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Liability Insurance under the Negligence Rule

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Abstract:
We analyze the efficiency properties of the negligence rule with liability insurance, when the tort-feasor's behavior is imperfectly observable both by the insurer and the court. Efficiency is shown to depend on the extent to which the evidence is informative, on the evidentiary standard for finding negligence, and on whether insurance contracts can condition directly on the same evidence as used by courts to assess behavior. When evidence is not directly contractible, the negligence rule with compensatory damages is generally inefficient and can be improved by decoupling liability from the harm suffered by the victim.

Keywords: Negligence, liability insurance, evidentiary standard, moral hazard, judicial error, decoupling, prudence

JEL Classification: D82, K41, K42

1 Introduction

We consider the negligence rule combined with liability insurance when courts, as well as insurers, have imperfect information about injurers' behavior. Assessing precautionary behavior is often difficult, for instance when professional liability is involved. In the much discussed case of medical malpractice, it has been suggested that court error induces "defensive medicine" (e.g., Kessler and McClellan, 1995), conversely that the combination of court error and liability insurance leads to underprecaution (e.g., Danzon, 1985, and Harrington and Danzon, 2000). We analyze the conditions under which the negligence rule together with liability insurance is consistent with efficient risk sharing and precaution levels.

Shavell (1982) showed that liability insurance is socially beneficial under the strict liability rule even with moral hazard. Victims are then fully compensated and are therefore indifferent to the probability of accident. Since it increases the utility of the insured, liability insurance is socially desirable whether or not precautionary behavior is observable. There are no comparable results regarding the effect of liability insurance under the negligence rule. Indeed, the usual assumption has been that courts could ex post perfectly ascertain the injurer's precautions. If the injurer is found to have exerted less than due care, he is held liable for full compensatory damages, otherwise he escapes liability. With due care set at the socially efficient level, potential injurers will undertake the appropriate precautions and be sure of avoiding liability. Accordingly, there is no demand for liability insurance.

We extend the analysis of the negligence rule to situations where the injurer's level of care is imperfectly observable. Court error has been discussed in this context but only for the case of risk neutral injurers, thereby precluding a demand for liability insurance (Diamond, 1974; Calfee and Craswell, 1986; Shavell, 1987; Kolstad, Ulen and Johnson, 1990; Edlin, 1994). Even in this simple set-up, however, no simple conclusion seems to emerge from the literature. One reason is the failure to introduce explicitly the legal concept of evidentiary standard, which refers to the "weight of evidence" for establishing negligence and differs from the notion of due care. An exception is

Fluet (2006) who characterizes the efficient evidentiary standard for inducing due care.

In the present paper, we also discuss evidentiary standards. Our setting, however, is different. As courts may err, risk averse injurers will purchase liability insurance. The consequence is a two-level incentive problem: tort rules impose liability risks on the injurer-insurer pair, resulting in a liability insurance contract; the contract itself imposes "penalties" on the injurer (through deductibles and the like), which induces precautionary behavior. The issue is whether the arrangement is socially efficient in terms of risk sharing and of precautions to prevent harm. Since under the negligence rule third parties are not always compensated, Shavell's argument on the efficiency of liability insurance does not apply. We show that efficiency depends on the extent to which the evidence is informative, on the evidentiary standard for establishing negligence, and on whether insurance contracts can condition directly on the same information as used by courts to assess behavior.

We first consider the case of contractible evidence. The ex post evidence about care, on which court decisions are based, is then assumed to be directly contractible under the liability insurance policy, irrespective of the form of the liability rule. A separation result then obtains between the role of legal liability and that of insurers. Given the liability risk, the insurance contract provides the optimal trade-off, under moral hazard, between risk-bearing and incentives to take precautions. Legal liability, by contrast, serves to provide the injurer-insurer pair with the incentives to design a contract inducing an appropriate level of care. We characterize the set of efficient liability rules and show that the negligence rule with the appropriate evidentiary standard belongs to this set, thereby extending Fluet (2006) to risk aversion. Moreover, the characterization of the efficient evidentiary standard, in terms of the relation between due care and evidence about care, is the same as in the risk-neutral case.

Next we consider the more realistic situation where the detailed evidence used by courts to reach a decision is not fully contractible. For instance, courts typically weigh many different testimonies to assess whether the injurer exerted due care. In practice, it is not feasible to write down ex ante into the insurance contract all possible evidentiary outcomes. For simplicity, we assume that the liability insurance policy can condition only on court decisions. We show that, generally speaking, the negligence rule then yields an inefficient allocation. However, efficiency can be restored through a modified negligence rule whereby court imposed damages differ from the victim's loss, i.e., if "decoupling" is allowed. In an efficient rule, punitive damages are then associated with a relatively demanding evidentiary standard, undercompensatory damages with a relatively weak one. We provide a partial characterization of when punitive rather than undercompensatory damages are best. This depends on the nature of the likely evidence and on the injurers' attitude with respect to risk, e.g., their risk aversion and prudence.

In the literature, "decoupled" liability has been justified on many grounds, usually in the context of the strict liability rule. A well known result is the need for punitive damages when injurers are not always identified or when victims do not always sue (Polinsky and Shavell, 1998, inter alia). Other reasons include limited liability problems (Lewis and Sappington, 1999) or the trade-off between incentives to sue, with the resulting litigation costs, and the injurer's incentives to exert care (Polinsky and Che, 1991). In our analysis, the benefits from decoupling stem from the interaction between the provision of appropriate incentives to the injurer-insurer pair and the provision of a "useful" signal for designing the liability insurance contract.

We stress that court error in our model is due solely to imperfect information about the injurer's behavior. In the risk-neutral literature referred to above, court error under the negligence rule is also often ascribed to mistakes about the due care level, the injurer's action being itself observable without error. This is the approach followed by Sarath (1991) in her analysis of liability insurance under the negligence rule. In that paper, the injurer's precautions are revealed ex post during litigation, hence moral hazard is not an issue in designing the liability insurance contract. It may be added that the emphasis in Sarath's paper is the trade-off between litigation costs (through incentives to sue) and incentives to exert care, given some liability rule. By contrast, our focus is the characterization of efficient rules in a context where litigation costs are negligible.

The paper develops as follows. Section 2 presents the basic set-up. Section 3 describes the equilibrium under arbitrary damage rules. In section 4, we assume the evidence is contractible, provide a characterization of efficient damage rules, and discuss the negligence rule. Section 5 considers the case where the liability insurance contract can condition only on court decisions. Section 6 concludes. The proofs of propositions are in the appendix unless the argument is obvious from the text.

2 The model

The basic framework is borrowed from Shavell (1982). There is a large population of identical potential injurers and an equal population of identical potential victims. An injurer can accidentally harm at most one victim. The following notation is used (subscripts denote partial derivatives, e.g., $U_w = \partial U/\partial w$):

l	=	victims' monetary loss if there is an accident;
p	=	probability of accident;
$e(p)$	=	injurers' effort on accident prevention, $e' < 0$, $e'' > 0$;
v	=	wealth of victims;
w	=	wealth of injurers;
$U(w, e)$	=	utility of injurers, $U_w > 0$, $U_{ww} < 0$, $U_e < 0$, $U_{ee} \geq 0$;
t	=	unconditional transfer from injurers to victims.

Both injurers and victims are risk averse. The precautions taken by a potential injurer — his level of care — are reflected in p or equivalently $e(p)$ and are private information. We assume $e'(1) = 0$, $e'(0) = -\infty$ to ensure interior solutions with $0 < p < 1$. Our formulation for the injurer's utility function encompasses both the separable and non-separable forms with either $U(w, e) = u(w) - e$ or $U(w, e) = u(w - e)$. We assume the validity of the first-order approach in deriving the optimal insurance contracts.

Injurers have sufficient wealth to pay for a victim's loss, that is, there is no limited liability problem. While an injurer's care is not directly verifiable, some information about his behavior becomes available ex post following the occurrence of an accident. This is represented by a signal x with density function $f(x,p)$ on the support $[0,1]$ and corresponding cumulative $F(x,p)$, i.e., precautions determine the distribution of the signal. The random variable x should be interpreted as a "summary" of all the detailed evidence available ex post.[1]

Assumption 1: *$f_p(x,p)/f(x,p)$ is strictly decreasing in x.*

The assumption is the monotone likelihood ratio property (MLRP) with the convention that large values of x constitute "favorable" evidence in the sense of suggesting high care. The condition implies $F_p(x,p) > 0$ except at the boundaries of the support where the derivative is nil. In words, the probability of unfavorable evidence (realizations below any given threshold x) increases as precautions decrease. Restricting the support of f to the unit interval simplifies notation and is without loss of generality.

Assumption 2: *$pF(x,p)$ is convex in p.*

Assumption 1 implies that the probability $pF(x,p)$ of "accident and unfavorable evidence" increases in p, i.e., with lower levels of precautions. Assumption 2 means that this is so at an non decreasing rate.[2]

Injurers may be held liable for the harm imposed on third-parties. A liability regime can in all generality be represented by a damage rule $D(x)$. This specifies the amount of damages paid to the victim by the injurer — or his liability insurer — when an accident occurs and the ex post evidence is x. Strict liability and negligence are specific forms of damage rules.

Under strict liability, the injurer is always held liable for full compensatory damages upon the occurrence of harm, which amounts to $D(x) = l$ for all x. Under the negligence rule, the injurer is liable for full compensatory

[1] If multidimensional evidence about the injurer's precautions can be ranked in terms of "more favorable than", there exists an exhaustive scalar statistic x satisfying MLRP (Milgrom 1981).

[2] The assumption amounts to the Convexity of the Distribution Function Condition (or CDFC, see Rogerson, 1985) with respect to the event "accident and unfavorable evidence". A sufficient condition is $F_{pp} \geq 0$.

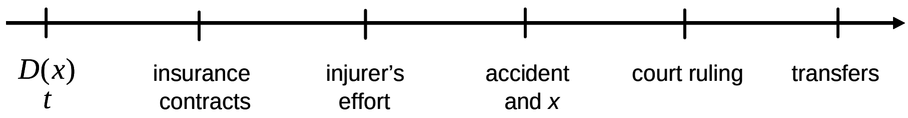

Figure 1: Time line

damages only if he is found to have exerted less than due care. We assume that, in applying this rule, the court is provided with some $\widehat{p}$ as due-care standard. One interpretation is that $\widehat{p}$ reflects a social norm regarding conduct and corresponds to what would normally be expected from potential injurers. Care being unobservable, the court must decide whether sufficient care was exerted on the basis of imperfectly informative evidence. It will find negligence — that is, will rule that the defendant's behavior led to $p > \widehat{p}$ — if the evidence is sufficiently unfavorable. Denoting by $\widehat{x}$ the court's evidentiary standard for finding negligence, the damage function is then of the form

$$D(x) = \begin{cases} l & \text{if } x < \widehat{x}, \\ 0 & \text{if } x \geq \widehat{x}, \text{ where } \widehat{x} \in (0,1). \end{cases} \tag{1}$$

The set-up is henceforth as follows. Society chooses the liability regime. It can also impose a lump-sum unconditional transfer t from potential injurers to potential victims.[3] Given the liability regime, potential injurers and potential victims contract with risk-neutral insurers. Insurance markets are competitive and coverage is sold at a fair price. Injurers purchase liability insurance, victims purchase first-party coverage against the risk that an injurer is not held liable in full. Injurers then choose effort. When an accident occurs, the evidence is revealed. There are no litigation costs and parties have symmetric information about the ex post evidence. They can therefore perfectly anticipate the court's decision and it is indifferent whether they settle or go to trial. For expository convenience, we assume that a trial always takes place. Following the trial, transfers are made according to the damage

[3] The transfer could be negative. As in Shavell (1982), when injurer and victim are in a producer-consumer relationship, $-t$ can also be interpreted as the price paid by the consumer for a unit of service.

rule and the insurance contracts. Figure 1 summarizes the time line.

3 Market equilibria

Denote by w_0 and v_0 the initial wealth of injurers and victims respectively. In equilibrium, victims expect to suffer the loss l with probability p and to be compensated according to $D(x)$. Since they are risk-averse, they purchase complete first-party coverage against the risk of uncompensated losses, therefore paying the insurance premium

$$\Delta(p) = pl - p \int_0^1 D(x) f(x,p)\, dx. \tag{2}$$

Given the lump-sum transfer, their equilibrium net wealth is

$$v \equiv v_0 + t - \Delta(p). \tag{3}$$

Consider now the injurers' problem. The liability insurance contract entails a premium π and a transfer schedule $I(\cdot)$. Following an accident and the realization of the evidence, the injurer-insurer pair pays the victim the damages $D(\cdot)$ imposed by the court, $I(\cdot)$ being the amount supported by the insurer. We write the transfer from the insurer as $I(z(x))$ where the function $z(x)$ depends on the contractibility of the ex post evidence. We consider two possibilities. In the first case, the evidence x is fully contractible under the liability insurance policy and $z(x) \equiv x$. In the second case, transfers from the insurer can only be contingent on court decisions, a constraint that is captured by $z(x) \equiv D(x)$.

Contractible evidence refers to a situation where x directly becomes available to the liability insurer, irrespective of the liability regime, and consists of verifiable facts with respect to which a contract can be written. When a lawsuit is filed, the same facts are made available to the court if the prevailing tort rule requires it to establish damages. By contrast, under non-contractible evidence, the liability insurance contract cannot directly condition payments on the same evidence as used by courts to reach a decision. The motivation is that the evidence is often very detailed, e.g., the particular testimony of a particular witness with a particular credibility. While courts routinely assess the weight that should be given to such evidence and rule accordingly, some evidentiary outcomes are presumably too complex to be described ex ante in the insurance policy.[4] For simplicity, we consider only extreme cases: the evidence is either fully contractible or non contractible at all.

Since insurance markets are competitive, insurers earn zero profits in equilibrium and the liability insurance contracts maximize the injurers' utility subject to the zero profit condition. Given the damage rule, the contract solves

$$\begin{aligned}\max_{p,\pi,I} EU &= (1-p)U\left(w_0 - t - \pi, e(p)\right) \\ &\quad + p\int_0^1 U\left(w_0 - t - \pi - D(x) + I(z(x)), e(p)\right) f(x,p)\, dx. \quad (4)\end{aligned}$$

subject to

$$\pi - p\int_0^1 I(z(x)) f(x,p)\, dx \geq 0, \quad (5)$$

$$EU_p = 0. \quad (6)$$

Equation (5) is the insurer's non negative profit condition, equation (6) is the incentive compatibility condition. The solution to this problem yields p, π and $I(\cdot)$ such that (5) holds as an equality.

We now turn to the allocations induced by a market equilibrium under some given damage rule. Let $w \equiv w_0 - t - \pi$ and $S(\cdot) \equiv D(\cdot) - I(\cdot)$. The injurer's expected utility becomes

$$EU = (1-p)U\left(w, e(p)\right) + p\int_0^1 U\left(w - S(z(x)), e(p)\right) f(x,p)\, dx. \quad (7)$$

Substituting from (2) and (3), the non negative profit condition (5) rewrites as

$$v + \left(w - p\int_0^1 S(z(x)) f(x,p)\, dx\right) \leq w_0 + v_0 - pl. \quad (8)$$

[4] A standard justification (e.g., Hart and Moore, 1999) is that it would be too costly to write a contingent contract with respect to all possible "states of the world", hence the insurance contract is "incomplete".

An equilibrium yields an allocation that is completely characterized by $(v, w, S(\cdot), p)$ where p is the risk of loss, v is the victims' net wealth, injurers' expected utility is given by (7) and the expected total wealth per victim-injurer pair satisfies (8). The latter is the economy's resource constraint on an average per-capita basis. The right-hand side is the average net wealth per pair of injurer-victim, taking accident costs into account. The left-hand side is the sum of the victims' guaranteed wealth and of what a potential injurer is allocated on average.

Observe that $S(\cdot)$ amounts to a penalty imposed on the injurer. The allocation generated by a damage rule could therefore also be considered as resulting from a "direct penalty scheme" chosen by a regulator, provided penalties are non insurable. Clearly, any equilibrium allocation can be replicated by a direct penalty scheme. A key question is whether a regulator can do better. In this view, the set of feasible allocations is defined by the resource constraint (8) and the incentive compatibility constraint (6) with the injurer's expected utility defined as in (7). Obviously, one must also specify whether the direct penalty functions considered are constrained by some given $z(x)$.

When evidence is contractible, direct penalty schemes are of the form $S(x)$. The relevant benchmark set of feasible allocations then does not depend on the damage rule. An equilibrium is efficient if no direct penalty scheme Pareto dominates it. In the next section we characterize efficient damage rules when the evidence is contractible. The case of non contractible evidence is analyzed in section 5.

4 Efficient damage rules under contractible evidence

From a welfare point of view, damage rules can be compared on the basis of the equilibria they generate. We define a damage rule to be efficient if it is not Pareto dominated by another damage rule under an appropriate transfer t. Note that we define efficiency by comparison with other damage rules.[5] Note also that damage rules can be compared in terms of the Pareto criterion only by allowing appropriate modifications of the transfer: rules have redistributive implications in addition to providing incentives to exert care.

We first provide a condition for a damage rule to yield an equilibrium at least as good as some given feasible allocation, where the latter may result either from a direct penalty scheme or from some arbitrary damage rule.

Proposition 1 *Any feasible allocation with* $(\widehat{p}, \widehat{v})$ *is weakly Pareto dominated by the equilibrium under the damage rule* $D(x)$ *and transfer* t*, where* $D(x)$ *is any rule such that* $\widehat{p}$ *maximizes* $\Delta(p)$ *and* $t = \Delta(\widehat{p}) + \widehat{v} - v_0$*.*

To see the intuition, suppose that the injurer chooses $\widehat{p}$ in the equilibrium under the rule $D(x)$. In this equilibrium, the victim fully insures against non compensated losses. Recalling (2), his premium for first-party coverage is therefore $\Delta(\widehat{p})$. It follows that the victim's final wealth is $\widehat{v}$ if the unconditional transfer t satisfies

$$\widehat{v} = v_0 + t - \Delta(\widehat{p}).$$

Now, consider the possibility that in equilibrium the injurer in fact chooses $p \neq \widehat{p}$. Since the damage rule is such that $\Delta(p) \leq \Delta(\widehat{p})$, the victim will then pay a smaller premium for full coverage, so that his final wealth will be larger than $\widehat{v}$. Hence, any deviation from $\widehat{p}$ on the part of the injurer benefits the victim. Since no arrangement between injurer and liability insurer can make the victim worse off than $\widehat{v}$, a liability insurance contract that maximizes the injurer's expected utility, subject to $D(x)$ and the insurer's non negative profit condition, yields an allocation at least as good as the initial one.

In particular, if the initial allocation is itself optimal (within the set of feasible allocations under a direct penalty scheme), then it will be implemented by a damage rule and transfer as specified in the proposition. In other words, if $\widehat{p}$ is part of an optimal allocation, then with an appropriate

[5] This is not quite the same as defining a damage rule to be efficient only if it yields an efficient allocation within the set of feasible allocations under a direct penalty scheme.

transfer the optimum can be "decentralized" through a damage rule $D(x)$ satisfying

$$\widehat{p} \in \arg\max_p \Delta(p) = pl - p \int_0^1 D(x) f(x,p)\, dx. \tag{9}$$

Obviously, if a damage rule implements an optimum, it is also efficient in terms of our definition of efficient rules. Proposition 1 therefore yields a sufficient condition for a damage rule to be efficient, in the sense of allowing implementation of an optimum as an equilibrium.[6]

A damage rule satisfying (9) always exists. In fact, one such rule is strict liability with $D(x) = l$ for all x. The victims' uncompensated expected loss, $\Delta(p)$, is then identically zero. Hence, it is maximized by any optimal probability $\widehat{p}$. Strict liability is therefore an efficient rule. Moreover, since any optimal allocation is implementable by a damage rule, we also have the following:

Corollary 1 *A damage rule is efficient if, and only if, it implements an efficient allocation.*

One may ask whether the "premium maximization condition" in proposition 1 is necessary. Specifically, can a damage rule be efficient if the equilibrium probability of loss under the rule does not maximize the victim's premium? We show necessity under a minor additional condition.

Proposition 2 *Let $\widehat{p}$ be part of an equilibrium allocation under a non increasing damage rule $D(x)$. If the equilibrium is efficient, then $\widehat{p}$ maximizes $\Delta(p)$.*

At equilibrium, the victim's premium $\Delta(\widehat{p})$ is the gap, as a function of the injurer's precautions, between the true expected loss and expected damages. The gap describes the extent to which the damage rule does not internalize upon the injurer-insurer pair the full expected harm suffered by third parties.

[6] Our result bears a similarity with Schweizer (2004) who examines sufficient conditions for post-law payoff functions to induce an efficient Nash equilibrium in the bilateral care problem with observable effort.

In the proof of the proposition we show that, if the equilibrium is efficient, then $\Delta'(\widehat{p}) = 0$, that is,

$$\frac{\partial (pl)}{\partial p} = \frac{\partial \left(p \int_0^1 D(x) f(x,p)\, dx \right)}{\partial p}, \text{ for } p = \widehat{p}. \tag{10}$$

In words, even though compensatory damages need not always be paid by the injurer-insurer pair, the damage rule must provide the same marginal incentives as under complete internalization.[7] When $D(x)$ is non increasing, $\Delta(p)$ is concave, (10) therefore ensures that $\Delta(p)$ is maximized at $\widehat{p}$.

The condition that $D(x)$ is non increasing in x has a straightforward interpretation. By assumption large values of x constitute "favorable" evidence suggesting high care. When a liability rule takes into consideration the injurer's likely level of care, the court should presumably not impose larger damages as more favorable evidence is obtained.

The negligence rule. The foregoing propositions fully characterize the set of efficient damage rules when the evidence is contractible. Strict liability has been shown to be one such rule. We henceforth discuss whether the negligence rule also belongs to the efficient set.

Under the negligence rule, recalling (1),

$$\Delta(p) = p\,(1 - F(\widehat{x}, p))\, l.$$

From proposition 1, if $\widehat{p}$ is part of an optimal allocation, the allocation can be implemented under the evidentiary standard $\widehat{x}$ solving

$$\widehat{p} = \arg\max_p \, p\,(1 - F(\widehat{x}, p))\,. \tag{11}$$

Conversely, proposition 2 applies since damages are non increasing in x under a negligence rule. Thus, if $\widehat{p}$ is part of an equilibrium under the rule with evidentiary standard $\widehat{x}$, the equilibrium is efficient only if $\widehat{x}$ solves (11). Note, however, that the standard must satisfy $\widehat{x} < 1$, otherwise the rule would amount to strict liability.

[7] Under appropriate convexity conditions, the condition in proposition 2 is also sufficient for an equilibrium to be efficient (see our discussion of the negligence rule).

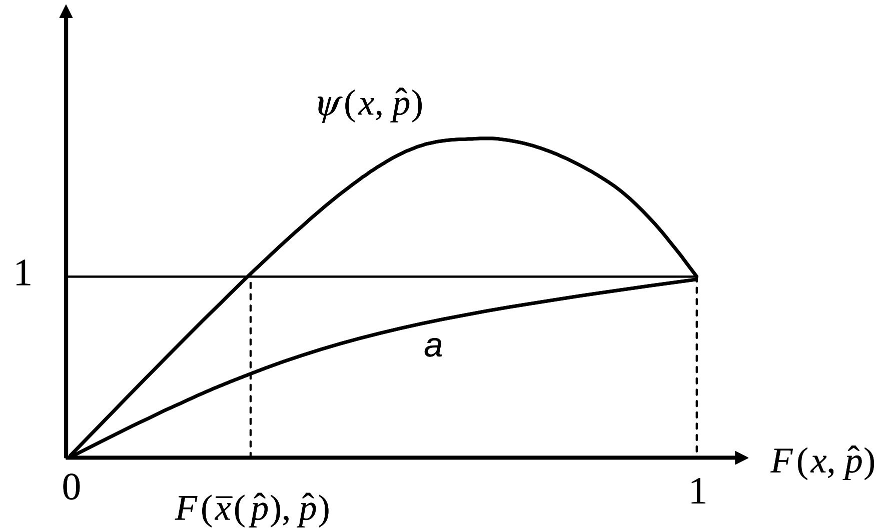

Figure 2: Evidentiary standards

Indeed, an important consideration for assessing the efficiency of the negligence rule is the fact that such a standard may not exist. To see this, define

$$\psi(x,\widehat{p}) \equiv F(x,\widehat{p}) + \widehat{p}F_p(x,\widehat{p}) = \frac{\partial}{\partial p}\left(pF(x,p)\right)\bigg|_{p=\widehat{p}}. \tag{12}$$

Due to assumption 2, (11) is equivalent to the first-order condition $\psi(\widehat{x},\widehat{p}) = 1$. Obviously, $\psi(0,p) = 0$ and $\psi(1,p) = 1$. Thus, (11) is solved by $\widehat{x} = 1$. The issue is whether there are other solutions. In figure 2, ψ is drawn as a function of $F(x,\widehat{p})$, a positive monotonic transformation of x. The curve is concave since

$$\frac{d\psi(x,\widehat{p})}{dF(x,\widehat{p})} = 1 + \widehat{p}\frac{f_p(x,\widehat{p})}{f(x,\widehat{p})},$$

$$\frac{d^2\psi(x,\widehat{p})}{dF(x,\widehat{p})^2} = \frac{\widehat{p}}{f(x,\widehat{p})}\frac{\partial}{\partial x}\left(\frac{f_p(x,\widehat{p})}{f(x,\widehat{p})}\right) < 0.$$

The figure depicts two possibilities. If ψ is as drawn, then a solution $\widehat{x} < 1$ exists and it is clearly unique. We denote this solution by $\overline{x}(\widehat{p})$, i.e., $\overline{x}(\widehat{p})$ solves (11) and is less than unity. $F(\overline{x}(\widehat{p}),\widehat{p})$ is then the probability, under the efficient evidentiary standard, that an injurer exerting due care is erroneously found negligent and may be referred to as the type 1 error. By contrast, $\overline{x}(\widehat{p})$ does not exist if ψ is a curve such as a. The implication is then that the optimum cannot be implemented by the negligence rule.

The difference between the two curves in the figure has to do with the informational quality of the evidence. In a situation such as depicted by curve a, the evidence provides relatively poor information about the injurer's level of care.[8] By Holmström's (1979) information principle, an optimal direct penalty scheme should nevertheless condition on the information, i.e., $S(x)$ should not be constant. However, the evidence is then too poor for a negligence rule to provide appropriate incentives, although this is always feasible under the strict liability rule as shown above. Thus, there is a difference between the value of information in the incentives-risk allocation trade-off under moral hazard and its value for assigning liability. The next proposition summarizes our results.

Proposition 3 *When the post accident evidence is directly contractible, any optimum is implementable, given adequate transfers, (i) by the strict liability rule, (ii) by the negligence rule under appropriate due care and evidentiary standards, provided the evidence is sufficiently informative.*

Note that implementing an efficient allocation is nevertheless always feasible under a modified negligence with decoupling, i.e., one that allows non compensatory damages. From proposition 1, given an arbitrary standard $\widehat{x}$, an efficient $\widehat{p}$ is implementable with the punitive or undercompensatory damages $\overline{D}$ satisfying

$$\widehat{p} \in \arg\max_{p} \Delta(p) = pl - pF(\widehat{x}, p)\overline{D}.$$

From the first-order condition, the appropriate amount of damages solves $\psi(\widehat{x}, \widehat{p})\overline{D} = l$. In particular, when the evidence is insufficiently informative so that $\psi(\widehat{x}, \widehat{p}) < 1$ for all $\widehat{x} < 1$, efficiency under any arbitrary evidentiary

[8] The evidence in some situation A is more informative than in situation B if the ψ curve in A is above that in B. This ranking of "information systems" follows from Demougin and Fluet (2001). See the proof of proposition 3.

standard is possible with some level of punitive damages $\overline{D} > l$, as long as we do not run into limited liability problems.[9]

In proposition 3, the strict liability and negligence rules (the latter with some proviso) are shown to be efficient because they can implement any given optimal allocation, subject to appropriate transfers. One may also ask whether the equilibrium under such rules is an efficient allocation irrespective of transfers, i.e., for any arbitrary t. It is trivial to show that the equilibrium under strict liability is efficient irrespective of t. Whether this is also true under the negligence rule, however, requires additional conditions even when the evidence is sufficiently informative.

Specifically, take t as given, and suppose $\widehat{p}$ is part of the equilibrium under the negligence rule with evidentiary standard $\overline{x}(\widehat{p})$. The necessary condition of proposition 2 is satisfied, but can we conclude that the equilibrium is efficient? The following provides sufficient conditions. Denote by $\widehat{\mathcal{U}}$ the injurer's expected utility in this equilibrium. Now, consider direct penalty schemes and let $W(p, \widehat{\mathcal{U}})$ be the minimum net wealth that must be given to the injurer, subject to his utility being $\widehat{\mathcal{U}}$ when some arbitrary p is to be implemented. From the resource constraint, the victim's net wealth in such schemes is easily seen to equal

$$w_0 + v_0 - pl - W(p, \widehat{\mathcal{U}}).$$

The equilibrium described above is therefore efficient if $\widehat{p}$ minimizes $pl + W(p, \widehat{\mathcal{U}})$. The latter can be shown to hold if $W(p, \widehat{\mathcal{U}})$ is differentiable and convex in p.[10] Observe that $W(p, \widehat{\mathcal{U}})$ would be convex in p if effort were perfectly observable. However, under moral hazard, the cost of implementing some p depends on the property of the signal x at that level of effort. Since the signal may be more or less "informative" at different effort levels, non convexity is a possibility.

[9] The punitive part may be assumed to be retained by the victim's insurer as compensation for a reduced first-party insurance premium. Alternatively, it can be paid to the state and redistributed as non-conditional transfers.

[10] The conditions imply that p is part of an efficient allocation if it solves $l = -W_p(p, \widehat{\mathcal{U}})$. The proof of proposition 2 shows that the latter holds at the equilibrium $\widehat{p}$.

To conclude the section, we remark that the evidentiary standard for establishing negligence can be given an interesting interpretation. Suppose courts view the efficient $\widehat{p}$ as reflecting the legal due care standard, i.e., the minimum level of precautions an injurer should have taken to escape a ruling of negligence. From the above argument, we know that the court should rule that less than due care was exerted if the evidence satisfies $x < \overline{x}(\widehat{p})$. Now, consider an outsider who does not know the detailed evidence but is informed of the court's decision. For this outsider, and using standard statistical terminology, $p\,(1 - F(\overline{x}(\widehat{p}), p))$ is the *likelihood* of care level p knowing that an accident occurred and that the injurer was not found negligent. Thus, the evidentiary standard is efficient if the outsider's maximum likelihood estimate of p is then precisely $\widehat{p}$. Moreover, the evidence is sufficiently informative for the negligence rule to implement $\widehat{p}$ if, and only if, such an evidentiary standard exists.

5 Non contractible evidence

In the above analysis, an efficient damage rule leads the injurer-insurer pair to behave as if it supported the full social costs of accidents. What matters is the expected damages as a function of the injurer's precautions. In particular, court decisions play no informational role in the design of liability insurance contracts since the ex post evidence is verifiable irrespective of the liability regime. Indeed, given appropriate transfers, the same allocation can be obtained under very different tort rules, e.g., the negligence rule or strict liability.

The latter does not hold when the evidence is not directly contractible. With insurance contracts constrained by court decisions, the transfer from the liability insurer is then of the form $I[D(x)]$. In addition to providing incentives to exert care, court decisions now generate relevant information for contracting purposes. If the damage rule is a generalized $D(x)$ strictly increasing in x, all underlying information is of course indirectly revealed and the results are the same as before. We therefore consider the case where $D(x)$ does not span the detailed evidence.

Our motivation is the negligence rule. Apart from the mere occurrence of an accident, contractible information under such a rule consists of the signal "liable-not liable" defined by the evidentiary standard for a ruling of negligence. Such a signal belongs to the family of binary signals of the form

$$z_{\widehat{x}}(x) = \begin{cases} 0 & \text{if } x < \widehat{x}, \\ 1 & \text{if } x \geq \widehat{x}, \text{ where } \widehat{x} \in [0,1]. \end{cases} \tag{13}$$

While a binary signal is motivated by the negligence rule, other damage rules are consistent with a similar information structure. Our aim is to assess how well the negligence rule fares within this restricted set of rules. We therefore consider the set of "binary damage rules", now written as

$$D(z_{\widehat{x}}(x)) = \begin{cases} \overline{D} & \text{if } z_{\widehat{x}}(x) = 0, \\ \underline{D} & \text{if } z_{\widehat{x}}(x) = 1. \end{cases}$$

The notation makes explicit that a rule is characterized by a contractible signal and by the damages imposed on the injurer for each realization of the signal. The negligence rule corresponds to a signal with $\widehat{x} < 1$ and damages $\underline{D} = 0$, $\overline{D} = l$; strict liability is the degenerate case with $\widehat{x} = 1$. Of particular interest, as will become clear, is the modified negligence rule with decoupling, i.e., with $\widehat{x} < 1$, $\underline{D} = 0$ and $\overline{D} \neq l$.

As before, a damage rule is efficient if it is not dominated by another rule, but now within the set of binary damage rules.[11] In discussing the efficiency of an allocation, we accordingly also limit consideration to direct penalty schemes based on binary information structures. The efficiency of a damage rule now implies two things: (i) the equilibrium allocation is Pareto undominated within the set of direct penalty schemes based on the signal associated with the rule; (ii) there exists no other damage rule, with a different binary signal, supporting a better allocation.

Concerning point (i), it should be clear that, for a given signal $z_{\widehat{x}}$, propositions 1 and 2 still hold: it suffices to replace the original signal x by $z_{\widehat{x}}$. In this view, the propositions compare optimal and equilibrium allocations

[11] Rules generating a richer signal can do better.

with the same information structure. Taking $z_{\widehat{x}}$ as given, consider an optimal direct penalty scheme with respect to this signal and denote by $\widehat{p}$ the probability induced by the scheme. According to proposition 1, the same allocation can be implemented by a damage rule $D(z_{\widehat{x}})$ such that

$$\widehat{p} \in \arg\max_{p} \Delta(p) = pl - pF(\widehat{x}, p)\overline{D} - p\left(1 - F(\widehat{x}, p)\right)\underline{D}. \tag{14}$$

Conversely, according to proposition 2, if the equilibrium under the rule $D(z_{\widehat{x}})$ is undominated within the set of allocations associated with the same signal $z_{\widehat{x}}$, then the equilibrium probability maximizes $\Delta(p)$.

While the foregoing clarifies point (i), it says nothing about (ii) which refers to the possibility that one damage rule may be better than another because it generates a "better" signal. Before addressing what "better" means, it is useful to discuss the relation between efficient rules and optimal direct penalty schemes, considering all possible binary signals. From the foregoing results, the allocation obtained under any optimal scheme is implementable by a damage rule with respect to the same signal if it satisfies (14). We remark that such a rule always exists, the argument being the same as in section 4. In particular, condition (14) is satisfied by the modified negligence rule with $\overline{D}$ solving $\psi(\widehat{x}, \widehat{p})\overline{D} = l$, where $\psi(\widehat{x}, \widehat{p})$ is defined as in (12). It follows that corollary 1 also applies in the present context, i.e., a damage rule is efficient if, and only if, it yields an efficient allocation.

Moving to the comparison of signals, we next show that there is no global ordering of binary signals. We prove this for the case where the injurers' utility function is separable, i.e., $U(w, e) = u(w) - e$.[12] In the proposition that follows, a signal is said to be preferred to another, for inducing some probability $\widehat{p}$, if it can sustain Pareto superior allocations. Define

$$A(w) = -\frac{u''(w)}{u'(w)}, \quad P(w) = -\frac{u'''(w)}{u''(w)}.$$

A is the injurer's coefficient of absolute risk aversion, P his degree of prudence as defined in Kimball (1990). An agent is *prudent* if u''' is positive.

[12]The literature on the comparison of information structures in agency problems has also dealt exclusively with the separable case.

Proposition 4 *Assume $U(w,e) = u(w) - e$ and consider the set of direct binary penalty schemes inducing $\widehat{p}$. Generically, for any $x' \in (0,1)$ there exists $x'' \neq x'$ such that: (i) if $P < 3A$, the signal defined by $\widehat{x} = \min(x', x'')$ is preferred to the one defined by $\widehat{x} = \max(x', x'')$; (ii) the converse holds if $P > 3A$.*

When the agent is risk averse, a more informative signal sustains strictly superior allocations because it improves the trade-off between risk-sharing and the provision of incentives (e.g., Holmström, 1990, and Kim, 1995). Clearly, a signal generated by some $\widehat{x} \in (0,1)$ is more informative than the degenerate signal with $\widehat{x} = 1$ and is therefore preferred. The proposition shows, by contrast, that the non degenerate signals themselves cannot be ranked in terms of the information criterion: they merely partition differently the same underlying information. As a result, preferences over signals depend on characteristics of the utility function other than risk aversion.[13]

Strict liability yields the least informative signal about the injurer's behavior. One would therefore expect that rules generating a more informative signals, such as the negligence rule, fare better. However, there is an additional problem here since a damage rule must also provide the appropriate social incentives to the injurer-insurer pair. From this perspective, we know that strict liability fully internalizes the externality. On the other hand, for reasons similar to the ones underlying proposition 3, it may not be feasible to provide appropriate incentives with the negligence rule unless the underlying evidence is sufficiently informative. In addition, we show that, even when the negligence rule fares better than strict liability, it is usually not an efficient rule.

Proposition 5 *When evidence is non contractible, strict liability is inefficient. In particular, it is dominated by the negligence rule provided the evidence is sufficiently informative. The negligence rule itself is generally dominated by a modified negligence rule allowing decoupling.*

[13] To illustrate, when the utility function is of the form $u(w) = \operatorname{sgn}(\gamma)\, w^{\gamma}$ with $\gamma < 1$, $P \lesseqgtr 3A$ if $\gamma \lesseqgtr \frac{1}{2}$.

By corollary 1, a damage rule is efficient only if it implements an efficient allocation (now within the set of allocations supported by binary signals). The inefficiency of strict liability therefore follows trivially from the foregoing discussion.[14]

The second claim concerns the possibility that the negligence rule constitutes an improvement with respect to strict liability. The argument is as follows. Suppose the equilibrium under strict liability is characterized by $\widehat{p}$ and provides the victim with the net wealth $\widehat{v}$. If the evidence is sufficiently informative (recall figure 2), there exists $\overline{x}(\widehat{p}) < 1$ solving (11). Consider now the best direct binary penalty scheme for implementing $\widehat{p}$, subject to the victim earning $\widehat{v}$, under the signal generated by $\overline{x}(\widehat{p})$. By the information principle, the resulting allocation is strictly better than the initial allocation.[15] Now, from proposition 1, the resulting allocation is itself weakly dominated by the equilibrium implemented by the negligence rule with the signal defined by $\overline{x}(\widehat{p})$, since (11) implies that $\widehat{p}$ maximizes $\Delta(p)$.

The third claim is that the negligence rule itself is generally not efficient. Suppose it is and assume the evidentiary standard is some $\widehat{x} < 1$, with the resulting equilibrium characterized by some $\widehat{p}$. By corollary 1, this equilibrium must be an efficient allocation within the set of all "binary allocations". In addition, by proposition 2, we must have $\widehat{x} = \overline{x}(\widehat{p})$, which does not depend on the injurer's utility function. However, proposition 4 shows that the best signal for implementing $\widehat{p}$ depends on the properties of the utility function, hence a contradiction since an improvement is then typically possible through a modified negligence rule based on the "better" signal. The sequel of this section illustrates this result.

Decoupling damages from harm. Consider an optimal allocation under a direct penalty scheme with signal defined by $\widehat{x}$ and inducing some $\widehat{p}$. As discussed above, the allocation can be implemented by the modified

[14] The proof in the appendix completes the argument in the text by extending Holmstrom's information principle to non separable utility functions.

[15] The injurer's expected utility can be increased because a more informative signal is now used.

negligence rule with the same signal and with damages $\overline{D}$ satisfying

$$\overline{D} = \frac{l}{\psi(\widehat{x}, \widehat{p})}.$$

The threshold $\widehat{x}$ in the optimal direct scheme depends *inter alia* on the injurer's utility function. It may therefore be below or above $\overline{x}(\widehat{p})$, assuming the latter exists. Recalling figure 2, when $\widehat{x} > \overline{x}(\widehat{p})$, $\psi(\widehat{x}, \widehat{p}) > 1$ so that $\overline{D} < l$, i.e., damages must be undercompensatory. Conversely, $\overline{D} > l$ when $\widehat{x} < \overline{x}(\widehat{p})$. Obviously, punitive damages are also needed when $\overline{x}(\widehat{p})$ does not exist, i.e., when the evidence is insufficiently informative so that $\psi(\widehat{x}, \widehat{p}) < 1$ for all $\widehat{x} < 1$.

To illustrate, we present a family of cases where the optimal $\widehat{x}$ differs generically from $\overline{x}(\widehat{p})$. Suppose the underlying signal x is exponentially distributed, with $F(x, p) = 1 - \exp\left(-xp^{\beta}\right)$ where $x \geq 0$ and β is some positive constant.[16] It is easily checked that

$$\beta \widehat{p}^{\beta} \overline{x}(\widehat{p}) = 1. \tag{15}$$

The proof of proposition 4 compares information structures with the same Fisher index for implementing some given $\widehat{p}$. As shown in the appendix, the index depends on the threshold $\widehat{x}$ defining a binary signal through the function

$$h(\widehat{x}, \widehat{p}) \equiv \frac{F_p^2(\widehat{x}, \widehat{p})}{F(\widehat{x}, \widehat{p})\left(1 - F(\widehat{x}, \widehat{p})\right)} = \frac{\beta^2 \left(\widehat{x}\widehat{p}^{\beta}\right)^2}{\widehat{p}^2 \left(\exp\left(\widehat{x}\widehat{p}^{\beta}\right) - 1\right)}.$$

It is easily seen that $h(0, \widehat{p}) = h(\infty, \widehat{p}) = 0$ and that $h(x, \widehat{p})$ is maximized by $x^*(\widehat{p})$ solving

$$\widehat{p}^{\beta} x^*(\widehat{p}) = k \equiv \arg\max_t \left(\frac{t^2}{\exp t - 1}\right) \simeq 1.59.$$

In figure 3, the signals with thresholds x' and x'' have the same Fisher index. When the injurer's utility function satisfies $P < 3A$, the signal with

[16] The support of x is then not the unit interval, contrary to the assumption made so far. However, one could define the equivalent signal $y \equiv F(x, p_0)$ where $p_0 \in (0, 1)$ is some arbitrary probability.

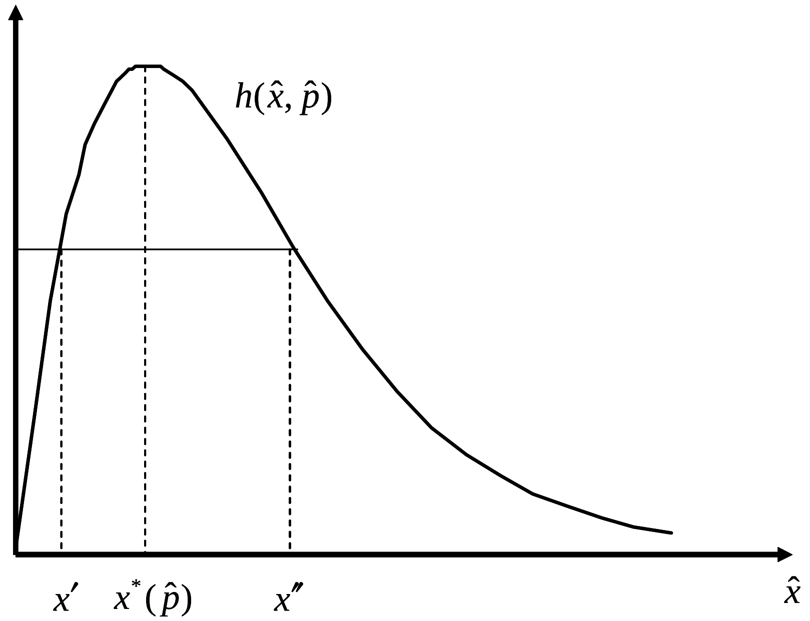

Figure 3: Preferences over signals

the smaller threshold x' is preferred. When $P > 3A$, the signal defined by x'' is preferred. It follows that the optimal signal for implementing $\widehat{p}$ is characterized by a threshold to the right of $x^*(\widehat{p})$ when $P > 3A$ and by a threshold to the left of $x^*(\widehat{p})$ when $P < 3A$.

Now, it is straightforward to verify that $x^*(\widehat{p}) \gtrless \overline{x}(\widehat{p})$ if $\beta k \gtrless 1$, i.e, irrespective of the value of $\widehat{p}$. Thus, if $\beta k < 1$ and $P < 3A$, an optimal signal is defined by some threshold smaller than $x^*(\widehat{p})$, implying that punitive damages are always required. When $\beta k > 1$ and $P > 3A$, we get the opposite case where an efficient rule is always characterized by undercompensatory damages.[17]

[17] The need for punitive damages because of insufficiently informative evidence does not arise here: for any β, there is a finite $\overline{x}(\widehat{p})$ solving (15).

6 Concluding remarks

Tort rules are relatively straightforward mechanisms. A court's role is to assess certain things, e.g., the amount of harm suffered by the victim, the injurer's precautions, the precautions he should have taken given the circumstances, and to assign liability according to the prevailing rule. In simple cases, this provides potential tort-feasors with appropriate incentives to prevent harm.

We assumed that the actual amount of harm was observable without error and that the efficient due care level was known to the court. We then proceeded to verify whether the standard negligence rule with compensatory damages, given liability insurance, yields an efficient outcome even though the injurer's precautions are imperfectly observable. Because evidence was imperfect, our formulation of the negligence rule needed to be complemented by the notion of evidentiary standard, a basic legal construct. We showed that, if liability insurers and courts can both condition directly on the same evidence, efficient allocations can be implemented by the negligence rule provided the evidence is sufficiently informative. The required "weight of evidence" for establishing whether the injurer exerted due care was shown to depend only on the relation between care and evidentiary outcomes. An efficient decentralized set-up is therefore feasible where courts do what they are meant to do and insurers maximize profits by providing policyholders with the best contracts, subject to the liability risks they face.

This separation result breaks down if, as in practice is often the case, court decisions — rather than the detailed evidence available following an accident — constitute relevant information for contracting purposes. The problem arises because the court's evidentiary standard now does two things. First, it determines the liability risk imposed on the injurer-insurer pair as a function of the injurer's effort. For instance, a very demanding standard (a small $\widehat{x}$) would impose little liability risk, resulting in insufficient incentives to take precautions. Secondly, the evidentiary standard also determines the properties of the signal represented by court decisions. It therefore also affects the risk sharing-incentives trade-off in the liability insurance policy.

As a consequence, one instrument is missing for the resulting equilibrium to be efficient.

Decoupling damages from harm adds the missing instrument. Depending on the tort-feasor's risk preferences and the informational properties of the evidence, an efficient rule is then generally characterized either by punitive damages together with a demanding evidentiary standard or by undercompensatory damages and a relatively weak standard. Obviously, the optimal modified negligence rule described in the paper is fine-tuned. Even if courts know the correct due-care standard, they need to solve a complex optimal mechanism problem to determine the appropriate evidentiary standard and level of damages, which presumably is not what courts or legal rules are meant to do. Indeed, the task is computationally as complex as finding the optimal direct penalty scheme. Nevertheless, our results suggest that damage caps and weak evidentiary standards or the converse could constitute an improvement in situations where risk preferences and the nature of the likely evidence are well understood.

Appendix

Proof of proposition 1. Let $(\widehat{v}, \widehat{w}, \widehat{S}, \widehat{p})$ be a feasible allocation, i.e., satisfying (8) and (6) with the injurer's expected utility written as in (7). Denote by $\overline{p}$ the equilibrium probability under the transfer and damage rule satisfying the conditions in the proposition. The victim then pays the premium $\Delta(\overline{p})$ and his net wealth is

$$\overline{v} = v_0 + t - \Delta(\overline{p}) = \widehat{v} + \Delta(\widehat{p}) - \Delta(\overline{p}) \geqslant \widehat{v}.$$

Consider now the injurer. The equilibrium insurance contract and probability of loss maximize (4) subject to (5) and (6). Write $w \equiv w_0 - t - \pi$ and $S \equiv D - I$. The non negative profit constraint (5) becomes

$$w_0 - t - w - p \int_0^1 (D - S) f \, dx \geqslant 0. \tag{A1}$$

Denote by $(\overline{\pi}, \overline{I})$ the equilibrium contract, so that the resulting allocation is $(\overline{v}, \overline{w}, \overline{S}, \overline{p})$ with $\overline{w} \equiv w_0 - t - \overline{\pi}$ and $\overline{S} \equiv D - \overline{I}$. Writing (4), (5) and (6) in terms of w and S shows that $(\overline{w}, \overline{S}, \overline{p})$ maximizes (7) subject to (A1) and the incentive constraint (6). Thus, if the initial $(\widehat{w}, \widehat{S}, \widehat{p})$ satisfies the same constraints, the injurer's equilibrium utility cannot be smaller than with the initial allocation. The latter satisfies (6). We therefore only need to show that it also satisfies (A1). Replacing t by $\widehat{v} + \Delta(\widehat{p}) - v_0$ and using (2), the constraint (A1) is equivalent to

$$w_0 + v_0 - pl \geqslant \widehat{v} + w - p \int_0^1 Sf\,dx + \Delta(\widehat{p}) - \Delta(p). \tag{A2}$$

Since $(\widehat{w}, \widehat{S}, \widehat{p})$ satisfies the resource constraint (8), it also satisfies (A2). *Q.E.D.*

Proof of proposition 2. We first prove that efficiency of the equilibrium implies $\Delta'(\widehat{p}) = 0$. Let $v = v_0 + t - \Delta(p)$, $w \equiv w_0 - t - \pi$, $S \equiv D - I$, and assume the equilibrium allocation $(\widehat{v}, \widehat{w}, \widehat{S}, \widehat{p})$ is efficient. Denote by $\widehat{\mathcal{U}}$ the injurer's expected utility in equilibrium. Since the allocation is efficient, it must maximize the victim's net wealth in the set of feasible allocations providing the injurer with at least $\widehat{\mathcal{U}}$. From the resource constraint (8), the victim's net wealth satisfies

$$v = w_0 + v_0 - pl - w + p \int Sf\,dx.$$

Consider the problem of maximizing the victim's net wealth, given $\widehat{\mathcal{U}}$ and for some arbitrary p, i.e.,

$$\min_{w,S} \left(w - p \int Sf\,dx \right) \text{ such that } EU \geqslant \widehat{\mathcal{U}},\ EU_p = 0. \tag{A3}$$

For future reference, call this program $P(p, \widehat{\mathcal{U}})$. Denote the solution value by $W(p, \widehat{\mathcal{U}})$ and assume the function is differentiable with respect to p.[18] The victim's wealth is then $w_0 + v_0 - pl - W(p, \widehat{\mathcal{U}})$. If $\widehat{p}$ is part of an efficient allocation, it must therefore be that

$$l = -W_p(\widehat{p}, \widehat{\mathcal{U}}). \tag{A4}$$

[18] A proof which does not require this assumption is available upon request.

The liability insurer's expected profit is

$$\pi - p\int If\,dx = w_0 - t - p\int Df\,dx - w + \int Sf\,dx$$

In equilibrium, the insurer proposes a contract that maximizes expected profit subject to the injurer obtaining the equilibrium utility $\widehat{\mathcal{U}}$. Thus, the insurer considers w and S solving (A3), for any p that might be induced by the contract, and chooses p to maximize $w_0 - t - p\int D(x)f(x,p)dx - W(p,\widehat{U})$. Since $\widehat{p}$ is part of the equilibrium,

$$\partial\left(p\int Df\,dx\right)/\partial p\bigg|_{p=\widehat{p}} = -W_p(\widehat{p},\widehat{\mathcal{U}}). \tag{A5}$$

Combining (A4) and (A5) yields $\Delta'(\widehat{p}) = 0$.

We now show that the latter implies that $\Delta(p)$ is maximized at $\widehat{p}$ when D is non increasing. Denote by $x_i, i = 1,..,n$, the points of discontinuity of D, with

$$x_0 = 0 < x_1 < .... < x_n < x_{n+1} = 1.$$

Integrating by parts,

$$\int_{x_i}^{x_{i+1}} Df\,dx = D(x_{i+1}^-)F(x_{i+1},p) - D(x_i^+)F(x_i,p) - \int_{x_i}^{x_{i+1}} D'F\,dx.$$

We thus have

$$\begin{aligned} p\int_0^1 Df\,dx &= p\sum_{i=0}^{n}\left(\int_{x_i}^{x_{i+1}} Df\,dx\right) \\ &= pD(1) + \sum_{i=1}^{n}(D(x_i^-) - D(x_i^+))pF(x_i,p) \\ &\quad - \int_0^1 D'pF(x,p)\,dx. \end{aligned} \tag{A6}$$

When $D(x)$ is non increasing, $D(x_i^-) - D(x_i^+) \geqslant 0$ and $D'(x) \leq 0$. Using assumption 2, (A6) is therefore convex, leading to $\Delta''(p) \leq 0$. *Q.E.D.*

Proof of proposition 3. Assume $(\widehat{p},\widehat{v})$ belongs to an optimal allocation. Part (i) follows from proposition 1 and required transfer satisfies $\widehat{v} = v_0 + t$.

The first part of (ii) also follows from proposition 1 in the case where $\overline{x}(\widehat{p})$ exists. The due care standard is then $\widehat{p}$, the evidentiary standard is $\overline{x}(\widehat{p})$ and the required transfer satisfies $\widehat{v} = v_0 + t - \Delta(\widehat{p})$. When $\overline{x}(\widehat{p})$ does not exist, the sufficient conditions of proposition 1 are not met by the negligence rule. However, proposition 2 implies that the existence of $\overline{x}(\widehat{p})$ is necessary. The argument is by contradiction. Suppose a negligence rule implements $\widehat{p}$. Since the equilibrium is an efficient allocation, proposition 2 implies that the evidentiary standard $\widehat{x}$ solves (11). Moreover, if the rule is one of negligence, we need $\widehat{x} < 1$. Hence, the standard must be $\overline{x}(\widehat{p})$.

The second part of (ii) relates the existence of $\overline{x}(\widehat{p})$ to the informativeness of the evidence. Compare two situations, one represented by the distribution $F(x,p)$, the other by the distribution $G(x,p)$, both satisfying MLRP. The "integral criterion" in Demougin and Fluet (2001) states that F is more informative than G if, for all p, $F_p(x',p) \geq G_p(x'',p)$ whenever $F(x',p) = G(x'',p)$. Intuitively, the more informative the evidence, the more the probability of unfavorable evidence is sensitive to changes in the level of care. The criterion is equivalent to Kim s' (1995) mean preserving spread condition on likelihood ratios. Thus, for x' and x'' such that $F(x',p) = G(x'',p)$,

$$F(x',p) + pF_p(x',p) \geq G(x'',p^*) + p^*G_p(x'',p). \tag{A7}$$

With respect to figure 2, this means that the ψ-curve for F is above the one for G. If $F(\widehat{x},p) + pF_p(\widehat{x},p) < 1$ for all $\widehat{x} < 1$, then by (A7) the same holds for any less informative G. Conversely if there exists $\widehat{x} < 1$ such that $G(\widehat{x},p) + pG_p(\widehat{x},p) = 1$, then the same is true with a more informative F. *Q.E.D.*

Before proving proposition 4, we introduce an intermediate result for comparing random variables. A variable $\widetilde{Y}$ is said to have more downside risk than a variable Y if any prudent decision-maker prefers Y to $\widetilde{Y}$ (Menezes, Geiss and Tressler, 1980).

Lemma 1. Let Y and $\widetilde{Y}$ be two random variables with support in the interval $[a,b]$ and cumulative distribution functions H and $\widetilde{H}$. Assume that Y and $\widetilde{Y}$ have the same mean and the same variance. If $H(y)$ and $\widetilde{H}(y)$ cross twice and $\widetilde{H}(y) - H(y) > 0$ for small values of $y \in [a, b]$, then $\widetilde{Y}$ has more downside risk than Y.

Proof of lemma 1. Let $\Gamma(\cdot)$ be a thrice differentiable VNM utility function.

$$E[\Gamma(Y)] - E[\Gamma(\widetilde{Y})] = \int_a^b \Gamma'(y)(\widetilde{H}(y) - H(y))\, dy.$$

Note that this also holds when the variables have discrete support. Integrating by parts twice yields

$$\begin{aligned} E[\Gamma(Y)] - E[\Gamma(\widetilde{Y})] &= \int_a^b \Gamma''(y) \int_a^y (H(t) - \widetilde{H}(t))\, dt\, dy \\ &= \int_a^b \Gamma'''(y) \int_a^y \int_a^z (\widetilde{H}(t) - H(t))\, dt\, dz\, dy. \quad \text{(A8)} \end{aligned}$$

The first step follows from $\int_a^b (H(t) - \widetilde{H}(t)) dt = 0$ when Y and $\widetilde{Y}$ have the same mean, the second step from the fact that the same mean and same variance implies

$$\int_a^b \int_a^z (\widetilde{H}(t) - H(t))\, dt\, dz = 0. \quad \text{(A9)}$$

Since Y and $\widetilde{Y}$ have the same mean, if $\widetilde{H}(y)$ and $H(y)$ cross twice and the difference is positive for small values of y, then $\int_a^z \widetilde{H}(t)\, dt$ and $\int_a^z H(t)\, dt$ cross once and the difference is positive for small values of z. It follows that

$$\int_a^y \int_a^z (\widetilde{H}(t) - H(t))\, dt\, dz$$

is at first increasing in y and then decreasing. Given (A9), the expression is therefore always positive. Thus, (A8) is positive when $\Gamma''' > 0$, implying that a prudent decision-maker prefers Y to $\widetilde{Y}$. *Q.E.D.*

Proof of proposition 4: The proof borrows from Fagart and Sinclair-Desgagné (forthcoming). They compare signals with the same Fisher index for implementing a given level of effort from an agent with utility function $U(w, e) = u(w) - e$. If u satisfies $P < 3A$, a signal is preferred if the distribution of the likelihood ratio of e has more downside risk, i.e., the risk-neutral principal then pays a smaller expected wage. The converse holds when $P > 3A$. In our setting, an efficient direct scheme for implementing p, equivalently the effort level $e(p)$, minimizes the injurer's expected wealth (i.e., maximizes the victim's net wealth) subject to some "reservation utility" for the injurer. In order for likelihood ratios to have the same signs as in Fagart and Sinclair-Desgagné, we take the inverse of $e(p)$ and write the probability of accident as $p(e)$. Note that $p' < 0$.

For any threshold $\widehat{x} \in [0,1]$ defining the binary signal discussed in the text, a direct penalty scheme conditions payments on the events "accident and $x < \widehat{x}$", "accident and $x \geq \widehat{x}$", and "no accident". We represent this information structure by the ternary signal

$$z_{\widehat{x}} = \begin{cases} 0 \text{ with probability } \alpha_0(\widehat{x}, e) = p(e)F(\widehat{x}, p(e)), \\ 1 \text{ with probability } \alpha_1(\widehat{x}, e) = p(e)\left(1 - F(\widehat{x}, p(e))\right), \\ 2 \text{ with probability } \alpha_2(\widehat{x}, e) = 1 - p(e). \end{cases} \tag{A10}$$

The likelihood ratio associated with the signal is the random variable

$$y_{\widehat{x}} \equiv \frac{\partial \ln \alpha_{z_{\widehat{x}}}(\widehat{x}, e)}{\partial e}, \text{ where } z_{\widehat{x}} \in \{0, 1, 2\}.$$

Its distribution is

$$y_{\widehat{x}} = \begin{cases} y_0(\widehat{x}, e) \equiv \dfrac{p'(e)}{p(e)} + \dfrac{p'(e)F_p(\widehat{x}, p(e))}{F(\widehat{x}, p(e))} \text{ with probability } \alpha_0(\widehat{x}, e), \\ y_1(\widehat{x}, e) \equiv \dfrac{p'(e)}{p(e)} - \dfrac{p'(e)F_p(\widehat{x}, p(e))}{1 - F(\widehat{x}, p(e))} \text{ with probability } \alpha_1(\widehat{x}, e), \\ y_2(\widehat{x}, e) \equiv -\dfrac{p'(e)}{1 - p(e)} \text{ with probability } \alpha_2(\widehat{x}, e). \end{cases} \tag{A11}$$

For any $\widehat{x}$, the variable $y_{\widehat{x}}$ has zero mean. The Fisher index of the signal $z_{\widehat{x}}$ is the variance of $y_{\widehat{x}}$, the value of which is

$$\varphi(\widehat{x}) = \frac{p(p')^2}{1 - p} + (p')^2 p \left[\frac{F_p^2(\widehat{x}, p)}{F(\widehat{x}, p)\left(1 - F(\widehat{x}, p)\right)}\right]. \tag{A12}$$

Obviously, $\varphi(0) = \varphi(1)$ since the second term in (A12) differs from zero only for $\widehat{x} \in (0,1)$. Therefore, for any $x' \notin \arg\max_x \varphi(x)$, there exists $x'' \neq x'$ such that $\varphi(x'') = \varphi(x')$. Moreover, when $\arg\max_x \varphi(x)$ is not unique, the same is true for any x'. Thus, given some x', there generically exists $x'' \neq x'$ such that $y_{x'}$ and $y_{x''}$ have both the same mean and the same variance.

We next show that the cumulative distributions of $y_{x'}$ and $y_{x''}$ satisfy the other conditions of lemma 1. Given MLRP and omitting reference to e for simplicity, $y_0(\widehat{x})$ and $y_1(\widehat{x})$ are both increasing in $\widehat{x}$ and $y_0(x') < y_1(x'')$ for any x', $x'' \in (0,1)$. Let $x' < x''$. Then

$$y_0(x') < y_0(x'') < y_1(x') < y_1(x'').$$

Denoting the cumulative of $y_{\widehat{x}}$ by $H(y,\widehat{x})$, the difference in the cumulatives of $y_{x'}$ and $y_{x''}$ is then easily seen to satisfy

$$H(y,x') - H(y,x'') = \begin{cases} 0 \text{ if } y < y_0(x'), \\ pF(x',p)) > 0 \text{ if } y \in [y_0(x'), y_0(x'')), \\ p(F(x',p) - F(x'',p)) < 0 \text{ if } y \in [y_0(x''), y_1(x')), \\ p(1 - F(x'',p)) > 0 \text{ if } y \in [y_1(x'), y_1(x'')), \\ 0 \text{ if } y \geq y_1(x''). \end{cases}$$

The conditions of lemma 1 are therefore satisfied, so that $y_{x'}$ has more downside risk than $y_{x''}$. Thus, if $\varphi(x') = \varphi(x'')$, we can apply the Fagart and Sinclair-Desgagné criterion, which yields that the signal $z_{x'}$ is preferred to $z_{x''}$ when the injurer's utility function satisfies $P < 3A$. The converse holds when $P > 3A$. *Q.E.D.*

Proof of proposition 5. We complete the argument in the text by extending Holmstrom's information principle to non separable utility functions. For some arbitrary threshold $\widehat{x}$, let $z_{\widehat{x}}$ be defined as in (A10). Consider the program defined in (A3) but now with S constrained by $z_{\widehat{x}}$. Call this program $P(p, z_{\widehat{x}}, \widehat{\mathcal{U}})$ with solution value $W(p, z_{\widehat{x}}, \widehat{\mathcal{U}})$. For simplicity, write $w(i) \equiv w - S(i)$, where $i = 0,1,2$ and $S(2) \equiv 0$. For any threshold $\widehat{x}$ and for the given effort level $e = e(\widehat{p})$, program $P(\widehat{p}, z_{\widehat{x}}, \widehat{\mathcal{U}})$ can be written as:

$$\min_{w(\cdot)} \sum_{i=0}^{2} \alpha_i(\widehat{x}, e) w(i)$$

$$\text{such that} \sum_{i=0}^{2} \alpha_i(\widehat{x}, e) U(w(i), e) \geqslant \widehat{\mathcal{U}},$$

$$\sum_{i=0}^{2} y_i(\widehat{x}, e)\alpha_i(\widehat{x}, e) U(w(i), e) + \sum_{i=0}^{2} \alpha_i(\widehat{x}, e) U_e(w(i), e) = 0,$$

where the $\alpha_i(\widehat{x}, e)$'s are defined as in (A10) and the $y_i(\widehat{x}, e)$'s as in (A11). Denote by λ the multiplier of the participation constraint and by μ that of the incentive constraint. From the necessary conditions for solving program $P(\widehat{p}, z_{\widehat{x}}, \widehat{\mathcal{U}})$,

$$\lambda U_w(w(i), e) + \mu \left[y_i(\widehat{x}, e) U_w(w(i), e) + U_{we}(w(i), e) \right] = 1,\ i = 0, 1, 2. \quad \text{(A13)}$$

The best penalty scheme (from the victim's point of view) with respect to the degenerate signal with $\widehat{x} = 1$ solves $P(\widehat{p}, z_1, \widehat{\mathcal{U}})$. Now consider any threshold $\widehat{x} \in (0, 1)$ and the associated program $P(\widehat{p}, z_{\widehat{x}}, \widehat{\mathcal{U}})$. The scheme solving $P(\widehat{p}, z_1, \widehat{\mathcal{U}})$ satisfies the constraints of $P(\widehat{p}, z_{\widehat{x}}, \widehat{\mathcal{U}})$ with $w(0) = w(1)$. Therefore, $W(\widehat{p}, z_1, \widehat{\mathcal{U}}) \geq W(\widehat{p}, z_{\widehat{x}}, \widehat{\mathcal{U}})$ and $W(\widehat{p}, z_1, \widehat{\mathcal{U}}) > W(\widehat{p}, z_{\widehat{x}}, \widehat{\mathcal{U}})$ if the solutions differ. To show that they do, we prove that $w(0) \neq w(1)$ in the solution of $P(\widehat{p}, z_{\widehat{x}}, \widehat{\mathcal{U}})$. Suppose the contrary. From (A13), $w(0) = w(1)$ implies $\mu y_0(\widehat{x}, e) = \mu y_1(\widehat{x}, e)$, yielding $\mu = 0$ for any $\widehat{x} \in (0, 1)$. As $\lambda > 0$, $\mu = 0$ implies $w(0) = w(1) = w(2)$, which in turn contradicts the incentive constraint because $U_e < 0$. Since strict liability solves $P(\widehat{p}, z_1, \widehat{\mathcal{U}})$ for the equilibrium $\widehat{\mathcal{U}}$, by corollary 1 strict liability is not an efficient rule. *Q.E.D.*

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