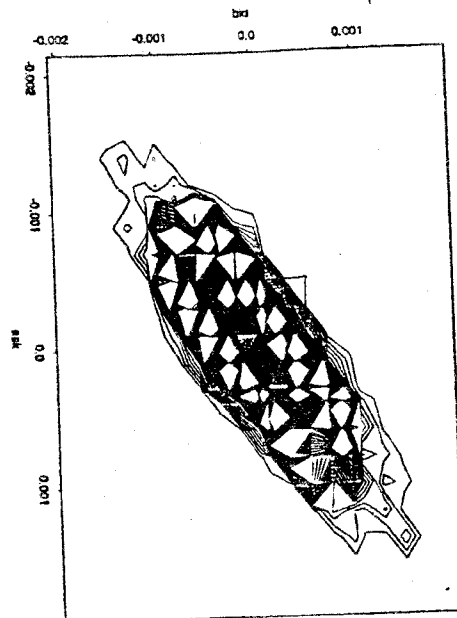
JPY_USD dif ln(ask) dif ln(bid) 20min
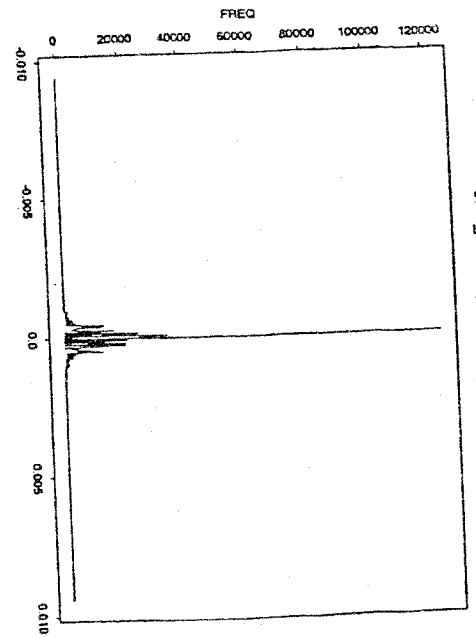
JPY_USD ln(ask(t)) - ln(ask(t-1)) 20min

29
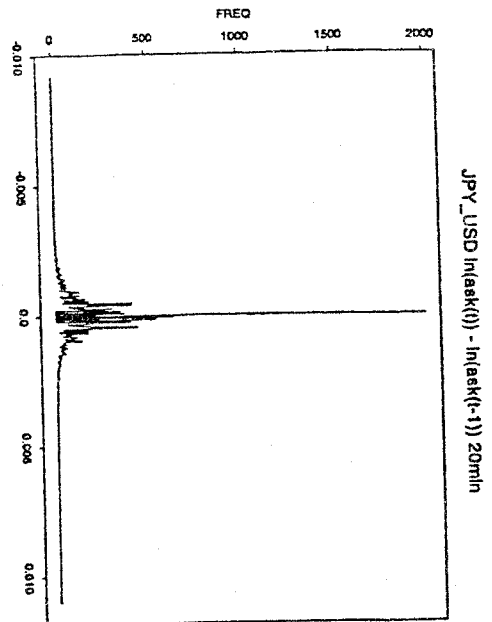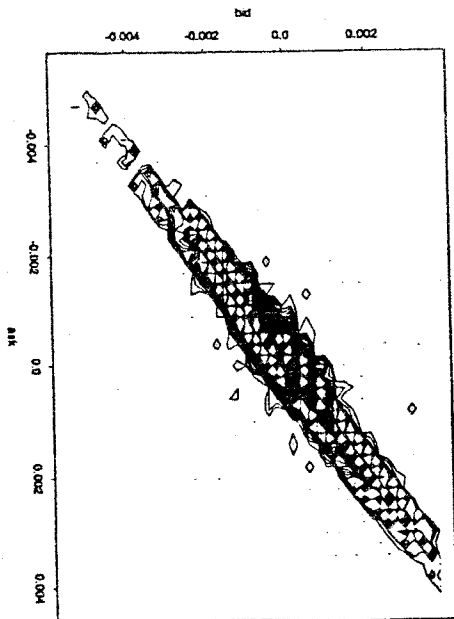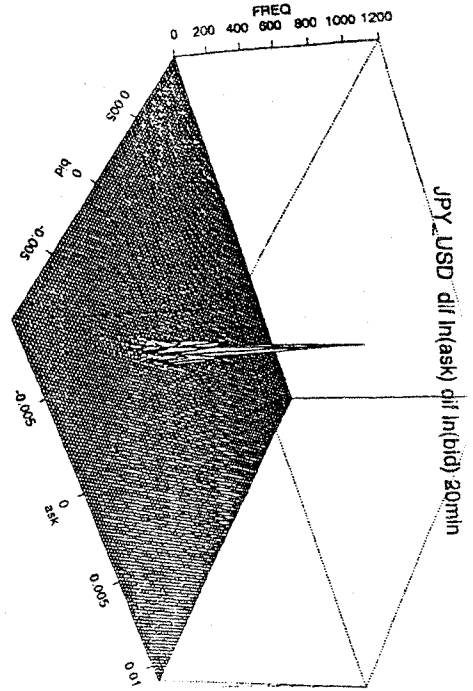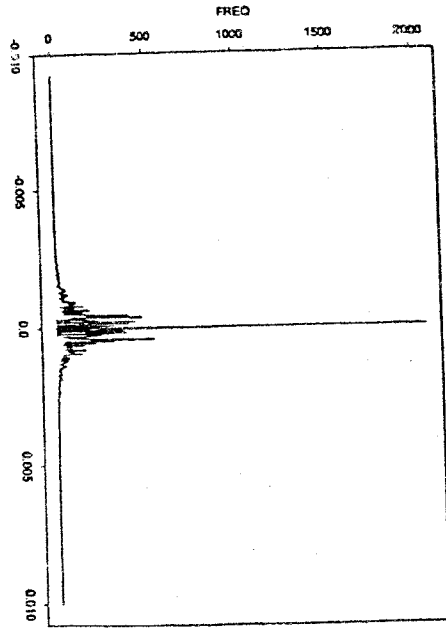
Figure 2.4

Bivariate Monthly Histograms of JPY/USD Quotes (Real Time)

january

march

may

july

october

december
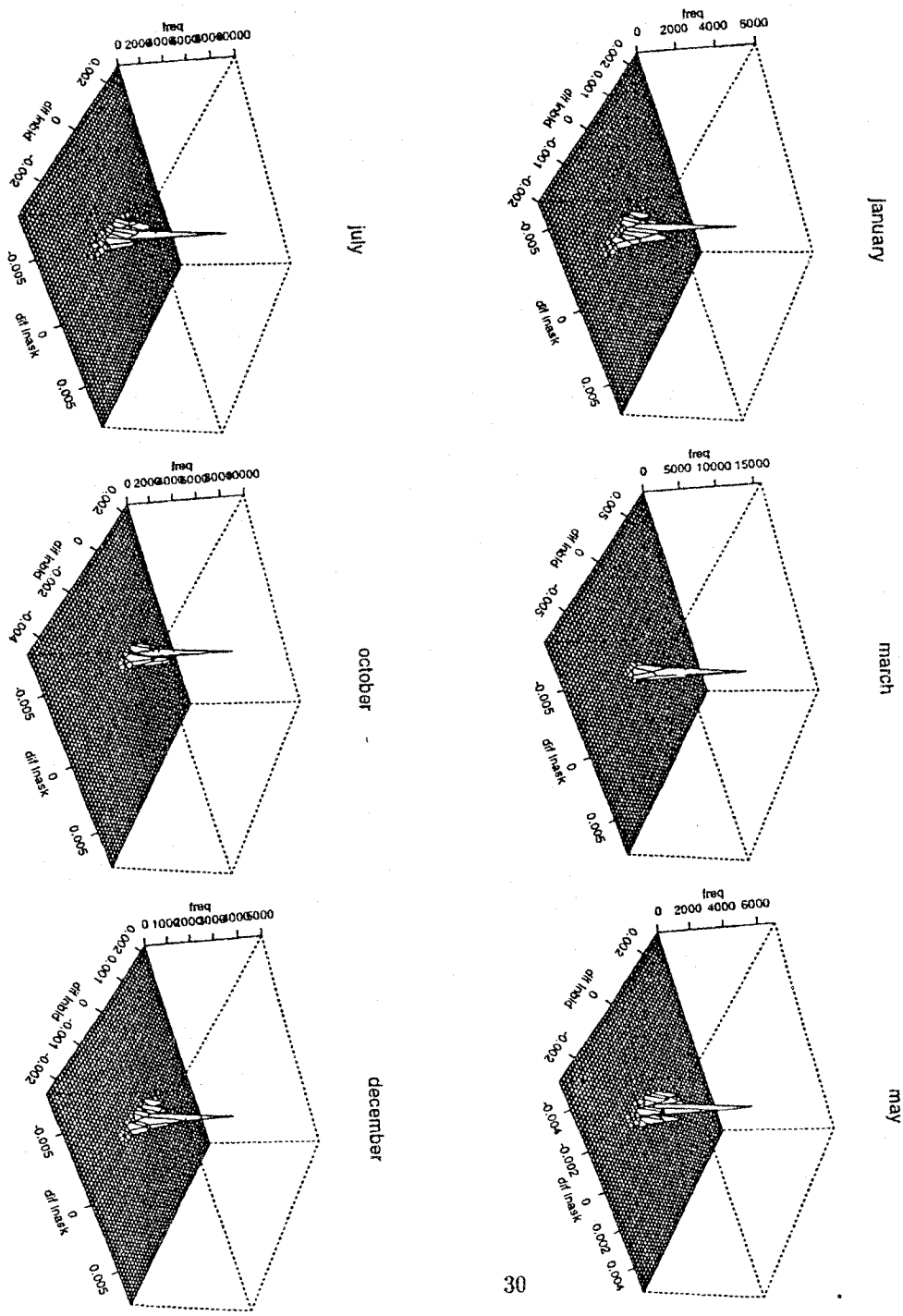
Figure 2.5

Bivariate Monthly Histograms of JPY/USD Quotes (20 Minutes)
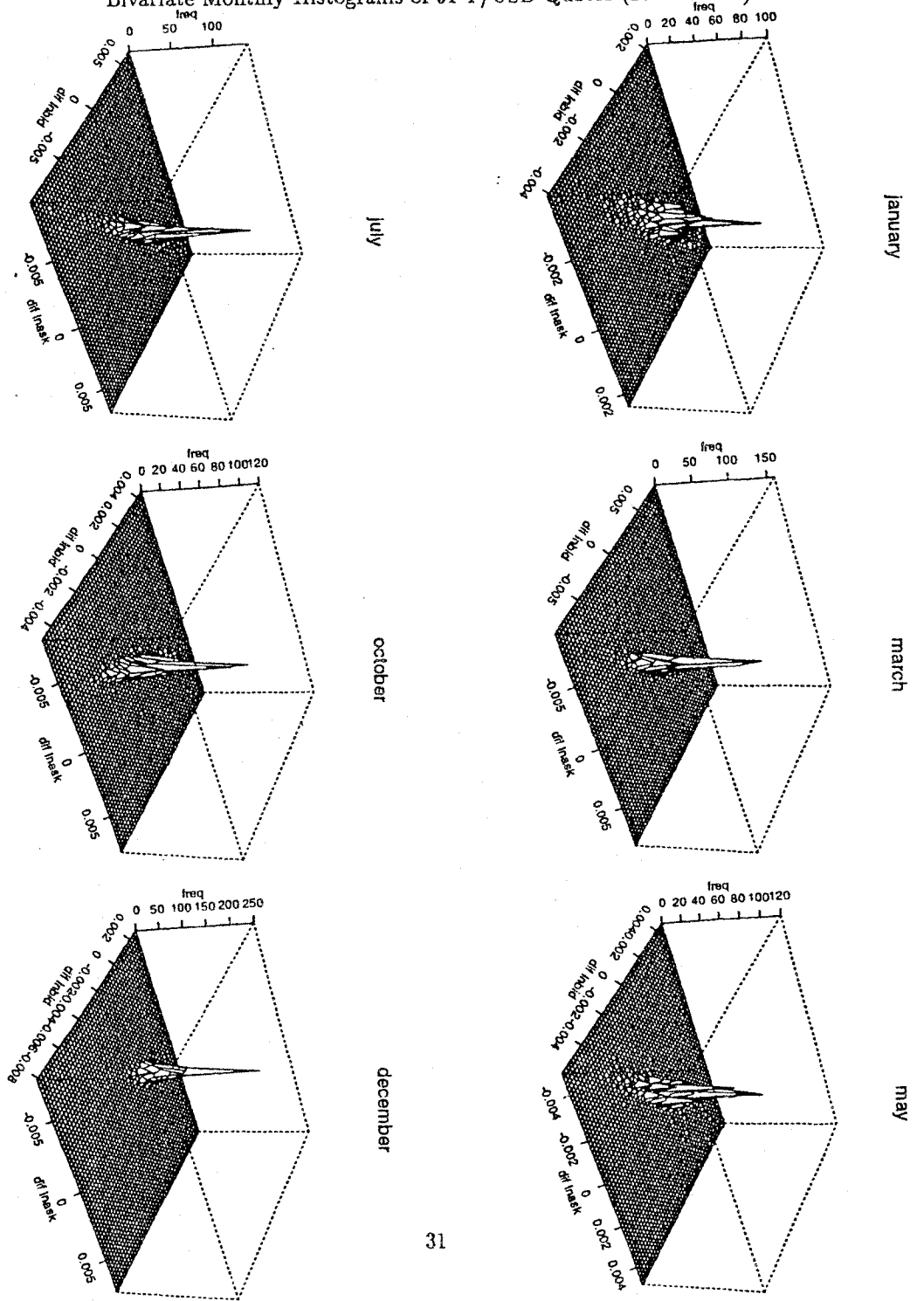
Figure 2.6

Bivariate Monthly Histograms of DEM/USD Quotes (Real Time)
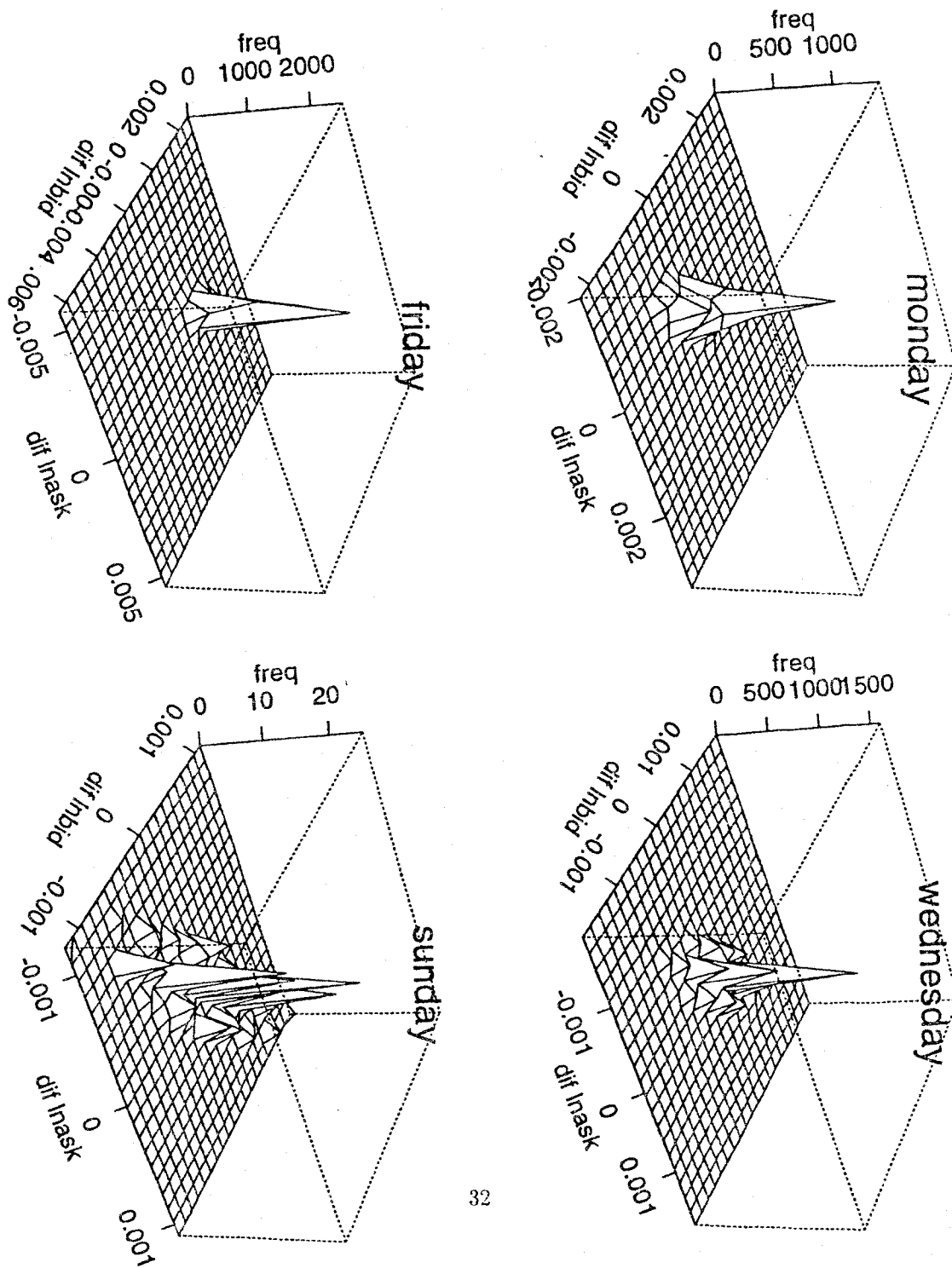
Figure 2.7

Bivariate Monthly Histograms of JPY/USD Quotes (20 Minutes)

Figure 2.8

ACF x(t) - x(t-1), squares, JPY_DEM

ACF ln(bid(t)/bid(t-1)), squares, JPY_DEM

ACF ln(ask(t)/ask(t-1)), squares, JPY_DEM

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Figure 2.9

ACF ln(ask(t)/ask(t-1))|, squares, JPY_DEM 20min

ACF ln(bid(t)/bid(t-1)), squares, JPY_DEM 20min

ACF x(t) - x(t-1), squares, JPY_DEM 20min

35

Appendix 2

Figure 4.1

Weekly Average Number of Quotes at 5 Min. Intervals
DEM_USD Market

JPY_DEM Market

JPY_USD Market

Weekly Average Number of Quotes at 20 Min. Intervals
DEM_USD Market

JPY_DEM Market

JPY_USD Market

Figure 4.2

Weekly Average Bid-Ask Spreads at 5 Min. Intervals
DEM_USD Market

JPY_DEM Market

JPY_USD Market

Weekly Average Bid-Ask Spreads at 20 Min. Intervals
DEM_USD Market

JPY_DEM Market

JPY_USD Market

Figure 4.3

Weekly Average Absolute Returns at 5 Min. Intervals
DEM_USD Market

JPY_DEM Market

JPY_USD Market

Weekly Average Absolute Returns at 20 Min. Intervals
DEM_USD Market

JPY_DEM Market

JPY_USD Market

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Figure 4.4

Deviations from Averages: Quotes 20 Min. Intervals - ACF
DEM_USD Market

JPY_DEM Market

JPY_USD Market

39
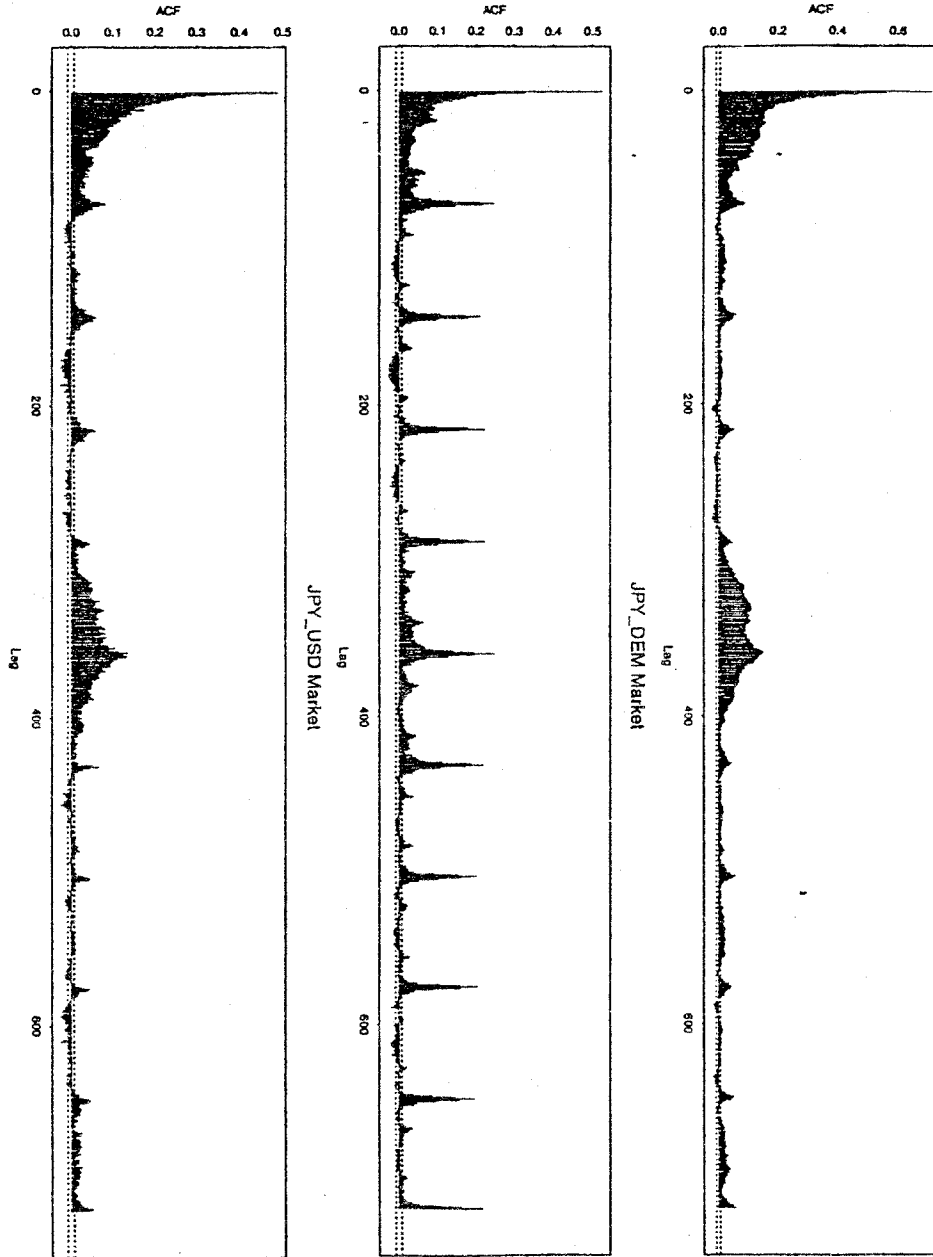
Figure 4.5

Deviations from Averages: Bid-Ask Spreads 20 Min. Intervals - ACF

40
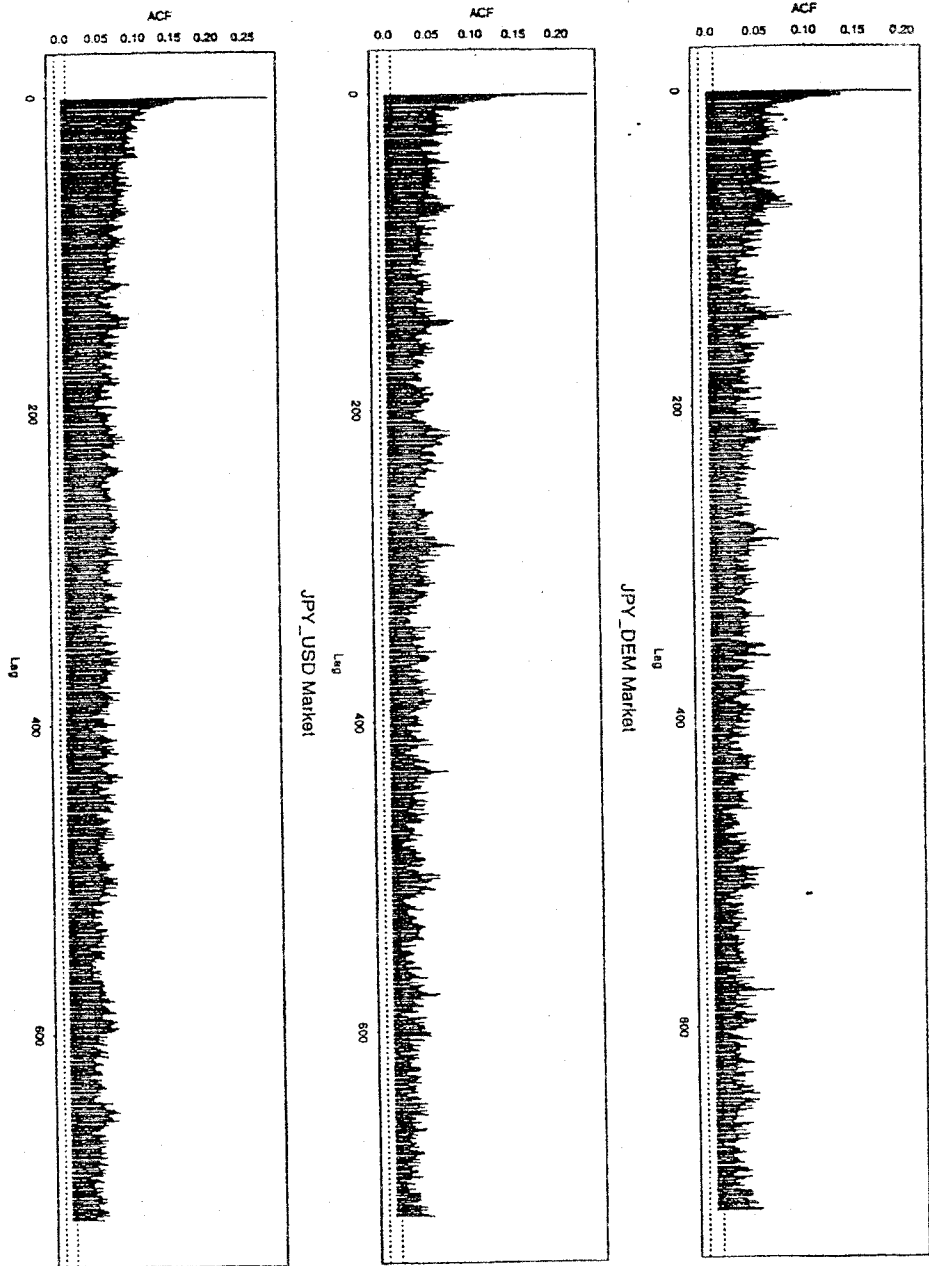
Figure 4.6

Deviations from Averages: Absolute Returns 20 Min. Intervals - ACF
DEM_USD Market

41

Appendix 3

Figure 5.1

US/DM
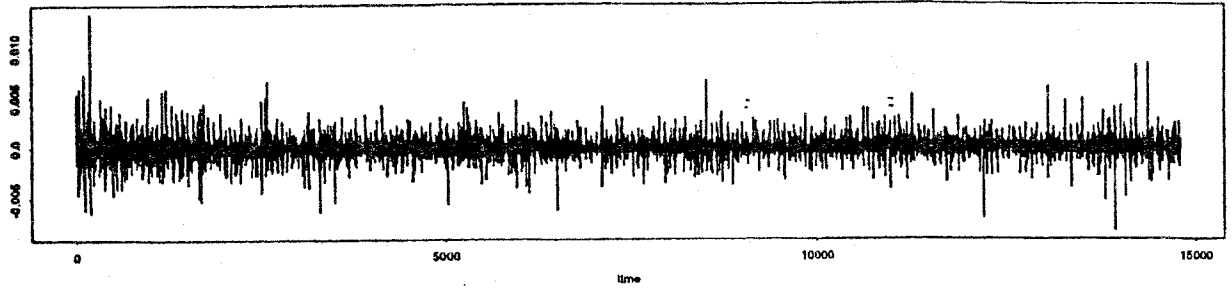
operational time (ave. quotes)
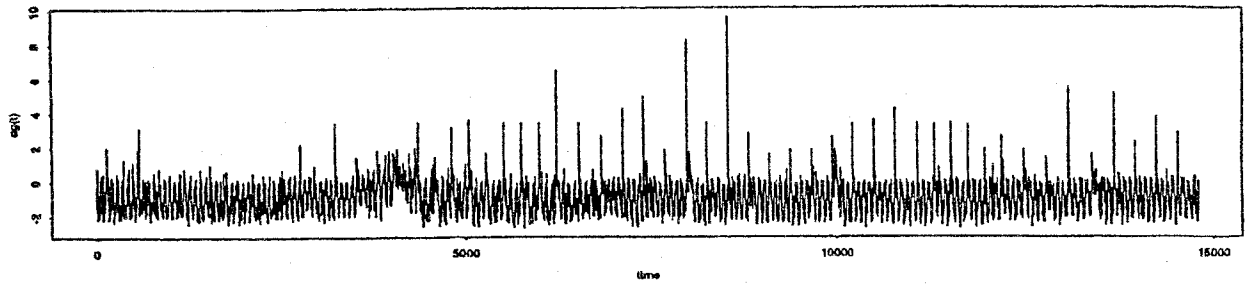
operational time (ave. spread)
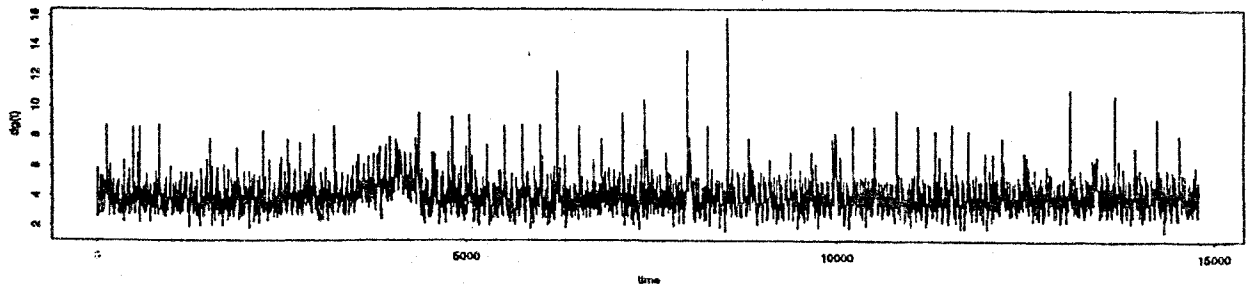
operational time (ave. abs ret.)

Figure 5.2

ACF for squared returns over twenty business minutes - O&A activity scale
DEM_USD Market
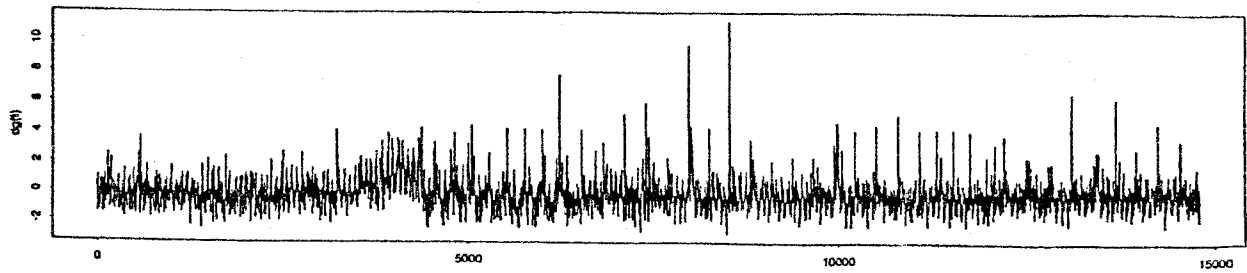
ACF return volatility
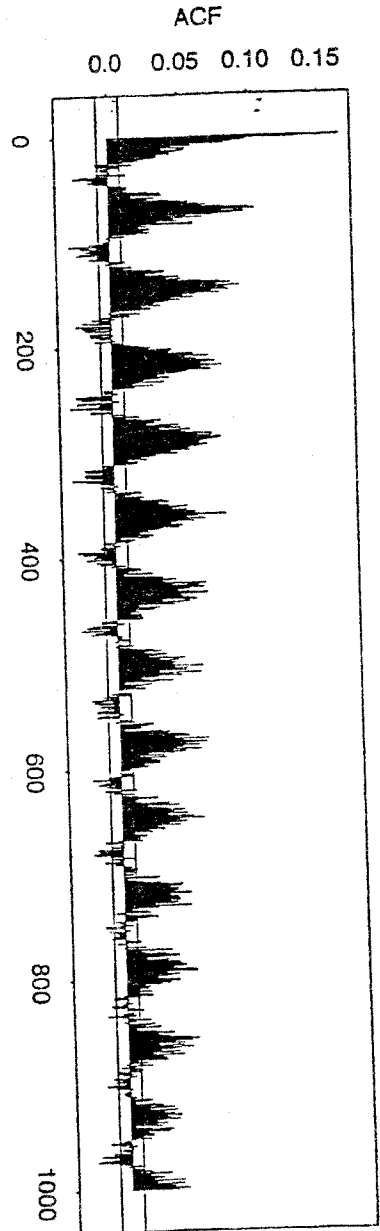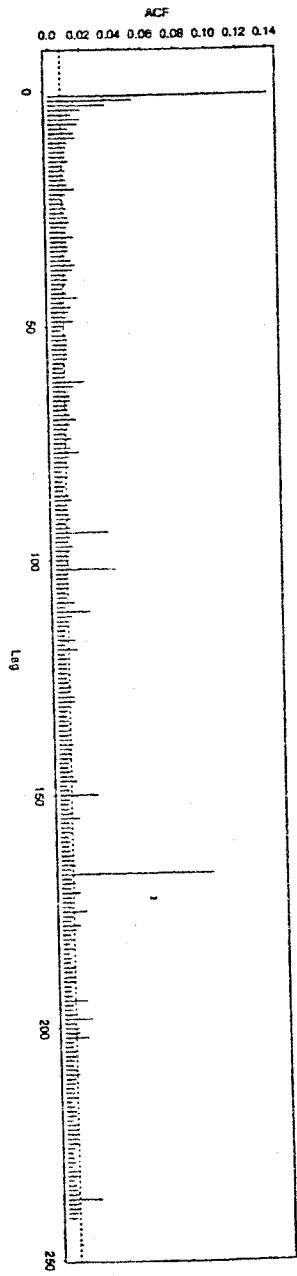
Figure 5.3

ACF return volatility in operational time
(ave.quotes number)
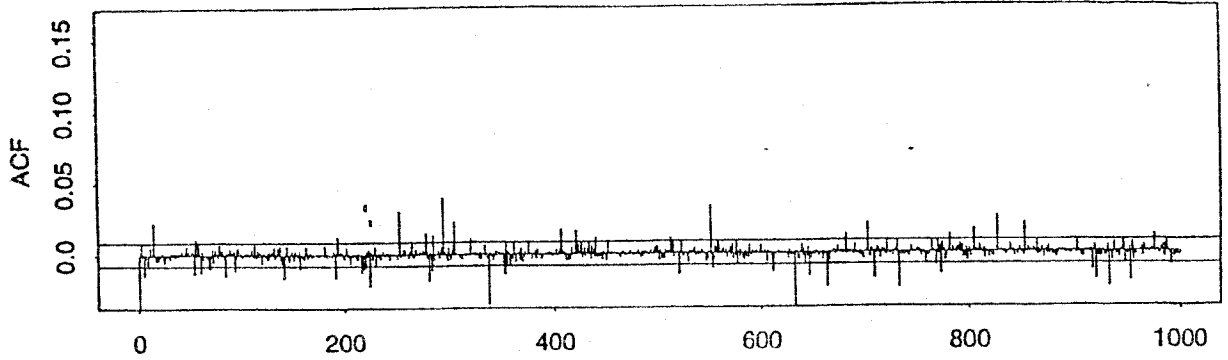
ACF return volatility in operational time
(ave.spread)
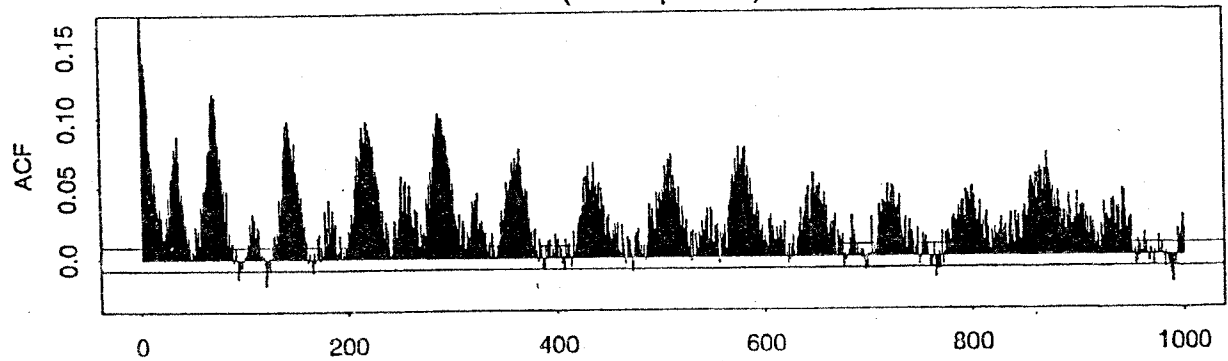
ACF return volatility in operational time
(ave. abs. return)
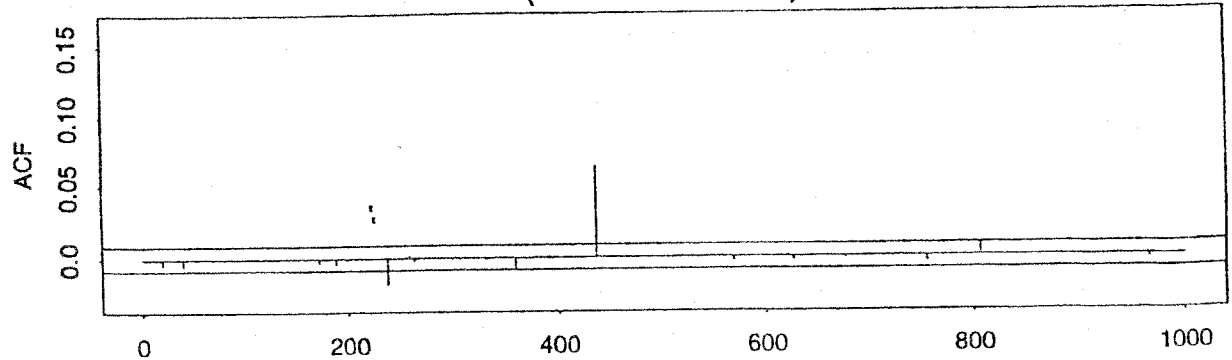
References

Andersen, T. and T. Bollerslev (1994), "Intraday Seasonality and Volatility Persistence in Financial Markets," Discussion Paper, Nothwestern University.

Baillie, R.T. and T. Bollerslev (1991), "Intra-Day and Inter-Market Volatility in Foreign Exchange Rates," *Review of Economic Studies* 58, 565–585.

Bessembinder, H. and P.J. Seguin (1993), "Price Volatility, Trading Volume and Market Depth: Evidence from Futures Markets," *Journal of Financial and Quantitative Analysis* 28(1), 21–39.

Bollerslev, T. and I. Domowitz (1993), "Trading Patterns and Prices in the Interbank Foreign Exchange Market," *The Journal of Finance* Vol. XLVIII, No. 4, pp. 1421–1443.

Bollerslev, T. and E. Ghysels (1994), "Periodic Autoregressive Conditional Heteroskedasticity," *Journal of Business and Economic Statistics* (forthcoming).

Chesney, M. and L. Scott (1989), "Pricing European Currency Option: A Comparison of the Modified Black-Scholes Model and a Random Variance Model," *Journal of Financial and Quantitative Analysis* 24, 267–284.

Clark, P.K. (1973), "A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices," *Econometrica* 41(1), 135–156.

Dacorogna, M.M., C.L. Gauvreau, U.A. Müller, R.B. Olsen and O.V. Pictet (1992), "Short Term Forecasting Models of Foreign Exchange Rates," Presentation at the IBM Summer Research Institute in Oberlech, Austria on "Advanced Applications in Finance, Investment and Banking," July 27 to 31, 1992.

Dacorogna, M.M., U.A. Müller, R.J. Nagler, R.B. Olsen and O.V. Pictet (1993), "A Geographical Model for the Daily and Weekly Seasonal Volatility in the Foreign Exchange Market," *Journal of International Money and Finance* 12, 413–438.

Drost, F.C. and T.E. Nijman (1993), "Temporal Aggregation of GARCH Processes," *Econometrica* 61, pp. 909–998.

Easley, D. and M. O'Hara (1992), "Time and the Process of Security Price Adjustment," *Journal of Finance* XLVII(2), 577–605.

Feinstone, L.J. (1987), "Minute by Minute: Efficiency, Normality and Randomness in Intra-Daily Asset Prices," *Journal of Applied Econometrics* 2, 193–214.

French, K. (1980), "Stock Returns and the Weekend Effect," *Journal of Financial Economics* 8, 55–69.

French, K.R., G.W. Schwert and R.F. Stambaugh (1987), "Expected Stock Returns and Volatility," *Journal of Financial Economics* 19, 3–29.

Gallant, A.R., P.E. Rossi, and G. Tauchen (1992), "Stock Prices and Volume," *Review of Financial Studies* 5, 871–908.

Gallant, A.R. and G. Tauchen (1994), "Which Moments to Match?," *Econometric Theory* (forthcoming).

Ghysels, E., C. Gouriéroux and J. Jasiak (1995), "Market Time and Asset Price Movements: Theory and Estimation," Discussion Paper CIRANO and CREST.

Ghysels, E. and J. Jasiak (1994), "Stochastic Volatility and Time Deformation: An Application to Trading Volume and Leverage Effects," Working Paper C.R.D.E.

Ghysels, E. (1994), "On the Economics and Econometrics of Seasonality," in C.A. Sims (ed.), *Advances in Econometrics — sixth World Congress*, (Cambridge University Press).

Goffe, W.L., Ferrier, G.D. and J. Rogers (1994), "Global Optimization of Statistical Functions with Simulated Annealing," *Journal of Econometrics* 60, 65–100.

Goodhart, C.A.E., and L. Figliuoli (1988), "The Geographical Location of the Foreign Exchange Market: A Test of an 'Island' Hypothesis," LSE financial markets group, Discussion Paper No. 38.

Goodhart, C.A.E., and L. Figliuoli (1991), "Every Minute Counts in Financial Markets," *Journal of International Money and Finance* 10, 23–52.

Gouriéroux, C., A. Monfort and E.Renault (1993), "Indirect Inference," *Journal of Applied Econometrics* 8, Supplement, S85–S118.

Harris., L. (1987), "Transaction Data Tests of the Mixture of Distributions Hypothesis," *Journal of Financial and Quantitative Analysis* 88, 127–141.

Harvey, A.C., E. Ruiz and N. Shephard (1994), "Multivariate Stochastic Variance Models," *Review of Economic Studies* 61, 247–264.

Heston, S.L. (1993), "A Closed-form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options," *Review of Financial Studies* 6, 327–343.

Hull, J. and A. White (1987), "The Pricing of Options on Assets with Stochastic Volatilities," *Journal of Finance* 42, 281–300.

Ito, T., and V.V. Roley (1987), "News from the US and Japan: Which Moves the Yen/Dollar Exchange Rate?," *Journal of Monetary Economics* 19, 255–277.

Johnson, H. and D. Shanno (1987), "Option Pricing when the Variance Is Changing," *Journal of Financial and Quantitative Analysis* 22, 143–152.

Karpoff, J. (1987), "The Relation between Price Changes and Trading Volume: A Survey," *Journal of Financial and Quantitative Analysis* 22, 109–126.

Lamoureux, C. and W. Lastrapes (1990), "Heteroskedasticity in Stock Return Data: Volume versus GARCH Effect," *Journal of Finance* XLV(1), 221–229.

Lamoureux, C. and W. Lastrapes (1992), "Forecasting Stock Return Variance: Toward an Understanding of Stock Implied Volatilities," Working Paper, J.M. Ohlin School of Business, Washington University.

Lamoureux, C. and W. Lastrapes (1993), "Endogenous Trading Volume and Momentum in Stock Return Volatility," Working Paper, J.M. Ohlin School of Business, Washington University.

Lo, A.W. (1988), "Maximum Likelihood Estimation of Generalized Ito Processes with Discretely Sampled Data," *Econometric Theory* 4, 231–247.

47

Mandelbrot, B. and H. Taylor (1967), "On the Distribution of Stock Prices Differences," *Operations Research* 15, 1057–1062.

Melino, A. (1991), "Estimation of Continuous Time Models in Finance," Working Paper, University of Toronto.

Melino, A. and S.M. Turnbull (1990), "Pricing Foreign Currency Options with Stochastic Volatility," *Journal of Econometrics* 45(1/2), 239–267.

Miron, J.A. (1994), "The Economics of Seasonal Cycles," in C.A. Sims (ed.), *Advances in Econometrics — Sixth World Congress* (Cambridge University Press).

Moody, J. and L. Wu (1989), "Statistical Analysis and Forecasting of High Frequency Foreign Exchange Rates," Working Paper, Oregon Graduate Institute.

Müller, U.A., M.M. Dacorogna, R.B. Olsen, O.V. Pictet, M. Schwarz and C. Morgenegg (1990), "Statistical Study of Foreign Exchange Rates. Empirical Evidence of a Price Change Scaling Law and Intraday Analysis," *Journal of Banking and Finance* 14, 1189–1208.

Müller, U.A., M.M. Dacorogna, R.D. Davé, O.V. Pictet, R.B. Olsen and J.R. Ward (1993), "Fractals and Intrinsic Time — A Challenge to Econometricians," Discussion Paper O & A, Zürich.

Nelson, D.B. (1988), "Time Series Behavior of Stock Market Volatility and Returns," Ph.D. dissertation, MIT.

Nelson, D.B. (1989), "Modelling Stock Market Volatility Changes," *Proceedings of the American Statistical Association, Business and Economics Section*, 93–98.

Nelson, D.B. (1991), "Conditional Heteroskedasticity in Asset Returns: A New Approach," *Econometrica* 59, 347–370.

Pagan, A. and G.W. Schwert (1990), "Alternative Models for Conditional Stock Volatility," *Journal of Econometrics,* Cambridge University Press, Cambridge.

Schwert, G.W. (1990), "Indexes of U.S. Stock Prices from 1802 to 1987," *Journal of Business* 63, 399–426.

Scott, L.O. (1987), "Option Pricing when the Variance Changes Randomly: Theory, Estimation and an Application," *Journal of Financial and Quantitative Analysis* 22, 419–438.

Stein, E.M. and J.C. Stein (1991), "Stock Price Distributions with Stochastic Volatility: An Analytic Approach," *Review of Financial Studies* 4, 727–752.

Stock, J.H. (1988), "Estimating Continuous Time Processes Subject of Time Deformation," *Journal of Statistical Association* 83(401), 77–84.

Tauchen, G.E. and M. Pitts (1983), "The Price Variability Volume Relationship on Speculative Markets," *Econometrica* 51(2), 485–505.

Wasserfallen, W. (1989), "Flexible Exchange Rates: A Closer Look," *Journal of Monetary Economics* 23, 511–521.

Wasserfallen, W. and H. Zimmerman (1985), "The Behavior of Intra–daily Exchange Rates," *Journal of Banking and Finance* 9, pp. 55–72.

Wiggins, J. (1987), "Options Values under Stochastic Volatility: Theory and Empirical Evidence," *Journal of Financial Economics* 19, pp. 351–372.

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