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Default Risk in Corporate Yield Spreads

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Abstract: An important research question examined in the recent credit risk literature focuses on the proportion of corporate yield spreads which can be attributed to default risk. Past studies have verified that only a small fraction of the spreads can be explained by default risk. In this paper, we reexamine this topic in the light of the different issues associated with the computation of transition and default probabilities obtained with historical rating transition data. One significant finding of our research is that the estimated default-risk proportion of corporate yield spreads is highly sensitive to the term structure of the default probabilities estimated for each rating class. Moreover, this proportion can become a large fraction of the yield spread when sensitivity analyses are made with respect to recovery rates, default cycles in the economy, and information considered in the historical rating transition data.

Keywords: Credit risk, default risk, corporate yield spread, transition matrix, default probability, Moody's, Standard and Poor's, recovery rate, data filtration, default cycle.

JEL Classification: G11, G12, G13, G21, G23

Résumé: Une importante question de recherche dans la littérature récente sur le risque de crédit aborde la proportion de l'écart de rendement entre les obligations privées et publiques (prime de crédit) pouvant être attribuée au risque de défaut. Des études antérieures ont démontré que cette proportion est faible (maximum 25 %). Dans cette recherche, nous réexaminons cette question en considérant différentes problématiques associées au calcul des probabilités de transition et de défaut obtenues des données historiques des transitions de notation de crédit. Un résultat important de notre recherche est que la proportion attribuable au risque de défaut dans les primes de crédit est sensible à la structure par terme des probabilités de défaut de chaque classe de risque. De plus, cette proportion peut devenir importante lorsque des analyses de sensibilité sont réalisées en fonction des taux de recouvrement, des cycles de défaut et de l'information considérée dans les données historiques des transitions de notation de crédit.

Mots clés: Risque de crédit, risque de défaut, prime de crédit, matrice de transition, probabilité de défaut, Moody's, Standard and Poor's, taux de recouvrement, filtre de données, cycle de défaut.

1 Introduction

An important research question studied in the recent credit risk literature examines the proportion of corporate yield spreads explained by default risk. This question is not only important for the pricing of bonds and credit derivatives but also for computing banks' optimal economic capital for credit risk (Crouhy, Galai, and Mark, 2000; Gordy, 2000). For example, in CreditMetrics (1997), all corporate yield spreads are assumed to be explained by default risk, so the implied capital charge for default risk can be too high if other factors enter in the composition of the spreads.

Elton, Gruber, Agrawal and Mann (Elton et al., 2001) have verified that only a small fraction of corporate yield spreads can be attributed to default risk or expected default loss. They got their result from a reduced form model and have shown that the expected default loss explains no more than 25% of corporate spot spreads. The remainder is attributed to a tax premium and a risk premium for systematic risk. More recently, Huang and Huang (2003) reached a similar conclusion with a structural model. They verified that, for investment-grade bonds (Baa and higher ratings), only 20% of the spread is explained by default risk.

One of the key inputs needed for such assessments is an estimate of the term structure of default probability, that is, the probability of defaulting for different time horizons. To get these quantities, databases on historical default frequencies from Moody's and Standard and Poor's can be used. For example, one can first come up with an estimate of transition probabilities between rating classes and then use them to compute the term structure of the default probability. This is the approach used in Elton et al. (2001). Although this method appears straightforward, obtaining probability estimates with such a procedure is not a trivial exercise. Many important issues arise in the process and the different choices made by the analyst might lead to different results.

A first choice to make when computing default and transition probabilities is the period over which the estimation is to be performed. As shown in Bangia et al. (2002), transition-matrix estimates are sensitive to the period in which they are computed. Business and credit cycles might have a serious impact on the estimated transition matrices and might lead to highly different estimates for the default-risk proportion.

A second issue calling for close attention concerns the statistical approach (Altman, 1998). Because defaults and rating transitions are rare events, the typical cohort approach used by Moody's and Standard and Poor's will produce transition probabilities matrices with many cells equal to zero. This does not mean that the probability of the cell is nil but that its estimate is nil. Such a characteristic could lead economic agents to underestimate the default-risk fraction in corporate yield spreads. Lando and Skodeberg (2002) have shown that a continuous-time analysis of rating transitions using generator matrices improves the estimates of rare transitions even when they are not observed in the data, a result that cannot be obtained with the discrete-time cohort approach of Carty and Fons (1993) and Carty (1997).

Finally, a third important issue arising in the computation of default and transition probabilities is the data filtering process which determines the information considered about issuers' movements in the database. For example, we must decide whether or not to consider issuers that are present at the beginning of the estimation period but leave for reasons other than default (withdrawn rating or right censoring). Another choice is whether or not to consider issuers that enter the database after the starting date of estimation.

Again, these choices might have non-negligible impacts on the final estimates.

In this article, we revisit the topic of corporate rate spread decomposition in light of the above considerations. In a first step, we introduce a simple discrete-time model for estimating the proportion of the corporate rate spread attributable to default risk. This model is then implemented using estimates of transition probabilities from the cohort method and computed for different periods of the default cycle. The results show that matching the periods over which the probabilities are computed with the period over which the spreads are examined can raise the proportion of the corporate yield spreads attributable to default risk. For example, for A rated bonds, this proportion jumps from 9% to 23% (Table 3b, panel B, 1987-1991). For Baa bonds, this proportion jumps from 32% to 65% (Table 3b, panel C, 1987-1991). As we shall see in more detail, the 1987-1991 sub-period corresponds to a high default cycle in the bond market. The above results are confirmed by a the continuous-time analysis of transition ratings in line with Lando and Skodeberg (2002) (Table 3b, MDC columns). The analysis also highlights the importance of using the proper recovery rates when estimating the proportion of the yield spread attributed to default risk. For Baa bonds, for example, the above proportion of 65% corresponds to a recovery rate of 49%. This proportion jumps to 74% when the recovery rate is cut to 40%, a more plausible rate in a high default risk period (Table 5).

In a second step, we look to see how sensitive our results are to the modelling approach taken and to the information considered in the database. For this purpose, we introduce an alternative continuous-time model. We also use this model to compute the approximate confidence intervals, in the spirit of Christensen, Hansen, and Lando (2004). Three data filtering processes for different types of information are examined : the first includes issuers that enter after the starting date of estimation (entry firms hereafter) as well as withdrawn-rating observations; the second one excludes entry firm observations; and the third excludes both entry firms observations and withdrawn-rating observations. Our results —when excluding withdrawn-rating and entry firm observations— show that the average spread proportion attributed to default risk for a maturity of ten years climbs to 71% for Baa rated bonds during the 1987-1991 period, with a 95% approximate interval of 56% and 86% when the recovery rate is 49% (Table 8b).

The rest of the paper is organized as follows. In Section 2, we describe how the empirical bond-spread curves are estimated. In Section 3, we present the discrete-time model used to estimate the default proportion of the corporate yield spread for different rating categories and maturities. This section also presents the results on the default-risk proportion obtained with this model and examines their sensitivity to assumptions about default cycles, estimation methodology of probabilities and recovery rates. The sensitivity of our modelling approach is then examined in Section 4 with the use of a continuous-time model. This section also presents results including inference obtained by Monte Carlo simulations and a detailed sensitivity analysis about the information considered in the databases. Section 5 concludes.

2 Empirical Bond-Spread Curves

2.1 Database Description

The data come from the Lehman Brothers Fixed Income Database (Warga, 1998). We choose this data to enable comparisons with other articles in this literature using the same database. Moreover, the database covers two default cycles an issue that will become important in the analysis. The data contains information on monthly prices (quote and matrix), accrued interest, coupons, ratings, callability and returns on all investment-grade corporate and government bonds for the period from January 1987 to December 1996. All bonds with matrix prices and options were eliminated; bonds not included in Lehman Brothers' bond indexes and bonds with an odd frequency of coupon payments were also dropped.[1] Appendix A.1, provides details on the treatment of accrued interest.

As in Elton et al. (2001), month-end estimates of the yield-spread curves on zero-coupon bonds for each rating class are needed to implement the models. These yield-spread curves are obtained by first estimating the parameters associated with the Nelson and Siegel curve fitting approach. Appendix A.2 provides a brief summary of this approach. All bonds with a pricing error higher than \$5 are dropped. We then repeat this estimation and data removal procedure until all bonds with a pricing error larger than \$5 have been eliminated. Using this procedure, 776 bonds were eliminated (one Aa, 90 A and 695 Baa) out of a total of 33,401 bonds found in the industrial sector, which is the focus of this study.

2.2 Empirical Analysis of Bond-Spread Curves

For each of the 120 months of this study, we first estimate the forward rate curve associated with government bonds.[2] The industrial corporate bonds are then grouped in three categories: Aa, A, and Baa. For each category, we estimate the corporate forward rate curves. Our results are coherent, in that all of our estimated empirical bond-spread curves are positive. Moreover, the bond-spread curves between a high rating class and a lower rating class are also positive. Throughout this article, we shall use the expression "spot rate" to indicate the yield to maturity on a zero-coupon bond. Figure 1 shows the empirical spreads on industrial bonds of six years maturity for the ratings Aa, A, and Baa. We observe that the spreads are higher during the January 1987-December 1991 sub-period.

(Figure 1 here)

Table 1 reports our results for corporate yield spreads for two to ten years of maturity. The results are very close to those presented in Table 1 of Elton et al. (2001) for the industrial sector. The small discrepancies might be explained by differences in data sets and estimation algorithms. In panel A, the results cover the entire 10-year period, while in panels B and C they refer to two sub-periods of five years. It

[1] We did, however, keep three categories from the list of eliminations in Elton et al.(2001), because we lacked the information needed to identify them: government flower bonds, inflation-indexed government bonds, and bonds with floating rate debt. As we shall see, this makes no difference in the results.

[2] Recent studies (see, for example, Hull et al. 2005) argued that the Treasury rate is not the appropriate risk-free-rate. We shall return to this issue in Section 4.3.

is important to observe that the average spreads are higher in the 1987-1991 sub-period than in the 1992-1996 sub-period. This difference will matter in the next sections where we explain the proportions of the corporate yield spreads associated with default risk. The two sub-periods represent two different default cycles. The 1987-1991 sub-period is usually associated with a high default cycle while the sub-period 1992-1996 tends to coincide with a low default cycle. The 1987-1991 period contains a macroeconomic recession and the US loan crisis.

Figure 2 presents default rates extracted from Moody's database (Moody's, 2005). We observe that the distribution of these rates over time has a shape similar to that of the empirical spreads for six years maturity in Figure 1. This suggests that the link between the default-risk proportion and the default risk should vary with the default cycles (see also Manning, 2004, for a similar conclusion).

(Figure 2 here)

Another interesting graph shown in Figure 3 examines how the average recovery rate varies with time. These recovery rates were obtained from Moody's database (Moody's, 2005) and are defined as the ratio of the defaulted bond's market price, observed 30-days after its default date, to its face value (par). The average recovery rates vary significantly with the default cycles defined above. For example, the average recovery rate for all bonds during the 1987-1991 sub-period is equal to 40.8% while that of the 1992-1996 sub-period is equal to 45.5%. Those for senior unsecured bonds vary between 45.5% and 50.66% for the same sub-periods. It is also documented in Moody's (2005) that the recovery rates are even lower for industrial bonds. Altman et al. (2003) present an interesting review of the literature on the link between recovery rate and default probability.

(Figure 3 here)

Table 2 compares the average squared-root mean errors of the difference between theoretical bond prices computed using the Nielson-Siegel model and the actual bond prices for treasuries and industrial corporate bonds. Again our results are similar to those of Elton et al. (2001).

3 Discrete-Time Model

3.1 Model

The corporate yield spread is defined as the difference between the yield curves of the risky zero-coupon bond and the risk-free, zero-coupon bond. Therefore, to characterize corporate yield spreads, one need only model the values of a risk-free and a corporate zero-coupon bond. The model developed here, unlike that of Elton et al. (2001), avoids specifying a coupon rate that might absorb effects unrelated to default risk. The model we propose thus focuses on zero-coupon bonds and assumes that a corporate yield spread might be totally explained by the recovery rate and the possibility of default. The model will be used to measure how much the observed corporate yield spread is explained by these two components.

The model is built on the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$. In this section, time is measured in discrete periods. Let $f(t,T)$ denote the risk-free, continuously compounded forward rate (annualized) at time t for a loan that starts at period T and ends one period later. The discount factor for the period t to T is $\beta(t,T) = \exp\left(-\sum_{s=t}^{T-1} f(s,s)\,\Delta_t\right)$ where Δ_t is the length (in years) of one period of time. In the following, it is assumed that :

(i) There exists at least one martingale measure Q under which the discounted value of any risk-free, zero-coupon bond is a martingale.

(ii) Under the martingale measure Q, the default time τ of the corporate bond is independent of the risk-free, forward interest rates $f(t,T)$, $t \leq T$.

(iii) In case of default, a fraction ρ_τ of the market value of an equivalent risky bond is recovered at the default time. This fraction is a deterministic function of the default time.

(iv) Under the martingale measure Q, the probability q_s that the default will arise in s periods from now will remain constant through time, that is, for $s = 1, 2, 3, \ldots$,

$$\begin{aligned} q_s &= Q\left[\tau = s \,|\tau > 0\right] \\ &= Q_t\left[\tau = t + s \,|\tau > t\right] \text{ for any } t, \end{aligned} \tag{1}$$

where Q_t denotes the conditional probability indicated by the information (the $\sigma-$field $\mathcal{F}_t$) available at period t.

(v) Investors are risk neutral with respect to default risk.

Assumption (i) is needed to price a bond at its expected discounted payoff. Assumption (ii) is used to simplify the bond-value computations. We did not consider using the $Q-$ forward measure to relax this hypothesis since there are as many forward measures as there are zero-coupon bonds. Assumption (iii) differs from the ones found in different studies. The recovery of a fraction of the face value of the bond is not suitable for zero-coupon bonds because it would allow disproportionate cash flows with respect to the bond price in case of early default. This, in turn, might give rise to negative spreads. When working with coupon-paying bonds, however, the recovery of a fraction of the face value can be used as a proxy for the recovery of a fraction of the market value because the latter is not too far from its face value. Assumption (iv) is not crucial to the model's application, and the model we propose can easily be extended to non-homogeneous default probabilities. This assumption is required only in the estimation stage of the model since there would otherwise be too many parameters to estimate. Finally, Assumption (v) implies that the law of the default time τ is the same under the objective measure P and the martingale measure Q. This assumption justifies using databases containing information about default probabilities under the objective measure P to estimate the parameters of the default distribution under the martingale measure Q which is required in the model. Since, in practice, risk-averse investors usually require a default-risk premium, the impact of this hypothesis tends to underestimate the proportion of the spread that may be explained by default risk. Therefore, all our results should be interpreted as lower bounds of the true default spread proportions.

Theorem 1 Under Assumptions (i), (ii), (iii), (iv) and (v) the time t value $\widetilde{P}(t,T)$ of a corporate zero-coupon bond with a face value of one dollar and a time to maturity of $(T-t)$ periods may be expressed as

$$\widetilde{P}(t,T) = P(t,T)\, p_{t,T}, \tag{2}$$

where $P(t,T)$ is the time t value of an equivalent risk-free, zero-coupon bond, $p_{T,T} = 1$,

$$p_{t,T} = \left\{ \sum_{u=1}^{T-t} \rho_{u+t} p_{t+u,T} q_u + \left(1 - \sum_{u=1}^{T-t} q_u\right) \right\}, \; t \in \{1, 2, ..., T-1\}, \tag{3}$$

and q_s is the probability, under the risk-neutral measure Q, that the default will occur in exactly s periods from now.

The result is established by using induction on the time to maturity. A complete proof may be found in Appendix B.1. The default-spread curve at time t is therefore

$$S(t,T) = \frac{\ln P(t,T)}{\Delta_t(T-t)} - \frac{\ln \widetilde{P}(t,T)}{\Delta_t(T-t)} = -\frac{\ln p_{t,T}}{\Delta_t(T-t)}. \tag{4}$$

3.2 Parameter Estimation

To compute the corporate yield spreads implied by Equation (4), one needs estimates of the recovery rates ρ_t and the probabilities q_s, $s = 1, 2, 3, ...$, i.e. the term structure of default probabilities. As argued in the introduction, the statistical approach adopted for the estimation of the default probabilities can influence the estimated proportion of the spreads attributable to default.

A first approach, which imposes little structure on the data, requires forming a cohort at a point in time and counting the defaults after one period, two periods, and so on. The drawback of such an approach stems from the large standard errors associated with the estimates. Generating accurate estimates needs the observation of many defaults, an unlikely possibility when working with investment grade bonds. For such a case, many estimated probabilities would simply be zero. This approach would also make it difficult to include the information provided by new firms entering the database.

Another approach found in the literature uses estimates of periodic transition matrices available from Moody's or Standard and Poor's via the cohort method of Carty and Fons (1993) and Carty (1997). The transitions from one credit rating class to another are counted and estimates of transition probabilities are calculated using the number of bonds in the cohort at the start of the period. Probabilities of defaulting for more than one period can then be conveniently computed from this transition matrix using simple matrix multiplications. This convenience comes at the cost of imposing a Markovian structure on the data and it is not clear if such a structure fits. As with the preceding approach, there are also several drawbacks associated with such estimates of default probabilities. Defaults and rating transitions are rare events and these transition matrices contain many cells with estimated probabilities equal to zero. This might lead to an underestimation of the default-spread. Again, as with the preceding approach, if one tries to build confidence intervals around these estimates, the results turn out to be unsatisfactory.

Recently, Lando and Skodeberg (2002) have suggested estimating a Markov-process generator rather than the one-year transition matrix. As with the cohort approach, this method also adds a Markovian structure not needed by the model we propose. Lando and Skodeberg (2002) have shown that this continuous-time analysis of rating transitions using generator matrices improves the estimates of rare transitions even when they are not observed in the data, a result that cannot be obtained with the discrete-time analysis of Carty and Fons (1993) and Carty (1997). A continuous-time analysis of defaults permits estimates of default probabilities even for cells that have no defaults. This is possible because the approach draws on the information contained in the transition from one class to another to infer better estimates of the default probabilities. Finally, as shown in Christensen, Hansen, and Lando (2004), inference in such a framework is informative and can be conveniently computed.

The next section will examine the proportion of the corporate spreads which can be attributed to default risk using the last two approaches described above for the estimation of the term structure of default probabilities. The recovery rate $\rho_\tau = \rho$ will be assumed constant through time. In Elton et al. (2001), ρ is about 60% for Aa and A bonds and about 50% for Baa bonds. This section also illustrate how sensitive corporate default spreads are to the value of the recovery rate.

3.3 Empirical Results with Discrete-Time Model

Table 3a summarizes the different sources of information about the transition matrices used in this section. Table 3b summarizes our results on default spread proportions generated with the discrete-time model. In the Elton et al. (2001) panel, we estimate their model using transition matrices drawn from five different sources.[3] We repeat the same sensitivity analysis for the default-spread proportions, using our model estimated with zero-coupon bonds and recovery rates as a fraction of the market value at the default time (Dionne et al., 2005 panel). The results are similar to those in the Elton (2001) panel, the latter being obtained with a coupon rate set so that a 10-year bond with that coupon would be selling close to par for all periods.

The first column of Table 3b in both panels presents the default-spread proportions obtained with a transition matrix calculated from Standard and Poor's data (SPEG). This matrix was estimated with the cohort approach over the 1981-1995 period (Altman, 1995). Notice that Elton et al. (2001) used this matrix to measure default risk over the entire 1987-1996 period.[4] Using the same transition matrix over the 1987-1991 and 1992-1996 sub-periods produces larger default-spread proportions in the latter case simply because the empirical corporate yield spreads are smaller during those years (see Table 1). Moreover, because this transition matrix underestimates the defaults during the high default cycle period of 1987-1991, it also underestimates the default-spread proportions for that period.

[3] Table A1 in Appendix reproduces the default spreads analysis of Elton et al. (2001) using our data and their transition matrix. We see that the two data sets produce equivalent results.

[4] Elton et al. (2001) were aware of this fact: "... our transition matrix estimates are not calculated over exactly the same period for which we estimate the spreads. However there are three factors that make us believe that we have not underestimated default spreads." (p. 263)

Column MDEG repeats the analysis using a matrix estimated over the 1970-1992 period with Moody's data. This matrix is also used in Elton et al. (2001). We observe that the corresponding default-spread proportions are, in general, lower than for the Standard and Poor's matrix, most likely because the two matrices were estimated over two different periods. The results presented in Table 1 indicate clearly that the estimated corporate yield spreads are very different in the two sub-periods. It might thus be more appropriate to use different transition matrices estimated with data corresponding to these two different sub-periods.

The MDB1 column reports the estimates obtained with transition matrices computed over the corresponding sub-periods with Moody's database. The matrices are one-year transition-matrix estimates obtained with the cohort approach described in, for example, Christensen et al. (2004) equation (1). It should be noticed that splitting the sample in high and low default periods might amplify (or shrink) the proportions in the high (low) default sub-period. For example, in a high default sub-period, we assume that a ten-year bond is priced with probabilities from the high default period even if this period is not expected to last for ten years. The reverse effect might also be obtained for the low default period. Low default probabilities are used to price a ten-year bond even if the low default cycle is not expected to last for ten years.

The fourth column (MDB2) also reports estimates made with cohort transition matrices computed with Moody's database on the corresponding sub-periods but now considering different information than in the MDB1 case. The data considered here is obtained using the filtering approach suggested in Christensen et al. (2004) [5]. The specific details regarding the information considered here are presented in Appendix A3.

The results in columns MDB1 and MDB2 are similar. It is interesting to observe that the default-risk proportion increases from 31.57% with the MDEG procedure to more than 60% with both MDB1 and MDB2 procedures for Baa bonds during the 1987-1991 sub-period. We also observe that the default risk proportion is significantly lower in the 1992-1996 sub-period which is associated with a much lower default cycle. The main difference with previous studies is that appropriate transition matrices are matched with the different default cycles.

The fifth column (MDC) uses the same data and filtering technique as for the MDB2 column and applies the estimation method of Lando et al. (2002) using continuous-time credit migration. The estimated generators are presented in Table A2. The default-spread proportions do not differ significantly with those computed for Aa bonds using the discrete cohort method (MDB2) for a ten-year period (Table 3b) but are slightly lower for A and Baa bonds. Some important differences appear also when we consider shorter periods and higher credit ratings. Table 3c shows that the average default-spread proportions are higher with the MDC method of estimation when we look at industrial Aa and A ratings for two-year periods. These results are confirmed in Table 4a. We observe that, for these rating categories, the implied default probabilities are much higher in the 1987-1991 sub-period with the continuous-time transition matrix approach (MDC_1). The main reason is that traditional methods of estimation yield zero estimates of short term default probabilities for higher rating classes whatever the default cycle.

Finally, Table 5 presents a sensitivity analysis of default-spread proportions with respect to recovery

[5] With one exception that does not yield a significant difference. They used the senior unsecured bond rating to construct the issuer rating while we used the estimated senior unsecured issuer rating given by Moody's database.

rates. We observe that the results are very sensitive to the different values chosen. For example, with a recovery rate of 40% (which is the rate often observed in high default cycles for industrial bonds), the default-spread proportion can increase up to about 74% in the industrial Baa rating class of corporate bonds when a discrete-time analysis of the data is done for the 1987-1991 sub-period. With the continuous-time analysis, it goes up to 70%. A sensitivity analysis with respect to coupon rates (not reported here) also shows that using bonds paying coupons does affect the results, which supports our approach of using zero-coupon bonds.

4 Continuous-Time Model

4.1 Model

To test the robustness of the results derived from our theoretical model, we examine here an alternative model specification where default can occur on a continuum of dates. This departs from the analysis of the previous section which relies on estimates of default probabilities for different discrete horizons.

In this section, time is no longer expressed in terms of the number of periods but as a continuous function. As shown in Theorem 2, the model presented here depends on a recovery rate ρ and the intensity $\{\lambda_t : t \geq 0\}$ associated with the distribution of τ, the default time. The risk-free discount factor for the time interval $(t, T]$ is $\beta(t,T) = \exp\left(-\int_t^T f(s,s)\ ds\right)$ where $f(t,T)$ denotes the instantaneous continuously compounded risk-free forward rate. In the following, it is assumed that:

(i) There exists at least one martingale measure Q under which the discounted value of any risk-free, zero-coupon bond is a martingale.

(ii) Under the martingale measure Q, the default time τ of the corporate bond is independent of the risk-free, forward interest rates $f(t,T)$, $t \leq T$.

(iii) In case of default, a constant fraction ρ of the market value of an equivalent risky bond is recovered at the default time.

(iv*) Under the martingale measure Q, the intensity process $\{\lambda_t : t \geq 0\}$ of the default time τ is deterministic but may vary through time.

(v) Investors are risk neutral with respect to default risk.

The first three assumptions are identical to those used in the discrete-time model. Assumption (iv*) can be relaxed easily, since any intensity process that satisfies the requirements of the Duffie-and-Singleton (1999) model is acceptable. However, the expression given in Theorem 2 will be modified. Finally, assumption (v), which implies that the distribution of τ will remain the same under the empirical probability measure P and the martingale measure Q, is again required for the use of databases containing information about default probabilities.

Theorem 2 Under the Assumptions (i), (ii), (iii), (iv*) and (v), the time t value of a corporate zero-coupon bond is

$$\widetilde{P}(t,T) = P(t,T)\exp\left(-(1-\rho)\int_t^T \lambda_s ds\right). \tag{5}$$

This result is a particular case of the Duffie-and-Singleton model (1999) and the proof is in Section B.2. In this particular case, the corporate yield spread curve at time t is given by

$$S(t,T) = \frac{\ln P(t,T)}{T-t} - \frac{\ln \widetilde{P}(t,T)}{T-t} = \frac{1-\rho}{T-t}\int_t^T \lambda_s ds. \tag{6}$$

For practical reasons such as the estimation of the model, we need to impose some additional structure on the distribution of τ. This is summarized in the following additional hypothesis:

(iv)** Under the martingale measure Q, the default time τ is driven by a time-homogeneous Markov process X modelling the credit rating migrations of the firm. This Markov process X is characterized by the generator matrix $\mathbf{\Lambda}$ and we assume that $\mathbf{\Lambda}$ is diagonable.

In this context, the intensity is

$$\lambda_t = \frac{\sum_{k=1}^m a_k d_k \exp(d_k t)}{1-\sum_{k=1}^m a_k \exp(d_k t)} \tag{7}$$

where the constants $d_1, ...d_m$ are the eigen values associated with the generator matrix $\mathbf{\Lambda}$ and the constants $a_1, ..., a_m$ are functions of the components of the eigen vectors of $\mathbf{\Lambda}$ and are described explicitly in the proof found in Appendix B.3.

4.2 Parameter Estimation

To perform comparisons similar to those made for the discrete-time model, we must have available estimates of the generators associated with the different transition matrices used in the analysis. However, as shown in Israel et al. (2001), the existence of such a generator for a given transition probability matrix is not guaranteed. We therefore borrow the procedure suggested in Israel et al. (2001) to verify the existence of an underlying generator for the transition matrices used in the previous sections. The results show that such a generator does not exist for the transition matrices from Standard and Poor's and Moody's used in Tables 3 and 4 (SPEG and MDEG) nor for the one computed from Moody's database (MDB).

As proposed in Israel et al. (2001), a possible solution to this problem is to obtain a generator that will produce a transition matrix close to the original transition matrix. We therefore follow their procedure to obtain the estimated generators. Using these estimates, we next compute the intensities with equation (7). The spreads are then computed using the following discrete approximation of equation (6) :

$$\frac{1-\rho}{T-t}\int_t^T \lambda_s ds \cong \frac{1-\rho}{n}\sum_{j=1}^n \hat{\lambda}_{j\Delta t} \tag{8}$$

with $\Delta t = (T-t)/n = 10^{-6}$.

4.3 Empirical Results with Continuous-Time Model

Table 6 presents the results associated with the continuous-time model and compares them to those obtained with our discrete-time model. As shown in this table, the results are robust to the model specification. We should mention that these results and those obtained with the discrete-time model should be interpreted in light of the risk neutrality assumption made to justify using databases with information about default probabilities under the objective measure. Relaxing this hypothesis would require, as reported in some studies (Duffie et al., 2003, and Jarrow and Purnanandam, 2004), scaling up the martingale intensity. This scaling would yield even higher proportions of estimated default present in the spread. Unfortunately, the correct scaling factors are unknown and model dependent. However, the effect that an uniform scaling would produce can be assessed by noticing that the spread is a linear function of the intensity in the continuous-time model. Hence, a scaling factor of two would double the theoretical spreads.

We now report the sensitivity of our results to the data filtering process and the information considered when computing the transition matrix. Such an analysis is important for financial institutions that are building their own internal rating system for Basel II and for the regulators who will have to monitor these systems. Approximate confidence intervals obtained by simulation are also reported in these tables. These intervals are approximate because the simulation procedure uses the estimated generator as if it were the true generator. The statistical uncertainty associated with this estimate is not taken into account. Hence, the intervals presented in this table are smaller than genuine confidence intervals. Appendix C gives the details of the simulation procedure and Table 7 reports the distribution of issuers by rating at the starting date of the simulation period.

Tables 8a, b, and c report results for the default spread proportions while Tables 9a, b, and c reports those for the estimated default probabilities. The main difference between Tables 8a, b, and c is in the treatment of the data. Table 8a uses censored data (withdrawn rating) and issuers that enter after the starting date (entry firms) of estimation. In Table 8b, we excluded withdrawn-rating and entry firm data. In Table 8c, we excluded only entry firm data. The same treatments are applied for Tables 9a, b, and c. As the results show, important differences are observed. Tables 8b and 9b report higher default proportions and higher default probabilities. As already mentioned, these tables exclude withdrawn-rating and entry firms and are more in the spirit of the standard cohort analysis of Moody's while Tables 8c and 9c include withdrawals. Moody's also produces statistics including withdrawals (right censoring). We observe, from Table 10, that the number of defaults is the same in panels b and c while the numbers of issuers and rating observations are higher in panel c. Inclusion of the withdrawals reduces default probabilities and default risk proportions in yield spreads. The same conclusion is obtained when entry firms are added (Tables 8a and 9a). Default-risk proportions and implied default probabilities are even lower. Two alternative explanations are possible: either the treatment of firms that enter and leave is not free of bias or the censored and entry issuers represent different default risks. Additional research is needed to discriminate between the two explanations.

Finally, Table 11 examines possible explanations for the low default-risk proportion in yield spreads for the 1992-1996 period. As recently argued in Hull, Predescu and White (2005), Treasury rates might not be proper benchmarks. Regulation on this market might artificially raise the demand for these bonds and reduce their yields.

Since, in theory, Credit Default Spreads (CDS) should be close to the credit spread of the bonds issued by the reference entity over the risk-free rate, Hull et al. (2005) used the credit-default swap data to estimate the benchmark risk-free rate used by participants in credit markets. Their main result is that the benchmark five-year, risk-free rate is on average about 83% of the way from the Treasury rate to the swap rate. We use this result to generate the benchmark risk-free rate for all maturities during the 1992-1996 period and to assess whether the low default-risk proportions we got for that period can be attributed to our Treasury benchmark. The results are reported in Table 11. Although the estimated proportions are higher, they are far below those anticipated.

Some authors have suggested liquidity risk as a possible explanation of yield spreads (see, for example, Longstaff, 2003). As argued by Duffie and Singleton (2003), liquidity risk should not be an important factor when using quote prices. Another benchmark could also be the Aaa rating to eliminate tax and some of the possible residual liquidity explanations. The results in Table 11 show that this does not substantially affect the results.

5 Conclusion

We have revisited the estimation of default-risk proportions in corporate yield spreads. Past studies have found that only a small proportion of the spreads can be attributed to default risk. We find here that such results do not hold for all periods of the default cycle. It has been documented that the 1987-1991 period corresponds to a high default cycle, while the 1992-1996 period corresponds to a low default cycle. When the transition matrices are appropriate for the default cycle under examination, substantial differences in the results are observed. The estimated proportions can reach 71% of the estimated spread for maturities of ten years for Baa bonds during the 1987-1991 period, with an approximate confidence interval of 56% and 86% when the recovery rate is 49% (Table 8b). This conclusion is important for financial institutions planning to use internal rating systems and for the regulators that will have to monitor these systems

We have also shown that the results can vary substantially when changing the recovery rate assumption. When the recovery rate is cut to 40%, a rate that seems more appropriate for industrial bonds in high default cycles, the above proportion for Baa bonds goes up to 83%, with approximate confidence intervals of 67% to 98% (computation details are available upon request). The default-spread proportions are multiplied by 1.5 on average when the recovery rate drops from 60% to 40% (Table 5).

Our study could be extended in several directions by relaxing some of the restrictive assumptions that have been used. First, the assumption of risk neutrality could be relaxed with preference-based models. The computation of risk-neutral probabilities different from the default probabilities under the objective measure could then be obtained. In this framework, the risk-neutral probabilities would simply become the default probabilities under the objective measure weighted by some function of the risk aversion. Building confidence intervals around such estimates might produce results that would leave a tight fit for taxes once liquidity premia are taken into account. This would produce results consistent with the vast and successful literature on derivative securities in which the inclusion of taxes has been found to be of little help.

Finally, it should be noticed that we have observed substantial increases in the estimated proportion in the high default cycle only. The results in the low default cycle maintain that a small proportion of the spread is attributable to the default risk. As a result an important question remains: Why are the proportions of yield spread associated to credit risk so low in low credit-risk cycles? This puzzle is discussed in recent contributions from the literature. It is now labelled the "credit spread level puzzle" (Chen, Collin-Dufresne, and Goldstein, 2005). It has been suggested that macroeconomic or Fama-French factors could explain the common variations of credit spreads. Time variation in risk premia is another possible explanation (Dionne et al., forthcoming), as well as non-Markov effects associated with hidden exited states (Christensen et al., 2004).

A Appendix: Technical Notes and Additional Tables

A.1 Treatment of Accrued Interest

First, when the date of the next coupon payment is the same as the transaction date and if it is not an odd coupon payment, the accrued interest in the Warga (1998) database is equal to zero before February 1991 and equal to the accrued interest of one day after February 1991. Second, when the date of the next coupon payment is the day following the transaction date and if it is not an odd coupon payment, the accrued interest in the database is equal to the coupon amount minus the accrued interest of one day before February 1991 and equal to zero after February 1991. Finally, for the other bonds, the accrued interest in the database is equal to the theoretical accrued interest (defined below) before February 1991 and equal to the theoretical accrued interest plus the accrued interest of one day after February 1991.

In order to get a similar accrued interest for the entire period, we calculated the theoretical accrued interest which is the same as that in the database for the first period (01-1987 to 02-1991) and we corrected the accrued interest for the second period (03-1991 to 12-1996). The theoretical accrued interest (AI) is given by this formula:

$$\text{AI} = \frac{nC}{N}$$

where n is the number of interest-bearing days, N is the number of days between two successive coupons[6] and C is the semiannual coupon amount.

In the database there are two methods of calculating the parameters n and N. For government bonds it is the actual/actual method. For corporate bonds it is the 30/360 method. Let $(d_1, m_1, y_1), (d_2, m_2, y_2)$ and (d_3, m_3, y_3) represent, respectively, the date from which accrued interest is calculated, the settlement date, and the relevant interest payment date with d_i the day's number from 1 to 31, m_i the month number from 1 and 12 and y_i the year number. The parameters n and N are given by the next formula for the actual/actual method:

$$\begin{aligned} n &= (d_2, m_2, y_2) - (d_1, m_1, y_1) \\ N &= (d_3, m_3, y_3) - (d_1, m_1, y_1) \end{aligned}$$

[6] Or the emission date and the first coupon date if the transaction date falls before the first coupon payment date.

and by the next formula for the 30/360 method:

$$n = \left(\tilde{d}_2 - \tilde{d}_1\right) + 30\left(m_2 - m_1\right) + 360\left(y_2 - y_1\right)$$
$$N = 180$$

with

$$\tilde{d}_1 = \begin{cases} 30 & : \quad m_1 \neq 2 \text{ and } d_1 = 31 \\ 30 & : \quad m_1 = 2 \text{ and } d_1 \geq 28 \\ d_1 & : \quad \text{otherwise} \end{cases}$$

and

$$\tilde{d}_2 = \begin{cases} 30 & : \quad m_2 \neq 2 \text{ and } d_2 = 31 \\ 30 & : \quad m_1 = m_2 = 2, \; d_2 \geq 28 \text{ and } d_1 \geq 28 \\ d_2 & : \quad \text{otherwise} \end{cases} .$$

A.2 The Nelson-Siegel (1987) Model

The empirical corporate yield spreads are obtained using Nelson and Siegel's approach which parameterize the instantaneous forward rate as

$$f_{NS}\left(t,T\right) = a_t + b_t \exp\left(-\frac{T-t}{\eta_t}\right) + c_t \frac{T-t}{\eta_t} \exp\left(-\frac{T-t}{\eta_t}\right)$$

where a_t, b_t, c_t and η_t are constants to be estimated for every month and rating class. The time t value of a zero-coupon bond paying one dollar at maturity can then be written as

$$P_{NS}\left(t,T\right) = \exp\left\{-\alpha_t\left(T-t\right) - \beta_t \eta_t \left(1 - \exp\left(-\frac{T-t}{\eta_t}\right)\right) - \gamma_t\left(T-t\right)\exp\left(-\frac{T-t}{\eta_t}\right)\right\},$$

where $\alpha_t = a_t$, $\beta_t = b_t + c_t$ and $\gamma_t = -c_t$.

For a given date t, we observe n bond prices $V_{obs}\left(t,T_1\right), ..., V_{obs}\left(t,T_n\right)$. Using Nelson-Siegel parametrization and the fact that a coupon bond can be expressed as a portfolio of zero-coupon bonds, we can write the ith bond price as :

$$V_{NS}\left(t,T_i\right) = C_i \sum_{k=1}^{K_i} P_{NS}\left(t,T_{i,k}\right) + P_{NS}\left(t,T_{i,K_i}\right) \tag{9}$$

where C_i is the coupon rate of the i^{th} bond, T_i is its maturity date, K_i is its number of remaining coupons and $T_{i,1}, ..., T_{i,K_i}$ are the coupon dates with $T_{i,K_i} = T_i$. Note that $V_{NS}\left(t,T_i\right)$ is a function of the known quantities C_i, K_i, $T_{i,1}, ..., T_{i,K_i} = T_i$ and of the unknown quantities α_t, β_t, γ_t and η_t. For each date t in our sample, we choose α_t, β_t, γ_t and η_t by minimizing the objective function

$$g\left(\alpha_t, \beta_t, \gamma_t, \eta_t\right) = \sum_{i=1}^{n} \left(V_{NS}\left(t,T_i\right) - V_{obs}\left(t,T_i\right)\right)^2$$

using the Levenberg-Marquardt algorithm (Marquardt, 1963). To minimize the chances of converging to a local rather than a global minimum, a grid search of $20^4 = 160\ 000$ points is performed to find a suitable starting point for the numerical minimization.

A.3 Data Description for Transition Matrix Estimation

The rating transition histories used to estimate the generator are taken from Moody's Corporate Bond Default Database (January, 09, 2002). We consider only issuers domiciled in United States and having at least one senior unsecured estimated rating. We started with 5,719 issuers (in all industry groups) with 46,305 registered debt issues and 23,666 ratings observations. For each issuer we checked the number of default dates in the Master Default Table (Moody's, January, 09, 2002). We obtained 1,041 default dates for 943 issuers in the period 1970-2001. Some issuers (91) had more than one default date. In the rating transition histories, there are 728 withdrawn ratings that are not the last observation of the issuer. Theses irrelevant withdrawals were eliminated and so we obtained 22,938 ratings observations. Table A3 compares our data set with that of Christensen et al. (2004).

The most important and difficult task is to get a proper definition of default. In order to compare our results with recent studies, we treat default dates as do Christensen et al. (2004). First, all the non withdrawn-rating observations up to the date of default have typically been unchanged. However, the ratings that occur within a week before the default date were eliminated.

Rating changes observed after the date of default were eliminated unless the new rating reached the B3 level or higher and the new ratings were related to debt issued after the date of default. In theses cases we treated theses ratings as related to a new issuer. It is important to emphasize that the first rating date of the new issuer is the latest date between the date of the first issue after default and the first date we observe an issuer rating higher than or equal to B3.

The same treatment is applied for the case of two and three default dates. Finally, few issuers have a registered default date before the first rating observation in the Senior Unsecured Estimated Rating Table (Moody's, January, 09, 2002). In theses cases, we considered that there was no default. With this procedure we got 5821 issuers with 965 default dates.

We aggregated all rating notches and so we got the nine usual ratings Aaa, Aa, A, Baa, Ba, B, Caa-C, Default and NR (Not Rated) with 15,564 rating observations.

B Proofs

B.1 Proof of Theorem 1

In this section, we determine the value $\widetilde{P}(t,T)$ of a corporate zero-coupon bond in the discrete-time setting described in Section 3. The value $\widetilde{P}(t,T)$ of the corporate bond is expressed as the expectation, under the martingale measure Q, of its discounted cash flows :

$$\begin{aligned}
&\widetilde{P}(t,T) \\
&= \mathrm{E}_t^Q\left[\sum_{s=t+1}^{T-1}\beta(t,s)\rho_s\widetilde{P}(s,T)\mathbf{1}_{\tau=s}+\beta(t,T)\left(\mathbf{1}_{\tau>T}+\rho_T\mathbf{1}_{\tau=T}\right)\right] \\
&= \sum_{s=t+1}^{T}\rho_s\mathrm{E}_t^Q\left[\beta(t,s)\widetilde{P}(s,T)\mathbf{1}_{\tau=s}\right]+\underbrace{\mathrm{E}_t^Q\left[\beta(t,T)\right]}_{=P(t,T)}\underbrace{\mathrm{E}_t^Q\left[\mathbf{1}_{\tau>T}\right]}_{Q_t[\tau>T|\tau>t]} \\
&= \sum_{s=t+1}^{T}\rho_s\mathrm{E}_t^Q\left[\beta(t,s)\widetilde{P}(s,T)\mathbf{1}_{\tau=s}\right]+P(t,T)\left(1-\sum_{u=1}^{T-t}q_u\right).
\end{aligned}$$

where the second equality hinges on the independence between the default time τ and the risk-free forward rate f.

The remainder of the proof is based on the induction on the time to maturity. Indeed, if there is only one period before maturity, then the value of the corporate zero-coupon bond is

$$\begin{aligned}
\widetilde{P}(T-1,T) &= \rho_T\mathrm{E}_{T-1}^Q\left[\beta(T-1,T)\mathbf{1}_{\tau=T}\right]+P(T-1,T)\quad(1-q_1) \\
&= \rho_T P(T-1,T)\,q_1+P(T-1,T)\quad(1-q_1) \\
&= P(T-1,T)\underbrace{\left\{1-(1-\rho_T)\,q_1\right\}}_{=p_{T-1,T}},
\end{aligned}$$

and for $t<T-1$, the value of the corporate zero-coupon bond is

$$\begin{aligned}
\widetilde{P}(t,T) &= \sum_{s=t+1}^{T}\rho_s\mathrm{E}_t^Q\left[\beta(t,s)\widetilde{P}(s,T)\mathbf{1}_{\tau=s}\right]+P(t,T)\left(1-\sum_{u=1}^{T-t}q_u\right) \\
&= \sum_{s=t+1}^{T}\rho_s\mathrm{E}_t^Q\left[\beta(t,s)P(s,T)p_{s,T}\mathbf{1}_{\tau=s}\right]+P(t,T)\left(1-\sum_{u=1}^{T-t}q_u\right) \\
&= \sum_{s=t+1}^{T}\rho_s p_{s,T}\mathrm{E}_t^Q\left[\beta(t,s)P(s,T)\right]\mathrm{E}_t^Q\left[\mathbf{1}_{\tau=s}\right]+P(t,T)\left(1-\sum_{u=1}^{T-t}q_u\right) \\
&= \rho\sum_{s=t+1}^{T}\rho_s p_{s,T}P(t,T)\,q_{s-t}+P(t,T)\left(1-\sum_{u=1}^{T-t}q_u\right) \\
&= P(t,T)\underbrace{\left\{\sum_{u=1}^{T-t}\rho_{u+t}p_{t+u,T}q_u+\left(1-\sum_{u=1}^{T-t}q_u\right)\right\}}_{=p_{t,T}}
\end{aligned}$$

where the second equality comes from the use of the induction hypothesis and the third one is justified by the independence between the default time τ and the risk-free forward rate f. ¥

B.2 Proof of Theorem 2

In this section, the value of a corporate zero-coupon bond is determined in the continuous-time setting described in Section 4. Recall that, in case of default, the bondholder recovers, at time τ, a fraction of the market value of an equivalent bond. The value of the corporate zero-coupon bond is expressed as the expectation, under the martingale measure Q, of its discounted payoff :

$$\begin{aligned}\widetilde{P}(t,T) &= \mathrm{E}_t^Q\left[\beta(t,T)\,\mathbf{1}_{\tau>T} + \beta(t,\tau)\,\rho\widetilde{P}(\tau,T)\,\mathbf{1}_{\tau\leq T}\right] \\ &= \mathrm{E}_t^Q\left[\exp\left(-\int_t^T \left[f(s,s) + (1-\rho)\,\lambda_s\right] ds\right)\right] \text{(from Duffie and Singleton (1999))} \\ &= \underbrace{\mathrm{E}_t^Q\left[\exp\left(-\int_t^T f(s,s)\,ds\right)\right]}_{=P(t,T)} \exp\left(-(1-\rho)\int_t^T \lambda_s ds\right). \text{ ¥}\end{aligned}$$

B.3 Proof for the form of λ_t under assumption (iv**)

If the generator matrix $\mathbf{\Lambda}$ is diagonable, then one can write $\mathbf{\Lambda} = \mathbf{PDP}^{-1}$ where the columns of the matrix $\mathbf{P}$ contain the eigen vectors of $\mathbf{\Lambda}$ and $\mathbf{D} = (d_i)$ is a diagonal matrix filled with the eigen values of $\mathbf{\Lambda}$. Let $\mathbf{Q}_t = (Q[X_t = j\,|X_0 = i])_{i,j=1,\ldots,m}$ denotes the transition matrix of the Markov process X. Then

$$\begin{aligned}\mathbf{Q}_t = \exp(\mathbf{\Lambda}t) &= \sum_{k=1}^{\infty}\frac{(\mathbf{\Lambda}t)^k}{k!} = \sum_{k=1}^{\infty}\frac{\mathbf{PD}^k\mathbf{P}^{-1}t^k}{k!} = \mathbf{P}\exp(\mathbf{D}t)\,\mathbf{P}^{-1} \\ &= \left(\sum_{k=1}^{m} p_{ik}\exp(d_k t)\,p_{kj}^{-1}\right)_{i,j=1,\ldots,m}\end{aligned}$$

where p_{ij} are the components of $\mathbf{P}$, p_{ij}^{-1} are the components of $\mathbf{P}^{-1}$, and the first equality is justified by the definition of the generator of a time-homogenous Markov process. Let τ_i be the default time of a firm initially rated i and note that the default state corresponds to state m. The cumulative distribution of τ_i is $Q[\tau_i \leq t] = Q[X_t = m\,|X_0 = i]$. Therefore, the intensity associated with τ_i is

$$\lambda_{i,t} = \frac{\frac{\partial}{\partial t}Q[X_t = \text{ default}\,|X_0 = i]}{1 - Q[X_t = \text{default}\,|X_0 = i]} = \frac{\sum_{k=1}^m p_{ik}p_{km}^{-1}d_k\exp(d_k t)}{1 - \sum_{k=1}^m p_{ik}p_{km}^{-1}\exp(d_k t)}. \text{ ¥}$$

C Approximate Confidence Interval Computation

The first step of our simulation procedure computes a generator matrix for a given period using the maximum likelihood estimator given in Lando and Skodeberg (2002). Let the length of this period be T. The estimated generator is considered to be the true parameters governing the data generating process. This does not take into account the statistical uncertainty associated with this estimate. Hence, this procedure produces approximate confidence intervals that will underestimate genuine confidence intervals that would account for this variability.

The second step uses the estimated generator obtained in the first step and the sample of issuers at the beginning of the period to simulate one rating history for each issuer. For each issuer with initial rating i, we simulate the waiting time for leaving this state with an exponential distribution having a mean equal to $\frac{1}{|\lambda_{ii}|}$, where λ_{ii} are the elements of the generator matrix when $j = i$. If the waiting time is longer than period T, the issuer stays in its current rating for all the period. If the waiting time is shorter than T, we simulate a uniform distributed random variable between 0 and 1 to determine the issuer's next rating, using the migration intensities $\frac{\lambda_{ij}}{|\lambda_{ii}|}$ for all j different from i so that the migration intensity is different from zero. Then, we repeat the same task with the new rating until the cumulative of waiting times is greater than T or the issuer gets default as a new rating. This procedure is carried out for each issuer having a rating at the beginning of the period. Using these rating histories for all issuers, a generator is estimated to obtain a term structure of default probabilities and an estimate of the average default-risk proportion in yield spreads for each of the maturities.

The second step is repeated 10,000 times to get 10,000 estimates of average default risk proportion in yield spreads. We then compute different statistics (mean, median, percentiles 2.5 and 97.5 used as our approximate confidence intervals, and minimum and maximum) of average default proportion for each rating and maturity

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Table 1: Measured Corporate Bond Spreads

This table reports the average corporate bond spreads from government bonds for industrial Aa, A and Baa corporate bonds and for maturities from two to ten years. Spot rates were computed using the Nelson-and-Siegel model with the Levenberg-Marquardt algorithm. Treasuries spot rates are annualized. Corporate bond spreads are calculated as the difference between the corporate spot rates and treasury spot rates for a given maturity. Panel A contains the average treasury spot rates and corporate bond spreads over the entire 10-year period. Panel B contains the averages for the first five years and panel C contains the averages for the second five years. Our results are compared to those of Elton et al. (2001). It is easily seen that results from both studies are substantially similar.

	Elton & al.(2001)				Dionne & al.(2005)			
Maturity	Treasuries	Aa	A	Baa	Treasuries	Aa	A	Baa
Panel A: 1987-1996								
2.00	6.414	0.414	0.621	1.167	6.454	0.413	0.612	1.180
3.00	6.689	0.419	0.680	1.205	6.709	0.416	0.672	1.206
4.00	6.925	0.455	0.715	1.210	6.920	0.447	0.722	1.229
5.00	7.108	0.493	0.738	1.205	7.090	0.477	0.752	1.237
6.00	7.246	0.526	0.753	1.199	7.226	0.502	0.769	1.234
7.00	7.351	0.552	0.764	1.193	7.337	0.526	0.776	1.224
8.00	7.432	0.573	0.773	1.188	7.426	0.548	0.779	1.210
9.00	7.496	0.589	0.779	1.184	7.500	0.569	0.778	1.193
10.00	7.548	0.603	0.785	1.180	7.562	0.590	0.776	1.174
Panel B: 1987-1991								
2.00	7.562	0.436	0.707	1.312	7.601	0.512	0.737	1.421
3.00	7.763	0.441	0.780	1.339	7.775	0.495	0.802	1.400
4.00	7.934	0.504	0.824	1.347	7.928	0.515	0.845	1.402
5.00	8.066	0.572	0.853	1.349	8.054	0.545	0.869	1.400
6.00	8.165	0.629	0.872	1.348	8.157	0.579	0.882	1.391
7.00	8.241	0.675	0.886	1.347	8.241	0.617	0.889	1.379
8.00	8.299	0.711	0.897	1.346	8.309	0.656	0.893	1.363
9.00	8.345	0.740	0.905	1.345	8.364	0.697	0.895	1.346
10.00	8.382	0.764	0.912	1.344	8.410	0.738	0.896	1.328
Panel C: 1992-1996								
2.00	5.265	0.392	0.536	1.022	5.306	0.315	0.487	0.939
3.00	5.616	0.396	0.580	1.070	5.643	0.336	0.543	1.012
4.00	5.916	0.406	0.606	1.072	5.912	0.379	0.599	1.056
5.00	6.15	0.415	0.623	1.062	6.126	0.409	0.635	1.074
6.00	6.326	0.423	0.634	1.049	6.296	0.425	0.655	1.077
7.00	6.461	0.429	0.642	1.039	6.433	0.434	0.664	1.070
8.00	6.565	0.434	0.649	1.030	6.544	0.439	0.665	1.057
9.00	6.647	0.438	0.653	1.022	6.636	0.441	0.661	1.040
10.00	6.713	0.441	0.657	1.016	6.713	0.442	0.655	1.019

Table 2: Comparison of Average Root Mean Squared Errors

This table presents the average root mean squared error (ARMSE) of the difference between theoretical bond prices computed using the Nelson-and-Siegel model and the actual bond prices for treasuries and industrial Aa, A and Baa corporate bonds. The estimation procedure is described in Section 2. Root mean squared error is measured in cents per dollar. For a given class of bonds, the root mean squared error is calculated once per period (month). The number reported is the average of all root mean squared errors within a given class over the months of the corresponding period. Our results are compared to those of Elton et al. (2001). Again, the results are similar. The ARMSE formula is given below:

$$ARMSE = \frac{\sum_{j=1}^{m} \sqrt{\frac{\sum_{i=1}^{n_j} (\hat{P}_i - P_i)^2}{n_j}}}{m}$$

where:

m: Number of months in the period.

n_j: Number of bonds in month j.

$\hat{P}_i$: The theoretical price of bond i.

P_i: The actual price of bond i.

	Elton & al.(2001)				Dionne & al.(2005)			
Period	Treasuries	Aa	A	Baa	Treasuries	Aa	A	Baa
1987-1996	0.210	0.728	0.874	1.512	0.220	0.525	0.812	1.458
1987-1991	0.185	0.728	0.948	1.480	0.304	0.555	0.876	1.387
1992-1996	0.234	0.727	0.800	1.552	0.136	0.496	0.748	1.529

Table 3a: Transition Matrix Information

This table describes the matrix information used in different estimations. All cohort methods exclude withdrawn rating and entry firms information. The corresponding matrices were estimated by using annual cohorts. The MDC transition matrix includes entry firms and right censored information. With this estimation method, the transition matrices were continuously estimated over the corresponding five- or ten-year period. SPEG means Standard & Poor's and Elton-Gruber. MDEG means Moody's and Elton-Gruber. MDB1 is for data available on CD's from Moody's. T identifies the 1987-1996 period while 1 is for 1987-1991 and 2 for 1992-1996. MDB2 uses raw data from Moody's. MDC is for the continuous-time method estimation.

Matrix	Estimation Method	Period of estimation	Database	Data description
SPEG	Cohort	1981-1995	Standard & Poors 1996	No entry and no right censoring
MDEG	Cohort	1970-1993	Moody's 1993	No entry and no right censoring
MDB1_T	Cohort	1987-1996	CD Moody's 9-01-2003	No entry and no right censoring
MDB1_1	Cohort	1987-1991	CD Moody's 9-01-2003	No entry and no right censoring
MDB1_2	Cohort	1992-1996	CD Moody's 9-01-2003	No entry and no right censoring
MDB2_T	Cohort	1987-1996	Raw Moody's 9-01-2002	No entry and no right censoring
MDB2_1	Cohort	1987-1991	Raw Moody's 9-01-2002	No entry and no right censoring
MDB2_2	Cohort	1992-1996	Raw Moody's 9-01-2002	No entry and no right censoring
MDC_T	Continuous time	1987-1996	Raw Moody's 9-01-2002	Entry firms and right censoring
MDC_1	Continuous time	1987-1991	Raw Moody's 9-01-2002	Entry firms and right censoring
MDC_2	Continuous time	1992-1996	Raw Moody's 9-01-2002	Entry firms and right censoring

Table 3b: Sensitivity of Ten-Year Average Default-Spread Proportion with Respect to Transition Matrices, Discrete- vs Continuous-Time Transition Probabilities and Discrete Theoretical Models

This table reports the average of the ten-year default-spread proportion for different transition matrices and models. It also compares results between the discrete-time vs continuous-time analysis of rating transition probabilities. The two theoretical models are the Elton et al. (2001) and the model proposed in section 3.1. Both models are in discrete time. The first two transition matrices are Standard and Poor's (SPEG), and Moody's (MDEG), both as used in Elton et al. (2001) over the entire 10-year period. We also consider the Moody's matrices extracted (MDB1) and estimated (MDB2) for the entire 10-year period and the two 5-year periods from Moody's data base, and the continuous-time transition matrix estimated for the same periods (MDC). For the Elton et al. (2001) theoretical model, the spreads are estimated with bonds paying coupons and the same coupon rates as in their article. For our discrete model, the spreads were estimated with zero-coupon bonds. The recovery rates in both models are the same as in the Elton et al. (2001) paper: 59.59% for Aa, 60.63% for A and 49.42% for Baa. Panel A contains the average default-spread proportions for industrial Aa bonds. Panel B contains the averages for industrial A bonds and panel C contains the averages for industrial Baa bonds. For each class of bonds we report the results for the entire 10-year period and the sub-periods 1987-1991 and 1992-1996 in order to emphasize the default cycle effect on the average default-spread proportions.

	Elton & al.(2001)					Dionne & al.(2005)				
Period/Matrix	SPEG	MDEG	MDB1	MDB2	MDC	SPEG	MDEG	MDB1	MDB2	MDC
Panel A : Industrial Aa Bonds										
1987-1996	9.23	4.71	5.01	6.59	6.59	8.42	4.29	4.56	6.00	6.00
1987-1991	7.35	3.75	12.03	12.60	13.28	6.61	3.37	10.82	11.34	11.96
1992-1996	11.11	5.67	0.34	1.52	1.54	10.22	5.21	0.31	1.40	1.42
Panel B : Industrial A Bonds										
1987-1996	19.45	12.23	10.87	13.50	11.01	17.76	11.12	9.88	12.28	10.00
1987-1991	16.80	10.56	25.34	26.09	23.35	15.13	9.47	22.85	23.54	21.05
1992-1996	22.11	13.90	1.26	4.53	3.82	20.40	12.77	1.15	4.15	3.50
Panel C : Industrial Baa Bonds										
1987-1996	37.90	36.94	33.26	35.84	31.01	36.37	35.38	31.81	34.36	29.64
1987-1991	34.12	33.25	66.81	63.96	60.39	32.45	31.57	64.92	62.04	58.44
1992-1996	41.68	40.62	7.76	15.72	13.25	40.29	39.20	7.36	14.99	12.61

Table 3c: Sensitivity of Two-Year Average Default-Spread Proportion with Respect to Transition Matrices, Discrete- vs Continuous-Time Transition Probabilities and Discrete Theoretical Models

This table reports the average of two-year, default-spread proportion for different transition matrices and models. It also compares results between the discrete-time vs continuous-time analysis of rating transition probabilities. The two theoretical models are the Elton et al. (2001) and the model proposed in section 3.1. Both models are in discrete time. The first two transition matrices are Standard and Poor's (SPEG), and Moody's (MDEG), both as used in Elton et al. (2001) over the entire 10-year period. We also consider the Moody's matrices extracted (MDB1) and estimated (MDB2) for the entire 10-year period and the two 5-year periods from Moody's data base, and the continuous-time transition matrix estimated for the same periods (MDC). For the Elton et al. (2001) theoretical model, the spreads are estimated with bonds paying coupons and the same coupon rates as in their article. For our discrete model, the spreads were estimated with zero-coupon bonds. The recovery rates in both models are the same as in the Elton et al. (2001) paper: 59.59% for Aa, 60.63% for A, and 49.42% for Baa. Panel A contains the average default-spread proportions for industrial Aa bonds. Panel B contains the averages for industrial A bonds and panel C contains the averages for industrial Baa bonds. For each class of bonds we report the results for the entire 10-year period and the sub-periods 1987-1991 and 1992-1996 in order to emphasize the default-cycle effect on the average default-spread proportions.

	Elton & al.(2001)					Dionne & al.(2005)				
Period/Matrix	SPEG	MDEG	MDB1	MDB2	MDC	SPEG	MDEG	MDB1	MDB2	MDC
Panel A : Industrial Aa Bonds										
1987-1996	1.03	0.22	0.34	0.45	1.17	0.94	0.20	0.31	0.40	1.06
1987-1991	0.83	0.18	0.83	0.89	2.50	0.74	0.16	0.74	0.79	2.22
1992-1996	1.24	0.27	0.00	0.01	0.10	1.13	0.24	0.00	0.01	0.09
Panel B : Industrial A Bonds										
1987-1996	10.53	1.36	1.37	1.77	2.45	9.42	1.23	1.24	1.60	2.01
1987-1991	8.54	1.11	3.09	3.23	4.04	7.54	0.98	2.74	2.86	3.57
1992-1996	12.52	1.62	0.04	0.65	1.02	11.30	1.47	0.04	0.59	0.93
Panel C : Industrial Baa Bonds										
1987-1996	15.49	10.37	10.14	14.31	13.59	14.39	9.64	9.43	13.29	12.63
1987-1991	11.90	7.97	16.55	16.98	17.60	10.95	7.34	15.18	15.58	16.15
1992-1996	19.08	12.78	1.72	11.17	10.84	17.82	11.94	1.62	10.46	10.15

Table 4a: Implied Default Probability for Industrial Aa Bonds

This table reports the implied default probabilities in percentages for industrial Aa bonds from different transition matrices. The implied default probability is the probability of default in year *n* conditional on a particular initial rating. The first two transition matrices are Standard and Poor's (SPEG), and Moody's (MDEG), both as used in Elton et al. (2001) over the entire 10-year period (1987-1996). We also consider the Moody's matrices extracted (MDB1) and estimated (MDB2) from Moody's data base (MDB1_T and MDB2_T for the entire 10-year period, MDB1_1 and MDB2_1 for the first 5-year period and MDB1_2 and MDB2_2 for the second 5-year period) and the continuous time transition matrix estimated for the same periods (MDC_T, MDC_1 and MDC_2). The implied default probability is given by the following formula: $q[\tau = n] = q[\tau \le n] - q[\tau \le n-1]$, where τ is the time of default.

Year / Matrix	SPEG	MDEG	MDB1_T	MDB1_1	MDB1_2	MDB2_T	MDB2_1	MDB2_2	MDC_T	MDC_1	MDC_2
1.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00434	0.01157	0.00017
2.00	0.01673	0.00360	0.00558	0.01685	0.00001	0.00723	0.01800	0.00017	0.01467	0.03872	0.00120
3.00	0.03739	0.01085	0.01404	0.04309	0.00015	0.01875	0.04668	0.00163	0.02736	0.07120	0.00321
4.00	0.06105	0.02159	0.02544	0.07837	0.00053	0.03432	0.08483	0.00433	0.04235	0.10889	0.00614
5.00	0.08712	0.03562	0.03977	0.12227	0.00123	0.05365	0.13146	0.00821	0.05961	0.15192	0.00991
6.00	0.11521	0.05268	0.05695	0.17414	0.00231	0.07640	0.18554	0.01321	0.07907	0.20022	0.01447
7.00	0.14499	0.07251	0.07682	0.23308	0.00383	0.10220	0.24596	0.01923	0.10063	0.25351	0.01975
8.00	0.17615	0.09480	0.09918	0.29798	0.00579	0.13065	0.31155	0.02621	0.12415	0.31123	0.02568
9.00	0.20839	0.11924	0.12374	0.36764	0.00822	0.16133	0.38109	0.03403	0.14946	0.37268	0.03220
10.00	0.24143	0.14550	0.15022	0.44078	0.01110	0.19382	0.45336	0.04262	0.17634	0.43702	0.03925

Table 4b: Implied Default Probability for Industrial A Bonds

This table reports the implied default probabilities in percentages for industrial A bonds from different transition matrices. The implied default probability is the probability of default in year *n* conditional on a particular initial rating. The first two transition matrices are Standard and Poor's (SPEG), and Moody's (MDEG), both as used in Elton et al. (2001) over the entire 10-year period (1987-1996). We also consider the Moody's matrices extracted (MDB1) and estimated (MDB2) from Moody's data base (MDB1_T and MDB2_T for the entire 10-year period, MDB1_1 and MDB2_1 for the first 5-year period and MDB1_2 and MDB2_2 for the second 5-year period) and the continuous-time transition matrix estimated for the same periods (MDC_T, MDC_1 and MDC_2). The implied default probability is given by the following formula: $q[\tau = n] = q[\tau \leq n] - q[\tau \leq n-1]$, where τ is the time of default.

Year / Matrix	SPEG	MDEG	MDB1_T	MDB1_1	MDB1_2	MDB2_T	MDB2_1	MDB2_2	MDC_T	MDC_1	MDC_2
1.00	0.10300	0.00000	0.00163	0.00327	0.00000	0.00000	0.00000	0.00000	0.01280	0.02691	0.00521
2.00	0.15435	0.03354	0.03215	0.09025	0.00086	0.04365	0.09765	0.01353	0.04228	0.09498	0.01601
3.00	0.20388	0.07442	0.06830	0.19102	0.00332	0.09184	0.20538	0.02797	0.07670	0.17993	0.02724
4.00	0.25321	0.12086	0.10895	0.30226	0.00717	0.14331	0.32068	0.04308	0.11506	0.27688	0.03877
5.00	0.30273	0.17126	0.15296	0.42013	0.01219	0.19688	0.44035	0.05867	0.15643	0.38174	0.05053
6.00	0.35223	0.22417	0.19920	0.54090	0.01819	0.25151	0.56124	0.07457	0.19994	0.49100	0.06244
7.00	0.40119	0.27833	0.24662	0.66130	0.02500	0.30625	0.68055	0.09062	0.24483	0.60162	0.07447
8.00	0.44905	0.33264	0.29432	0.77858	0.03243	0.36033	0.79596	0.10668	0.29039	0.71108	0.08656
9.00	0.49526	0.38622	0.34154	0.89059	0.04034	0.41308	0.90567	0.12263	0.33597	0.81729	0.09866
10.00	0.53935	0.43834	0.38763	0.99573	0.04859	0.46399	1.00830	0.13837	0.38104	0.91864	0.11073

Table 4c: Implied Default Probability for Industrial Baa Bonds

This table reports the implied default probabilities in percentages for industrial Baa bonds from different transition matrices. The implied default probability is the probability of default in year *n* conditional on a particular initial rating. The first two transition matrices are Standard and Poor's (SPEG), and Moody's (MDEG), both as used in Elton et al. (2001) over the entire 10-year period (1987-1996). We also consider the Moody's matrices extracted (MDB1) and estimated (MDB2) from Moody's data base (MDB1_T and MDB2_T for the entire 10-year period, MDB1_1 and MDB2_1 for the first 5-year period and MDB1_2 and MDB2_2 for the second 5-year period) and the continuous-time transition matrix estimated for the same periods (MDC_T, MDC_1 and MDC_2). The implied default probability is given by the following formula: $q[\tau = n] = q[\tau \le n] - q[\tau \le n-1]$, where τ is the time of default.

Year / Matrix	SPEG	MDEG	MDB1_T	MDB1_1	MDB1_2	MDB2_T	MDB2_1	MDB2_2	MDC_T	MDC_1	MDC_2
1.00	0.21200	0.10300	0.10401	0.19970	0.00831	0.18697	0.23364	0.14764	0.19801	0.28598	0.15003
2.00	0.34972	0.27384	0.26452	0.57829	0.04286	0.33216	0.56439	0.18253	0.29526	0.54118	0.17036
3.00	0.49065	0.43892	0.41309	0.92090	0.07638	0.46989	0.87617	0.21729	0.39681	0.80378	0.19142
4.00	0.62546	0.59276	0.54798	1.22160	0.10849	0.59698	1.15690	0.25110	0.49844	1.05700	0.21313
5.00	0.74871	0.73207	0.66802	1.47770	0.13882	0.71155	1.40080	0.28330	0.59664	1.28940	0.23514
6.00	0.85758	0.85517	0.77275	1.68930	0.16699	0.81270	1.60620	0.31335	0.68877	1.49460	0.25699
7.00	0.95105	0.96158	0.86232	1.85850	0.19274	0.90029	1.77400	0.34090	0.77304	1.66940	0.27824
8.00	1.02920	1.05160	0.93737	1.98880	0.21592	0.97471	1.90660	0.36574	0.84841	1.81350	0.29851
9.00	1.09290	1.12610	0.99885	2.08430	0.23647	1.03670	2.00730	0.38777	0.91441	1.92810	0.31751
10.00	1.14330	1.18620	1.04790	2.14950	0.25441	1.08720	2.07970	0.40702	0.97105	2.01530	0.33501

Table 5: Sensitivity of Average Default-Spread Proportion in Our Discrete Theoretical Model With Respect to Recovery Rates

This table reports the average of ten-year, default-spread proportions obtained by our discrete theoretical model for different transition matrices and recovery rates. The default spreads are estimated with zero-coupon bonds. The first two transition matrices are Standard and Poor's (SPEG) and Moody's (MDEG) both as used in Elton et al. (2001) over the entire 10-year period. We also consider the Moody's matrices estimated for the entire 10-year period and the two 5-year periods from Moody's data base (MDB2) and the continuous-time transition estimated for the same periods (MDC). We present three recovery rates: 40%, 50%, and 60%. Panel A contains the average of default-spread proportions for industrial Aa bonds. Panel B contains the averages for industrial A bonds and panel C contains the averages for industrial Baa bonds. For each class of bonds we report the results for the entire 10-year period and the sub-periods 1987-1991 and 1992-1996 in order to emphasize the default-cycle effect on the average of default-spread proportions.

	Recovery Rate of 40%				Recovery Rate of 50%				Recovery Rate of 60%			
Period / Matrix	SPEG	MDEG	MDB2	MDC	SPEG	MDEG	MDB2	MDC	SPEG	MDEG	MDB2	MDC
Panel A : Industrial Aa Bonds												
1987-1996	12.51	6.37	8.92	8.92	10.42	5.30	7.43	7.43	8.33	4.24	5.94	5.94
1987-1991	9.82	5.00	16.87	17.78	8.18	4.17	14.04	14.81	6.54	3.33	11.23	11.83
1992-1996	15.19	7.73	2.07	2.11	12.66	6.44	1.73	1.76	10.12	5.15	1.38	1.40
Panel B : Industrial A Bonds												
1987-1996	27.12	16.97	18.74	15.27	22.58	14.13	15.61	12.71	18.05	11.30	12.47	10.16
1987-1991	23.10	14.45	36.04	32.20	19.23	12.03	29.97	26.79	15.37	9.62	23.92	21.39
1992-1996	31.15	19.49	6.33	5.34	25.93	16.23	5.27	4.45	20.73	12.97	4.22	3.56
Panel C : Industrial Baa Bonds												
1987-1996	43.24	42.08	40.84	35.22	35.95	34.97	33.96	29.29	28.69	27.91	27.11	23.39
1987-1991	38.58	37.54	73.94	69.64	32.08	31.20	61.31	57.76	25.60	24.90	48.80	45.99
1992-1996	47.90	46.61	17.79	14.97	39.83	38.74	14.81	12.47	31.79	30.91	11.84	9.97

Table 6: Comparison of Ten-Year Average Default Spread Proportions between Discrete and Continuous-Time Theoretical Models

This table reports the average of ten-year default spread proportions for different transition matrices and models. It compares results between our discrete-time and continuous-time theoretical models. Recovery rates used are the same as in Elton et al. (2001): 59.59% for Aa. 60.63% for A, and 49.42% for Baa. The default spreads are estimated with zero-coupon bonds. The first transition matrix is Moody's (MDEG) used in Elton et al. (2001) over the entire 10-year period. We also consider the Moody's matrices extracted (MDB1) and estimated (MDB2) for the entire 10-year period and the two 5-year periods from Moody's data base and the continuous-time transition matrix estimated for the same periods (MDC). Panel A contains the average default spread proportions for industrial Aa bonds. Panel B contains the averages for industrial A bonds and panel C contains the averages for industrial Baa bonds. We observe that the continuous time model gives similar proportions of default risk in corporate bond spreads as the discrete-time model does for continuous time transition matrix.

	Discrete Time Model				Continuous Time Model			
Period / Matrix	MDEG	MDB1	MDB2	MDC	MDEG	MDB1	MDB2	MDC
	Panel A : Industrial Aa bonds							
1987-1996	4.29	4.56	6.00	6.00	4.97	5.05	6.79	6.00
1987-1991	3.37	10.82	11.34	11.96	3.90	12.13	12.78	11.92
1992-1996	5.21	0.31	1.40	1.42	6.03	0.41	1.83	1.42
	Panel B : Industrial A bonds							
1987-1996	11.12	9.88	12.28	10.00	11.79	10.32	13.03	9.97
1987-1991	9.47	22.85	23.54	21.05	10.04	23.80	24.78	20.87
1992-1996	12.77	1.15	4.15	3.50	13.54	1.34	4.58	4.00
	Panel C : Industrial Baa bonds							
1987-1996	35.38	31.81	34.36	29.64	34.78	31.64	33.78	29.07
1987-1991	31.57	64.92	62.04	58.44	31.03	62.33	59.71	56.14
1992-1996	39.20	7.36	14.99	12.61	38.53	7.65	14.95	12.50

Table 7: Rating Distributions Used to Compute Descriptive Statistics

This table reports the distribution of issuers, by rating, at the starting date of the estimation period used to construct the confidence sets of average default-spread proportions. There are two periods of simulations: 1987-1991 and 1992-1996.

Period/Rating	Aaa	Aa	A	Baa	Ba	B	CC-C
1987-1991	58	210	456	287	343	179	6
1992-1996	51	160	416	321	275	197	12

Table 8a: Descriptive Statistics of Average Default-Spread Proportions in the Continuous-Time Model Using Withdrawn-Rating and Entry Firm Observations

This table reports the mean, the standard error, the median, the percentiles 2.5 and 97.5, and the minimum and the maximum of average default-spread proportions for different maturities (2, 5, 8 and 10 years) obtained from Monte Carlo simulations using the continuous-time model and continuous-time transition matrix estimated for the two 5-year periods from Moody's data base (MDC). The transition matrix is estimated using withdrawn rating and entry firm observations. The recovery rates are the same as in the Elton et al. (2001) paper: 59.59% for Aa, 60.63% for A, and 49.42% for Baa. Panel A contains the results for industrial Aa bonds. Panel B contains the results for industrial A bonds and panel C contains the results for industrial Baa bonds.

	Period 1987-1991				Period 1992-1996			
Maturity	2	5	8	10	2	5	8	10
Panel A: Industrial Aa Bonds								
Mean	2.24	6.19	9.39	11.95	0.09	0.42	0.93	1.41
Standard Error	1.28	2.43	2.64	2.79	0.04	0.17	0.34	0.49
Percentile 2.5	0.42	2.44	5.06	7.25	0.02	0.14	0.35	0.58
Median	2.02	5.88	9.11	11.68	0.09	0.40	0.90	1.36
Percentile 97.5	5.14	11.66	15.22	18.02	0.19	0.82	1.72	2.54
Minimum	0.17	1.18	2.71	4.09	0.00	0.03	0.10	0.19
Maximum	9.46	20.09	24.16	27.07	0.32	1.36	2.84	4.15
Panel B: Industrial A Bonds								
Mean	3.58	8.96	15.81	20.91	0.93	1.76	2.72	3.48
Standard Error	0.79	1.49	2.22	2.71	0.40	0.66	0.91	1.09
Percentile 2.5	2.15	6.26	11.70	15.88	0.27	0.64	1.17	1.61
Median	3.54	8.90	15.73	20.81	0.89	1.70	2.65	3.40
Percentile 97.5	5.27	12.10	20.41	26.53	1.82	3.25	4.76	5.92
Minimum	1.15	4.43	8.99	12.12	0.04	0.18	0.45	0.70
Maximum	6.68	15.08	24.69	31.46	3.02	5.14	7.43	9.16
Panel C: Industrial Baa Bonds								
Mean	16.18	30.89	46.11	56.21	10.12	9.75	11.10	12.44
Standard Error	4.08	4.60	5.58	6.30	5.46	3.99	3.65	3.64
Percentile 2.5	9.27	22.54	35.82	44.50	1.45	3.15	4.96	6.27
Median	15.84	30.63	45.91	55.97	9.39	9.36	10.81	12.15
Percentile 97.5	25.19	40.60	57.35	68.81	22.92	18.78	19.27	20.50
Minimum	5.57	15.47	25.69	32.53	0.40	1.27	2.29	3.09
Maximum	34.79	50.90	69.16	81.20	38.20	29.26	28.20	29.08

Table 8b: Descriptive Statistics of Average Default-Spread Proportions in the Continuous-Time Model Excluding Withdrawn-Rating and Entry Firm Observations

This table reports the mean, the standard error, the median, the percentiles 2.5 and 97.5, and the minimum and the maximum of average default-spread proportions, for different maturities (2, 5, 8 and 10 years) obtained from Monte Carlo simulations using the continuous-time model and continuous-time transition matrix estimated for the two 5-year periods from Moody's data base (MDC). The transition matrix is estimated by excluding withdrawn-rating and entered firm observations. The recovery rates are the same as in the Elton et al. (2001) paper: 59.59% for Aa, 60.63% for A, and 49.42% for Baa. Panel A contains the results for industrial Aa bonds. Panel B contains the results for industrial A bonds and panel C contains the results for industrial Baa bonds.

	Period 1987-1991				Period 1992-1996			
Maturity	2	5	8	10	2	5	8	10
Panel A: Industrial Aa Bonds								
Mean	3.18	7.89	11.30	14.08	0.16	0.73	1.60	2.41
Standard Error	1.89	3.34	3.38	3.43	0.06	0.25	0.50	0.71
Percentile 2.5	0.43	2.71	5.80	8.36	0.06	0.31	0.76	1.21
Median	3.08	7.58	10.96	13.78	0.15	0.71	1.56	2.36
Percentile 97.5	7.49	15.40	18.87	21.63	0.30	1.28	2.70	3.96
Minimum	0.23	1.81	4.13	6.16	0.01	0.09	0.28	0.48
Maximum	12.90	24.64	27.65	30.13	0.48	2.02	4.12	5.92
Panel B: Industrial A Bonds								
Mean	4.66	11.43	19.72	25.70	1.72	3.26	4.93	6.24
Standard Error	0.99	1.82	2.63	3.17	0.62	1.02	1.37	1.62
Percentile 2.5	2.88	8.06	14.81	19.77	0.66	1.50	2.54	3.39
Median	4.59	11.34	19.63	25.62	1.67	3.18	4.84	6.14
Percentile 97.5	6.77	15.20	25.14	32.15	3.08	5.46	7.89	9.73
Minimum	1.70	4.91	9.41	12.89	0.14	0.47	0.97	1.42
Maximum	9.62	19.15	30.86	39.08	4.87	8.35	11.71	14.19
Panel C: Industrial Baa Bonds								
Mean	21.47	40.50	58.90	70.56	15.39	15.94	18.57	20.85
Standard Error	4.72	5.48	6.64	7.44	6.32	4.80	4.50	4.56
Percentile 2.5	13.24	30.36	46.37	56.45	4.76	7.76	10.74	12.82
Median	21.16	40.33	58.78	70.42	14.84	15.60	18.32	20.62
Percentile 97.5	31.56	51.66	72.41	85.63	29.36	26.45	28.21	30.53
Minimum	8.48	22.87	35.74	44.02	1.57	4.00	6.73	8.38
Maximum	44.94	62.55	82.73	97.71	47.66	40.29	41.05	43.26

Table 8c: Descriptive Statistics of Average Default-Spread Proportions in the Continuous-Time Model Excluding Only Entry Firm Observations

This table reports the mean, the standard error, the median, the percentiles 2.5 and 97.5, and the minimum and the maximum of average default-spread proportions for different maturities (2, 5, 8 and 10 years) obtained from Monte Carlo simulations using the continuous-time model and continuous-time transition matrix estimated for the two 5-year periods from Moody's data base (MDC). The transition matrix is estimated by excluding only entry firm observations. The recovery rates are the same as in the Elton et al. (2001) paper: 59.59% for Aa, 60.63% for A, and 49.42% for Baa. Panel A contains the results for industrial Aa bonds. Panel B contains the results for industrial A bonds and panel C contains the results for industrial Baa bonds.

	Period 1987-1991				Period 1992-1996			
Maturity	2	5	8	10	2	5	8	10
Panel A: Industrial Aa Bonds								
Mean	2.42	6.58	9.89	12.53	0.04	0.14	0.33	0.45
Standard Error	1.37	2.57	2.76	2.89	0.02	0.06	0.13	0.17
Percentile 2.5	0.43	2.60	5.42	7.74	0.01	0.05	0.12	0.18
Median	2.29	6.27	9.57	12.24	0.04	0.14	0.32	0.44
Percentile 97.5	5.59	12.43	15.99	18.83	0.08	0.27	0.62	0.83
Minimum	0.14	1.06	2.55	3.93	0.00	0.01	0.02	0.04
Maximum	10.45	20.74	23.96	26.66	0.15	0.48	1.10	1.47
Panel B: Industrial A Bonds								
Mean	4.18	10.24	17.72	23.15	1.39	2.63	3.99	5.07
Standard Error	0.86	1.61	2.37	2.88	0.52	0.85	1.16	1.37
Percentile 2.5	2.63	7.35	13.44	17.91	0.52	1.18	2.00	2.68
Median	4.12	10.16	17.61	23.02	1.35	2.55	3.90	4.95
Percentile 97.5	6.00	13.60	22.67	29.14	2.52	4.45	6.49	8.02
Minimum	1.62	4.87	9.48	13.08	0.08	0.35	0.79	1.21
Maximum	8.24	17.51	28.68	36.47	4.07	7.22	10.22	12.39
Panel C: Industrial Baa Bonds								
Mean	18.24	34.01	49.69	59.85	13.20	13.24	15.22	17.03
Standard Error	4.26	4.81	5.80	6.50	5.92	4.40	4.07	4.09
Percentile 2.5	10.75	25.15	38.76	47.61	3.04	5.66	8.19	9.92
Median	17.96	33.86	49.56	59.68	12.80	12.92	14.92	16.74
Percentile 97.5	27.35	44.07	61.48	73.05	26.13	22.78	23.88	25.70
Minimum	6.74	19.57	30.40	37.58	0.95	2.62	4.48	5.77
Maximum	38.05	54.83	75.52	88.53	52.00	40.90	39.66	40.72

Table 9a: Descriptive Statistics of Implied Default Probability Using Withdrawn-Rating and Entry Firm Observations

This table reports the mean, the standard error, the median, the percentiles 2.5 and 97.5, and the minimum and the maximum of implied default probability q_s for different maturities (s = 2, 5, 8 and 10 years) obtained from Monte Carlo simulations using the continuous-time model and continuous-time transition matrix estimated for the two 5-year periods from Moody's data base (MDC). The transition matrix is estimated using withdrawn-rating and entry firm data. Panel A contains the results for industrial Aa bonds. Panel B contains the results for industrial A bonds and panel C contains the results for industrial Baa bonds.

	Period 1987-1991				Period 1992-1996			
Maturity	2	5	8	10	2	5	8	10
Panel A: Industrial Aa Bonds								
Mean	0.039	0.153	0.312	0.437	0.001	0.010	0.026	0.039
Standard Error	0.021	0.048	0.063	0.071	0.001	0.004	0.009	0.013
Percentile 2.5	0.008	0.074	0.202	0.310	0.000	0.003	0.011	0.018
Median	0.036	0.147	0.307	0.433	0.001	0.009	0.025	0.038
Percentile 97.5	0.087	0.260	0.446	0.586	0.002	0.019	0.046	0.067
Minimum	0.003	0.038	0.116	0.190	0.000	0.001	0.003	0.007
Maximum	0.160	0.424	0.632	0.764	0.004	0.031	0.075	0.108
Panel B: Industrial A Bonds								
Mean	0.095	0.383	0.712	0.919	0.016	0.050	0.086	0.110
Standard Error	0.020	0.056	0.085	0.100	0.007	0.017	0.025	0.030
Percentile 2.5	0.059	0.279	0.554	0.733	0.005	0.021	0.042	0.058
Median	0.094	0.380	0.709	0.916	0.015	0.049	0.084	0.108
Percentile 97.5	0.138	0.499	0.886	1.121	0.031	0.090	0.143	0.175
Minimum	0.033	0.214	0.421	0.561	0.001	0.007	0.020	0.031
Maximum	0.176	0.610	1.036	1.287	0.051	0.141	0.219	0.264
Panel C: Industrial Baa Bonds								
Mean	0.542	1.291	1.813	2.014	0.170	0.234	0.297	0.333
Standard Error	0.109	0.148	0.169	0.169	0.083	0.071	0.067	0.067
Percentile 2.5	0.351	1.016	1.491	1.689	0.035	0.112	0.179	0.213
Median	0.535	1.287	1.810	2.012	0.160	0.229	0.293	0.329
Percentile 97.5	0.778	1.587	2.147	2.347	0.359	0.390	0.440	0.472
Minimum	0.221	0.745	1.152	1.335	0.010	0.051	0.098	0.127
Maximum	1.008	1.862	2.467	2.682	0.582	0.566	0.575	0.603

Table 9b: Descriptive Statistics of Implied Default Probability
Excluding Withdrawn-Rating and Entry Firm Observations

This table reports the mean, the standard error, the median, the percentiles 2.5 and 97.5, and the minimum and the maximum of implied default probability q_s for different maturities (s = 2, 5, 8 and 10 years) obtained from Monte Carlo simulations using the continuous-time model and continuous-time transition matrix estimated for the two 5-year periods from Moody's data base (MDC). The transition matrix is estimated by excluding withdrawn-rating and entry firm observations. Panel A contains the results for industrial Aa bonds. Panel B contains the results for industrial A bonds and panel C contains the results for industrial Baa bonds.

	Period 1987-1991				Period 1992-1996			
Maturity	2	5	8	10	2	5	8	10
Panel A: Industrial Aa Bonds								
Mean	0.055	0.185	0.360	0.497	0.002	0.017	0.044	0.066
Standard Error	0.031	0.062	0.073	0.078	0.001	0.006	0.013	0.018
Percentile 2.5	0.009	0.085	0.233	0.355	0.001	0.008	0.022	0.035
Median	0.053	0.179	0.355	0.493	0.002	0.017	0.043	0.065
Percentile 97.5	0.126	0.325	0.517	0.661	0.004	0.030	0.071	0.105
Minimum	0.005	0.058	0.174	0.270	0.000	0.002	0.009	0.016
Maximum	0.215	0.487	0.670	0.821	0.006	0.046	0.106	0.151
Panel B: Industrial A Bonds								
Mean	0.124	0.481	0.863	1.089	0.030	0.092	0.153	0.191
Standard Error	0.025	0.067	0.097	0.111	0.011	0.027	0.037	0.042
Percentile 2.5	0.078	0.357	0.680	0.878	0.012	0.046	0.087	0.116
Median	0.122	0.479	0.861	1.087	0.029	0.090	0.151	0.189
Percentile 97.5	0.177	0.619	1.058	1.312	0.053	0.150	0.233	0.282
Minimum	0.046	0.223	0.455	0.611	0.003	0.017	0.039	0.056
Maximum	0.248	0.763	1.261	1.531	0.083	0.223	0.332	0.391
Panel C: Industrial Baa Bonds								
Mean	0.722	1.650	2.186	2.337	0.270	0.399	0.501	0.549
Standard Error	0.129	0.174	0.187	0.179	0.098	0.089	0.086	0.086
Percentile 2.5	0.491	1.321	1.823	1.985	0.103	0.240	0.342	0.390
Median	0.716	1.648	2.184	2.335	0.262	0.394	0.497	0.545
Percentile 97.5	0.994	1.997	2.558	2.689	0.487	0.588	0.683	0.727
Minimum	0.336	1.020	1.476	1.647	0.039	0.155	0.242	0.282
Maximum	1.288	2.275	2.952	3.054	0.764	0.818	0.866	0.901

Table 9c: Descriptive Statistics of Implied Default Probability Excluding Entry Firm Data

This table reports the mean, the standard error, the median, the percentiles 2.5 and 97.5, the minimum and the maximum of implied default probability q_s for different maturities (s = 2, 5, 8 and 10 years) obtained from Monte Carlo simulations using the continuous-time model and continuous-time transition matrix estimated for the two 5-year periods from Moody's data base (MDC). The transition matrix is estimated by excluding only entry firm data. Panel A contains the results for industrial Aa bonds. Panel B contains the results for industrial A bonds and panel C contains the results for industrial Baa bonds.

	Period 1987-1991				Period 1992-1996			
Maturity	2	5	8	10	2	5	8	10
Panel A: Industrial Aa Bonds								
Mean	0.042	0.161	0.326	0.455	0.002	0.015	0.038	0.057
Standard Error	0.023	0.051	0.064	0.072	0.001	0.005	0.011	0.016
Percentile 2.5	0.009	0.080	0.214	0.324	0.001	0.006	0.019	0.030
Median	0.040	0.155	0.321	0.451	0.002	0.014	0.037	0.056
Percentile 97.5	0.095	0.274	0.463	0.606	0.003	0.026	0.063	0.092
Minimum	0.003	0.035	0.112	0.188	0.000	0.002	0.007	0.013
Maximum	0.175	0.421	0.622	0.773	0.006	0.046	0.106	0.151
Panel B: Industrial A Bonds								
Mean	0.111	0.432	0.781	0.990	0.024	0.075	0.124	0.156
Standard Error	0.022	0.060	0.089	0.103	0.009	0.022	0.032	0.036
Percentile 2.5	0.071	0.324	0.616	0.797	0.009	0.036	0.069	0.092
Median	0.109	0.429	0.778	0.987	0.023	0.073	0.122	0.154
Percentile 97.5	0.158	0.558	0.966	1.202	0.043	0.123	0.192	0.234
Minimum	0.044	0.223	0.464	0.628	0.002	0.013	0.034	0.051
Maximum	0.214	0.709	1.186	1.443	0.070	0.196	0.291	0.342
Panel C: Industrial Baa Bonds								
Mean	0.609	1.393	1.892	2.062	0.227	0.325	0.406	0.447
Standard Error	0.114	0.153	0.170	0.167	0.091	0.080	0.076	0.076
Percentile 2.5	0.404	1.105	1.567	1.739	0.071	0.185	0.269	0.308
Median	0.604	1.391	1.891	2.061	0.221	0.320	0.403	0.443
Percentile 97.5	0.853	1.702	2.230	2.397	0.425	0.494	0.563	0.603
Minimum	0.272	0.872	1.277	1.449	0.024	0.103	0.177	0.216
Maximum	1.106	2.079	2.578	2.747	0.802	0.766	0.760	0.799

Table 10: Data Composition of Tables 8 and 9

This table reports the number of firms, transitions, and defaults used to estimate the continuous-time transition matrices. There are three cases. First, we excluded entry firm and right censored data (panel b). Second, we excluded only entry firm data (panel c). Finally, we included entry firm and withdrawn-rating data (panel a). The analysis was done for the 1987-1996 period and for both the 1987-1991 and 1992-1996 sub-periods. We observe that the proportion of default issuers (Defaults/Issuers) decreases when we include censored and entry firms data and so do the average default-spread proportions (Tables 8a, 8b, and 8c) and the corresponding implied default probabilities (Tables 9a, 9b, and 9c).

	Including Withdrawals and Entry Firm Data (a)			Standard (b)			Including Withdrawals (c)		
	1987-1996	1987-1991	1992-1996	1987-1996	1987-1991	1992-1996	1987-1996	1987-1991	1992-1996
Issuers	3,879	2,656	3,090	1,239	1,539	1,432	1,977	1,977	1,867
Rating observations	7,652	4,690	4,829	2,731	2,672	2,236	4,590	3,667	3,213
Defaults	399	267	132	250	196	92	250	196	92
Defaults/ Issuers	10.29%	10.05%	4.27%	20.18%	12.74%	6.42%	12.65%	9.91%	4.93%

Table 11: Average Default-Spread Proportions for the 1992-1996 Period with Three Different Benchmarks

Panel A reports the default-spread proportion using the Hull et al. (2005) risk-free rate benchmark. The latter is equal to the Treasury rate plus 83% of the difference between the interest-rate swap and the Treasury rate, as suggested by the authors. The interest-rate-swap term structure is constructed using interest-rate swap rates of 2, 4, 5, 7, and 10 years from Datastream database and Libor rates for 3 and 6 months from the Federal Reserve historical data. We applied our discrete theoretical model with the continuous-time transition matrix estimated for the 5-year period 1992-1996 and the recovery rates from the Elton et al. (2001) paper: 59.59% for Aa, 60.63% for A, and 49.42% for Baa. Panel B reports the average default-spread proportions using the industrial Aaa benchmark. This method was suggested in the literature to isolate the credit risk from the tax and liquidity factors in observed spreads. The results in the two panels are compared with those in Table 3a with the MDC transition matrix for the 1992-1996 period (Panel C). Both new benchmarks generate higher average default-spread proportions for that period but still lower than those for the 1987-1991 period.

	A: Hull *et al*. (2005) Benchmark				B: Aaa Benchmark				C: Treasury Rate Benchmark
	T=7	T=8	T=9	T=10	T=7	T=8	T=9	T=10	T=10
Industrial Aa Bonds	2.08	2.68	3.50	4.71	3.42	4.40	4.75	7.28	1.42
Industrial A Bonds	4.12	4.78	5.52	6.37	4.81	5.49	6.36	7.57	3.50
Industrial Baa Bonds	14.56	15.43	16.44	17.62	15.73	16.48	17.47	18.76	12.61

Table A1: Mean, Minimum, and Maximum Default Spreads and Default Spread Proportions

This table reports the average, minimum and maximum of default spreads (panels A, B and C) and the proportion of average default spreads (in percentage, panel D) obtained using the Elton et al. (2001) model with the same assumptions for recovery rates, coupons rates, and Standard and Poor's transition matrix, as described in the text. The recovery rates are 59.59% for Aa, 60.63% for A, and 49.42% for Baa. Our results are compared to those of Elton et al. (2001). We observe that they are equivalent. The proportion of average default spread (DSP) is obtained from the average of default spreads (DS) and the average of corporate bond spreads (CBS) by the following formula: $DSP = 100 \frac{DS}{CBS}$.

	Elton & al.(2001)			Dionne & al.(2005)		
Years	Aa	A	Baa	Aa	A	Baa
	Panel A: Mean Default Spreads					
1.00	0.000	0.043	0.110	0.000	0.046	0.117
2.00	0.004	0.053	0.145	0.004	0.057	0.154
3.00	0.008	0.063	0.181	0.008	0.068	0.192
4.00	0.012	0.074	0.217	0.013	0.078	0.229
5.00	0.017	0.084	0.252	0.018	0.089	0.265
6.00	0.023	0.095	0.286	0.023	0.100	0.300
7.00	0.028	0.106	0.319	0.029	0.111	0.333
8.00	0.034	0.117	0.351	0.036	0.122	0.364
9.00	0.041	0.128	0.380	0.042	0.133	0.394
10.00	0.048	0.140	0.409	0.049	0.144	0.421
	Panel B: Minimum Default Spreads					
1.00	0.000	0.038	0.101	0.000	0.045	0.115
2.00	0.003	0.046	0.132	0.004	0.055	0.151
3.00	0.007	0.055	0.164	0.008	0.066	0.188
4.00	0.011	0.063	0.197	0.012	0.076	0.225
5.00	0.015	0.073	0.229	0.017	0.086	0.260
6.00	0.020	0.083	0.262	0.023	0.097	0.294
7.00	0.025	0.093	0.294	0.028	0.107	0.326
8.00	0.031	0.104	0.326	0.034	0.118	0.356
9.00	0.038	0.116	0.356	0.041	0.129	0.385
10.00	0.044	0.128	0.385	0.047	0.140	0.412
	Panel C: Maximum Default Spreads					
1.00	0.000	0.047	0.118	0.000	0.047	0.119
2.00	0.004	0.059	0.156	0.004	0.058	0.157
3.00	0.009	0.071	0.196	0.008	0.069	0.195
4.00	0.014	0.083	0.235	0.013	0.080	0.233
5.00	0.019	0.094	0.273	0.019	0.092	0.270
6.00	0.025	0.106	0.309	0.024	0.103	0.306
7.00	0.031	0.117	0.342	0.030	0.114	0.340
8.00	0.038	0.129	0.374	0.037	0.126	0.372
9.00	0.044	0.140	0.403	0.043	0.137	0.401
10.00	0.051	0.151	0.431	0.050	0.148	0.429
	Panel D: Default-Spread Proportions					
2.00	0.966	8.535	12.425	0.969	9.314	13.051
3.00	1.909	9.265	15.021	1.923	10.119	15.920
4.00	2.637	10.350	17.934	2.908	10.803	18.633
5.00	3.448	11.382	20.913	3.774	11.835	21.423
6.00	4.373	12.616	23.853	4.582	13.004	24.311
7.00	5.072	13.874	26.739	5.513	14.304	27.206
8.00	5.934	15.136	29.545	6.569	15.661	30.083
9.00	6.961	16.431	32.095	7.381	17.095	33.026
10.00	7.960	17.834	34.661	8.305	18.557	35.860

Table A2: Estimated Generators

Table A3a: Estimated Generator for the 1987-1991 Period

	Aaa	Aa	A	Baa	Ba	B	CCC-C	D
Aaa	-0.0608	0.0582	0.0000	0.0000	0.0026	0.0000	0.0000	0.0000
Aa	0.0134	-0.1355	0.1168	0.0018	0.0018	0.0018	0.0000	0.0000
A	0.0008	0.0138	-0.0946	0.0678	0.0100	0.0023	0.0000	0.0000
Baa	0.0006	0.0044	0.0404	-0.1356	0.0758	0.0122	0.0006	0.0017
Ba	0.0004	0.0008	0.0041	0.0382	-0.1729	0.1174	0.0016	0.0103
B	0.0000	0.0012	0.0012	0.0047	0.0507	-0.2081	0.0461	0.1043
CCC-C	0.0000	0.0000	0.0139	0.0418	0.0139	0.0279	-0.9341	0.8365
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table A3b: Estimated Generator for the 1992-1996 Period

	Aaa	Aa	A	Baa	Ba	B	CCC-C	D
Aaa	-0.0752	0.0716	0.0036	0.0000	0.0000	0.0000	0.0000	0.0000
Aa	0.0035	-0.1071	0.1036	0.0000	0.0000	0.0000	0.0000	0.0000
A	0.0000	0.0113	-0.0552	0.0418	0.0011	0.0011	0.0000	0.0000
Baa	0.0005	0.0014	0.0737	-0.1120	0.0327	0.0014	0.0009	0.0014
Ba	0.0000	0.0000	0.0068	0.0610	-0.1395	0.0683	0.0000	0.0034
B	0.0005	0.0005	0.0036	0.0047	0.0738	-0.1616	0.0410	0.0374
CCC-C	0.0000	0.0000	0.0000	0.0000	0.0124	0.1244	-0.3443	0.2074
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table A3c: Estimated Generator for the 1987-1996 Period

	Aaa	Aa	A	Baa	Ba	B	CCC-C	D
Aaa	-0.0669	0.0639	0.0015	0.0000	0.0015	0.0000	0.0000	0.0000
Aa	0.0091	-0.1232	0.1111	0.0010	0.0010	0.0010	0.0000	0.0000
A	0.0004	0.0125	-0.0741	0.0543	0.0053	0.0017	0.0000	0.0000
Baa	0.0005	0.0028	0.0585	-0.1228	0.0524	0.0063	0.0008	0.0015
Ba	0.0002	0.0004	0.0053	0.0487	-0.1576	0.0949	0.0009	0.0071
B	0.0003	0.0008	0.0025	0.0047	0.0629	-0.1835	0.0434	0.0690
CCC-C	0.0000	0.0000	0.0032	0.0096	0.0128	0.1023	-0.4794	0.3516
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table A3
Comparison with Christensen et al (2004) database

Christensen et al. (2004) used issuers having senior unsecured bonds rated by Moody's only, while we used issuers having an "Estimated senior unsecured rating" by Moody's, even if they have not issued senior unsecured bonds. In both studies, the data source is Moody's Corporate Bond Default Database (January 9, 2002).

	Christensen et al. (2004)	Dionne et al. (2005)
Initial number of issuers	3,405	5,719
Initial ratings (when irrelevant withdrawals were eliminated)	13,390	22,938
Final number of issuers	3,446	5,821
Defaults	305	943
Refreshed firms	41	102
Final ratings	9,991	15,564

Figure 1: Empirical spreads on industrial bonds of six years maturity

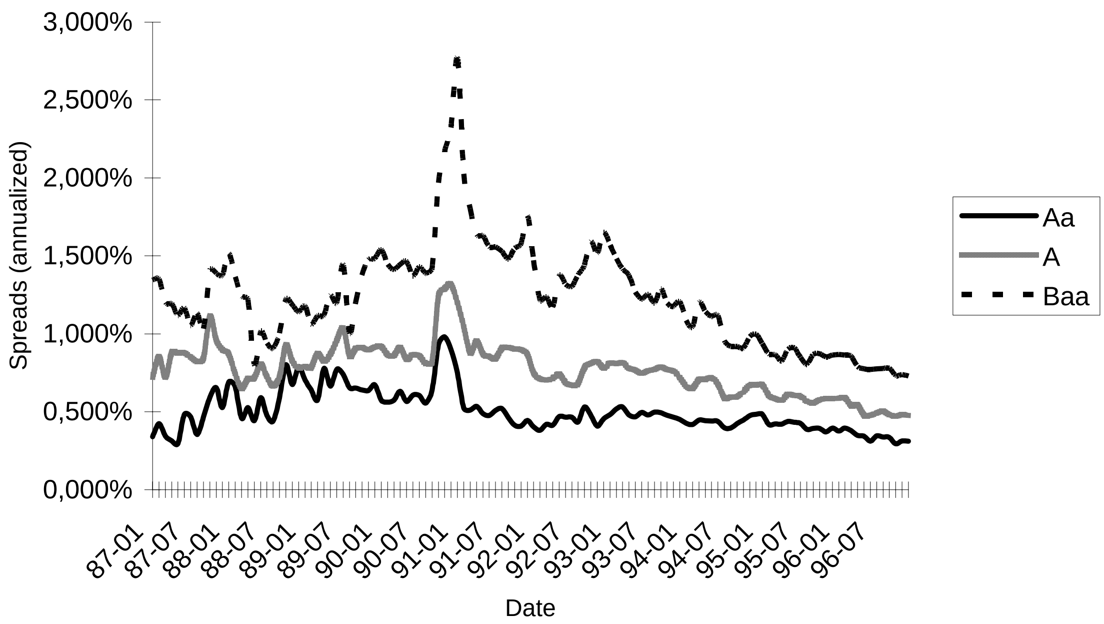
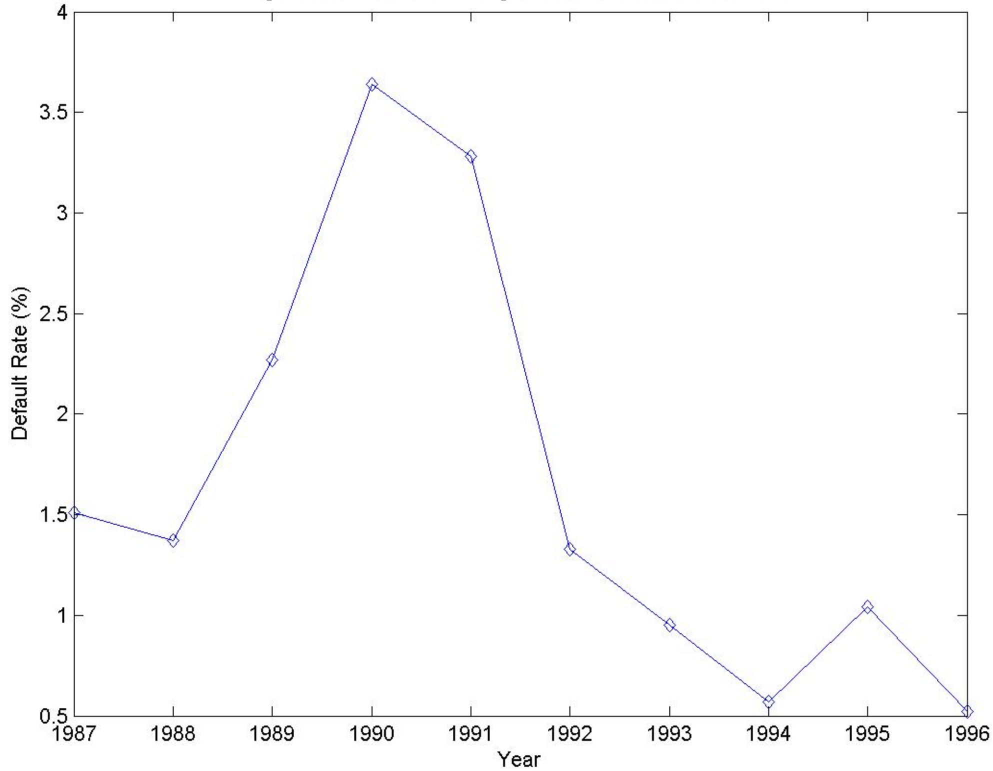


Figure 2: Annual Issuer Weighted Default Rates, 1987-1996



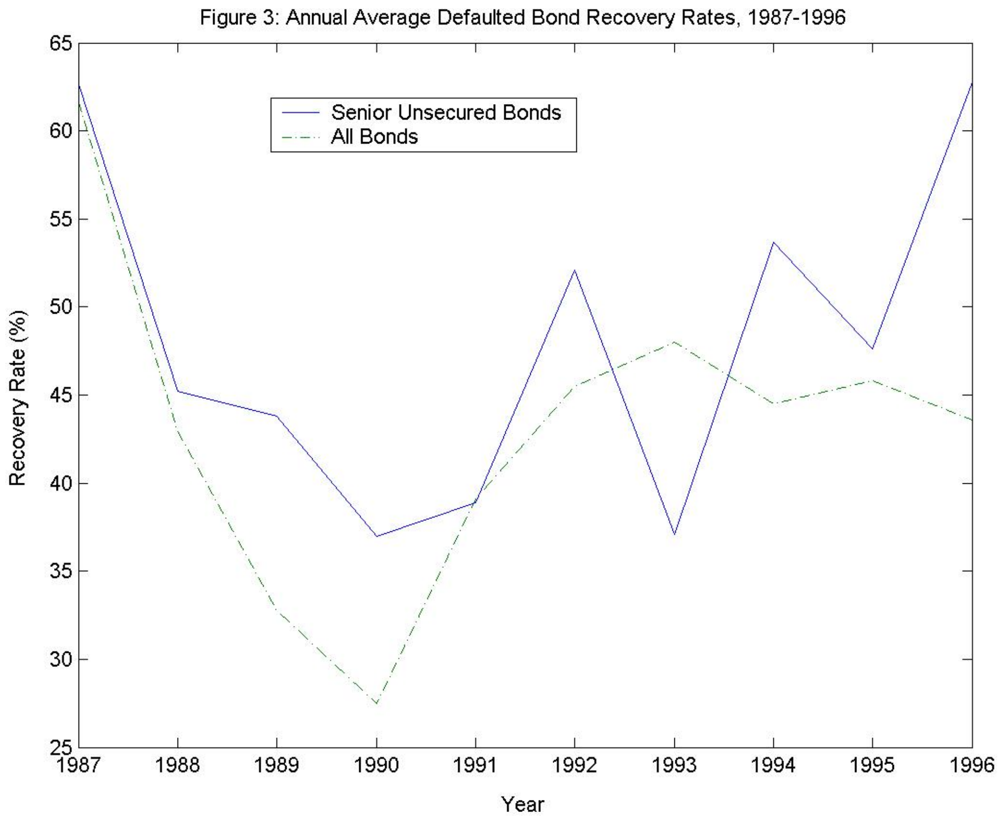


Figure 3: Annual Average Defaulted Bond Recovery Rates, 1987-1996
Senior Unsecured Bonds
All Bonds
Recovery Rate (%)
Year
65
60
55
50
45
40
35
30
25
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996