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Equilibrium Excess Demand in the Rental Housing Market

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Abstract:
We develop a model of a competitive rental housing market with endogenous default due to income uncertainty. There is a large number of identical, potential suppliers who each face a fixed cost of entering the rental housing market. Those suppliers who choose to enter decide how many rental units to supply and the rental price to charge. Potential tenants who differ in their income and face an uninsurable income shock choose whether to engage in a costly search for rental housing. If they find a rental unit, then they must commit to a rental agreement before the income uncertainty is resolved. Consequently, some tenants may default on their rental payments. We show that tenancy default can explain persistent excess demand in the rental housing market without any government price regulations. With excess demand in equilibrium, some individuals are simply unable to find rental housing. We study both government regulations affecting the cost of default to the housing suppliers and the quality of rental units, and the imposition of rent control. We show that rent control can have non-standard effects on the access to rental housing and on welfare.

Keywords: Tenancy Default, Excess Demand, Rental Housing Policies

JEL Classification: R21, R31, R38, D41

1. Introduction

Government regulations are often blamed for the persistent shortage of rental housing.[1] In this paper, we show that if some tenants default on their rent, an equilibrium with excess demand (i.e. with shortage) can exist in the rental housing market in the absence of any government regulations. For households at the bottom of the income distribution, the lack of access to financial means can severely restrict their ability to self-insure against adverse shocks.[2] Such households may therefore face a very real risk of defaulting on their rent and being evicted from their homes in the event of an adverse income shock. Our contribution is to explain how the possibility of involuntary rental default can translate into a shortage of rental housing in a competitive housing market.

How important an issue is default in the rental housing market? Unfortunately, there is very little direct evidence on actual defaults of rental payment. Most of the evidence comes indirectly through observations on rental housing evictions. However, because there is no central registry for maintaining information about actual evictions,[3] the evidence is generally from records of forceful evictions where a court-order was applied for and/or issued.[4] From these records, we know that the majority of (forceful) evictions result from the non-payment of rent.[5] For example, in Montreal close to 85% of all complaints filed by landlords cite non-payment of rent as the reason for the complaint (UN-Habitat Urban Indicators, 2005). As to how many tenants are

[1] Price regulations and other forms of rent control that restrict the rental price could generate excess demand in the rental housing market. Land-use and zoning regulations could also inhibit the development of new construction and thereby cause an undersupply of rental housing (Arnott, 1995). Other government interventions could also create an undersupply of rental housing. For example, Quigley and Raphael (2004) argue that government building standards reduced the stock of affordable rental housing in the US during the 1980s and 1990s.

[2] As noted in the Canadian Housing Observer (2006): "Due to their limited incomes, low- and moderate-income households face greater challenges in addressing their housing needs and in balancing housing costs against other household expenses."

[3] We are unaware of any such registry in North America.

[4] Of course, some tenants might choose to voluntarily leave their homes before being forcefully removed and thus the evidence on evictions might understate the true number of evictions (Hartmann and Robinson, 2003).

[5] See, for example, CMHC (2005) for Canadian evidence and Hartmann and Robinson (2003) for US evidence. Tenants may also be legally evicted if they are engaging in some form of anti-social behaviour or if the rental unit is to be renovated or converted to an alternative use.

being evicted each year, this study indicates that an eviction ratio of about 3% of the rental stock is standard for most Canadian cities.[6] The available evidence suggests that tenants do default on their rent and are subsequently evicted as a result of rent arrears. It is also worth noting that the available survey data suggests that the main reason for non-payment of rent is some unanticipated change in the financial circumstances of the household. For example, over 80% of tenants facing eviction in Toronto in 2002 claimed to be in rent arrears due to loss of employment income, medical problems, or family breakdowns (Lapointe, 2004). Not surprisingly, these tenants facing evictions had incomes 30% lower than the average income of rental households in the city (Lapointe, 2004). Tenants who are not paying their rent as a result of some unanticipated adverse financial shock and who are evicted are lower income households. Finally, we note that the available evidence suggests that evictions are costly for both the tenant and the landlord. For example, the average cost to a landlord of evicting a tenant was over $3000 in Canada (CMHC, 2005). Evictions may also have longer term social costs.

In this paper, we incorporate the very real possibility that low-income rental households who receive an adverse income shock may default on their rent. To do this, we build a model of the rental housing market in which individuals face some income uncertainty and must commit to a housing decision before this uncertainty is resolved. Therefore, once their final income is known, some individuals with a rental unit will be unable to pay their rent and will be evicted. Further, we assume that income is private information. We show that with the possibility of tenancy default, excess demand can be an equilibrium phenomenon. Increasing supply to eliminate the excess demand (or, equivalently increasing the rental price) increases the average probability of default in the rental housing market and may reduce the supplier's expected profit. Therefore, it may be in the interest of each profit-maximizing supplier not to increase supply. As a result, some individuals who choose to look for a rental unit will be unable to find one. Given tenancy default can generate an equilibrium with excess demand, we then examine the impact of government policies on the housing market. We show that changes in government policies (e.g. regulation on the quality of housing or regulation impacting the cost of default for the housing suppliers) may well exacerbate the problem of the lack of affordable rental housing. We also explain how rent control can have a significantly different impact when the economy rests in an equilibrium with excess demand due to tenancy default as opposed to in a standard

[6] Similar numbers have been determined for large American cities, see Hartmann and Robinson (2003). In terms of absolute numbers, for example, 12,300 eviction orders were issued in Toronto in 2002 (Lapointe, 2004).

market clearing equilibrium without tenancy default.

That competitive suppliers or demanders may find it in their interest to charge an *efficiency price* different from the market-clearing price was established, among others, by Weiss (1980), Stiglitz and Weiss (1981), and Shapiro and Stiglitz (1984). The paper which is the closest to ours is Stiglitz and Weiss (1981) in which it is shown that banks may prefer to keep interest rates on loans at a low level to avoid attracting only high-risk borrowers. In our paper, a similar "sorting" effect is also present, but it works in the opposite direction for suppliers (landlords): by charging a low rent, suppliers reduce the probability of default for a given set of tenants, but they also worsen the pool of tenants as lower rents attract relatively poorer tenants. Our analysis also differs from Stiglitz and Weiss (1981) by focusing on the rental housing market and examining the impact of housing market regulations and policies.[7] Finally, as will be shown, in our analysis the level of excess demand affects demand and supply, a phenomenon which is absent in Stiglitz and Weiss (1981).[8] As we show, the responses of demand and supply to excess demand is important in establishing the impact of policies like rent control on the key variables of interest. Interestingly, Becker (1991) has also noted, but in the rather different context of restaurant pricing, that aggregate demand could affect individual decisions and subsequently yield excess demand in equilibrium. In his model, individuals are simply assumed to care about the aggregate demand for a particular good. We instead provide a rationale for why individuals care about aggregate demand and supply and subsequently why suppliers care about excess demand as well.

The remainder of the paper is as follows: The next section outlines our model of the rental housing market. We characterize the equilibrium with excess demand arising in this market in Section 3. We then examine the impact of government policies affecting the cost of default to suppliers and the quality of rental housing as well as rent control in Section 4. In Section 5, we present a numerical example illustrating that an equilibrium with excess demand can exist as well as some comparative static results. Section 6 briefly concludes.

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[7] In a pair of papers examining tenancy rent control, Basu and Emerson (2000, 2003) demonstrate that a similar 'efficiency rent' can exist when there is private information about how long a tenant will stay in the rental unit and there is a monopolistic landlord. In their model, individuals are not allowed to be evicted and there is no tenancy default.

[8] Note that in Shapiro and Stiglitz (1984), labour supply (effort) depends on excess supply (unemployment). However, Shapiro and Stiglitz (1984) is a moral hazard model while our model and that of Stiglitz and Weiss (1981) are adverse selection models.

2. The Model

The model we construct should be envisioned as reflecting the rental housing market for its lowest quality segment and for individuals at the bottom of the income distribution, for example those belonging to the lowest decile of the income distribution. We assume that there are N individuals who differ in terms of *ex ante* income y distributed on $[\underline{y}, \overline{y}]$ with $\underline{y} > 0$ according to the cumulative distribution $G(y)$ with density $G'(y) = g(y) > 0$ for all y, and with $G(\overline{y}) = 1$ and $G(\underline{y}) = 0$.[9] Individuals extract utility v from the consumption of housing H and of some composite good c where

$$v(H, c) = u(H) + g(c)$$

with $u' > 0 \geq u''$, $g(c) = c$ for $c \geq c_o$, and $g(c) \to -\infty$ for $c < c_o$ where $c_o \geq 0$ is some minimal consumption level of the composite good needed for survival.[10] Utility v is assumed to be bounded below for non-negative levels of housing, that is, $v(H, c \geq c_o) \geq \underline{v} > -\infty$, $\forall\ H \geq 0$. It follows that in choosing their consumption bundle all individuals ensure that $c \geq c_o$.

Final or *ex post* income is uncertain. *Ex ante* income y is affected by an *i.i.d.* shock s drawn from the cumulative distribution $F(s)$ with density $F'(s) = f(s) > 0$ for all s, unit mean $E(s) = 1$, and support $[\underline{s}, \overline{s}]$ with $\underline{s} > 0$. Thus, *post*-shock final income of an individual with *ex ante* income y who has drawn shock s is simply sy, so expected income is $E(sy) = y$. We assume that this income shock is uninsurable.[11] We also assume that both *ex ante* and *ex post* income are private information to the individual.

Individuals must decide whether to look for rental housing. Rental housing can only be consumed in a single discrete amount h. Individuals who do not have access to a rental unit

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[9] Total population can therefore be written as $N \int_{\underline{y}}^{\overline{y}} dG$.

[10] Assuming separability simplifies the analysis but the same qualitative results should be obtained without separability provided $v_{cH} > 0$.

[11] As we are considering individuals at the bottom of the income distribution, it is reasonable to assume these individuals have limited ability to self-insure against adverse income shocks, that is, they have no savings and lack the means to borrow money. Adverse income shocks could result from loss of employment income due to injury or illness, from any increases in the price of necessities, or from the loss of income from a supporting person or government program. Some of these adverse shocks may be publicly insured. But even so, there are often waiting times before public benefits are received and it is during this period with no benefits that default could occur. These issues are discussed further in Boadway *et al.* (forthcoming).

either because they did not look for housing, couldn't not find any, or defaulted on their rent receive a level of housing $h_o \geq 0$ which is strictly less than h and is costless.[12] Therefore, $H \in \{h_o, h\}$ with $h > h_o \geq 0$. The actual value of h represents both quality features, such as location and state of disrepair, and quantity features, such as square footage, of the rental unit. In what follows, we will assume that the government, through regulation, can affect the value of h.[13] Indeed, as was often done in the past in Canada or in the U.S., a government can increase the value of h by introducing modifications to the construction or building code.

The price of rental housing or rent is denoted by r and the price of the composite good is unity (that is, c is the numeraire). There is a fixed utility cost of looking for housing denoted by $k > 0$ which could be interpreted as the time it takes to look for housing.[14] If individuals search for a rental unit and are successful in finding one, then they must sign a lease before shock s is realized (i.e. before income uncertainty is resolved). We assume that there is no voluntary default, that is, we assume that the difference between $u(h)$ and $u(h_o)$ is large enough so that $u(h) - k + sy - r > u(h_o) - k + sy$ or $u(h) - r > u(h_o)$.[15] Individuals only default *ex post* because they are forced to. For individuals who find rental housing and are lucky enough to draw a good shock (i.e. those for which $sy \geq r + c_o$), there is no need to adjust H and the final consumption of c is *ex post* income minus spending on housing: $c = sy - r$, so final utility is $v = u(h) - k + sy - r$. However, some individuals draw a bad shock and have to default *ex post* on their rent to ensure that they satisfy $c \geq c_o$. This happens if $sy < r + c_o$. In this case, H is

[12] This alternate housing could be living with family/friends, in a shelter or on the street. The utility derived from such housing would obviously depend on the specific alternative.

[13] Having the parameter h determined by some form of regulation is not crucial for obtaining an equilibrium with excess demand. We are, however, interested in determining how such a policy would affect the rental housing market when there is equilibrium excess demand in the market. We discuss this issue in more detail later in the paper.

[14] Incorporating a positive search cost generates an elastic demand for housing. Alternatively, a secondary rental housing market with rental units of higher quality/quantity could be assumed. Having rental units of differing quality, however, would greatly complicate the analysis without changing the possibility of a market equilibrium with excess demand. Another way to generate elastic demand would be to assume that there is some utility cost from defaulting on one's rent. In this case, both demand and supply would be independent of excess demand in the market but an equilibrium with excess demand could still be obtained.

[15] Also note that there is no moral hazard: individuals cannot affect the value of shock s that they draw.

revised to h_o,[16] final consumption is $c = sy$ and final utility is $v = u(h_o) - k + sy$. The worst shock $\underline{s}$ is assumed to be large enough so that c_o can always be purchased yet not so large as to preclude the possibility that *any* individual may default on their rent.[17]

It is possible that there will be excess demand in the rental market. If there is excess demand, then we assume that rental units are allocated randomly according to a pure Bernouilly mechanism. Let μ denote the probability that an individual will not be allocated a rental unit given that he has opted to look for housing. This probability will be equal to the proportion of total demand for rental housing that is not met in equilibrium, i.e. the ratio of excess demand for rental housing to the total demand for rental housing or the *rate of excess demand*. The rate of excess demand, μ, is determined endogenously in equilibrium. Those that look for housing or demand rental housing, but are unsuccessful at obtaining a rental unit, simply end up consuming h_o and receive final utility $u(h_o) - k + sy$.

On the other side of the market, there is a large number of potential risk-neutral suppliers. All of these potential suppliers are identical, and face a fixed positive cost ϕ of entering the housing market.[18] A supplier will enter the housing market if their maximized expected profits are greater than the fixed cost of entry. Once a supplier enters the rental housing market, the supplier chooses how many rental units to supply and the rental price to charge that maximizes expected profits. The total cost of producing n housing units of quality/quantity h for any supplier is $hC(n)$, where $C' > 0$, $C'' \geq 0$.

Since both *ex ante* income y and *final* income sy are private information, suppliers can only charge a single rental price to all of their tenants.[19] Further, since all suppliers are identical there will only be one price in the rental housing market, denoted by r. Suppliers know the distributions of both income and income shocks. Therefore, suppliers know the average probability

[16] Defaulting automatically leads to eviction. Provided defaulting leads to eviction with some probability between zero and one, our results would continue to hold.

[17] The worst shock $\underline{s}$ is such that the poorest *ex post* individual will always be able to afford c_o: $\underline{sy} \geq c_o$ and the individual will the highest *ex ante* income who receives this shock will default on their rent: $\underline{s}\overline{y} < c_o + r$.

[18] We could allow for heterogeneous entry costs without changing our qualitative results.

[19] Re-negotiation of the rent *ex post* is not possible even if that would be beneficial for the owner and the tenant given neither income nor the income shock is verifiable. Such information is likely to be difficult and costly to observe.

of default in the rental housing market. Each supplier recognizes that the probability that their tenants will default on their rent will depend on the rental price they charge, and take this into account when they choose their rental price. Suppliers, however, view themselves as having a negligible effect on total market supply and therefore on excess demand in the market. In other words, firms take μ or the rate of excess demand in the market as given.[20]

The timing of events is as follows:

1. Individuals choose whether to look for housing taking the rental price and the probability of finding a rental unit as given. Housing suppliers decide whether to enter the housing market and upon entry decide on both the number of rental units to supply and the rental price to charge taking as given the rate of excess demand.

2. Equilibrium in the housing market is obtained which determines the number of suppliers operating in the housing market, the rental price, the quantity of rental housing supplied and consumed, and the rate of excess demand in the market.

3. Income shock is realized and individuals who succeeded in finding a rental unit will default if $sy < c_o + r$.

2.1 Demand Side of the Rental Housing Market

An individual with *ex ante* income y must decide whether to look for a rental unit. If he chooses to look for a rental unit and finds one, then he will default on his rent if $sy < c_o + r$, or if

$$s < \frac{c_o + r}{y} \equiv \hat{s}(r, y, c_o) \tag{1}$$

and he will not default if $s \geq \hat{s}(r, y, c_o)$. The cut-off income shock $\hat{s}$ is increasing in both r and c_o, and decreasing in y. For a given income level, individuals are more likely to default the higher the rental price or the higher the minimum consumption level. For a given rental rate, higher income individuals are less likely to default.

[20] Effectively suppliers act as if they can rent as many units as they want at their chosen rental price.

An individual's expected utility if he looks for rental housing is given by:

$$\begin{aligned} v_h =&(1-\mu)\left\{\int_{\underline{s}}^{\hat{s}(r,y,c_o)}[u(h_o)-k+sy]dF+\int_{\hat{s}(r,y,c_o)}^{\overline{s}}[u(h)-k+sy-r]dF\right\} \\ &+\mu\left\{\int_{\underline{s}}^{\overline{s}}[u(h_o)-k+sy]dF\right\}. \end{aligned} \tag{2}$$

The first term on the right-hand side of the above expression is the expected utility of an individual when he finds a housing unit, an event which occurs with probability $(1-\mu)$. In this case, the individual defaults on his rent for realizations of the shock below $\hat{s}$ and does not consume any rental housing. For realizations of the shock larger than $\hat{s}$, the individual consumes the rental unit. The second term on the right-hand side of (2) represents the expected utility of the individual when he cannot find a unit, an event which occurs with probability μ.

If the individual chooses not to look for rental housing, then his expected utility is given by:

$$v_o=\int_{\underline{s}}^{\overline{s}}[u(h_o)+sy]dF=u(h_o)+y. \tag{3}$$

Therefore, an individual with a given expected income y will look for rental housing if the expected utility differential between looking and not looking for rental housing is non-negative, $v_h-v_o\geq 0$, or if

$$(1-\mu)[1-F(\hat{s}(r,y,c_o))][u(h)-r-u(h_o)]-k\geq 0. \tag{4}$$

We assume there exists some income level in $[\underline{y},\overline{y})$ such that left-hand side of (4) is equal to zero. Denote this level of income by $\hat{y}(r,\mu;h)$.[21] Differentiating the expected utility differential given by (4) with respect to y yields:

$$\frac{\partial(v_h-v_o)}{\partial y}=(1-\mu)[u(h)-r-u(h_o)]f(\hat{s})\frac{\hat{s}}{y}>0. \tag{5}$$

It follows from (5) that all individuals with $y\geq\hat{y}(r,\mu;h)$ will look for rental housing and all individuals with $y<\hat{y}(r,\mu;h)$ will not search for rental housing. Total demand for rental housing will be given by $D(r,\mu;h)=[1-G(\hat{y}(r,\mu;h))]N$ and will depend on the endogenous variables

[21] Our focus is on government policies affecting the quality of rental housing h and the cost of default to landlords z. Therefore, we suppress the parameters representing the search cost k and the minimum consumption level c_o. A full analysis with these parameters is given in the Appendix.

$\{r, \mu\}$ and the exogenous parameter h. We can now obtain the following partial derivatives (see Appendix for details):

$$\hat{y}_r > 0, \quad \hat{y}_\mu > 0, \quad \hat{y}_h < 0 \tag{6}$$

$$D_r < 0, \quad D_\mu < 0, \quad D_h > 0 \tag{7}$$

Market demand is decreasing in the rental price. For a given rental rate, individuals are more likely to look for housing the higher the quality/quantity of the rental housing units, h, and the lower the probability that an individual does not find rental housing given they searched for rental housing, μ.

2.2 Supply Side of the Rental Housing Market

To solve the problem of the representative housing supplier, we first need to establish the default rate in the rental market. To do this, we simply have to work out the average probability of default of those individuals who demand rental housing. This average probability of default, which we denote by π, can be determined by summing up the probability of default for all individuals who demand rental housing and dividing by the total demand for rental housing.[22] Doing this, we obtain:

$$\begin{aligned} \pi(r, \mu; h) &= \frac{1}{1 - G(\hat{y}(r, \mu; h))} \int_{\hat{y}(r,\mu;h)}^{\overline{y}} \int_{\underline{s}}^{\hat{s}(r, c_o, y)} dF dG \\ &= \frac{1}{1 - G(\hat{y}(r, \mu; h))} \int_{\hat{y}(r,\mu;h)}^{\overline{y}} F\left(\frac{c_o + r}{y}\right) dG \end{aligned} \tag{8}$$

Differentiating (8) yields

$$\pi_r = B_1 + B_2 \hat{y}_r, \quad \pi_\mu = B_2 \hat{y}_\mu < 0, \quad \pi_h = B_2 \hat{y}_h > 0 \tag{9}$$

where

$$B_1 = \frac{1}{1 - G(\hat{y})} \int_{\hat{y}}^{\overline{y}} f\left(\frac{c_o + r}{y}\right) \frac{1}{y} dG > 0,$$

$$B_2 = \frac{g(\hat{y})}{[1 - G(\hat{y})]^2} \left[\int_{\hat{y}}^{\overline{y}} F\left(\frac{c_o + r}{y}\right) dG - F\left(\frac{c_o + r}{\hat{y}}\right) (1 - G(\hat{y})) \right] < 0.$$

[22] The actual size of the population N does not affect the average probability of default as N appears in both the numerator and denominator of expression on the right-hand side of (8).

A change in the rental price r has both a direct and an indirect or selection effect on the average probability of default. These two effects work in opposite directions. The direct effect, denoted by B_1, is positive. An increase in r increases the probability of default for a given set of tenants. Only r has a direct effect. Any change in the probability of not finding rental housing, μ, or the quality of rental housing, h, only has an indirect effect on the average probability of default. The indirect or selection effect works through changes in the demand for rental housing or in the set of individuals looking for rental housing, $\hat{y}$. The higher is $\hat{y}$, the higher is the average income of those looking for rental housing and the lower is the average probability of default as given by the negative term B_2. An increase in μ reduces the demand for housing whereas an increase in h increases the demand for housing. Therefore, the average probability of default is decreasing in μ and increasing in h. An increase in r reduces the demand for rental housing or increases $\hat{y}$ which reduces the average probability of default. Therefore, the net effect of a change in r of the average probability of default is ambiguous.

The interesting case we consider in this paper is the one in which the direct effect on the average probability of default of an increase in the rental price dominates the selection effect.[23] In this case, the average probability of default will be increasing in the rental price. We assume this to be the case for the remainder of the paper:

Assumption 1: $\pi_r > 0$

The cost to the supplier of having a tenant default on their rent is denoted by $z > 0$.[24] This could be interpreted as time and/or money cost of evicting tenants, and in what follows, we assume that the government can influence this cost through housing market regulations, for example, by lengthening the procedure for eviction. Consider now a representative supplier who has chosen to enter the housing market. The representative supplier's expected profit is given by:

$$nR(r, \mu; h, z) - hC(n) \tag{10}$$

[23] If the average probability of default was decreasing in the rental rate, then expected revenue per rental unit would always be increasing in the rental rate and there would never be an equilibrium with excess demand. Housing suppliers would always want r to be as large as possible.

[24] It is possible that z is less than zero so housing suppliers recover some portion of the rent from tenants who default. In this case, we would have to account for where this money is coming from.

where

$$R(r, \mu; h, z) = r - \pi(r, \mu; h)(r + z) \tag{11}$$

is the supplier's expected revenue per rental unit. Thus, with probability π, the tenant of a rental unit defaults thereby causing a loss of $(r + z)$ for the supplier of the rental unit. The representative supplier chooses the number of rental units n to supply and the rental price r to charge that maximize expected profits. The first-order conditions for n and r are

$$R(r, \mu; h, z) - hC'(n) = 0, \tag{12}$$

$$nR_r(r, \mu; h, z) = 0, \tag{13}$$

respectively. Together the above first-order conditions yield the supplier's supply $n(\mu; h, z)$ and rental rate $r(\mu; h, z)$.[25] From (13), it is clear that the rental price that maximizes expected profits is simply the rental price that maximizes expected revenue per rental unit, that is, $r(\mu; h, z)$ is *independent* of the number of rental units supplied. It is also instructive to note that the expected revenue per rental unit will be decreasing in r for any rental price larger than $r(\mu; h, z)$.

To simplify the exposition for the rest of the paper, we assume that the marginal expected revenue per rental unit is increasing in μ:[26]

Assumption 2: $R_{r\mu} > 0$.

Then, totally differentiating the above first-order conditions, we obtain (see Appendix)

$$n_\mu(\mu; h, z) > 0, \quad n_h(\mu; h, z) < 0, \quad n_z(\mu; h, z) < 0, \tag{14}$$

$$r_\mu(\mu; h, z) > 0, \quad r_h(\mu; h, z) < 0, \quad r_z(\mu; h, z) < 0. \tag{15}$$

Supply for a given firm is increasing in the (endogenous) probability of not finding a rental unit, μ, and decreasing in the other exogenous parameters. Increases in μ make it more costly to look for housing so only those with higher incomes will look for rental units. The average

[25] To ensure that the second-order conditions are satisfied for a maximum, we assume that $R_{rr} < 0$.

[26] This assumption is not crucial for our main results and is only required to sign r_μ and r_h in (15). See Appendix for details.

probability of default will be lower and the supplier's expected revenue per rental unit will be higher giving the supplier incentive to produce more rental units. Therefore, supply is increasing in μ. An increase in h will have the opposite effect. Higher quality housing induces more people to look for housing which increases the average probability of default, and reduces the supplier's expected revenue per rental unit. In addition, higher quality housing is more expensive to produce. Supply will also be decreasing in z since a higher cost of default reduces the expected return from each rental unit. The expected profit-maximizing rental price is increasing in the probability of not finding a rental unit, and decreasing in the quality of rental housing and the cost of default.

The supplier's maximized expected profits will be given by

$$\Pi(\mu; h, z) = n(\mu; h, z)R(r(\mu; h, z), \mu; h, z) - hC(n(\mu; h, z)) \tag{16}$$

where, from the envelope theorem, we have

$$\Pi_\mu(\mu; h, z) > 0, \quad \Pi_h(\mu; h, z) < 0, \quad \Pi_z(\mu; h, z) < 0 \tag{17}$$

A supplier will enter the rental housing market provided

$$\Pi(\mu; h, z) - \phi \geq 0 \tag{18}$$

We now turn to characterizing the equilibrium in the rental housing market.

3. Market Equilibrium with Excess Demand

3.1 Fixed Number of Suppliers

Consider first the market equilibrium with a fixed number of suppliers, denoted by m.[27] Each supplier is identical and so chooses the same number of rental units to supply and the same rental price to charge. Total supply in the market (for a given μ) will be equal to $S(m, \mu; h, z) = mn(\mu; h, z)$. We now show that there may exist a rental price in which there is excess demand, $\mu > 0$, in the rental housing and that no firm has an incentive to charge another price. To save on notation, we suppress h and z and write market demand as $D(r, \mu)$, rental units per supplier

[27] One could think about the rental housing market in the very short-run.

as $n(\mu)$, market supply as $S(m,\mu)$, and the expected profit maximizing rental price as $r(\mu)$. We now state and prove the following proposition.

Proposition 1: *Given a fixed number of firms m in the rental housing market, there is an equilibrium with excess demand if $D(r(0),0)$ is greater than $S(m,0)$.*

Proof: Consider the case in which there is excess demand at $r(0)$. From (15), the expected profit maximizing rental price is increasing in μ. Hence, for any $\mu > 0$, the expected revenue per housing unit $R(r,\mu)$ is decreasing in r for all $r > r(\mu)$. We can pick some $\bar{r}$ such that $\bar{r} > r(\mu)$ and let $\bar{n}(\bar{r},\mu) = \arg\max_n nR(\bar{r},\mu) - hC(n)$. Then, for some

$$n^o = \frac{R(\bar{r},\mu)}{R(r(\mu),\mu)}\bar{n}(\bar{r},\mu) < \bar{n}(\bar{r},\mu)$$

we have

$$n^o R(r(\mu),\mu) - hC(n^o) = \bar{n}(\bar{r},\mu)R(\bar{r},\mu) - hC(n^o) > \bar{n}(\bar{r},\mu)R(\bar{r},\mu) - hC(\bar{n}(\bar{r},\mu))$$

where the inequality follows from $n^o < \bar{n}(\bar{r},\mu)$. So firms are better off producing n^o rental units at $r(\mu)$ then any number of rental units at $\bar{r} > r(\mu)$. Of course, if the rental price is $r(\mu)$, then each firm supplies $n(\mu)$=$\arg\max_n nR(r(\mu),\mu) - hC(n)$. Thus, by definition

$$n(\mu)R(r(\mu),\mu) - hC(n(\mu)) \geq n^o R(r(\mu),\mu) - hC\left(n^o\right) > \bar{n}(\bar{r},\mu)R(\bar{r},\mu) - hC(\bar{n}(\bar{r},\mu))$$

The supplier earns a larger expected unit return and therefore larger expected profits by charging $r(\mu)$ rather than charging any higher rental price even with excess demand in the rental market. Charging $r(\mu)$ and having excess demand in the rental market is consistent with competitive firm behaviour.

An equilibrium with excess demand for a fixed number of suppliers is given by some positive level of μ, denoted by μ^f, such that:

$$(1-\mu^f)D(r(\mu^f),\mu^f) - S(m,\mu^f) = 0 \tag{19}$$

where $\mu^f(m)$ is the equilibrium probability that someone looking for a house does not find one and in which all suppliers charge price $r(\mu^f(m))$. Note that given $r(\mu^f)$ is increasing in μ^f, the left-hand side of the equilibrium condition (19) is strictly decreasing in μ. Therefore, there should be some $\mu^f(m) \in (0,1)$ such that the above condition (19) is exactly satisfied.[28] This completes the proof.[29] ■

If the condition in Proposition 1 does not hold and supply is greater than demand at $\mu = 0$, then there will not be an equilibrium with excess demand. Suppliers would be forced to compete with one another for tenants and the market equilibrium rental price would be competed down below $r(0)$.

3.2 Endogenous Number of Suppliers

Now suppose the number of suppliers is determined endogenously as is the case in the long run. Recall that suppliers enter the market provided their maximized expected profits is greater than the fixed cost of entry. The equation determining the equilibrium number of suppliers (bringing back the suppressed notation for h and z) is

$$\Pi(\mu^f(m;h,z);h,z) = \phi \tag{20}$$

which yields $m^*(h,z,\phi)$.[30] The equilibrium number of suppliers then determines the equilibrium rate of excess demand given by $\mu^*(h,z,\phi) = \mu^f(m^*(h,z,\phi);h,z)$, which in turns determines the equilibrium rental rate $r^*(h,z,\phi) = r(\mu^*(h,z,\phi);h,z)$ and the equilibrium market supply $S^*(h,z,\phi) = S(m^*(h,z,\phi),\mu^*(h,z,\phi))$. Finally, μ^* and r^* determine the equilibrium market demand given by $D^*(h,z,\phi) = D(r^*(h,z,\phi),\mu^*(h,z,\phi),h)$.

We now state and prove the following proposition.

Proposition 2: *Given entry into the rental housing market, there can be a competitive equilibrium with excess demand provided the fixed cost of entry is sufficiently high.*

[28] Note also that if $\mu^f(m)$ exists it will be unique. For the case when Assumption 2 doesn't hold and $r(\mu)$ is decreasing in μ, a sufficient (non-necessary) condition for excess demand (i.e. the left-hand side of equation (19)) to be decreasing in μ is that the direct effect of an increase of μ (i.e. $(1-\mu)D_\mu - mn_\mu < 0$) outweighs the indirect effect of the increase in μ through r (i.e. $[(1-\mu)D_r]r_\mu > 0$).

[29] The above proof is similar in nature to the argument presented in Weiss (1980). Weiss showed that if the expected revenue from hiring a working is an increasing function of the wage, then competitive firms may choose not to lower wages (hire more workers) even when there is excess supply in the labour market.

[30] As shown in the Appendix, how the equilibrium number of suppliers changes with h or z is ambiguous.

Proof: From Proposition 1, it has already been shown for any fixed number of firms m there can be an equilibrium with excess demand provided $D(r(0),0) > S(m,0)$.

If maximized expected profits were greater than ϕ when $\mu = 0$, that is, $\Pi(0;h,z) > \phi$, it would be profitable for suppliers to continually enter the market and all demand would necessarily be served. Therefore, such a low ϕ cannot lead to an equilibrium with excess demand.

Suppose instead maximized expected profits is less than ϕ when evaluated at $\mu = 0$. Then, given this large fixed cost of entry, no firm wants to enter the market. With no firms in the market, there is certainly excess demand, which contradicts $\mu = 0$. Therefore, for ϕ large enough such that $\Pi(0;h,z) < \phi$ an equilibrium must entail $\mu^* > 0$, i.e. there is an equilibrium with excess demand. ■

Remark: A competitive equilibrium with excess demand can exist in our model without government regulation. In the above analysis we have assumed that the quality of rental housing, h, and the cost of default to suppliers, z, are exogenous and can be influenced by government policies. There is no need, however, for specific values of h or z for Propositions 1 and 2 to hold. Regarding z, it is easily seen that the analysis above would go through if $z = 0$. Turning now to h, the model could be re-formulated so as to endogenize h, but this would have been at the cost of simplicity and heavy exposition. Alternatively we could re-write the model without h. All that is required for Propositions 1 and 2 are the following three elements: (i) Two discrete levels of housing consumption; (ii) The low level of housing is free — or at least cannot lead to default — while the high level of housing is costly and can lead to default; and (iii) a strictly positive utility differential between consuming the low level of housing and consuming the high level of housing. Thus, if the above analysis was re-written with $u(h_o) = u_0$, $u(h) = u_1 > u_0$, and the supplier's cost was independent of h, then the above analysis would go through. We are interested, however, in determining the effect of government policies on a rental housing equilibrium with excess demand. Therefore, we have chosen to include the exogenous parameters h and z in our analysis, but again, their inclusion is not required for our Propositions.[31]

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[31] A fixed cost of entry is required, however, to obtain an equilibrium with excess demand when there is entry of suppliers into the rental housing market.

4. Impact of Government Policies

We now examine how changes in government regulations affecting the quality of housing, h, and the cost of default, z, affect the competitive equilibrium. We begin with the following result:

Result 1: *The equilibrium rate of excess demand, $\mu^*(h, z)$, is increasing in both the quality of housing h and the default cost z.*

Thus, our analysis is in line with that of Quigley and Raphael (2004): an increase in the quality of housing can make it more difficult for households to find affordable housing. To understand Result 1, suppose the rental housing market is in an equilibrium with excess demand. From Propositions 1 and 2, the equilibrium probability that someone looking for a rental unit does not find one, $\mu^*(h, z)$ can be determined from the following expression $\Pi(\mu^*; h, z) = \phi$. Determining how μ^* is affected by changes in the quality of rental housing, h, and in the cost of default, z, simply amounts to determining how these parameters (and μ) affect the maximized expected profits of the housing suppliers. As shown in (17), maximized expected profits are increasing in μ and decreasing in both h and z. Result 1 follows directly.[32]

Although we obtain unambiguous comparative statics on the equilibrium value of the probability of not finding rental housing, the effect of changes in policies on the equilibrium values of the rental rate, quantity demanded and quantity supplied are all ambiguous.[33] To further illustrate these comparative statics, we consider changes in the values of z and h in a numerical example in Section 5. Before doing so, we briefly discuss the impact of rent control.

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[32] Note also that an increase in the fixed cost will increase the equilibrium rate of excess demand in the rental housing market which increases the equilibrium number of rental units per supplier. Not surprisingly an increase in the fixed cost of entry reduces the equilibrium number of suppliers in the market. This reduction in the number of suppliers more than offsets the increase in the number of rental units per supplier and total supply of rental units in equilibrium decreases with the fixed cost of entry. See Appendix for details.

[33] The comparative statics on the number of rental units per supplier are unambiguous. An increase in either z and h has a (negative) direct effect on n and a (positive) indirect effect on n through μ. In the case of an increase in z, the two effects exactly offset one another and there is no change in the equilibrium number of rental units per supplier. In the case of an increase in h, the (negative) direct effect outweighs the (positive) direct effect and the equilibrium number of rental units per supplier falls. In both cases, the effect on the equilibrium number of suppliers in the rental housing market is ambiguous and thus total quantity supplied in the market could go up or down with an increase in either h or z.

4.1 Rent Control

By definition, rent control imposes a maximal rental price or rent ceiling in the rental housing market. If rent control is binding, then the rental price ceiling, $\bar{r}$, will be less than the equilibrium rental price, r^*. The following can be shown.

Result 2: *Under a binding rental price ceiling, the rate of excess demand is decreasing in the regulated rental price, and increasing in both the quality of housing h and the default cost z.*

Thus, starting from some equilibrium (with excess demand), imposing rent control will increase the level of excess demand, but the closer $\bar{r}$ is to r^*, the smaller this increase in excess demand. Interestingly, and as was noted earlier, an increase in the rate of excess demand μ reduces the rate of default and increases the number of rental units per supplier. Any increase in $\bar{r}$ will also increase rental units per supplier. It turns out that this direct (positive) effect of an increase in $\bar{r}$ on n will exactly offset the (negative) indirect effect through μ. Consequently, the number of rental units per supplier does not change. The effect of an increase in $\bar{r}$ on the equilibrium number of suppliers is ambiguous and thus, paradoxically, rent control and its associated increase in μ can lead to an increase in market supply and to an increase in the quantities exchanged. These effects are absent in the standard microeconomic textbook analysis of rent control since it ignores the possibility of tenancy default. Such an analysis can therefore lead to an incorrect assessment of the full (welfare) impact of rent control. We come back to this in our numerical example.

5. Numerical Example

We construct a numerical example to confirm that an excess demand equilibrium can be obtained in our competitive economy and further to illustrate the impact government policies have on both the rental housing market and social welfare.

We consider a very simple economy of the type described in previous sections. The following specific functional forms are assumed:

- ◇ The total number of individuals in the economy is $N = 100$;
- ◇ The utility of housing is given by $u(h) = \alpha h$;

- ◇ The distribution of income is uniform on the interval $[\underline{y}, \overline{y}]$;
- ◇ The distribution of shocks is uniform on the interval $[\underline{s}, \overline{s}]$;
- ◇ The cost of producing n housing units of quality h is $hn^2/2$.
- ◇ Social welfare is given by the following direct utilitarian function:[34,35]

$$\begin{aligned} W(r,\mu;h) =& N\int_{\underline{y}}^{\hat{y}}(u(h_o)+y)dG + N\int_{\hat{y}}^{\overline{y}}(1-\mu)\left[F(\hat{s})u(h_o)+(1-F(\hat{s}))(u(h)-r)\right]dG \\ &+ N\int_{\hat{y}}^{\overline{y}}\left[\mu u(h_o)+y-k\right]dG \end{aligned} \tag{21}$$

The parameters are set as follows: $\underline{y} = 2$; $\overline{y} = 4$; $\underline{s} = 0.8$; $\overline{s} = 1.2$; $c_o = 0.5$; $k = 0.1$; $h = 0.8$; $h_o = 0$; $z = 0.1$; $\phi = 1.982$; and $\alpha = 4.5$.

The level of excess demand is denoted by $E = D - S$. The rate of homelessness (or, the proportion of individuals without rental housing) is denoted by M. There are three reasons individuals are without rental housing: they did not look for rental housing $[N - D]$, they looked for rental housing but were unable to find any $[\mu D = E]$, or they looked for and found rental housing but defaulted on their rent and were evicted $[D(1-\mu)\pi]$. Thus, $M = 1 - (D/N) + (D/N)(\mu + (1-\mu)\pi)$.[36] One could interpret this measure as reflecting the extent of the lack of access to rental housing.

Over the past several years public attention has increasingly focused on the issue of 'affordable rental housing' and its presumed shortage. It is worth distinguishing between the lack of access to 'affordable' rental housing and the lack of access to rental housing as described above. In the former case, individuals might be able to afford to rent homes at the current

[34] In equilibrium, the suppliers' expected profits from operating exactly offset their fixed costs of entering the housing market. Therefore, suppliers do not receive any weight in the social welfare function. If we had heterogeneous fixed costs, then all the infra-marginal suppliers would be earning positive returns and would have to be given some weight in the social welfare function.

[35] As shown in the Appendix, we are unable to obtain unambiguous analytical comparative statics on social welfare and therefore rely on the numerical examples to illustrate how the various parameters affect social welfare. A more general social welfare function could have been used without much gain in terms of understanding.

[36] Since $D = (1 - G(\hat{y}))N$, the rate of homelessness will be independent of the size of the population since, as shown in the Appendix, the equilibrium levels of μ and r which affect $\hat{y}$ do not change with N.

prices but doing so puts a financial strain on them. For example, Quigley and Raphael (2004) document the trends in the affordability of rental housing in the US over the last 40 years by calculating both the ratio of the median rent to the median income of rental households and the proportion of rental households spending more than 30% of their income on rent.[37] To relate our model to the discussion about access to 'affordable' housing, we calculate the ratio of the average income of those who look for rental housing to the equilibrium rent and denote this ratio by $A = [(\hat{y} + \bar{y})/2]/r$. Clearly, the larger is A, the smaller the financial strain on those who consume rental housing and the more affordable is the available rental housing. Of course, the average rate of default π also reflects to some extent the 'unaffordability' of rental housing.

Table 1

Benchmark, Impact of Government Policies and Rent Control §

Variable	μ	$r^{\dagger}$	D	S	m	n	π	M	A	W
(a) Benchmark										
	0.083	1.916	97.110	89.044	40	2.226	0.067	0.169	1.587	430.182
(b) Impact of Government Policies, Changes in h and z										
1% ↑ in h	0.260	1.934	95.793	70.837	31.98	2.215	0.071	0.342	1.575	402.354
	+	+	—	—	—	—	+	+	—	—
1% ↑ in z	0.084	1.915	97.130	88.910	39.94	2.226	0.066	0.170	1.587	430.061
	+	—	+	—	—	n/c	—	+	+	—
(c) Impact of Imposing Rent Control, Maximal Rent Set at Level $\bar{r}$										
$\bar{r} = 99\%\ r^*$	0.085	1.896	97.944	89.534	40.22	2.226	0.058	0.156	1.595	433.835
	+	—	+	+	+	n/c	—	—	+	+
$\bar{r} = 97\%\ r^*$	0.108	1.858	99.569	88.755	39.87	2.226	0.039	0.147	1.618	438.474
	+	—	+	—	—	n/c	—	—	+	+

§: A "+" indicates an increase in the variable, a "—" indicates a decrease, and "n/c" indicates no change, relative to the initial values in the benchmark.

†: For Parts (a) and (b), the reported r is the equilibrium r, i.e. r^*. For Part (c), the reported r corresponds to $\bar{r}$ set at a proportion of the benchmark r^* given in the corresponding line.

Before we proceed, recall that one should view what follows as an example of the rental housing

[37] Similar statistics are also used to measure affordability in the Canadian rental housing market (CHMC, 2007).

market for the lowest quality of housing available in a given market — that for which regulated minimal quality requirements would be a binding constraint. Also, the set of individuals we are focusing on is the segment of the population with the lowest income, for example, those belonging to the lowest decile of the income distribution.

In Part (a) of Table 1, we report the values of several variables in our benchmark equilibrium. One of the main message of this paper is that excess demand, i.e. a lack of access to rental housing, can be an equilibrium phenomenon in the housing market. Part (a) of Table 1 confirms that such an equilibrium can exist. Indeed, our benchmark equilibrium is characterized by a rate of excess demand (μ^*) of 8.3%. Note that pinning down a competitive equilibrium with excess demand was not difficult — such equilibria result for a large set of parameter values. Thus, the example we report is simply an illustration. In this example, there are 40 suppliers operating and each of them provide 2.226 housing units.[38] The maximal demand is for 100 units,[39] but because of the search cost, only 97.110 units are demanded. Out of those 97.110 units demanded, only 89.044 will be served (hence a rate of excess demand of 8.3%). Further, for the 89.044 units for which a transaction takes place, there is a possibility of default. With a rate of default of 6.7%, there will be default in 5.965 units. Thus, the rate of homelessness is at 16.9% [= 2.890 (no demand) + 8.066 (excess demand) + 5.965 (default)]. As for our affordability measure A of housing expenditures, note that the equilibrium rent represents 63.2% of the average income of those who demand housing as given by $1/A$.[40]

In Part (b) of Table 1, we report the impact of government policies affecting housing quality and the cost of default for an economy initially resting in the benchmark. Thus, Part (b) indicates the direction of the change in a variable from its original value in Part (a). The first row in Part (b) shows that an increase in housing quality h, as observed in the 1980s and the 1990s when more stringent regulation of the housing construction industry was enacted by governments (Quigley and Raphael, 2004) could have exacerbated the problem of access to rental housing

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[38] As in the model, we are allowing for the number of rental units to be a continuous variable.

[39] As shown in the Appendix, the size of the population N only affects the equilibrium number of suppliers in the market. If population was increased by x%, the equilibrium number of landlords, m, would also increase by x%, while the other variables would remain at the equilibrium values reported in Part (a) of Table 1.

[40] For the very poorest segment of the population, this number is plausible. For example, in 2004, the average housing cost to income ratio of the 20 per cent of households in Canada with the lowest income was over 40% (CHMC, 2007).

as measured by the increase in π (defaulting) and M (no consumption of housing) and the reduction in A (affordability).[41] These facts, together with the fact that an increase in h leads to an increase in the equilibrium rental price, explain why welfare goes down as the quality of rental housing goes up. Thus, a regulated increase in the minimal quality of housing could have led to a deterioration of the housing market for the relatively poor segment of the population.

Consider now the impact of increased protection of tenants from eviction which increases the cost of default to suppliers, z. Increased protection could come about from the imposition of a legal delay before a tenant can be evicted, or by forbidding eviction in the winter. Such policies are often observed in practice. The numerical analysis shows that increasing the protection of tenants from eviction lowers the equilibrium rent. In terms of access to affordable housing, the results are mixed: the probability of default (π) goes down and affordability (A) improves, but the rate of homelessness (M) increases as a result of the contraction in supply. An increase in tenant protection, however, ultimately leads to a decrease in welfare.

In Part (c) of Table 1, we report the impact, relative to the benchmark in Part (a), of implementing a rental price ceiling equal to 99% of the equilibrium rent. As was explained in Section 4, rent control leads to an increase in μ — this is expected when rent control is implemented. What is more surprising, and completely contrary to the standard analysis, are the other columns of Part (c) in Table 1. In this example, rent control leads to more rental units being supplied (and thus exchanged), to a lower probability of default, to less homelessness, to increased affordability, and, ultimately, to an increase in welfare. Further, note that an increase in total supply is not necessary for rent control to be welfare improving. Indeed, when the maximal rent is set at 97% of equilibrium rent, total supply goes down, but because of the lower price, reduced homelessness and improved affordability, welfare increases. Our key message here is that rent control should be analyzed taking into account the possibility that tenants may default on their rent and, consequently, that demand and supply will both adjust to the excess demand in the rental housing market. Further, the standard textbook prescriptions — that rent control is detrimental to tenants because it creates excess demand — may not be valid in a competitive housing market equilibrium when one introduces the possibility of tenancy default. As we have just illustrated, any increase in the rate of excess demand in the market can be more than compensated for by the lower rental rate and the accompanying improvement in terms of

[41] Similar links between the minimum quality of rental housing and homelessness have been empirically documented (Quigley *et al.*, 2001).

affordability.

6. Concluding Remarks

We have shown that if some tenants involuntarily default on their rent, then a competitive market equilibrium could exhibit excess demand in the absence of government interventions. Tenant default could explain the shortage of rental housing. Further, we have shown that a regulated increase in the minimal quality of rental housing, observed in the 1980s and the 1990s when more stringent regulation of the housing construction industry was enacted by governments, could have exacerbated the lack of access to affordable housing. Similarly, an increase in the protection of tenants from eviction, by imposing a delay before a tenant can be evicted, or by forbidding eviction in the winter, leads to less households finding a rental unit. However, such an increase in protection also leads to a lower equilibrium rent, which in turn translates into better affordability according to measures like the probability of default or the financial strain put on those who do rent.

Introducing the possibility of tenancy default leads to surprising implications regarding the impact of rent control. Recall that in the standard textbook world, under perfect competition and with no possibility of default, rent control is generally presented as a bad policy as it generates excess demand and it reduces the number of rental units exchanged. As our analysis showed, things are not so simple when tenancy default is introduced in an otherwise competitive environment. The main reason is that in our framework, the rate of excess demand in the market directly affects demand and indirectly affects the supply through the average probability of default. Thus, introducing rent control creates excess demand, which in turn increases supply. As we showed in our numerical example, rent control can lead to a larger quantity supplied (and thus exchanged), to a lower probability of default, to less homelessness, to better affordability, and, ultimately, to an increase in welfare. So rent control may be a good policy in the presence of tenancy default.

There are several housing policies we did not consider in this paper. For example, the framework we developed could be used to study the provision of public housing, rental allowances, and rental subsidies. Some of these policies would require financing, so extra ingredients would have to be added to our model to account for this requirement. We leave these investigations for future research.

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Appendix

Section 2.1: Comparative Statics of Market Demand

To determine how market demand changes with the various parameters, we can differentiate $v_h - v_o$ given by the expression on the left-hand side of (4) with respect to the various parameters.

$$\begin{aligned}
\frac{\partial(v_h - v_o)}{\partial r} &= -(1-\mu)[u(h) - r - u(h_o)]f(\hat{s})\frac{1}{y} - (1-\mu)[1-F(\hat{s})] < 0, \\
\frac{\partial(v_h - v_o)}{\partial \mu} &= -[1-F(\hat{s})][u(h) - r - u(h_o)] < 0, \\
\frac{\partial(v_h - v_o)}{\partial c_o} &= -(1-\mu)[u(h) - r - u(h_o)]f(\hat{s})\frac{1}{y} < 0, \\
\frac{\partial(v_h - v_o)}{\partial k} &= -1 < 0, \quad \frac{\partial(v_h - v_o)}{\partial h} = (1-\mu)[1-F(\hat{s})]u'(h) > 0.
\end{aligned} \tag{A1}$$

Using (A1) and (5), we obtain the following partial derivatives for $\hat{y}(r, \mu; c_o, k, h)$

$$\begin{aligned}
&\hat{y}_r = \frac{1}{\hat{s}} + \frac{1-F(\hat{s})}{[u(h) - r - u(h_o)]f(\hat{s})(\hat{s}/y)} > 0, \quad \hat{y}_\mu = \frac{1-F(\hat{s})}{(1-\mu)f(\hat{s})(\hat{s}/y)} > 0, \quad \hat{y}_{c_o} = \frac{1}{\hat{s}} > 0, \\
&\hat{y}_k = \frac{1}{(1-\mu)[u(h) - r - u(h_o)]f(\hat{s})(\hat{s}/y)} > 0, \quad \hat{y}_h = -\frac{[1-F(\hat{s})]u'(h)}{[u(h) - r - u(h_o)]f(\hat{s})(\hat{s}/y)} < 0
\end{aligned} \tag{A2}$$

which yields (6). Then, from the definition of market demand $D = [1 - G(\hat{y})]N$ it follows that

$$\begin{aligned}
&D_r = -g(\hat{y})\hat{y}_r N < 0, \quad D_\mu = -g(\hat{y})\hat{y}_\mu N < 0, \quad D_N = 1 - G(\hat{y}) > 0 \\
&D_{c_o} = -g(\hat{y})\hat{y}_{c_o} N < 0, \quad D_k = -g(\hat{y})\hat{y}_k N < 0, \quad D_h = -g(\hat{y})\hat{y}_h N > 0
\end{aligned} \tag{A3}$$

which yields (7).

Section 2.2: Comparative Statics of the Supplier's Problem

To determine how the average probability changes with the various parameters, we differentiate (8) to obtain:

$$\pi_r = B_1 + B_2\hat{y}_r, \quad \pi_{c_o} = B_1 + B_2\hat{y}_{c_o}, \quad \pi_\mu = B_2\hat{y}_\mu < 0, \quad \pi_k = B_2\hat{y}_k < 0, \quad \pi_h = B_2\hat{y}_h > 0 \tag{A4}$$

where

$$\begin{aligned}
B_1 &= \frac{1}{1-G(\hat{y})}\int_{\hat{y}}^{\overline{y}} f\left(\frac{c_o + r}{y}\right)\frac{1}{y}dG > 0, \\
B_2 &= \frac{g(\hat{y})}{[1-G(\hat{y})]^2}\left[\int_{\hat{y}}^{\overline{y}} F\left(\frac{c_o + r}{y}\right)dG - F\left(\frac{c_o + r}{\hat{y}}\right)(1 - G(\hat{y}))\right] < 0
\end{aligned}$$

which yields (9). We now state and prove the following Lemma.

Lemma 1: If $\pi_r > 0$, then $\pi_{c_o} > 0$.

Proof:

From ($A2$), $\hat{y}_r > \hat{y}_{c_o} > 0$ which implies from ($A4$) that $\pi_r < \pi_{c_o}$. Therefore, if $\pi_r > 0$ it follows that $\pi_{c_o} > 0$. Therefore, we have Lemma 1. ■

To determine how the supplier's optimal choice of n and r changes with the various parameters, we first differentiate the supplier's expected revenue per rental unit $R(r, \mu; c_o, k, h, z)$ given by (11). Doing this and using ($A4$), Assumption 1 and Lemma 1, we obtain

$$\begin{aligned} &R_r = (1 - \pi) - (r + z)\pi_r \gtreqless 0, \quad R_\mu = -(r + z)\pi_\mu > 0, \\ &R_{c_o} = -(r + z)\pi_{c_o} < 0, \quad R_k = -(r + z)\pi_k > 0, \\ &R_h = -(r + z)\pi_h < 0, \quad R_z = -\pi < 0 \end{aligned} \tag{A5}$$

It is also instructive to determine how the marginal expected revenue per rental unit R_r changes with the various parameters. Using the expression for $R_r(r, \mu, c_o, k, h, z)$ from (A5) and differentiating, we have

$$R_{rr} = -2\pi_r - (r + z)\pi_{rr}, \quad R_{rz} = -\pi_r < 0, \quad R_{rj} = -\pi_j - (r + j)\pi_{rj} \tag{A6}$$

for $j = \mu, c_o, k, h$.

We now state and prove the following Lemma.

Lemma 2: If $R_{r\mu} > 0$, then $R_{rk} > 0$ and $R_{rh} < 0$.

Proof:

Using ($A2$) and ($A4$) and given that the order of differentiation is irrelevant for cross-partials, we have that

$$\pi_{r\mu} = \pi_{\mu r} = \frac{dB_2}{dr}\hat{y}_\mu + B_2\hat{y}_{\mu r}, \quad \pi_{rk} = \pi_{kr} = \frac{dB_2}{dr}\hat{y}_k + B_2\hat{y}_{kr}, \quad \pi_{rh} = \pi_{hr} = \frac{dB_2}{dr}\hat{y}_h + B_2\hat{y}_{hr}$$

We can write the expression for $\hat{y}_r$ as follows:

$$\hat{y}_r \equiv Y(h, r, \hat{s}, \hat{y}) = \frac{1}{\hat{s}} + \frac{H(\hat{s})}{\hat{s}} \frac{1}{u(h) - r - u(h_o)}\hat{y}$$

where $H(\hat{s}) = (1 - F(s))/f(s)$, $\hat{s} = (c_o + r)/y$ and $\hat{y}(r, \mu, c_o, k, h)$. We do not make any assumptions on the sign of $H'(s)$. Differentiating Y, we obtain

$$\begin{aligned} Y_h &= -\frac{H(\hat{s})}{\hat{s}} \frac{u'(h)}{[u(h) - r - u(h_o)]^2} < 0 \\ Y_r &= \frac{H(\hat{s})}{\hat{s}} \frac{1}{[u(h) - r - u(h_o)]^2} > 0 \\ Y_{\hat{s}} &= -\frac{1}{\hat{s}} + \frac{H'(\hat{s})}{\hat{s}} \frac{1}{u(h) - r - u(h_o)} - \frac{H(\hat{s})}{\hat{s}^2} \frac{1}{u(h) - r - u(h_o)} \\ Y_{\hat{y}} &= \frac{H(\hat{s})}{\hat{s}} \frac{1}{u(h) - r - u(h_o)} > 0 \end{aligned}$$

We can then determine the following second partial derivatives:

$$\hat{y}_{r\mu} = \hat{y}_{\mu r} = \left(Y_{\hat{s}}\hat{s}_y + Y_{\hat{y}}\right)\hat{y}_\mu$$
$$\hat{y}_{rk} = \hat{y}_{kr} = \left(Y_{\hat{s}}\hat{s}_y + Y_{\hat{y}}\right)\hat{y}_k$$
$$\hat{y}_{rh} = \hat{y}_{hr} = \left(Y_{\hat{s}}\hat{s}_y + Y_{\hat{y}}\right)\hat{y}_h + Y_h$$

Using the above, we have

$$\begin{aligned}\pi_\mu + (r+z)\pi_{r\mu} &= B_2\hat{y}_\mu + (r+z)\left(\frac{dB_2}{dr}\hat{y}_\mu + B_2\hat{y}_{\mu r}\right)\\ &= \hat{y}_\mu\left[B_2 + (r+z)\left(\frac{dB_2}{dr} + B_2\frac{\hat{y}_{r\mu}}{\hat{y}_\mu}\right)\right]\end{aligned}$$

$$\begin{aligned}\pi_k + (r+z)\pi_{rk} &= B_2\hat{y}_k + (r+z)\left(\frac{dB_2}{dr}\hat{y}_k + B_2\hat{y}_{kr}\right)\\ &= \hat{y}_k\left[B_2 + (r+z)\left(\frac{dB_2}{dr} + B_2\frac{\hat{y}_{rk}}{\hat{y}_k}\right)\right]\end{aligned}$$

$$\begin{aligned}\pi_h + (r+z)\pi_{rh} &= B_2\hat{y}_h + (r+z)\left(\frac{dB_2}{dr}\hat{y}_h + B_2\hat{y}_{hr}\right)\\ &= \hat{y}_h\left[B_2 + (r+z)\left(\frac{dB_2}{dr} + B_2\frac{\hat{y}_{rh}}{\hat{y}_h}\right)\right]\\ &= \hat{y}_h\left[B_2 + (r+z)\left(\frac{dB_2}{dr} + B_2\left(Y_{\hat{s}}\hat{s}_y + Y_{\hat{y}} + Y_h/\hat{y}_h\right)\right)\right]\\ &= \hat{y}_h\left[B_2 + (r+z)\left(\frac{dB_2}{dr} + B_2\left(Y_{\hat{s}}\hat{s}_y + Y_{\hat{y}}\right)\right)\right] + (r+z)B_2Y_h\end{aligned}$$

Since $\hat{y}_{r\mu}/\hat{y}_\mu = \hat{y}_{rk}/\hat{y}_k$, $\hat{y}_\mu > 0$ and $\hat{y}_k > 0$, it follows that $\pi_\mu + (r+z)\pi_{r\mu} < 0$ if and only if $\pi_k + (r+z)\pi_{rk} < 0$. Given $\hat{y}_\mu > 0$ and $\hat{y}_h < 0$, the condition $\pi_\mu + (r+z)\pi_{r\mu} < 0$ implies that $\pi_h + (r+z)\pi_{rh} > 0$. Therefore, we have Lemma 2. ∎

In what follows, we focus, without loss of generality, on the case in which the effect on the average probability of default dominates the effect on the marginal probability of a change in μ.

Assumption 2: $R_{r\mu} > 0$.

By Lemma 2, we can then sign R_{rk} and R_{rh}. The sign of R_{rc_o}, however, remains ambiguous.

We can now turn to determining how a representative supplier's choice of rental units and rental price change with the various parameters. The first order conditions are of the representative supplier's problem are

$$R(r,\mu;c_o,k,h,z) - hC'(n) = 0$$
$$nR_r(r,\mu;c_o,k,h,z) = 0$$

Together, the above conditions yield $\tilde{n}(\mu; c_o, k, h, z)$ and $\tilde{r}(\mu; c_o, k, h, z)$. Totally differentiating the above system of equations yields

$$\begin{bmatrix} R_r & -hC''(n) \\ nR_{rr} & R_r \end{bmatrix} \begin{bmatrix} d\tilde{r} \\ d\tilde{n} \end{bmatrix} =$$

$$\begin{bmatrix} -R_\mu & -R_{c_o} & -R_k & -R_h + C'(\tilde{n}) & -R_z \\ -\tilde{n}R_{r\mu} & -\tilde{n}R_{rc_o} & -\tilde{n}R_{rk} & -\tilde{n}R_{rh} & -\tilde{n}R_{rz} \end{bmatrix} \begin{bmatrix} d\mu \\ dc_o \\ dk \\ dh \\ dz \end{bmatrix}$$

Applying Cramer's Rule and assuming that $R_{rr} < 0$ we obtain the following total partial derivatives

$$\begin{aligned} &\tilde{r}_\mu = -\frac{R_{r\mu}}{R_{rr}} > 0, \quad \tilde{r}_{c_o} = -\frac{R_{rc_o}}{R_{rr}} \gtrless 0, \\ &\tilde{r}_k = -\frac{R_{rk}}{R_{rr}} > 0, \quad \tilde{r}_h = -\frac{R_{rh}}{R_{rr}} < 0, \quad \tilde{r}_z = -\frac{R_{rz}}{R_{rr}} < 0 \end{aligned} \tag{A7}$$

$$\begin{aligned} &\tilde{n}_\mu = \frac{R_\mu}{hC''(\tilde{n})} > 0, \quad \tilde{n}_{c_o} = \frac{R_{c_o}}{hC''(\tilde{n})} < 0, \\ &\tilde{n}_k = \frac{R_k}{hC''(\tilde{n})} > 0, \quad \tilde{n}_h = \frac{R_h - C'(\tilde{n})}{hC''(\tilde{n})} < 0, \quad \tilde{n}_z = \frac{R_z}{hC''(\tilde{n})} < 0 \end{aligned} \tag{A8}$$

where we have used the first-order condition $\tilde{n}R_r = 0$ and Assumption 2. The signs follow from $(A5)$, $(A6)$ and Lemma 2.

The supplier's maximized expected profit is given by

$$\Pi(\mu, c_o, k, h, z) = \tilde{n}(\mu, c_o, k, h, z)R(\tilde{r}(\mu, c_o, k, h, z), \mu, c_o, k, h, z) - hC(\tilde{n}(\mu, c_o, k, h, z)) \tag{A9}$$

Differentiating and using $(A5)$, we obtain

$$\Pi_\mu = \tilde{n}R_\mu > 0, \quad \Pi_{c_o} = \tilde{n}R_{c_o} < 0, \quad \Pi_k = \tilde{n}R_k > 0, \quad \Pi_h = \tilde{n}R_h - C(\tilde{n}) < 0, \quad \Pi_z = \tilde{n}R_z < 0 \tag{A10}$$

Section 3: Comparative Statics of Market Equilibrium

Section 3.1: Fixed Number of Suppliers

The equilibrium with excess demand and a fixed number of firms, denoted by m, can be represented by the following condition:

$$(1 - \mu^f)D(\tilde{r}(\mu^f, c_o, k, h, z), \mu^f, N, c_o, k, h) - m\tilde{n}(\mu^f, c_o, k, h, z) = 0$$

which yields $\mu^f(m, N, c_o, k, h, z)$. Totally differentiating the above, and using Assumption 2, (A3), (A7) and (A8), we obtain

$$\begin{aligned}
\mu^f_m &= -\frac{\tilde{n}}{\varPhi} < 0 \\
\mu^f_N &= \frac{(1-\mu)D_N}{\varPhi} > 0 \\
\mu^f_{c_o} &= \frac{(1-\mu)[D_r\tilde{r}_{c_o} + D_{c_o}] - m\tilde{n}_{c_o}}{\varPhi} \gtreqless 0 \\
\mu^f_k &= \frac{(1-\mu)[D_r\tilde{r}_k + D_k] - m\tilde{n}_k}{\varPhi} > 0 \\
\mu^f_h &= \frac{(1-\mu)[D_r\tilde{r}_h + D_h] - m\tilde{n}_h}{\varPhi} > 0 \\
\mu^f_z &= \frac{(1-\mu)D_r\tilde{r}_z - m\tilde{n}_z}{\varPhi} > 0
\end{aligned} \tag{A11}$$

where

$$\varPhi = D - (1-\mu)[D_r r_\mu + D_\mu] + m\tilde{n}_\mu > 0.$$

Section 3.2: Endogenous Number of Suppliers

The condition determining the equilibrium number of suppliers is given by

$$\varPi(\mu^f(m, N, c_o, k, h, z); c_o, k, h, z) = \phi$$

which yields $m^*(N, c_o, k, h, z, \phi)$. Differentiating the above condition, we obtain

$$\begin{aligned}
&m^*_N = \frac{\varPi_\mu \mu^f_N}{\varPsi} = \frac{(1-\mu)D_N}{\tilde{n}} > 0, \quad m^*_{c_o} = \frac{\varPi_\mu \mu^f_{c_o} + \varPi_{c_o}}{\varPsi} \gtreqless 0, \quad m^*_k = \frac{\varPi_\mu \mu^f_k + \varPi_k}{\varPsi} > 0, \\
&m^*_h = \frac{\varPi_\mu \mu^f_h + \varPi_h}{\varPsi} \gtreqless 0, \quad m^*_z = \frac{\varPi_\mu \mu^f_z + \varPi_z}{\varPsi} \gtreqless 0, \quad m^*_\phi = -\frac{1}{\varPsi} < 0
\end{aligned} \tag{A12}$$

where $\varPsi = -\varPi_\mu \mu^f_m > 0$

The equilibrium number of suppliers $m^*(N, c_o, k, h, z, \phi)$ determines the proportion of demand not being met in equilibrium given by $\mu^*(N, c_o, k, h, z, \phi) = \mu^f(m^*(\cdot), N, c_o, k, h, z)$ which in turn determines the equilibrium rental price $r^*(c_o, k, h, z, \phi) = \tilde{r}(\mu^*(\cdot); c_o, k, h, z)$ and the equilibrium number of rental units each supplier produces, $n^*(c_o, k, h, z, \phi) = \tilde{n}(\mu^*(\cdot); c_o, k, h, z)$. Finally, the equilibrium rental rate $r^*(\cdot)$ and $\mu^*(\cdot)$ determines the demand for rental housing $D^*(N, c_o, k, h, z, \phi) = D(r^*(\cdot), \mu^*(\cdot); N, c_o, k, h)$.

Section 4: Comparative Statics of Government Policies

The proportion of demand not being met in equilibrium is given by

$$\mu^*(N, c_o, k, h, z) = \mu^f(m^*(N, c_o, k, h, z), N, c_o, k, h, z).$$

Totally differentiating the above and using (A10) and (A12), yields

$$\mu_N^* = \mu_m^f m_N^* + \mu_N^f = 0, \quad \mu_{c_o}^* = \mu_m^f m_{c_o}^* + \mu_{c_o}^f = -\frac{\Pi_{c_o}}{\Pi_\mu} > 0, \quad \mu_k^* = \mu_m^f m_k^* + \mu_k^f = -\frac{\Pi_k}{\Pi_\mu} < 0$$
$$\mu_h^* = \mu_m^f m_h^* + \mu_h^f = -\frac{\Pi_h}{\Pi_\mu} > 0, \quad \mu_z^* = \mu_m^f m_z^* + \mu_z^f = -\frac{\Pi_z}{\Pi_\mu} > 0, \quad \mu_\phi^* = \mu_m^f m_\phi^* > 0 \tag{A13}$$

Therefore, we have Result 1. Note, also that the size of the population N has no effect on the equilibrium rate of excess demand and therefore will also not affect either the equilibrium rental price or the equilibrium number of rental units per supplier. Given total supply is equal to mn, it follows from (A3) and (A12) that any increase in N increases the equilibrium number of suppliers to exactly meet the increase in demand. We can also determine how the equilibrium number of rental units per supplier and the equilibrium rental rate changes with the various parameters.

$$n_N^* = \tilde{n}_\mu \mu_N^* = 0, \quad n_{c_o}^* = \tilde{n}_\mu \mu_{c_o}^* + \tilde{n}_{c_o} = 0, \quad n_k^* = \tilde{n}_\mu \mu_k^* + \tilde{n}_k = 0$$
$$n_h^* = \tilde{n}_\mu \mu_h^* + \tilde{n}_h = \frac{1}{hC''(\tilde{n})}\left[\frac{C(\tilde{n})}{\tilde{n}} - C'(\tilde{n})\right] < 0, \quad n_z^* = \tilde{n}_\mu \mu_z^* + \tilde{n}_z = 0, \quad n_\phi^* = \tilde{n}_\mu \mu_\phi^* > 0 \tag{A14}$$

$$r_N^* = \tilde{r}_\mu \mu_N^* = 0, \quad r_{c_o}^* = \tilde{r}_\mu \mu_{c_o}^* + \tilde{r}_{c_o} \gtrless 0, \quad r_k^* = \tilde{r}_\mu \mu_k^* + \tilde{r}_k = 0$$
$$r_h^* = \tilde{r}_\mu \mu_h^* + \tilde{r}_h \gtrless 0, \quad r_z^* = \tilde{r}_\mu \mu_z^* + \tilde{r}_z \gtrless 0, \quad r_\phi^* = \tilde{r}_\mu \mu_\phi^* > 0 \tag{A15}$$

From (A12) and (A14), it follows that the equilibrium quantity of rental housing supplied in the market is increasing in k. It can also be shown that it is decreasing in the fixed cost of entry, ϕ.

Section 4.1: Rent Control

We assume that the rental price $\bar{r}$ imposed is such that $\bar{r} < r^*(c_o, k, h, z, \phi)$. Given the properties of the expected revenue per rental unit, $R_r > 0$ at $r = \bar{r}$. A supplier's first-order condition on n given a maximal rental price $\bar{r}$ is given by

$$R(\bar{r}, \mu; c_o, k, h, z) - hC'(n) = 0$$

which yields $\bar{n}(\bar{r}, \mu; c_o, k, h, z)$ where $\bar{n}_j = \tilde{n}_j$ for $j = \mu, c_o, k, h, z$ and $\bar{n}_{\bar{r}} = R_r/(hC''(\bar{n})) > 0$.

The supplier's maximized expected profit is given by

$$\bar{\Pi}(\bar{r}, \mu, c_o, k, h, z) = \bar{n}(\cdot)R(\bar{r}, \mu; c_o, k, h, z) - hC(\bar{n}(\cdot))$$

where $\bar{\Pi}_{\bar{r}} = \bar{n}R_r > 0$ and $\bar{\Pi}_j = \Pi_j$ for $j = \mu, c_o, k, h, z$.

The equilibrium with excess demand and a fixed number of firms, denoted by m, with rent control can be represented by the following condition:

$$(1-\bar{\mu}^f)D(\bar{r},\bar{\mu}^f,c_o,k,h) - m\bar{n}(\bar{r},\bar{\mu}^f;c_o,k,h,z) = 0$$

which yields $\bar{\mu}^f(m,\bar{r},c_o,k,h,z)$ where

$$\begin{aligned}
&\bar{\mu}^f_m = -\frac{\bar{n}}{\Phi'} < 0, \quad \bar{\mu}^f_{\bar{r}} = \frac{(1-\mu)D_{\bar{r}} - m\bar{n}_{\bar{r}}}{\Phi'} < 0, \quad \bar{\mu}^f_{c_o} = \frac{(1-\mu)D_{c_o} - m\tilde{n}_{c_o}}{\Phi'} \gtrless 0 \\
&\bar{\mu}^f_k = \frac{(1-\mu)D_k - m\bar{n}_k}{\Phi'} > 0, \quad \bar{\mu}^f_h = \frac{(1-\mu)D_h - m\bar{n}_h}{\Phi'} > 0, \quad \bar{\mu}^f_z = \frac{-m\bar{n}_z}{\Phi'} > 0
\end{aligned} \tag{A16}$$

where

$$\Phi' = D - (1-\mu)D_\mu + m\bar{n}_\mu > 0.$$

The condition determining the equilibrium number of suppliers is given by

$$\bar{\Pi}(\bar{\mu}^f(m,\bar{r},c_o,k,h,z);c_o,k,h,z) = \phi$$

which yields $\bar{m}^*(\bar{r},c_o,k,h,z,\phi)$. Differentiating the above condition, we obtain

$$\begin{aligned}
&\bar{m}^*_{\bar{r}} = \frac{\bar{\Pi}_\mu\bar{\mu}^f_{\bar{r}} + \bar{\Pi}_{\bar{r}}}{\Psi'} \gtrless 0, \quad \bar{m}^*_{c_o} = \frac{\bar{\Pi}_\mu\bar{\mu}^f_{c_o} + \bar{\Pi}_{c_o}}{\Psi'} \gtrless 0, \quad \bar{m}^*_k = \frac{\bar{\Pi}_\mu\bar{\mu}^f_k + \bar{\Pi}_k}{\Psi'} > 0, \\
&\bar{m}^*_h = \frac{\bar{\Pi}_\mu\bar{\mu}^f_h + \bar{\Pi}_h}{\Psi'} \gtrless 0, \quad \bar{m}^*_z = \frac{\bar{\Pi}_\mu\bar{\mu}^f_z + \bar{\Pi}_z}{\Psi'} \gtrless 0, \quad \bar{m}^*_\phi = -\frac{1}{\Psi'} < 0
\end{aligned} \tag{A17}$$

where $\Psi' = -\bar{\Pi}_\mu\bar{\mu}^f_m > 0$

The equilibrium number of suppliers $\bar{m}^*(\bar{r},c_o,k,h,z,\phi)$ determines the proportion of demand not being met in equilibrium given by $\bar{\mu}^*(\bar{r},c_o,k,h,z,\phi) = \bar{\mu}^f(\bar{m}^*(\cdot),c_o,k,h,z)$ which in turn determines the equilibrium number of rental units each supplier produces, $\bar{n}^*(\bar{r},c_o,k,h,z,\phi) = \bar{n}(\bar{r},\bar{\mu}^*(\cdot);c_o,k,h,z)$ and the equilibrium demand for rental housing $\bar{D}^*(\bar{r},c_o,k,h,z,\phi) = D(\bar{r},\bar{\mu}^*(\cdot);c_o,k,h)$.

Totally differentiating the equilibrium proportion of demand not being met in equilibrium yields

$$\begin{aligned}
&\bar{\mu}^*_{\bar{r}} = \bar{\mu}^f_m\bar{m}^*_{\bar{r}} + \bar{\mu}^f_{\bar{r}} = \frac{-\bar{\Pi}_{\bar{r}}}{\bar{\Pi}_\mu} < 0, \quad \bar{\mu}^*_{c_o} = \bar{\mu}^f_m\bar{m}^*_{c_o} + \bar{\mu}^f_{c_o} = \frac{-\bar{\Pi}_{c_o}}{\bar{\Pi}_\mu} > 0 \\
&\bar{\mu}^*_k = \bar{\mu}^f_m\bar{m}^*_k + \bar{\mu}^f_k = \frac{-\bar{\Pi}_k}{\bar{\Pi}_\mu} < 0, \quad \bar{\mu}^*_h = \bar{\mu}^f_m\bar{m}^*_h + \bar{\mu}^f_h = \frac{-\bar{\Pi}_h}{\bar{\Pi}_\mu} > 0 \\
&\bar{\mu}^*_z = \bar{\mu}^f_m\bar{m}^*_z + \bar{\mu}^f_z = \frac{-\bar{\Pi}_z}{\bar{\Pi}_\mu} > 0, \quad \bar{\mu}^*_\phi = \bar{\mu}^f_m\bar{m}^*_\phi > 0
\end{aligned} \tag{A18}$$

Therefore, we have Result 2.

It follows from the above expressions for $\bar{n}_{\bar{r}}$ and $\bar{\Pi}_{\bar{r}}$, and from $(A8)$ and $(A18)$ that $\bar{n}^*$ does not change with $\bar{r}$, i.e. $\bar{n}^*_{\bar{r}} = \bar{n}_{\bar{r}} + \bar{n}_{\bar{\mu}}\bar{\mu}^*_{\bar{r}} = 0$. From (A17), $\bar{m}^*$ may increase or decrease in $\bar{r}$. This implies that the total supply of rental housing could be increasing or decreasing in the maximal rental ceiling.

Section 5: Social Welfare

Social welfare is given by

$$W(r,\mu;h) = N\int_{\underline{y}}^{\hat{y}}(u(h_o)+y)dG + N\int_{\hat{y}}^{\overline{y}}(1-\mu)\left[F(\hat{s})u(h_o)+(1-F(\hat{s}))(u(h)-r)\right]dG$$
$$+ N\int_{\hat{y}}^{\overline{y}}\left[\mu u(h_o)+y-k\right]dG$$

By the definition of $\hat{y}$ given by (4), the change in social welfare with respect to $\hat{y}$ is equal to zero,

$$\frac{\partial W}{\partial \hat{y}} = -N\left[(1-\mu)\left(1-F\left(\frac{c_o+r}{\hat{y}}\right)\right)[u(h)-r-u(h_o)]-k\right]g(\hat{y}) = 0.$$

Consider now the partial effect of a change in the rental price r and the probability of not finding rental housing μ. We have

$$\frac{\partial W(r,\mu;h)}{\partial r} = -(1-\mu)N\int_{\hat{y}}^{\overline{y}}\left[(1-F(\hat{s}))+f(\hat{s})(1/y)(u(h)-r-u(h_o))\right]dG < 0 \qquad \text{(A19)}$$

$$\frac{\partial W(r,\mu;h)}{\partial \mu} = -N\int_{\hat{y}}^{\overline{y}}(1-F(\hat{s}))\left[u(h)-r-u(h_o)\right]dG < 0 \qquad \text{(A20)}$$

An increase in r or μ reduces social welfare. From Result 1, we know that an increase in h or z will increase μ and thus reduce social welfare. The impact on the equilibrium rental rate r^* of an increase in either h or z is ambiguous and therefore we are unable to determine the indirect effect of a change in h or z on social welfare. We can also derive the direct effect of policies on social welfare.

$$\frac{\partial W}{\partial h} = N\int_{\hat{y}}^{\underline{y}}(1-F(\hat{s}))u'(h)dG > 0 \qquad \text{(A21)}$$

$$\frac{\partial W}{\partial z} = 0 \qquad \text{(A22)}$$

Given the above analysis, it is clear that we will be unable to say anything conclusive about the total welfare effects of a change in policies when the economy is in an equilibrium with excess demand. Therefore, we rely on numerical examples to illustrate these comparative statics.