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Price Formation in a Sequential Selling Mechanism

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Abstract:
This paper analyzes the trade of an indivisible good within a two-stage mechanism, where a seller first negotiates with one potential buyer about the price of the good. If the negotiation fails to produce a sale, a second-price sealed-bid auction with an additional buyer is conducted. The theoretical model predicts that with risk neutral agents all sales take place in the auction rendering the negotiation prior to the auction obsolete. An experimental test of the model provides evidence that average prices and profits are quite precisely predicted by the theoretical benchmark. However, a significant large amount of sales occurs already during the negotiation stage. We show that risk preferences can theoretically account for the existence of sales during the negotiation stage, improve the fit for buyers' behavior, but is not sufficient to explain sellers' decisions. We discuss other behavioral explanations that could account for the observed deviations.

1 Introduction

Traditionally, outcomes of negotiations have been analyzed separately from those of one or double sided auctions. More recently, attention has been diverted towards the analysis of strategic interaction in hybrid environments which combine both negotiation and auction. A prototype of such an environment presents a seller who negotiates the sale of a good with a small number of buyers before, in case of disagreement, auctioning off the good to a broader set of agents. Such two-stage mechanism is the focus of the current paper.

One of the reasons for the growing interest in such environments is their increasing use by successful internet auction sites like eBay, Yahoo, and QXL,[1] all of which offer sellers the possibility to sell their products at a fixed price directly before or during the auction. eBay, for example, offers sellers the possibility to announce additionally to their call for bids a "Buy It Now" price, where the sale is concluded once a buyer accepts this "Buy It Now" offer. If the price finds no buyer, then the good is auctioned off. Shortly after the introduction of the "Buy It Now" feature in November 2000, eBay reported that 30%, 35% and later even 45% (eBay Q1,Q2 and Q4 2001) of all listings in eBay auctions made use of the "Buy It Now" feature. Supporting this number, recent investigations in eBay's "Buy It Now" feature have observed 40% of listings with a "Buy It Now" offer (Reynolds and Wooders, 2003). The economic relevance of such hybrid mechanisms is reflected in the magnitude of fixed price trades which in the first half of 2004 comprised only on eBay 4.4 billion USD, which translates to 28% of the total Gross Merchandize Volume of the company during this period.[2]

Early theoretical analysis fails to explain these stylized facts. For example, Bulow and Klemperer (1996) compare auctions to negotiations emphasizing the benefit to sellers in waiving any type of bargaining power. They consider also the sequential combination of negotiation and auction and conjecture that "... if a seller could negotiate with N bidders while maintaining the right to subsequently hold an English auction without a reserve price and with an additional bidder, the seller would always do better to proceed directly to the auction."(page 182), a conjecture in clear contrast to the behavior observed in reality.

More recently, Kirkegaard (2004) reconsiders the results by Bulow and Klemperer and shows that bargaining is more profitable than an English auction if de-

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[1]http://www.ebay.com; http://www.yahoo.com; http://www.qxl.com

[2]According to eBay, acceptance of the "Buy It Now" feature is the primary contributor to the fixed price trades.

mand is discrete and agents are sufficiently patient. He also demonstrates that the English auction can be improved by negotiations prior to the auction if buyers are asymmetric or the marginal revenue is non-monotonic.

Two further implicit assumptions made by Bulow and Klemperer (1996), risk neutrality of agents and high discount rates, are also believed to hamper a successful prediction of the existence and the magnitude of sales during the negotiation phase. Mathews (2004) shows that the probability of successful settlements in the negotiation stage increases with the impatience of either sellers or buyers. eBay, for example, indicates that the average auction duration decreased by almost 10%, a result which the company reports to be caused by an increased adoption of the "Buy It Now" feature (eBay, Q4 2002). This provides some evidence, although not conclusive, supporting the impatience hypothesis. But also risk preferences can account for successful trades already in the negotiation stage. The riskiness of the auction as an outside option can contrive risk averse buyers to accept price offers in the negotiation that are higher than the expected price from the auction (see, Reynolds and Wooders, 2003).[3] Alternatively, risk averse sellers might favor an agreement in the negotiation stage by asking for lower prices than those expected from the auction (Mathews and Katzman, 2006).

In this study, we introduce and experimentally test a model based on the situation considered by Bulow and Klemperer: A seller can make a take-it-or-leave-it offer to a buyer who might either accept or reject the offer. In the latter case, a second buyer joins and a second–price sealed–bid auction takes place (see Bulow and Klemperer, 1996, section II.C, page 189). In our setup, contrary to the models mentioned above (Reynolds and Wooders, Mathews, Mathews and Katzman), the price offer is not made simultaneously to all bidders and the buyer joining the auction is not informed about the price offer from the negotiation.[4] We provide the theoretical closed form solution supporting the conjecture of Bulow and Klemperer and use experimental tests to explore whether actual behavior in a controlled environment exhibits similar qualitative properties as the theoretical prediction: In the

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[3]Risk preferences are also seen to cause successful sales in similar hybrid mechanisms, where a seller negotiates the sale of the good while simultaneously auctioning it off: Budish and Takeyama (2001) show that risk neutral sellers can increase their profits offering a permanent buy price during the auction to risk averse buyers, whereas Hidvégi, Wang and Whinston (forthcoming) propose a model where risk aversion of both, sellers and buyers, are combined to yield sales in the negotiation.

[4]We believe that such procedure resembles reality very closely. For instance, the "Buy It Now" price at eBay disappears once some potential buyer opened the auction by placing a bid. Additional bidders are not informed about the former "Buy It Now" price.

negotiation stage sellers should make sufficiently high price offers that will always be rejected by the buyers and all sales will be achieved in the auction.

Average prices and profits in the experiment are very close to the theoretical prediction. However, we observe a substantial amount of sales in the negotiation phase. Our experimental design allows us to exclude time preferences and asymmetry of the bidders as possible explanation for the deviation of the actual behavior from the theoretical predictions. We investigate whether and to which extend individual heterogeneity in risk preferences can account for the success of negotiations. We show that by relaxing the assumption of risk neutrality for both, sellers and buyers, successful sales during the negotiation stage can occur in equilibrium. Using existing population estimates of risk preferences we provide quantitative predictions for the distribution of sellers' price offers. Our experimental results show that relaxing the assumption of risk neutrality can only partly explain agents' behavior: It improves the fit for buyers' behavior, but it is not sufficient to explain sellers' deviations from equilibrium prices. More than one third of all observed price offers lie outside the predicted price range with heterogeneous agents indicating systematically under and over pricing. Therefore, we discuss other behavioral concepts that might be considered to understand and to explain agents' decisions.

The remainder of the paper is organized as follows. In section 2 we derive the theoretical solution. The experimental design and procedure are presented in section 3. Section 4 contains the analysis of the experimental results. In section 5 we investigate whether and to what extent incorporating risk attitudes into the model can explain our experimental data. In section 6 we discuss alternative behavioral concepts. Section 7 concludes.

2 Model and Predictions

In the model a seller who offers a single indivisible item for sale faces two potential buyers. Buyers' valuations of the product, v_i with $i = 1, 2$, are private information and independently drawn from a uniform distribution with support $[0, 1]$. We denote the cumulative distribution function (cdf) of v_i by $F(x) = x$ and its probability distribution function (pdf) by $f(x) = 1$. The distribution of buyers' valuations as well as seller's value of the object, which is zero, are common knowledge. All agents are assumed to risk-neutral.

First, the seller makes a take-it-or-leave-it price offer to one of the buyers, who may accept or reject. If the buyer accepts, he receives the product for the agreed

upon price. If the negotiation fails to produce a sale, a second–price sealed–bid auction without a reservation price is conducted with both potential buyers. Buyers place their bids simultaneously. The bidder who submits the highest bid is the winner of the auction and pays the second highest bid. The buyer, who does not participate in the negotiation, is not informed about the price offered at the negotiation stage.

In a second–price sealed–bid auction truthful bidding is an equilibrium in weakly dominant strategies. The buyer's expected profit from the auction is $\int_0^v (v - x) f(x) dx$. Facing a price offer, p, a negotiating buyer has to decide whether he prefers the profit in the negotiation, in which case he receives $(v - p)$, or to take a chance in the auction. Therefore, solving

$$v - p = \int_0^v (v - x) f(x) dx \tag{1}$$

results in $v^*(p)$, the threshold value, returning the valuation at which a buyer will be indifferent between accepting and rejecting a given price offer. Given our distributional assumptions on the buyers' valuations,

$$v^*(p) = \begin{cases} 1 - \sqrt{1 - 2p} & \text{if} \quad p < \frac{1}{2} \\ 1 & \text{if} \quad p \geq \frac{1}{2}. \end{cases} \tag{2}$$

Contrary to the buyer, the seller has no choice between one or the other institution as he enters the combined mechanism with his price offer. Nevertheless, the seller can increase the probability to sell in the auction by raising his price offer towards $p = 1$. Similarly, by decreasing the price offer, the seller increases the likelihood that the price offer is accepted and the auction will not take place.

The seller receives his price offer if the buyer accepts, i.e., if the value of the buyer he is negotiating with is in the interval $(v^*(p), 1]$. If the buyer rejects, which happens when the buyer's valuation is in the range $[0, v^*(p)]$, the seller's profit is determined by the outcome of the auction which is the minimum of the valuations of the two bidders. Note, that either only the first buyer's value or both buyers' values might lay in the interval $[0, v^*(p)]$, which is taken into account by the density of the first order statistic below. The maximization problem a seller faces is choosing a price offer such that

$$\max_{p \in [0,1]} \Pi^S(p) = p \int_{v^*(p)}^1 f(x) dx + \int_0^{v^*(p)} y g_{(1)}(y, v^*(p)) dy \int_0^{v^*(p)} f(x) dx \tag{3}$$

where $g_{(1)}(y, v^*(p)) = (1 + v^*(p) - 2y) / v^*(p)$ is the density function of the minimum value (the first order statistic) for cdf $G(x) = x / v^*(p)$ when either one or both

bidders' valuations lay in the interval $[0, v^*(p)]$ (for the derivation of $g_{(1)}(y, v^*(p))$ see appendix A).

The resulting first order condition (FOC) is

$$\Pi_p^S(p) = (1 - v^*(p)) + (v_p^*(p)/2)\,(v^*(p)\,(2 - v^*(p)) - 2p) \tag{4}$$

where $z_p(\cdot)$ denotes the derivative of a function $z(\cdot)$ at p.

There exists no price in $0 < p < (1/2)$ at which the FOC, eq. (4), becomes zero. This result, combined with (i) $\Pi_p^S(p) > 0$ for $p \in (0, 1/2)$, (ii) $\Pi_p^S(p) = 0$ as the expected profit function is a constant for prices $p \in (1/2, 1)$, and (iii) $\Pi^S(1/2) = \Pi^S(1) = \Pi^S(1/2 < p < 1) = 1/3$, leads to the conclusion that the expected profit function of the seller reaches its maximum for prices $p^* \in [1/2, 1]$. Figure 1 plots the seller's expected profit in the $[0, 1]$–range of price offers.

Thus, taking the threshold value of the buyer in the negotiation into account, a seller's expected profit for the whole game is maximized by choosing a rather high price offer, $p^* \geq 1/2$, at the negotiation stage which in return will be rejected by the buyer, $v^*(p^*) = 1$, and the final price is determined by the auction. Therefore, no price offer will be accepted in the negotiation and sales will always take place at the auction stage.[5] Expected earnings are $1/3$ for the seller as well as $E[v_i^2/2] = 1/6$ and $v_i^2/2$ for the two buyers $i \in \{1, 2\}$ before and after knowing the realization of the own valuation, respectively.

3 Experiment

In order to test our theoretical predictions and to investigate behavior in such a sequential mechanism we conducted an experiment. In the experiment each subject was either a buyer or a seller. One seller and two buyers constituted a trading group in which they interacted as described by the model. The composition of the trading groups was changed between periods as each period sellers and buyers were rematched randomly. An experimental session consisted of four cycles of eight trading periods. A buyer was assigned the right to negotiate either in the odd or the even cycles, so that each buyer negotiated 16 times out of the 32 periods (eight times in the first and eight times in the second part of the experiment). Buyers' private reselling values for the product were randomly and independently drawn from

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[5]Myerson (1981) shows that any two mechanisms that always lead to the same allocation of the good (and meet one further trivial condition) would yield the same expected revenue. In the auction the product will always be allocated to the buyer with the highest value, this is not necessarily so in the presented combined mechanism. It follows that the auction is better.

the set $V = \{0, 1, 2, ..., 99, 100\}$ with all $v_i \in V$ being equally likely. Subjects could choose integer bids and price offers between 0 and 100. All values were denoted in a fictitious currency termed ECU for Experimental Currency Unit.[6]

In the beginning of a single period the trading groups were formed and sellers were asked to submit their price offer. Buyers who attended the negotiation stage were informed about their private values and the seller's price offer, which they could accept or reject. At the end of the negotiation stage each group member was told whether or not a sale had been reached, i.e., whether the price offer was accepted by the buyer or not. If the price offer was accepted, the sale was accomplished in the negotiation stage and there was no auction. If the price offer was rejected, the other buyer, who did not negotiate, was now informed about his private value (but not about the price offered at the negotiation stage) and a second–price sealed–bid auction with both buyers took place. At the end of the auction stage all group members were informed about the outcome in this stage: who won the auction, the price paid by the winner, and their own payoff in the current period. In addition, they were given an account of their total profit up to this period. After the experiment participants answered a post–experimental questionnaire. Besides standard demographic questions, they were additionally asked to comment briefly on their reasoning during the experiment.

The experiment comprised 10 sessions with a total of 90 participants. We conducted 5 sessions with 6 and 5 sessions with 12 participants each. We pooled the data from the sessions of different sizes since we did not find any differences between them. All experimental sessions were computerized and the software system was created with z-Tree (Fischbacher, 1999). The experiment was conducted at Humboldt-University Berlin, Germany, and most participants were students of economics, business administration, law, and physics. One session lasted on average 90 min. The conversion rate of ECU earned by each participant into cash was: 1 ECU = 0.0125 EUR. Participants' total earnings ranged between 8.05 EUR to 16.86 EUR with a mean of 11.82 EUR (as a seller: 13.14 EUR, as a buyer: 11.15 EUR).[7]

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[6]See appendix B for a shortened and translated version of the instructions. Complete sets of the original instructions (in German) are available upon request to the authors.

[7]These numbers include a starting capital for buyers of 5 EUR.

4 Results

Our results show that average experimental outcomes are close to the theoretical prediction. Table 1 reports descriptive statistics for all periods and, in order to illustrate the changes over time, for all cycles. In total we have 960 trades. Average prices (column 1) with $p = 0.51$ as well as average earnings for sellers and buyers with 0.33 and 0.15 (columns 3) are stable over time and in line with the theoretical benchmark.[8]

The second–price auction is efficient since truthful bidding is an equilibrium in dominant strategies.[9] However, applying this combined mechanism might result in a loss of efficiency compared to a pure auction. Column 4 of Table 1 lists the share of efficient outcomes achieved in the experiment. The efficiency of observed sales is with 85% quite comparable to those reported in second–price sealed–bid auction experiments. For instance, the share of efficient allocations by Kagel and Levin (1993) is 79%. Güth, Ivanova-Stenzel and Wolfstetter (2005) report 88% and Pezanis-Christou (2002) about 91% of efficient allocations.

The theoretical relation between prices and expected profits (see eq. (3) and Figure 1) is reflected in the experimental data. The solid line in Figure 2 presents the nonparametric estimate of sellers' profits earned in the experiment conditional on prices: $\Pi^S = E(\Pi^S | P = p) + \varepsilon = m(p) + \varepsilon$ where the error term ε has the properties $E(\varepsilon|p) = 0$ and $E(\varepsilon^2|p) = \sigma^2(p)$. The surrounding dotted lines are the 95% (pointwise) confidence bounds of the expected profit estimate around a given price.

The expected profit reaches the predicted outcome of $E(\Pi^S) = 0.33$ already at a price $p = 0.44$, much earlier than the predicted price $p = 0.5$. We observe further, that for prices $p \geq 0.35$ the expected profit in the experiment is not significantly different from 0.33, the theoretical profit.

Furthermore, contrary to the theoretical prediction, we observe successful trades in the negotiation phase comprising one third of all sales (column 2 of Table 1), an amount which remains stable over time.[10] In order to understand these contradicting findings, we continue with an elaborate analysis of individual price setting and acceptance behavior. Prices depend on the behavior of buyers at the negotiation stage, which in turn relates to buyer's expected outcome from the auction. There-

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[8]For ease of comparison to the theoretical model we report our results for normalized valuations, i.e., all experimental outcomes are transformed from the $[0, 100]$ to the $[0, 1]$–range.

[9]Efficiency requires that the buyer with the highest valuation purchases the product.

[10]Considering only price offers which were below the valuation of buyers, 63% of all sales occur during the negotiation stage.

fore, we will first investigate buyers' bidding behavior in the auction and continue with their acceptance behavior during the negotiation stage before we turn our attention to sellers' price setting behavior.

Bidding should (theoretically) not be influenced neither by the negotiation nor by buyers' risk attitudes since bidding the own value is a weakly dominant strategy. The experience from the negotiation does not seem to change behavior in the auction. Half of all observed bids are equal to subjects' valuations. Moreover, truthful bidding rises from 31% in the first to 64% in the last period. These numbers increase to 56% and 74%, respectively, considering bids within a range of 0.05 around the valuation. Despite some slight over- and underbidding, we do not find significant differences between bids and the predicted equilibrium bidding strategy.[11] Given these observations, truthful bidding turns out to be a reasonably good prediction for buyers' behavior in the auction.

At the negotiation stage, buyers have to choose between accepting the price offer or joining the auction. In the experiment, each buyer was confronted half of the time with a price offer of different sellers, which leaves us with information about the acceptance behavior of each buyer for 16 periods. We find that only five out of 60 buyers (8%) behave according to the theoretical benchmark, i.e., accept a price offer when their valuation lays above the corresponding threshold value of eq. (2) and reject the offer when their valuation is below this threshold. The majority of buyers either accept offers below the threshold (57%) or reject offers above the threshold (18%), which obviously violates the game theoretic prediction.[12]

Given their expectations about buyers' behavior, sellers have to make only one decision: to choose a price offer. Figure 3 presents the density estimate of price offers which range from 0.05 to 1. Half of the price offers (48.4%) are conform with the theory, i.e., are greater or equal to 0.50. However, the other half (51.6%) of observed prices is below 0.50, which is clearly not in line with the theoretical benchmark. Price offers remain stable over time.

All these results show that the game theoretic model assuming homogeneous and risk-neutral agents seems to be a good predictor for average prices and prof-

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[11]22% of all bids lay above the dominant strategy, the own valuation, which is rather low compared to other sealed–bid second–price auction experiments (see Kagel, 1995, for a survey). A t-test cannot reject the hypothesis that the relative bid deviation is equal to zero (p=0.365 and and p=0.789 in the first and second part of the experiment).

[12]The remaining 10 buyers (17%) could not be classified as the deviation of their behavior was not consistent, i.e., price offers were accepted (rejected) a risk neutral person would have rejected (accepted).

its. Nevertheless, it cannot very well rationalize observed individual decisions that appear to drive the contradictory finding of successful sales during the negotiation stage.

This leaves us with the possibility that behavior might be based upon unobserved heterogeneity in subjects' preferences. One feasible explanation for the observed deviations might be heterogeneity in risk preferences of both, sellers and buyers. Risk aversion might, for instance, explain, why the majority of buyers accept offers, which yield them a lower profit then their expected profit from the auction and which a risk neutral buyer would have rejected. Another indication for possible risk aversion of buyers can be found in the substantial increase in sellers' profits compared to the theoretical prediction for the case of high price offers. On the other hand, price offers below the theoretical prediction might be driven by risk aversion of sellers.

5   Heterogeneous Risk Attitudes

In this section we investigate whether and to what extent incorporating individual risk preferences into the model can explain our experimental data. Therefore, all agents are assumed to be expected utility maximizers and have preferences that can be represented by utility function $u(\cdot)$ which is twice differentiable, strictly increasing and satisfies $u''(\cdot) \leq 0$ everywhere on its support. More precisely, for our analysis we restrict risk preferences to belong to the class of constant relative risk aversion, $u(x) = x^{(1-\alpha)}/(1-\alpha)$, where α is the Arrow-Pratt measure of relative risk preferences.[13] Constant relative risk aversion (hereafter CRRA) can provide an excellent fit for data patterns, a reason why this utility function has been commonly applied in the experimental literature examining risk preferences.

Buyer

Allowing for heterogeneous risk preferences leads to a threshold value that depends not only on the price but also on the individual risk attitude. The threshold value can be derived from the decision problem of the buyer, i.e., $u(v - p) = \int_0^v u(v - x)f(x)dx$. However, given our distributional assumptions and functional form, the

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[13]This specification implies risk loving behavior for $\alpha < 0$, risk neutrality for $\alpha = 0$ and risk aversion for $\alpha > 0$. When $\alpha = 1$, the natural logarithm, $u(x) = ln(x)$, is used.

threshold can only implicitly be defined by

$$p = v^* - \left(\frac{v^{*(2-\alpha_i^B)}}{2 - \alpha_i^B} \right)^{\left(\frac{1}{1-\alpha_i^B} \right)}$$

Figure 4 plots the threshold values for buyers with different levels of risk prefer-ences. Note that if a buyer is risk-averse the monotonicity of the threshold value is not maintained. For example a buyer with $\alpha^B = 0.7$ will not only reject a price offer of 0.64 if his valuation is lower than 0.79 but also if his valuation is higher than 0.88, meaning that a buyer with a very high valuation is more likely to accept the risky outcome from the auction rather than the certain outcome from the nego-tiation. Note also that if a buyer is sufficiently risk averse, i.e. $\alpha^B \geq 1$, he would be willing to accept a price equal to his valuation.[14] The threshold is shifted up-wards with increasing levels of α^B implying that higher prices are more likely to be accepted when a buyer is more risk averse.

We can estimate buyers' risk preferences directly from the experimental data based on their decision rule. In the model a buyer accepts a price offer if

$$u(v - p) + \varepsilon_1 \geq \int_0^v u(v - x) f(x) dx + \varepsilon_2 \tag{5}$$

where we assume the unobservable error terms, ε_i with $i \in \{1, 2\}$, to follow a normal distribution $\varepsilon_i \sim N(0, \sigma_i^2)$. Assuming that risk preferences can be represented by $u(x) = x^{1-\alpha}/(1-\alpha)$, given the distributional assumptions and the decision rule in eq. (5), we estimate the risk preference parameter in the buyers' population by maximum likelihood.[15]

The parameter estimate for buyers' risk preferences is $\hat{\alpha}^B = 0.45$ with a loglike-lihood function value of -274.48. The model assuming risk neutral buyers (fixing $\alpha^B = 0$), results in a likelihood function value of -297.35. A likelihood ratio–test with a test value of 45.74 (5% χ^2 critical value of 3.84) corroborates that allowing for risk preferences improves significantly the fit of the data. Overall, buyers seem to react in a risk averse manner, which might explain acceptance of high price offers.

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[14]This is somewhat counterintuitive since in this case such a buyer would get zero payoff whereas by rejecting he would have a chance of a positive net payoff. On the other hand, given the assumption of CRRA and the specification of the utility function, a buyer with $\alpha^B > 1$ and a valuation between 0 and 1 will always have negative net utility from entering the auction.

[15]For $v \geq p$ ($N = 482$) the choice probabilities are given by

$$Pr \left(\frac{1}{1-\alpha} \left((v-p)^{(1-\alpha)} - \frac{v^{(2-\alpha)}}{2-\alpha} \right) \geq \varepsilon \right)$$

with $\varepsilon = \varepsilon_2 - \varepsilon_1$ and $\varepsilon \sim N(0, 1)$.

The estimated level of buyers' risk attitudes is in line with risk preference estimates in the literature. Numerous estimates of risk attitudes for student subjects imply the average level of risk preferences to be around $0.3 - 0.7$ and to be quite robust to different decision environments, e.g., gambles, other individual decision tasks, games, and auctions. For example, Cox and Oaxaca (1996), Goeree, Holt and Palfrey (2002), Chen and Plott (1998), and Ivanova-Stenzel and Salmon (2004) estimate relative risk preferences in private value auction experiments to be $\alpha = 0.67, 0.52, 0.48$, and 0.34, respectively. Goeree, Holt and Palfrey (2003) and Holt and Laury (2002) use experimental data from single decision tasks to estimate individual risk parameters. They find the average risk attitudes (for payment levels used in laboratory experiments) to be around 0.28 and 0.32, respectively.

Seller

Contrary to the risk neutral case, we could not derive an explicit solution for the case of general risk preferences for sellers. Nevertheless, we can solve numerically for optimal prices sellers would choose and derive the distribution of prices for given distribution of agents' risk attitudes. This also implies that we can only indirectly investigate whether risk preferences improve the fit of the model for sellers. We will do this by comparing predicted price distributions to the prices observed in the experiment.

To derive quantitative price predictions using "reasonable" levels of risk preferences, we rely on estimated risk preference distributions in the literature. In order to check for the robustness of the predictions, we will use four different frequency distributions of estimated individual risk preferences provided by the studies mentioned above (Cox and Oaxaca (henceforth C&O) and Ivanova-Stenzel and Salmon (henceforth I&S) for auctions as well as Goeree, Holt and Palfrey (2003, henceforth G,H&P) and Holt and Laury (henceforth H&L) for individual decision tasks). Estimated individual risk attitudes of those four studies lay in the interval $-1.48 \leq \alpha \leq 1.37$ and characteristics of the distribution ($10\%, 50\%$ and 90% quantiles) are reported in column 3 of Table 2.[16] Based on these distributions we simulate outcomes allowing for varying risk preferences of both, buyers and sellers.[17] We in-

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[16]H&L classify their participants in 9 categories. We assign all subjects within a category the mean of this category as individual risk parameter. Subjects in the outer categories $\alpha < -0.95$ and $\alpha > 1.37$ were assigned $\alpha = -0.95$ and $\alpha = 1.37$. We do the same for G,H&P who distinguish between 7 risk categories, with $\alpha < -0.56$ and $\alpha > 0.93$ as lower and upper bound.

[17]We assume common knowledge about the population distribution of risk attitudes, which is the

vestigate the quantitative change in the decision variables and the expected amount of successful sales during the negotiation stage.

Buyers will accept prices up to 1 when they are strongly risk averse, i.e., $\alpha^B \geq 1$, and only up to 0.45 when they are risk loving, i.e., $\alpha^B = -1.48$, which is the lowest level of the preference parameter estimates reported by the four studies. This translates into an increase (decrease) of the maximum price a risk neutral agent would accept by 100% (-10%). In order to determine the impact of risk on sellers' price setting behavior, we simulate prices offered by sellers with different risk attitudes. Sellers can anticipate the reaction of buyers with different risk preferences towards prices and, knowing the distribution of buyer's risk preferences, calculate their "optimal price offer" given their own risk attitude. Figure 5 presents predicted price frequencies for each of the estimated population distributions of risk preferences. The price range spanned by all predictions is $p \in [0.43, 0.69]$. Compared to the risk neutral benchmark, this price range is smaller in size and is shifted downwards. However, the lowest bound is with 0.43 still close to the prediction for risk neutral agents of 0.5. The stronger decrease of the upper bound from 1.0 to 0.69 can be explained by the fact that when buyers are risk averse even high prices have a chance of still being accepted. The seller can therefore increase his profit by keeping prices high yet affordable to risk averse buyers. For instance, a risk neutral seller ($\alpha^S = 0$) who meets a strongly risk averse buyer ($\alpha^B \geq 1$) maximizes his profit by asking for a price of 0.73.

The price distributions are strikingly similar with medians at 0.55 (G,H,&P), 0.53 (H&L), 0.54 (C&O) and 0.51 (I&S), reported in Table 2. Predictions based on parameter estimates elicited via lottery choices (G,H&P and H&L) are more widely spread than those based on auction data (C&O and I&S). Columns 4a in Table 2 report quantiles of the predicted price distribution. 10% quantiles are with 0.48 up to 0.51 below but quite close to the risk neutral benchmark. The 90% quantiles lay like wise below the upper bound of the risk neutral prediction, however, they are with 0.56 up to 0.68 further away. Column 4b of Table 2 reports the expected amount of sales during the negotiation phase for each distribution. Predicted acceptance rates in the negotiation vary between 17.6% and 36.9%.

Thus, allowing for general risk preferences opens a price floor within which agreements are possible already during the negotiation stage. The exact magnitude depends, however, on the particular preference distribution.

The acceptance rate of 33% observed in our experiment is within the range of

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same for both, buyers and sellers.

predicted acceptance rates with heterogeneous agents. Observed median prices are with 0.49 close to the predicted median prices $(0.51 - 0.55)$. However, allowing for heterogeneous risk preferences seems to improve the explanatory power of the model only little for sellers. The prediction $p \in [0.43, 0.69]$ covers more than half of the price offers (56%) in the experiment, which is an improvement by 8% with regard to the (risk neutral) benchmark prediction. Still, almost half of the offers remain unexplained. Price offers in lower and higher ranges are much more dispersed than predicted. Figure 6 plots two price distributions. The light bars present the average predicted price offer frequencies of the four studies. The dark bars show the price frequencies observed in the experiment. A Pearson Goodness of fit test strongly rejects the prediction of the model for each of the estimated distributions separately as well as the combination of them.[18]

Furthermore, whereas price offers above 0.69 can be explained by the risk neutral benchmark, price offers below 0.43 are not captured by neither model. What seems to be puzzling is that those low offers comprise 29% of all prices. This observation is not only stable over time but is generally caused by the same sellers: One third of all sellers offer prices below 0.43 more than half of the time.

6 Discussion

In the previous section we have shown that allowing for heterogeneous risk preferences can theoretically explain the existence of successful negotiations in equilibrium. Participants' comments in the post–experimental questionnaire confirm that risk attitudes might indeed account for sales during the negotiation stage. For example, some participants in the seller's role were concerned with the auction because it generated "too volatile prices" and mentioned that they favored an agreement during the negotiation phase. Participants in the role of buyers emphasized that they preferred to negotiate as "the chance of buying the item was higher." The experimental results show, however, that real behavior can only partly be explained by agents' risk attitudes. Even though, relaxing the assumption of risk neutrality improves significantly the fit for buyers' behavior, it is not sufficient to account for sellers' behavior. Observed prices vary much more than predicted with a substantial amount of over and under pricing.

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[18]The Pearson Goodness of Fit test, also known as χ^2-Test, requires independent observations. As different price offers of an individual seller might not fulfil this requirement, we use mean price offers of individual sellers for the test.

One might be tempted to explain the excess variance in price offers with noise in behavior. There are two problems with this argument. First, the observed distribution of price offers is far from uniform, as would be required by a model where sellers randomly choose across all possible price offers. Second, noise would presumably decline over time as participants have the opportunity to learn and adjust their price offers during the 32 periods of the experiment. Nevertheless, the observed price offer distribution remains stable over time, with systematically under and over pricing compared to the theoretical predictions.

Another possible way to look at our results is in light of bargaining literature investigating environments with asymmetric information. For example, Samuelson and Bazerman (1985) show that subjects systematically deviate from the predicted behavior and fall prey to the "winner's curse," in the sense that they either enter into loss–making purchases or forgo profit–making opportunities. The latter might apply also to our experimental situation where the seller in the negotiation (the uninformed party) has to condition his behavior on the strategic action of the buyer (his informed opponent).

In our experiment a seller faces a cognitively very demanding decision task. First, he has to consider buyers' acceptance threshold values. Second, conditional on these threshold values, he has to form expectations about his utility for different prices and find the price offer that maximizes this utility. Such reasoning requires that sellers not only optimize but also condition correctly their price offer on the buyer's reaction.

Let's assume for a moment that there are sellers who do not optimize and do not condition. Then from the view of such (bounded rational) seller any accepted price offer, p, which generates utility above the expected utility from the auction will increase his profit. Such seller has to ensure that his price offer is above the expected auction profit and will be accepted with positive probability such that $E(\pi_A) \leq p \leq E(v)$, for a seller whose utility is equal to his expected profit. For example, with $E(\pi_A) = 1/3$ and $E(v) = 1/2$, a non–optimizing non–conditioning seller might offer prices between $1/3 \leq p \leq 1/2$.

Following this argument we can not only explain the existence of low prices but also the fact that the observed under pricing remains stable over time. Suppose a seller neglects the strategic reaction of a buyer towards his own price offer and uses solely his experience to build his expectation about the prospects of selling either in the negotiation or the auction. Such seller might form false expectations about the prospects of the auction. A seller who offers low prices in the negotiation is more

likely to experience lower profits in the auction: buyers with relatively high values might accept low prices, but buyers who cannot even afford those low price offers, reject and go to the auction. This leads to the selection of low value buyers into the auction and consequently to low auction prices, reinforcing seller's expectation about low prospects of the auction. Sellers who reason this way forgo therefore profit opportunities.

Our data do not allow us to test directly this conjecture. However, it is very likely that subjects in our experiment fall pray to such 'seller's curse.' Some sellers indeed argued in the post–experimental questionnaire that the "auction generated too low prices" and that this was the reason why they preferred to reach an agreement during the negotiation stage. Experimental investigations testing this conjecture and other behavioral explanations might yield promising answers and further interesting insights in behavior in such hybrid mechanisms. We plan to pursue such investigations in our future research.

7   Summary

In this paper, we presented a model of a sequential mechanism combining negotiation and auction. A seller can first negotiate with one potential buyer to sell a single indivisible good and, in case the negotiation did not lead to a sale, he conducts an auction with the current and an additional buyer. We derived the closed form solution for this model and show that with risk neutral agents sales will always take place in the auction rendering the negotiation prior to the auction obsolete. An experimental test of the theory suggested that the theoretical benchmark can very well predict average prices and profits in such combined mechanism. However, individual buyers and sellers deviate from the theoretical prediction bringing about a substantial amount of successful negotiations. This last finding led us to explore other behavioral explanations in order to account for the observed deviations.

First, we investigated whether and to which extend heterogeneity in preferences can account for successful negotiations by incorporating risk attitudes into our model. We showed that allowing for individual heterogeneity in risk preferences of both market sides leads to sales already during the negotiation phase. This result seems to be driven rather by risk aversion of buyers than of sellers as risk averse buyers accept more often higher offers but sellers lower their price offer only marginally below the price prediction for risk neutral sellers. By using existing population estimates of risk preference parameters we were able to make quantitative

predictions for the distribution of sellers' price offers and acceptance behavior of buyers. When we compared these predictions to the experimental data, we found, however, that the experimental behavior can only partly be explained with agents' risk preferences. Relaxing the assumption of risk neutrality improves the fit of the model for buyers, but cannot account for a big part of individual seller's decisions. A great deal of sellers offer prices which are too low as well as too high to be explained by heterogeneity in risk preferences alone. This is especially striking for low price offers. Even though participants experienced the mechanism for several periods, they did not increase their price offers, thus their profit opportunities, during the experiment.

We, therefore, discussed an alternative explanation for individual deviations of sellers to the theoretical benchmark. An important issue seems to be whether or not uninformed agents take into account the strategic behavior of their informed opponents. If sellers fail to anticipate buyers' strategic reaction to their price offer they might choose prices which are too low and forgo profit–making opportunities, a fallacy which resembles the winner's curse in negotiations.

8 Appendix

A Derivation of the first order statistic

For simplicity in notation we will continue to write v^* instead of $v^*(p)$, leaving implicit the dependance on the price p. If an auction takes place, either only the first buyer's value or both buyers' values are in the interval $[0, v^*]$. The probability that only the first buyer's value is below v^* is $\pi(1) = P(v_2 > v^*) = (1 - v^*)$ and that both values are below v^* is $\pi(2) = P(v_2 \leq v^*) = v^*$. Where $\pi(k)$ denotes the probability that k values lay in the interval $[0, v^*]$.

Following Rohatgi (1987) we determine the cumulative distribution function (cdf) and the probability distribution function (pdf) of the first order statistic under the constraint that the number of random draws, N, in the range $[0, v^*]$ might be one or two. Let $\pi_i = P(N \geq i) = \sum_{k=i}^{\infty} \pi(k)$. For $|s| < 1$ let $\phi(s)$ denote the probability generating function of N, $\phi(s) = \sum_{n=1}^{\infty} s^n \pi(n)$. The pdf and cdf of v in the range $[0, v^*]$ are $g(x) = 1/v^*$ and $G(x) = x/v^*$, respectively.

The cdf of the ith order statistic in random sampling from a cdf $G(x)$ with random sample size is

$$G_{(i)}(x) = \frac{1}{(i-1)!\pi_i} \int_0^{G(x)} t^{i-1} \phi^{(i)}(1-t) dt$$

in the case when $G(x)$ has a pdf $g(x)$ this function of the ith order statistic is given by

$$g_{(i)}(x) = \frac{1}{(i-1)!\pi_i} G^{i-1}(x) g(x) \phi^{(i)}(1 - G(x)).$$

In our application the probability generating function reduces to $\phi(s) = (1 - v^*)s + v^* s^2$. Let $\phi^{(1)}(s)$ denote the first derivation of $\phi(s)$, which is $\phi^{(1)}(s) = 1 + (2s - 1)v^*$ and $\pi_1 = P(N \geq 1) = 1$. The cdf and pdf of the first order statistic of our application $G_{(1)}$ and $g_{(1)}$ are therefore

$$G_{(1)}(x) = \frac{x(1 + v^* - x)}{v^*}$$

and

$$g_{(1)}(x) = \frac{1 + v^* - 2x}{v^*} \qquad .$$

B Instructions

The experiment was conducted in German language, and the original experimental instructions were also in German (available upon request). This is a shortened translated version of the instructions. Participants read the paper instructions before the computerized experiment started. In the beginning of the instructions, everybody was informed that instructions were the same for every participant, that any decision made would be anonymous and could

not be related to a person. At the end of the paper instructions, participants were also informed that wins and losses from all periods would be added, that the exchange rate from ECU (Experimental Currency Units) to EURO was: 40 *ECU = EURO* 1, *and that buyers would receive an initial endowment of 5 EURO. The main instructions were as follows:*

In every period one person (a seller) offers two other persons (buyers) a fictitious commodity for sale. At the beginning of the experiment each participant is randomly assigned to a role (seller or buyer) and keeps this role throughout the entire experiment.

All valuations are denoted in a fictitious Experimental Currency Unit (ECU). In each period the private value for the product of each buyer, v, is independently drawn from the interval $0 \leq v \leq 100$, with every integer number between 0 and 100 being equally likely. Each buyer is informed only about his own private value and will not get to know the private value of the other buyer. The seller is not informed about the private values of the buyers.

Each period consists of either one or two stages and proceeds as follows:
In the first stage, the seller negotiates with one of the two buyers. He makes a price offer (in the range from 0 to 100) to this buyer. The buyer can either accept or reject this price offer.
1.) If he accepts, then he pays the price and receives the product. The period is terminated. The buyer's profit is the difference between his private value for the product and the price. The seller receives the price. The other buyer (who has not participated in the negotiation) does not receive anything and does not pay anything, i.e., he makes a zero profit.
2.) If he rejects then the period proceeds to the second stage. In the second stage, an auction takes place with the seller and the two buyers. Both buyers submit simultaneously their bids. The bidder with the highest bid buys the commodity. The price he has to pay is equal to the second highest bid. His profit is the difference between his private value for the product and the price. The seller receives the price. The bidder who submits the second highest bid does not receive anything and does not pay anything, i.e., he makes a zero profit. If both bids are equal, the buyer is chosen randomly. In this case the second highest bid is equal to the highest bid.

Each participant receives the following information: After the first (negotiation) stage the seller and both buyers are informed whether the sale takes place.
1.) In case of a sale, parties involved in the negotiation (the seller and one of the buyers) are informed about the price and own profits in this period. In addition all participants are informed about their own total profit up to this period.
2.) In case of a second (auction) stage the seller and both buyers are informed about the winner of the auction, the price which has to be paid by the buyer, the own profit in this period, and the own total profit up to this period.

In each period trading groups (one seller and two buyers) are formed randomly. Altogether, there will be 32 periods, which consist of 4 cycles of 8 trading periods. After each cycle buyers who participated in the first (negotiation) stage will change and participate in the auction only and vice versa.
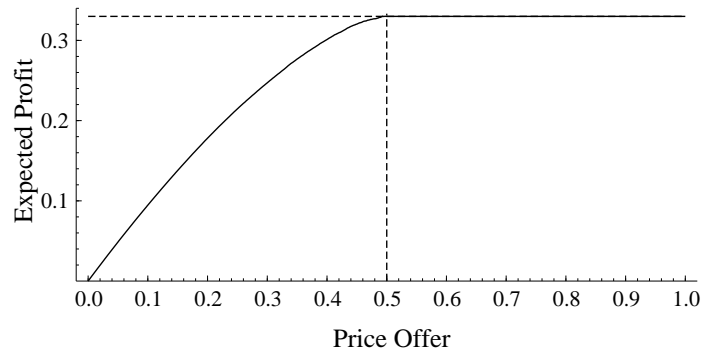
C Figures and Tables

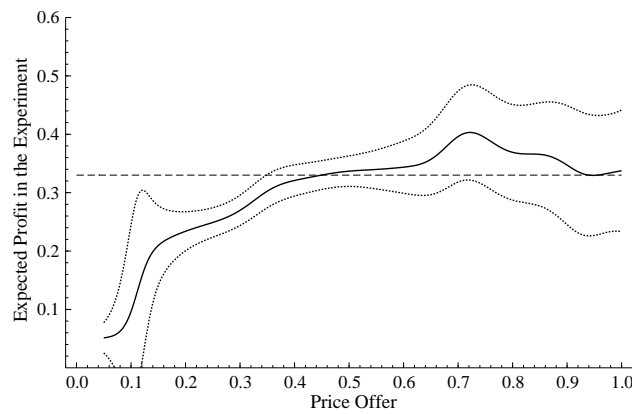Figure 1: Sellers' expected profit in the combined mechanism for risk neutral agents

Figure 2: Expected profits in the experiment for sellers. The thick line represents the nonparametric regression of expected profits on price offers using a gaussian kernel and bandwidth chosen by cross-validation. Dotted lines represent 95% pointwise confidence intervals. The dashed line is at 0.33, the seller's expected profit in a second–price sealed–bid auction.
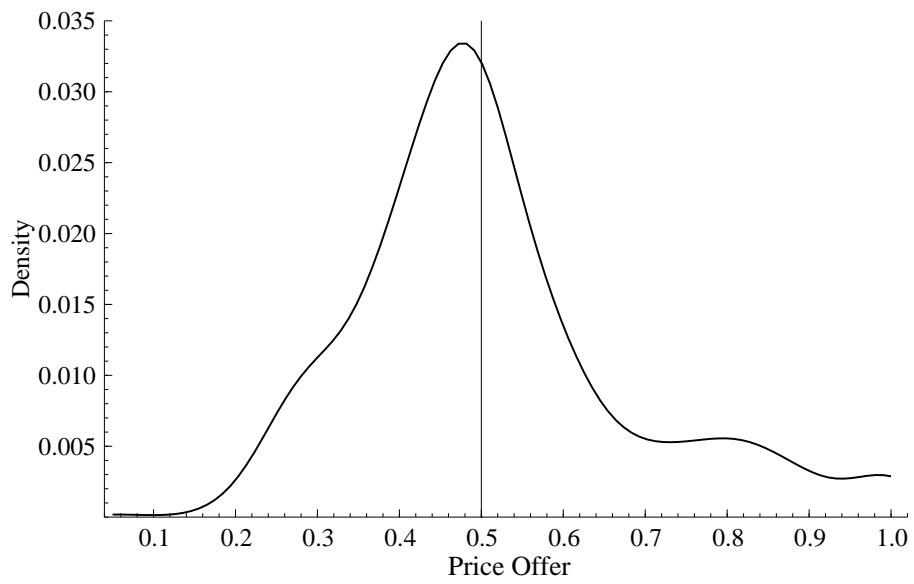
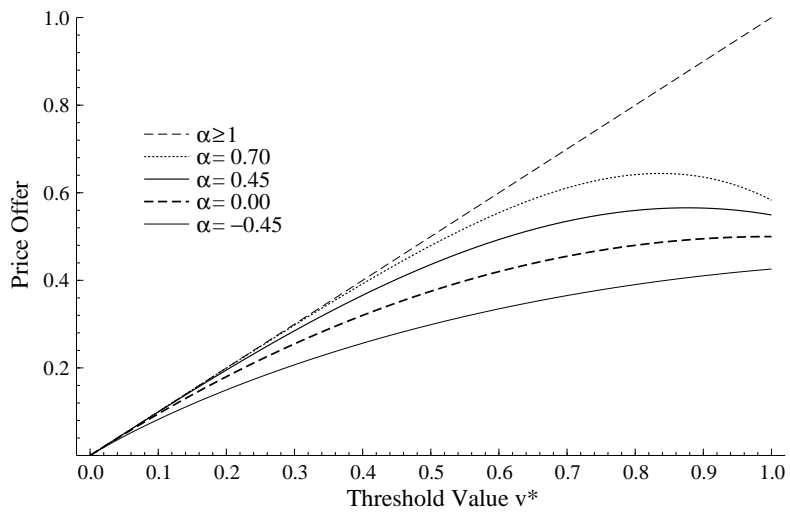Figure 3: Posted price density estimation, gaussian kernel

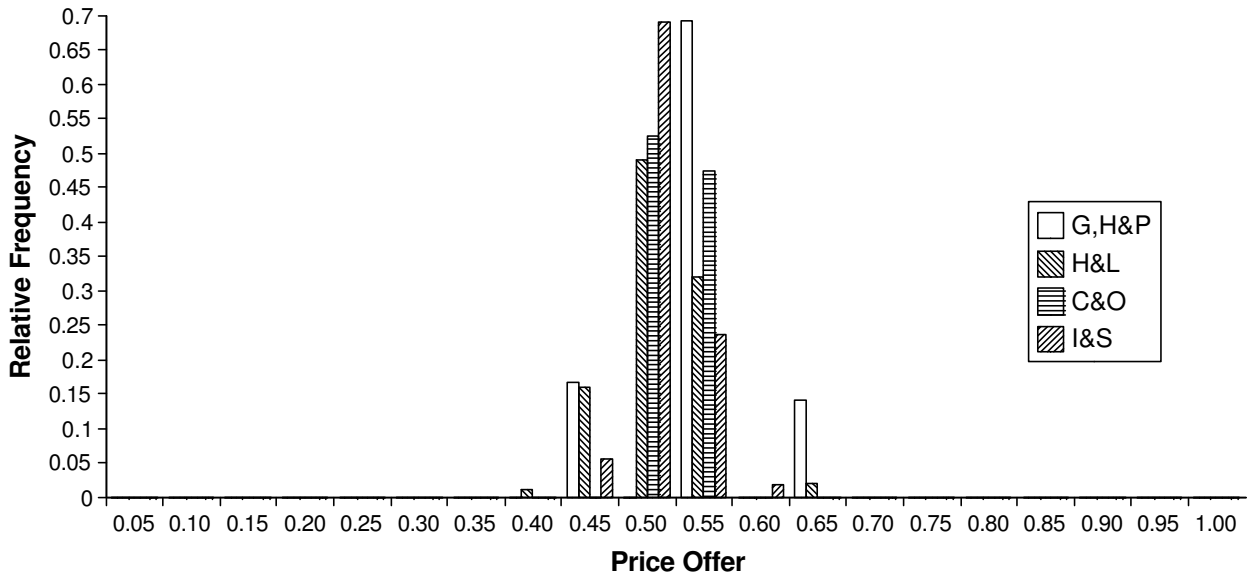Figure 4: Threshold value for different levels of risk aversion

Figure 5: Predicted price offers based on risk preference distribution estimates of four studies
"G, H & P"-Goeree, Holt and Palfrey (2003), "H & L"–Holt and Laury (2002),
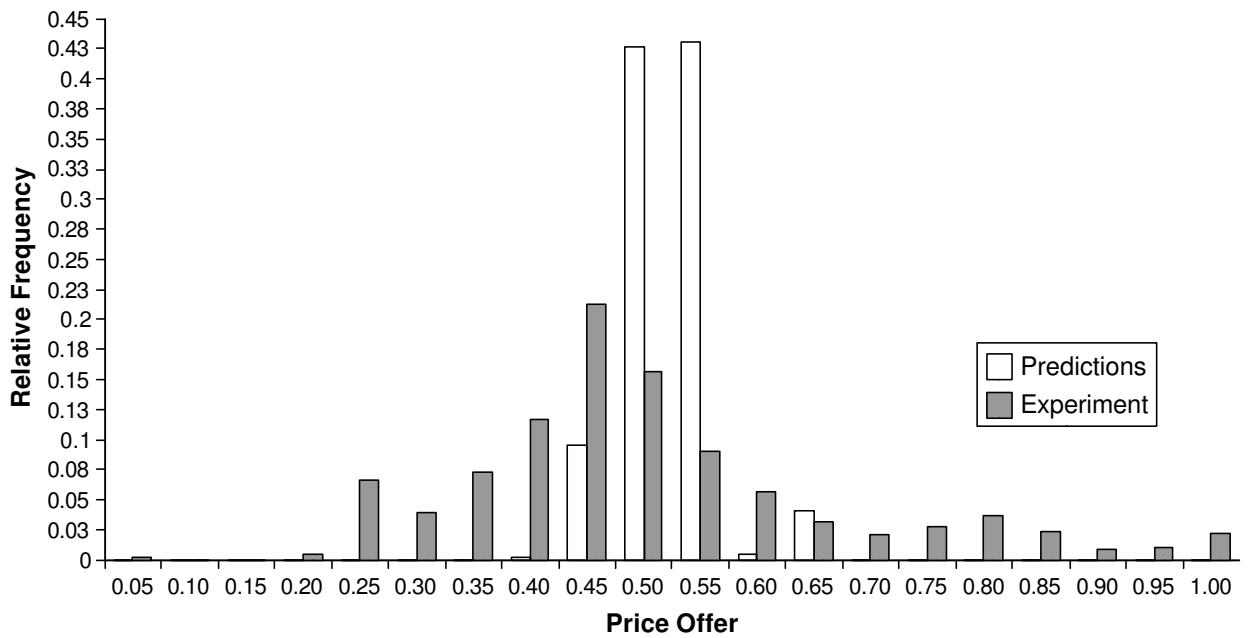"C & O"–Cox and Oaxaca (1996), "I & S"–Ivanova-Stenzel and Salmon (2004).

Figure 6: Predicted* and experimental price offers
*based on the average of different risk preference distribution estimates

cycle	Nobs	(1) Price offer		(2) Accept-ance rate (in %)	(3) Profits				(4) Effi-ciency (in %)
					Seller		Buyer		
		Mean	(StD.)		Mean	(StD.)	Mean	(StD.)	
1	240	0.53	(0.17)	35	0.34	(0.19)	0.14	(0.11)	80.4
2	240	0.51	(0.18)	35	0.32	(0.20)	0.16	(0.12)	85.0
3	240	0.52	(0.18)	28	0.33	(0.19)	0.16	(0.12)	87.9
4	240	0.50	(0.16)	36	0.32	(0.18)	0.16	(0.11)	86.7
all	960	0.51	(0.17)	33	0.33	(0.19)	0.15	(0.11)	85.0
Theory:		$0.5 \leq p \leq 1$		0	0.33		0.17		100

Table 1: Descriptive Statistics: Number of observations, price offers, acceptance rates, profits reported for sellers and buyers, and share of efficient sales for all periods and per cycle (1 cycle = 8 periods) and summary of the theoretical prediction

(1) Study	(2) Nobs	(3) Distribution of α–Estimates Quantiles			(4a) Prediction of Prices (p) Quantiles			(4b) Accept-ance rate (in %)
		10%	50%	90%	10%	50%	90%	
G, H & P	42	−0.38	0.14	0.89	0.48	0.55	0.68	17.6
H & L	175	0	0.28	0.83	0.48	0.53	0.56	21.2
C & O	40	0.37	0.72	0.92	0.53	0.54	0.56	36.9
I & S	55	−0.17	0.46	0.72	0.51	0.51	0.56	25.8

Table 2: Summary of different studies (col. 1) with varying sample sizes (col. 2) of their reported distribution of individual risk attitude (α) estimates (10%, 50% and 90% quantiles in col. 3), predicted price distributions (10%, 50% and 90% quantiles in col. 4 a) as well as sales in the negotiation phase based on those estimates (col. 4b). "G,H&P"-Goeree, Holt and Palfrey (2003), "H&L"–Holt and Laury (2002), "C&O"–Cox and Oaxaca (1996), "I&S"–Ivanova-Stenzel and Salmon (2004).

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