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The Response of Hours to a Technology Shock: a Two-Step Structural VAR Approach

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Abstract:
The response of hours worked to a technology shock is an important and a controversial issue in macroeconomics. Unfortunately, the estimated response is generally sensitive to the specification of hours in SVARs. This paper uses a simple two-step approach in order to consistently estimate technology shocks from a SVAR model and the response of hours that follow this shock. The first step considers a SVAR model with a set of relevant stationary variables, but excluding hours. Given a consistent estimate of technology shocks in the first step, the response of hours to this shock is estimated in a second step. When applied to US data, the two-step approach predicts a short-run decrease of hours after a technology improvement followed by a hump-shaped positive response. This result is robust to the specification of hours, different sample periods, measures of hours and output and to the variables included in the VAR in the first step.

Keywords: SVARs, long-run restriction, technology shocks, hours worked

JEL Classification: C32, E32

Introduction

The response of hours to a technology shock is the subject of many controversies in quantitative macroeconomics. The contributions of Galí (1999), Basu, Fernald and Kimball (2006) and Francis and Ramey (2005a) show that the short–run response of hours worked to a technology shock is significantly negative in the US economy. Galí (1999) and Francis and Ramey (2005a) obtain this result using a Structural Vector Autoregression (SVAR) of labor productivity growth and hours in first difference (DSVAR) with long–run restrictions (see Blanchard and Quah, 1989). Basu, Fernald and Kimball (2006) use a direct measure of aggregate technology change, controlling for imperfect competition, varying utilization of factors and aggregation effects, and find that hours fall significantly on impact after a technology improvement. Moreover, Galí (1999) and (2004) shows that the level of hours significantly decreases in the short run in all G7 countries and the euro area as a whole, with the exception of Japan. These results are in contradiction with Christiano, Eichenbaum and Vigfusson (2004). Using a SVAR with a level specification of hours (LSVAR), they find a positive and hump–shaped response of hours after a technology shock. Moreover, they show that the LSVAR specification encompasses the DSVAR specification.

The specification of hours in level or in difference appears to be the core issue of the controversies. Galí (1999), Galí and Rabanal (2004) and Christiano, Eichenbaum and Vigfusson (2004) perform various unit root tests, but it becomes hard to obtain clear–cut evidence in favor of level or difference specification. Furthermore, recent contributions proceeding with simulation experiments point out that the specification of hours in SVARs using long–run restrictions can alter significantly the estimated effect of a technology shock on hours. For example, Chari, Kehoe and McGrattan (2007) simulate a business cycle model estimated by Maximum Likelihood on US data with multiple shocks. They show that the DSVAR specification leads to a negative response of hours under a RBC model in which hours respond positively. As pointed by Christiano, Eichenbaum and Vigfusson (2004), the DSVAR specification may induce strong distortions if hours worked are stationary in level.

In this paper, we use a simple method that allows to consistently estimate technology shocks and thus the responses of hours to a technology improvement. In contrast to existing LSVAR and DSVAR specifications, we choose to exclude hours worked series from SVARs to identify technology shocks.[1] The proposed approach consists in the following two steps. In a first step, a SVAR model with long–run restriction that includes well-chosen covariance stationary variables

[1]Simulation experiments based on DGSE models in Fève and Guay (2007) show clear evidence that the uncertainty about the right specification of hours in the SVAR model is more detrimental for the estimation of technology shocks and their impacts on hours than the information loss resulting from the omission of this variable in the SVAR model.

allows to properly identify the technology shock series. Among these variables, the consumption to output ratio seems to be a promising candidate. Two reasons motivate this choice. First, as argued by Cochrane (1994), this ratio may help to better predict the permanent and transitory components of output. Indeed, using a simple permanent income argument, permanent (technology) shocks can be separated from other (non–technology) shocks because these latters do not modify the consumption. The joint observation of output growth and consumption to output ratio allows then the econometrician to properly identify permanent and transitory shocks. Second, on the empirical side, our findings show that the consumption to output ratio displays less persistence than hours. When a SVAR model with long–run restrictions includes variables characterized by a highly persistent process (typically hours with the level specification), the identification of the responses of hours to technology shocks can be seriously disturbed. Gospodinov (2006) using a near-unit root process for the hours and Christiano, Eichenbaum and Vigfusson (2004) using a unit root process have shown that the LSVAR specification for such highly persistent processes leads an inconsistent estimator of the technology shocks. Respective to this result, a less persistent variable such as consumption to output ratio should improve the identification of the technology shocks. Moreover, the specification of this ratio is not subject to controversies in quantitative macroeconomics and the cointegration between consumption and output is usually imposed in SVARs (see Cochrane, 1994, Christiano, Eichenbaum and Vigfusson, 2004, Francis and Ramey, 2005a, King, Plosser, Stock and Watson, 1991, among others).

In the second step, the Impulse Response Functions (IRFs) of hours at different horizons are obtained by a simple regression of hours on the estimated technology shocks for different lags. In this latter step, according to the debate about the right specification of hours, we consider hours worked in level and in difference in this regression. Our method can be seen as a combination of a SVAR approach in the line of Blanchard and Quah (1989), Galí (1999) and Christiano, Eichenbaum and Vigfusson (2004) and the regression equation used by Basu, Fernald and Kimball (2006) in their growth accounting exercice.

One advantage of our approach is that the specification of hours does not matter in either in the identification step and the estimation step. Indeed, when hours are characterized by a stationary process, the proposed two-step strategy delivers a consistent estimator of the response of hours to a technology shock. Since hours worked per capita are by definition bounded, this variable cannot asymptotically have a unit root. However, to better understand the behavior of our approach in small sample, we investigate the case for which hours are modeled as a nearly integrated process given that standard unit root tests do not reject the null for the observed series. In this setting, when hours are projected in level in the second step, we show that the OLS estimator does not asymptotically converge in probability to the true IRFs. More precisely, this estimator is asymptotically centered to the true value but characterized by sampling uncertainty which does not asymptotically vanish. On the other hand, the estimator obtained when hours are projected in first difference is asymptotically consistent.[2] According to these results, the specification with hours in first difference should be preferred if we suspect a highly persistent process for this variable as observed in the actual data sets.

We then apply the two–step approach to US data.[3] We obtain that hours worked decrease significantly in the short–run after a positive technology shock but display a positive hump–shaped response. The latter is also precisely estimated for the specification in difference. Our results are in line with previous empirical findings which show that hours fall on impact (see Galí, 1999, Basu, Fernald and Kimball, 2006, Francis and Ramey, 2005a, 2005b) and display a positive hump pattern during the subsequent periods (see Christiano, Eichenbaum and Vigfusson, 2004 and Vigfusson, 2004). So, our approach allows to bridge the gap between the LSVAR and DSVAR specifications. These results appear robust to the sample period considered, measures of hours and output, bivariate VARs, relevant larger VARs and breaks in labor productivity. Interestingly, the results obtained in all cases are in accordance with the asymptotic distribution derived in the case of highly persistent process for hours. First, the level and difference specifications of hours provide similar IRFs in all our estimations. Second, the level specification of hours delivers uninformative IRFs characterized by wide confidence intervals. Third, the dynamic responses when hours are taken in first difference in the second step are precisely estimated for our selected horizon.

The paper is organized as follows. In a first section, we present our two-step approach. Section 2 is devoted to the exposition of the empirical results. The last section concludes.

1 The Two-Step Approach

The goal of our approach is to accurately identify technology shocks in a first step using adequate covariance–stationary variables in the VAR model. A large part of the performance of the two-step approach depends on the time series properties of these variables, which can be interpreted as instruments allowing to estimate with more precision the true technology shocks.

The objective of the first step is then to include a set of variables in the SVAR model to properly identify the technology shocks series. Among these variables, a promising candidate is

[2]In fact, the estimator of the response of hours to technology shocks is given by the cumulative sum of the OLS estimator for the regression of hours in difference on the contemporaneous and lagged values of identified technology shocks.

[3]The paper focusses only on the empirical debate about the response of hours to a technology shock using actual data. In Fève and Guay (2007), we perform various simulation experiments in the line of Erceg, Guerrieri and Gust (2005), Chari, Kehoe and Mc Grattan (2007) and Christiano, Eichenbaum and Vigfusson (2006). In particular, we find that our two–step approach clearly outperforms the DSVAR and LSVAR specifications.

the log of consumption to output ratio.[4] There is both structural and empirical evidence that supports the selection of this variable.

First, following Cochrane (1994), we argue that the consumption to output ratio contains useful econometric information to disentangle the permanent to the transitory component. Indeed, this ratio helps to identify transitory shocks as those that have no effect on consumption. The argument of Cochrane (1994) is based on a structural interpretation using a simple permanent income model. This model implies that consumption is a random walk and that consumption and total income are cointegrated. Consequently, it follows from the intertemporal decisions on consumption that any shock to aggregate output that leaves consumption constant is necessarily a transitory shock. The joint observation of output growth and the log of consumption to output ratio allows then the econometrician to decompose aggregate shocks into permanent and transitory shocks, as perceived by consumers.

Second, as shown in the empirical section, the unit root can be rejected for this ratio at a conventional level and the empirical autocorrelation function indicate a less persistent process than the one of hours. Gospodinov (2006) using a near-unit root process for the hours and Christiano, Eichenbaum and Vigfusson (2004) using an exact unit root process show that a SVAR model which includes such highly persistent processes leads to a weak instrument problem. This weak instrument problem implies that technology shocks and their impacts are inconsistently estimated. Consequently, the introduction of a less persistent variable in the VAR, as the consumption-output ratio, should improve the identification of the technology shocks by avoiding the weak instrument problem. The impact of these shocks on the variable of interest (hours worked) is evaluated in the second step. To do so, hours are projected in level and in difference on the identified technology shocks series. In the applications, we also consider in the first step larger SVARs that have been used in the relevant literature (see for example, Galí, 1999, Francis and Ramey, 2005a and Christiano, Eichenbaum and Vigfusson, 2004) to check the robustness of our two-step strategy. We now present in more details the two-step approach.

Step 1: *Identification of technology shocks*

Consider a VAR(p) model which includes productivity growth $\Delta(y_t - h_t)$ and consumption to output ratio $c_t - y_t$ (in logs).[5]:

$$X_t = \sum_{i=1}^{p} B_i X_{t-i} + \varepsilon_t \tag{1}$$

[4] Another promising candidate in our sample is the log of investment to output ratio. We show in the empirical section that these two ratios deliver very similar results.

[5] Notice that we use labor productivity growth rather than output growth, as in Blanchard and Quah (1989) and Cochrane (1994). Galí (1999) shows that labor productivity growth must be preferred to output growth if there exists shocks that permanently and jointly shift the output and the labor input.

where $X_t = (\Delta (y_t - h_t), c_t - y_t)'$ and $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t})'$ with $\mathrm{E}(\varepsilon_t \varepsilon_t') = \Sigma$.[6] Under usual conditions, this VAR(p) model admits a VMA(∞) representation

$$X_t = C(L)\varepsilon_t$$

where $C(L) = (I_2 - \sum_{i=1}^{p} B_i L^i)^{-1}$ and L is the backshift operator. The structural VMA(∞) representation is given by

$$X_t = A(L)\eta_t$$

where $\eta_t = (\eta_t^T, \eta_t^{NT})'$. η_t^T is period t technology shock, whereas η_t^{NT} is period t composite non–technology shock.[7] By normalization, these two orthogonal shocks have zero mean and unit variance. The identifying restriction implies that the composite non–technology shock has no long–run effect on labor productivity. This means that the upper triangular element of $A(L)$ in the long run must be zero, *i.e.* $A_{12}(1) = 0$. In order to uncover this restriction from the estimated VAR(p) model in equation (1), the matrix $A(1)$ is obtained by the Choleski decomposition of $C(1)\Sigma C(1)'$. The structural shocks are then directly deduced up to a sign restriction by

$$\begin{pmatrix} \eta_t^T \\ \eta_t^{NT} \end{pmatrix} = C(1)^{-1}A(1)\begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}$$

Step 2: *Estimation of the response of hours to a technology shock*

The structural infinite moving average representation for hours worked as a function of the technology shock and the composite non–technology shock[8] is given by:

$$h_t = a_1(L)\eta_t^T + a_2(L)\eta_t^{NT}. \tag{2}$$

The coefficient $a_{1,k}$ ($k \geq 0$) measures the effect of the technology shock at lag k on hours worked, *i.e.* $a_{1,k} = \partial h_{t+k}/\partial \eta_t^T$.

The identifying restriction in Step 1 implies that non–technology shocks are orthogonal to technology shocks by construction, *i.e.* $\mathrm{E}(\eta_{t-i}^T, \eta_{t-j}^{NT}) = 0 \ \forall \, i,j$ and that the technology and non–technology shocks are serially uncorrelated which implies $\mathrm{E}(\eta_t^T, \eta_{t-i}^T) = 0$ and $\mathrm{E}(\eta_t^{NT}, \eta_{t-i}^{NT}) = 0 \ \ \forall i \neq 0$. These properties allow to obtain consistent estimates of the dynamic responses.

[6] To simplify the presentation, the deterministic components are omitted. Asymptotic results presented in the section are still valid with slight modifications. For example, the Ornstein-Uhlenbeck process considered below have to be replaced by a demeaned Ornstein-Uhlenbeck process when a constant term is included in the second step.

[7] See Blanchard and Quah (1989) and Faust and Leeper (1997) for a discussion on the conditions for valid shock aggregation in the small SVAR models.

[8] In typical DSGE models, non–technology shocks correspond to preference, taxes, government spending, monetary policy shocks and so on. When the number of stationary variables in the SVAR model is small respective to the number of these shocks and without additional identification schemes, these shocks are not identifiable. For our purpose, this identification issue does not matter since we only focus on the dynamic response of hours to a (permanent) technology shock.

According to the debate on the right specification of hours worked, we examine two specifications to measure the effect of a technology shock. In the first specification, hours are projected in level on the identified technology shocks while in the second specification, hours are projected in difference.

Let us now examine in more details both specifications. In the first one, the log of hours worked is regressed on the current and past values of the identified technology shocks $\widehat{\eta}_t^T$ in the first-step:

$$h_t = \sum_{i=0}^{q} \theta_i \widehat{\eta}_{t-i}^T + \nu_t \tag{3}$$

where $q < +\infty$ and $\hat{\eta}_t^T$ denotes the estimated technology shocks obtained from the SVAR model in the first step. ν_t is an error term that accounts for non–technology shocks and the remaining technology shocks. A standard OLS regression provides the estimates of the population responses of hours to the present and lagged values of the technology shocks, namely: $\widehat{a}_{1,k} = \widehat{\theta}_k$.

The log of hours worked is also regressed in first difference on the current and past values of the identified technology shocks. The response to a technology shock is now estimated from the regression:

$$\Delta h_t = \sum_{i=0}^{q} \tilde{\theta}_i \widehat{\eta}_{t-i}^T + \tilde{\nu}_t. \tag{4}$$

As hours are specified in first difference, the estimated response at horizon k is obtained from the cumulated OLS estimates: $\widehat{\tilde{a}}_{1,k} = \sum_{i=0}^{k} \widehat{\tilde{\theta}}_i$.

The two estimators $\widehat{a}_{1,k}$ and $\widehat{\tilde{a}}_{1,k}$ obtained from equations (3) and (4) are consistent estimators of $a_{1,k}$ in equation (2). The consistency is a direct consequence of the properties of technology and composite non–technology shocks, since they are mutually independent and serially uncorrelated.[9] The consistency property is obtained under the assumption that hours is a stationary process. Hours worked per capita are by definition bounded and therefore the stochastic process of this variable cannot asymptotically have a unit root even though a unit process could provide a good statistical approximation in a small sample . To derive the consistency property, only the asymptotic behavior of hours worked matters. Consequently, the consistency of the estimators $\widehat{a}_{1,k}$ and $\widehat{\tilde{a}}_{1,k}$ for both specifications is derived under the assumption that hours worked per person is a stationary process. This property of both estimators implies that the specification of hours (level or first difference) does not asymptotically matter for the estimation of the effect of a technology improvement on this variable. However, the small sample behavior of the estimators associated to the two specifications can differ, especially when hours display hight persistence. Since this is the case with actual data, it seems legitimate to further investigate this issue.

[9]See Fève and Guay (2007) for more details on this property.

In this respect, consider that the hours worked are characterized by a near-unit root process. Assume also for the ease of exposition the following simple representation of hours worked with a local-to-unity parametrization

$$h_t = \left(1 + \frac{c}{T}\right) h_{t-1} + \psi^T \left(1 - \left(1 + \frac{c}{T}\right) L\right) \eta_t^T + \psi^{NT} \eta_t^{NT}, \tag{5}$$

where the constant $c < 0$ and T denotes the number of observations.[10] The simple framework allows for a transitory effect of a technology shock on hours and a highly persistent effect of a non-technology shock in small sample. In particular, when $T \to \infty$, a non-technology shock has a permanent effect. One can easily see this by rewritting equation (5) as:

$$h_t = \psi^T \eta_t^T + \psi^{NT} \sum_{i=1}^{t} \left(1 + \frac{c}{T}\right)^{t-i} \eta_i^{NT}.$$

This simplified formulation is in line with arguments in Francis and Ramey (2005a). First, technology shocks have a transitory effect on hours worked. Second, shifts in the share in government spending, preference shocks, labor income tax rates and capital income tax rates can produce a dynamic of the labor input well approximated by a unit root process for many samples of interest (see also Gali, 2005, for similar arguments).

Now, let us examine the second step linear projection of hours in level on the contemporaneous technology shocks:

$$h_t = \theta_0 \hat{\eta}_t^T + \nu_t.$$

The OLS estimator is simply given by

$$\widehat{\theta}_0 = \frac{\frac{1}{T}\sum_{t=2}^T h_t \widehat{\eta}_t^T}{\frac{1}{T}\sum_{t=2}^T \widehat{\eta}_t^{T\ 2}}$$

and combining with equation (5) yields

$$\widehat{\theta}_0 = \psi^T - \psi^T \frac{\left(1 + \frac{c}{T}\right) \frac{1}{T}\sum_{t=2}^T \widehat{\eta}_{t-1}^T \widehat{\eta}_t^T}{\frac{1}{T}\sum_{t=2}^T \widehat{\eta}_t^{T\ 2}} + \frac{\frac{1}{T}\sum_{t=2}^T \left(1 + \frac{c}{T}\right) h_{t-1} \widehat{\eta}_t^T}{\frac{1}{T}\sum_{t=2}^T \widehat{\eta}_t^{T\ 2}} + \psi^{NT} \frac{\frac{1}{T}\sum_{t=2}^T \widehat{\eta}_t^{NT} \widehat{\eta}_t^T}{\frac{1}{T}\sum_{t=2}^T \widehat{\eta}_t^{T\ 2}}. \tag{6}$$

We can show that the asymptotic distribution of the OLS estimator is given by:

$$\widehat{\theta}_0 - \psi^T \Rightarrow \psi^{NT} \int_0^1 \widetilde{J}_c(r) dW(r),$$

where $W(r)$ and $\widetilde{W}(r)$ are two independent Brownian motions, $\widetilde{J}_c(s) = \exp(cs) \int_0^s \exp(-cr) d\widetilde{W}(r)$ is an Ornstein-Uhlenbeck process and $\Rightarrow$ signifies weak convergence.[11] The OLS estimator $\widehat{\theta}_0$ has

[10] This specification could be easily augmented by introducing lag polynomials $\psi^T(L)$ and $\psi^{NT}(L)$ with all roots outside the unit circle for technology and non–technology shocks without altering the conclusions presented here.

[11] The derivation uses the property that the variance of technology shock is normalized to unity.

then a non trivial asymptotic distribution. This distribution can be more precisely characterized knowing that the expression $\int_0^1 \widetilde{J}_c(r)dW(r)$ is distributed as a $\mathcal{N}\left(0, \int_0^1 \widetilde{J}_c(r)^2 dr\right)$ (see Stock, 1991). The estimator $\widehat{\theta}_0$ is then asymptotically centered to the true value but with a non degenerated distribution. This implies that the OLS estimator does not converge in probability to the true IRFs. Moreover, the estimator is characterized by sampling uncertainty resulting from the fact that the term $\psi^{NT} \int_0^1 \widetilde{J}_c(r)dW(r)$ is a random variable whatever the number of observations. The range of this uncertainty depends on the size of the highly persistent non–technology shock in the hours process parameterized in this simple framework by ψ^{NT}.

This problem does not occur when hours are taken in first difference. To understand why, consider the second step linear projection of hours in difference on the contemporaneous technology shocks:

$$\Delta h_t = \tilde{\theta}_0 \hat{\eta}_t^T + \tilde{\nu}_t.$$

Using equation (5), the OLS estimator of $\tilde{\theta}_0$ is given by

$$\widehat{\tilde{\theta}}_0 = \psi^T - \psi^T \frac{\left(1+\frac{c}{T}\right)\frac{1}{T}\sum_{t=2}^T \widehat{\eta}_{t-1}^T \widehat{\eta}_t^T}{\frac{1}{T}\sum_{t=2}^T \widehat{\eta}_t^{T\ 2}} + \frac{\frac{c}{T^2}\sum_{t=2}^T h_{t-1}\widehat{\eta}_t^T}{\frac{1}{T}\sum_{t=2}^T \widehat{\eta}_t^{T\ 2}} + \psi^{NT} \frac{\frac{1}{T}\sum_{t=2}^T \widehat{\eta}_t^T \widehat{\eta}_t^{NT}}{\frac{1}{T}\sum_{t=2}^T \widehat{\eta}_t^{T\ 2}}.$$

The last three terms on the RHS asymptotically vanish as $T \to \infty$. In particular, for the last two terms, this result follows from that $\sum_{t=2}^T h_{t-1}\widehat{\eta}_t^T$ is $\mathcal{O}_p(T)$ and $\frac{1}{T}\sum_{t=2}^T \widehat{\eta}_t^T \widehat{\eta}_t^{NT}$ converge in probability toward zero. The OLS estimator of ψ^T is thus asymptotically consistent.

The discussion above suggests that hours in difference must be preferred to a specification in level in the second step if we suspect a near unit root in their time series process. Indeed, when hours do not display too much persistence, so that no doubt exits about their stationarity, the two estimators are consistent. Conversely, when hours are function of a sizeable and highly persistent component and thus display a near unit root process, the estimator obtained with hours in level is asymptotically centered to the true value but does not converge in probability. Additionally, this estimator is characterized by wide confidence intervals. On the other hand, the estimator is consistent when hours are taken in first difference.

Finally, the two–step procedure is not only used to measure the effect of technology shocks on hours worked but also for hypothesis testing the significance of these responses. The approach raises two practical econometric issues. First, confidence intervals in the second step must account for the uncertainty resulting from the first step estimation. This is usually called the *generated regressors problem.*[12] Second, the residuals in the second step can be serially correlated in practice. This is especially true for the regression (3) with hours in level. Confidence intervals of IRFs are computed using a consistent estimator of the asymptotic variance-covariance of the

[12] Basu, Fernald and Kimball (2006) face the same problem of generated regressors and correct for it.

second step parameters. The consistent estimator that we use is borrowed from Newey (1984). Indeed, our two step procedure can be represented as a member of the method of moments estimators. With this representation in hand, we can derive the asymptotic variance-covariance matrix of the second step estimator.[13]

2 Empirical Results

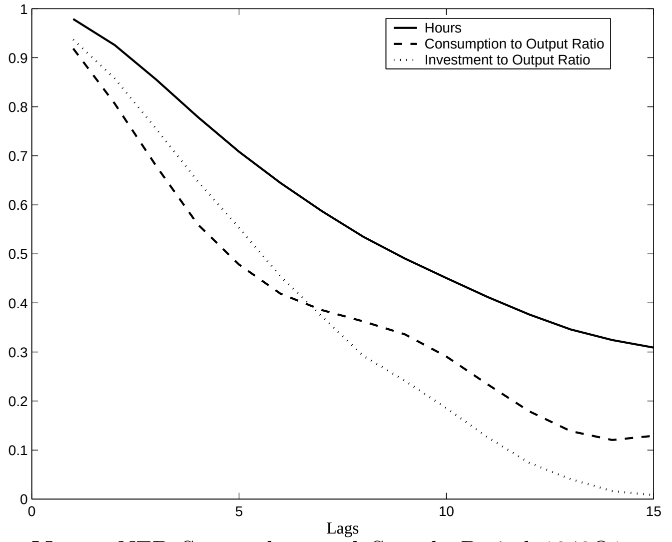
We now apply the two-step methodology to US data. The data used in the SVARs are reported in Figure 9 in the Appendix. Except for the Federal Fund rate, the data cover the sample period 1948Q1-2003Q4. We consider different measures of hours and output, bivariate VARs and larger VAR specifications, different sample periods and breaks in labor productivity.

We first present results based on a simple bivariate VAR model in the first step. This VAR model includes the growth rate of labor productivity and the log of consumption to output ratio. Labor productivity is measured as the non farm business output divided by non farm business hours worked. Consumption is measured as consumption on nondurables and services and government expenditures. The ratio is obtained by dividing these nominal expenditures by nominal GDP. In the second step, the log level h_t (see equation (3)) and the growth rate of hours Δh_t (see equation (4)) are projected on the estimated technology shocks. Hours worked in the non farm business sector are converted to per capita terms using a measure of the civilian population over the age of 16. The period is 1948Q1-2003Q4 and we will therefore refer to this as the long sample.

We also compare the estimations results with our two–step approach to those obtained from the estimation of SVAR models. As a benchmark, these SVAR models include growth rate of labor productivity and either the log level of hours (LSVAR) or the growth rate of hours (DSVAR). We have also investigated larger LSVAR and DSVAR models. In each of the SVAR models, we identify technology shocks as the only shocks that can affect the long-run level of labor productivity. The lag length p for each VAR model (1) is obtained using the Hannan–Quinn criterion. For each estimated model, we also apply a LM test to check for serial correlation. The number of lags p is 3 or 4 depending on the data and the sample. In the second step, we include the current and past twelve values of the identified technology shocks in the first step, *i.e.* $q = 13$ in equations (3) and (4). In order to assess the dynamic properties of hours worked and consumption to output ratio (in logs), we first compute their autocorrelation functions (ACFs). Figure 1 reports these ACFs for lags between 1 and 15. As this figure makes clear, the autocorrelation functions of hours worked always exceed those of the consumption to output

[13] In Appendix A, we provide more details on the implementation and computation of the consistent estimator adapted from Newey (1984).

Figure 1: ACFs



Note: NFB Sector data and Sample Period 1948Q1–2003Q4. All variables in logs.

ratio and decay at a slower rate. Additionally, we perform Augmented Dickey Fuller (ADF) test of unit root. For each variable, we regress the growth rate on a constant, lagged level and four lags of the first difference. The ADF test statistic is equal to -2.74 for hours and -2.93 for the consumption to output ratio. This hypothesis cannot be rejected at the 5 percent level for hours, whereas it is rejected at the 5 percent level for the consumption to output ratio. The ACFs and the ADF test suggest that this latter variable is less persistent than hours.

The estimated IRFs of hours after a technological improvement are reported in Figure 2. The upper left panel shows the well known conflicting results of the effect of a technology shock on hours worked between LSVAR and DSVAR specifications. The LSVAR specification displays a positive hump–shaped response whereas the DSVAR specification implies a decrease in hours. We obtain wide confidence intervals (not reported) in the LSVAR specification, such that the estimated IRFs of hours are not significantly different from zero at any horizon. For the DSVAR specification, the impact response is significant, but as the horizon increase the negative response is not significantly different from zero. The conflicting result between LSVAR and DSVAR specifications is virtually unaffected if these specifications include the log of the consumption to output ratio together with the growth rate of labor productivity and the log (level or first difference) of hours (see Figure 10 in appendix). In SVARs, the consumption to output ratio does not help to reconcile the two specifications.[14]

[14]Christiano, Eichenbaum and Evans (2004) also obtain conflicting results in larger SVARs. Furthermore, we have considered a six–variable DSVAR and LSVAR models and we still find opposite results for the two specifications. The six–variable SVAR includes labor productivity growth, hours (level or difference), consumption to output ratio, investment to output ratio, the inflation rate and the Federal Fund rate. The data concern Non Farm Business Sector and the sample Period is 1959Q1–2003Q4.

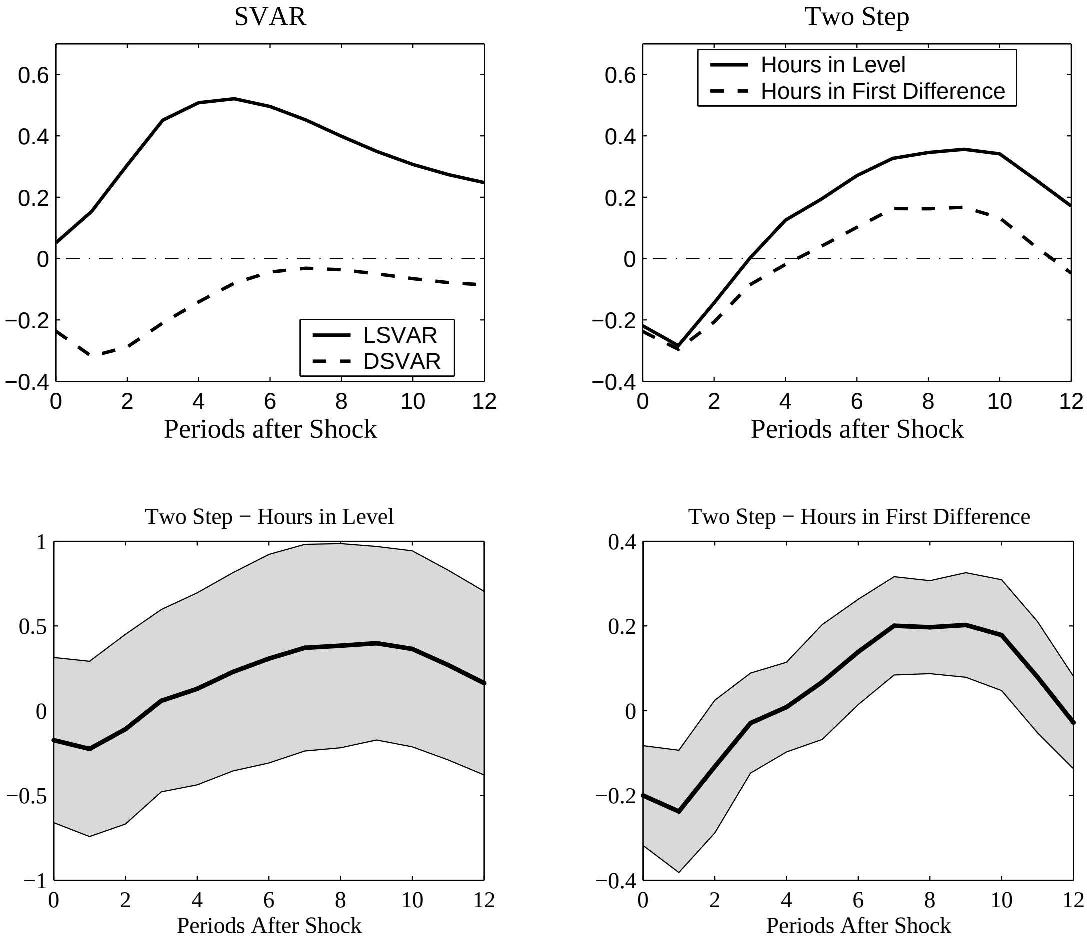
In contrast, the two-step approach delivers almost the same picture whether hours are specified in level or first difference (see the upper right panel of Figure 2). In the very short run, the IRFs of hours are identical and when the horizon increases the positive response is a bit more pronounced when hours are taken in level rather than in first difference. On impact, hours worked decrease, but after five periods the response becomes persistently positive and hump–shaped. The bottom panel of Figure 2 also reports the 95 percent asymptotic confidence interval. As previously mentioned, these confidence intervals account for the *generated regressor problem* and the serial correlation of the errors term in equations (3) and (4). The confidence interval is wide when we consider hours in level. Consequently, these responses cannot be used, for instance, to discriminate among business cycle theories. In contrast, when hours are projected in first difference, the dynamic response are very precisely estimated. On impact, hours significantly decrease. Moreover, the positive hump–shaped response after 8 quarters is precisely estimated. Notice that these findings are in accordance with the asymptotic distributions of estimators derived under the assumption that hours worked are characterized by a near-unit root process. Our empirical results are in line with those of previous empirical papers which obtain that hours fall significantly on impact (see Galí, 1999, Basu, Fernald and Kimball, 2006, Francis and Ramey, 2005a, 2005b), but display a hump–shaped positive response during the subsequent periods (see Vigfusson, 2004).

We now check the robustness of our first results to different measures of hours and output, bivariate VARs and larger VAR specifications, different sample periods and breaks in labor productivity. The results are reported in Figures (3)–(7). Figure 12 in the Appendix compares results for all cases according to the specification of hours in the second step (level and first difference) and Figure 8 at the end of this section summarizes our findings.

We first consider an alternative measure of output (labor productivity) and hours with the long sample. The alternative measure of productivity and hours is based on business sector data. Figure 3 shows that the IRFs are similar to those reported in Figure 2, especially for hours worked in first difference. Hours decrease in the short run but increase after four quarters. While the shape of the IRFs is similar[15] for both specifications (see Panel (a) in Figure 12), the estimated values differ more than the ones obtained with non farm business sector data. To understand this difference, Figure 11 in the Appendix reports the estimated response of hours from the LSVAR and DSVAR specifications for both data sets: non farm business data and business sector data. Although the DSVAR specification delivers the same response for both sets of data, the positive estimated response from LSVAR specification for the business sector is almost three times larger than for the non farm business sector. The difference between

[15]The positive IRFs of hours after two years obtained from equation (3) is more pronounced than the ones obtained using equation (4).

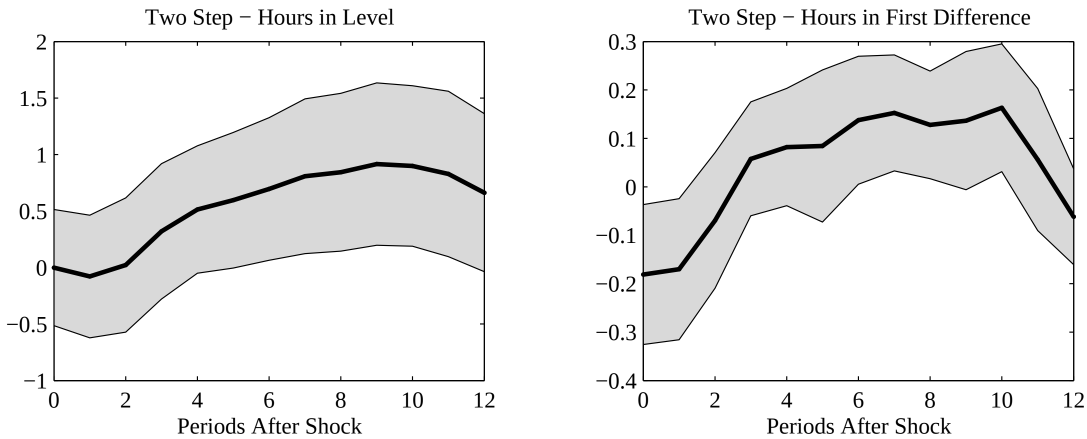
Figure 2: IRFs of Hours to a Technological Improvement (NBF data)



Note: DSVAR, LSVAR and two–step identification. The DSVAR model includes labor productivity growth and the log of hours in first difference. The LSVAR model includes labor productivity growth and the log of hours. For the two–step procedure, the SVAR model in the first step includes labor productivity growth and the log of consumption to output ratio. In the second step, the dynamic responses of hours are obtained from equations (3) and (4). Top left panel, IRFs computed from DSVAR and LSVAR specifications. Top right panel, IRFs computed from two–step procedure (equations (3) and (4)). Bottom left panel, IRFs obtained with the log of hours in level in the second step. Bottom right panel, IRFs obtained with the log of hours in first difference in the second step. Non Farm Business Sector data and sample period 1948Q1–2003Q4. The selected horizon for IRFs is 13. 95 percent asymptotic confidence interval shown.

the response of hours from the LSVAR and DSVAR specifications is then exacerbated for the business sector data. This can be explained by the time series properties of business hours worked compared to non farm business hours. To assess the persistence of these series, we test the null hypothesis of a unit root for these two measures of hours using the ADF test. The ADF test statistic is equal to -1.95 for business hours and -2.74 for non farm business hours. The null hypothesis of a unit root cannot be rejected at the 10 percent level for business hours, whereas it is rejected at the 10 percent level for the non farm business hours. As previously shown, the nearly nonstationary behavior of business hours probably drives the difference between both specifications. Notice again that the dynamic responses are not precisely estimated when hours are projected in level on the technology shocks.

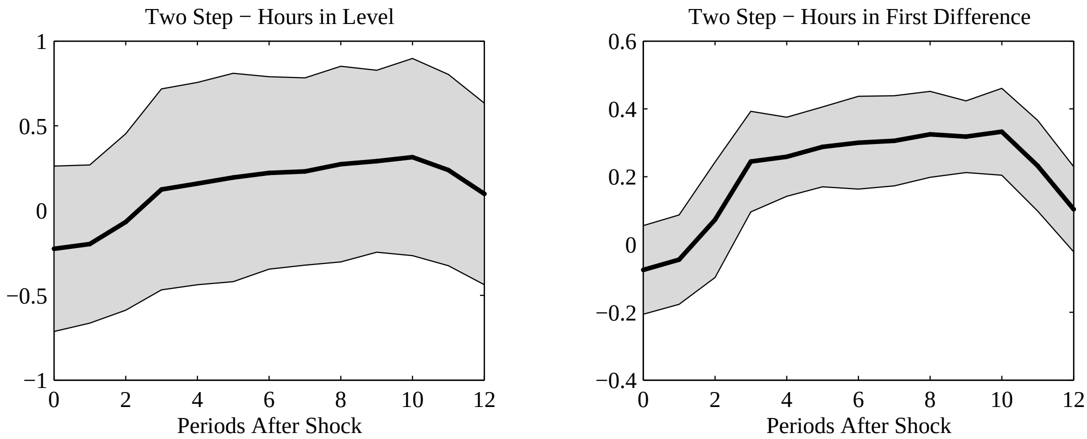
Figure 3: IRFs of Hours with Business Sector Data



Note: Two–step identification. The SVAR model in the first step includes labor productivity growth and the log of consumption to output ratio. In the second step, the dynamic responses of hours are obtained from equations (3) and (4). Left panel, IRFs obtained with the log of hours in level. Right panel, IRFs obtained with the log of hours in first difference. Business Sector data and sample period 1948Q1–2003Q4. The selected horizon for IRFs is 13. 95 percent asymptotic confidence interval shown.

We now maintain the bivariate SVAR model in the first step but replace the log of the consumption to output ratio by the log of the ratio of nominal investment expenditures to nominal GDP. Investment is measured as expenditures on consumer durables and private investment. This ratio is another promising candidate in the SVAR model, since it displays lower serial correlation than hours. Indeed, Figure 1 shows that the ACFs of the ratio are substantially lower than the ones of hours for any lag. These ACFs are very similar to the ones for the consumption to output ratio. In addition, we perform ADF test of unit root including four lags and a constant term. The ADF test statistic is equal to -3.50 for the investment to output ratio. The null hypothesis of unit root is rejected at the 1 percent level. We consider again non farm business data and the long sample.[16] Figure 4 displays the IRFs. The replacement of consumption to output ratio by the investment to output ratio does not modify the previous findings and the response of hours displays the same pattern. The two specifications yield very similar IRFs

Figure 4: IRFs of Hours using Investment to Output Ratio



Note: Two–step identification. The SVAR model in the first step includes labor productivity growth and the log of investment to output ratio. In the second step, the dynamic responses of hours are obtained from equations (3) and (4). Left panel, IRFs obtained with the log of hours in level. Right panel, IRFs obtained with the log of hours in first difference. Non Farm Business Sector data and sample period 1948Q1–2003Q4. The selected horizon for IRFs is 13. 95 percent asymptotic confidence interval shown.

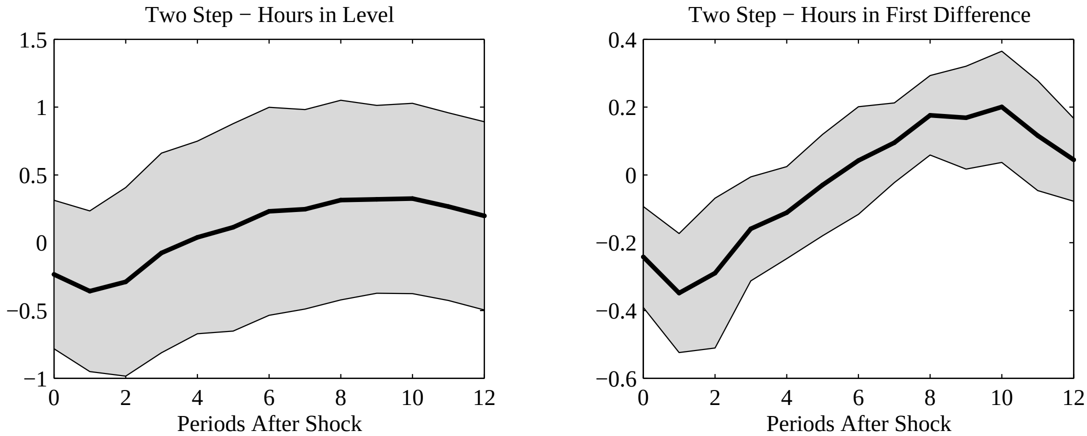
for hours (see Panel (b) in Figure 12) and again the confidence intervals are wide when hours are considered in level. Notice that the negative impact response in not significantly different from zero with hours in first difference. Moreover, the positive hump–shaped pattern of hours is precisely estimated.

We now examine the robustness of the two-step strategy using a larger VAR system in the first step. We maintain non farm business data for labor productivity and hours and we use the long sample. The SVAR model in the first step includes labor productivity growth, consumption to output ratio, investment to output ratio and the rate of inflation. The measure of inflation is obtained using the growth rate of the GDP deflator. Results are reported in Panel (a) of Figure 5. The IRFs are very similar to those of Figure 2. Moreover, IRFs are close for both specifications (see Panel (c) in Figure 12). Again the specification with hours in difference in the second step delivers precise estimates of the IRFs: hours significantly decrease in the short–run, but positively increases after two years. Conversely, the confidence interval with hours in level is so wide that results obtained with this specification are not very informative. Using this larger system, the exercise is repeated with a shorter sample. Since much of business cycle

[16]We obtain similar results (not reported) with business sector output and hours.

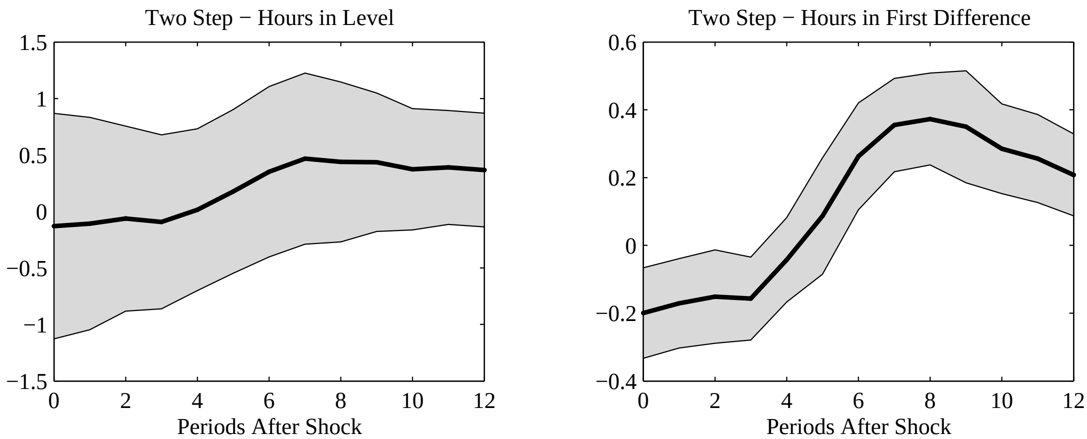
Figure 5: IRFs of Hours with a Four Variable System

Panel (a). NFB Sector data and Sample Period 1948Q1–2003Q4



Note: Two–step identification. The SVAR model in the first step includes labor productivity growth, the log of consumption to output ratio, the log of investment to output ratio and the rate of inflation. In the second step, the dynamic responses of hours are obtained from equations (3) and (4). Left panel, IRFs obtained with the log of hours in level. Right panel, IRFs obtained with the log of hours in first difference. Non Farm Business Sector data and sample period 1948Q1–2003Q4. The selected horizon for IRFs is 13. 95 percent asymptotic confidence interval shown.

Panel (b). NFB Sector data and Sample Period 1959Q1–2003Q4

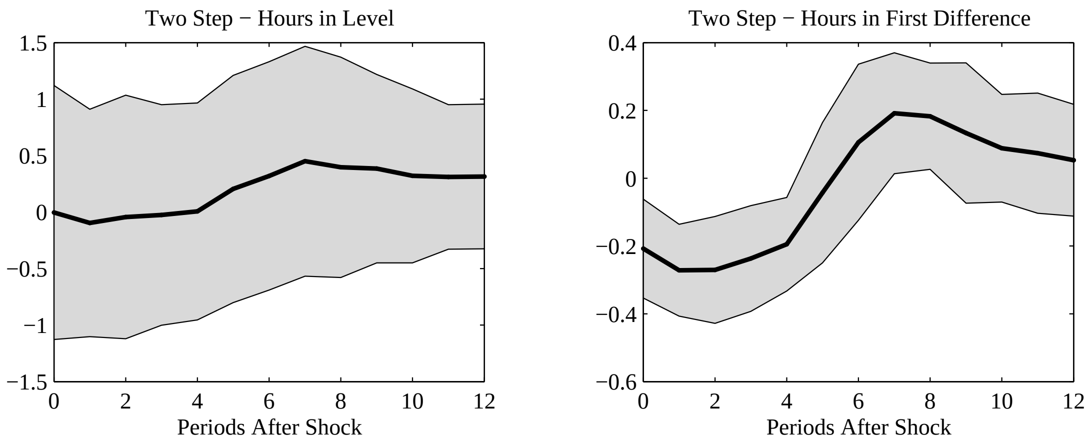


Note: Two–step identification. The SVAR in the first step includes labor productivity growth, the log of consumption to output ratio, the log of investment to output ratio and the rate of inflation. In the second step, the dynamic responses of hours are obtained from equations (3) and (4). Left panel, IRFs obtained with the log of hours in level. Right panel, IRFs obtained with the log of hours in first difference. Non Farm Business Sector data and sample period 1959Q1–2003Q4. The selected horizon for IRFs is 13. 95 percent asymptotic confidence interval shown.

literature is concerned with post–1959 data, we follow Christiano, Eichenbaum and Vigfusson (2004) and therefore consider a second sample period given by 1959Q1–2003Q4. Panel (b) of Figure 5 reports the estimated responses. We obtain again the same shape for the IRFs previously obtained from a level and a first difference specification of hours (see Panel (d) in Figure 12). The negative responses in the short–run differ slightly according to the specification of hours, but the two IRFs become positive and very close after five periods. The difference in the two IRFs can be explained by the higher persistence of the hours series for this shorter sample. Indeed, the ADF test statistic is equal to -2.47 for the short sample compared to -2.74 for the long sample. Again, the response of hours is precisely estimated when hours are taken in difference. This is not the case for the level specification which appears less informative.

We also add the federal fund rate in the larger system and consider the short sample 1959Q1–2003Q4. The results are reported in Figure 6. The negative response of hours is more pronounced in the short run compared to the previous cases (when hours are taken in first difference), but we still find a persistent increase in the subsequent periods. Notice that the response of hours differs according to their specification, but the shapes of the two IRFs remain very similar (see Panel (e) in Figure 12). As for other cases, the confidence intervals for the level specification are larger but the difference in the confidence intervals between both specifications is here amplified.

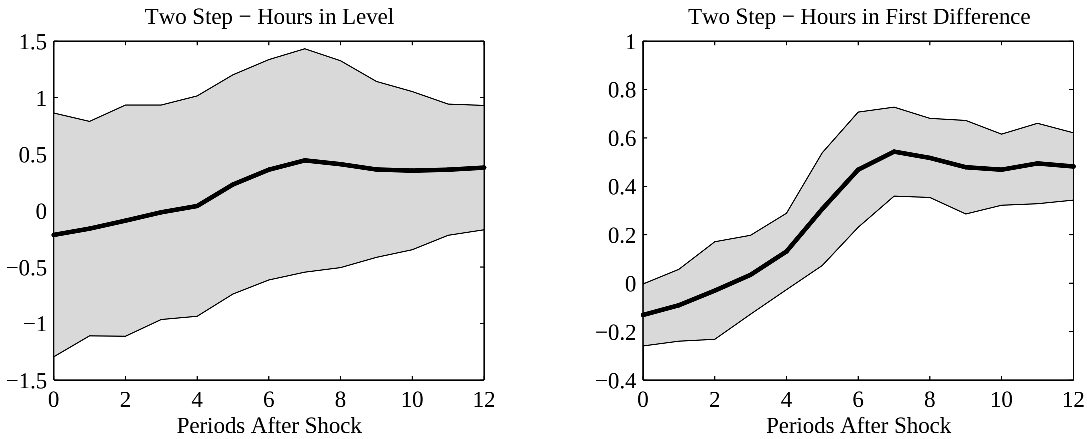
Figure 6: IRFs of Hours with a Five Variable System



Note: Two–step identification. The SVAR model in the first step includes labor productivity growth, the log of consumption to output ratio, the log of investment to output ration, the inflation rate and the Federal Fund rate. In the second step, the dynamic responses of hours are obtained from equations (3) and (4). Left panel, IRFs obtained with log of hours in level. Right panel, IRFs obtained with log of hours in first difference. NFB Sector data and sample period 1959Q1–2003Q4. The selected horizon for IRFs is 13. 95 percent asymptotic confidence interval shown.

As last experiment, we investigate the sensitivity of our results to structural breaks in labor productivity. We consider this issue in the context of the latter experiment. Fernald (2005) shows that once we allow for trend breaks in labor productivity, the response of hours to a technology shock in the LSVAR model becomes persistently negative.[17] The breaking dates identified by Fernald are 1973Q1 and 1997Q2. Labor productivity growth is first regressed on a constant, a pre–1973Q1 dummy variable and a pre–1997Q1 dummy variable. The residuals of this regression are then used as a new measure of labor productivity growth in the first step. The responses of hours are reported in Figure 7. The response appears unaffected as the negative response on impact is around -0.2 (see Figure 6 for a comparison). Moreover, the hump–shaped and delayed–positive response is maintained for both specifications (see Panel (f) in Figure 12) and is significant for the specification in difference. A possible explanation of the robustness to

Figure 7: IRFs of Hours with Breaks in Labor Productivity



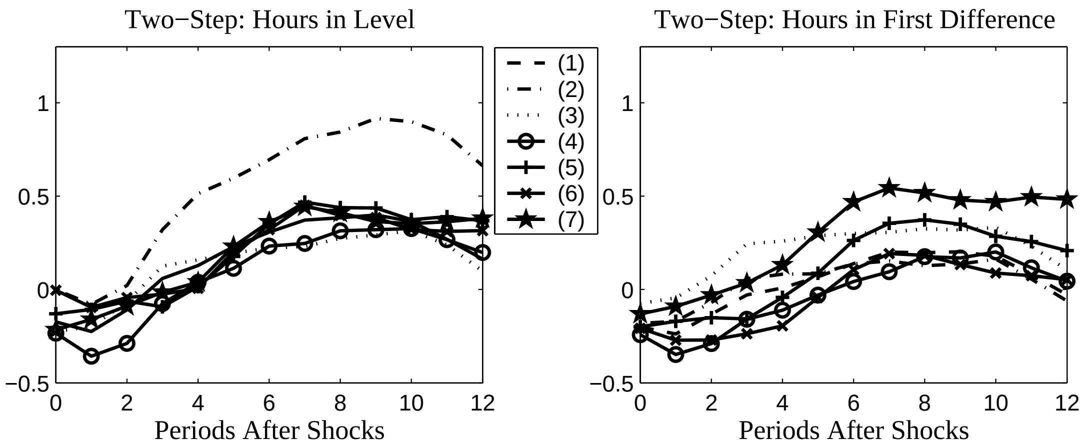
Note: Two–step identification. The SVAR model in the first step includes labor productivity growth, the log of consumption to output ratio, the log of investment to output ratio, the inflation rate and the Fed Fund rate. The breaking dates are 1973Q1 and 1997Q2. The new measure of labor productivity growth is obtained as the residual of the regression of the original measure on dummy variables associated to breaks. In the second step, the dynamic responses of hours are obtained from equations (3) and (4). Left panel, IRFs obtained with the log of hours in level. Right panel, IRFs obtained with the log of hours in first difference. NFB Sector data and Sample Period 1959Q1–2003Q4. The selected horizon for IRFs is 13. 95 percent asymptotic confidence interval shown.

potential breaks is the following. The response of hours to a technology shock in the LSVAR specification is sensitive to time variations, *i.e.* breaks in labor productivity. These breaks alter the low frequency correlation between hours and labor productivity, but does not modify the one between consumption to output ratio and labor productivity. Since hours are eliminated from the VAR model in the first step, our approach seems to be more immune to structural time

[17]Gambetti (2005) finds similar results in a Time–Varying Coefficients Bayesian VARs.

variations.

Figure 8: Summary of the Results



Note: Two–step procedure. The SVAR model in the first step includes: (1) labor productivity growth and the log of consumption to output ratio; Non Farm Business Sector data and sample period 1948Q1–2003Q4; (2) labor productivity growth and the log of consumption to output ratio; Business Sector data and sample period 1948Q1–2003Q4; (3) labor productivity growth and the log of investment to output ratio; Non Farm Business Sector data and sample period 1948Q1–2003Q4; (4) labor productivity growth, the log of consumption to output ratio, the log of investment to output ratio and the rate of inflation; Non Farm Business Sector data and sample period 1948Q1–2003Q4; (5) labor productivity growth, the log of consumption to output ratio, the log of investment to output ratio and the rate of inflation; Non Farm Business Sector data and sample period 1959Q1–2003Q4; (6) labor productivity growth, the log of consumption to output ratio, the log of investment to output ratio, the rate of inflation and the Federal Fund rate; Non Farm Business Sector data and sample period 1959Q1–2003Q4; (7) labor productivity growth with breaks, the log of consumption to output ratio, the log of investment to output ratio, the rate of inflation and the Federal Fund rate; Non Farm Business Sector data and sample period 1959Q1–2003Q4. In the second step, the dynamic responses of hours are obtained from equations (3) and (4). The selected horizon for IRFs is 13. Asymptotic confidence interval not reported.

Finally, Figure 8 compares the dynamic responses of hours worked for all cases examined above. The results when hours are specified in level are reported in the left panel of this Figure, while the ones with a specification of hours in first difference are in the right panel. As this figure shows, the dynamic responses of hours in all cases and for both specifications are remarkably similar. In the very short–run, hours decrease after a technology improvement. After some period, hours gradually increase and display a hump–shaped pattern. This finding does not vary too much with different sample periods, variables included in the VAR model at the first step and structural breaks in labor productivity.[18] That seems to confirm the robustness of our proposed two-step strategy and the appeal of this alternative simple approach for further empirical investigations.

[18]One exception concerns the dynamic responses with Business Sector data and the level specification of hours. However, the results with the difference specification is not sensitive to the measure of output (labor productivity) and hours. Notice that this result is already present in SVARs.

3 Concluding Remarks

This paper uses a simple two-step approach to consistently estimate technology shocks and the responses of hours worked after a technology improvement. In a first step, a SVAR model with labor productivity growth and the log of consumption to output ratio (or a set of relevant covariance–stationary variables) allows us to identify and estimate technology shocks. In a second step, the response of hours is obtained by a simple regression of hours on the estimated technology shocks. When hours worked are characterized by a stationary process, we obtain that the dynamic responses of hours are consistently estimated, whatever their specification (level and first difference) in the second step. We also show that when hours are characterized by a highly persistent process (near unit–root process with a local to unity parametrization) the level specification leads to an estimator which does not asymptotically converge in probability to the true dynamic responses. Conversely, the estimator obtained with a first difference specification is asymptotically consistent. The two-step approach, when applied to US data, predicts a short–run decrease of hours after a technology improvement, as well as a delayed and hump–shaped positive response. The dynamic responses of hours are precisely estimated with a first difference specification, whereas their confidence intervals are wide with a level specification. These differences in the estimated confidence intervals can be explained by the asymptotic distribution derived when hours are characterized by a near-unit root process. Theses findings appear robust to different sample periods,measures of hours and output and to the variables included in the VAR model in the first step. The proposed approach is devoted here to the estimation of the responses of hours worked. However, this empirical strategy can easily be used to evaluate the effect of a technology shock on other persistent aggregate variables.

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Appendix

A Computation of the estimator for the asymptotic covariance matrix in our two-step approach

Following Newey (1984), our sequential two step estimators can be rewritten as a set of moment conditions with a recursive structure. First consider a method of moment estimator based on the population moment conditions

$$E\left[f(x_t, \beta_0)\right] = 0.$$

The corresponding empirical moment conditions

$$\frac{1}{T}\sum_{t=1}^{T}\left[f(x_t, \beta)\right],$$

can be used to obtain a method of moments estimator $\widehat{\beta}$ by setting these sample moments as close as possible to zero (see Hansen, 1982). Now, consider the partition of the parameter vector β as $\beta = (\theta', \lambda')'$ so that

$$f(x_t, \beta) = \left(g(x_t, \theta)', h(x_t, \theta, \lambda)'\right)'$$

where $g(x_t, \theta)$ and $h(x_t, \theta, \lambda)$ are respectively the corresponding population moment conditions of the first and the second step estimations. In our application, $g(x_t, \theta)$ is given by the orthogonality conditions of the VAR model (1), namely:

$$g(x_t, \theta) = \mathcal{Z}_{t-1} \otimes \varepsilon_t(\theta)$$

where $\mathcal{Z}_{t-1}$ is a vector which includes a constant and the lagged values up to order p of labor productivity in difference and the set of relevant stationary variables (consumption to output ratio, investment to out ratio,...). The second set of moment conditions $h(x_t, \theta, \lambda)$ corresponds to the orthogonality conditions of the OLS estimation (equations (3) and (4) in our setup) given by

$$h(x_t, \theta, \lambda) = W_t(\theta) \times \nu_t(\theta, \lambda)$$

where the vector $W_t(\theta)$ contains a constant and the identified technology shocks in the first-step which depends on θ.

Let now defines $F = E\left[f_\beta(x_t, \beta_0)\right]$ as the derivative of the population moment conditions respective the the true parameter vector β_0 and $V = E\left[f(x_t, \beta_0)f(x_t, \beta_0)'\right]$ as the covariance matrix of the population moment conditions evaluated at the true value β_0. Let partition F and V be conformable with β and $f(x_t, \beta)$, so that,

$$F = \begin{pmatrix} G_\theta & 0 \\ H_\theta & H_\lambda \end{pmatrix}$$

and

$$V = \begin{pmatrix} V_{gg} & V_{gh} \\ V_{hg} & V_{hh} \end{pmatrix},$$

with, for example, $H_\theta = E\left[\partial h(x_t, \theta_0, \lambda_0)/\partial\theta\right]$ and $V_{gh} = [g(x_t, \theta_0)h(x_t, \theta_0, \lambda_0)']$.

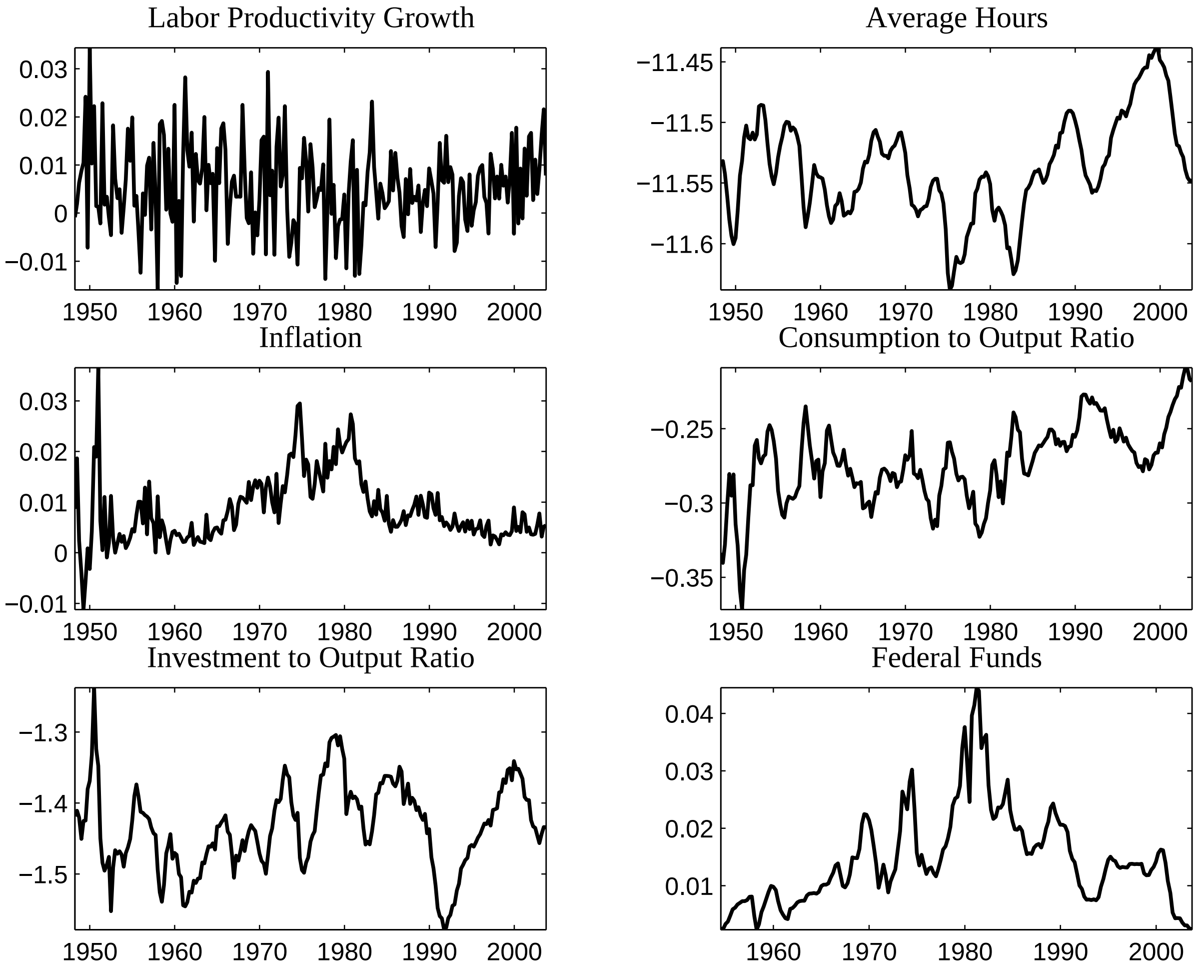
Newey (1984) shows that the asymptotic covariance matrix of the second step estimator is given by the following expression:

$$\Omega_\lambda = H_\lambda^{-1}V_{hh}H_\lambda^{-1\prime} + H_\lambda^{-1}H_\theta\left(G_\theta^{-1}V_{gg}G_\theta^{-1\prime}\right)H_\theta' H_\lambda^{-1\prime} - H_\lambda^{-1}\left(H_\theta G_\theta^{-1}V_{gh} + V_{hg}G_\theta^{-1\prime}H_\theta'\right)H_\lambda^{-1\prime}.$$

The first term of this expression corresponds to the usual covariance matrix of second step estimators. The second and the third terms correct for the generated regressors problem involved in the first step estimation.

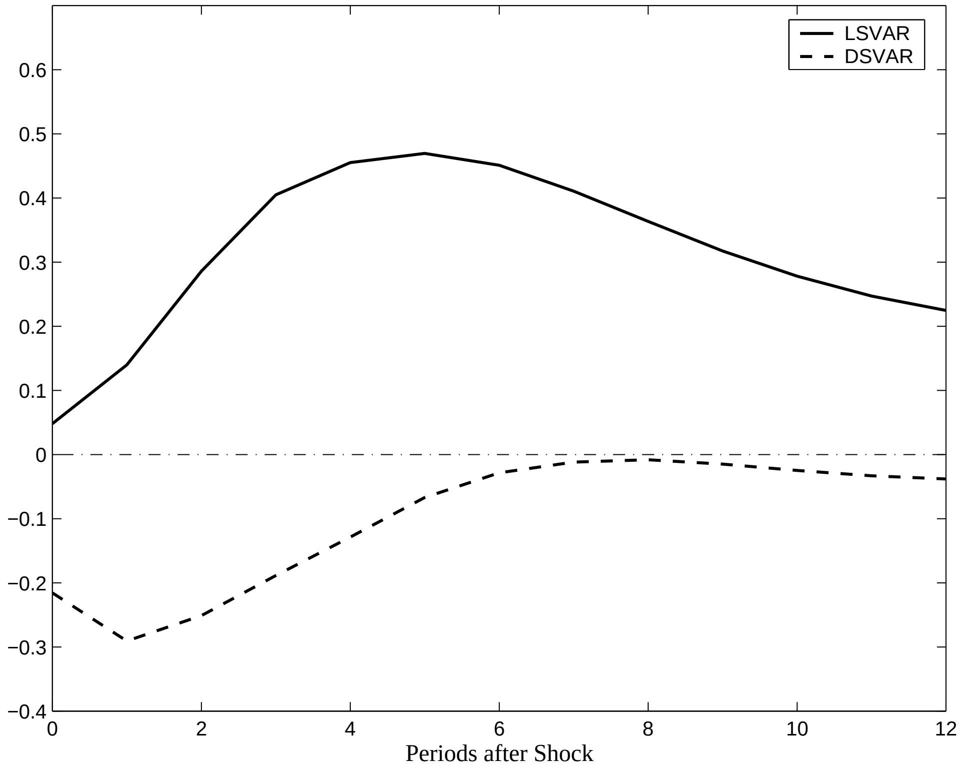
A consistent estimator of the asymptotic covariance matrix can be obtained with a consistent estimator of each terms. For the VAR model at the first step with a sufficient number of lags, the moment conditions corresponding to this step are serially uncorrelated, the variance covariance matrix is thus given by an estimator of $\Sigma \otimes \mathcal{Z}_{t-1}'\mathcal{Z}_{t-1}$. We can also easily show that the estimator of the terms V_{gh} and V_{hg} does not need be adjusted for serial correlation. A consistent estimator of the asymptotic covariance matrix of the second step moments conditions V_{hh} which are probably serially correlated can be obtained with the usual Newey and West (1994) estimator.

Figure 9: Variables used in SVARs



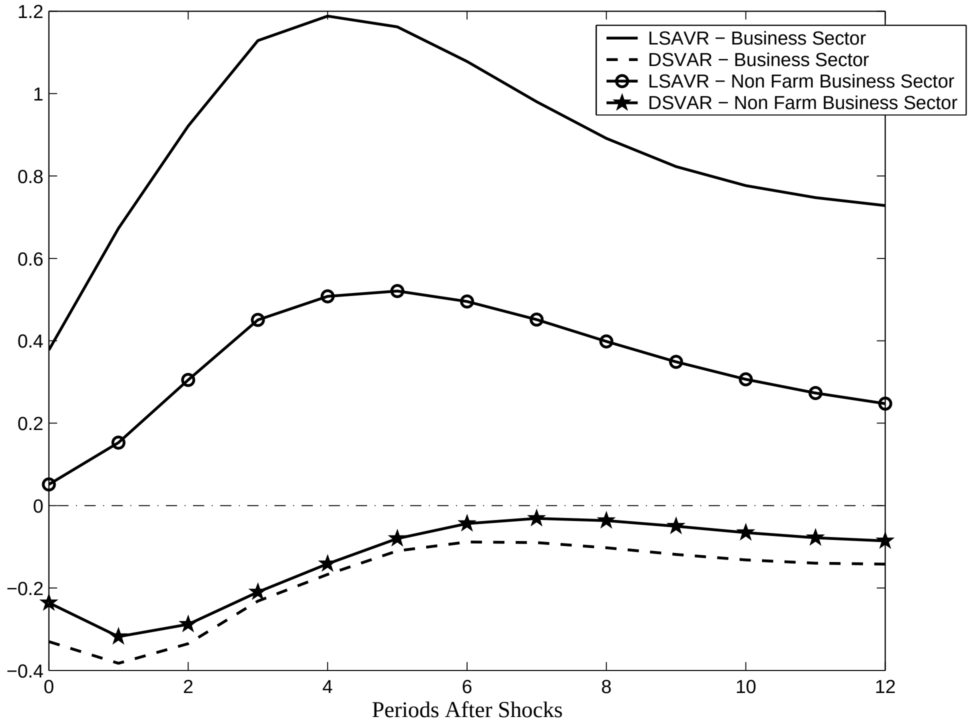
Note: NFB Sector data and Sample Period 1948Q1–2003Q4, except for Federal Fund rate.

Figure 10: Three–Variables SVARs



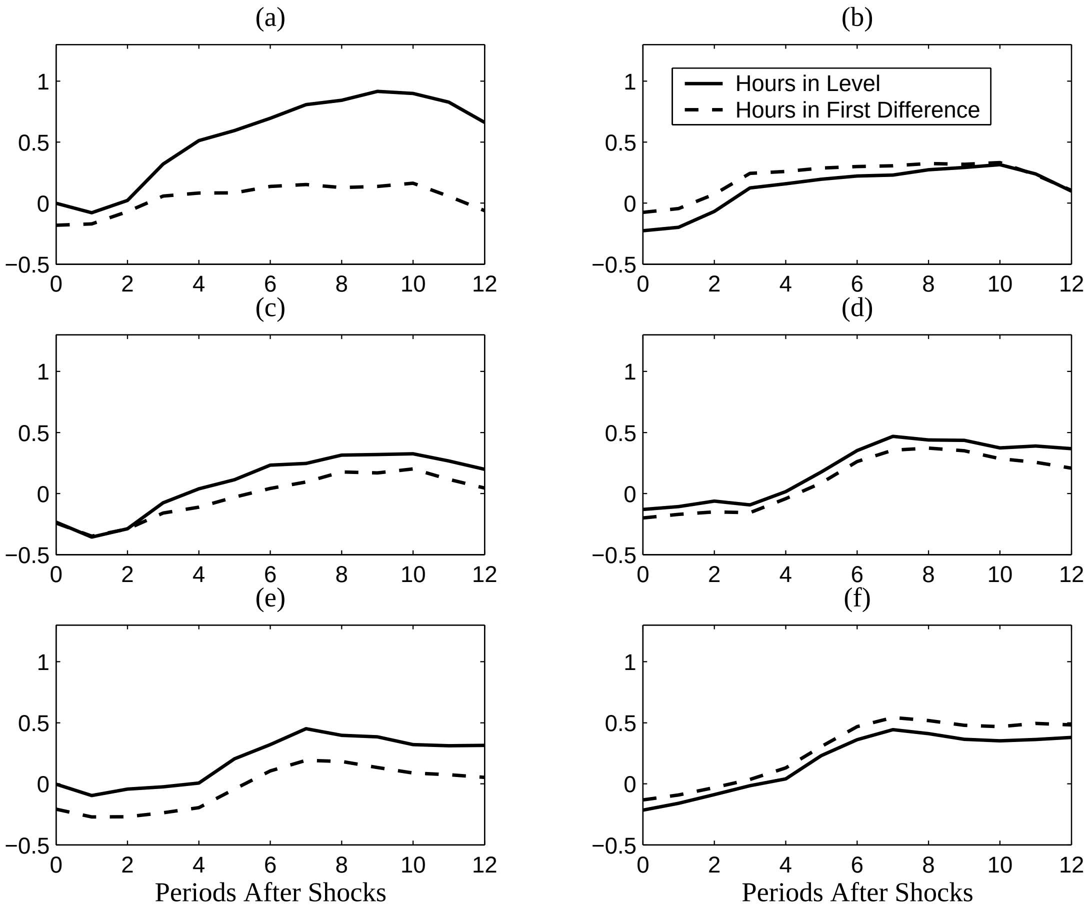
Note: The DSVAR model includes labor productivity growth, the log of consumption to output ratio and the log of hours in first difference. The LSVAR model includes labor productivity growth, the log of consumption to output ratio and the log of hours in level. Non Farm Business Sector data and sample period 1948Q1–2003Q4. The selected horizon for IRFs is 13. Asymptotic confidence interval not reported.

Figure 11: SVARs with Business Sector Data



Note: The DSVAR model includes labor productivity growth and the log of hours in first difference. The LSVAR model includes labor productivity growth and the log of hours in level. Business Sector data, Non Farm Business Sector data and sample period 1948Q1–2003Q4. The selected horizon for IRFs is 13. Asymptotic confidence interval not reported.

Figure 12: IRFs of Hours for Different Estimations



Note: Two–step procedure. The SVAR model in the first step includes: Panel (a) labor productivity growth and the log of consumption to output ratio; Business Sector data and sample period 1948Q1–2003Q4; Panel (b) labor productivity growth and the log of investment to output ratio; Non Farm Business Sector data and sample period 1948Q1–2003Q4; Panel (c) labor productivity growth, the log of consumption to output ratio, the log of investment to output ratio and the rate of inflation; Non Farm Business Sector data and sample period 1948Q1–2003Q4; Panel (d) labor productivity growth, the log of consumption to output ratio, the log of investment to output ratio and the rate of inflation; Non Farm Business Sector data and sample period 1959Q1–2003Q4; Panel (e) labor productivity growth, the log of consumption to output ratio, the log of investment to output ratio, the rate of inflation and the Federal Fund rate; Non Farm Business Sector data and sample period 1959Q1–2003Q4; Panel (e) labor productivity growth with breaks, the log of consumption to output ratio, the log of investment to output ratio, the rate of inflation and the Federal Fund rate; Non Farm Business Sector data and sample period 1959Q1–2003Q4. In the second step, the dynamic responses of hours are obtained from equations (3) and (4). The selected horizon for IRFs is 13. Asymptotic confidence interval not reported.