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RÉSUMÉ

On évalue dans cet article la performance empirique d'un modèle dynamique d'évaluation d'options qui fournit une formule de prix fondée sur des processus latents de variables d'état. Cette formule est une généralisation de la formule dite de Hull et White (1987) qui évalue une option européenne écrite sur un actif à volatilité stochastique. On propose dans un premier temps de fonder sur cette formule une procédure empirique ad hoc permettant l'évaluation d'une option à partir du calcul de deux paramètres implicites extraits sur les prix d'options observés la veille. Appliquée à la prévision des prix d'options sur l'indice S&P 500, cette procédure offre un gain de précision significatif par rapport à la pratique usuelle de prévision des prix à travers une volatilité implicite conformément à ce que suggère la formule de Hull et White. Dans un second temps, on propose de particulariser le modèle dans un contexte d'équilibre intertemporel avec utilité récursive. On fournit alors des résultats d'expériences de Monte Carlo montrant que les statistiques de prix d'options produisent des estimateurs des paramètres structurels de l'équilibre beaucoup plus précis que les données de prix de l'actif sous-jacent. Ceci suggère en retour que les paramètres structurels devraient jouer un rôle important dans l'évaluation d'options. Cette affirmation est validée empiriquement sur les données d'options sur l'indice S&P 500 en montrant que la formule de Hull et White est dominée, en termes de prévision des prix d'options hors-échantillon par une formule dépendant explicitement de paramètres de préférence, a fortiori si ceux-ci prennent en compte de façon non contrainte à la fois l'aversion pour le risque et l'élasticité de substitution intertemporelle.

Mots clés : évaluation d'options, facteur d'actualisation stochastique, volatilité stochastique, volatilité implicite de Black-Scholes, effet de sourire, modèle d'équilibre d'évaluation d'options

ABSTRACT

This paper assesses the empirical performance of an intertemporal option pricing model with latent variables which generalizes the Hull-White stochastic volatility formula. Using this generalized formula in an ad-hoc fashion to extract two implicit parameters and forecast next day S&P 500 option prices, we obtain similar pricing errors than with implied volatility alone as in the Hull-White case. When we specialize this model to an equilibrium recursive utility model, we show through simulations that option prices are more informative than stock prices about the structural parameters of the model. We also show that a simple method of moments with a panel of option prices provides good estimates of the parameters of the model. This lays the ground for an empirical assessment of this equilibrium model with S&P 500 option prices in terms of pricing errors.

Key words : option pricing, stochastic discount factor, stochastic volatility, Black-Scholes implied volatility, smile effect, equilibrium option pricing

1. Introduction

The search for option pricing models that will outperform the market makers' method of using implied volatilities across moneyness and maturity is never ending, but has failed to produce a clear winner. The ad-hoc strawman as dubbed by Dumas, Fleming and Whaley (1998) is based of course on an internally inconsistent procedure since volatilities that vary with time and maturity are extracted from the constant-volatility Black and Scholes (1973) model. Yet, Dumas, Fleming and Whaley (1998) show that it performs marginally better than the theoretically-consistent deterministic volatility approach based on implied binomial trees.

This ad-hoc procedure which is applied in practice to predict option prices can be rationalized by a Hull and White (1987) stochastic volatility model where we deliberately forget the Jensen effect associated with the conditional expectation of the Black-Scholes formula with respect to the stochastic volatility process. In other words, we assimilate the Hull-White formula, which is a prediction of Black-Scholes given the volatility process, with a Black-Scholes formula of the predicted volatility.

In this paper, we pursue a similar idea with an option pricing model developed in Garcia, Luger and Renault (2000) which generalizes the Black and Scholes (1973) and Hull and White (1987) formulas. In this model, we specify the joint dynamics of the stochastic discount factor and the stock returns as a function of a set of latent state variables. Along with the implied volatility, we can extract an implicit factor that modifies the value of the stock. This additional implicit element proves crucial in predicting option prices. Substantial gains are made with respect to the implied volatility strawman for near-the-money and in-the-money options, while similar performances are obtained for out-of-the-money options. In Garcia, Luger and Renault (2000), we show that this factor explains the various asymmetries in shapes of smiles, smirks and frowns of the observed implied volatility curves. Our prediction exercise is conducted with S&P 500 index option prices from 1988 to 1997.

One of the features of this generalized option pricing model is that it is not in general preference-free. Preference parameters appear precisely in the factor modifying the stock price, which depends on the characteristics of the stochastic discount factor. In so-called preference-free formulas, it happens that these parameters are eliminated from the option pricing formula through the observation of the bond price and the stock price. In our case, the bond pricing formula and the stock pricing formula provide two dynamic restrictions relating the SDF characteristics to the bond and stock price processes. Therefore, it appears natural to investigate the informativeness of option prices.

First, based on simulations, we show that option prices are more informative than stock prices about the structural parameters of the asset pricing model. The model is cast within the recursive utility framework of Epstein and Zin (1989) in which the respective roles of discounting, risk aversion and intertemporal substitution are disentangled. More precisely, we show that a set moment conditions based on the mean, variance and autocovariance of order one of stock returns does not provide good estimates of the preference parameters. Therefore, one can possibly question the empirical tests of intertemporal asset pricing models that have been based mostly on bond and stock returns. On the contrary, similar moment conditions with option prices recover with great accuracy the preference parameters. Part of the explanation lies probably in the better spanning of the stochastic discount factor (or the underlying risk neutral probability distribution) by a panel of option prices. The nonlinear nature of the option payoffs could also help given the nonlinearity in parameters in the model.

We further show that a simple method of moments with a panel of option prices provides good estimates of all the parameters of the model, that is parameters associated with the fundamentals in the economy along with the preference parameters. This lays the ground for an empirical assessment of the model with S&P 500 option prices in terms of out-of-sample pricing errors and a comparison with usual stochastic volatility and expected utility models which appear as particular cases of our general framework. Our results indicate clearly that the explicit incorporation of preferences improves the performance of the option pricing model and that time nonseparable preferences improve the results further.

The interplay between preferences and latent factors that affect the stochastic discount factor have been explored recently. David and Veronesi (1999) show that option prices are affected by investors' beliefs about the drift of a firm's fundamentals. In particular, they show how investors' beliefs and their degree of risk aversion affect stock returns and hence option prices. The importance of preference parameters in explaining fluctuations in equity prices has also been explored by Mehra and Sah (1998) who show that small changes in investors' subjective discount factors and attitudes towards risk can induce volatility in equity prices. The main thesis of the paper is that some instantaneous causality effects between state variables and asset prices can capture the stylized facts of interest without having to introduce any fluctuation in beliefs or preferences.

The rest of the paper is organized as follows. Section 2 presents a generalized option pricing formula with latent variables developed in Garcia, Luger and Renault (2000) and draws its empirical implications. The performance of an implied stock index is compared to the performance of the usual implied volatility to price options on the S&P 500. Section 3 explores the implications of the option pricing formula with a specific stochastic discount factor based on recursive utility. We make explicit the information about preference parameters contained in option prices. Pricing errors with respect to competing models are assessed for S&P 500 option prices. Section 4 concludes.

2. A Generalized Option Pricing Formula with Latent Variables

2.1. Conditioning Information and Log-Normality

To define a general pricing functional $\pi_t(.)$ which characterizes the price at time t of a single payoff p_T occurring at time $T > t$, a cornerstone is the no-arbitrage condition. Hansen and Richard (1987) show that, in a very general setting, this condition amounts to the existence of a positive stochastic discount factor (SDF) $m_{t,T}$ such that, for any payoff $p_{T:}$

$$\pi_t(p_T) = E[m_{t,T} p_T | J_t] \tag{2.1}$$

where J_t is the relevant conditioning information at time t. Moreover, Hansen and Richard (1987) emphasize that if all the finite variance (given J_t) random variables are feasible payoffs, one and only one among them is a correct SDF. We will therefore refer to *the* SDF[1] and think about it as a payoff. In addition, since agents observe typically more than the econometrician, the information set J_t at time t may contain not only past values of prices and payoffs, but also some latent state variables.

Extending the Hansen and Richard (1987) setting to an intertemporal framework and applying the law of iterated expectations, the log-SDFs necessarily fulfill:

[1]Notice that this unicity property does not refer to any completeness property which would be unrealistic in discrete time.

$$\log m_{t,T_2} = \log m_{t,T_1} + \log m_{T_1,T_2} \text{ , for } t < T_1 < T_2. \tag{2.2}$$

and therefore: $m_{t,T} = \prod_{\tau=t}^{T-1} m_\tau$, with: $m_\tau = m_{\tau-1,\tau}$. Following Constantinides (1992), rather than specifying the SDF sequence through a given specification of preferences, we will directly specify the time-series properties of the stochastic process[2] m_t, $t = 1, 2, ..T$. The key feature of the asset pricing model with latent variables developed in Garcia, Luger and Renault (2000) is an assumption about the sequence $(m_\tau)_{1\leq\tau\leq T}$ of unit period SDFs which amounts to a factor analysis in the longitudinal dimension: there exists a number of state variables which summarize their stochastic dependence, in the sense that, given the state variables, consecutive SDFs are mutually conditionally independent. The same assumption is made about the sequence of consecutive returns of the primitive asset of interest on which options are written. Therefore, in terms of the joint distribution of m_t and returns on a given asset price S_t, we maintain the following assumption.

Assumption 1: The variables $(m_{\tau+1}, \frac{S_{\tau+1}}{S_\tau})_{1\leq\tau\leq T-1}$ are conditionally serially independent given the path $U_1^T = (U_t)_{1\leq\tau\leq T}$ of a vector U_t of state variables.

The relevant conditioning information at time t will be: $J_t = \sigma[m_{\tau,}S_\tau, U_\tau, \tau \leq t]$. This model provides two extensions relative to Constantinides (1992). In the latter, since the focus of interest was the term structure of interest rates and options written on bonds, Assumption 1 was only maintained for the SDF sequence (m_t). Resulting bond prices were therefore deterministic functions of the state variables, and Assumption 1 becomes trivial with S_t viewed as a bond price. The second extension relates to the processes considered for the state variables. While Constantinides (1992) considers only AR(1) processes, our general setting accommodates any process. In particular, we have in mind general Markov switching models which can capture any kind of stochastic volatility and jumps in the return process as well as in the volatility.

The simplicity of the Black and Scholes option pricing methodology stems from the joint log-normality of the SDF and the primitive asset return, but the corresponding normal probability distribution is unconditional and degenerate. We extend it in a conditional and bivariate setting:

[2] As stressed by Constantinides (1992), this alternative approach makes it unnecessary to assume an economy with a representative consumer with von Neumann-Morgenstern preferences. Actually we will consider in section 3 more general non time-separable preferences.

Assumption 2: The conditional probability distribution of $(\log m_{t+1}, \log \frac{St+1}{S_t})$ given U_1^T is, for $t = 1, ..., T-1$, a bivariate normal[3]:

$$\aleph \left[\begin{pmatrix} \mu_{mt+1} \\ \mu_{st+1} \end{pmatrix}, \begin{bmatrix} \sigma^2_{mt+1} & \sigma_{mst+1} \\ \sigma_{mst+1} & \sigma^2_{st+1} \end{bmatrix} \right].$$

Assumption 2 is somehow a consequence of a standard central limit argument which can be applied thanks to the additivity property (2.2) through an arbitrary time scale. Given these two quite standard assumptions, to repeat, a Hansen and Richard (1987) setting and a factor analysis-type assumption about the joint series of returns and the SDF, one obtains the generalized Black-Scholes (GBS) option pricing formula derived in Garcia, Luger and Renault (2000) and reported below.

However, it should be recognized that this setting is really useful only if the required set of state variables is sufficiently reduced to be considered as stationary, Markov and exogenous. We therefore add the following assumption.

Assumption 3: The conditional moments $\mu_{m\tau+1}, \mu_{s\tau+1}, \sigma^2{}_{m\tau+1,} \sigma^2{}_{s\tau+1,} \sigma_{ms\tau+1}$ are fixed functions $\mu_m, \mu_s, \sigma^2{}_{m,} \sigma^2{}_{s,} \sigma_{ms}$ of the current (U_{t+1}) and lagged (U_t) state variables.

As usual for dynamic exogeneity in econometrics, we maintain a non-causality assumption from the processes ($m_{t+1}, \frac{St+1}{S_t}$) to the state variables in order to ensure that future values of the state variables are irrelevant in the conditioning of Assumption 2. In the next section, we explore the empirical implications of this general model, which does not impose any restrictions about preferences.

2.2. Empirical Implications of the Generalized Black and Scholes Option Pricing Model

According to the generalized Black and Scholes option pricing formula derived by Garcia, Luger and Renault (2000) based on the set of assumptions summarized in subsection 2.1 above, the price π_t at time t of an European call maturing at time T with strike price K is given by:

$$\pi_t = E_t \left\{ Q_{ms}(t,T) S_t \Phi(d_{1t}) - K \widetilde{B}(t,T) \Phi(d_{2t}) \right\} \quad (2.3)$$

[3]Notice that, due to Assumption 1, this bivariate normal distribution is also the conditional probability distribution of $(\log m_{t+1}, \log \frac{St+1}{S_t})$ given U_1^T <u>and</u> J_t.

where E_t denotes the conditional expectation operator given J_t, Φ the cumulative distribution function of the standard normal,

$$\begin{aligned} d_{1t} &= \frac{1}{\overline{\sigma}_{t,T}} Log\left[\frac{Q_{ms}(t,T)S_t}{K\widetilde{B}(t,T)}\right] + \frac{\overline{\sigma}_{t,T}}{2} \\ d_{2t} &= d_{1t} - \overline{\sigma}_{t,T} \\ \overline{\sigma}^2_{t,T} &= \sum_{\tau=t+1}^{T} \sigma^2_{s\tau}. \end{aligned}$$

and:

$$\begin{aligned} \widetilde{B}(t,T) &= \exp(\sum_{\tau=t+1}^{T} \mu_{m\tau} + \frac{1}{2}\sum_{\tau=t+1}^{T} \sigma^2{}_{m\tau}), \\ Q_{ms}(t,T) &= \widetilde{B}(t,T)\exp(\sum_{\tau=t+1}^{T} \sigma_{ms\tau})E[\frac{S_T}{S_t}|U_1^T]. \end{aligned} \tag{2.4}$$

This general pricing formula provides a fortiori a pricing equation for the underlying asset (say a stock) and for a bond. These equations can be written respectively:

$$E_t[Q_{ms}(t,T)] = 1, \text{and} \tag{2.5}$$

$$E_t[\widetilde{B}(t,T)] = B(t,T) \tag{2.6}$$

if $B(t,T)$ denotes the price at time t of a pure discount bond maturing at time T.

Garcia, Luger and Renault (2000) document the particular case where $Q_{ms}(t,T) = 1$ and $\widetilde{B}(t,T) = \prod_{\tau=1}^{T-1} B(\tau,\tau+1)$. In this case, the option price (2.3) is nothing but the conditional expectation of the Black-Scholes price, where the expectation is computed with respect to the joint probability distribution of the rolling-over interest rate $\overline{r}_{t,T} = -\sum_{\tau=t}^{T-1}\log B(\tau,\tau+1)$ and the cumulated volatility $\overline{\sigma}_{t,T}$. This framework nests three well-known models. First, the most basic ones, the Black and Scholes (1973) and Merton (1973) formulas, when interest rates and volatility are deterministic. Second, the Hull and White (1987) stochastic volatility extension, since $\overline{\sigma}^2_{t,T} = Var\left[\log\frac{S_T}{S_t}|U_1^T\right]$ corresponds to the integrated volatility $\int_t^T \sigma_u^2 du$ in the Hull and White continuous-time setting. Third, the formula allows for stochastic interest rates as in Turnbull and Milne (1991) and Amin and Jarrow (1992).

It is often argued that the interest rate risk can be neglected for pricing relatively short maturity options written on a stock. Whatever the argument may be, the simple version of (2.3) where $\widetilde{B}(t,T) = \exp[-\overline{r}_{t,T}]$ is computed with a rolling-over interest rate $\overline{r}_{t,T}$ considered as known at time t, is sufficient to get a fruitful extension of the standard Black and Scholes and Hull and White settings. In this case, formula (2.3) can be rewritten:

$$\pi_t = E_t BS_K[\widetilde{S}_{t,T}, \overline{\sigma}_{t,T}] \tag{2.7}$$

where $\widetilde{S}_{t,T} = S_t Q_{ms}(t,T)$ and $BS_K[S,\sigma]$ denotes the Black-Scholes option price for a volatility parameter σ and a current price of the underlying asset equal to S (for a given strike price K and a maturity $(T-t)$). Notice that (2.7) generalizes the Hull and White option pricing formula in two respects. First, the implicit stock price value $\widetilde{S}_{t,T}$ does not coincide with the current value S_t. It is generally random and only its conditional expectation given current information coincides with S_t ($E_t\ \widetilde{S}_{t,T} = S_t$). Renault (1997) has documented the effect of a fixed implicit value S_t^* different from S_t plugged in the Hull and White option pricing formula. It is shown that, while the standard Hull and White implies a symmetric volatility smile when Black and Scholes volatility is plotted as a function of the log-strike price (Renault and Touzi (1996)), a small discrepancy between S_t and S_t^* ($|S_t - S_t^*| = 10^{-3} S_t$) introduces a dramatic skewness in the smile, opening the door to smirks and frowns as inferred more frequently from market data. As far as the option pricing formula (2.7) is concerned, the convexity[4] of the Black-Scholes option price with respect to the underlying asset price will generally imply:

$$E_t BS_K[\widetilde{S}_{t,T}, \overline{\sigma}_{t,T}] = E_t BS_K[S_t^*(K), \overline{\sigma}_{t,T}] \tag{2.8}$$

with: $S_t^*(K) = E_t S_t^*(K) \neq S_t = E_t\ \widetilde{S}_{t,T}$.

The second extension regards the risk neutral measure versus the objective probability measure. While the Hull and White option pricing model leads to a conditional expectation of the Black-Scholes price with respect to the risk neutral probability distribution of the integrated volatility $\overline{\sigma}_{t,T}$, the probability distributions and the expectations in this paper are under the true probability mea-

[4]Note that if $\overline{\sigma}_{t,T}$ were not random, the Jensen inequality applied to the convex increasing function $BS(.,\overline{\sigma}_{t,T})$ would imply that: $S_t^*(K) > S_t$. However the random nature of $\overline{\sigma}_{t,T}$, which may be correlated with $\widetilde{S}_{t,T}$ may even reverse the inequality between S_t and $S_t^*(K)$.

sure (and not the equivalent martingale measure). As a by-product of the direct specification of the bivariate stochastic process which governs the SDF and the underlying asset price we get a pricing of volatility risk. The market price of this volatility risk is captured by $Q_{ms}(t,T)$ which makes the difference between $\widetilde{S}_{t,T}$ and S_t. As already noticed by Amin and Ng (1993), even when the interest rate risk is neglected, preference parameters appear in the option pricing formula through the term $\sum_{\tau=t+1}^{T}\sigma_{ms\tau}$ in the SDF. They are not hidden in the stock price (and the bond price) when the stock pricing formula (2.5) does not imply that $Q_{ms}(t,T)\equiv 1$, that is: $\widetilde{S}_{t,T}\equiv S_t$. Garcia, Luger and Renault (2000) have shown that the difference between $\widetilde{S}_{t,T}$ and S_t (the fact that $Q_{ms}(t,T)$ is a nondegenerate random variable) is produced by several kinds of leverage effects, that is instantaneous causality relationships between state variables and asset prices. They document the various smile asymmetries which may result from these effects.

In order to draw in this section some empirical implications which are free from any particular specification of the SDF, we will develop an empirical methodology which captures the difference between the "implied stock price" S_t^* and its current value S_t. We will address the issue of pricing with a smile, which will be a smirk when the implied stock price $S_t^*(K)$ does not coincide with S_t. In the same way that standard Hull and White option pricing model may provide a rationale for using BS implied volatilities (see Renault and Touzi (1996)), the generalized BS pricing formula (2.7) suggest to define an ad-hoc pricing methodology based on both BS implied volatility and implied stock price. The following result, proved in the Appendix, justifies such an approach.

Proposition 2.1. *If $K_1 \neq K_2$ then the mapping:*

$$\begin{pmatrix} S \\ \sigma \end{pmatrix} \rightarrow \begin{pmatrix} BS_{K_1}(s,\sigma) \\ BS_{K_2}(s,\sigma) \end{pmatrix}$$

is locally invertible in a neighborhood of (S,σ) for any positive S and σ.

Then, when one considers at the same date t two option contracts written on the same asset with two different strike prices, the two option prices, if conformable to Black and Scholes, allow one to recover simultaneously an implied volatility parameter and an implied stock price as well. We can therefore build a class of empirical methodologies first explored by Longstaff (1995), who inverts the Black and Scholes model to estimate both the implied index value and the implied volatility. However, his approach for estimating the two parameters consists in minimizing by grid search the sum of square deviations between the theoretical and the actual option prices for the set of option prices available each day, since it is generally not possible to find a single implied index value and volatility estimate that exactly fit all the call prices. This approach appears too global to capture not only the well-documented volatility smile but also the index value smile which both result from (2.7). The implied values for S and σ extracted from (2.7) have two important features. When $\widetilde{S}_{t,T} \equiv S_t$, the BS implied volatilities plotted against the log-strike price have a symmetric U-shaped pattern [5]. The BS implied index $S_t^*(K)$ will depend heavily on K since the degree of convexity of the BS price with respect to the stock price heavily depends on the moneyness of the considered option.

The theoretical exploration of these two smiles in the spirit of Renault and Touzi (1996) is not the purpose of the current paper. Here, we want to explore the empirical relevance of the following very simple ad-hoc procedure. Faced with the pitfall that the inversion of a couple of option prices conformable to proposition 2.1 will generally provide implied values $S(K_1, K_2)$ and $\sigma(K_1, K_2)$ which both depend upon the particular couple (K_1, K_2) of considered strike prices, we propose to overcome this difficulty by setting $K_1 = S_t$, that is one of the two options is always at the money. This is because it is generally admitted that the Black-Scholes model performs relatively better for at-the-money options than for out-of or in-the-money options. Therefore, we proceed with the inversion in two steps: (i) look for an implied volatility parameter $\sigma_{imp,t}$ such that for $K_1 = S_t$: $\pi_t(K_1) = BS_{K_1}(S_t, \sigma_{imp,t})$; (ii) look for an implied index value $S_t^*(K)$ such that for $K_2 = K$:

$$\pi_t(K_2) = BS_{K_2}(S_t^*(K), \sigma_{imp,t}). \tag{2.9}$$

In other words, we choose the standard reference of the BS implied volatility parameter for the at-the-money option and we complete this information by an implied index value $S_t^*(K)$ which depends upon the moneyness. To show that the generalized BS formula in (2.3) is relevant for pricing options, we will show that this ad-hoc procedure applied for any strike price K works better than the straw-man defined in Dumas, Fleming and Whaley (1998), which consists in smoothing

[5] See Renault and Touzi (1996) and Renault (1997). The latter provides a simplified proof which makes more explicit the role of Jensen's inequality.

the implied volatility across exercise prices. We will therefore compare in the next subsection the prediction errors of the pricing model using both implied volatility and stock price $BS(\widetilde{S}^*_{t+1}, \sigma_{imp,t})$, where $\widetilde{S}^*_{t+1} = S_{t+1} \times \frac{S^*_t(K)}{S_t}$, to the pricing errors obtained with the standard way used by market makers to price options with the volatility smile.

2.3. Pricing with an Implied Index Call Options on the S&P 500

From daily S&P 500 option price data, we first compute averaged Black and Scholes implied volatilities for three classes of maturities (less than 60 days, between 60 days and 180 days, more than 180 days) and six classes of sufficiently liquid moneynesses $x_i = Log\frac{S}{K_i}, i = 1$ to 6 $(x_1 < x_2 < x_3 \cdots < x_6)$. Then we use at-the-money BS implied volatilities in each class of maturity for extracting daily implied index values $S^*(K_i)$, $i = 1$ to 6 by inversion of the Black-Scholes price[6] $BS(S, \sigma_{imp,t})$ with respect to the spot value of the index S. We gauge the out-of-sample pricing performance of our ad-hoc GBS-based procedure by comparing it to the usual implied volatility procedure (the strawman).

The implied volatility performance is summarized in Tables I and III, corresponding respectively to absolute and relative pricing errors. In each case, the pricing error corresponds to the discrepancy between the observed price and the BS price computed with the observed spot value of the index and the BS implied volatility computed the day before in the same class of moneyness and maturity.

The ad-hoc GBS-based procedure is assessed through pricing errors provided in Tables II and IV. These pricing errors are then computed from the difference between the observed price and the BS price obtained with the BS at-the-money implied volatility (extracted the day before in the same class of maturity) and the implied index value $\widetilde{S}^*_{t+1}$ for the relevant class of both moneyness and maturity, based on $S^*_t(K)$ also extracted the day before.

The comparison of Tables I and II shows without any doubt that the pricing performance of the ad-hoc GBS procedure is better than the strawman for in-the-money options (columns $i = 4, 5, 6$ corresponding respectively to the largest moneynesses $x_4 < x_5 < x_6$). The dominance is almost without exceptions (less

[6] In all this section, Black and Scholes option pricing formulas are computed with the value of the continuous time interest rate observed at the date of pricing, as if the term structure was flat and deterministic.

than 3 per cent of the cases) for short maturity options and remains true to a large extent for longer-term options, particularly if we forget the year 1997 which is very special and leads to a bad performance of any standard option pricing model. Let us notice that it is not surprising that, even if dominated, the BS model performs relatively better for longer-term options. When the time to maturity (T-t) is large, a law-of-large-number effect pushes the random variables $Q_{m,}(t,T)$ and $\bar{\sigma}_{t,T}$ towards constant values (corresponding to unconditional expectations of the stochastic moments $\mu_m, \mu_s, \sigma_m^2, \sigma_s^2$ and σ_{ms}). Therefore, the BS model becomes more and more correct.

Notice also that the GBS pricing formula performs better than the BS implied volatility procedure for slightly out-of-the-money options ($i = 2$) in 70% of the cases. Actually the only cases where the strawman wins the competition is for deep out-of-the-money options ($i = 1$) while the performance of the two models is similar for near-the-money options ($i = 3$). These two cases do not invalidate the GBS approach. We have applied the GBS method in an ad-hoc way which uses BS implied volatilities computed at the money, because it is known that the BS model performs quite well for at-the-money options. Indeed, we have just seen that this approach is fruitful for a number of categories of options. However, it should be emphasized that this method does not draw advantage of the well-documented volatility smile. If we do this by extracting implied index values out of the money by inversion of the function BS $(\cdot, \sigma_{imp})$ where σ_{imp} correspond to BS implied volatilities which were also computed out of the money, we are able to do better than the strawman for deep out-the-money options as well. This can be checked without ambiguity by comparing column 1 of Table V with the same column in Table I.

The comparison of relative pricing errors (Tables II, IV and VI) leads to the same type of conclusion. In terms of the sign of the errors, notice that, while BS generally underprices the option contracts, it is less often the case with GBS, in particular for deep in-the-money options. For these options, the strawman often leads to a significant pricing error of several percentage points while the GBS performance is ten times better.

To conclude, let us stress that in order to run a fair competition between BS and GBS, we have in Tables I to IV only compared the pricing model $BS\left[S_t^*(K), \sigma_{imp,t}\right]$ to the standard practitioners' method of pricing with the volatility smile. This does not mean that we recommend to forget about the volatility smile. On the contrary, we have shown that we can do even better for deep out-of the money options by using simultaneously both the volatility smile and the implied index value. But the main message remains that, even if one wants to follow the Dumas, Fleming and Whaley (1998) recommendation that "simpler is better", the simple information provided by the implied index value (the mapping $K \longrightarrow S_t^*$) is richer than the most usual volatility smile (the mapping $K \longrightarrow \sigma$), except for deep out-of-the money options.

In our opinion, this opens the door to more sophisticated uses of our generalized Black and Scholes formula, which would be able to draw advantage simultaneously of the volatility smile and the implied index value as well. An ad-hoc way to do this would be to use the result of proposition 2.1 to invert couples of option prices which are symmetric with respect to the money. In other words, our implied index value information will be used as a complement to a postulated symmetric volatility smile, in order to capture its well-documented asymmetries. We have run some experiments with this methodology and obtained improvements of the basic performance of Tables II and IV similar to the ones documented in Tables V and VI. However, the results may be flawed by the empirical difficulty of finding couples of option contracts which are exactly symmetric with respect to the money.

The previous methods remain ad-hoc procedures. Theoretically speaking, the most efficient use of our GBS model to capture asymmetric volatility smiles should go through the exact option pricing formula (2.3) applied with well-calibrated preference parameters. We will document in Section 3 what could be the gain of using such a sophisticated option pricing methodology with respect to the standard ones.

3. A Stochastic Discount Factor Based on Recursive Utility

Our results until now have been independent of any specification of preferences. Yet, as we mentioned before, our setting makes the option pricing function dependent of preference parameters, which means that option prices can be informative about these parameters. In this section, we explore the latter issue by using both simulations and the same S&P 500 call data as in the last section. We choose the recursive utility framework of Epstein and Zin (1989). The advantage of this particular utility specification is that we can investigate time separability of preferences in the context of option data since until now this issue has been looked at mainly with stock and bond returns. As we will see, an important result of the simulations is that option prices are much more informative than stock returns about preference parameters.

In a recursive utility model, the stochastic discount factor is given by:

$$E_t\left[\beta^{\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma(\rho-1)} M_{t+1}^{\gamma-1} R_{j,t+1}\right] = 1 \tag{3.1}$$

with $\beta = 1/(1+\delta)$, $\delta > 0$. In (3.1) the expectation is taken conditional on information available at time t, M_{t+1} is the return on the market portfolio, and $R_{j,t+1}$ is the return on asset j. The equation in (3.1) defines the relationship between aggregate consumption and rates of return on any asset in the economy that must hold in equilibrium. When future consumption is deterministic, the Epstein and Zin recursive utility function specializes to an intertemporal constant elasticity of substitution utility function with elasticity of substitution $1/(1-\rho)$ and rate of time preference δ. Thus the parameter ρ is interpreted as reflecting intertemporal substitution. Epstein and Zin (1989) show that $\alpha = \gamma\rho$ may be interpreted as a relative risk aversion parameter with the degree of risk aversion increasing as α falls ($\alpha \leq 1$).

When $\rho = 1$, the equation in (3.1) yields the market-based CAPM. Due to the perfect intertemporal substitutability of consumption, the market return becomes the only factor by which asset payoffs are discounted. This is also the case when preferences towards risk are captured by a logarithmic utility function ($\gamma = 0$). At the other extreme, when $\gamma = 1$ asset payoffs are discounted only by consumption growth as in the consumption CAPM. For all other values of γ and ρ, both consumption growth and the market return enter the stochastic discount factor. In this framework the representative agent has preferences, in the Kreps and Porteus (1978) sense, for early resolution of uncertainty when $\alpha < \rho$, preferences for late resolution when $\alpha > \rho$, and is indifferent when $\alpha = \rho$ (the expected utility case).

3.1. The Generalized Black and Scholes Option Pricing Model with recursive utility

The equilibrium price of the market portfolio is given by $P_t^M = \lambda(U_1^t)C_t$, where the payoff on the market portfolio is equal to aggregate consumption C_t. As for the stock, in equilibrium its price is given by $S_t = \varphi(U_1^t)D_t$ where D_t is the corresponding dividend payment. The price-earning ratios $\lambda(U_1^t)$ and $\varphi(U_1^t)$ are functions of an exogenous state variable U_t, whose history up to time t we denote U_1^t. This variable can be thought of as reflecting the state of the economy in the sense that conditional on U_1^t the growth rates of economic fundamentals are independent and identically distributed. Under some regularity assumptions, it can be shown [see Garcia and Renault (1998)] that these price-earning ratios are determined as a fixed point solution of:

$$\lambda(U_1^t)^\gamma = E\left[\beta^\gamma \left(\frac{C_{t+1}}{C_t}\right)^{\gamma\rho} (\lambda(U_1^{t+1})+1)^\gamma \mid U_1^t\right] \tag{3.2}$$

and

$$\varphi(U_1^t) = E\left[\beta^\gamma \left(\frac{C_{t+1}}{C_t}\right)^{\alpha-1} \left(\frac{\lambda(U_1^{t+1})+1}{\lambda(U_1^t)}\right)^{\gamma-1} (\varphi(U_1^{t+1})+1)\frac{D_{t+1}}{D_t} \mid U_1^t\right] \tag{3.3}$$

The dynamic behavior of the market portfolio and stock price are entirely determined by the distribution of (X_t, Y_t, U_t) where we defined $X_t = \log(C_t/C_{t-1})$ and $Y_t = \log(D_t/D_{t-1})$ as the growth rates of consumption and dividends respectively. We assume that

$$\begin{pmatrix} X_t \\ Y_t \end{pmatrix} | U_1^t \sim N\left[\begin{pmatrix} m_{Xt} \\ m_{Yt} \end{pmatrix}, \begin{bmatrix} \sigma_{Xt}^2 & \sigma_{XYt} \\ \sigma_{XYt} & \sigma_{Yt}^2 \end{bmatrix}\right] \tag{3.4}$$

where m_{Xt}, m_{Yt}, σ_{Xt}^2, σ_{Xt}^2, σ_{XYt} are fixed functions m_x, m_y, σ_x^2, σ_y^2 and σ_{xy} of the current (U_t) and lagged (U_{t-1}) state variables.

In order to price a European call option maturing at time T, substitute its payoff $\max(0, S_T - K)$ into the Euler condition in (3.1). One then arrives at the following generalized Black-Scholes pricing formula:[7]

$$\frac{\pi_t}{S_t} = E_t\left\{Q_{XY}(t,T)\Phi(d_1) - \frac{K\widetilde{B}(t,T)}{S_t}\Phi(d_2)\right\}, \tag{3.5}$$

where:

$$d_1 = \frac{Log\left[\frac{S_t Q_{XY}(t,T)}{K\widetilde{B}(t,T)}\right]}{(\sum_{\tau=t+1}^{T}\sigma_{Y\tau}^2)^{1/2}} + \frac{1}{2}(\sum_{\tau=t+1}^{T}\sigma_{Y\tau}^2)^{1/2}, \text{and } d_2 = d_1 - (\sum_{\tau=t+1}^{T}\sigma_{Y\tau}^2)^{1/2}.$$

[7] Of course, this formula and the role of the quantities Q_{XY} and $\tilde{B}$ can be deduced from the GBS formula (2.3) with the similar quantities Q_{ms} and $\tilde{B}$ defined in (2.4). However, for expositional simplicity, we present a set of self-contained comments about (3.5), (3.6) and (3.7). See Garcia and Renault (1998) for a detailed derivation of the formula.

and:

$$\widetilde{B}(t,T) = \beta^{\gamma(T-t)} a_t^T(\gamma) \exp((\alpha-1)\sum_{\tau=t+1}^{T} m_{X\tau} + \frac{1}{2}(\alpha-1)^2 \sum_{\tau=t+1}^{T} \sigma^2{}_{X\tau}), \quad (3.6)$$

with: $a_t^T(\gamma) = \prod_{\tau=t}^{T-1}\left[\frac{(1+\lambda(U_1^{\tau+1})}{\lambda(U_1^{\tau})}\right]^{\gamma-1}$, and

$$Q_{XY}(t,T) = \widetilde{B}(t,T) b_t^T \exp((\alpha-1)\sum_{\tau=t+1}^{T} \sigma_{XY\tau}) E[\frac{S_T}{S_t}|U_1^T]. \quad (3.7)$$

with: $b_t^T = \prod_{\tau=t+1}^{T} \frac{(1+\varphi(U_1^{\tau+1}))}{\varphi(U_1^{\tau})}$.

The quantities $Q_{XY}(t,T)$ and $\widetilde{B}(t,T)$ appearing in the pricing formula (3.5) are particularly determinant. The first, $Q_{XY}(t,T)$ is related to the time t equilibrium stock price S_t and satisfies

$$E_t[Q_{XY}(t,T)] = 1 \quad (3.8)$$

Notice that $Q_{XY}(t,T)$ can be factorized as $Q_{XY}(t,T) = \prod_{\tau=t}^{T} Q_{XY}(\tau,\tau+1)$. When $Q_{XY}(\tau,\tau+1)$ is known at time τ, we have that $Q_{XY}(t,T) = 1$ by (3.8) and by the law of iterated expectations. Garcia, Luger and Renault (2000) show that this will be the case when there is absence of a generalized leverage effect in the sense that the joint distribution of consumption and dividend growth rates does not depend on the contemporaneous value of the state variable. The random variable $\widetilde{B}(t,T)$ determines the equilibrium price of the discount bond $B(t,T)$ as

$$B(t,T) = E_t[\widetilde{B}(t,T)] \quad (3.9)$$

Here also we have a factorization $\widetilde{B}(t,T) = \prod_{\tau=t}^{T-1} \widetilde{B}(\tau,\tau+1)$ such that when $\widetilde{B}(\tau,\tau+1)$ is known one period in advance the price of the discount bond becomes $B(t,T) = E_t \prod_{\tau=t}^{T-1} B(\tau,\tau+1)$. Such predictability obtains when there is absence of a leverage effect through the market risk, that is when the marginal distribution of the consumption growth rate does not depend on the contemporaneous value of the state variable. Another case of interest occurs when m_{Xt} and σ^2_{Xt} are constant, and thus $\lambda(U_1^t)$ in turn is constant, such that the consumption growth rates X_t are iid and the discount factor $B(t,T)$ is deterministic.

The simple gross return on stocks is given by:

$$R_t = \frac{S_t + D_t}{S_{t-1}} = \frac{(\varphi_t + 1)}{\varphi_{t-1}} \frac{D_t}{D_{t-1}} \quad (3.10)$$

From a comparison of the option pricing formula (3.5) and the stock return in (3.10) we can see intuitively why option prices might be more informative than stock prices about the preference parameters. In (3.10) the preference parameters only appear indirectly through the stock price-earnings ratio, which in equilibrium are determined as solution of the Euler conditions in (3.3). On the other hand this ratio also appears in the option price through the term $Q_{XY}(t,T)$. This term in (3.5) along with $\widetilde{B}(t,T)$ depends directly on the preference parameters in addition to the price-earnings ratios for the stock and the market portfolio.

To make the model estimable, we choose a Markov-chain setup for the state variables. The process describing the joint evolution of X_t and Y_t is parameterized as follows:

$$\begin{aligned} X_t &= m_X(U_t) + \sigma_X(U_t)\varepsilon_{Xt} \\ Y_t &= m_Y(U_t) + \sigma_Y(U_t)\varepsilon_{Yt} \end{aligned}$$

The vector $(\varepsilon_{Xt}, \varepsilon_{Yt})'$ follows a standard bivariate normal distribution with correlation coefficient ρ_{XY} and serial independence. The time-varying mean and variance parameters are a function of the state variable process $\{U_t\}$, which is assumed to be a two-state discrete first-order Markov chain. The transition probabilities between the two states are given by $p_{ij} = \Pr(U_t = j | U_{t-1} = i)$ for $i, j = 1, 2$. The unconditional probability of being in state 1 is denoted π_1 and is equal to $(1 - p_{22})/(2 - p_{11-}p_{22})$ and $\pi_2 = 1 - \pi_1$.

Our first goal is to compare the informational content of stock returns and option prices with respect to the preference parameters. That is, we wish to see from which series can one better infer the values of the preference parameters of the structural model. In order to make a fair comparison the same estimation method should be applied in both cases. To start the estimation in the simplest way, we apply an exact method of moments to recover jointly the three preference parameters β, ρ and α. It should be noticed that option prices allow for more flexibility in the sense that we observe more than one option at each date, but only one price for the underlying stock.

The moments for the stock returns that we consider are:

$$E[r_t] = \sum_{i=1}^{2}\sum_{j=1}^{2} \pi_i p_{ij} \left(\log \frac{\varphi_j + 1}{\varphi_i} + m_{Yj} \right), \tag{3.11}$$

$$Var[r_t] = \sum_{i=1}^{2}\sum_{j=1}^{2}\pi_i p_{ij}\left[\left(\log\frac{\varphi_j+1}{\varphi_i}\right)^2 + 2m_{Yj}\left(\log\frac{\varphi_j+1}{\varphi_i}\right) + m_{Yj}^2 + \sigma_{Yj}^2\right] - E[r_t]^2, \tag{3.12}$$

$$Cov[r_t, r_{t-1}] \tag{3.13}$$
$$= \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\pi_i p_{ij} p_{jk}\left[\left(\log\frac{\varphi_j+1}{\varphi_i}\right)^2 + m_{Yj}^2\right]\left[\left(\log\frac{\varphi_k+1}{\varphi_j}\right)^2 + m_{Yk}^2\right] - E[r_t]^2.$$

Let $C_t(U_t = i, \kappa, \tau)$ denote the ratio $\frac{\pi_t}{K}$ where π_t is the price of a European call option as given by the generalized Black-Scholes pricing formula (3.5) when state i is operative at time t and the option's moneyness is equal to $\kappa = S_t/K$ and time to maturity is $\tau = (T - t)$. Given the nonstationarity of S_t it is not surprising that option prices will also be nonstationary since S_t is one of the arguments of the option pricing formula. However the variable S_t/K is stationary as strike prices bracket the underlying asset price. This suggests estimating the parameters of interest by matching the moments of $C_t(U_t = i, \kappa, \tau)$. For instance, the following moment:

$$E\left[\frac{\pi_t}{K}\right] = \sum_{i=1}^{2}\pi_i C_t(U_t = i, \kappa, \tau) \tag{3.14}$$

forms the basis for the method of moments applied to option prices of different moneynesses and maturities. Insofar as we consider observed option prices for a given set of values of the moneyness (possible values of S_t/K), option prices are deterministic functions of the current state variable. They are therefore much less random than stock prices which involve at each date the noise ε_{Yt} of the dividend process. Therefore, in order to make the comparison between estimation results based on option prices and those based on stock returns as fair as possible, we added noise to the ratio $\log(S_t/K)$ as $\log(S_t/K) + \sigma_Y(U_t)\varepsilon_t$ where ε_t is a standard normal white noise. Note that the added error term is proportional to the state-contingent standard error of the dividend process. One may also consider higher moments for option prices. We also compute the moments based on the mean, variance and covariance of a particular option price (normalized by a given moneyness for stationarity).

In order to estimate simultaneously all the structural parameters we shall consider combining moment conditions from the stock returns and option price series. In particular this will allow us to infer the values of the mean and variance of consumption growth from financial market data. We also need to implicitly estimate the λ and φ parameters from Euler equations.

3.2. Simulation results

In this section we compare the empirical performances of the estimates based on option prices and on stock returns. The experiment was carried out as follows. For given values (β, γ, α) characterizing preferences and $(p_{11}, p_{22}, m_{X1}, m_{X2}, \sigma_{X1}, \sigma_{X2}, m_{Y1}, m_{Y1}, \sigma_{Y1}, \sigma_{Y2}, \rho_{XY})$ describing the endowment process, we first obtain the equilibrium values of the price-dividend ratios $(\lambda_1, \lambda_2, \varphi_1, \varphi_2)$ by numerically solving the following set of simultaneous equations:

$$\lambda_i^\gamma = \sum_{j=1}^{2} p_{ij} \left[\beta^\gamma \exp\left\{ \alpha m_{Xj} + \frac{1}{2}(\alpha\sigma_{Xj})^2 \right\} (\lambda_i + 1)^\gamma \right]$$

$$\varphi_i = \sum_{j=1}^{2} p_{ij} \left[\beta^\gamma A_j \left(\frac{\lambda_1 + 1}{\lambda_1} \right)^{\gamma - 1} (\varphi_i + 1) \right]$$

where

$$A_j = \exp\left\{ (\alpha - 1)m_{Xj} + m_{Yj} + \frac{1}{2}\left((\alpha - 1)^2\sigma_{Xj}^2 + \sigma_{Yj}^2 + 2(\alpha - 1)\rho_{XY}\sigma_{Xj}\sigma_{Yj} \right) \right\}$$

The stock returns $\{r_t, t = 1, ..., N\}$ are obtained as follows

$$r_t = \log \frac{\varphi_t + 1}{\varphi_{t-1}} + Y_t \tag{3.15}$$

with $Y_t = \log \frac{D_t}{D_{t-1}}$ given by

$$Y_t = m_{Yt}(U_t) + \sigma_{Yt}(U_t)\varepsilon_{Yt}$$

where $\varepsilon_{Yt} \sim N(0, 1)$. The process $\{U_t, t = 1, ..., T\}$ is a first-order Markov chain such that U_t takes values in the set {1,2} with $\Pr(U_t = j) = \sum_{i=1}^{2} p_{ij} \Pr(U_{t-1} = i)$ and transition probability $p_{ij} = \Pr(U_t = j | U_{t-1} = i)$. For each path of the state variable U_t from time 1 through T normalized option prices $C_t(U_t = i, \kappa, \tau)$ given that state i is operative at time t, for a given moneyness $\kappa = S_t/K$ and time to maturity $\tau = (T - t)$ are generated according to the generalized Black-Scholes pricing formula given in (3.5) and division by the strike price.

For reasons explained in the previous section the option-based series of interest is $\{C_t(U_t = i, \kappa, \tau), t = 1, ..., T\}$ where the option prices are divided by their strike price. This transformation results in a binary process in the sense that for given values of κ and τ the transformed option prices take one of two values depending on which state is operative at time t. Estimating parameters on the basis of this simulated series would have resulted in a perfect fit as the generalized option pricing model has more parameters that there are sources of randomness driving the transformed option price series. However, as explained above, we added noise to the ratio $\log(S_t/K)$ as $\log(S_t/K) + \sigma_Y(U_t)\varepsilon_t$ where ε_t is an $i.i.d\ N(0,1)$ process. The additional error term makes for a fair comparison of the informational content of stock returns vis-à-vis option prices.

We investigate the properties of the estimators for the preference parameters while holding the other parameters of the model fixed at their true values[8]. In tables VII, VIII and IX we report the results of this simulation experiment in terms of mean, median, standard error and root mean square error for the three parameters. We report the results for the method-of-moments estimators based on option prices (from a time series and an across-moneyness perspective), stock returns, and price-dividend ratio[9] respectively. First, we notice that the estimators based on stock returns are more biased than the estimators based on moment conditions for options. It is the case even if we use comparable moments computed on the time series of one particular option. The bias is more pronounced for the parameters ρ and α than for the subjective discount factor β. A possible reason for this finite sample bias could be the nonlinearity in parameters present in the model[10]. It is possible that the nonlinear nature of the option payoffs helps in this

[8]The values of the endowment process are similar to those estimated from actual data by Bonomo and Garcia (1996). Other values (such as the ones used in David and Veronesi (1999)) yield the same conclusions.

[9]The informational content of price-dividend ratios was suggested by Bansal and Lundblad (1999).

[10]This is not a numerical issue. In fact, we gave an advantage to the stock returns conditions in the sense that we started the optimization at the true parameter values, while for the options the initial values were taken in a random neighborhood of the true values.

regard. Improvements in terms of RMSE can be obtained in two directions, one for options, the other for the stock.

First, by using a set of three options with different moneyness, we can see that the RMSE is reduced at least for ρ and α. The main difference in the information base of the sets of estimators is that in one case we use a time series of a unique asset, while in the other we use a panel of option prices. To estimate well the preference parameters, it is necessary to recover well the stochastic discount factor[11] or the underlying risk neutral probability distribution. This is easier with a panel of option prices than with a time series on the underlying asset or one particular option. The second direction of improvement is to use moments on the price-dividend ratio of the stock instead of stock returns to estimate the parameters. The RMSE is reduced for the three parameters compared to the estimates obtained with the stock returns. It should be emphasized that in the true model used to simulate the prices, the price-dividend ratio takes two values, one for each state, as it is the case for option prices. However the RMSEs remain higher than the RMSEs obtained with option prices.

These simulation results tell us that using a set of option prices allows us to recover well the preference parameters if the formula is not preference free. It therefore suggests that option price data could be better to test different models of preferences than bond and stock returns as it is usually done. Additionally, price-dividend ratios might also be more informative than stock returns.

In table X, we proceed to estimate jointly all the parameters of the model, again with an exact method of moments. We use enough moment conditions from option prices and stock returns to estimate the 12 parameters of interest. We get 9 moment conditions on options by considering 3 different moneynesses (1.1, 1 and 0.9) and time to maturity (1, 2 and 3 periods) and three moment conditions from the stock returns (mean, variance and covariance). The results indicate that apart from the means of the consumption process the parameters are generally estimated without a large bias.

[11] α is a risk aversion parameter while $1/(1-\rho)$ is an intertemporal elasticity of substitution. However, as acknowledged by Epstein and Zin (1989), this interpretation should be mitigated since the position of ρ with respect to α determines the preference vis-a-vis time of resolution of uncertainty. In this respect, ρ is also a risk parameter.

3.3. Is There Evidence of Preference Parameters in S&P 500 Option Prices?

The Monte Carlo experiments of section 3.2 lay the ground for a general estimation of the model with option price data. We will use daily S&P 500 option price data to estimate the parameters and assess the out-of-sample pricing performance of the model and its main competitors. We used the following method of estimation. At time t, the GBS model is estimated by the method of moments using the moments defined in the simulation study. A three-month window prior to the time of estimation is used to compute the empirical moments. To make sure we explore well the preference parameter space in the optimization, we set a grid of initial values for γ and ρ as follows: ρ= -0.5, -1, -2 ,-4 ,-8 and γ = 0.01, 0.05, 0.1, 0.5, 1.0. Overall, we have 25 pairs of starting values which correspond to various degrees of risk aversion and elasticity of intertemporal substitution. The model with the set of initial values that yields the lowest average absolute pricing error over the following week $(t, t+5)$ is retained for pricing over the next week $(t+5, t+10)$. Before pricing, the model is reestimated using information up to time $t+5$. The same strategy is also applied to the expected utility model where γ is constrained to a value of 1. In this case only the elements of the grid for ρ are relevant. The initial values for all the other parameters are the same for both of these models as well as for the preference-free model[12]. We conduct this experiment for five years, from 1991 to 1995.

Using the estimates obtained each week we forecast the prices for all the options of the following week separated in long, medium and short maturities and in different moneyness categories. We average the weekly forecast errors over each year for the corresponding categories and compare the performance of three models: the most general option model for the non-separable recursive utility model given by formula (3.5), the expected utility model obtained by setting γ equal to one in (3.5) to judge the importance of non-separabilities, and finally the Hull and White

[12] We only conduct a grid search over part of the preference parameter space since we want to illustrate the relative importance of the preference parameters in option pricing at least compared to the expected utility model and the preference-free model à la HW. Further reductions in the absolute pricing errors could be achieved by a more extensive search over the space of preference parameters and those associated with the endowment processes. However, given our estimation and forecasting procedure, the computational cost would be too high.

stochastic volatility model which results from (3.5) when $Q_{XY}(t,T) = 1$ to gauge the importance of preferences for option prices. It should be emphasized that the objective of this forecasting exercise is to assess the relative performance of the three models and not to select a model that will be implementable in practice. The use of unconditional moments is not appropriate for this purpose. Conditional information needs to be incorporated in some way to hope achieving performances that are comparable to the ad-hoc models explored in the previous section. We will explore the added forecasting value of this conditioning information in future research.

The results are clear. For short and medium-term options, GBS does better that the specification where γ is equal to one which is turn is better than the Hull and White specification. Compared to Hull and White, the relative error for GBS is reduced from 20 to 50 per cent for short-term and medium-term options. This shows that preferences are important in pricing options on the index. Moreover, the data seem to indicate that preferences are of the non-separable type since the restricted value of γ generally increases the relative error. Of course, as we advance in maturity, the relative error falls since the smile effect flattens and pricing tends to approach Black-Scholes. For long-term options, where the smile is the flattest, the differences between the three models are small. These results parallel the simulation results reported in Garcia, Luger and Renault (2000) about the smile. First, it was shown that a non-preference free framework was able to reproduce the various asymmetries observed in the implied volatility curve inferred from option price data. Second, the parameter γ was perceived to be more important than the risk aversion parameter α in calibrating the smile.

Table XII reports the average values of the preference parameters that we obtained over the five years. These values imply a coefficient of relative risk aversion of 0.5688 on average and a mean value of 0.7363 for the elasticity of intertemporal substitution. As the standard errors indicate the values obtained are not very variable. Not only these values imply considerable differences in terms of pricing errors compared with the expected utility model, but they also appear much more reasonable than the values obtained with the latter model. Indeed, when the parameter γ is constrained to be equal to 1, we obtain a high average value of around 8 for the coefficient of relative risk aversion and a very low value for β. Relaxing this constraint results in an elasticity of intertemporal substitution lower than the inverse of the relative risk aversion coefficient, in accordance with the generally accepted stylized fact.

In terms of state variables we find average values of 0.80 and 0.40 for the transition probabilities in state 1 and 2 respectively. The first state is a state with a low volatility of dividends.

4. Conclusion

The main message of this paper is that, even though the strawman of BS implied volatilities has proved its efficiency and robustness among practitioners for a while, it should not be considered as impossible to improve upon. To support this claim, we provide two pieces of empirical evidence which, in our opinion, are all the more convincing that they are independent and reinforce each other. On the one hand, we show that a very general option pricing model only restricted by an assumption of joint conditional log-normality of returns and stochastic discount factor (SDF), given a set of state variables, leads to out-of-sample pricing errors smaller than the strawman, without any increase in complexity. This simplicity results from an ad-hoc procedure, comparable in spirit with standard pricing with a volatility smile. Moreover, it opens the door for more flexible empirical procedures, which would adjust the skewness of the volatility smile through an implied spot value. On the other hand, we also assess option pricing errors which result from a fully specified equilibrium model which corresponds to a specific SDF. We check that the performance of this model is significantly better than a standard Hull and White option pricing, which neglects the price of volatility risk. In this respect, recursive preferences which disentangle the risk aversion and the elasticity of intertemporal substitution parameters appear to be particularly relevant for option pricing. Conversely, option price data are shown to be very informative about these preference parameters.

Of course, a great deal of additional work remains to be done before claiming that equilibrium models including explicitly some preference parameters should be used for option pricing in practice. However, we consider that both our Monte Carlo experiments and the estimation performed with S&P 500 option price data prove without ambiguity that preference parameters are important in option pricing. Among the required extensions, one should of course discuss the specifications of both preferences and the distribution of the state variables. Finally, an assessment of hedging performance is also needed.

Appendix

We have to show that the Jacobian matrix of the mapping of interest is non-singular. But the gradient of the mapping: $(S, \sigma) \to BS_K(S, \sigma)$ is the vector of the coefficients delta and vega of the call, that is (see e.g. Hull (1993), pages 314 and 329):

$$[\phi(d_K), S_t\sqrt{T-t}\varphi(d_K)] \tag{A1}$$

where $\varphi(d) = \phi'(d)$ is the density function of the standard normal and:
$d_K = \log \frac{S_t}{KB(t,T)} + \sigma\sqrt{T-t}$.

In other words, we have to show that for $K_1 \neq K_2$, the two vectors:

$$\begin{gathered}[\phi(d_{K1}), S_t\sqrt{T-t}\varphi(d_{K1})] \\ [\phi(d_{K2}), S_t\sqrt{T-t}\varphi(d_{K2})]\end{gathered} \tag{A2}$$

are linearly independent, that is:

$$\frac{\varphi(d_{K1})}{\phi(d_{K1})} \neq \frac{\varphi(d_{K2})}{\phi(d_{K2})} \tag{A3}$$

This is a direct consequence of the strict monotonicity of the Mills ratio:
$x \to f(x) = \frac{\varphi(x)}{\phi(x)}$.

Actually:

$$f'(x) = -\frac{\varphi(x)}{\phi^2(x)}[x\phi(x) + \varphi(x)] < 0 \tag{A4}$$

since: $g(x) = x\phi(x) + \varphi(x) > 0$, given that: $g'(x) = \phi(x) > 0$ and $\lim g(x)_{x \to -\infty} = 0$.

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Table I
Absolute Pricings Errors - BS

	M < 60						60 < M < 180						M > 180					
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
1988	0.8243	0.5635	0.5741	1.0928	1.0597	1.5254	0.5118	0.8664	1.0179	1.0758	1.0897	1.7987	0.7340	-	-	0.6042	0.5555	1.8013
1989	0.4380	0.6094	0.5219	0.8093	1.3119	1.7251	0.4883	0.5154	0.6153	0.8063	1.1093	1.2805	0.4070	1.4199	0.5035	1.2428	0.5125	1.3367
1990	1.0176	0.5264	0.5715	1.0894	1.2463	1.8153	0.6896	0.7921	0.8926	1.1879	1.3865	1.6949	0.9467	0.7472	1.1467	0.9493	2.5026	2.9619
1991	0.5250	0.5146	0.5224	0.9938	1.2323	1.8404	0.5363	0.6724	0.8405	1.2740	1.3519	1.3840	0.6546	0.6452	1.0785	1.8921	1.8058	1.4824
1992	0.3048	0.4425	0.4259	0.9355	1.2815	2.5912	0.5400	0.5235	0.7196	1.1818	1.2263	1.3042	0.6524	0.8362	0.8845	0.9207	1.1404	1.2713
1993	0.3375	0.2174	0.4892	0.8633	1.1344	2.3278	0.4233	0.4107	0.6276	1.1687	1.1622	1.4856	0.5879	0.6925	0.7160	1.3183	0.5839	1.6219
1994	0.2000	0.1509	0.2255	0.9610	1.4188	2.1385	0.1969	0.4175	0.6127	1.1385	1.4137	1.3919	0.5145	0.8007	0.8462	1.2007	1.5110	1.1752
1995	0.1200	0.1968	0.2183	0.9952	1.5826	2.6817	0.2332	0.3073	0.4611	1.2548	1.2107	2.0517	0.3689	0.6284	0.8005	1.2270	1.4989	1.4146
1996	0.3568	0.3680	0.4578	1.8026	1.8990	3.6946	0.5609	0.8109	1.0581	2.0931	2.1122	3.1546	1.1346	1.1684	1.4707	1.9769	2.1499	2.5425
1997	1.3729	1.2408	1.4530	2.9460	3.6383	5.4628	3.0132	6.5033	5.1386	5.5736	5.2681	9.2125	6.3800	8.8467	11.4401	10.1661	6.8337	8.2851
Mean	0.5497	0.4830	0.5460	1.2489	1.5805	2.5803	0.7194	1.1820	1.1984	1.6755	1.7331	2.4759	1.2381	1.7539	2.0985	2.1498	1.9094	2.3893

Table II

Absolute Pricings Errors - GBS

	M < 60						60 < M < 180						M > 180					
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
1988	0.8414	0.5430	0.5668	1.0241	1.0936	1.1692	0.5551	0.8557	0.9949	1.0585	1.0895	1.8162	0.7511	-	-	0.6010	0.5316	1.3319
1989	0.4903	0.5269	0.5440	0.7425	0.9679	0.8199	0.4785	0.5724	0.6088	0.8141	1.0629	1.3072	0.4174	1.4023	0.5143	1.1883	0.6147	1.3297
1990	1.0602	0.5218	0.5777	1.0873	1.1389	1.2336	0.7679	0.8000	0.9026	1.1962	1.3549	1.5200	1.0041	0.7212	1.1547	0.9819	2.5150	1.3781
1991	0.5375	0.4920	0.5186	0.9652	1.1685	1.2091	0.5692	0.6556	0.8397	1.2586	1.4275	1.1560	0.6581	0.6313	1.1059	1.8613	1.7736	1.6137
1992	0.4167	0.4144	0.4215	0.8825	1.0647	1.2573	0.5641	0.5156	0.7102	1.1656	1.1827	1.1693	0.6286	0.8082	0.8429	0.9075	1.0960	1.1762
1993	0.3943	0.3130	0.4457	0.8108	0.8205	0.9911	0.4301	0.4048	0.6231	1.1508	1.1159	1.0307	0.6157	0.6818	0.7239	1.3280	0.6294	1.7749
1994	0.1981	0.1543	0.2384	0.8696	1.2279	1.5671	0.2138	0.4069	0.6014	1.1595	1.3587	1.3525	0.5873	0.8002	0.8437	1.2244	1.5188	1.2577
1995	0.1295	0.1860	0.2224	0.9126	1.2047	1.0765	0.2369	0.3051	0.4653	1.2416	1.2618	1.3059	0.3836	0.6304	0.8081	1.2221	1.5076	1.5165
1996	0.3481	0.3750	0.4644	1.6318	1.6967	1.8567	0.5862	0.8164	1.0666	2.0380	2.1259	2.2657	1.1864	1.1651	1.4716	1.9543	2.1371	2.1660
1997	1.2609	1.1940	1.3412	2.8437	3.3205	3.9235	2.9300	6.2965	5.0167	5.5876	5.6389	7.9382	6.1669	8.6589	11.2689	10.3326	7.4460	8.6761
Mean	0.5677	0.4720	0.5208	1.1405	1.4151	1.5104	0.7332	1.1629	1.1829	1.6671	1.7619	2.086	1.2399	1.7222	2.0816	2.1601	1.9770	2.2221

Table III

Relative Pricings Errors - BS

	$M < 60$						$60 < M < 180$						$M > 180$					
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
1988	-0.3730	-0.0762	-0.0536	-0.0079	-0.0017	-0.0149	-0.0568	-0.0393	-0.0204	-0.0107	-0.0074	0.0030	-0.0057	-	-	0.0035	-0.0341	0.0200
1989	-0.0702	-0.2232	-0.0596	-0.0049	-0.0391	-0.0389	-0.1242	-0.0243	-0.0316	-0.0051	-0.0107	0.0029	-0.0085	-0.2816	0.0581	-0.0695	0.0136	0.0090
1990	-0.4424	-0.0639	-0.0370	-0.0052	-0.0104	-0.0269	-0.0819	-0.0580	0.0159	-0.0072	-0.0048	-0.0129	-0.0295	-0.0796	0.0041	-0.0055	-0.0008	-0.0440
1991	-0.0725	-0.0479	-0.0527	-0.0106	-0.0171	-0.0213	-0.0451	-0.0800	-0.0254	-0.0135	-0.0047	-0.0042	-0.0177	0.0130	-0.0040	0.0120	0.0041	0.0077
1992	-0.1435	-0.0332	-0.0375	-0.0079	-0.0170	-0.0507	-0.1315	-0.0563	-0.0467	-0.0051	0.0063	0.0030	-0.0985	0.0041	-0.0155	-0.0118	-0.0096	-0.0021
1993	0.1138	0.0697	-0.0897	-0.0105	-0.0106	-0.0406	0.0827	-0.0341	-0.0706	-0.0116	-0.0081	-0.0279	-0.1021	-0.0784	0.0068	-0.0099	0.0001	-0.0039
1994	0.1499	-0.0124	-0.0179	-0.0065	-0.0229	-0.0125	-0.0170	-0.0611	-0.0358	-0.0071	0.0032	-0.0103	-0.0769	-0.0080	0.0059	-0.0068	-0.0166	0.0043
1995	-0.1043	-0.0602	-0.0177	-0.0089	-0.0160	-0.0348	-0.0276	-0.0194	-0.0033	-0.0054	0.0081	-0.0083	-0.0019	0.0001	-0.0126	-0.0004	0.0033	-0.0001
1996	-0.0091	-0.0438	-0.0229	-0.0099	-0.0062	-0.0310	-0.0333	-0.0084	-0.0152	-0.0069	0.0038	-0.0090	0.0038	-0.0015	0.0114	-0.0056	-0.0097	0.0024
1997	-0.2452	-0.0821	-0.0840	-0.0146	-0.0101	-0.0290	-0.1425	-0.0962	-0.0527	-0.0194	-0.0007	-0.0216	-0.1643	-0.0675	-0.0734	-0.0203	0.0162	-0.0094
Mean	-0.1197	-0.0573	-0.0473	-0.0087	-0.0151	-0.0301	-0.0577	-0.0477	-0.0286	-0.0092	-0.0015	-0.0085	-0.0501	-0.0555	-0.0108	-0.0114	-0.0034	-0.0016

Table IV
Relative Pricings Errors - GBS

	M < 60						60 < M < 180						M > 180					
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
1988	-0.1570	-0.0653	-0.0490	-0.0025	0.0044	0.0066	-0.0724	-0.0379	-0.0199	-0.0077	-0.0081	0.0027	-0.0019	-	-	0.0040	-0.0326	0.0020
1989	-0.1713	-0.1288	-0.0656	0.0013	-0.0137	0.0035	-0.1273	-0.0349	-0.0336	-0.0048	-0.0079	0.0071	-0.0102	-0.2781	0.0585	-0.0663	0.0203	0.0085
1990	-0.4321	-0.0456	-0.0332	-0.0020	-0.0007	-0.0008	-0.0890	-0.0517	-0.0122	-0.0041	-0.0025	0.0004	-0.0289	-0.0764	0.0054	-0.0079	-0.0011	-0.0007
1991	-0.1118	-0.0137	-0.0436	-0.0085	-0.0097	0.0026	-0.0329	-0.0760	-0.0239	-0.0105	-0.0037	0.0015	-0.0147	0.0137	-0.0034	0.0118	0.0054	0.0037
1992	-0.2936	-0.0275	-0.0179	-0.0038	-0.0027	-0.0071	-0.1366	-0.0521	-0.0430	-0.0048	0.0048	0.0104	-0.0938	0.0052	-0.0154	-0.0118	-0.0107	-0.0070
1993	0.2721	-0.0949	-0.0412	-0.0043	0.0068	0.0024	0.0578	-0.0275	-0.0657	-0.0126	-0.0088	-0.0133	-0.1285	-0.0748	0.0038	-0.0096	-0.0025	-0.0031
1994	0.1453	-0.0103	-0.0063	0.0005	-0.0069	0.0085	-0.0113	-0.0588	-0.0338	-0.0075	0.0054	-0.0054	-0.0905	-0.0066	0.0051	-0.0070	-0.0181	0.0023
1995	-0.1083	-0.0529	-0.0139	-0.0013	0.0015	0.0061	-0.0266	-0.0187	0.0001	-0.0048	0.0064	0.0115	-0.0019	-0.0018	-0.0107	0.0001	0.0038	0.0012
1996	-0.0083	-0.0383	-0.0126	-0.0024	-0.0009	0.0086	-0.0122	-0.0034	-0.0125	-0.0060	0.0058	0.0062	0.0122	0.0015	0.0124	-0.0043	-0.0094	0.0026
1997	-0.1504	-0.0670	-0.0502	-0.0062	-0.0036	0.0013	-0.0806	-0.0745	-0.0442	-0.0203	-0.0046	-0.0008	-0.1003	-0.0590	-0.0668	-0.0223	0.0196	-0.0065
Mean	-0.1015	-0.0544	-0.0334	-0.0029	-0.0026	0.0032	-0.0531	-0.0436	-0.0251	-0.0083	-0.0013	0.0020	-0.0459	-0.0529	-0.0012	-0.0113	-0.0025	0.00037

Table V
Absolute Pricings Errors - GBS

	M < 60					
	1	2	3	4	5	6
1988	0.8246	0.5587	0.5704	1.0761	1.0833	1.5803
1989	0.4232	0.5909	0.5250	0.8010	1.2803	1.6921
1990	0.9801	0.5231	0.5702	1.0787	1.2207	1.7706
1991	0.4982	0.4933	0.5188	0.9570	1.2116	1.9602
1992	0.3002	0.4299	0.4165	0.9030	1.2819	2.5857
1993	0.3639	0.2169	0.4506	0.8516	1.1596	2.3058
1994	0.1995	0.1508	0.2233	0.9080	1.4006	2.0450
1995	0.1200	0.1957	0.2183	0.9684	1.5541	2.9471
1996	0.3563	0.3634	0.4574	1.7642	1.8595	3.9171
1997	1.2247	1.1614	1.3296	2.8491	3.6307	5.6438
Mean	0.5301	0.4684	0.5280	1.2157	1.5682	2.6448

Table VI
Relative Pricings Errors - GBS

	M < 60					
	1	2	3	4	5	6
1988	-0.3573	-0.0715	-0.0534	-0.0053	0.0011	-0.0071
1989	-0.0530	-0.1973	-0.0578	0.0002	-0.0330	-0.0321
1990	-0.3932	-0.0565	-0.0365	-0.0039	-0.0060	-0.0196
1991	-0.0348	-0.0184	-0.0427	-0.0091	-0.0145	-0.0077
1992	-0.1307	-0.0122	-0.0248	-0.0036	-0.0125	-0.0497
1993	0.0663	0.0721	-0.0446	-0.0056	-0.0046	-0.0262
1994	0.1508	-0.0110	-0.0090	-0.0008	-0.0177	-0.0097
1995	-0.1043	-0.0585	-0.0122	-0.0024	-0.0117	-0.0139
1996	-0.0015	-0.0363	-0.0185	-0.0053	-0.0015	-0.0086
1997	-0.1064	-0.0494	-0.0476	-0.0072	-0.0040	-0.0067
Mean	-0.0964	-0.0439	-0.0347	-0.0043	-0.0071	-0.0182

Tables VII, VIII and IX: Descriptive statistics for the method of moments estimator of preference parameters. The moments used in the estimation are the mean, the variance and the autocovariance of the respective series. For options, we also used the mean of the three options with different moneyness. The true values are $\rho = -10$, $\alpha = -5$ and $\beta = 0.95$ for the preferences and $p_{11} = 0.9$, $p_{22} = 0.6$, $m_{X1} = 0.0015$, $m_{X2} = -0.0009$, $\sigma_{X1} = \sigma_{X2} = .003$, $m_{Y1} = m_{Y2} = 0$, $\sigma_{Y1} = 0.02$, $\sigma_{Y2} = 0.12$ and $\rho_{XY} = 0.6$ for the endowment process. The results are reported for options with maturity of one period . The results are based on 1000 replications of the experiment.

Table VII

Options Prices (time series)	ρ	α	β
Mean	-10.1585	-4.6162	0.9445
Median	-10.2131	-4.7979	0.9445
Std Err	1.0524	1.8975	0.0093
RMSE	1.0638	1.9350	0.0108

Options Prices (across moneyness)	ρ	α	β
Mean	-10.1421	-4.6770	0.9504
Median	-10.2171	-4.7927	0.9500
Std Err	1.0117	1.2921	0.0159
RMSE	1.0212	1.3312	0.0159

Table VIII

Stock Returns	ρ	α	β
Mean	-11.0711	-2.4557	0.9950
Median	-10.9812	-1.8966	0.9955
Std Err	1.0457	1.6153	0.0035
RMSE	1.4965	3.0134	0.0451

Table IX

Price-dividend ratio	ρ	α	β
Mean	-10.5537	-3.5051	0.9501
Median	-10.0003	-4.9861	0.9497
Std Err	1.2742	2.1530	0.0017
RMSE	1.3887	2.6202	0.0017

Table X: Descriptive statistics for the joint estimation of the structural parameters by the method of moments. The true values are $\rho = -10$, $\alpha = -5$ and $\beta = 0.95$ for the preferences and $p_{11} = 0.9$, $p_{22} = 0.6$, $m_{X1} = 0.0015$, $m_{X2} = -0.0009$, $\sigma_{X1} = \sigma_{X2} = .003$, $m_{Y1} = m_{Y2} = 0$, $\sigma_{Y1} = 0.02$, $\sigma_{Y2} = 0.12$ and $\rho_{XY} = 0.6$ for the endowment process. The results are based on 1000 replications of the experiment.

Table X

	β	ρ	α	p11	p22	ρ_{XY}
Mean	0.9164	-10.0517	-4.9728	0.8983	0.5916	0.5954
Median	0.9504	-9.9903	-5.0177	0.9010	0.5983	0.5997
Std Err	0.1119	1.4381	1.3672	0.0507	0.0749	0.0980
RMSE	0.1168	1.4383	1.3667	0.0507	0.0753	0.0981
	m_{X1}	m_{X2}	σ_{X1}	m_{Y1}	σ_{Y1}	σ_{Y2}
Mean	0.0520	0.0500	0.0068	-0.0780	0.0462	0.1849
Median	0.0013	-0.0052	0.0031	-0.0088	0.0193	0.1249
Std Err	1.0176	0.8822	0.0267	0.5529	0.3704	0.2028
RMSE	1.0183	0.8832	0.0269	0.5581	0.3711	0.2128

Table XI: Yearly Relative Pricing Errors for Short, Medium and Long-Term Call Options Averaged Over Moneyness. GBS refers to the generalized Black-Scholes formula in (3.5);EU to the same formula special case where the parameter γ is equal to 1; HW to the Hull and White formula (special case of (3.5) with $Q_{XY}(t,T) = 1$).

Table XI

Short-Term	GBS	EU	HW
1991 (3072)	0.8165	1.0986	1.4482
1992 (2834)	0.9680	0.9844	1.3366
1993 (2856)	0.6331	0.9526	1.1968
1994 (3310)	0.7796	1.0001	1.8166
1995 (3967)	1.4601	1.8301	2.0499

Medium-Term	GBS	EU	HW
1991 (2090)	0.3916	0.5487	0.6351
1992 (2271)	0.6815	0.7403	0.7912
1993 (2097)	0.3679	0.5814	0.9359
1994 (2854)	0.5657	0.9627	1.6389
1995 (2926)	0.9337	1.2140	1.1438

Long-Term	GBS	EU	HW
1991 (706)	0.0075	0.1166	0.0100
1992 (543)	0.0005	0.0024	-0.0011
1993 (482)	-0.2818	-0.1314	0.1120
1994 (911)	-0.2617	-0.1338	0.2420
1995 (1053)	0.1228	0.2982	0.2359

Table XII: Means and Standard Errors of Weekly Estimated Preference Parameters from S&P 500 Option Price Data over the Period 1991-1995

Table XII

GBS Model			
	ρ	γ	β
Mean	-0.3582	-1.2038	0.8889
Std Err	0.0214	0.0735	0.0044
Expected Utility Model			
	ρ	γ	β
Mean	-7.3728	1	0.5800
Std Err	1.8005	-	0.0554