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HETEROGENEOUS EXPECTATIONS, SHORT SALES RELATION AND THE RISK-RETURN RELATIONSHIP

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Heterogeneous Expectations, Short Sales Regulation and the Risk-Return Relationship*

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Abstract / Résumé

This paper examines, in a Canadian context, the effect of short sales regulation on the risk-return relationship. Drawing from Jarrow's work (1980), we derive an equilibrium risk-return relationship that accounts for both heterogeneous expectations and short sales regulation. We conclude that the required rate of return on risky assets in a world where short sales are forbidden is equal to the required rate which would prevail in a world free of short sales restrictions, minus an opportunity cost induced by short sales regulation. We show that, theoretically, this opportunity cost is positively related to the dispersion of agents' beliefs and negatively related to the security's liquidity level. We test the model over the sixty-month period from January 1985 through December 1989 and use 13079 observations (220 companies on average). We pool all the observations into a time series cross-sectional model and use Litzenberger and Ramaswamy's methodology (1979) to address three econometric problems: heteroscedasticity, cross-correlation of disturbance terms and beta measurement errors. The results permit us to establish that a negative linear relationship links expected risky asset returns and the divergence of agents' beliefs. This negative relationship is consistent with the presence of opportunity costs resulting from short sales regulation when return beliefs are heterogeneous. We find that the negative relationship between security returns and dispersion of beliefs is essentially confined to illiquid securities, that is, those monitored by a small number of analysts. Finally, these results are not modified when tested on two sub-periods nor when we introduce two control variables (size, as measured by the number of analysts monitoring the stock, and January effect).

L'étude traite de l'effet de la réglementation des ventes à découvert sur la relation rendement-risque, au Canada. À partir du cadre développé par Jarrow (1980), nous développons une expression de la relation rendement-risque lorsque les anticipations des agents sont hétérogènes et les ventes à découvert sont restreintes. Il apparaît alors que les restrictions sur les ventes à découvert induisent un coût d'opportunité qui réduit le taux de rendement anticipé. Ce coût d'opportunité devrait être une fonction positive de la dispersion des anticipations et une fonction négative du niveau de liquidité du titre. Ces hypothèses sont vérifiées à l'aide de données mensuelles, qui couvrent la période de janvier 1985 à décembre 1989. La méthodologie de Litzenberger et Ramaswamy (1979), est utilisée afin de résoudre les divers problèmes économétriques. Les résultats montrent une relation linéaire négative entre le rendement des titres et le niveau d'hétérogénéité des anticipations, mesuré par la dispersion des prévisions des analystes financiers. Cette relation est surtout observable pour les titres les moins liquides, qui sont ici les moins suivis par les analystes financiers. Ces résultats valent pour chaque sous période et résistent à l'introduction de variables de contrôle.

Key words: heterogeneous expectations, short sales regulation, dispersion of analysts' forecasts.

Mots clé : anticipation, hétérogènes, réglementation des ventes à découvert, dispersion, prévision des analystes.

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This paper examines the effects of short sales regulation on the risk-return relationship, in a world in which agents have heterogeneous beliefs over future asset returns[1]. Short sales regulation is an important element of capital market micro-structure and has already caught the attention of several researchers. Lintner (1969), Miller (1977), Jarrow (1980), Figlewski (1981), Peterson and Peterson (1982b), and Mayshar (1983) studied the valuation effects of restricting pessimistic investors' opportunity to sell securities short. They argue that short sales regulation has significant effects on information aggregation due to its asymmetric impact on investors with favorable and unfavorable information. Risky asset prices do not reflect average beliefs since the transactions of optimistic investors outweigh those of pessimistic investors in the formation of asset prices. In these Walrasian equilibrium models based on prior beliefs, risky asset prices are higher than those that would prevail in a similar economy in which short sales were unrestricted. This systematic overvaluation of risky assets induced by market institutions is inconsistent with the existence of rational investors who should eventually adjust their expectations with respect to market imperfections. In fact, the overvaluation effect of short sales regulation on risky asset prices would be absent in a fully revealing rational expectations equilibrium model. However, only noisy rational expectations equilibrium models are consistent with the dynamics of financial markets. In such models, due to various sources of noise, investors can only partially adjust expectations for one fraction of the short sales regulation effect. Diamond and Verrecchia (1987) show that short sales regulation eliminates some of the transactions and reduces adjustment of prices to private information.

Very little empirical work has been done to validate the overvaluation hypothesis and, up to now, the results are inconclusive. Figlewski (1981) finds a significant negative relationship between risky asset returns and short interest, with the latter used as a proxy for the amount of negative information which would otherwise results in short sales were there no restrictions. Repeating the same tests on another period, Figlewski and Webb (1993) still show the relationship to be negative, albeit non-significant. The use of the recorded short interest as a proxy for the amount of adverse information excluded from the market price requires that observed short positions be proportional to short sales to be undertaken in the absence of restrictions. However, empirical results from Peterson and Waldman (1984) and Brent, Morse and Stice (1990) contradict this relationship. Constraints on short sales are different among

[1] When agents' beliefs are homogeneous, no short positions are held at equilibrium (Bamberg and Spremann, 1986). Thus, the analysis of the effect of short sales regulation on risky asset returns has to be conducted in a world in which investors have heterogeneous beliefs. Nevertheless, in some special cases, the short selling of risky assets may be deemed optimal by some investors even when agents' beliefs are homogeneous: if future endowments are stochastic or if agents have state dependent utility functions (Detemple, 1990).

securities (margin requirements, the security's liquidity level) and among investors (legal or contractual prohibitions on institutional investors).

This paper aims to test the hypothesis of a short sales regulation effect on the risk-return relationship. First, basing our theoretical argument on Jarrow's work (1980), we show in a mean-variance framework that the positive price differential due to short sales regulation results in a negative return differential. We derive an equilibrium relationship in which the constrained expected rate of return on risky assets is equal to the unconstrained rate of return minus an opportunity cost due to short sales regulation. Second, we show that this opportunity cost is a positive function of the number of constrained investors and of the importance of the investors' individual opportunity costs. These two variables are directly related to the divergence of investors' beliefs. This formalization enables us to reach a proper econometric model specification. Third, we use the Litzenberger and Ramaswamy's methodology (1979) to test the model. We pool all the observations into a time series cross-sectional model. We test the model over the sixty-month period from January 1985 through December 1989 and use 13079 observations (220 firms on average), a much larger sample than those employed in previous studies. Finally, this empirical test permits us to establish that a negative linear relationship links expected risky asset returns and the opportunity costs induced by short sales regulation as measured by the divergence of agents' beliefs.

The paper is organized as follows. In the first section, we present the theoretical model and a version of the CAPM that accounts both for heterogeneous expectations and short sales regulation. The methodology and different model specifications are described in the second section. In the last section, we present the data and discuss the results.

1 THE MODEL

1.1 Definitions and assumptions

Contrary to the CAPM, the analytical framework used here rests upon a double set of assumptions concerning agent utility functions (A1) and the distribution of risky asset prices (A2). The heterogeneity of agent beliefs is addressed in assumption (A3). Finally, short sales regulation is formalized through assumption (A4).

(A1) Investors behave as risk-averse expected utility maximizers of end-of-period wealth. Each investor displays constant absolute risk aversion (A_k>0) and his utility function can be rewritten as a negative exponential function of the type: $U_k(W_{k1}) = \exp(-A_k W_{k1})$ where W_{k1} stands for the wealth of the k^{th} investor at time t=1.

(A2) Prices of the J risky assets (j=1,...,J) are multivariate stochastic normally-distributed variables. The risky asset price vectors at time t=0 and time t=1 are respectively denoted **P** and **X**.

(A3) Investors (k=1,...,N) behave as price-takers and have partially heterogeneous beliefs[2]:
a) each investor has his own estimate of the expected price vector $E_k(\mathbf{X})$.
b) all investors share the same covariance matrix of risky assets prices, $\mathbf{\Omega_k}=\mathbf{\Omega}$, $\forall$k.

(A4) Short sales are forbidden[3]. Consequently, the quantity of risky assets of the j^{th} firm held by the k^{th} investor at time t=0, is either positive or null.

We shall now determine the risky asset price equilibrium relationship for an economy with heterogeneous agent beliefs, assuming short sales are forbidden (A4).

1.2 Heterogeneous beliefs, restrictions on short sales and security prices

We consider a single period economy composed of J risky assets and one risk-free asset. Since W_{k1} is normally distributed, with mean $E_k(W_{k1})=\mu_k$ and variance[4] $Var_k(W_{k1})=\sigma_k^2$, the constrained portfolio selection problem for the k^{th} investor is the following:

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[2] The overvaluation effect of short sales on security prices only holds true when agents' beliefs as to $E_k(\mathbf{X})$ differ from the average expected prices and when the expected price covariance matrices are homogeneous, diagonal or identical up to a positive factor (Bamberg and Spremann, 1986). In the case where expected covariance matrices differ, the impact on security prices is ambiguous due to possible substitution effects (Jarrow, 1980).

[3] Reality is in fact an intermediate case, for short sales are simply discouraged. No short sale is permitted except on a rising price (*uptick rule*) or if the last previous change price was upward (*zero plus tick rule*). The main impediment to short sales, however, is the withholding of the sale proceeds from the investor. The proceeds from the sale are held by the broker as collateral for the borrowed stock (Figlewski, 1981).

[4] The expected future wealth, $E_k(W_{k1})$, and the variance of future wealth, $Var_k(W_{k1})$, are given by:

$$E_k(W_{k1}) = \mathbf{q}_k^t E_k(\mathbf{X}) + R_f(W_{k0} - \mathbf{q}_k^t \mathbf{P}) \text{ and } Var(W_{k1}) = \mathbf{q}_k^t \mathbf{\Omega} \mathbf{q}_k$$

$$\underset{\mathbf{q}_k}{\text{Max}}\ E_k[-\exp(-A_k W_{k1})] \equiv \underset{\mathbf{q}_k}{\text{Max}}\ \mu_k - (A_k/2)\sigma_k^2 \equiv \underset{\mathbf{q}_k}{\text{Max}}\ \mathbf{q}_k^t(E_k(\mathbf{X}) - R_f\mathbf{P}) + R_f W_{k0} - (A_k/2)\mathbf{q}_k^t\mathbf{\Omega}\mathbf{q}_k \quad (1)$$

subject to the constraint on short sales: $\mathbf{q}_k \geq \mathbf{0}$

where $\mathbf{q}_k = (q_{k1},...,q_{kJ})^t$ stands for the vector of risky asset quantities sought by the k^{th} investor and where r_f stands for the rate of return on the risk-free asset (R_f=1+r_f). $\mathbf{q}_k^*$ = $(q_{k\ 1}^*,...,q_{k\ J}^*)^t$ is the optimal solution of this non-linear problem if and only if there exist J Lagrange multipliers u_j (j=1,...,J) such that the Kuhn-Tucker conditions are satisfied:

$$E_k(\mathbf{X}) - R_f\mathbf{P} - A_k\mathbf{\Omega}\mathbf{q}_k + \mathbf{u}_k = \mathbf{0}, \quad (2)$$

$$\mathbf{u}_k^t\mathbf{q}_k = 0,\ \mathbf{u}_k \geq \mathbf{0},\ \mathbf{q}_k \geq \mathbf{0} \quad (3)$$

The k^{th} investor's demand for risky assets is:

$$\mathbf{q}_k^* = (A_k\mathbf{\Omega})^{-1}(E_k(\mathbf{X}) - R_f\mathbf{P}) + (A_k\mathbf{\Omega})^{-1}\mathbf{u}_k \quad (4)$$

The first term on the right-hand side of (4) represents the demand for risky assets in a world without restrictions on short sales. The second term represents the differential demand due to the forbidding of short sales.

Market equilibrium requires that aggregate demand equal aggregate supply of risky assets $\mathbf{Q} = (Q_1,...,Q_J)^t$. Under this equilibrium condition, we can solve for the risky asset equilibrium price vector, $\mathbf{P}$, (Jarrow, 1980, eq.14):

$$\mathbf{P} = R_f^{-1}\left\{\left(\textstyle\sum_{k=1}^{N}\alpha_k E_k(\mathbf{X})\right) - \tau^{-1}\mathbf{\Omega}\mathbf{Q}\right\} + R_f^{-1}\left\{\textstyle\sum_{k=1}^{N}\alpha_k\mathbf{u}_k\right\} \quad (5)$$

$$\mathbf{u}_k^t\mathbf{q}_k = 0,\ \mathbf{u}_k \geq \mathbf{0},\ \mathbf{q}_k \geq \mathbf{0} \ \text{ for } k=1,...,N$$

where $\alpha_k = A_k^{-1}/\left(\sum_{k=1}^{N}A_k^{-1}\right) = \tau_k/\left(\sum_{k=1}^{N}\tau_k\right) = \tau_k/\tau$ is the ratio of the k^{th} investor's risk tolerance to the sum of investors' risk tolerances.

The first term on the right-hand side of (5) represents the equilibrium price vector in a market without restrictions on short sales. The second term represents the price differential due to restrictions on short sales. It is equal to the present value of a weighted mean of implicit price vectors related to constraints on short sales, denoted $\mathbf{u}_k$. Each weight is equal to α_k, that is, the ratio of the k^{th} investor's risk tolerance to the sum of investors' risk tolerances. Since the implicit prices, u_{kj}, are either positive or null, the second term is also positive or null. If the constraint on short sales is binding for at least one investor, the equilibrium price of risky assets when short sales are forbidden is always higher than it would be in a market without such restrictions. Equation (5) can be restated as:

$$\mathbf{P}^C = \mathbf{P}^U + R_f^{-1} \left\{ \sum_{k=1}^{N} \alpha_k \mathbf{u_k} \right\} \tag{6}$$

where $\mathbf{P}^C$ and $\mathbf{P}^U$ respectively represent the risky asset equilibrium price vectors in a world where short sales are forbidden and in an identical world where short sales are unrestricted. The positive price differential due to short sales regulation (the market shadow price) will result in a negative return differential (the market shadow return). The purpose of the next section is to formalize the relationship between the market required rate of return when short sales are forbidden, $E(\mathbf{R})^C$, and when short sales are unlimited, $E(\mathbf{R})^U$.

The price equilibrium relationship (5) can be restated as a return equilibrium relationship (see appendix 1 for details):

$$\sum_{k=1}^{N} \alpha_k E_k(\mathbf{R}) = R_f \mathbf{i_J} + \mathbf{ß} \left\{ \left(\sum_{k=1}^{N} \alpha_k E_k(R_M) \right) - R_f \right\} - \left\{ \sum_{k=1}^{N} \alpha_k \mathbf{v_k} \right\} \tag{7}$$

For the j^{th} security, the effect of short sales regulation results in an expected security return lower than that would prevail in a world free of short sales restrictions. The expected return is reduced by a factor equal to the following implicit return: $\sum_{k=1}^{N} \alpha_k v_{kj}$.[5] This factor represents the marginal return investors would expect to earn, on average, for the j^{th} security if the regulator were to relax the constraint by one unit. We can rewrite (7) as:

$$E(\mathbf{R}) = R_f \mathbf{i_J} + \mathbf{ß}\ [E(R_M) - R_f] - \mathbf{v} \tag{8}$$

where $E(\mathbf{R})$ and $E(R_M)$ stand for respectively the average expected rate of return vector and the average expected market rate of return. Let us further define $\mathbf{v}$ as the average market implicit return vector due to short sales regulation. The equilibrium relationship (8) becomes:

$$E(\mathbf{R})^C = E(\mathbf{R})^U - \mathbf{v} \tag{9}$$

Hence, the result of short sales regulation is presumably to decrease the market required rate of return on risky assets. In the next section, we proceed to formulate more precise hypotheses, to further investigate the determinants of v_j, the implicit return on the j^{th} security.

[5] The vector of implicit prices $\mathbf{u_k}$ is equal to $\mathbf{D_P v_k}$, where $\mathbf{v_k}$ and $\mathbf{D_P}$ denote the vector of implicit returns and a diagonal matrix whose elements are risky asset prices, respectively. $u_{kj} = p_j v_{kj}$ represents the k^{th} agent's expected marginal utility pursuant to releasing the constraint on the j^{th} security. Because of the particular form of the agents' utility functions (A1), u_{kj} and v_{kj} are the expected marginal increase of the k^{th} agent's wealth and portfolio return, respectively.

1.3 The effects of short sales regulation on expected returns

For the j^{th} security, the marginal expected return for the k^{th} investor is equal v_{kj} if the constraint is binding. This return is a function of the difference between the average return belief[6] and the individual belief. The more pessimistic the investor's beliefs, the greater the opportunity cost induced by short sales regulation: $y_{kj}=g(t)$ where t stands for the investor's belief and g(.) is a decreasing function of t ($\partial g/\partial t<0$ and g(.) bounded to the left, as asset prices cannot go below zero). The effect of short sales regulation on expected returns can be interpreted as the sum of probabilities of being constrained multiplied by the corresponding implicit return. If return beliefs are normally distributed around the consensus beliefs, μ_j, and with a dispersion of beliefs $\sigma_j=DISP_j$, the effect of short sales regulation on returns, $-v_j$, is equal to:

$$-v_j = -\sum_{k=1}^{N} \frac{v_{kj}}{N} = -\left[\int_{-\infty}^{\mu_j} g(t)\ f(t)\ dt + \int_{\mu_j}^{\infty} g(t)\ f(t)\ dt \right] \qquad (10)$$

Where f(t) stands for the density function of a normal distribution with mean μ_j and standard deviation $\sigma_j=DISP_j$.

If we consider two belief distributions X and Y with means $\mu_X=\mu_Y$ and standard deviations $\sigma_X > \sigma_Y$, we can show that (see appendix 2):

$$v_X - v_Y = \int_{-\infty}^{\mu_Y=\mu_X} g(t)\ [f_X(t) - f_Y(t)]dt > 0 \qquad (11)$$

When we assume a linear cost $g(t)=\mu_j-t$, we have (see appendix 3):

$$-v_j = -\sum_{k=1}^{N} \frac{v_{kj}}{N} = -\frac{\sigma_j}{\sqrt{2\pi}} = -\frac{DISP_j}{\sqrt{2\pi}} \qquad (12)$$

Thus, v_j is a positive function of the dispersion of beliefs about the j^{th} security, $DISP_j$. When short sales are forbidden, the required rate of return on risky assets is an inverse function of the divergence of agent beliefs regarding an asset's return due to the resulting overvaluation. Consequently, we predict a negative relationship between the expected rate of return on a risky asset and the divergence of investors' beliefs.

[6] When short sales are unrestricted and beliefs are heterogeneous, the expected rate of return of risky assets are representative of the average investor: see the first term of (5).

2 EMPIRICAL TESTS

2.1 Model specification

The effect of short sales regulation on the expected returns of the j[th] security, $-v_j$, is a negative function of the divergence of investor beliefs: $-v_j = \gamma_2 . DISP_j$, where $\gamma_2 < 0$. The structural form of the expected theoretical relationship is the following:

$$E(R_{jt}) - R_{ft} = \gamma_0 + \gamma_1 \beta_j + \gamma_2 \, DISP_j \quad (13)$$

where the unknown parameters, γ_0, γ_1 and γ_2 are assumed to be constants that are respectively null, positive (the risk premium on the market portfolio) and negative ($\gamma_0=0$, $\gamma_1>0$ and $\gamma_2<0$). The econometric model used to test this relationship is based upon an *ex post* equivalent of (13):

$$R_{jt} - R_{ft} = \gamma_0 + \gamma_1 \beta_{jt} + \gamma_2 \, DISP_{jt} + \epsilon_{jt}, \ j=1,...,J_t; t=1,...,T \quad (14)$$

where ϵ_{jt} represents the disturbance term for the j[th] security at time t.

Estimation of the model parameter vector $\mathbf{\Gamma}=(\gamma_0, \gamma_1, \gamma_2)^t$ in (14) must address several econometric problems. Indeed, it is likely that the disturbance term variances differ among risky assets and that returns within a given period are cross-correlated. Furthermore, since true betas are not observable, the betas are subject to measurement errors. We resort to Litzenberger and Ramaswamy's procedure (1979, hereafter LR) to handle heteroscedasticity, cross-correlation of disturbance terms and beta measurement errors. First, LR show that to correct for heteroscedasticity and cross-correlation, we can deflate the variables by the standard deviation of residual risk. Under such conditions, the GLS estimator, $\mathbf{\Gamma_{GLS}}$ is equivalent to the weighted least squares (WLS) estimator, $\mathbf{\Gamma_{WLS}}$. Second, LR show that the variance of measurement errors in betas is proportional to the residual variance. To simultaneously correct the problem of beta measurement errors, the variables can be deflated by the standard deviation of the measurement error in betas, rather than by the residual standard deviation. LR show that this WLS estimator is not consistent in the presence of measurement errors. They propose a correction and show that the corrected WLS estimator corresponds to the maximum likelihood (ML) estimator. Though our results analysis centres on the ML estimators, the other estimators (OLS and WLS) are reported for comparison purposes. Before we describe the data and analyze results, we re-examine model (14) to account for a control variable likely to modify the risk level of short sales: the security's liquidity level. In addition, we introduce different control variables.

2.2 Complementary model specifications

The analysis of section 1.3 is based on the number of investors for which short sales regulation is binding. It implies an identical risk level for all short sales because only $DISP_j$ enters into the risk associated with the short sale of the security. It neglects the fact that an unhedged short sale is a speculative transaction whose riskiness is amplified if the security is illiquid. Indeed, short sales are more attractive when the date on which the seller must close his position is distant. However, the short seller may be required to close his position should the owner of the underlying securities decide to sell them and the broker is unable to borrow them elsewhere. The probability that such a situation occurs is inversely related to the security's liquidity level. We measure it by the number of analysts monitoring a security, as this not only considers the number of shares outstanding and the firm's ownership structure of the firm, but also includes the interest of institutional investors in the security (Bhushan, 1989) and the presence of the security on their lending lists[7]. Besides, the number of analysts following a security appears to be one of the major explanatory variables of its short interest, along with the existence of options[8] (Peterson and Waldman, 1984; Brent, Morse and Stice, 1991). To test this complementary hypothesis, which posits that the effect of short sales regulation on risky asset prices is greater for illiquid securities, we have partitioned the sample into two groups based on the number of analysts[9]. The less (more) liquid securities appear in the first (last) group. A dichotomous variable reflects group membership of a security and is denoted D_{jt}. It equals 0 if the security belongs to the first group (less than nine analysts) and 1 otherwise (at least nine analysts). As we want to determine whether the average effect of short sales regulation on security returns, as captured by the γ_2 coefficient from model (14), in fact conceals differences related to the security's liquidity level, we have estimated the coefficients of the following model:

$$R_{jt} - R_{ft} = \gamma_0' + \alpha_0 D_{jt} + \gamma_1' ß_{jt} + \gamma_2' DISP_{jt} + \gamma_3 D_{jt} DISP_{jt} + \xi_{jt}, j=1,...,J; t=1,...,T \qquad (15)$$

This specification enables us to test for differences in the intercept and slope coefficients and particularly to test the null hypothesis $\gamma_3 = 0$ against the alternative one: $\gamma_3 > 0$.

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[7] The lending of securities by institutional investors facilitates short sales of highly capitalized securities that usually figure prominently in their portfolios.

[8] The entire set of those securities for which option contracts are available also belongs to the group of the most widely monitored securities. In addition, these securities are those for which the margin requirements are lowest. These securities are the most liquid and those for which short sales regulation is least binding.

[9] We do not explicitely test for differences in short sales regulation with respect to individual securities, because the estimation of the short sales restriction level is impossible for individual securities.

To validate our results, we examine if the variable DISP can be a proxy for missing factors. We check to what extent our results are robust to the introduction of control variables. Recently, Abarbannel, Lanen and Verrecchia (1994) extensively examined the problems induced by the measurement of investors' expectations by analysts' forecasts. They stressed that failure to control for the number of analysts in empirical tests may lead to mispecified models. We consequently examine the sensitivity of the coefficients in model (14) to the introduction of a control variable: the coverage level, measured by the logarithm of the number of analysts monitoring the stock, $LnNBA_{jt}$ (model 16).

$$R_{jt} - R_{ft} = \gamma_{0t}'' + \gamma_1'' \beta_{jt} + \gamma_2'' DISP_{jt} + \gamma_4 Ln\ NBA_{jt} + \zeta_{jt}, j=1,...,J; t=1,...,T \qquad (16)$$

Controlling for the number of analysts monitoring a security is very similar to controlling for size[10]. As this last effect is also closely linked to the January effect, we re-examine model (14) to account also for this control variable . Finally, we test whether there is a structural change around the market crash of October 1987. In the next section, we describe the data used to test the various hypotheses. Results are then discussed.

3 DATA AND RESULTS

3.1 Data

The study covers the period from January 1985 through December 1989. Monthly security returns were computed using monthly closing prices (or the average between the bid and ask prices if no transactions took place) of the daily *TSE/Western Data Base*. The risk-free rate was computed using the rate of return on 90-day Treasury bills reported in the Bank of Canada Review. The market portfolio returns were based on the TSE Total Return Index monthly returns. Betas, β_{jt}, residual variances, and variances of measurement errors, were computed over the sixty-month period prior to the t^{th} month considered. Descriptive monthly statistics on distributions of security risk premia and betas appear in table 1. The first two moments of those distributions are relatively stable over the 1985-1989 period. The security risk premia and beta distributions are slightly asymmetric to the right. Whereas the former distribution is mesokurtic, the latter is platikurtic over the entire period. Due to the market crash, we omit October 1987 from the sample; its average monthly excess return is -.242.

[10] The R^2 of a regression of market capitalisation against the number of analysts is almost 70%.

Since analysts' earnings forecasts are one of the major products of the financial analysis industry (Givoly and Lakonishok, 1984), one can reasonably assume they are instrumental in the formulation of investors' beliefs with respect to returns. Examining analysts' forecasts as a possible source of delayed stock price responses to earnings, Abarbanell and Thomas (1992) found that analysts' forecasts share properties that are consistent with the stock price behavior. Using Granger causality tests, Forbes and Skerratt (1992) showed *the presence of instantaneous feedback from analysts' forecasts to stock price movements and conversely from stock price movements to the revision of analysts' forecasts*. Then, the dispersion in financial analyst forecasts of annual earnings per share is commonly used as a surrogate of the unobservable dispersion in beliefs surrounding a firm's future stock return (Abarbannel, Lanen and Verrecchia, 1994). A few agencies, such as the *Institutional Brokers Estimate System* (*I/B/E/S*), provide statistics on the distribution of analysts' earnings per share forecasts. These data are published monthly[11]. We therefore use the coefficient of variation of analysts' earnings forecasts, represented by CV,[12] as an *ex ante* measure of the dispersion of investors' expected returns[13]. Monthly descriptive statistics relative to the distributions of coefficients of variation (table 1) show that on average the coefficient of variation is about .35, that the distribution is bounded by zero to the left, is asymmetric to the right and highly leptokurtic. It is almost impossible to infer any time pattern from these statistics. However, Brown, Foster and Noreen (1985) pointed out that the coefficient of variation is sensitive to the length of time until the fiscal year-end; they found that it tends to diminish along the fiscal year. We investigate the sensitivity of our results to this problem and repeat the tests using only companies which have a fiscal year-end in December.

We retained all firms monitored by *I/B/E/S* during the 1985-1989 period. Nevertheless, we impose the condition that the firm be followed by at least three analysts[14]. The number of firms monitored by *I/B/E/S* has greatly increased over time: from 111 in January 1985 to 244 in December 1989 (13079 observations, table 1). The number of participating analysts has also increased. While in 1985, 75.9% of

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[11] See L'Her and Suret (1991) for further details regarding this database.

[12] We prefer this measure of relative divergence of beliefs to the standard deviation of the distribution of forecasted earnings per share, which is a measure of absolute divergence. The coefficient of variation is expressed in percentage and measures the degree of homogeneity of the distribution of analysts' forecasts. Givoly and Lakonishok (1988), Varaiya (1988), Pari, Carvell and Sullivan (1989) and Atiase and Bamber (1994) amongst others used this measure of dispersion.

[13] In fact, analysts do not revise their forecasts daily, resulting in "out-of-date" proxies for the divergence of beliefs (Forbes and Skerratt: 1992). By putting together up to date and out-of-date forecasts, summary measures of dispersion of opinion overstate the true dispersion of beliefs (Stickel: 1991).

[14] Computing the coefficient of variation with as few as 3 observations may be problematic, so we repeat the analysis with companies monitored by at least 5 analysts.

securities were monitored by less than ten analysts, in 1989, this percentage went down to 54,3% (table 2). In 1985, only 10,6% of the sample companies were monitored by more than 13 analysts. In 1989, 35.2% of the sample companies are monitored by more than 13 analysts. The sectorial representation has remained stable, at 28%, 34% and 38% on average for the primary, secondary and tertiary sectors, respectively. Companies for which at least three analysts provide forecasts make up almost 95% of the TSE 300 index. The correlation coefficients between the coefficients of variation of analysts' forecasts (CV) the betas and the logarithm of the number of analysts (LnNBA) are respectively the following: .19 (CV and beta), .20 (beta and LnNBA) and -0.029 (CV, LnNBA). The first correlation coefficients are significantly different from zero at a 1% level, but their magnitudes are too low to create serious multicollinearity problems. The last one is significant only at the 15% level.

3.2 Results

Panel A of table 3 reports the estimated coefficients of (14) using OLS, WLS and ML estimation methods and the corresponding t values, when all the observations are used in the estimation. Panel B reports the coefficients and t values when the more lightly followed stocks are omitted. The main results regarding the null and alternative hypotheses ($\gamma_0 = 0$, $\gamma_1 > 0$ and $\gamma_2 < 0$) follow. First, one can not reject the null hypothesis $\gamma_0 = 0$ for the OLS, WLS and ML estimation methods. The estimates of the intercept, γ_0, are all positive. The estimated monthly coefficients γ_{ML0} are respectively equal to .3964% and .2867% if we impose the restrictions that the companies be followed by at least 3 or 5 analysts.

Secondly, the γ_{OLS1}, γ_{GLS1} and γ_{ML1} estimates are all positive, but we can not reject the null hypothesis, $\gamma_1 = 0$. The γ_{ML1} estimates which take account of the attenuation due to measurement errors in betas are greater than the γ_{OLS1} and γ_{GLS1} estimates, but do not differ statistically from zero. Depending on the restriction we impose on the number of analysts monitoring the stocks, the estimated coefficients γ_{ML1} are respectively equal to .6818% and .7437%. The existence a null risk premium contradicts the concept of risk aversion. So, this *a priori* puzzling result requires further analysis. The $\gamma_1 > 0$ hypothesis is an *ex ante* hypothesis which does not exclude the possibility that the *ex post* slope be null, or even negative, in short bearish market periods. In fact, Fama and French (1992) and Kothary, Shanken and Sloan (1995) report a positive but not significant risk premium over the post-1963 period in the United States. In Canada, Calvet and Lefoll (1985) have drawn similar conclusions. Finally, note that the γ_1 estimates do not differ very much from the average risk premium measured *ex post* for the 1985-89 period, that is, 1.018% with a standard deviation of 4.5%.

Thirdly, we reject the null hypothesis, $\gamma_2 = 0$. All three γ_{OLS2}, γ_{WLS2} and γ_{ML2} coefficient estimates are negative and significantly different from zero at a 1% confidence level. The γ_{ML2} coefficient estimates are -.501% (t value: -2.808) for stocks followed by at least 3 analysts and -.4898% (t value: -2.356) for stocks followed by at least 5 analysts. These results suggest that the required rate of return on a security with a large dispersion of beliefs is lower than on a security characterized by a small dispersion of beliefs. This finding is consistent with the expected effect of short sales regulation on expected returns of risky assets.

We report in table 4 the estimation results of model (15). We can not reject the null hypothesis $\gamma_3 = 0$ in favour of the alternative hypothesis $\gamma_3 > 0$ except for the OLS estimation method. The negative effect of divergence of beliefs on expected returns of risky assets is lower (in absolute value) for securities monitored by more analysts. This effect is still negative (γ_2'$+\gamma_3$ = -.3511%), but is not significantly different from zero (t value = -1.209)[15]. These results show that on average expected returns of risky assets are negatively related to the divergence of investors' beliefs. The magnitude of this effect is more important and statistically significant for illiquid assets. These findings corroborate the general hypothesis according to which short sales regulation induces opportunity costs which are priced by the market. Moreover, the findings are consistent with the complementary hypothesis which stipulates that the effect is less important for liquid assets characterized by a low probability of call-back by the brokers.

Finally, we report in table 5 the estimation results obtained when we control for the number of analysts, the January effect or structural changes in the two sub-periods surrounding the October market crash. Only ML estimators are reported, because the results are unsensitive to the estimation method. The introduction of the control variable, LnNBA, does not modify the conclusions drawn from the estimation of model (14). The signs and magnitudes of the coefficients associated with the betas and the coefficient of variation of analyst forecasts are almost the same as in table 3 (Panels A and B). Only the intercept is affected by the introduction of LnNBA. As expected, the coefficient associated with LnNBA, a measure of size, is negative but not significantly different from zero. When January returns are excluded, the coefficient associated with the dispersion of forecasts is equal to -.5451% and is similar to the one observed in table 3. On the contrary, the risk premium is much lower: 0.192% as compared to .6818%. The estimated risk premium is also very different among the two sub-periods analyzed: 1.2288% during the 1985-October 1987 sub-period against 0.0314% during the October 1987-1989 sub-period. The estimated coefficient associated with the dispersion of forecasts is less negative during the second sub-period, but is still significant (-.6281% against -.4333%). Finally, we

[15] Barring methodological problems mentioned in the introduction, this result likely explains part of the ambiguity in the conclusions drawn in previous studies. Indeed, Swidler (1988) retains in his sample all firms monitored by at least three analysts and Peterson and Peterson (1982a,b) only retain firms monitored by eight analysts. The former examines the dispersion of beliefs independently of the number of analysts, while the latter *de facto* rules out effects possibly related to the security's liquidity level.

repeat the tests on a sub-sample composed only of companies having a fiscal year-end in December (63% of the sample) in order to eliminate the noise induced by considering all fiscal year-ends together. However, the results are not reported as they are very similar to the previous ones.

CONCLUDING REMARKS

Drawing from Jarrow's work (1980), we derive a version of the CAPM that accounts for both heterogeneous beliefs and short sales regulation. The relevance of this CAPM extension becomes all the more evident in that heterogeneous agent beliefs appear to be one of the major factors behind capital market activity (Varian, 1989), whereas short sales regulation is an important feature of market micro-structure.

The main conclusion that follows this CAPM extension is that, in the presence of positive risk premia, the required rates of return on risky assets are a positive linear function of systematic risk and a negative function of opportunity costs induced by short sales regulation. We show that in principle those opportunity costs are positively related to the dispersion of agents' beliefs and negatively related to the security's liquidity level. In an economy where short sales are restricted, the required rate of return on securities with a large dispersion of beliefs is consequently lower than that would prevail in an identical economy where short sales are not restricted.

We use Litzenberger and Ramaswamy's methodology (1979) to test the hypothesis that short sales regulation induces an opportunity cost for constrained investors and whether this cost is priced by the market. Our study covers the 60-month 1985-1989 period and uses 13079 observations. Results indicate a significant negative relationship between security returns and the dispersion of beliefs. This negative relationship is consistent with the presence of opportunity costs resulting from short sales regulation when return beliefs are heterogeneous, such that the required rate of return on risky assets is lower than would prevail in a economy without restrictions on short sales. Our results also corroborate the hypothesis that the average effect of short sales regulation on the expected returns of risky assets conceals differences in security liquidity levels. Measuring this liquidity level by the number of analysts monitoring the security, we find that the negative relationship between security returns and dispersion of beliefs is essentially confined to illiquid securities, that is, those monitored by a small number of analysts. Finally, these results are not sensitive to the introduction of different control variables: the number of analysts monitoring the stock, the exclusion of January returns, the sub-period analyzed or companies' fiscal year-ends.

TABLE 1
Descriptive statistics on security risk premia distributions, systematic risk levels and coefficients of variation of analysts' earnings forecasts for every year over the period 1985-1989

	Observations	Average	Standard Deviation	Skewness	Kurtosis
			R_j - R_f		
1985	1967	0.0128	0.0886	0.0528	1.9741
1986	2552	0.0007	0.0966	0.2103	1.9869
1987	2579	0.0179	0.1032	0.4931	2.3204
1988	2957	0.0011	0.0877	0.5911	3.2903
1989	3024	0.0017	0.0783	0.5068	3.4702
			Beta		
1985	1967	0.9159	0.4389	0.3259	-0.5947
1986	2552	0.9176	0.4285	0.1764	-0.3704
1987	2579	0.9337	0.4257	0.4257	-0.4191
1988	2957	0.9948	0.3658	0.1319	-0.3243
1989	3024	1.0069	0.3536	0.0056	-0.2155
			Coefficient of Variation		
1985	1967	0.3687	0.8525	6.2446	48.3415
1986	2552	0.4085	0.8158	4.8445	30.0703
1987	2579	0.3707	0.8701	6.4946	51.8123
1988	2957	0.2877	0.6684	7.1839	66.8179
1989	3024	0.3524	0.8681	6.0513	44.9015

TABLE 2
Frequencies and cumulative frequencies of the number of financial analysts on a yearly basis

	1985		1986		1987		1988		1989	
Nba	Freq	C.Freq	Freq	C.Freq	Freq	C.Freq	Freq	C.Freq	Freq	C.Freq
3-5	38.9	38.9	25.2	25.2	30.5	30.4	31.8	31.8	30.1	30.1
6-9	37.0	75.9	31.1	56.3	27.4	57.9	24.3	56.1	24.2	54.3
10-12	13.5	89.4	20.0	76.3	15.6	73.5	11.0	67.1	10.5	64.8
more than 13	10.6	100.0	23.7	100.0	26.5	100.0	32.9	100.0	35.2	100

TABLE 3
Estimates and tests of model (14) relating excess security return to systematic risk level and the dispersion of beliefs

		Intercept	Beta	Disp
Panel A: Companies followed by at least 3 analysts				
OLS	Coefficients	3.771	5,746	-6,095
	t values	0.897	0.979	-3.705**
WLS	Coefficients	5.628	4.677	-4856
	t values	1.544	0.841	-2.732**
ML	Coefficients	3.964	6.818	-5.001
	t values	0.823	0.837	-2.808**
Panel B: Companies monitored by at least 5 analysts				
OLS	Coefficients	2.249	7.179	-4.289
	t values	0.583	1.211	-2.523**
WLS	Coefficients	4.551	5.313	-4.801
	t values	1.219	0.931	-2.378**
ML	Coefficients	2.867	7.437	-4.898
	t values	0.553	0.902	-2.356**

This pooled time series cross-sectional model corresponds to the following regression:

$$R_{jt} - R_{ft} = \gamma_0 + \gamma_1 \beta_{jt} + \gamma_2 DISP_{jt} + \epsilon_{jt}; j=1,...,J_t; t=1,...,T$$

$R_{jt} - R_{ft}$	:	excess return of the j^{th} security for the t^{th} month.
β_{jt}	:	systematic risk level of the j^{th} security for the t^{th} month.
$DISP_{jt}$	:	divergence of analysts' earnings forecasts measured by the coefficient of variation of the j^{th} security for the t^{th} month.

A separate regression is computed for each month of the period. The coefficient estimates (γ_{0t}, γ_{1t}, γ_{2t}) are then grouped and the reported coefficient estimates are averages of the time series coefficients γ_{kt} (k=0,...2)

In table 3 to 5, ** indicates that the test is significant at the 1% level and the coefficient estimates are multiplied by 10^3.

TABLE 4

Estimates and tests of model (15) relating excess security returns to their systematic risk level and the dispersion of beliefs when permitting for different slope effects depending on the security liquidity level.

		Intercept	D	Beta	Disp	D.Disp
Total sample						
OLS	Coefficients	4.127	1.604	5.964	-9.709	8.994
	t values	0.903	0.717	1.029	-3.254**	1.647*
WLS	Coefficients	5.955	-1.778	5.194	-6.895	4.956
	t values	1.621	-0.916	0.942	-2.634**	1.023
ML	Coefficients	4.121	-2.051	7.881	-6.999	3.484
	t values	0.843	-1.052	0.951	-2.643**	0.846

TABLE 5

Estimates and tests of model (16) relating excess security returns to their systematic risk level and the dispersion of beliefs when controlling for missing factors, the January effect or different sub-periods.

		Intercept	Beta	Disp	LnNBA
Panel A: Total sample					
ML	Coefficients	8.412	8.691	-5.391	-3.006
	t values	1.511	1.023	-3.039**	-1.472
Panel B: Sub-sample (all January returns are excluded)					
ML	Coefficients	5.241	1.912	-5.451	
	t values	1.138	0.261	-3.302**	
Panel C: from January 1985 to September 1987					
ML	Coefficients	4.102	12.288	-6.289	
	t values	0.556	1.032	-2.387**	
Panel D: from November 1987 to December 1989					
ML	Coefficients	6.031	0.314	-4.313	
	t values	1.062	0.029	-1.972**	

These pooled cross-sectional time-series models correspond to the following regressions:

$$R_{jt} - R_{ft} = \gamma_{0t}' + \alpha_0 D_{jt} + \gamma_1' ß_{jt} + \gamma_2' DISP_{jt} + \gamma_3 D_{jt} DISP_{jt} + \xi_{jt}, j=1,...,J; t=1,...,T \quad (15)$$

$$R_{jt} - R_{ft} = \gamma_{0t}'' + \gamma_1'' ß_{jt} + \gamma_2'' DISP_{jt} + \gamma_4 Ln NBA_{jt} + \zeta_{jt}, j=1,...,J; t=1,...,T \quad (16)$$

where:

$Ln NBA_{jt}$: logarithm of the number of analysts monitoring the j^{th} security for the t^{th} month.

$D_{jt} DISP_{jt}$: the dichotomous variable crossed with the dispersion of the j^{th} security for the t^{th} month. D_{jt} equals 1 if the number of analysts monitoring the j^{th} security is at least 9 and 0 otherwise.

A separate regression is computed for each month of the period. The coefficient estimates are then grouped and the reported coefficient estimates are averages of the time series coefficients.

APPENDIX 1

Transformation of the price equilibrium equation (5) into the return equilibrium equation (7)

First, we find an expression of the inverse of the sum of investors' risk tolerances, τ^{-1} (in a world without restrictions on short sales); second, we substitute this expression of τ^{-1} in the equation of the risky asset equilibrium price vector (5).

Step 1 : Were short sales unrestricted, the equilibrium price vector would be the following:

$$R_f\mathbf{P} = \left(\Sigma_{k=1}^{N}\alpha_k E_k(\mathbf{X})\right) - \tau^{-1}\mathbf{\Omega Q}$$

Prior multiplication of this expression by $\mathbf{Q}^t$ yields the inverse value of the sum of investors' risk tolerances[16]:

$$\tau^{-1} = \left[\left(\Sigma_{k=1}^{N}\alpha_k E_k(R_M)\right) - R_f\right]/[P_M.\sigma^2(R_M)]$$

Step 2 : $E_k(\mathbf{X}) = \mathbf{D}_P E_k(\mathbf{R})$, where $E_k(\mathbf{R})$ represents the vector of risky asset expected returns and $\mathbf{D}_P$ represents the diagonal matrix with time 0 risky asset prices along the main diagonal. Furthermore, the vector of implicit prices $\mathbf{u_k}$ is equal to $\mathbf{D_P v_k}$ where $\mathbf{v_k}$ stands for the vector of implicit returns. Hence, equation (7) can be rewritten as:

$$R_f\mathbf{P} = \left\{\left(\Sigma_{k=1}^{N}\alpha_k\mathbf{D}_P E_k(\mathbf{R})\right) - \tau^{-1}\mathbf{\Omega Q}\right\} + \left\{\Sigma_{k=1}^{N}\alpha_k\mathbf{D_P v_k}\right\} \quad (5')$$

Upon prior multiplication of (7') by $\mathbf{D_P}^{-1}$, the equilibrium equation becomes:

$$R_f\mathbf{i_J} = \left\{\left(\Sigma_{k=1}^{N}\alpha_k E_k(\mathbf{R})\right) - \tau^{-1}\mathbf{D_P}^{-1}\mathbf{\Omega Q}\right\} + \left\{\Sigma_{k=1}^{N}\alpha_k\mathbf{v_k}\right\} \quad (5'')$$

The j^{th} element of the vector $\mathbf{\Omega Q}$ is equal to: $\Sigma_{i=1}^{J} Q_i Cov(X_j, X_i) = Cov(X_j, \Sigma_{i=1}^{J} Q_i X_i) = Cov(X_j, \Sigma_{i=1}^{J} V_i) = Cov(X_j, V_M) = P_j P_M Cov(R_j, R_M)$ where V_j stands for the market value of j.

If we multiply the j^{th} element of the vector $\mathbf{\Omega Q}$ by the inverse of the sum of investors' risk tolerances τ^{-1}, this element becomes: $\left[\left(\Sigma_{k=1}^{N}\alpha_k E_k(R_M)\right) - R_f\right].P_j.\beta_j$.

Hence, vector $\tau^{-1}\mathbf{\Omega Q}$ can be written as: $\left[\left(\Sigma_{k=1}^{N}\alpha_k E_k(R_M)\right) - R_f\right]\mathbf{D_P \beta}$

The substitution of $\tau^{-1}\mathbf{\Omega Q} = \left[\left(\Sigma_{k=1}^{N}\alpha_k E_k(R_M)\right) - R_f\right]\mathbf{D_P \beta}$ in (7'') yields an expression of the risk-return relationship prevailing in an economy where short sales are forbidden.

$$R_f\mathbf{i_J} = \left\{\left(\Sigma_{k=1}^{N}\alpha_k E_k(\mathbf{R})\right) - \mathbf{\beta}\left\{\left(\Sigma_{k=1}^{N}\alpha_k E_k(R_M)\right) - R_f\right\}\right\} + \left\{\Sigma_{k=1}^{N}\alpha_k\mathbf{v_k}\right\} \quad (7)$$

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[16] The first equation becomes: $R_f\mathbf{Q^t P} = \left(\Sigma_{k=1}^{N}\alpha_k\mathbf{Q^t}E_k(\mathbf{X})\right) - \tau^{-1}\mathbf{Q^t\Omega Q}$

that is $R_f P_M = \left(\Sigma_{k=1}^{N}\alpha_k E_k(V_M)\right) - \tau^{-1}\sigma^2(V_M)$

where P_M and V_M respectively stand for the value of the market portfolio at time t=0 and t=1. $E_k(V_M)=P_M.E_k(R_M)$, where R_M denotes one plus the rate of return on the market portfolio. Consequently, the preceding equation becomes: $R_f P_M = \left(\Sigma_{k=1}^{N}\alpha_k P_M E_k(R_M)\right) - \tau^{-1}P_M^2\sigma^2(R_M)$ and one can solve for τ^{-1}.

APPENDIX 2

Consider two distributions of investors beliefs X and Y with means $\mu_X=\mu_Y$ and standard deviations $\sigma_X > \sigma_Y$. The average opportunity costs induced by short sales in each case are respectively:

$$v_X = \int_{-\infty}^{\mu_X} g(t) f_X(t)\, dt \tag{2.1}$$

$$v_Y = \int_{-\infty}^{\mu_Y} g(t) f_Y(t)\, dt \tag{2.2}$$

We can show that[17]:

$$v_X - v_Y = \int_{-\infty}^{\mu_Y=\mu_X} g(t)\ [f_X(t) - f_Y(t)]\, dt > 0 \tag{2.3}$$

Thus, v is a positive function of σ=DISP.

Proof:

Define:

$u(t)=g(t)$ and $u'(t)=g'(t)$
$s(t)=F_X(t)-F_Y(t)$ and $s'(t)=f_X(t)-f_Y(t)$

Integration by parts gives:

$$v_X - v_Y = [g(t)\ (F_X(t) - F_Y(t))]_{-\infty}^{\mu_X} - \int_{-\infty}^{\mu_X} g'(t)\ [F_X(t) - F_Y(t)]\, dt \tag{2.4}$$

The first term is equal to zero, for $F_X(\mu_X)=F_Y(\mu_X)=1/2$ and $F_X(-\infty)=F_Y(-\infty)$.
We know that $g'(t)<0$ and $F_X(t)-F_Y(t)>0$, consequently, the second term is positive.

[17] Given the form of this expression, the proof will be somewhat analoguous to that of the second degree stochastic dominance.

APPENDIX 3

When short sales are forbidden, the opportunity cost the k^{th} investor whose beliefs are lower than the consensus beliefs with respect to the j^{th} security is equal to the difference between the consensus expected rate of return, μ_j, and the rate the k^{th} agent expects, $E_k(R_j)$. If we assume that investor beliefs are normally distributed with mean μ_j and a dispersion σ_j, the density of probabilities f(t) is equal to:

$$f(t) = \frac{1}{\sigma_j \sqrt{2\pi}} \cdot \exp\left[-\frac{(t - \mu_j)^2}{2\sigma_j^2}\right] \tag{3.1}$$

and the effect of short sales regulation on risky asset returns can be written as:

$$-v_j = -\sum_{k=1}^{N} \frac{v_{kj}}{N} = \frac{1}{\sigma_j \sqrt{2\pi}} \int_{-\infty}^{\mu_j} (\mu_j - t) \cdot \exp\left[-\frac{(t - \mu_j)^2}{2\sigma_j^2}\right] dt \tag{3.2}$$

After the variable change: $z = (t - \mu_j)/\sigma_j$, the above integral becomes:

$$-\sum_{k=1}^{N} \frac{v_{kj}}{N} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} [\mu_j - (\sigma_j . z + \mu_j)] \cdot \exp\left[-\frac{z^2}{2}\right] dz \tag{3.3}$$

$$-\sum_{k=1}^{N} \frac{v_{kj}}{N} = \sigma_j \cdot \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} z \cdot \exp\left[-\frac{z^2}{2}\right] dz \tag{3.4}$$

$$-\sum_{k=1}^{N} \frac{v_{kj}}{N} = -\sigma_j \cdot \left[\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)\right]_{-\infty}^{0} \tag{3.5}$$

$$\text{That is,} \quad -\sum_{k=1}^{N} \frac{v_{kj}}{N} = -\frac{\sigma_j}{\sqrt{2\pi}} = -\frac{DISP_j}{\sqrt{2\pi}} \tag{3.6}$$

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