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Search Frictions in Physical Capital Markets as a Propagation Mechanism

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Abstract:
We build a Dynamic General Equilibrium model with search frictions for the allocation of physical capital and investigate its implications for the business cycle. While the model is in principle capable of generating substantial internal propagation to small exogenous shocks, the quantitative effects are modest once we calibrate the model to fit firm-level capital flows. We then extend the model with credit market frictions that lead to countercyclical default and countercyclical risk premia as in the data. Countercyclical default directly affects capital reallocation and has important general equilibrium income effects on labor supply. Yet, for calibrations in line with observed consumption dynamics, we find that even in this extended model, search frictions in physical capital markets play only a small role for business cycle fluctuations.

Keywords: Capital allocation frictions, Search and matching, Credit frictions, Business cycles, Dynamic general equilibrium

JEL Classification: E22, E32, E44

1 Introduction

Physical capital is often specific to a certain task and/or fixed to a particular location. These specificities imply that physical capital markets are subject to potentially important allocation frictions. Most of the modern macro literature has ignored these market imperfections and examined instead the effects of aggregate investment constraints such as time-to-build delays (e.g. Kydland and Prescott, 1982) or convex adjustment costs (e.g. Cogley and Nason, 1995). The general conclusion from this literature is that in general equilibrium, such aggregate investment constraints have relatively small business cycle effects on their own. In this paper, we investigate whether the same holds true for market imperfections. In particular, we introduce search frictions for the allocation of physical capital into an otherwise standard real business cycle (RBC) model and ask whether these imperfections help generate more amplified and persistent responses to small exogenous shocks.

Our investigation is motivated by empirical evidence from industry- and firm-level data, discussed in detail in Section 2, that lead to three stylized observations. First, depending on the degree of specificity, a substantial amount of physical capital remains unmatched in any given period. Second, congestion in the physical capital market is countercyclical from the point of view of the supplier; i.e. the probability of (re-)allocating a given unit of capital to a firm increases in business cycle upturns and inversely decreases in downturns. Third, the distribution of investment rates across individual firms is wide, even in narrowly defined sectors and independent of aggregate conditions. The three observations suggest that physical capital markets are characterized by similar frictions than labor markets and thus, our modelization draws on the now widely employed search approach for the labor market, pioneered by Blanchard and Diamond (1990) and Mortensen and Pissarides (1994), and introduced into the DGE context by Merz (1995), Andolfatto (1996) and Den Haan, Ramey and Watson (2000).

The model we develop in Section 3 is populated by representative households and firms. Firms must post projects at a cost to search for available physical capital that is supplied endogenously by households.[1] The probability of a match varies with the state of the economy

[1] As opposed to most labor search models where the supply of available workers is fixed, we endogenize the supply of available capital for the model to be consistent with balanced growth properties of aggregate capital stocks.

and depends on the ratio of available capital to the total number of posted projects. Once matched, households keep lending their capital to the same firm until separation, which is assumed to occur with exogenous probability in the baseline model. Once separated, the capital returns to the household for reallocation.

Under relatively weak conditions, the proposed search environment implies countercyclical congestion in physical capital markets, as in the data. This mechanism has potentially important aggregate consequences. In the wake of a positive technology shock, for example, the decrease in allocation frictions together with the presence of readily available unmatched capital means that the reaction of *productive* matched capital stocks and indirectly labor demand is more important than in the RBC benchmark. This effect continues over several periods after the shock and may lead to more amplified and persistent output dynamics.

To assess the quantitative importance of the search friction, Section 4 calibrates the model to fit long-run averages of firm-level capital flows using Compustat data and compares its business cycle characteristics with the ones of the RBC benchmark. The main result is that capital flows in and out of production are not important enough for search frictions to have a significant impact. Only when we increase separation and reallocation to counterfactually large flows does the model generate more amplified and persistent output dynamics.

Based on this result, Section 5 extends the baseline model with credit market frictions. Following Townsend (1979), firms are subject to idiosyncratic productivity shocks that occur after all optimal decisions are taken and that households (the lenders) can observe only after incurring a monitoring cost. This costly state verification problem implies an optimal debt contract that results in endogenous capital separation through default. In particular, households monitor all loss-making firms and sever the lending relationship with those whose productivity level is below some threshold that makes refinancing more expensive than reallocating the capital to another firm.

The extension is motivated by the observation that different measures of financial distress and related capital sales / liquidations are countercyclical. Similar to Den Haan, Ramey and Watson's (2000) argument that countercyclical job destruction generates substantial internal propagation in labor search models, countercyclical capital separations in our model may magnify and prolong the effects of exogenous shocks as more (less) capital gets separated in downturns (upturns) and needs to go through a time-consuming reallocation process.[2] As an interesting by-product, the extended model also allows us to assess the importance of taking into account costly capital reallocation when quantifying the business cycle effects of credit frictions. In fact, existing DGE models with costly state verification such as Carlstrom and Fuerst (1997) or Bernanke, Gertler and Gilchrist (1999) only investigate the effects of net worth on investment and output but ignore the reallocation of capital from bankrupt firms. With the exception of a few special cases, these net worth effects alone have relatively small consequences for business cycle fluctuations. It is therefore interesting to see how the addition of costly capital reallocation changes this result.[3]

As the quantitative analysis reveals, the extended model indeed generates countercyclical capital separations as well as countercyclical risk premia, in line with the data. This latter result constitutes an improvement over the credit friction models of Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999) where risk premia are either procyclical or acyclical.[4] The extended model also implies more volatile and persistent output fluctuations. Closer inspection reveals, however, that the increased internal propagation is mostly a general equilibrium effect brought about by a smaller (or even inverse) reaction of household consumption and thus labor supply to exogenous shocks. Once we calibrate the model such as to match the consumption dynamics in the data, the extended model implies only modest amplification and persistence. The conclusion of the paper thus remains that capital separation and reallocation flows on their own are too small for search frictions in physical capital markets to play an important role for business cycle fluctuations.

The results of our paper mostly concur with existing studies on the business cycle effects of physical capital specificities. Ramey and Shapiro (1998), for example, examine the aggregate effects of large military spending shocks in a world where moving capital from one sector to

[2] By contrast to Den Haan, Ramey and Watson (2000) where job destruction is an efficient outcome, capital separations in our model are the consequence of an information friction and thus socially inefficient. As we discuss in Section 5, this assumption is based on firm-level evidence indicating that capital separations due to (presumably efficient) sales and mergers are mildly procyclical rather than countercyclical.

[3] Section 5 provides more details about the business cycle effects of the net worth channel of credit frictions.

[4] As we discuss in Section 5, the countercyclical risk premium is a direct consequence of the time-varying costs of incomplete contracting in a world with ex-post factor specificity that Willamson (1979) or more recently Caballero and Hammour (1996) term the *fundamental transformation problem.*

another is subject to a time-delay and a fixed cost. For certain specifications, they report some output amplification effects. However, these effects are based on unusually important sectoral shifts and the model is not analyzed in a full-blown DGE context. Boldrin, Christiano and Fisher (2001), in turn, consider a model with habit persistence and one-period inflexibilities for both labor and capital. While their focus is mostly on asset pricing implications, their model is capable of generating substantial persistence in output growth. However, this result seems to be due mostly to the imposed adjustment delay on hours worked. Finally, Veracierto (2002) examines the effects of investment irreversibilities and concludes that they do not matter for the business cycle.[5] The main contribution of our paper compared to these studies is that we focus more squarely on the *time-varying* nature of the market imperfections involved in the allocation of physical capital. First, we document that congestion in the physical capital market is countercyclical. Second, we introduce search frictions to capture the state-dependent nature of this congestion and show that it has interesting consequences in general equilibrium, mostly through its indirect effect on labor supply.[6] Third, we are, to our knowledge, the first to explicitly calibrate a DGE model to gross capital flows from firm-level data. The relative unimportance of these capital flows (compared to, say, labor flows) is the main reason for our conclusion that search frictions in physical capital markets play only a modest role for business cycle fluctuations.

2 Empirical evidence

To motivate our extension of the RBC benchmark, this section first provides evidence on the time-varying nature of market imperfections in the allocation and reallocation of physical capital. Second, we review empirical studies on the wide distribution of investment rates across firms.

[5] A recent literature examines the role of nonconvexities in plant-level adjustment costs for aggregate investment dynamics, which can be considered as a combination of costs to both allocation of new capial and reallocation of used capital. See for example Kahn and Thomas (2006a) and the references therein. As in Veracierto (2002), these costs are found to have only small general equilibrium effects.

[6] Related to our model, Den Haan, Ramey, and Watson (2003) and Wasmer and Weil (2004) propose search frictions for the allocation of financing from lenders to firms. While relevant for new entrepreneurs and small firms, such frictions seem less obvious for large firms that account for the bulk of capital accumulation in the economy. Furthermore, their analysis is not carried out in a full-blown quantitative DGE context.

2.1 Allocation frictions for physical capital

Most physical capital is specific to a certain task and/or fixed to a particular location. The market imperfections brought about by these specificities are likely to imply substantial costs for the allocation and reallocation of physical capital. Similar to the labor market, one can think of these costs as search frictions that depend on the degree of specificity and potentially vary with business conditions. Unlike for the labor market where we observe aggregate unemployment and job advertisement rates, however, there is no comprehensive direct evidence on "unemployed" capital or unfilled investment projects.[7] Nevertheless, a substantial amount of indirect evidence exists that allows at least a partial characterization of the frictions involved.

We start by considering the market for leased non-residential property, which is one of the capital types most comparable to labor in the sense that similar to unemployment, vacant space is directly observable. Figure 1 shows the evolution of the average U.S. vacancy rate for industrial and office space in competitively leased multi-tenant buildings between 1988 and 2006. We obtained these data series from Torto Wheaton Richard Ellis, a large commercial real estate firm that surveys all major U.S. property markets on a quarterly basis.

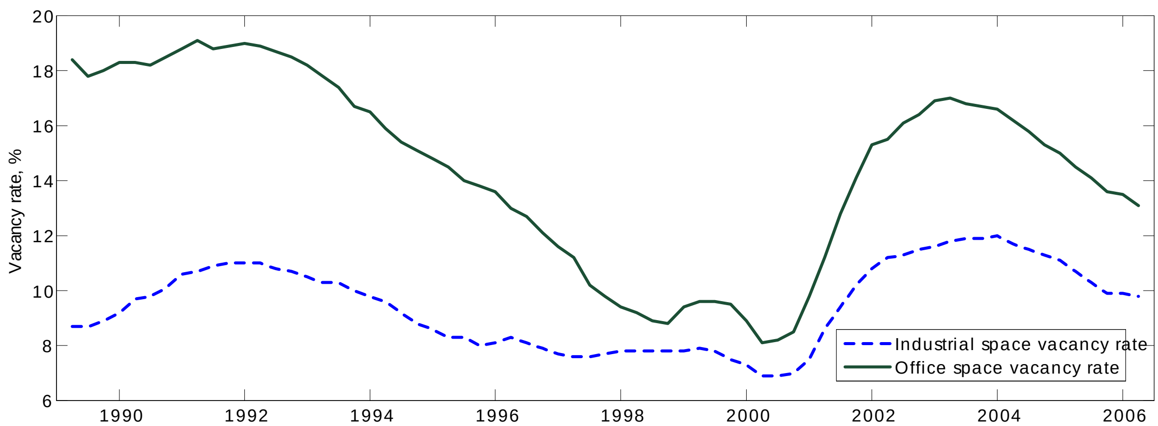

Figure 1: Vacancy rate for multi-tenant industrial and office space; average over 56 metropolitan U.S. markets. Source: Torto Wheaton Richard Ellis.

Vacancies were at a record high at the end of the 1990-1992 recession, with the rate for office space approaching 20%. Vacancies then gradually decreased over the rest of the 1990s before

[7] See Davis, Faberman and Haltiwanger (2006) for a recent survey of the relevant data for labor markets.

jumping up again at the onset of the 2001 recession. On average, these vacancy rates are substantial (9.5% for industrial space and 14.5% for office space) and their time-varying nature suggests that congestion in the non-residential property market (from the point of view of the proprietor) varies inversely with the business cycle.[8]

Industrial and office space is, of course, a very specific type of capital because it is bound to a particular location and can hardly be converted for alternative usage. On the other end of the spectrum are newly finished, relatively mobile capital goods. Here, the BEA's Survey of Current Business (2000) allows us to observe detailed time series on inventories and output from capital goods producing industries. Using this information, we can compute the hazard rate q_{it} with which a new unit of capital good i is allocated as follows:

$$v_{it} = (1 - q_{it})(v_{it-1} + y_{it}),$$

with v_{it} and y_{it} denoting end-of-period t inventories and output during period t of capital good i, respectively. Table 1 reports the results for three large categories of finished capital goods over the sample 1977 to 1999.

Table 1: Allocation rates of finished capital goods

	Average q	$corr(q_t, gdp_t)$
Industrial machinery and equipment	0.70	0.40
Motor vehicles and equipment	0.83	0.16
Electronic and other electric equipment	0.90	0.16
Average	0.86	0.36

Notes: Second moments relate to Hodrick-Prescott filtered data

As expected, the allocation rate for these capital goods is closer to unity (no friction) as production can be adjusted to accommodate demand and none of the capital types is bound to a specific location. Nevertheless, it is interesting to observe that industrial machinery – presumably a more specific capital good – takes on average longer to be allocated (i.e. a lower

[8]Unfortunately, Torto Wheaton does not provide information on newly vacated space and, to our knowledge, none of the U.S. statistical agencies provides comparable data on the non-residential property market. Hence, we cannot compute hazard rates for the transition out of vacancy as it possible for the labor market where we have separate time series on newly unemployed individuals (e.g. Shimer, 2005).

q) and congestion in that market reacts more inversely with the business cycle (i.e. the allocation rate q is more procyclical).[9]

Aside from these direct measures, there is a host of indirect evidence about the importance and the countercyclical nature of the frictions in physical capital markets, especially what the reallocation of used capital is concerned. Eisfeldt and Rampini (2006), for example, use Compustat data to show that reallocation of used capital (measured as sales of plant, property and equipment plus acquisitions as a fraction of gross investment) is highly procyclical, with a Hodrick-Prescott filtered correlation coefficient with GDP of 0.64 for the sample 1971-2004.[10] By contrast, different measures of the benefits from reallocation (dispersion in firm level Tobin's Q, firm level investment rates, total factor productivity growth rates, and capacity utilization) are all countercyclical. If there were no reallocation frictions or if the degree of congestion in the used capital market was constant, we would expect most reallocations to take place when the benefits are greatest. Yet, exactly the opposite is the case.

Another piece of indirect evidence about reallocation frictions for used capital comes from a case study by Ramey and Shapiro (2001) who measure the resale value of equipment after the closure of three aeronautical plants. They find that other aerospace companies are overrepresented among buyers, and that even after taking into account age-related depreciation, the average resale value of equipment is only 28% of the replacement cost.[11] Although some of these losses may be due to unaccounted obsoleteness, Ramey and Shapiro's results suggest that the frictions involved in the reallocation of used capital are substantial. Otherwise, the used capital would not sell at such a large discount below its productive value.

[9] The traditional explanation for the existence of inventories relies on the assumption that production is costly to adjust. As a result, firms use inventories to smooth production when faced with fluctuating sales (e.g. Blinder and Maccini, 1991). An alternative explanation relies on the existence of fixed delivery costs, inciting firms to hold inventory stocks. Firms thus make adjustments only when stocks are sufficiently far from their target (e.g. Kahn and Thomas, 2006b). Our argument of congestion differs from these explanations in the sense that we interpret the variation in hazard rates for inventory exit across goods as evidence of different degrees of market imperfections.

[10] Compustat collects a wide range of data, including information on physical capital, for all publicly traded firms in the U.S. We discuss this dataset in more detail in the calibration part of Section 4.

[11] Even for machine tools, which typically have a better resale value than specialized aerospace equipment, the resale value is only about 40% relative to the replacement cost.

Besides market imperfections in general, the specificity embodied in most physical capital can lead to an additional equilibrium effect that Shleifer and Vishny (1992) call *asset illiquidity* and that may explain part of the surprisingly low resale prices reported in Ramey and Shapiro's case study. Shleifer and Vishny argue that when firms sell assets or liquidate to meet financial constraints, the specific nature of capital means that the buyers who value these assets most are likely to be firms in the same industry. But financial distress often affects industries as a whole, which means that these buyers are likely to be financially constrained as well. As a result, the assets are sold at a steep discount within the same industry or to less constrained industry outsiders who have a lower valuation because the characteristics of the sold asset are suboptimal for their line of business or because they cannot value the asset appropriately.[12] Pulvino (1998) provides evidence about the countercyclical nature of asset illiquidity from the used aircraft market. Based on U.S. data of commercial aircraft transactions, Pulvino finds that financially constrained airlines sell aircrafts at a 14% discount to the average market price, but that these discounts exist only in times when the airline industry is depressed and not when it is booming. Furthermore, aircraft leasing institutions pay a discount of 30% during industry recessions because they themselves value aircrafts much lower than the actual airlines and because the risk associated with finding another lessee during recessions is much higher than in upturns.

A final, more aggregate piece of evidence about the frictions involved in the reallocation of physical capital comes from Becker et al. (2005) who use data from the Annual Capital Expenditure Survey (ACES). In existence since 1993, ACES is a representative dataset of U.S. firms that can be used to compute the capital stock of firms that disappear, either because they cease to be active or because they continue to operate under a different firm. The resulting series of total separated capital can then be compared with the following year's series of aggregate used capital expenditures. For the period 1993-1999, the resulting ratio of separated capital to used expenditures equals on average 64%, suggesting that reallocation frictions are substantial.[13]

[12] Ramey and Shapiro (2001) advance a telling example about a wind tunnel that was constructed to test aeronautical parts at high air speeds and that was leased out afterwards to test bycicle helmet designs.

[13] As other datasets on capital expenditures, ACES comes with several caveats. See Becker et al. (2005) for a detailed discussion. Also, the 64% absorption rate could be biased either upwards or downwards. On the one hand, expenditures in used capital include assets sold by continuing firms, which makes the effective absorption

In sum, the evidence presented here leads us to the following two stylized characterizations of physical capital markets. First, allocation frictions for physical capital can be sizable depending on the degree of specificity of the capital good and whether it is new investment or a reallocation to another firm. Second, congestion in the physical capital market varies inversely with the business cycle; i.e it is more costly and time-consuming to (re-)allocate physical capital to a firm in business cycle downturns than it is in upturns.

2.2 Distribution of investment rates across firms

Further evidence suggesting that the allocation of physical capital is probabilistic in nature comes from the well-documented wide distribution of investment rates across firms. Studies by Caballero, Engel and Haltiwanger (1995), Doms and Dunne (1998), Cooper, Haltiwanger and Power (1999) or Cooper and Haltiwanger (2005) show that investment at the plant level is characterized by a wide distribution. At any given point in time, there is a substantial mass of establishments with zero investment that coexists with establishments that have investment rates above 20% of their capital stock (i.e. investment spikes).[14]

Most of the literature has interpreted this large distribution of investment rates across establishments as the result of plant-specific productivity and non-convex adjustment costs that lead to (S,s) type investment rules (e.g. Khan and Thomas, 2006a and references therein). While this approach is certainly capable of rationalizing the observed data, the wide distribution of investment rates – even in narrowly defined sectors – affords another, potentially complementary explanation; one that focuses on *market imperfections* in the allocation of physical capital. In fact, there is plenty of circumstantial evidence suggesting that in expansionary periods, firms face sometimes substantial difficulties in securing a reliable supplier of capital goods.[15]

rate for separated capital from firm death even lower. On the other hand, some of the separated capital may be exported abroad in which case the effective absorption rate is higher.

[14] Becker et al. (2005) reconfirm these findings in their summary using plant-level data from the Annual Survey of Manufacturers (ASM).

[15] Interestingly, Statistics Canada collects information on intended capital purchases in one of their firm-level surveys that could be compared over time to actual expenditures. Unfortunately, this information is not publicly available at the moment.

3 Model

As in the frictionless RBC benchmark, our model is populated by two agents: firms that produce using capital and labor; and households who decide on optimal consumption, leisure and investments in productive capital. But instead of instantaneous allocation, the matching of capital from households with firms involves a costly and time-consuming search process. This search process is in principle very similar to the standard labor search environment (e.g. Andolfatto, 1996), with the exception that we endogenize the supply of available capital. This complication is necessary because depreciated capital needs to be replaced and, more importantly, because we want our model to be consistent with the stylized fact that output and capital grow on average at the same rate.

At the same time, our model retains a number of other simplifications that facilitate comparison with the RBC benchmark. First, there is no distinct sector for capital allocation. Instead, households directly act as capital lenders. Second, the same matching friction applies to the allocation of both new and used (i.e. previously separated) capital. This renders the analysis considerably easier as we do not need to keep track of different types of capital. Third, production is constant-returns-to-scale. Firms therefore choose the same optimal capital-labor ratio, independent of firm size, which allows us to abstract from firm heterogeneity.

3.1 Search and matching in the capital market

Capital is either in a productive state or in a liquid state. We define by K_t the capital stock that enters the production function of a representative firm in period t. Liquid capital L_t, in turn, is made up of two components: used capital that has been separated previously from other firms and new capital made available by households.

To undertake investments, firms must post projects and search for liquid capital at cost κ per project. We denote by V_t the total number of posted projects in period t. Total capital additions to production in period t is the result of a matching process $m(L_t, V_t)$, with $\partial m(\cdot)/\partial L_t > 0$ and $\partial m(\cdot)/\partial V_t > 0$. A firm's probability to find capital is therefore given by $p(\theta_t) = \frac{m(V_t, L_t)}{V_t}$ with $\partial p(\theta_t)/\partial \theta_t > 0$, where $\theta_t = \frac{L_t}{V_t}$ is a measure of congestion in the physical capital market from the household's point of view (i.e. the capital supplier). Likewise, the probability of liquid capital being matched to a firm equals $q(\theta_t) = \frac{m(V_t,L_t)}{L_t}$ with $\partial q(\theta_t)/\partial\theta_t < 0$.[16] Firms and households are assumed to be sufficiently small to take $p(\theta_t)$ and $q(\theta_t)$ as exogenous.

Capital matched to a firm in period $t-1$ enters production in period t. This match between firm and capital continues into period $t+1$ with probability $(1-s)$ and so on for the periods thereafter. Hence, the evolution of the capital stock is described by[17]

$$K_{t+1} = (1-\delta)(1-s)K_t + m(L_t, V_t). \tag{1}$$

With probability s, the match is terminated, in which case a fraction φ net of depreciation δ of the capital is returned to the household; i.e. the household receives $\varphi(1-\delta)sK_t$. The remainder $(1-\varphi)(1-\delta)sK_t$ is a deadweight loss incurred during the separation process. Note that in this baseline formulation of our model, we keep the separation rate s exogenous. In Section 5 below, we introduce credit market frictions to endogenize the separation rate.

3.2 Firms and households

At the beginning of each period, firms and households observe exogenous aggregate technology X_t. Given the existing capital stock K_t, the representative firm then posts new projects V_t at unit cost κ and hires labor N_t to produce output Y_t with constant-returns-to-scale technology

$$Y_t = f(X_t N_t, K_t), \tag{2}$$

with f_N, $f_K > 0$ and f_{NN}, $f_{KK} < 0$. The resulting profit maximization problem is described by

$$\begin{aligned} J(K_t) &= \max_{N_t,V_t}\left\{f(X_tN_t,K_t) - \rho_tK_t - W_tN_t - \kappa V_t + \beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}J(K_{t+1})\right\} \\ \text{s.t. } K_{t+1} &= (1-\delta)(1-s)K_t + p(\theta_t)V_t, \end{aligned}$$

where ρ_t is the rental rate of capital; W_t is the wage per unit of labor; and $\beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}$ is the discount factor of future cash flows. This discount factor is a function of the marginal utility of consumption Λ because the firm transfers all profits to the households. The firm takes W_t and ρ_t as exogenous. The exogeneity of W_t is a direct consequence of our assumption of competitive

[16] In addition, to ensure that $p(\theta_t)$ and $q(\theta_t)$ are between 0 and 1, we require that $m(L_t, V_t) \leq min[L_t, V_t]$

[17] Since firm size is indeterminate, the separation rate s describes either the probability that a firm disappears in a given period or the fraction of capital that gets separated from a given firm (aside from depreciation). In either case, the evolution of the aggregate capital stock is described by (1).

labor markets. The exogeneity of ρ_t, in turn, implies that firms do not internalize the effects of their capital stock on the marginal productivity of capital and thus on the negotiation of ρ_t (see below).

The first-order conditions of the optimization problem are

$$(N_t) : f_N(X_t N_t, K_t) = w_t \tag{3}$$

$$(V_t) : \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J_K(K_{t+1}) = \frac{\kappa}{p(\theta_t)}. \tag{4}$$

Equation (3) is the standard labor demand. Equation (4) states that the expected discounted marginal value of an additional unit of matched capital has to equal its average cost $\kappa/p(\theta_t)$, with the marginal value of an additional matched unit of capital $J_K(K)$ being defined as

$$J_K(K_t) = f_K(X_t N_t, K_t) - \rho_t + (1-\delta)(1-s)\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J_K(K_{t+1}). \tag{5}$$

This equation states that the value to the firm of an additional unit of capital is worth today's marginal product of capital net of the rental rate plus the expected future value net of depreciation in case the match is continued.

Households maximize the expected discounted flow of utility $u(C_t, 1-N_t)$ over consumption C_t, leisure $1-N_t$ and the amount of liquid capital L_t destined for matching with firms. Time spent working yields revenue $W_t N_t$, capital matched last period yields revenue $\rho_t K_t$, while unmatched capital is carried into the present period with zero net return. Formally, this problem is described by

$$\begin{aligned} V(U_t, K_t) = \max_{C_t, N_t, L_t} & \left[u(C_t, 1-N_t) + \beta E_t V(U_{t+1}, K_{t+1}) \right] \\ & + \Lambda_t [W_t N_t + \rho_t K_t + \varphi(1-\delta) s K_t + U_t + D_t - C_t - L_t] \end{aligned}$$

$$\text{s.t. } K_{t+1} = (1-\delta)(1-s)K_t + q(\theta_t)L_t$$

where $U_t = (1-q(\theta_{t-1}))L_{t-1}$ is the quantity of unmatched capital in the beginning of t; D_t are firm profits transferred to households, and $\varphi(1-\delta)sK_t$ is the amount of separated capital returned into the budget constraint. Similar to the firm's optimization problem, we assume that the household considers W_t and ρ_t as exogenous.

The first-order conditions of the optimization problem are

$$(C_t) : u_C(C_t, 1 - N_t) = \Lambda_t \tag{6}$$

$$(N_t) : u_N(C_t, 1 - N_t) = \Lambda_t W_t \tag{7}$$

$$(L_t) : \beta E_t[V_U(U_{t+1}, K_{t+1})(1 - q(\theta_t)) + V_K(U_{t+1}, K_{t+1})q(\theta_t)] = \Lambda_t \tag{8}$$

The first two conditions are standard. The third condition states that the discounted expected utility of a marginal unit of liquid capital L_t must equal the marginal utility of an additional unit of consumption. With probability $(1-q(\theta_t))$ liquid capital remains unmatched and is worth $V_U(U_{t+1}, K_{t+1})$ to the household, while with probability $q(\theta_t)$ it is matched with a project and turned into productive capital with marginal value $V_K(U_{t+1}, K_{t+1})$. From the above Bellman equation, we can derive these marginal values as

$$V_U(U_t, K_t) = \Lambda_t \tag{9}$$

$$V_K(U_t, K_t) = \Lambda_t \left[\rho_t + \varphi(1 - \delta)s\right] + (1 - \delta)(1 - s)\beta E_t V_K(U_{t+1}, K_{t+1}). \tag{10}$$

3.3 Rental rate of capital and equilibrium

To close the model, we follow much of the labor search literature and assume that once matched, households and firms determine the rental rate of capital by Nash bargaining over the surplus of the match. The relevant surplus is the sum of marginal benefits to each party: $S_t = J_K(K_t) + \frac{V_k(U_t,K_t)-V_U(U_t,K_t)}{\Lambda_t}$. Defining η as the household's relative bargaining power, the household thus receives $\frac{V_k(U_t,K_t)-V_U(U_t,K_t)}{\Lambda_t} = \eta S_t$, while the firm's share is $J_K(K_t) = (1 - \eta)S_t$. After some algebraic manipulations that are detailed in the appendix, we obtain the following expression for the rental rate[18]

$$\rho_t = \eta \left[f_K(X_t N_t, K_t) + (1 - \delta)(1 - s)\frac{\kappa}{\theta_t} \right] + (1 - \eta)[\delta + (1 - \varphi)(1 - \delta)s]. \tag{11}$$

The first term in brackets is the maximum amount the firm is willing to pay per unit of capital. It equals the marginal product of capital plus the average cost that is saved by entering the proposed capital match rather than continuing to search. The second term in brackets is the household's opportunity cost of entering the proposed capital match, which equals the fraction

[18] The appendix is available on the authors' website (http://www.er.uqam.ca/nobel/r16374)

not lost to depreciation when capital remains liquid, δ, plus the deadweight loss in case the capital gets separated $(1-\varphi)(1-\delta)s$.

As mentioned before, the constant-returns-to-scale assumption for technology implies that all firms choose the same optimality conditions. The equilibrium of the economy is thus defined by the system of equations (1)-(11) and the definition of aggregate dividends $D_t = Y_t - W_t N_t - \rho_t K_t - \kappa V_t$ (see appendix for details). Dividends are positive because the search friction gives rise to a surplus for each unit of matched capital that the firm and household split as specified above.

3.4 Comparison with the RBC benchmark and qualitative considerations

In the following analysis, it will be useful to compare our capital search model with the RBC benchmark where capital can be allocated costlessely and instantaneously (see for example King and Rebelo, 2000). In fact, our model collapses to the RBC benchmark for the case where the cost of project postings κ and the deadweight loss from separations $1-\varphi$ are both zero. Firms then post an infinity of projects and all capital is reallocated in the beginning of each period; i.e. $s=1$, $q(\theta_t)=1$ and $U_t=0$. Under these assumptions, it can be shown that the repayment on liquidity equals the marginal product of capital: $\rho_t = f_K(X_t N_t, K_t)$.[19] Furthermore, choosing liquid capital L_t amounts to directly choosing the new stock of capital K_{t+1}. This implies a value of matched liquidity $V_K(U_t, K_t) = \Lambda_t[\rho_t + (1-\delta)]$, and the optimality condition for the choice of liquidity (i.e. new investment) reduces to the standard Euler equation: $\beta E_t \Lambda_{t+1}[\rho_{t+1} + (1-\delta)] = \Lambda_t$. Finally, by combining the household's budget constraint with the firm's first-order conditions and the capital accumulation equation, we recover the familiar national accounting identity of the RBC benchmark $Y_t = C_t + K_{t+1} - (1-\delta)K_t$.

The national accounting identity of our capital search model is quite different. Specifically, the household's budget constraint together with the definition of dividends yields

$$Y_t = C_t + [L_t + \kappa V_t] - [\varphi(1-\delta)sK_t + U_t]. \tag{12}$$

The first term in brackets on the right-hand side represents the total resources devoted to *gross investment* by households and firms. The second term in brackets denotes *idle capital* in the

[19] The bargaining power η is irrelevant in this case because perfect competitition in the capital market draws the surplus between firms and lenders to zero.

form of newly separated capital and unmatched capital from the previous period. The difference between the two quantities defines *net new investment*. Idle capital thus drives a wedge in the economy's resource constraint that increases the amount effectively made available to firms without affecting consumption. Akin to unemployment in models with labor market frictions, the presence of these additional resources may magnify and prolong the economy's reaction to shocks.

The second potential source of internal propagation in our model is the state-dependent nature of the search friction. In response to a persistent increase in aggregate productivity X_t, the marginal value of future matched capital increases. By virtue of conditions (4) and (8), firms and households thus find it optimal to increase V_t and L_t, respectively. Which of the two responses is larger depends on the exact specification of the model and thus, we cannot say in general whether congestion in the physical capital market is procyclical or countercyclical. However, by combining (4) and (8) with the definition of the division of the surplus, we can show that the following proposition holds.

Proposition 1 *Congestion in the physical capital market – defined as the ratio of liquidity to project postings $\theta_t \equiv L_t/V_t$ – is increasing in the expected growth rate of the marginal utility of consumption.*

Proof: see appendix. □

Under relatively weak conditions, this proposition implies that congestion is countercyclical, as evidenced in the data. For example, if preferences are additive and concave in consumption, θ_t is inversely related to consumption growth. Since consumption reacts gradually to persistent changes in aggregate productivity (e.g. Fig. 10 in King and Rebelo, 2000), congestion decreases in business cycle upturns and inversely increases in downturns. This countercyclical behavior of congestion has two effects. First, capital stocks react proportionally more after impact than if no search frictions were present. Second, the decrease in congestion implies that household's devote a relatively smaller share of their resources to liquid capital and consume relatively more. As a result, the income effect on labor supply is larger and depresses the response of equilibrium hours on impact. But because the subsequent shift in labor demand is larger (as capital stocks accumulate faster), equilibrium hours may respond more in the periods after the shock. These effects together have the potential to generate amplified yet hump-shaped (i.e. persistent) responses of hours and output to technology shocks.

4 Quantitative evaluation

We explore the quantitative implications of search frictions in the allocation of capital by comparing the business cycle performance of our capital matching model to the RBC benchmark in terms of impulse response functions (IRFs) and unconditional second moments.

4.1 Shocks and functional forms

Following much of the RBC literature, we assume that the exogenous labor-augmenting shock X_t has both a deterministic trend part $\bar{X}_t$ and a stochastic transitory part A_t. In particular $X_t \equiv A_t^{1/(1-\alpha)}\bar{X}_t$. The deterministic trend part evolves according to $\bar{X}_t = g\bar{X}_{t-1}$, with $g > 1$, and the stochastic transitory part evolves according to

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A,$$

with $\varepsilon_t^A \sim (0, \sigma_A^2)$.[20]

For household preferences, we follow King and Rebelo's (2000) baseline specification and define the family's period utility as $u(C, 1-N) = \log C + \frac{\omega}{1-\xi}(1-N)^{1-\xi}$. For production, we assume a Cobb-Douglas function with constant returns to scale of the form $f(XN, K) = A(\bar{X}N)^{1-\alpha}K^{\alpha}$ with $0 < \alpha < 1$. Finally, we follow much of the labor search literature and specify the matching technology as a Cobb-Douglas $m(V, L) = \chi V^{\epsilon} L^{1-\epsilon}$ with $0 < \epsilon < 1$. This constant returns to scale assumption implies that $p(\theta_t) = \theta_t q(\theta_t)$, which turns out to simplify the steady state computations in our model.

[20] The assumption of a deterministic trend in labor productivity implies that we need to normalize all aggregates by $\bar{X}_t$ so as to obtain a stationary system that we then simulate using the log-linear rational expectations solution algorithm of King and Watson (1998). We thank Bob King for providing us with the relevant Matlab code. Alternatively, we could have specified a stochastic technology shock that is difference stationary. Our results are robust to such an alternative specification of the shock process.

4.2 Calibration

We calibrate our model to U.S. quarterly data. For the parameters that are common with the RBC benchmark, we use calibrations that are standard in the literature (e.g. King and Rebelo, 2000). We set $g = 1.004$ and $\beta = 0.992$ so as to match an annual mean trend growth rate of 1.6% and an average annual real yield on a riskless 3-month treasury bill of 4.95%. For the labor supply, we fix the parameter ω such that the average fraction of hours worked equals $n = 0.2$. Together with $\xi = 4$, this results in a Frisch elasticity of labor supply of 1. Furthermore, we set the share of capital in the production function to $\alpha = 1/3$, and the rate of depreciation of capital to $\delta = 0.025$. Finally, to calibrate the exogenous driving process for the temporary technology shock, we extract a Solow residual from the data and then subtract a linear trend with average growth rate g. Estimation of the above specified AR(1) process with this series yields $\rho_A = 0.979$ and $\sigma_A = 0.0072$.

For the non-standard parameters, we calibrate them to match long-run averages of *gross* aggregate capital flows. Unfortunately, the U.S. National Production and Income Accounts (NIPA) only measures investment flows of *new* capital goods and then infers aggregate capital stock as the sum of current and past investment flows less depreciation.[21] We thus need to look at firm-level data of capital flows. One of the first studies to do so is Ramey and Shapiro (1998) who use Compustat data to compute gross capital additions and substractions of all publicly traded firms in the U.S.[22] For their full sample 1959-1995, Ramey and Shapiro thus find that annual gross flows of capital additions average 17.3% of depreciated capital stocks, with 70% of these flows coming from expenditures in new property, plant and equipment (PP&E), 25% from

[21] In particular, new investment flows are measured as the total value of shipments from capital goods producing industries adjusted for imports and exports. See Becker et al. (2005) for a detailed discussion.

[22] Since Compustat covers publicly traded firms only, small and medium-size firms are likely to be underrepresented. It turns out, however, that as opposed to employment, most physical capital is concentrated in large publicly held firms. Compustat data should still therefore provide a useful approximation. If at all, the reported numbers understimate the extent of capital reallocation because smaller unlisted firms are more likely to undergo major changes (merger/acquisition, bankruptcy, structural reorganization) and invest larger fractions in used capital. See Eisfeldt and Rampini (2007) for evidence. Also note that other firm-level surveys such as the Longitudinal Research Database (LRD) or ACES may be more representative of the economy than Compustat. At the same time, these surveys provide less detailed information on capital additions and substractions, span over a smaller sample period and suffer from their own selection problems (e.g. Becker et al., 2005).

acquisitions of used capital, and the remaining 5% from entries of new firms. The aforementioned study by Eisfeldt and Rampini (2006) broadly confirms these findings. Based on a Compustat sample from 1971 to 2000, they find that reallocation of used capital makes up 24% of gross investment and that the average annual gross investment rate equals 22% of depreciated capital stocks.[23]

A second useful piece of information from the Compustat dataset are the direct measures of separation flows. In Ramey and Shapiro's study, for example, total separations make up an annual average of 7.3% in terms of undepreciated capital and 4.8% in terms of depreciated capital. By themselves, these numbers are not very revealing because depreciation during the life-cycle of a capital unit is not captured by an actual outflow of capital. What is more interesting is the fraction of capital separations due to reasons other than depreciation. Here, Ramey and Shapiro report that 71% come from retirements – which we interpret as the final step of depreciation – 21% come from sales, and the remaining 9% come from exits due to mergers and bankruptcies. Hence, capital separations are an important phenomenon above and beyond deprecation, with about 30% of all separations being due to reallocations to new firms.

Based on this evidence, we choose a quarterly separation rate of $s = 0.01$. Together with $\delta = 0.025$, this calibration implies that 71% of all separations are due to depreciation and 29% are due to sales and firm exits / acquisitions, as in Ramey and Shapiro (1998). Furthermore, using the capital accumulation equation (1), we can derive that these calibrations imply a quarterly steady state *gross* investment rate of

$$\frac{m(V,L)}{K} = [g - (1-\delta)(1-s)] = 0.03875,$$

which corresponds to a yearly rate of 15.5%. This number lies somewhat below the Compustat evidence reported in Ramey and Shapiro (1998) and Eisfeldt and Rampini (2006). One has

[23] Apart from the different sampling period, one of the reasons for the difference in investment rates is that Eisfeldt and Rampini (2006) use book values while Ramey and Shapiro (1998) apply artificial price deflators to convert their capital measures to current costs that should reflect changes in productive value. Furthermore, Eisfeldt and Rampini (2006) measure reallocation indirectly as sales of PP&E *plus* acquisitions, while Ramey and Shapiro (1998) measure reallocation directly as all additions of used capital. Both count purchases of existing firms, however, arguing that mergers and acquisitions not only represent a change of ownership but often involve important modifications to the composition and use of existing capital. See Jovanovic and Rousseau (2004) for a similar argument.

to keep in mind, however, that the gross investment rates in these two studies are likely to be exaggerated because part of the depreciation applied to capital stocks in Compustat represents accounting standards rather than actual decreases in the value-of-use. Finally, we set $\varphi = 0.95$ such that investment in used capital as a fraction of gross investment, $\varphi(1-\delta)s$, coincides with the 24% reported by Eisfeldt and Rampini (2006).

Consider next the steady state probability of capital allocation q. On the one hand, we know from Section 2 that the hazard rate for different (relatively liquid) finished capital goods averages $q = 0.86$. On the other hand, the vacancy rates for (less liquid) leased industrial and office space average 9.5% and 14.5% of total space, respectively. Defining the corresponding vacancy rate in our model as $U/(U+K) = (1-q)L/(U+K)$ and remembering that gross investment equals $m(V,L) = qL$, we can back out an average q. For the above gross rate of 0.03875, we obtain $q = 0.27$ if we use the vacancy rate of office space and $q = 0.19$ if we use the industrial vacancy rate. These numbers suggest that the average hazard rate is very different for different used and new capital goods. For the purpose of our model, we choose an average value of $q = 0.5$.

The remaining parameters to consider are the household's bargaining weight η and the elasticity of the matching function ϵ. It turns out that ϵ does not affect any of the steady state values. Furthermore, we have no particular long-run information to tie down η. In what follows, we set $\eta = 0.5$ and $\epsilon = 0.5$ and check afterwards whether the results are robust to alternative values.

4.3 Results

Panel A of Figure 1 plots the IRFs of output, productive capital and hours to a persistent, temporary technology shock for both our capital search model (solid lines) and the RBC benchmark (dotted lines). Panel B plots the IRFs of variables that are specific to our capital search model.

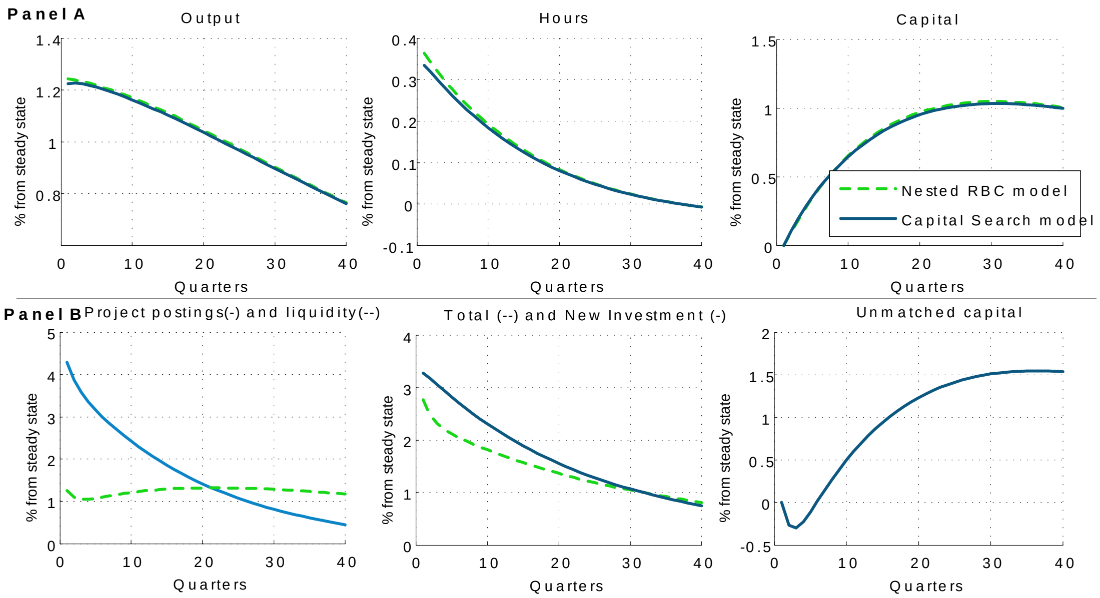

Figure 2: IRFs to a persistent technology shock for baseline specification

Consider first Panel B. In response to the technology shock, households devote more resources to liquidity and firms open up more vacancies. Hence, both total gross investment $m(L_t, V_t)$ and net new investment $I_t = [L_t + \kappa V_t] - [\varphi(1-\delta)sK_t + U_t]$ increase (since K_t and U_t are predetermined). Furthermore, since preferences are additive and concave in consumption and the technology shock is persistent, congestion in the capital market $\theta_t = L_t/V_t$ decreases by proposition 1. For the first few periods after the shock, this decrease in congestion in the capital market leads to a proportionally more important response of capital stocks than in the RBC benchmark. Yet, as Panel A of Figure 1 shows, the difference is quantitatively negligible and its effect on output is dwarfed by the smaller response of hours. This latter result is due to the larger income effect on labor supply as the decrease in congestion lets the households devote more resources to consumption. Overall, output thus responds slightly less than in the RBC benchmark.

As we document in the appendix, the lack of internal propagation of the capital search model is robust to alternative calibrations of q, φ, ϵ and η.[24] The principal reason for this result is that capital separation and allocation flows implied by our calibration of δ and s are too small

[24] Interestingly, an increase in the deadweight loss $1-\varphi$ slightly decreases the internal amplification of the model, thus replicating the result in Veracierto (2002, Table 1) that capital irreversibilities dampen rather than increase output fluctuations.

for the countercyclical congestion mechanism to have a sizable effect. To illustrate this point, we resimulate the model with a much larger separation rate of $s = 0.15$. This would have the counterfactual implication that almost 70% of all capital leaves production in each year (including depreciation) and that average investment flows are equally important. We simply choose this calibration here for expositional purposes and to draw a comparison with Andolfatto (1996) who calibrates his labor search model to the same quarterly separation rate of $s = 0.15$.[25]

As Figure 3 shows, when separation and investment flows are much larger, the countercyclical congestion mechanism starts to matter.

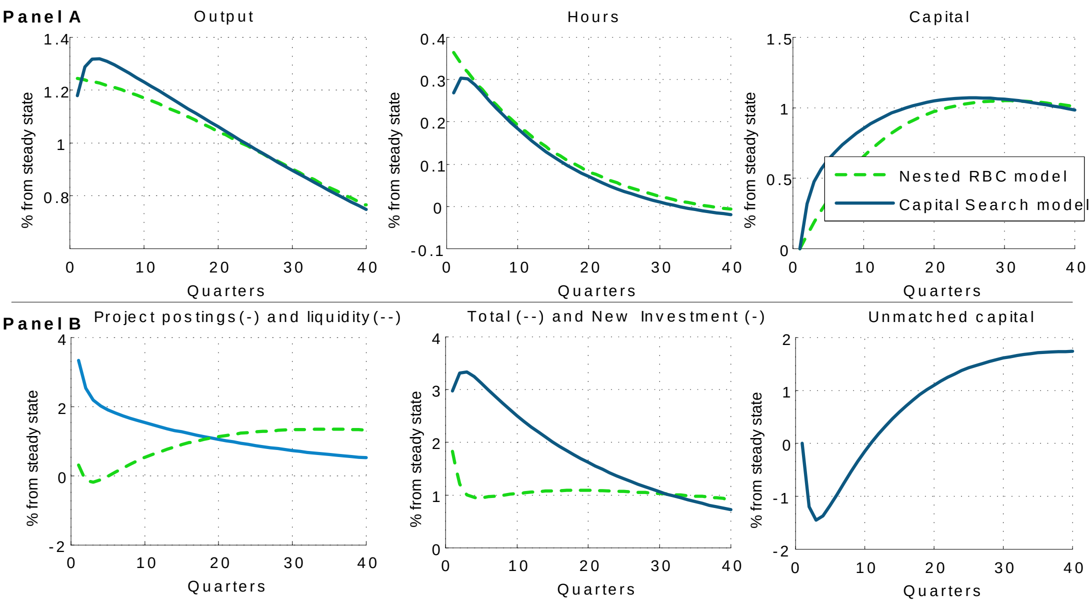

Figure 3: IRFs to a persistent technology shock for counterfactually high separation rate

Panel B explains the origin of these changes. Liquid capital L_t now hardly increases while the jump of project postings V_t is almost as large as before. Hence, the drop in congestion is more important, which explains why capital stocks now respond almost twice as much in the periods following the shock than in the RBC benchmark. Furthermore, households devote a proportionally larger share to consumption on impact, which result is an amplified and humpshaped

[25] For his calibration, Andolfatto (1996) finds that search frictions in the labor market yield significant output persistence in response to technology shocks. Den Haan, Ramey and Watson (2000) argue, however, that when the separation rate is calibrated to the more reasonable value of 10% per quarter, most of these effects disappear as long as separations are constant over the cycle (see their footnote 22). This is an interesting analogue to the point made here.

response of hours. The consequence is an amplified and more persistent reaction of output.

To sum up the quantitative evaluation, Table 2 compares the unconditional standard deviation of Hodrick-Prescott filtered output and autocorrelations of unfiltered output growth of our capital search model with U.S. data and the RBC benchmark.

Table 2: Unconditional second moments of baseline capital search model

	U.S. data	RBC benchmark	Capital search	
			$s = 0.01$	$s = 0.15$
$\sigma(y)$	1.66	1.17	1.16	1.22
$corr(\Delta y, \Delta y_{-1})$	0.264	0.004	0.010	0.102
$corr(\Delta y, \Delta y_{-2})$	0.227	0.003	0.005	0.035
$corr(\Delta y, \Delta y_{-3})$	0.057	0.002	0.003	0.010

Notes: Standard deviation of output is H-P filtered; autocorrelations of growth rates are unfiltered. U.S. data are from DRI Economics for 1953:2 - 2001:4 (see appendix for details).

As discussed in King and Rebelo (2000), the benchmark RBC model is incapable of generating sizable amplification of the exogenous technology shock and remains below the standard deviation reported in the data despite the counterfactually large fluctuations in the exogenous technology shock. Likewise, as Cogley and Nason (1995) document, the RBC model fails to generate the sizable positive autocorrelation of output growth over several quarters that we observe in the data. Our capital search model – when appropriately calibrated – fails equally to generate internal amplification and persistence. The principal reason is that separation and reallocation flows are too small for the countercyclical congestion of our model to have sizable effects. In this sense, the proposed search friction for capital allocation has similarly negligible general equilibrium effects than models with adjustment costs on investment (e.g. Cogley and Nason, 1995 or more recently Khan and Thomas, 2006a) or time-to-build delays (e.g. Kydland and Prescott, 1982) even though the qualitative implications of our model are quite different.

5 Endogenous capital separations due to credit frictions

Different empirical measures suggest that credit frictions and thus capital separations due to financial distress are countercyclical. Covas and Den Haan (2006), for example, document that default rates for U.S corporate bonds peak at the end of recessions. Likewise, we find that current liabilities of business failures taken from DRI (mnemonic: fail) are countercyclical.[26] Parallel to Den Haan, Ramey and Watson's (2000) argument that countercyclical job destruction implies substantial propagation in a labor search model, this suggests that extending our baseline model with credit frictions such as to generate countercyclical capital separations may, in fact, help our capital search model to generate more important business cycle effects.

As a by-product, the extension also allows us to assess the role of costly capital reallocation for the business cycle effects of credit frictions. In fact, existing DGE models with credit frictions such as Carlstrom and Fuerst (1997, CF henceforth) or Bernanke, Gertler and Gilchrist (1999, BGG henceforth) exclusively focus on the effects of net worth on investment and output. But since factors of production in these models can be moved costlessely from one firm to another, they abstract by definition from the effects of capital reallocation due to financial distress.

5.1 Model extension

As in CF and BGG, we introduce credit frictions through a costly state verification (CSV) mechanism originally proposed by Townsend (1979). Firms are subject to a idiosyncratic productivity shock that households (the capital lenders) can only observe after paying a monitoring cost. This assumption of asymmetric information implies that in the absence of monitoring, the firm would always want to underreport its productivity so as to avoid payment of the previously agreed upon rental rate. Households solve this agency problems with a debt contract that specifies monitoring and default if the idiosyncratic productivity level of the firm falls below some optimal threshold.

While we follow the same CSV approach, our model differs from CF and BGG in three important details. First, the optimal default threshold in our model is below the one in CF

[26] The H-P filtered contemporaneous correlation of Covas and Den Haan's (2006) default rate with real GDP is -0.33 for the period 1971-2004, and -0.77 for the period 1986-2004. The H-P filtered correlation coefficient of our liabilities series with real GDP is -0.33 for the sample 1948-1998 and -0.27 for the sample 1980-1998.

and BGG because capital reallocation is costly in our model while in CF and BGG, it is not. Second, we assume as in the baseline model that firms transfer all of their profits to households at the end of the period. Hence, net worth – the channel through which credit frictions affect investment in CF and BGG – is absent. Third, we retain the assumption that the rental rate is determined so as to split the surplus of the lending relationship. CF and BGG assume instead that the lending market is perfectly competitive and thus, all of the surplus goes to the firm.

The specifics of the extended model are as follows. The representative firm's technology becomes

$$Y_t = a_t f(X_t N_t, K_t), \tag{13}$$

where $f(X_t N_t, K_t)$ describes the same constant-returns-to-scale function as before, and a_t denotes the realization of the idiosyncratic productivity shock. Contrary to the aggregate shock X_t, which is known to all participants at the beginning of the period, the shock a_t occurs after all optimal decisions have taken place and is only observed by the firm. As in CF and BGG, we assume that a_t is independently and identically distributed over time and follows a lognormal distribution $\log(a) \sim N(-\frac{\sigma^2_{\log(a)}}{2}, \sigma^2_{\log(a)})$ so as to ensure $a_t \in [0, \infty]$ and $E(a) = 1$.[27]

To deal with the asymmetric information about firm productivity, households and firms negotiate the rental rate ρ_t per unit of matched capital *prior to* the realization of a_t. If the firm makes positive profits (i.e. if $a_t \geq \bar{a}_t$ where $\bar{a}_t$ is such that $\bar{a}_t f\ (X_t N_t, K_t) - W_t N_t - \rho_t K_t - \kappa V_t = 0$), the firm pays $\rho_t K_t$, the household refrains from monitoring and the capital match continues. If, on the other hand, $a_t < \bar{a}_t$ the firm is unable to pay the negotiated capital rental because we assume that the wage bill $W_t N_t$ and the cost of posting vacancies κV_t need to be covered first in order for the firm to continue operating next period. In this situation, the household pays the monitoring cost to verify the firm's production and decides on the continuation of the capital match. If a_t is above some optimal threshold $\underline{a}_t$ that we derive below, the household takes all of the firm's production and covers for the totality of $W_t N_t$ and κV_t so as to continue the capital match. If instead a_t is below the threshold $\underline{a}_t$, the household separates the match and takes back its capital stock without receiving nor paying anything. In

[27] The assumption that a_t is independently and identically distributed in conjunction with constant-returns-to-scale technology simplifies the analysis as we do not need to consider the history of shocks incurred by each firm. Firm size thus remains irrelevant, which is why our notation continues to abstract from firm subscripts.

this case, the firm is liquidated and the difference between production and the cost of $W_t N_t$ and κV_t is picked up by an insurance that is funded with the dividends from profit-making firms.[28]

Given these assumptions, endogenous separations s_t^e due to financial distress are defined as

$$s_t^e = H(\underline{a}_t),$$

where $H(a)$ denotes the cumulative density of a. Aside from this endogenous part, we still allow for exogenous (constant) separations that we denote by s^x. Hence, the total separation rate is defined as

$$s_t = s^x + s_t^e. \tag{14}$$

Furthermore, the household's expected gross revenue from matched capital equals

$$\begin{aligned} R_t^K \quad = \quad & \int_{\bar{a}_t}^{\infty} \rho_t K_t dH(a) + \int_{\underline{a}_t}^{\bar{a}_t} \left[a f(X_t N_t, K_t) - W_t N_t - \kappa V_t \right] dH(a) \\ & - \tau \int_0^{\bar{a}_t} \left[a f(X_t N_t, K_t) \right] dH(a) + (1-\delta)\varphi_t s_t K_t. \end{aligned} \tag{15}$$

The first two terms denote net revenues from continuing relationships. The third term denotes the expected total monitoring cost paid by the household, which we assume to be a fixed proportion $\tau > 0$ of the defaulting firms' output. The fourth term corresponds to the value of separated capital returned to the household's budget constraint. In this last term, we assume that the recovery rate of separated capital φ_t is time-varying and more specifically, that it is a convex function of total endogenous capital separations; i.e. $\varphi_t = \varsigma(s_t^e)$ with $\varsigma'(\bullet) < 0$ and $\varsigma''(\bullet) < 0$. Two considerations motivate this choice. First, we want to capture industry-specific asset illiquidity as proposed by Shleifer and Vishny (1992) that are otherwise absent in our representative agent model (see discussion in Section 2). Second, the additional flexibility afforded by this function allows us to match the business cycle dynamics of endogenous capital separations due to financial distress.

Consider now the household's optimal choice of $\underline{a}_t$. It obtains for the level of a_t below which refinancing a firm is more expensive than severing the lending relationship and incurring the cost of reallocating the capital to another firm. More formally, we can derive it from the household's

[28] See the appendix for the details on this insurance. Suffice to say here that we implicitly assume that firms or capital lenders on their own cannot contract a similar insurance on their own to prevent the firm from disappearing.

optimization problem as (see the appendix for a detailed description)

$$\Lambda_t(1-\delta)\varphi_t K_t = \Lambda_t \left[\underline{a}_t f(X_t N_t, K_t) - W_t N_t - \kappa V_t\right] + (1-\delta)K_t \beta E_t V_K(U_{t+1}, K_{t+1}). \qquad (16)$$

The left-hand side is the marginal value (in utility terms) of separating and returning the capital unit into the budget constraint for reallocation, where we assume that the representative household takes φ_t as exogenous. The right-hand side is the marginal revenue from matched capital plus the marginal value of continuing the match into the future.[29]

Conditional on selecting a debt contract, the proposed monitoring and separation scheme is optimal for both parties. The firm would not gain anything from reporting output below what it actually produced because in case of monitoring, it will loose all output anyway. Likewise, the household would not gain anything from negotiating a higher or lower auditing cutoff $\bar{a}_t$ or a separation threshold $\underline{a}_t$, by definition of the utility-maximizing condition in (16).

Since any revenues associated with productivity shocks below $\bar{a}_t$ are absorbed either by the capital lender (in case of continuation of the capital match) or by an insurance (in case of capital separation), firms now maximize only over the positive portion of revenue net of current costs; i.e. $\int_{\bar{a}_t}^{\infty} \left[a f(X_t N_t, K_t) - \rho_t K_t - W_t N_t - \kappa V_t\right] dH(a)$. As the appendix details, the first-order conditions resulting from this objective function would imply substantial overhiring of labor relative to the RBC benchmark and thus an unrealistically high labor share. We correct this implication by assuming, in addition, that the representative firm in the extended model applies a constant markup $1/\psi \geq 1$ on its optimal decision problem.[30]

To close the model, we assume as before that the rental rate is determined by Nash bargaining over the surplus of the capital relationship. This rental rate is now conditional on the optimal

[29] It can be shown that $\underline{a}_t f(X_t N_t, K_t) < W_t N_t + \kappa V_t$; i.e. the household is willing to refinance distressed firms up to a certain point so as to continue the capital match. This is because walking away from a relationship to reallocate capital with another firm is costly in the sense that separated capital yields zero return in the next period and comes with the risk that rematching takes time. By contrast, lenders in the CF and BGG models never refinance since liquidating a defaulting firm and reallocating the capital is costless.

[30] As proposed by Blanchard and Kiyotaki (1987), such a markup could result from a situation where otherwise identical firms produce imperfectly substitutable goods such that each firm faces a downward-sloping demand in its relative price.

$\underline{a}_t$ (see appendix)

$$\begin{aligned}\rho_t &= \eta\left[\bar{\mu}_t\psi f_K(X_tN_t,K_t)+(1-\delta)(1-s_t)\frac{\kappa}{\theta_t}[1-H(\bar{a}_t)]\right]+(1-\eta)\left[\delta+(1-\delta)(1-\varphi_t)s_t\right] \\ &\quad+\left[\rho_tH(\bar{a}_t)-(1-\eta)(\underline{\mu}_t-\bar{\mu}_t-\tau(1-\bar{\mu}_t))f_K(X_tN_t,K_t)\right], \qquad (17)\end{aligned}$$

where $\underline{\mu}_t = \int_{\underline{a}_t}^{\infty} adH(a)$ and $\bar{\mu}_t = \int_{\bar{a}_t}^{\infty} adH(a)$ denote partial expectations. Compared to the case with exogenous separation, the first term in brackets is altered to reflect the marginal product of capital and the saved search costs actually accruing to the firm. The third term in brackets represents the risk premium that arises because households do not receive the full contractual payment ρ_t (or even need to reinject money) and pay monitoring costs when the firm's idiosyncratic shock drops below $\bar{a}_t$.[31]

5.2 Calibration

To compare the extended model with the baseline model where all separations are constant, we keep the common parameters unchanged in a first time; i.e. $q = 0.5$, $s = 0.01$, $\eta = 0.5$, and $\epsilon = 0.5$. Further below, we perform robustness checks with respect to alternative calibrations. The additional parameters requiring calibration are the markup of price over marginal cost, $1/\psi$, the fraction of output expended on monitoring, τ, the fraction of capital separation due to financial distress, s^e/s, and the elasticity $(\partial\varphi/\partial s^e)/(s^e/\varphi)$ around steady state.[32]

The crucial dimensions we want to match with our calibration are the relative importance and business cycle dynamics of capital separations due to financial distress. Since the aforementioned studies on firm-level capital flows do not report such details, we compute the relevant series ourselves from Compustat data (see appendix for a detailed description of the data). Specifically, we treat the following categories as capital separations due to financial distress: (i) exits due to liquidation (chapter 7); (ii) sales during the years (-1 0 1 2) around bankruptcy filings (chapter 11); and (iii) sales during the years (-1 0 1 2) around drops of more than 2 credit ratings in long term debt. Compustat provides information on the reasons of exit for disappearing firms

[31] Broadly speaking, this risk premium is the consequence of incomplete contracting in a world with ex-post factor specificity that Williamson (1979) and more recently Caballero and Hammour (1996) term the *fundamental transformation problem*. The general equilibrium consquence is reduced flexibility of separation decisions and, in turn, a slower capital accumulation process.

[32] Since we loglinearize the model, the other functional characteristics of $\varphi = g(s^e)$ are irrelevant.

as well as information about debt ratings of continuing firms. To identify firm bankruptcies, we link the Compustat database with information on chapter 11 filings from the Bankruptcy Research Database.[33] Total separations (defined as sales and exits) and retirements, in turn, are computed as in Ramey and Shapiro (1998).[34] Table 3 provides the thus computed averages for the sample 1980-1993.[35]

Table 3: Capital separations

	Retirements	Sales (S)	Exits (E)	S & E Total	S & E due to Fin. Distress
Fraction of PP&E	4.94%	1.31%	1.11%	2.42%	0.15%
Correlation with output	0.30	0.48	0.37	0.15	-0.31
Standard dev rel output	21.82	4.82	99.5	39.52	2.46

Notes: Standard deviations and correlation coefficients apply to H-P filtered series;
Data source from Compustat 1980-1993 (see appendix for details).

In line with Ramey and Shapiro (1998), retirements make up roughly two thirds of all separations while sales and exits make up about one third.[36] Sales and exits due to financial distress make up only 6% of total capital separations (and only 4.6% for the 1980-2004 period), which amounts to 0.15% of average capital stocks. The series is countercyclical, in line with the aforementioned evidence on the cyclicality of financial distress, and about two and a half times as volatile as output. To roughly match these characteristics, we calibrate $s^e/s = 0.05$ and set

[33] The Bankruptcy Research Database (BRD) is compiled by Lynn M. LoPucki from UCLA Law. Of the 751 reported cases of bankruptcy filings by large publicly traded firms since October 1979, we were able to match 623 firms with the unique firm identifiers used by Compustat (mnemonic: gvkey).

[34] Ramey and Shapiro (1998) count as total exits the ones related to mergers and liquidations but do not count exits due to privatizations, leveraged buyouts and other reasons.

[35] We start the sample only in 1980 because, as Davis, Haltiwanger, Jarmin and Miranda (2006) document, the proportion of medium-size and smaller firms listed publicly increased importantly in the early 1980s. This makes the Compustat sample more representative – especially with regards exits due to financial distress. The end date 1993 is chosen because thereafter, firms no longer provide accurate numbers for retirements. As mentioned before, Compustat data should be more representative for capital than for employment because physical capital is concentrated in large firms, most of which are publicly traded (e.g. Eisfeldt and Rampini, 2007).

[36] As discussed before, the total numbers are small because depreciation during the life-cycle of a capital unit is not matched by an actual outflow of capital.

$(\partial\varphi/\partial s^e)/(s^e/\varphi)$ such that the relative volatility of s^e in the model coincides with the one in the data.

For the other two additional parameters, we choose $1/\psi = 1/0.8 = 25\%$ and set the monitoring cost parameter to $\tau = 0.05$.[37]. The resulting long-run ratios of interest are the following: the consumption-output ratio equals 73.13%, which is in line with King and Rebelo (2000); the labor share equals 74%, which corresponds to estimates reported by Gollin (2002); the average annualized risk premium equals 3.56%, which lies in-between the spread of the post-war average Aaa corporate bond yield over the 3-month Treasury bill (1.87%) and the post-war average equity risk premium for the U.S. (7.58%); and profits (dividends) relative to output equal 8.9%, which is somewhat too high compared to the evidence reported in Basu and Fernald (1997).[38]

Before continuing, we return to Table 4 to consider the overall behavior of sales and exits. Both series are procyclical and especially exits are highly volatile relative to output. This latter result is due to the large variations in mergers and acquisitions (M&A) that account for most of capital separations in the Compustat data.[39] Somewhat counterfactually, we omit these variations in our extended model and instead assume this part of capital separations to be constant. The reason for this omission is two-fold. First, as the below quantitative analysis shows, even small countercyclical capital separations due to credit frictions can have substantial effects in general equilibrium. Second, the procyclical nature of sales and M&A is likely to be the result of reallocation towards more efficient firms in the wake of technological change (e.g. Jovanovic and Rousseau, 2004). Our representative agent framework is designed, by definition, to quantify the effects of search frictions on their own but does not allow us to consider reallocation costs in conjunction with persistent productivity differences. As we discuss in the conclusion of

[37] A great deal of controversy surrounds the costs related to bankruptcy. In our model, this cost should only entail the direct costs related to monitoring and reorganization. We therefore set it to a value that is well below estimates of direct and indirect costs of bankruptcy that seem to lie between 20 and 35% of output. See Carlstrom and Fuerst (1997) for a discussion. As robustness checks in the appendix reveal, the value of τ has little influence on the dynamics of our model.

[38] Other values of interest implied by our calibration but for which we do not have any empirical counterparts are: an average cost of posting vacancies relative to output equal to $v\kappa/y = 2.22\%$, and a standard deviation of the idiosyncratic productivity shock equal to $\sigma_a = 0.33$.

[39] The procyclicality of M&A is consistent with evidence reported in Maksimovic and Phillips (2001). They use LRD data and find that change in ownership of large manufacturing plants is highly procyclical.

the paper, this is an interesting avenue for future research.

5.3 Quantitative evaluation

As in Section 4, we start our quantitative evaluation by considering IRFs to a persistent but temporary technology shock. As is immediately apparent from Figure 3, the extended capital matching model (solid lines) generates a substantially amplified response of output and hours compared to the RBC benchmark (dotted lines).

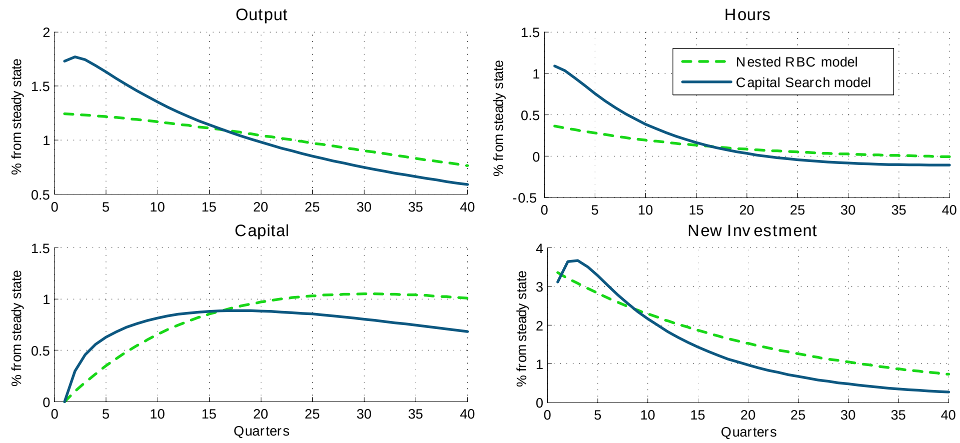

Figure 3: IRFs to a persistent technology shock for the extended model

The amplification has its origin in the state-dependent nature of the credit friction. To illustrate this, Figure 4 displays the IRFs of the variables related to changes in the stock of capital entering the production function. The positive technology shock shifts the firms' productivity distribution to the right, which means that bankruptcies and thus capital separations drop (top-right panel). Hence, less capital is separated from production and returned to the household's budget constraint for time-consuming rematching. This explains why productive capital stocks react more strongly than in the RBC benchmark.

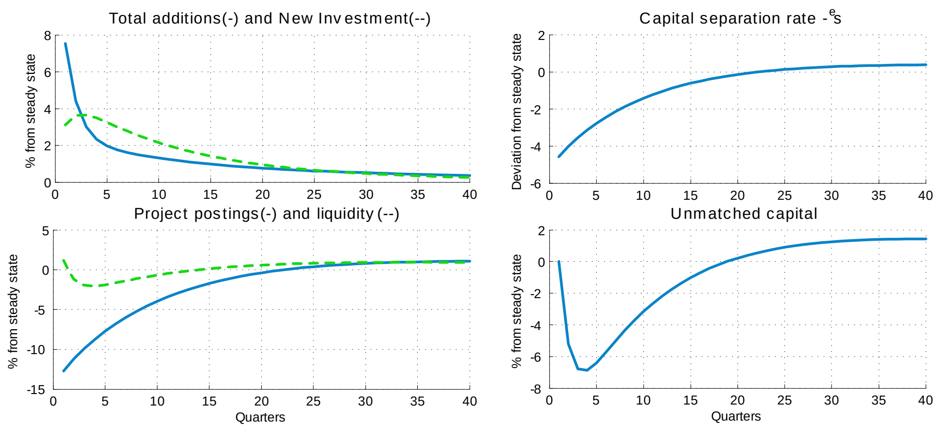

Figure 4: IRFs to a persistent technology shock for the extended model.

As an indirect effect of the drop in capital separations, households now find it optimal to allocate more resources to new investment than in the baseline model with constant separations. Compared to the RBC model, consumption thus reacts less on impact, which results in a smaller income effect on labor supply. In addition, the more important reaction of productive capital implies that the marginal product of labor and thus labor demand increases more rapidly in the periods after the shock than in the RBC model. The conjunction of these two general equilibrium effects leads to a substantially larger response of equilibrium hours and, as the ensuing analysis reveals, this is what explains most of the increased internal amplification of output relative to the RBC benchmark.

Table 4 presents prominent unconditional second moments for U.S. postwar data, the RBC benchmark, the baseline capital search model with exogenous separations as well as the extended capital search model with endogenous separation. For this last case, we report two cases: one for $\epsilon = 0.5$, as used so far, and one for $\epsilon = 0.25$. As we will see, the calibration of this parameter now has important implications.

Table 4: Second moments for baseline calibration

	U.S data		RBC		Capital search					
			benchmark		Exogenous		Endogenous			
							$\epsilon = 0.5$		$\epsilon = 0.25$	
	a	b	a	b	a	b	a	b	a	b
c	0.58	0.69	0.45	0.96	0.48	0.97	0.31	-0.38	0.33	0.85
n	0.95	0.87	0.29	0.97	0.27	0.98	0.63	0.97	0.40	0.97
k	-	-	0.28	0.10	0.28	0.14	0.30	0.44	0.25	0.10
i	2.89	0.87	2.68	0.99	2.68	0.99	2.14	0.99	2.53	0.99
s^e	2.46	-0.96	-	-	-	-	2.46	-0.96	2.46	-0.96
premium	0.54	-0.59	0.004	0.003	0.003	0.98	0.10	-0.97	0.03	-0.97
$\sigma(y)$	1.66		1.17		1.16		1.71		1.28	

Notes: (a) Standard deviation relative to output; (b) contemporaneous correlation with output. All moments are Hodrick-Prescott filtered. Data source from DRI Basic Economics 1953:2-2001:4 (see appendix for details).

Consider first the case where $\epsilon = 0.5$. As indicated by the IRFs, this version of the extended model generates substantial amplification of output relative to the RBC benchmark. As for persistence, however, the model still fails to generate the marked positive autocorrelation of output growth that we see in the data (see Table 5 below). The increase in internal amplification is rooted in the general equilibrium effects on labor supply and labor demand that result in more volatile dynamics of hours. Interestingly, both the zero profit threshold $\bar{a}_t$ and the separation threshold $\underline{a}_t$ are countercyclical, which implies, in turn, that the model generates a countercyclical risk premium. Although the fluctuations of this premium are not as volatile in the data, this result is a significant success of our extended model over the RBC benchmark as well as over standard credit friction models without costly capital reallocation (see below).

Closer inspection of Table 4 reveals that the more volatile dynamics of equilibrium hours come at the cost of countercyclical consumption, which is clearly at odds with the data. In fact, the negative income effect brought about by the drop in capital separations is so strong that households choose to decrease their consumption on impact. These consumption dynamics hinge crucially on the elasticity ϵ that links the matching probability $q(\theta_t)$ to the congestion measure θ_t.[40] For $\epsilon = 0.5$, the response of $q(\theta_t)$ is relatively large. We thus recalibrate $\epsilon = 0.25$ so as to roughly match the consumption dynamics in the data. The last column of Table 4 reports the results. Consumption is now procyclical and almost as volatile as in the data. The consequence of this adjustment is a much smaller income effect on labor supply, which reduces the standard deviation of output to 1.28 – a value just slightly above the RBC benchmark.

This exercise makes clear that the interplay between time-consuming capital (re-)allocation and countercyclical capital separation leads to amplification by affecting the response of hours supplied by households. Exogenous shocks not only affect the factor productivity as in the RBC benchmark, but also the *stock* of productive capital and the amount of resources that need to go through the time-consuming allocation process. The time-varying capital separation rate limits the income effect of rising returns to capital, thus inducing households to shift more resources away from consumption towards investment and supplying more hours. However, once we calibrate the model to yield reasonable consumption dynamics, we find that these effects are modest and result only in a small increase in internal amplification.

5.4 Assessing the effect of removing search frictions

To further illustrate the interplay between search frictions and countercyclical capital separations, we remove the search friction from the model. As in the RBC benchmark, this corresponds to a situation where $\kappa = 0$ and $\varphi_t = 1$. Firms thus post an infinity of new projects in every period and the probability of allocating a liquid unit of capital to a firm becomes $q(\theta_t) = 1$. Households still monitor firms that cannot make the negotiated rental payment and separate the lending relationship with firms whose revenues fall below their wage bill. In other words, households are no longer willing to reinject funds to keep a lending relationship alive because reallocating capital is now costless. Despite this change in optimal separation decision, the rental rate still involves a risk premium that takes into account the expected cost of monitoring.[41] As

[40] Recall from the first order condition (8) that the expected return from liquid capital is an average of the marginal values of matched and unmatched capital, weighted by the matching probability $q(\theta)$. A stronger cyclical response of $q(\theta)$ means the average return to liquid capital rises more quickly in an upturn.

[41] Specifically, we derive the intertemporal Euler equation as (see appendix for details)

$$\Lambda_t = \beta E_t \left[\Lambda_{t+1} \Psi_{t+1} f_k(X_{t+1} N_{t+1}, K_{t+1}) + (1 - \delta) \right],$$

in CF and BGG, this risk premium affects the price of capital and thus investment.

Table 5 provides unconditional second moments for the extended model without search frictions and contrasts them to the baseline model with both frictions and the RBC benchmark. To put these results in perspective, we report the same summary statistics for a non-monetary version of the BGG model as well as the CF model.[42]

Table 5: Comparison of second moments when frictions are removed

	Extended model with both frictions	Extended model without search	RBC benchmark	BGG	CF
$\sigma(y)$	1.28	1.20	1.17	1.1	1.1
$corr(s^e, y)$	-0.96	-0.72	-	0.47	0.32
$corr(premium, y)$	-0.97	-0.93	0.98	-0.03	0.32
$corr(\Delta y, \Delta y_{-1})$	-0.004	0.002	0.004	-0.01	0.2
$corr(\Delta y, \Delta y_{-2})$	-0.004	0.001	0.003	0.004	0.02
$corr(\Delta y, \Delta y_{-3})$	-0.005	0	0.002	0.003	0.01

Notes: Standard deviation of output is Hodrick-Prescott filtered.

Autocorrelations of growth rates are unfiltered

The extended model without search frictions generates less internal amplification than the model with both frictions. The reason for this decrease of internal amplification is that separated capital can now be reallocated costlessely and immediately. The only thing that sets apart the extended model without search frictions from the RBC benchmark is the countercyclical risk premium. After a positive shock, this premium decreases in our model because less firms are expected to default. This leads to a slightly more important investment boom at the expense of consumption. The resulting smaller income effect on labor supply implies that equilibrium hours and thus output react more than in the RBC model. However, this difference is minimal.

where $\Psi_{t+1} = \left[\psi\overline{\mu}_{t+1} + (\underline{\mu}_{t+1} - \overline{\mu}_{t+1}) - \tau(1 - \overline{\mu}_{t+1})\right]$ denotes the risk premium (which depends on conditional expectations $\overline{\mu}_{t+1}$ and $\underline{\mu}_{t+1}$ associated with the monitoring and separation thresholds, and the optimal separation threshold is now determined by $\underline{a}_t f(X_t N_t, K_t) - W_t N_t = 0$). In the RBC benchmark without credit market frictions, by contrast, the marginal product of capital equals $\beta E_t\{\Lambda_{t+1}\left[f_K(X_{t+1}N_{t+1}, K_{t+1}) + 1 - \delta\right]\} = \Lambda_t$.

[42] Details about the BGG model and the CF model, including their calibration, are provided to the interested reader upon request.

Compared to the BGG model and the CF model, our extended model with credit frictions succeeds in generating countercyclical default and countercyclical risk premia, independent of whether the search friction is present or not.[43] This difference is due to the absence of net worth in our model.[44] As shown in Covas and Den Haan (2006), firms in the BGG and CF models seek to increase their capital immediately in response to a positive technology shock even though their net worth adjusts only sluggishly. The resulting increase in their debt to net worth ratio implies that monitoring by lenders in an upturn actually increases, which in turn pushes up the external finance premium. This implies an increase in the monitoring and separation threshold, thus exerting upward pressure on the risk premium.

5.5 Volatility of separations and robustness to alternative calibrations

As highlighted by the above results, a crucial ingredient for the marked internal propagation of our extended model is the income effect on labor supply whereby households withhold current consumption to finance capital investments. The following robustness checks assess to what extent alternative calibrations affect the performance of the model. In all of these exercises, we keep $\epsilon = 0.25$ so as to roughly match the consumption dynamics in the data and adjust the elasticity $(\partial\varphi/\partial s^e)/(s^e/\varphi)$ such as to keep the relative volatility of s^e consistent with the Compustat data. Table 6 reports the results.

[43] Interestingly, both the BGG and CF model generate somewhat less amplified output dynamics with respect to technology shocks than the RBC benchmark. Meier and Muller (2005), Queijo (2006) and Christensen and Dib (2007) confirm these findings in more elaborate DGE models. The net worth channel may, however, have more important effects with respect to other shocks. Gertler, Gilchrist and Natalucci (2003) find, for example, that net worth can play an important role in a small open economy that combines liquidity shocks (i.e. an exogenous change to the foreign borrowing premium) with exchange rate targeting monetary policy. Furthermore, note that the CF model succeeds in generating some persistence in output growth. The reason for this result is that the CF model applies the credit friction only to the investment goods producing sector whereas the BGG model applies the friction to the entire economy.

[44] Another difference is that in the BGG and CF models, lenders who sever the financing relationship can walk away with any revenue net of wage payments whereas in our model, this is not the case. This is why the monitoring threshold coincides with the separation threshold in the BGG and CF models; i.e. there is no chapter 11.

Table 6: Sensitivity of model performance to alternative calibrations

	Baseline	Mean allocation rate		Bargaining power		Separation rates	
	calibration	$q(\theta) = 0.25$	$q(\theta) = 0.75$	$\eta = 0.45$	$\eta = 0.75$	$s^e/s = 0.01$	$s = 0.02$
$\sigma(y)$	1.28	1.29	1.28	1.28	1.26	1.29	1.62
$\sigma(s^e)/\sigma(y)$	2.46	2.46	2.46	2.46	2.46	2.46	2.46
$corr(s^e, y)$	-0.96	-0.97	-0.96	-0.96	-0.96	-0.96	-0.97
$corr(\Delta y, \Delta y_{-1})$	-0.004	-0.017	0.010	-0.004	-0.002	-0.004	0.004
$corr(\Delta y, \Delta y_{-2})$	-0.004	-0.013	-0.002	-0.005	-0.003	-0.004	-0.01
$corr(\Delta y, \Delta y_{-3})$	-0.005	-0.010	-0.004	-0.005	-0.004	-0.005	-0.016

Notes: Standard deviations and cross-correlations are Hodrick-Prescott filtered.

Autocorrelations of growth rates are unfiltered.

Changes in $q(\theta), \eta$ and s^e/s (keeping $s = 0.01$) have essentially no impact on the dynamics of the model.[45] This result would even hold if we didn't adjust $(\partial\varphi/\partial s^e)/(s^e/\varphi)$ so as to keep $\sigma(s^e)/\sigma(y) = 2.46$. The reason for this robustness is that income effects on labor supply remain small when $\epsilon = 0.25$ and capital separations on their own are too insignificant to affect output significantly.

The dynamics of the model are more sensitive to changes in the average separation rate s. For example, when we calibrate $s = 0.02$ per quarter (keeping s^e/s at 0.05), the standard deviation of output rises to $\sigma(y) = 1.62$. The mechanism for this increase in amplification is the same than before. The larger average s implies that the drop in separated capital after a positive technology shock is more important and thus, households divert more resources away from consumption in order to achieve the desired amount of liquid capital. The resulting negative income effect increases the volatility of hours, thus leading to an amplified output response. As before, however, this effect is accompanied by a negative correlation of consumption with output. If we correct this counterfactual implication by lowering ϵ even more, the amplification of output is reduced substantially.

Finally, it is interesting to note that there are several calibrations for which the extended capital search model generates both important amplification and persistence effects. For example, if

[45] For the given calibration, there is no rational expectations solution to the model for values of η below 0.45.

we set the elasticity $(\partial\varphi/\partial s^e)/(s^e/\varphi) = 0$ (i.e. φ is constant) and $\epsilon = 0.5$, we obtain $\sigma(y) = 1.52$, $corr(\Delta y, \Delta y_{-1}) = 0.28$ and $corr(\Delta y, \Delta y_{-2}) = 0.08$ without counterfactual consumption dynamics (see appendix for details). This marked improvement in internal propagation is due to an overly volatile endogenous separation rate (more than a 1000 times as volatile than output). This illustrates that the *combination* of search frictions for physical capital and countercyclical capital separations due to credit frictions leads at least in principle to more important business cycle fluctuations. The issue is simply that the flows of physical capital in and out of production are not large and not volatile enough for these effects to play a substantial role.

6 Conclusion

In this paper, we examined the business cycle consequences of search frictions for the allocation of physical capital. The investigation is motivated by firm- and industry-level evidence on market imperfections in the allocation of physical capital. Despite the fundamentally different nature of physical capital and labor, we argue that the market imperfections involved in the allocation of these two factors are quite similar. We thus consider our paper as a first step towards analyzing capital allocation with the same type of search frictions that have proven fruitful for our understanding of labor markets. By the same token, we propose a complementary view to existing models of investment that focus on aggregate adjustment costs and building delays in a world with perfect markets.

The capital search model that we develop generates countercyclical congestion in physical capital markets, in line with the data. Our analysis in a modern DGE context suggests, however, that for reasonable calibrations, the internal propagation effects of these search frictions are modest. The main reason for this lack of internal propagation is quantitative: separation and reallocation flows of physical capital are too small for the search friction to play a significant role. This conclusion remains intact when we extend the model with credit market frictions that result in countercyclical capital separations. While the combination of countercyclical separations and imperfect capital (re-) allocation increases internal propagation, almost all of these effects stem from a general equilibrium income effect that these frictions have on labor supply. Once we tie down the model to generate consumption dynamics in line with the data, we find that capital separations due to financial distress are simply not important and volatile enough for them to generate significant internal propagation.

Our results provide an interesting contrast to Den Haan, Ramey and Watson (2000) who show that the introduction of countercyclical job destruction in a labor search model substantially magnifies and prolongs the business cycle effects of small shocks. This difference in results is mainly due to the fact that labor is twice as important of an input to production as capital and that job destructions fluctuate on average much more over the business cycle than capital separations. Furthermore, job destructions overall are countercyclical while for capital separations, only the part linked to financial distress is countercyclical. This part makes up only a small fraction of all capital reallocations, which explains why its impact is so limited.

The comparison suggest that capital reallocations due to sales and M&A are a more important source of internal propagation. From our firm-level data, we know that most capital reallocations occur through these two channels and are substantially more volatile than capital reallocations due to financial distress. The problem is that sales and M&A are procyclical rather than countercyclical and thus, they would not generate more important business cycle dynamics in the proposed representative agent framework. At the same time, Jovanovic and Rousseau (2004) argue that sales and M&A of capital are often the consequence of reorganization in the aftermath of embodied technological progress. Hence, combining embodied technological progress in a heterogenous firm framework with search frictions for the reorganization of physical capital could entail important internal propagation effects as it takes time for firms and sectors to reallocate factors of production to their most productive use.[46]

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