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Addressing the Food Aid Curse

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Abstract:
In this paper, we build a model of agrarian economies in which a kleptocratic government taxes farmers to maximize its life-time utility. The model is a dynamic general equilibrium model in which the subsistence of farmers requires a minimum level of consumption. We analyze the effect that a benevolent food aid agency can have in such an environment. If it expects the food aid agency to intervene, the kleptocratic government will starve its farmers, in a clear case of the Samaritan's dilemma. We show that the likelihood of man-made famines, however, can be greatly reduced if the food aid agency intervenes with probability slightly lower than one. No aid agency devoted to saving lives, however, can commit to such policy. We propose a solution to this food aid curse.

Keywords: Food aid, famines, commitment

JEL Classification: F35, D72, C72

Introduction

The mere existence of aid agencies generates a need for aid. This fact is known as the Samaritan's dilemma and it was first highlighted by Buchanan (1975). An agency devoted to fighting poverty in developing countries, for example, is likely to unwillingly generate poverty as candidate recipient countries strive to qualify for aid (Pedersen, 2001; Hagen, 2006). The dilemma is particularly striking in the case of food aid agencies committed to saving lives in famine-stricken countries. We show, in this paper, that the availability of food aid is indeed an important determinant of man-made famines. Clearly, a withdrawal of aid agencies would solve the dilemma. But aid agencies cannot commit not to intervene. A humanitarian aid agency, whose objective is to minimize deaths from hunger, will always respond to famines (even man-made ones) if it has the means to do so. We also show that a simple policy in which the agency commits to intervene with a probability less than one could remove the temptation for an opportunistic ruler to starve his population. This policy, however, is not credible. The agency faces a time inconsistency problem à la Kydland & Prescott (1977), which makes commitment unachievable. Indeed, whenever hunger is detected ex post, the agency will intervene with probability one if its budget permits. We call this commitment problem the food aid curse.

We propose a solution to this curse, which involves gifts to well-behaved rulers and a punishment phase for a defiant ruler involving all other parties. Our solution therefore implies a general penal code in the sense of Abreu (1988). In the vast literature on the role of food aid,[1] we offer another possible explanation: food aid may serve as a necessary bribe to avoid man-made famines.

Several papers address the political economy of institutions in the presence of development aid. Drazen (1999) builds a case for withholding aid transfers as a means to trigger political change. Acemoglu, Robinson, & Verdier (2004) show how kleptocrats are capable of remaining in power in the absence of a powerful support base, and the importance of foreign aid in implementing their strategy. Svensson (2000) shows that rent-seeking is exacerbated by expectations of foreign aid, but that this effect can be mitigated if the aid agency can

[1] See Isenman & Singer (1977) Srinivasan (1989), Gupta, Clements, & Tiongson (2004), Abdulai, Barrett, & Hazell (2004), Barrett & Maxwell (2005) for important contributions to this literature.

commit to an optimal policy. In this paper, we take the institution as given and show how efficient behavior can be obtained even though the aid agency's humanitarian agenda makes it incapable of withholding aid.

Economists have long been interested in the origins of famines. Sen (1981) demonstrated the fact that famines do not necessarily imply a shortage of food. In our paper, they follow solely from human behavior, i.e. the greed of kleptocrats and their opportunistic behavior in the presence of aid agencies. Our analysis is in line with Ravallion (1997)'s claim that economics is essential in understanding famines.

We work within an infinite-horizon, dynamic model featuring a number of countries, each ruled by a kleptocrat who has the ability to expropriate its citizens' output. We analyze the effect of introducing into this world a humanitarian aid agency with a given food aid bank. There is no interaction among dictators in this paper. For a look at how dictators (or warlords) interact strategically with an aid agency while in conflict with each other see Blouin & Pallage (2007).

1 Benchmark model

The world is composed of N countries $i \in \{1, ..., N\}$, each ruled by an infinitely-lived kleptocratic dictator. Time is discrete. A country is made of a continuum of farmers of initial measure 1. Every farmer produces $y = 1$ of the unique consumption good. Farmers have no alternative than to work for the dictator, who taxes away their output. Their survival requires a minimal consumption of $\bar{c}$ every period. A dying farmer is never replaced. The cost of starving one's population is therefore borne over an infinite horizon.

A given dictator i cares solely about his consumption stream over time, $\{c_{it}\}_{t=0}^{\infty}$. He maximizes the following objective:

$$\sum_{t=0}^{\infty} \beta^t \ln(c_{it}) \tag{1}$$

where $\beta \in (0, 1)$ is a time discount factor.

Given his objective and the lack of outside options for farmers, there is no reason why the kleptocrat would pay farmers an amount in excess of $\bar{c}$. Hence if kleptocrat i allows a fraction s_{it} of his population to survive from time t to $t+1$, his consumption at t is:

$$c_{it} = s_{it}(1-\bar{c}) + (1-s_{it}) = 1 - s_{it}\bar{c}\ . \tag{2}$$

If n_{it} is country i's population at time t, the population next period will thus be $n_{i,t+1} = s_{it}n_{it}$.

Our benchmark environment is one in which no aid agency exists to provide relief to starving farmers. In this case, the typical kleptocrat solves the following Bellman equation in which we drop time subscripts and use primes to denote future states:

$$V^b(n_i) = \max_{s_i \in [0,1]} \ln[n_i(1-s_i\bar{c})] + \beta V^b(s_i n_i) \tag{3}$$

Problem (3) has the following solution:

Proposition 1 (Benchmark) *If $\beta \geq \bar{c}$, the kleptocrats do not starve anyone ($s_i(n_i) = 1$ for all i). If $\beta < \bar{c}$, the kleptocrats starve a constant fraction $1-\beta/\bar{c}$ of their population every period.*

Proof. The solution to Problem (3) is easily obtained using the guess-and-verify method of standard dynamic programming. □

Because the cost of starving a farmer has to be borne over an infinite horizon, if β is sufficiently high ($\beta \geq \bar{c}$), this cost outweighs the one-shot gain from consuming an additional $\bar{c}$. In that case, we have a corner solution $s_i(n_i) = 1$ and $V^b(n_i) = 1/(1-\beta)\ln[n_i(1-\bar{c})]$. Otherwise, the policy function is $s_i(n_i) = \beta/\bar{c}$ and the value function can be written as $V^b(n_i) = 1/(1-\beta)\ln(n_i) + K$, where $K = 1/(1-\beta)\ln(1-\beta) + \beta/(1-\beta)^2\ln(\beta/\bar{c})$ is a constant.

2 Food aid

The existence of a food aid agency committed to saving lives from starvation is likely to perturb the equilibrium choices of the kleptocrats. We assume hereafter that the aid agency has a constant budget B every period. We can think of B as a basket of perishable food or, as in most public administration offices, as a budget which expires at the end of the fiscal year. We assume that the aid agency always prefers spending less to achieve the same goal.

2.1 Samaritan's dilemma

We begin by considering the case where the agency acts non-strategically. By this we mean that the agency makes no threats: it merely reacts to kleptocrats' actions by saving all lives or expending its entire budget. With a per-period budget of B, the agency can save $\tilde{n} \equiv B/(N\bar{c})$ people per country per period, assuming each country gets an equal share. We shall in fact assume symmetric equilibrium throughout. If each country has a population no greater than $\tilde{n}$ (i.e. if the agency's budget is large enough to save everyone), then the Bellman equation for each ruler becomes

$$V(n_i) = \max_{s_i \in [0,1]} \ln[n_i(1 - s_i\bar{c})] + \beta V(n_i) \tag{4}$$

In this case, we face a pure Samaritan's dilemma:

Proposition 2a *If the aid agency acts non-strategically (see above) and $n_i = n \leq \tilde{n}$ for all i, then in equilibrium each kleptocrat $i \in \{1, ..., N\}$ chooses $s_i(n_i) = 0$.*

The mere presence of a food aid agency either generates or exacerbates man-made famines. Although well-intentioned, the agency's commitment relieves the dictator of the cost of starving his population. Hence the dictator's objective becomes a decreasing fraction of s_i.

If the agency has a limited budget, i.e. if $n_i = n > \tilde{n}$ for all i, then in a symmetric equilibrium each kleptocrat will find it optimal to set $s_i(n_i) > 0$. The formal result is

Proposition 2b *If the aid agency acts non-strategically and $n_i = n > \tilde{n}$ for all i, then in equilibrium each kleptocrat $i \in \{1, ..., N\}$ chooses*

$$s_i(n_i) = \begin{cases} 1 - \tilde{n}/n & \text{if } \beta \geq \bar{c}; \\ s^*(n_i) & \text{if } \beta < \bar{c}; \end{cases} \tag{5}$$

where $s^*(n_i)$ is defined in the appendix.

The result is derived in the appendix. If $\beta \geq \bar{c}$ the population remains constant, since dictators starve exactly the number of people that the agency can save. If $\beta < \bar{c}$, on the other hand, then the dictators' impatience makes them starve *more* people than the agency can save: population decreases as a result, until it reaches $\tilde{n}$, which it does in finite time. Once population reaches $\tilde{n}$, each dictator sets $s_i(\tilde{n}) = 0$, as expected from Proposition 2a.

The Samaritan's dilemma is the same as that facing most international aid agencies. Yet for most aid agencies, there is a simple solution to that dilemma. It involves a commitment not to intervene in case of a dictator's bad behavior. Although a commitment to non-action may be difficult to obtain, even in the context of development aid, it is arguably a lot easier than in the case of humanitarian aid. Indeed, if development aid agencies strive for the development of many recipients, their goal can always be attained if they substitute a well-behaved recipient for an ill-behaved one. Committing not to act is impossible for a humanitarian aid agency whose objective is to save lives.

Yet, solving the Samaritan's dilemma does not necessarily imply such a radical strategy as a commitment not to intervene. A commitment to stochastic intervention would typically do the trick. Indeed, if the agency could commit to saving all lives in each country with probability $p < 1$, then it could greatly reduce the extent of man-made famines. The Bellman equation of a dictator under such policy would be:

$$V(n_i) = \max_{s_i \in [0,1]} \ln[n_i(1 - s_i\bar{c})] + \beta \left(pV(n_i) + (1-p)V(s_i n_i)\right) \tag{6}$$

The policy function of such problem is given by:

$$s_i(n_i) = \begin{cases} \frac{\beta(1-p)}{(1-\beta p)\bar{c}} & \text{if } \frac{\beta(1-p)}{1-\beta p} < \bar{c} \\ 1 & \text{otherwise} \end{cases} \tag{7}$$

Clearly, if we were in a corner solution in the no-aid scenario, there would exist $p^\star$ such that the agency would not aggravate things if it could commit to intervene only with probability lower than $p^\star$. Unfortunately, the agency will always renege on its commitment ex post, if any kleptocrat chooses to call its bluff. The agency's objective to save lives will always make it impossible for it not to intervene if it has the budget to act. Such is the food aid curse we wish to address in this paper.

2.2 Addressing the food aid curse

It is far from trivial to solve the commitment problem facing an aid agency whose objective is to minimize the number of deaths from starvation. Clearly, if the aid agency and the kleptocrats agree to some form of cooperation at time t, there is no way the aid agency will deny intervention if a kleptocrat deviates from the agreement and generates a massive famine. The punishment for such deviation has to take place in the future and must involve all agreeing parties. The only way for the agency to commit not to intervene in the whole future in the deviating country is to divide, from $t+1$ onward, all its budget between the countries that did not deviate at t and for the latter to set s_i as low as possible given the agency's budget, i.e. $s_i = 1 - B/[(N-1)\bar{c}]$. Still this punishment profile will only be credible if the agency does not have a left-over budget once it feeds all starving agents in all $N-1$ non-deviating countries: $B \leq (N-1)\bar{c}$. The agency's strategy profile will involve a gift g delivered at the end of each period to all complying countries. The following proposition formalizes this intuition. Strategies are assumed to be implemented at the beginning of the game, i.e. at $n = 1$. A more general formulation, i.e. for $n < 1$, is possible.

Proposition 3 *The following strategy profile is a subgame perfect Nash equilibrium if and only if $B \in [Ng, (N-1)\bar{c}]$. The aid agency requests $s_i = 1$ for all i and extends complying parties a gift in the amount of g at the end of each period, and the kleptocrats choose $s_i = 1$ for all i, as long as no one deviates. In any subgame following a deviation, the agency and the $N-1$ non-deviants act as in Proposition 2b with $\tilde{n}$ now defined as $\frac{B}{(N-1)\bar{c}}$. The gift g is as follows:*

$$g = \begin{cases} \bar{c} + (1-\bar{c})^{\beta} - 1 & \text{if } \beta \geq \bar{c} \quad ; \\ \bar{c} + (1-\beta)^{\beta} \left(\frac{\beta}{\bar{c}}\right)^{\frac{\beta^2}{1-\beta}} - 1 & \text{otherwise} \quad . \end{cases} \tag{8}$$

The gift is chosen so that no kleptocrat has an incentive to deviate if the others do not, i.e. g is such that he is indifferent between complying today and not complying given that the future will unfold as in the benchmark. Formally, g solves:

$$\frac{1}{1-\beta} \ln[(1-\bar{c}) + g] = \max_{s_i \in [0,1]} \ln[(1 - s_i \bar{c})] + \beta V^b(1) \tag{9}$$

It can easily be shown, using the closed-form expression for the benchmark value function, that g must be as in equation (8). Both expressions in equation (8) are decreasing functions of β: the lower the discount factor, the lower the importance of the punishment in the kleptocrat's utility. Obviously, the agency needs a sufficient budget to expend such gifts. It must be that $Ng \leq B$ for this equilibrium to exist. This is captured by the lower bound of admissible budgets in the proposition.

However, we need to make sure that there exists an admissible budget, i.e. that the interval's upper bound is not lower than the lower bound. For the interval in Proposition 3 to exist, it must be that that $Ng \leq (N-1)\bar{c}$. In the case of $\beta \geq \bar{c}$, this is equivalent to finding conditions under which $1/N\bar{c} + (1-\bar{c})^{\beta} - 1 \leq 0$. With a little algebra, we can show that a sufficient condition is $\beta \geq 1/N$. In the case $\beta < \bar{c}$, it can be shown that $\beta \geq 1/2$ is sufficient. In infinite horizon games such as ours, conditions on the discount factor are quite typical to insure the existence of equilibria involving a threat of future punishment. For the kleptocrats to comply with the aid agency's plan, they must value the future enough to outweigh the benefits of not complying in the present.

It should be noted that in our proposed equilibrium, no kleptocrat finds it optimal to deviate from his strategy. The punishment will never be implemented, but the punishment phase is itself a Nash perfect equilibrium. Indeed, once the agency has bailed out a cheater, the game becomes an $(N-1)$-player replica of the food aid curse, in which each kleptocrat behaves according to Proposition 2b.

3 Discussion

What if the agency has a small budget, i.e. smaller than what is needed to extend the gift to all kleptocrats? Then the results above will hold in a subset of countries corresponding exactly to B/g. The others will be left in autarky. This result is somewhat uninteresting since the agency's whole budget will have to be used every period to bribe dictators not to starve their population.

The reader could argue that one of the policy implications of our analysis is for humanitarian aid agencies to self-destruct, at least when the discount factor is large. Clearly, such cannot be the case, however. Many famines have natural causes; our model does not encompass these. Yet all our qualitative results would carry through in a model in which the aid agency can determine whether the cause of the famine is natural or human. This is not a bad assumption, as aid agencies have become experts in the art of determining disaster causes and can easily draw the line between a natural disaster and one that is not so natural. See, for instance, USAID (2006) for an overview of its interventions that year.

Throughout the paper, we have been confronted with two scenarios with respect to the value of a kleptocrat's discount factor, β, relative to the survival consumption as a percentage of per capita output, $\bar{c}$. The case $\beta \geq \bar{c}$ is likely if the kleptocrats rule over relatively wealthy countries for which $\bar{c}$ is not a large fraction of y. The other case is more likely in extremely poor countries, for which output barely covers survival consumption. Both scenarios can be found simultaneously in different parts of the world. Rewriting the model to account for possible income inequality between possible aid recipients is feasible and would not affect the basic results of the paper. It would, however, make the exposition unnecessarily complex.

Our analysis suggests that one way to avoid man-made famines while solving the food aid curse is to bribe dictators on a regular basis, so to speak.[2] This highlights a new role for international aid, that of a famine-prevention mechanism. Doing so is cheaper in the end and more effective than acting non-strategically. Furthermore, it reduces the extent of kleptocracy. Of course, development aid is not typically meant to be a bribe. We do not claim that all of foreign aid needs to be used as such. Our paper simply makes the case that it may not be a bad thing if a fraction of foreign aid is explicitly used as a bribe to buy out incentives for future man-made famines.

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[2] Alesina & Dollar (2000) have already stressed that much of development aid is in fact allocated based on political considerations. Boone (1996) made it also quite explicit that a significant chunk of that aid ends up in government consumption rather than investment, a conclusion that can also be derived from Easterly (2003)'s review of the empirical evidence of the (lack of) impact of development aid on the growth of recipients' output.

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Appendix: Proof of Proposition 2b

Here we deal with the case where each country has a population n (initially equal to 1) and the agency, having a budget $B < Nn\bar{c}$, plays non-strategically, i.e. sends aid unconditionally wherever needed. We derive the dictator's optimal policy function $s_i^*(n_i)$ and value function $V^*(n_i)$.

We will look for a symmetric equilibrium to this problem. In equilibrium, therefore, each dictator chooses the same policy function $s_i^*(\cdot) = s^*(\cdot)$ and expects to receive B/N or whatever is needed to save his entire population from hunger, whichever is less. That is, he receives

$$a \equiv \bar{c} \cdot \min\{\tilde{n}, (1-s)n\} \quad . \tag{10}$$

where, we remind the reader, $\tilde{n} = B/(N\bar{c})$. Population evolves according to

$$n' = sn + \frac{a}{\bar{c}} = \min\{sn + \tilde{n}, n\} \quad . \tag{11}$$

Equations (10) and (11) have two immediate consequences. First, the population never falls below $\tilde{n}$, since that is how many people the agency can save even if the dictator sets $s = 0$. Second, $n' = n$ whenever $s \geq \bar{s}(n) \equiv 1 - (\tilde{n}/n)$; this means that it is never optimal for the dictator to set s above $\bar{s}(n)$. Thus

$$s^*(n) \in [0, \bar{s}(n)] \quad \text{for all } n. \tag{12}$$

The population level $\tilde{n}$ is particularly important. Since $\bar{s}(\tilde{n}) = 0$, the optimal policy function at this level must necessarily be $s^*(\tilde{n}) = 0$. This leads to $n' = \tilde{n}$, so the same solution must hold next period, and so on. The value function is therefore

$$V^*(\tilde{n}) = \ln(\tilde{n}) \ / \ (1-\beta) \quad . \tag{13}$$

So once a dictator finds it optimal to set $s = 0$, he will continue to do so forever, and the population will remain at $\tilde{n}$.

When $n > \tilde{n}$, however, the problem is less trivial. The value to be maximized is

$$V(n) = \max_s \ln[n(1 - s\bar{c})] + \beta V(n') \tag{14}$$

where n' is given by (11).

Claim 1. *If $\beta \geq \bar{c}$, then in equilibrium $s^*(n) = \bar{s}(n)$ for all $n > \tilde{n}$.*

Proof. The derivative of the right-hand side of (14) with respect to s is

$$\left(\frac{-\bar{c}}{1-s\bar{c}}\right) + \beta V'(n')\frac{\partial n'}{\partial s} \tag{15}$$

We submit that $s = \bar{s}(n)$ is the optimal policy function and that

$$\bar{V}(n) \equiv \left(\frac{1}{1-\beta}\right) \ln[n(1 - \bar{s}(n)\bar{c})] \tag{16}$$

is the associated value function. To verify this, use $V(n) = \bar{V}(n)$ in (15). The expression obtained is discontinuous at $s = \bar{s}(n)$, but negative for $s > \bar{s}(n)$ and non-negative for $s < \bar{s}(n)$ — in fact strictly positive for $s < \bar{s}(n)$ if β is strictly greater than $\bar{c}$. Moreover, $V(n) = \bar{V}(n)$ and $s = \bar{s}(n)$ together satisfy (14). □

That establishes the part of Proposition 2b dealing with the case where $\beta \geq \bar{c}$. The rest of this appendix assumes $\beta < \bar{c}$.

Claim 2. *In equilibrium $s^*(n) < \bar{s}(n)$ for all $n > \tilde{n}$.*

Proof. Suppose $\bar{s}(n)$ is optimal. Then $n' = n$ and the same situation recurs next period. The value function in this case is $\bar{V}$, defined in (16).

But consider what happens if instead the dictator sets $s(n) = \bar{s}(n) - \epsilon$ for one period [where $\epsilon < s(n)$] and then $s(n) = \bar{s}(n)$ thereafter. Then his value, which we denote by $\hat{V}$, is

$$\hat{V}(n, \epsilon) = \ln n + \ln[1 - \bar{c}\bar{s}(n) + \bar{c}\epsilon] + \beta\bar{V}(n') \tag{17}$$

where $n' = n\bar{s}(n) - n\epsilon + \tilde{n} = n(1 - \epsilon)$. It is straightforward to show that, for all n, $\hat{V}(n, \epsilon)$ approaches $\bar{V}(n)$ as ϵ goes to zero, and that $\hat{V}(n, \epsilon)$ is increasing in ϵ at $\epsilon = 0$. Consequently there exists $\epsilon > 0$ such that $\hat{V}(n, \epsilon) > \bar{V}(n)$ for all n, proving that $\bar{V}(n)$ is not optimal. □

As a result of this claim, $n' < n$ whenever $n > \tilde{n}$. The population declines, in other words. The question is, does it reach $n = \tilde{n}$ in finite time? The next claim shows that it does.

Claim 3. *Eventually $s^*(n) = 0$ becomes optimal.*

Proof. Suppose $s = 0$ is never optimal. Then we must have $0 < s^*(n) < \bar{s}(n)$ for all n. Therefore $s^*(n)$ must be the solution to

$$V(n) = \max_s \ln n + \ln(1 - s\bar{c}) + \beta V(n') \quad , \tag{18}$$

where $n' = n_s + \tilde{n}$ and s is unconstrained. But the solution to this problem is

$$V(n) = K + \left(\tfrac{1}{1-\beta}\right) \ln \left[n + \left(\frac{\tilde{n}\bar{c}}{1-\bar{c}} \right) \right] \quad ; \tag{19}$$

$$s(n) = \left(\tfrac{\beta}{\bar{c}}\right) - \left(\tfrac{1-\beta}{1-\bar{c}}\right)\left(\tfrac{\tilde{n}}{n}\right) \quad ; \tag{20}$$

where K is defined in the paragraph following Proposition 1. The policy function given by (20) generates the following population equation:

$$n' = \left(\tfrac{\beta}{\bar{c}}\right) n - \left(\tfrac{\bar{c}-\beta}{1-\bar{c}}\right) \tilde{n} \quad . \tag{21}$$

According to this, n must eventually fall below $\tilde{n}$, which can only happen if $s < 0$. This is impossible. The constraint $s \geq 0$ must therefore become binding at some point. □

The dictator will therefore set $s = 0$ at some point; and as we have seen, once he sets $s = 0$ he will continue to do so always. Let n_t denote the population at date t. Let T be the first period in which the dictator sets $s = 0$. Then we must have

$$V^*(n_T) = \ln n_T + \beta V^*(\tilde{n}) \quad . \tag{22}$$

We may work backwards from T. Note that T is the last period where $n_t > \tilde{n}$.

At $T - 1$ the solution satisfies $s > 0$. This must be an interior solution to

$$V(n_{T-1}) = \max_s \ln n_{T-1} + \ln(1 - s\bar{c}) + \beta V^*(n_T) \quad , \tag{23}$$

where $n_T = sn_{T-1} + \tilde{n}$. The optimal policy function is

$$s^*(n_{T-1}) = \frac{\beta n_{T-1} - \tilde{n}\bar{c}}{n_{T-1}\bar{c}(1-\beta)} \quad ; \tag{24}$$

and the corresponding value function is

$$V^*(n_{T-1}) = \quad (1+\beta)\ln(n_{i,T-1}+\tilde{n}\bar{c}) - (1+\beta)\ln(1+\beta)$$
$$+\beta\ln(\beta/\bar{c}) + \beta^2 V^*(\tilde{n}) \quad . \tag{25}$$

Substituting the policy function into the population equation, we see that $n_T > \tilde{n}$ implies $n_{T-1} > (\bar{c}/\beta)\tilde{n}$. This means that the results obtained for T, namely $s^*(n_T)$ and $V^*(n_T)$, are valid for $\tilde{n} < n_T \leq (\bar{c}/\beta)\tilde{n}$. Conversely, if $\tilde{n} < n_t \leq (\bar{c}/\beta)\tilde{n}$, then $t = T$ and the optimal action is $s = 0$.

This analysis may be repeated as many times as necessary. In general we have

$$s^*(n_{T-k}) = \frac{\sum_{j=1}^{k}[\beta^j n_{T-k} - \bar{c}^j\tilde{n}]}{\bar{c}n_{T-k}\sum_{j=0}^{k}\beta^j} \quad ; \tag{26}$$

$$V^*(n_{T-k}) = \quad \sum_{j=0}^{k}\beta^j\left[\ln\left(n_{T-k}+\tilde{n}\sum_{l=1}^{k}\bar{c}^l\right) - \ln\left(\sum_{l=0}^{k}\beta^l\right)\right]$$
$$+\sum_{j=1}^{k} j\beta^j\ln(\beta/\bar{c}) + \beta^{k+1}V^*(\tilde{n}) \quad . \tag{27}$$

The value of k, which is the number of periods until the dictator begins setting $s = 0$, depends on the current population. Specifically, $n_t = n_{T-k}$ if and only if $h(k) < n_t \leq h(k+1)$, where $h(k)$ is defined as follows:

$$h(k) \equiv \left[\left(\frac{c^k\sum_{j=0}^{k-1}\beta^j}{\beta^k}\right) - \sum_{j=1}^{k-1}\bar{c}^j\right]\tilde{n} \quad . \tag{28}$$

Consider the following example. Let $\bar{c} = 2/3$, $\beta = 1/2$, and $^{\text{B}}/_{\text{N}} = 1/4$. Then one can calculate $h(1) = 1/2$, $h(2) = 3/4$, $h(3) = 41/36$, and so on. Initially $n_1 = 1$, which falls between $h(2)$ and $h(3)$. Thus the dictator is two periods away from setting $s = 0$. In the first period (i.e. $T-2$) he sets $s = 2/7$, which leads to $n_2 = 37/56$. Naturally this falls between $h(1)$ and $h(2)$. The dictator then sets $s = 9/74$, which leads to $n_3 = 51/112$. This is less than $h(1)$, and so the dictator sets $s = 0$. He does this in every subsequent period. From period 4 onward the population is $\tilde{n} = 3/8$.