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PIECE-RATES, PRINCIPAL-AGENT MODELS, AND PRODUCTIVITY PROFILES: PARAMETRIC AND SEMI-PARAMETRIC EVIDENCE

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Piece-Rates, Principal-Agent Models, and Productivity Profiles: Parametric and Semi-Parametric Evidence*

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Abstract / Résumé

This paper is a revised version of cahier de recherche 9406, département d'économique, Université Laval.

This paper exploits the natural link between observed wages and productivity that is inherent in piece-rate wage data to estimate worker productivity profiles. Piece-rate wages are functions of the parameters of the compensation system and worker effort. Identifying productivity from such data requires separating out these effects. This can be accomplished by explicitly modelling the principal-agent relationship between the worker and the firm and deriving optimal decision rules for worker effort. This approach is applied to historical payroll data collected from a British Columbia copper mine. The salient aspects of the mine's production process are incorporated into the model, namely, asymmetric information, team production and heterogeneous workers. Solving the model for equilibrium worker effort implies a censored wage distribution which is estimated both parametrically and semi-parametrically. Methods to control for unobserved heterogeneity among workers are also used. Productivity profiles are then constructed from the resulting parameter estimates. Results suggest that productivity profiles were increasing concave functions of worker tenure.

Ce cahier est une version révisée du cahier de recherche 9406, département d'économique, Université Laval.

Cette étude utilise le lien naturel qui existe entre les salaire observés et la productivité des travailleurs lorsque les travailleurs sont payés à la pièce afin d'estimer les profils de productivité des travailleurs. Quand les travailleurs sont payés à la pièce, les salaires observés sont une fonction des paramètres du système de compensation et du niveau d'effort des travailleurs. L'identification de la productivité des travailleurs nécessite la séparation de ces deux effets dans les données. Ceci peut être accompli en modélisant la relation principal-agent qui existe entre la firme et le travailleur tout en trouvant les règles de comportement optimal de l'effort du travailleur. Cette approche est appliquée à des données de salaires historiques, colligées à partir des archives d'une mine en Colombie-Britannique. Les aspects importants de la technologie de la mine sont incorporés dans le modèle, *ie.* l'asymétrie d'information qui existe entre la firme et les travailleurs, la production en équipe et les travailleurs hétérogènes. Le modèle est résolu pour le niveau d'effort du travailleur en équilibre. Celui-ci implique une distribution de salaires censorés qui est estimée de façon paramétrique et semi-paramétrique tout en contrôlant pour l'hétérogénéité inobservable des travailleurs. Les profils de productivité sont construits à partir des paramètres estimés. Les résultats suggèrent que les profils de productivité étaient des fonctions croissantes et concaves de l'ancienneté du travailleur.

Keywords: Principal-Agent Models, Labour Productivity, Performance Pay, Censored Regression Models, Semi-Parametric Estimation.

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INTRODUCTION

Piece–rate[1] compensation schemes provide economists with a natural link between observed wages and worker productivity. Exploiting this link to recover productivity from piece–rate wages can provide direct evidence on worker productivity within the firm.[2] Yet piece–rate compensation schemes also have incentive effects on workers. The principal–agent literature emphasizes the use of such compensation systems to solve shirking problems in the presence of asymmetric information.[3] These incentive effects can have important implications for productivity studies using piece–rate data. In particular, observed wages are functions of variables influenced by both the worker–his or her effort, and the firm–the parameters of the compensation system. Identifying the productivity of workers requires separating out these two effects. The fact that worker effort is unobservable not only complicates the identification of the productivity profile, but may also bias the results. That is, wage regressions that ignore worker effort will be subject to an omitted variable bias if effort is correlated with the regression's explanatory variables.

In this paper I estimate worker productivity profiles from piece–rate data using methods which control for incentive effects. To take account of incentive effects I explicitly model the principal–agent relationship between the firm and its workers. Modelling the incentive system which generated the observed wages allows for the derivation of optimal decision

[1] I use the term piece-rate to refer to a payment scheme under which workers are paid according to their output.

[2] Gunderson (1975) used piece–rate data to control for productivity differences across workers in his study of male–female wage differentials. Similarly, Weiss (1992) used piece–rate data to estimate the learning curve for manufacturing workers.

[3] See Hart and Holmstrom (1985) for a review of this literature.

rules for worker effort as a function of observable worker characteristics. These optimizing restrictions can then be used to control for worker effort and to identify productivity profiles.

I apply this approach to piece–rate data collected from the Britannia Mining Company. During the 1920s workers at Britannia were paid according to a productivity based bonus system. Workers worked in teams and received a guaranteed base wage supplemented with a bonus in proportion to team output exceeding a company set production standard.

The model developed in this paper incorporates the important aspects of the production process at the Britannia mine: asymmetric information, team production, and heterogeneous workers. Output is a function of worker effort and a random shock that is observed by team members (but not the firm) before choosing an effort level. Effort is in turn a function of worker characteristics tenure and age. Solving the model for equilibrium worker effort implies a censored wage distribution, the parameters of which can be estimated using well-known censored regression techniques. These parameters can then be used to identify productivity profiles. Semi-parametric estimation allows for the relaxation of distributional assumptions of the parametric model.

The estimated productivity profiles provide unique evidence on the relationship between productivity and tenure. This relationship has been the focus of a large empirical literature in economics. Becker (1975) suggested that workers and firms would utilize a sharing contract in which wages were positively sloped in tenure, in order to protect investments in firm specific human capital which increase worker productivity. A related literature of learning-by-doing models considers the effect of productivity growth on observed wages in the absence of formal training. Estimates of rising wage profiles in tenure have traditionally been interpreted as supportive of these models. However, in most data sets, the link between wages and productivity is unknown. In fact, several well-known papers have derived positively sloped wage profiles in the absence of any productivity growth.[4] This casts doubt on the general ability of wage profiles to identify productivity profiles. An advantage of using the Britannia data set is the limited applicability of these alternative models. Base-wages and bonus rates were not directly linked to worker tenure so that observed changes in wages for individual workers reflect changes in productivity.

In general, firms can use piece rate compensation schemes to attain goals other than motivate workers. In particular, such schemes can be used to sort workers, both across and within firms.[5] Taking account of the aims of the firm is important to the interpretation and accuracy of empirical results. Firm records, which provide insight into these goals, suggest that the primary reason for introducing the compensation system at Britannia was to motivate workers. Yet, it is still important to take account of possible sorting effects in empirical work as they may bias estimates of the productivity profile. To accomplish this the model is extended to account for unobserved heterogeneity among workers within the mine.

The empirical results suggest that productivity profiles were increasing concave functions of tenure. The shape of the profile is robust to departures from normal heteroscedastic errors and to the presence unobserved heterogeneity among workers. However, the slope of the profile is upwardly biased when unobserved heterogeneity is ignored. Results suggest that a one month increase in miner tenure (from the sample average of 40 months) increased productivity per shift by approximately

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[4] See for example Salop and Salop (1976), Lazear (1979),and Jovanovic (1979).

[5] See Sitglitz (1975) or Lazear (1986).

2.0e-3 pounds of copper. Furthermore, effort is positively correlated with worker tenure. As workers gain more experience in the mine the cost of effort decreases. This suggests that ignoring the effort decision of the worker will lead to an upwardly biased estimate of the tenure effect on productivity.

The paper is organized as follows. A brief history of the bonus system at Britannia is given is Section 1. Section 2 presents and solves the model of the bonus system. Section 3 derives the wage distribution implied by the model and discusses estimation issues. Section 4 discusses the data and Section 5 gives the results.

1. THE BONUS SYSTEM

During the 1920s the Britannia Mining Company introduced a productivity based bonus system into its mining operations at Britannia Beach, British Columbia. Workers, who worked in teams, received a guaranteed base wage supplemented with a bonus in proportion to team productivity exceeding a company set standard.

There were several mines in operation at Britannia during the 1920s. This study concentrates on the Victoria mine which employed approximately one quarter of the mines total workforce which averaged 871 men in 1927. Within the Victoria mine several areas were worked simultaneously, with small groups of workers at each site. Bonuses were paid twice per month. Where the quality of rock was not important, as in the building of access tunnels, productivity was measured in terms of feet of advance. On the other hand, in ore removal operations, productivity was measured in terms of tons of ore removed. This prevented workers from loading up ore carts with waste rock.

Productivity and bonuses were calculated on a per-shift basis. In particular, a production standard was set for each area of the mine in terms of output per shift. At the end of each pay period, total output at each site was divided by the number of shifts worked at the site. If this measure of productivity per shift exceeded the production standard, then bonuses were paid to all workers who had worked shifts in that area of the mine. Each of these workers would receive an amount in proportion to the number of shifts they had worked in that area. Workers could receive bonuses from more than one area of the mine.

A bonus engineer was employed by the company to keep track of conditions and advise management on the setting of bonus rates and production standards. These were set once per month. As well, while the bonus system was started in 1923, the sample used in this study were collected from the period 1926–28. This period was chosen to minimize the effects of firm learning about worker productivity and appropriate piece–rates.

A number of different occupations were involved in the mining operations at Britannia. Those most directly involved in underground ore removal were miners and muckers. Miners drilled and blasted rock, and muckers shoveled away what the miners had blasted. Other occupations of timberman and timberman's helper reinforced the tunnels with timber supports; however variations in the hardness of rock and mining methods led to periods in which no timbermen were employed. While these occupations received different base wages, company reports suggest that all workers in a given work place received the same bonus regardless of occupation.[6] Finally, payroll records show that base wages were not di-

[6] Annual Report of the Mine Superintendent 1926, p.26. Britannia Records, Box

rectly linked to tenure. Similarly there is no mention in firm reports of production standards or bonus rates being linked to tenure either.

2. MODELLING THE BONUS SYSTEM

The model developed in this section is based on asymmetric information and team production. Output per team is assumed to be a function of each team member's effort λ_i and a productivity shock θ. The productivity shock captures variation in the quality of rock at the rock face. Teams are assumed to comprise one miner and one mucker. I ignore timbermen and timberman helpers since their absence from certain periods in the data suggest that they were not regular team members. Team members can observe the value of θ before they choose their effort level, however, the firm can only observe final team output. The miner is assumed to act as team leader, in that he chooses his effort level before the mucker does. This captures the temporal aspect of production in the mine where the miner drills and blasts rock and then the mucker shovels this rock away.

It is clear that the mucker cannot shovel away more rock than the miner has blasted, similarly, the miner will not receive credit for any rock that the mucker leaves on the ground. I therefore approximate technology with a Leontief production function. That is, the output of a team working in sector j of the mine is

$$Y_j = d_j \theta_j \min\{\lambda_a, \lambda_b\} + b_j,$$

where d_j and b_j represent sector specific fixed effects to productivity.

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Timing (within a pay period):

0. The firm chooses the parameters of the bonus system.
1. Nature chooses θ, where $\ln(\theta) \sim N(0, s^2)$.
2. The miner observes θ and chooses λ_a.
3. The mucker observes θ and λ_a and chooses λ_b.
4. The firm observes Y_j and pays wages.

Wages for individual i in occupation n and team j are a function of team output according to the piece-wise linear scheme

$$W_i = \beta_n + w_i,$$

where β_n is the base wage for occupation n, and w_i is individual i's bonus. The bonus is determined by

$$w_i = \begin{cases} \alpha_j(Y_j - x_j), & \text{if } Y_j > x_j; \\ 0 & \text{otherwise,} \end{cases}$$

where, x_j denotes the production standard set by the firm, α_j is the bonus rate and β_n is the base wage for occupation n.

The parameters of the wage system, α_j, x_j and β_n are chosen by the firm to maximize expected profits. Note that α_j and x_j are common among team members. This is consistent with practice in the mine. They are, however, subscripted by j to reflect the fact that they can potentially vary across regions of the mine. Firm reports suggest that they were varied in response to changing conditions in the mine. It is also evident from firm reports that if workers did not think these rates were set fairly in certain areas of the mine, they would resist working in those areas. Following Ferrall and Shearer (1994), I assume that the firm sets α_j and x_j to render miners ex-ante indifferent as to the area of the mine in which they work in any period.[7] This can be accomplished by setting $\alpha_j = \frac{\alpha}{d_j}$ and $x_j = d_j x + b_j$. That is, α_j and x_j cancel out fixed differences across areas of the mine. This allows consideration of the workers actions in response to the parameters α and x regardless of the area of the mine in which they worked. I therefore concentrate on the normalized output function, wage and bonus equations

$$y = \theta \min\{\lambda_a, \lambda_b\} \tag{1}$$

$$W_i = \beta_n + w_i \tag{2}$$

$$w_i = \begin{cases} \alpha(y-x), & \text{if } y > x; \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

It is possible to solve the firm's problem of choosing α, β and x to maximize profits using numerical methods. However, this is not required to identify productivity and will not be pursued here. In the empirical work that follows, α, β and x are allowed to change from period to period in response to changing market and mine conditions.[8] The contract the mine paid its workers is not the optimal contract that solves the classic principal agent model. Such a contract would depend on all observable characteristics which are related to productivity such as tenure and occupation. One possible explanation for the simplicity of the observed scheme is that more complicated contracts have prohibitively high transaction costs associated with them. Ferrall and Shearer (1994) investigate empirically the role of transaction costs in determining the firm's choice of the bonus system at Britannia.

[7] This assumption is necessitated by the fact that I do not observe the region of the mine in which a particular worker worked. It precludes, for example, the possibility that the firm located its most experienced miners in select areas of the mine.

[8] In general, the firm will solve a dynamic programming problem which determines the optimal rate of extraction from the mine. Therefore, they will maximize expected discounted profits subject to an incentive compatibility constraint, and a labour supply constraint.

Utility of individual i is defined as,

$$U_i = U\left(W_i - \frac{k(X_i)}{2}\lambda_i^2\right),$$

where $\frac{k(X_i)}{2}\lambda_i^2$ is the cost of effort for an individual with personal characteristics X_i. This form of utility function is common in incentive models.[9] Its main benefit is the absence of an income effect on effort, which implies that the base wage does not affect the worker's optimal choice of effort. While base wages contain information on productivity that is not sensitive to incentives, they can be ignored when estimating the worker's response to the bonus system. Estimation can concentrate solely on the distribution of bonuses, w_i.

For the purposes of this paper, X_i includes the variables tenure, age, and occupation. The cost of effort function captures the relationship between tenure and productivity in the model. If the partial derivative of $k(X_i)$ with respect to tenure is negative, then the cost of effort decreases in tenure, so productivity and wages increase with tenure. If workers become more productive as they acquire human capital (or experience), then it is reasonable to assume that it is less costly to provide a given level of effort. Alternatively, if little or no learning occurs, it may be that workers become less productive as tenure increases due to fatigue. Estimating the nature of this effect from the distribution of wages will be the focus of the empirical analysis in Section 5.[10]

[9] See, for example, Holmstrom and Milgrom (1987).

[10] The static nature of the model restricts the manner in which effort can affect productivity. In particular, it may be the case that investment in human capital and/or learning themselves require effort, in which case productivity may be a function both of current and past levels of effort.

The Effort Decision.

Because θ is observed before effort is chosen, the effort decision comprises two choices. Working independently, each worker would either choose a positive level of effort which equates his marginal benefit of effort to his marginal cost

$$\lambda_i = \frac{\alpha\theta}{k_i},$$

or set effort equal to zero and shirk. That is, because workers only receive a bonus when productivity exceeds the production standard x, and because workers observe θ before choosing effort, there are certain values of θ for which the effort cost of earning a bonus is too high. The value of θ that equates the indirect utilities of each option is

$$\theta_i^* = \sqrt{\frac{2k_i x}{\alpha}}.$$

This is the value of θ for which a worker is just indifferent between working and shirking.

Team production places further constraints on the effort decision. Namely, since the production function is Leontief, any amount of effort which exceeds a teammate's effort level is wasted.

The model is solved using backward induction. First, the mucker's best response function $\hat{\lambda}_b(\lambda_a, \theta)$ is derived. The miner then chooses his effort level to maximize his utility taking the mucker's best response function as given. The mucker's best response function solves

$$\hat{\lambda}_b(\lambda_a, \theta) = \arg\max_{\lambda_b} \alpha\big(\theta \min\{\lambda_a, \lambda_b\} - x\big) - \frac{k_b}{2}(\lambda_b)^2.$$

The mucker's best response function is therefore

$$\hat{\lambda}_b = \begin{cases} \min\{\frac{\alpha\theta}{k_b}, \lambda_a\} & \text{if } \theta > \theta_b^* \\ 0 & \text{otherwise.} \end{cases}$$

Where θ_b^* is now that level of θ which renders the mucker indifferent between providing effort $\min\{\frac{\alpha\theta}{k_b}, \lambda_a\}$ and shirking. Taking $\hat{\lambda}_b(\lambda_a, \theta)$ as given the miner will choose his effort level to solve

$$\hat{\lambda}_a(\theta, \hat{\lambda}_b) = \arg\max_{\lambda_a} \alpha\big(\theta \min\{\lambda_a, \hat{\lambda}_b\} - x\big) - \frac{k_a}{2}\big(\lambda_a\big)^2,$$

giving

$$\hat{\lambda}_a = \begin{cases} \min\{\frac{\alpha\theta}{k_a}, \frac{\alpha\theta}{k_b}\} & \text{if } \theta > \theta_a^* \\ 0 & \text{otherwise.} \end{cases}$$

In equilibrium, effort levels will be equal.[11] That is

$$\lambda_a = \lambda_b = \begin{cases} \frac{\alpha\theta}{\max\{k_a, k_b\}} & \text{if } \theta > \theta^* \\ 0 & \text{otherwise,} \end{cases}$$

where

$$\theta^* = \sqrt{\frac{2\max\{k_a, k_b\}x}{\alpha}}.$$

In equilibrium, each worker only supplies effort if his teammate supplies effort. Furthermore, when both team members supply effort, each will supply no more effort than the other. The value of θ^* in the team setting is the maximum of the θ_i^*'s that render each team member indifferent between effort and shirking when they act independently. Since both members of the team receive the same bonus, equilibrium effort is determined by their relative costs of effort.

[11] The equilibrium in a simultaneous move game is similar to that derived here in that effort levels will always be equal. However, in a simultaneous move game, multiple equilibria are possible. In particular, any positive effort level less than $\frac{\alpha\theta}{\max\{k_a, k_b\}}$ can be a Nash equilibrium in a simultaneous move game. These equilibria are ruled out in the sequential move game since the miner realizes that by increasing his effort level to $\frac{\alpha\theta}{\max\{k_a, k_b\}}$ the mucker will follow him.

Expected Bonuses and Productivity

Substituting equilibrium effort into the equation for bonuses, (3), gives the following equilibrium bonus distribution.

$$w = \begin{cases} \alpha\left(\frac{\alpha\theta^2}{\max\{k_a,k_b\}} - x\right) & \text{if } \theta > \theta^*; \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Note that when $\theta = \theta^*$

$$w(\theta^*) = \alpha x \quad \text{and} \quad y(\theta^*) = 2x$$

That is, bonuses are censored away from zero. Even though the firm is willing to pay bonuses for all levels of output greater than x, workers only provide effort if their output will be above $2x$ and the resulting bonus above αx. This censoring is due to workers observing θ before choosing their effort level and the fact that they only receive a bonus if output exceeds the production standard. Only for θ's leading to bonuses greater than αx will worker utility dominate that of shirking. This restriction will play an important role in estimating the model.

Expected bonuses can be derived using the properties of the log normal distribution.[12] In particular,

$$\begin{aligned} E[w_\tau] &= \frac{\alpha_\tau}{\max\{k_a, k_b\}} \int_{\theta_\tau^*}^{\infty} \theta^2 f(\theta)\, d\theta - \alpha_\tau x_\tau [1 - F(\theta_\tau^*)] \\ &= \frac{\alpha_\tau}{\max\{k_a, k_b\}} e^{2s^2} \left[1 - \Phi\left(\frac{ln(\theta_\tau^*)}{s} - 2s\right)\right] - \\ &\quad \alpha_\tau x_\tau [1 - F(\theta_\tau^*)]. \end{aligned} \tag{5}$$

Recall that workers earn a base wage as well as the bonus. While the base wage does not affect worker behaviour within the bonus system,

[12] See Maddala (1983).

it does contain information on worker productivity. To incorporate this information into the analysis, I assume that the firm earns zero expected profits in the labour market. That is, normalizing the price of output to one, expected profits per team can be written

$$E[\Pi] = E[Y] - W_a - W_b = 0.$$

The expected productivity per worker is then

$$\frac{E[Y]}{2} = \frac{\beta_a + \beta_b}{2} + E[w]$$

or, using (5)

$$\frac{E[Y_\tau]}{2} = \frac{\beta_a + \beta_b}{2} + \frac{\alpha_\tau}{\max\{k_a, k_b\}} e^{2s^2}\left[1 - \Phi\left(\frac{ln(\theta_\tau^*)}{s} - 2s\right)\right] - \alpha_\tau x_\tau [1 - F(\theta_\tau^*)]. \tag{6}$$

3. THE BONUS DISTRIBUTION AND ESTIMATION

The data are uninformative as to the composition of teams. Estimating the model therefore requires an assumption on the effort determination within teams. In particular, the personal characteristics determining the effort decision enter through the cost of effort parameter k_i. Thus, if teams are matched so that miners determine effort within teams i.e. $k_a > k_b$, then a mucker's bonus will be not be a function of his own tenure, but rather the tenure of the miner with whom he is matched. In the long run the most efficient composition of teams would be attained by matching workers such that $k_a = k_b$. However, this may be difficult to achieve in any period. Instead, I assume that $k_a > k_b$. That is miners are insured not only against regional differences within the mine but also with whom they are matched to work. This assumption implies that only the wages of miners can be analysed in relation to their tenure. Also note that the resulting estimates will still be consistent (although there will be an efficiency loss) if the firm did successfully match teams so that $k_a = k_b$. The estimates will be inconsistent if instead muckers determined team effort. Thus in Section 5, I check the robustness of the empirical results to changes in this assumption.

Given the bonus distribution (4), define

$$\psi_\tau(X_i) = \frac{k_a(X_i)}{\alpha_\tau^2},$$

where X_i is a vector of observations on individual characteristics. The subscript τ indicates periods and reflects the fact that the firm can change the parameters of the bonus system in each period. Positive bonuses for miner i in period τ can then be written as

$$w_{i,\tau} = \frac{\theta_{i,\tau}^2}{\psi_\tau(X_i)} - \alpha_\tau x_\tau,$$

Upon rearranging terms and taking logarithms,

$$ln(w_{i,\tau} + \alpha_\tau x_\tau) = -ln(\psi_\tau(X_i)) + u_{i,\tau}, \tag{7}$$

where $u_{i,\tau} = 2ln\theta_{i,\tau}$.

But note that under the assumption of log-normal production shocks with constant variance $u_{i,\tau} \sim N(0, 4s^2)$ so (7) is simply a censored regression model with normally distributed errors. Conditional on $\alpha_\tau x_\tau$, the parameters of $-ln(\psi_\tau(X_i))$ can be estimated using censored regression techniques. That is, define the latent variable

$$y^*_{i,\tau} = -ln(\psi_\tau(X_i)) + u_{i,\tau}, \quad \text{then}$$

$$ln(w_{i,\tau} + \alpha_\tau x_\tau) = \begin{cases} y^*_{i,\tau}, & \text{if } y^*_{i,\tau} > ln(2\alpha_\tau x_\tau); \\ 0, & \text{otherwise.} \end{cases}$$

The censoring point is $ln(2\alpha_\tau x_\tau)$ since the model imposes the restriction that $w_{i,\tau}$ is censored at $\alpha_\tau x_\tau$. The fact that $\alpha_\tau x_\tau$ is unknown adds a slight complication; however, the restriction that $w_{i,\tau} \geq \alpha_\tau x_\tau$ suggests estimating $\alpha_\tau x_\tau$ by the minimum observed positive bonus in period τ. This estimator is consistent and converges faster than $\sqrt{n}$.[13] This implies that $\alpha_\tau x_\tau$ can be treated as fixed for the purposes of estimation, and the estimates of $-ln(\psi_\tau(X_i))$ conditional on $\alpha_\tau x_\tau$ will be consistent and asymptotically normal.

The likelihood function for period τ is

$$L = \prod_{i=1}^{n} \left[\Phi \left(\frac{ln(\sqrt{2\psi_\tau(X_i)\alpha_\tau x_\tau})}{s} \right) \right]^{(1-d_i)} \left[\frac{1}{2s(w_{i,\tau} + \alpha_\tau x_\tau)} \, \phi \left(\frac{ln[\psi_\tau(X_i)(w_{i,\tau} + \alpha_\tau x_\tau)]}{2s} \right) \right]^{d_i} \tag{8}$$

where Φ is the cumulative standard normal distribution function, ϕ is the standard normal density and

$$d_i = \begin{cases} 1, & \text{if individual i receives a bonus;} \\ 0, & \text{otherwise.} \end{cases}$$

Substituting the estimated parameters from maximization of (8) into (6) and using information in the data on worker base wages permits estimation of worker productivity.

[13] See Flinn and Heckman (1982), Donald and Paarsch (1993), and Christensen and Keifer (1991).

4. DATA FOR THE BRITANNIA MINE

The data contain information on employees who worked in Britannia's Victoria mine during the period from the beginning of 1926 through the end of 1928. The workers in the sample are all blue-collar, non-unionized males. The payroll records of the Victoria mine provide information on wages received under the bonus system. Workers were paid twice per month. For each pay period data is available on each worker's occupation, base wage, number of shifts worked, and total bonus for the pay period. No information is available on the bonus rate or on output. Furthermore, there is no information on when (ie. time of day), where (ie. in which sector of the mine), or with whom (ie. the composition of his team) a worker was working.

Personnel records kept by the company provide data on individual characteristics such as date of birth and starting date at the mine. These records were kept by Britannia officials at least since the year 1913.

Workers were matched between the payroll records and the personnel files on the basis of name and payroll number. Matching proved difficult since payroll numbers often changed as did the spelling of names. As well, certain gaps have been noted in the alphabetic sequence of personnel files which suggests some records may have been lost. Approximately one quarter of the workers in the payroll records during the period 1926–1928 have been matched with the personnel files.

The matched sample consists of 4793 observations on 244 individuals in 4 occupations — miner, mucker, timberman, and timberman's helper. It is important to note that although the payroll data is restricted to the years 1926-28, the personnel files cover a much broader range of time.

Namely, there are workers in the sample who started work at Britannia before 1926. While workers were paid twice per month, the mine would alter the parameters of the pay system, at most, once per month. The data are therefore grouped into monthly periods with each pay period wage being treated as an independent record. For the purposes of empirical analysis, wages are converted into real values by dividing by the monthly New York price per pound of copper. Furthermore, since bonuses were paid on a per-shift basis, bonus per shift is used in the empirical analysis.

Of immediate concern is whether the matched sample is representative of the population (the mine's work force as a whole). Figures 1 and 2 compare the average positive bonus per shift with the proportion of workers receiving a bonus in the matched sample and the population for each period. The average positive bonus per shift is calculated by averaging bonus per shift over all workers who received a bonus in either of the two week periods within a month. The two series match up very well. The same general patterns are apparent in both the matched sample and the population, and the two series are generally very close to each other. This suggests that no systematic bias was introduced by the matching process.

– Figure 1 –

– Figure 2 –

Teams are assumed to consist of miners and muckers. Table 1 presents summary statistics for the sample of these occupations. The wage data in Table 1 is expressed in 1926 pennies. It is apparent that the bonus per shift was usually quite small. Base wages of miners were \$4.25 per shift while those of muckers were \$4.00. The average bonus per shift over the course of the sampling period was approximately \$.20. Thus on average workers were supplementing their income by approximately 5%

through bonuses.

– Table 1 –

A shift at Britannia was 8 hours in length. The average shifts worked per two week period is consistently high. It is over 12 in all periods and over 13 in many. Many workers were receiving only one or two days off per month. This is not surprising due to the fact that the miners were not unionized and they were working in an isolated community.

Worker tenure is measured in months. It is calculated as the total number of calendar months in which the worker has worked at Britannia. In particular, if a worker arrives at the mine on March 11, 1926 he is considered to have one months tenure in March of 1926. Age is measured in years, since the month and day of birth was often not available.

The work force at Britannia was quite experienced. The average age in the sample is 32.5 years while the median age is 30. Similarly, average tenure in the sample is 30.6 months, and the median level of tenure is 17 months. Table 2 presents average and median values for age and tenure by occupation. Note the workers in the occupation requiring more skill, namely miners, are generally older and have longer tenure. These results are consistent with human capital theory since workers are expected to have longer durations in jobs requiring a high degree of human capital investment where turnover is more costly. Note that tenure here is the length of time the individual has been at the firm. No measure of tenure within occupation is possible with this data set.

– Table 2 –

5. RESULTS

I first estimate the model under the assumptions of normally distributed errors, constant variance and no unobserved heterogeneity. I then relax these assumptions to test the model's specification.

Specification

Econometric analysis focuses on the specification and estimation of $\psi_\tau(X_i)$. Recall

$$\psi_\tau(X_i) = \frac{k_a(X_i)}{\alpha_\tau^2},$$

where k_a is the cost of effort function of the miner and X_i is a vector of miner characteristics. The specification of $\psi_\tau(X_i)$ must capture both the personal characteristics of the miners and period specific effects on α_τ. The period-specific effects allow for variation in α_τ as the firm adjusted the bonus rate in response to changing conditions within the mine. Taking logs gives

$$-ln(\psi_\tau(X_i)) = 2ln(\alpha_\tau) - ln(k_a(X_i)). \tag{9}$$

To specify $ln(k_a(X_i))$ let

$$-ln(k_a(X_i)) = \beta_0 + \beta_1 t_i + \beta_2 t_i^2 + \beta_3 a_i + \beta_4 a_i^2 + \beta_5 a_i * t_i, \tag{10}$$

where t_i and a_i denote respectively the tenure and age of individual i. Equation (9) also makes it clear that α_τ will not be separately identified from the constant term in $-ln(k_a(X_i))$.

To avoid problems of multicollinearity associated with including a dummy variable for each period, I approximate these dummies with an Almon polynomial. I started with a polynomial of degree seven and sequentially tested the restrictions of decreasing this degree. The preferred model has polynomial of degree five. The estimated equation is therefore,

$$ln(w_{i,\tau} + \alpha_\tau x_\tau) = \begin{cases} \sum_{j=1}^{5} \gamma_j A_j + \beta_0 + \beta_1 t_i \\ \quad +\beta_2 t_i^2 + \beta_3 a_i \\ \quad +\beta_4 a_i^2 + \beta_5 a_i * t_i & \text{if rhs} > ln(2\alpha_\tau x_\tau); \\ 0 & \text{otherwise,} \end{cases} \tag{11}$$

where the A_j are the variables of the Almon polynomial.

The results for this specification appear in Table 3(A). Note that the coefficients on tenure and age*tenure are both significant at the five percent level, with positive and negative signs respectively. The age, age squared and tenure squared coefficients are insignificant. After dropping age and age squared from the equation, the p-value on tenure is effectively zero, while those for age*tenure and tenure squared are .002 and .061 respectively. The results are presented in Table 3(B).

– Table 3 –

The sample of 1583 observations includes only miners working in the Victoria mine. Muckers are excluded since the assumption on effort determination within teams implies that mucker bonuses are independent of their tenure. The other extreme case is where muckers determine effort levels within teams, ie. $k_b < k_a$. A comparison of results under both assumptions is useful in gaging the robustness of the results. The results from estimation performed on the sample of muckers are given in Table 4. It is clear from Table 4 that the profile will retain its upward slope in tenure, however the second order term in tenure no longer has a significant affect and is dropped from the specification. While age and age squared are not individually significant they are jointly significant. Furthermore, while the exact nature of the estimates change, the overall form of the profile appears to be robust to changes in the assumption over effort determination within teams.

– Table 4 –

Productivity Profiles.

Productivity profiles are derived by substituting the estimate of $\psi_\tau(X)$ into (6). Differentiating (6) with respect to tenure t gives

$$\frac{\partial E[Y_\tau]}{\partial t} = -\frac{1}{\psi_\tau(X)^2}\frac{\partial \psi_\tau(X)}{\partial t}e^{2s^2}\Bigg[1 - \Phi\left(\frac{ln(\theta^*_\tau)}{s} - 2s\right) + \frac{\phi(\frac{ln(\theta^*_\tau)}{s} - 2s)}{2s} - \frac{\phi(\frac{ln(\theta^*_\tau)}{s})}{2s}\alpha_\tau x_\tau \psi_\tau(X)e^{-2s^2}\Bigg].$$

Estimates of this expression, evaluated at the sample means for age and tenure are given for each period in the first column of Table 5. These estimates suggest that a one month increase in tenure increased productivity per shift by approximately 5.8e-3 pounds of copper. The elasticity of per shift productivity with respect to tenure can also be calculated as $\frac{\partial E[Y_\tau]}{\partial t}\frac{t}{E[Y_\tau]}$. Estimates of this expression, evaluated at the sample means are listed in column 2 of Table 5. They suggest that a one percent increase in tenure led to a corresponding percentage increase in per shift productivity in the neighbourhood of 6.6e-3 percent. Productivity profiles can also be graphed for each period. One such profile, for period 1, is shown in Figure 3. The graph shows a miner with 40 months of tenure was producing approximately 33.2 pounds of copper per shift. Production data from company reports show that in June 1925 average output per miner shift was 20.5 tons of rock. This suggests that the ore concentration was approximately 33.2/(20.5*2000)*100 = .08%. This estimate appears reasonable, in 1926, for example, 1,187,632 tons of rock were mined at

Britannia producing 31,734,089 pounds of copper (Hovis 1986). The actual ore concentration was therefore 31,734,089/(1,187,632*2000)*100 = 1.3%.

– Table 5 –

– Figure 3 –

The increasing concave productivity profiles estimated here are similar in shape to those derived from studies on worker wages.[14] In this respect the results are supportive of the human capital and learning-by-doing interpretation of wage profiles. Namely, that the increasing concave wage profiles reflect changes in worker productivity over the course of tenure in the firm.

A comparison of the point estimates of the effect of tenure on productivity with other studies is less instructive due to the unique nature of the sample used here. In perhaps the most closely related study, Weiss (1992) uses data on workers who were paid piece–rates to estimate a learning curve in three electronics manufacturing plants. He reports median increases in productivity in the first two months of employment of between 10 and 45 percent at the three plants. Productivity gains decreased rapidly, however, and were between 0 and 1 percent by the fifth month. The fact that the miner's profile is increasing over a longer period of time probably reflects the nature of the job. In particular, the miner must become accomplished at mining under a variety of different conditions. The ability to recognize different kinds of rock and thus apply suitable techniques can only be gained through experience and training.

[14] See Hutchens (1989) for a review of this literature.

The Effect of Modelling Behaviour.

Before going on to discuss the specification of the model, it is of interest to consider the effect of explicitly modelling the effort decision of the worker. Consider a naive model of the bonus system in which behavior is not modelled. Let a latent variable

$$w^*_{i,\tau} = \alpha_0 + \sum_{j=1}^{5} \gamma_j A_j + \alpha_1 * t_i + \alpha_2 * t_i^2 + \alpha_3 * a_i * t_i + \epsilon_{i,\tau}.$$

Then

$$w_{i,\tau} = \begin{cases} w^*_{i,\tau} & \text{if } w^*_{i,\tau} > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

This is simply a standard statistical wage equation adjusted for the censoring of the bonus distribution. Comparing this specification to (11), it is clear that the failure to model effort implies an inability to separate the effects of worker choices on bonuses from the effects of firm choices, namely $(\alpha_\tau x_\tau)$. Intuitively, a regression between wages and tenure captures both firm and worker behavior. Modelling worker behavior, allows the separate identification of worker effects.

Results from estimating (12) are presented in Table 6. Note that the coefficient on tenure is larger than that of Table 3. The estimated marginal return to tenure derived from (12) is $\alpha_1 + 2 * \alpha_2 * t + \alpha_3 * a$. These results are presented in the fifth column of Table 5. While they differ from the values in column 1, there is no consistent relationship between the two estimates. Differences between these estimates may be caused by two factors. First, the effort decision is not modelled in (12) and therefore enters into the error term. If effort is changing over the course of tenure, then this will cause a bias in the estimated tenure coefficient. The fact that the tenure coefficient is larger when the effort decision is ignored suggests that unobserved effort is biasing the estimate upwards. In other words, effort is positively correlated with tenure. Second, it is clear from the model that the worker's effort decision is affected by α_τ and x_τ. However, since (12) cannot identify these parameters, their effect on the productivity profile is ignored.

While the naive model does not nest (11), the regressors in the two models are identical and the dependent variables are non linear transformations of each other. This allows for a comparison of the models based on the value of their respective log likelihood functions (taking account of the appropriate Jacobian terms).[15] Namely, the logarithmic transformation of the dependent variable in (11) creates a Jacobian term for the density of positive bonuses: the term

$$\frac{1}{(w_{i,\tau} + \alpha_\tau x_\tau)}$$

in (8). While this term is constant and will therefore not affect parameter estimates (and is not included in the reported value of the loglikelihood function in Table 3), it does affect the value of the loglikelihood. Summing the log of this term over all positive observations and adding the result to the value of the loglikelhood reported in Table 3, gives the correct value of the loglikelihood function, -2106.16. Comparing this value to value of the loglikelihood function from the naive model, -2251.59, suggests that (11), the specification in which effort is modelled, is preferred.

– Table 6 –

[15] See Davidson and Mackinnon (1993) p. 491.

6. TESTS OF MISSPECIFICATION

The results of the model presented in the previous section were estimated under the assumption of homoscedastic and normal errors. It is well known that if either of these assumptions are violated, then the resulting estimates from a censored regression model will be inconsistent.

Robust Estimation.

A general way in which to deal with possible inconsistency due to heteroscedasticity and non-normality is to use robust estimation methods. One such method is the symmetrically censored least squares (SCLS) technique developed by Powell (1986). Powell's estimator exploits the orthogonality condition that must hold when errors are symmetrically distributed about a mean of zero. In particular, given the censored regression model

$$y_i = \begin{cases} x_i'\beta + u_i & \text{if } x_i'\beta + u_i > 0 \\ 0 & \text{otherwise,} \end{cases}$$

the random variable $y_i^+ = \min\{y_i, 2x_i'\beta\}$, is symmetrically distributed around $x_i'\beta$ when $x_i'\beta > 0$.

Powell's estimator minimizes the objective function

$$S(\beta) = \sum_{i=1}^{n}(y_i - \max\{\frac{1}{2}y_i, x_i'\beta\})^2 + \sum_{y_i > 2x_i'\beta}[(\frac{1}{2}y_i)^2 - (\max\{0, x_i'\beta\})^2].$$

The corresponding first order condition is

$$0 = \sum_{i=1}^{n} 1(x_i'\hat{\beta} > 0)(\min\{y_i, 2x_i'\hat{\beta}\} - x_i'\hat{\beta})x_i',$$

where $1(x_i'\hat{\beta} > 0)$ denotes an indicator function for all observations satisfying $x_i'\hat{\beta} > 0$. Symmetry is therefore ensured by censoring y_i from above at $2x_i\hat{\beta}$ and deleting all observations for which $x_i\hat{\beta} < 0$.

Powell shows that this estimator is consistent and asymptotically normal. Furthermore, the asymptotic variance covariance matrix of $\hat{\beta}$ can be consistently estimated as

$$V(\hat{\beta}) = \hat{C}^{-1}\hat{D}\hat{C}^{-1},$$

where

$$\hat{C} = \frac{1}{T}\sum_{i=1}^{T} 1(0 < y_i < 2x_i'\hat{\beta})x_i x_i',$$

and

$$\hat{D} = \frac{1}{T}\sum_{i=1}^{T} 1(x_i'\hat{\beta} > 0)\min\{(y_i - x_i'\hat{\beta})^2, (x_i'\hat{\beta})^2\}x_i x_i'.$$

Results from implementing Powell's SCLS procedure on the model are presented in Table 7.[16] The coefficients on tenure, tenure squared and age*tenure are all of the same sign as the corresponding maximum likelihood estimates in Table 3 however the magnitudes are smaller in absolute value. The variables are no longer individually significant. Furthermore, a joint test fails to reject the null hypothesis that these coefficients are all equal to zero at standard levels of statistical significance.[17] The lack of statistical significance in the estimates may be due to the amount of information that is discarded during the censoring procedure.

– Table 7 –

The fact that the SCLS tenure coefficient is smaller than the corresponding maximum likelihood estimate suggests that the assumption

[16] Estimates were calculated using the trimmed least squares iteration algorithm given in Powell (1986). The algorithm converged after (108) iterations. At the final estimates, 785 observations are included in estimation.

[17] The test statistic is $n\,\hat{\beta}_{[1:3]}\,[V(\hat{\beta})]^{-1}_{[1:3]}\,\hat{\beta}_{[1:3]}$, which is distributed as $\chi^2(3)$ under the null hypothesis. The value of the test statistic is 4.97 which has a p-value of 0.17.

of constant variance may be unsatisfactory. There are two reasons why this assumption could be violated. First, conditions in the mine were changing from period to period as rock was removed and new rock faces were exposed. It therefore seems reasonable to allow the variance of productivity shocks to change from period to period to reflect these changing conditions. Second, heterogeneous workers may have been sorted through time. That is, a cohort of workers may receive a wide range of bonuses at the beginning of their tenure, reflecting their different abilities as miners. Through time, as the less able miners leave for other more suitable employment, the cohort becomes more homogeneous and the variance of the distribution of bonuses decreases.

To consider these effects I first estimate the model allowing for period specific variances. The results are presented in Table 8. Note that the value of the tenure coefficient is .023, similar to the tenure coefficient in Table 3(B), .020, derived from parametric estimation restricting s to be constant across periods. Yet, the estimates of sigma vary a great deal from period to period. To keep figures to a minimum Table 8 presents only the average (1.334), maximum (2.493), and minimum (0.88) values of the sigmas. The value of the likelihood function increases significantly to -1949.28. The likelihood ratio test statistic of the restrictions is equal to 60.17 with 36 degrees of freedom. The p-value is .007 which rejects the null hypothesis that s is constant at standard significant levels.

– Table 8 –

Next, I consider the issue of unobserved heterogeneity among workers. It is straight forward to alter the model to take account of heterogeneity in the cost of effort function. That is, let $k(X_i, \eta_i)$ represent the cost of effort of a worker with observed characteristics X_i and unobserved characteristics η_i. The distribution of wages conditional on X and η is

$$g(w|X,\eta) = \frac{1}{\sqrt{2\pi}}\frac{1}{2s}\frac{1}{(w+\alpha x)}exp\left\{-\frac{1}{8s^2}\left[ln(\psi(X,\eta)(w+\alpha x))\right]^2\right\}$$

where η now replaces the constant term β_0 in (10). The distribution of wages for *any* individual with characteristics X_i is

$$g(w|X_i) = \int_\eta g(w|X_i,\eta)f(\eta)d\eta.$$

For purposes of estimation, I restrict workers to be of two types That is, $\eta \in \{\eta_h, \eta_l\}$. The likelihood function can then be written as

$$\prod_{i=1}^{n}\Bigg[p_h\Phi\Big(\frac{ln(\sqrt{2\psi_\tau(X_i,\eta_h)\alpha_\tau x_\tau})}{s_\tau}\Big)+ (1-p_h)\Phi\Big(\frac{ln(\sqrt{2\psi_\tau(X_i,\eta_l)\alpha_\tau x_\tau})}{s_\tau}\Big)\Bigg]^{d_i}\Big[g(w_i|X_i)\Big]^{(1-d_i)} \quad (13)$$

where p_h is the probability that a miner is a high productivity type.

Maximizing (13) with respect to the parameters of $\psi_\tau(X), s_\tau, \eta_h, \eta_l$, and p_h gives maximum likelihood estimates which take account of unobserved heterogeneity. These estimates are presented in Table 9. The two constant terms correspond to the two values of η and the probability parameter corresponds to p_h. Notice that the estimated tenure coefficient (.010) is similar to that arrived at using Powell's semi-parametric estimator (.013). The estimated marginal effect of tenure based on the estimates of Table 9 are presented in the third column of Table 5. Controlling for unobserved heterogeneity reduces the estimate of the marginal effect of productivity to approximately 2.0e-3 pounds of copper per shift. The productivity profile based on these estimates is presented in Figure 4. [18]

[18] I derive the productivity profile from (13) rather than the robust estimates, since the parametric assumption allows the derivation of expected productivity whereas Powell's method only allows the derivation of median productivity.

While the shape of the profile is similar to the profile in Figure 3, it is considerably flatter.

– Figure 4 –

Figure 5 provides a more direct comparison of the two profiles. Here, both profiles are scaled to facilitate comparison. The fact that the unobserved heterogeneity controlled productivity profile is flatter than the parametric profile suggests that the parametric profile over-estimates the effect of tenure on productivity. This is consistent with the sorting argument. Namely, the steep slope of the productivity profile in Figure 3 does not reflect the rate at which tenure increases the productivity of a given worker, but rather that as tenure increases the average quality of the workforce increases as poor workers are selected out of the mine.

– Figure 5 –

6. CONCLUSION

The direct observation of worker productivity is very rare. In light of this, econometric studies often proxy productivity with wages. Wage regressions, with tenure as an independent variable, are then used to estimate worker productivity profiles. However, theoretical models have suggested that wages may change with tenure, independently of productivity. This causes problems in interpreting the results of these wage regressions. These problems of interpretation can be overcome by using wage data generated from a piece–rate compensation scheme. Inherent in such data is a natural link between wages and worker productivity which allows for the identification of productivity profiles from observed wages.

This paper has estimated productivity profiles from piece–rate data, controlling for the incentive effects that the compensation system has on worker productivity. Data was collected and analyzed from the payroll records of the Britannia Mining Company. Results suggest that miner productivity was an increasing, concave function of tenure. Tests of misspecification confirm the robustness of the shape of the productivity profile. The slope of the profile was found to be upwardly biased by unobserved heterogeneity among the workers. Controlling for unobserved heterogeneity in the data, a one month increase in tenure was estimated to increase productivity by 2.0e-3 pounds of copper per shift for a miner with 40 months of experience. This is compared to an equivalent estimate of 5.8e-3 pounds per shift when unobserved heterogeneity is ignored.

The increasing concave shape of the profile is consistent with studies which use wages as a proxy for productivity. This provides evidence in support of the human capital/learning-by-doing interpretation of these wage regressions. Namely, that the changes in wages reflect changes in worker productivity.

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TABLE 1
SUMMARY STATISTICS

MONTH	PROPORTION WITH BONUS	AVE POS BONUS/SHFT (cents)	STD DEV BONUS/SHFT	MIN POS BONUS/SHFT (cents)	AVE SHIFTS PER PAY PERIOD
1	63,00	8,56	6,35	0,65	13,67
2	0,80	9,30	5,35	0,38	12,45
3	0,75	10,92	5,68	0,42	12,60
4	0,72	12,59	14,45	1,69	11,35
5	0,63	10,35	7,25	1,46	12,49
6	0,64	11,13	7,26	1,78	12,98
7	0,59	13,92	12,75	1,78	13,64
8	0,57	14,98	20,98	1,77	13,24
9	0,62	12,82	13,90	1,69	13,31
10	0,54	12,75	10,06	1,70	13,33
11	0,53	18,53	18,85	1,83	13,13
12	0,33	23,94	30,82	2,04	12,45
13	0,33	19,51	19,36	3,20	12,20
14	0,44	13,23	12,48	3,42	12,13
15	0,52	13,51	12,25	3,21	12,95
16	0,45	17,79	18,18	3,43	11,78
17	0,42	15,14	14,89	3,18	12,53
18	0,57	15,15	11,30	3,38	13,27
19	0,53	20,58	55,60	3,17	12,51
20	0,56	14,80	15,36	3,18	12,53
21	0,49	22,36	20,26	3,68	13,13
22	0,59	13,45	13,53	3,43	14,27
23	0,38	22,99	18,79	3,51	12,56
24	0,43	18,83	15,47	3,43	12,77
25	0,39	22,33	19,39	3,23	13,32
26	0,66	26,43	36,08	3,38	12,99
27	0,47	24,39	23,26	3,41	13,19
28	0,45	35,04	54,90	3,39	12,73
29	0,41	26,92	24,93	3,65	12,97
30	0,31	24,99	23,96	3,44	12,82
31	0,53	25,31	34,91	3,47	12,26
32	0,47	23,57	21,22	0,70	13,13
33	0,35	36,73	38,01	4,03	12,61
34	0,53	28,46	31,92	3,28	13,73
35	0,47	29,89	22,64	3,51	13,69
36	0,53	34,53	29,83	3,52	13,11
AVERAGE	0,52	19,60	20,62	2,68	12,88

NOTE: Bonus per shift statistics are in January 1926 pennies

TABLE 2
AGE AND TENURE BY OCCUPATION

OCCUPATION	AVERAGE TENURE (months)	MEDIAN TENURE (months)	AVERAGE AGE (years)	MEDIAN AGE (years)
MINERS	39.64	30.00	35.04	33.00
MUCKERS	25.15	13.00	31.04	29.00

TABLE 3
MAXIMUM LIKELIHOOD ESTIMATES

(A) UNRESTRICTED

OBSERVATIONS	1583
LOGLIKELIHOOD	-1979.3751

	COEF	STD ERROR	P>|t|
TENURE	0.023	0.006	0.000
TENURE2	-1.93E-05	3.81E-05	0.611
AGE	-0.047	0.043	0.279
AGE2	0.001	0.001	0.234
TEN*AGE	-4.79E-04	2.11E-04	0.023
ALM1	0.407	0.150	0.007
ALM2	-0.072	0.024	0.003
ALM3	0.005	0.002	0.001
ALM4	-1.73E-04	4.91E-05	0.000
ALM5	1.94E-06	5.30E-07	0.000
CONS	-1.175	0.801	0.143
SIG	1.330	0.036	0.000

LEFT CENSORED	737
UNCENSORED	846

(B) RESTRICTED

OBSERVATIONS	1583
LOGLIKELIHOOD	-1980.1758

	COEF	STD ERROR	P>|t|
TENURE	0.020	0.005	0.000
TENURE2	-5.15E-05	2.74E-05	0.061
TEN*AGE	-2.77E-04	8.89E-05	0.002
ALM1	0.398	0.149	0.008
ALM2	-0.071	0.024	0.004
ALM3	0.005	0.002	0.001
ALM4	-1.69E-04	4.90E-05	0.000
ALM5	1.90E-06	5.29E-07	0.000
CONS	-1.879	0.306	0.000
SIG	1.331	0.036	0.000

LEFT CENSORED	737
UNCENSORED	846

TABLE 4
MUCKER CONSTRAINED TEAMS

(A) UNRESTRICTED

OBSERVATIONS	2618
LOGLIKELIHOOD	-3054.8473

	COEF	STD ERROR	P>|t|
TENURE	0.017	0.006	0.004
TENURE2	2.06E-05	1.53E-05	0.180
AGE	-0.030	0.030	0.331
AGE2	7.02E-04	4.63E-04	0.130
TEN*AGE	-4.94E-04	1.87E-04	0.009
ALM1	0.499	0.129	0.000
ALM2	-0.093	0.021	0.000
ALM3	0.007	0.001	0.000
ALM4	-2.12E-04	4.15E-05	0.000
ALM5	2.35E-06	4.45E-07	0.000
CONS	-1.826	0.558	0.001
SIGMA	1.416	0.032	0.000

LEFT CENSORED	1400
UNCENSORED	1218

(B) RESTRICTED

OBSERVATIONS	2618
LOGLIKELIHOOD	-3055.7493

	COEF	STD ERROR	P>|t|
TENURE	0.012	0.004	0.007
AGE	-0.018	0.029	0.538
AGE2	4.48E-04	4.23E-04	0.290
TENAGE	-2.76E-04	9.44E-05	0.003
ALM1	0.504	0.129	0.000
ALM2	-0.093	0.021	0.000
ALM3	0.007	0.001	0.000
ALM4	-2.11E-04	4.15E-05	0.000
ALM5	2.34E-06	4.45E-07	0.000
CONS	-1.980	0.546	0.000
SIGMA	1.417	0.032	0.000

LEFT CENSORED	1400
UNCENSORED	1218

TABLE 5
THE EFFECT OF TENURE ON PRODUCTIVITY

	MODELLED BEHAVIOUR				WAGE EQUATION
	HETEROGENEITY UNCONTROLLED		HETEROGENEITY CONTROLLED		
PERIOD	MARGINAL EFFECT POUNDS/SHIFT	ELASTICITY	MARGINAL EFFECT POUNDS/SHIFT	ELASTICITY	MARGINAL EFFECT POUNDS/SHIFT
1	3.60E-03	4.21E-03	7.68E-04	9.11E-04	6.24E-03
2	4.02E-03	5.14E-03	9.64E-04	1.25E-03	5.67E-03
3	4.69E-03	5.94E-03	1.10E-03	1.42E-03	5.87E-03
4	6.99E-03	6.10E-03	1.12E-03	1.00E-03	8.18E-03
5	7.35E-03	6.50E-03	1.46E-03	1.31E-03	8.30E-03
6	7.13E-03	7.19E-03	1.16E-03	1.19E-03	8.11E-03
7	7.66E-03	6.35E-03	1.85E-03	1.56E-03	8.96E-03
8	7.15E-03	6.18E-03	1.89E-03	1.66E-03	8.66E-03
9	6.54E-03	6.24E-03	1.38E-03	1.34E-03	8.09E-03
10	6.12E-03	6.29E-03	1.22E-03	1.28E-03	7.60E-03
11	5.89E-03	6.33E-03	1.17E-03	1.28E-03	7.32E-03
12	6.10E-03	6.35E-03	2.06E-03	2.16E-03	7.47E-03
13	5.86E-03	6.15E-03	9.08E-04	9.69E-04	7.19E-03
14	6.04E-03	6.00E-03	5.35E-04	5.42E-04	7.16E-03
15	6.48E-03	6.59E-03	1.06E-03	1.10E-03	7.30E-03
16	6.65E-03	7.09E-03	1.38E-03	1.50E-03	6.99E-03
17	7.02E-03	7.09E-03	1.04E-03	1.08E-03	6.95E-03
18	7.24E-03	7.55E-03	4.58E-04	4.90E-04	6.75E-03
19	7.13E-03	7.76E-03	1.65E-03	1.83E-03	6.35E-03
20	7.56E-03	8.69E-03	1.14E-03	1.34E-03	6.40E-03
21	9.20E-03	8.48E-03	1.23E-03	1.16E-03	7.45E-03
22	9.47E-03	8.30E-03	9.85E-04	8.88E-04	7.55E-03
23	8.99E-03	8.57E-03	1.22E-03	1.20E-03	7.21E-03
24	8.28E-03	9.01E-03	1.45E-03	1.62E-03	6.72E-03
25	6.27E-03	8.09E-03	9.71E-04	1.29E-03	5.25E-03
26	5.66E-03	7.58E-03	2.09E-03	2.84E-03	5.13E-03
27	5.25E-03	6.68E-03	1.34E-03	1.74E-03	5.28E-03
28	4.85E-03	6.57E-03	1.26E-03	1.73E-03	5.52E-03
29	4.45E-03	5.80E-03	1.01E-03	1.33E-03	5.84E-03
30	4.42E-03	5.61E-03	2.06E-03	2.63E-03	6.62E-03
31	3.70E-03	4.80E-03	7.33E-04	9.65E-04	6.28E-03
32	3.66E-03	4.80E-03	1.30E-03	1.72E-03	6.41E-03
33	2.93E-03	4.17E-03	7.34E-04	1.06E-03	5.70E-03
34	3.24E-03	4.11E-03	1.09E-03	1.39E-03	5.95E-03
35	3.54E-03	4.79E-03	9.95E-04	1.36E-03	5.69E-03
36	4.44E-03	6.12E-03	1.43E-03	2.00E-03	5.59E-03
AVERAGE	5.99E-03	6.48E-03	1.23E-03	1.39E-03	6.77E-03

TABLE 6
NEW ESTIMATES OF TOBIT MODEL

OBSERVATIONS	1583
LOGLIKELIHOOD	-2251.5861

	COEF	STD ERROR	P>|t|
TENURE	0.025	0.006	0.000
TENURE2	-5.36E-05	3.66E-05	0.143
TEN*AGE	-3.86E-04	1.20E-04	0.001
ALM1	0.434	0.203	0.033
ALM2	-0.083	0.033	0.011
ALM3	0.006	0.002	0.006
ALM4	-1.86E-04	6.61E-05	0.005
ALM5	2.02E-06	7.13E-07	0.005
CONS	-0.631	0.418	0.131
SIG	1.789	0.046	0.000

LEFT CENSORED	714
UNCENSORED	869

TABLE 7
ROBUST ESTIMATION

	COEF	STD ERROR	P>|t|
TENURE	0.013	0.011	0.233
TENURE2	-2.65E-05	5.99E-05	0.658
TEN*AGE	-2.37E-04	1.88E-04	0.209
ALM1	-0.299	0.617	0.628
ALM2	0.151	0.175	0.390
ALM3	-0.022	0.021	0.277
ALM4	0.001	0.001	0.223
ALM5	-2.49E-05	1.93E-05	0.197
CONS	-0.959	0.647	0.138

TABLE 8
PERIOD SPECIFIC VARIANCES

OBSERVATIONS	1583
LOGLIKELIHOOD	-1949.28

	COEF	STD ERROR	P>|t|
TENURE	0.023	0.005	0.000
TENURE2	-7.09E-05	2.78E-05	0.011
AGE*TEN	-2.85E-04	9.27E-05	0.002
ALM1	0.400	0.199	0.045
ALM2	-0.072	0.030	0.017
ALM3	0.006	0.002	0.005
ALM4	-1.82E-04	5.83E-05	0.002
ALM5	2.09E-06	6.29E-07	0.001
CONS	-1.888	0.470	0.000
AVESIG	1.334	0.295	0.000
MAXSIG	2.493	0.753	0.000
MINSIG	0.880	0.196	0.000

TABLE 9
UNOBSERVED HETEROGENEITY CONTROLLED ESTIMATES

OBSERVATIONS	1583
LOGLIKELIHOOD	-1834.44308517

	COEF	STD ERROR	P>|t|
TENURE	0.010	0.004	0.006
TENURE2	-2.66E-05	2.08E-05	0.200
AGE*TEN	-1.27E-04	6.97E-05	0.069
ALM1	0.315	0.093	0.001
ALM2	-0.052	0.017	0.002
ALM3	0.004	0.001	0.001
ALM4	-1.22E-04	3.81E-05	0.001
ALM5	1.36E-06	4.31E-07	0.002
CONS1	-3.863	0.247	0.000
CONS2	-0.863	0.173	0.000
PROB	0.405	0.017	0.000
AVESIG	0.774	0.160	0.000
MAXSIG	1.309	0.454	0.002
MINSIG	0.367	0.060	0.000