Centre Interuniversitaire sur le Risque, les Politiques Économiques et l'Emploi

Cahier de recherche/Working Paper **08-06**

On the Decomposition of Polarization Indices: Illustrations with Chinese and Nigerian Household Surveys

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Mai/May 2008

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I am grateful to the PEP programme of IDRC for financial support. Also, I thank Jean-Yves Duclos and Sami Bibi for their valuable comments and Erwin Corong for his valuable assistance.

Abstract:
This paper explores the link between polarization and inequality and proposes some analytical methods to decompose the Duclos, Esteban, and Ray (2004) polarization index by population groups or income sources. In some cases, the decomposition methods were extended to the Esteban and Ray (1994) one. The main aim of these decomposition methods is to extend the interpretation derived from polarization indices to that of contribution components. Results drawn from Chinese data conclude that even if inequality has increased sharply during the last two decades, the pure polarization component was remained constant or even decreased on average. On the other hand, results from the 2004 Nigerian survey conclude that the population is spatially polarized, and this, based on geo-ecological zones. Furthermore, the two income sources, namely, *Employment income* and *Non farm business income*, significantly contribute to total polarization.

Keywords: Polarization, Equity, Inequality, Decomposition

JEL Classification: D63, D64

1 Introduction

In addition to macroeconomic performance criteria, social economic performance has received growing interest from researchers and policymakers during the last few decades. Indeed, there is a consensus on the importance of studying the negative aspects of distribution of income and of taking into account social dimensions in the design of economic policies. In general, governmental interventions are necessary to boost the main and strategic economic sectors during times of crises; to correct the distributive failure of free market; and to ensure a worthy level of wellbeing to population and to prevent the socioeconomic conflicts.

Broadly, polarization refers to the disparity or divergence between objects, attracted by different forces toward the main masses or narrowly, the grouped objects. Such phenomenon can be transposed into socioeconomic framework. With polarization measurement, we aim to assess the importance of the more apparent social groups, which are composed of individuals that share the same socioeconomic characteristics. When interests of these apparent groups are different from each other, they can act to defend their interests and may thus lead to the so-called *social conflict*. The more groups are identified or their opposite interests are higher, the greater is the intensity of conflict. For distributive analysis, we simply focus on the importance of polarization in incomes[1]. Thus, this reduces our space to an important socioeconomic dimension, which is the distribution of wellbeing. Esteban and Ray (1994) argues about the relevance of studying polarization by what follows:

> "Why are we interested in polarization? It is our contention that the phenomenon of polarization is closely linked to the generation of tensions, to the possibilities of articulated rebellion and revolt, and to the existence of social unrest." (Esteban and Ray 1994, p. 820)

In general, there are tangible forces or factors that can explain the level of divergence between clustered groups. For usual distributions in developing countries, incomes of the poor and richer groups are attracted by two opposite forces. These forces are explained mainly by the set of individual characteristics - demographic, social, economic, etc - which determine the capacity of each individual to generate income. By limiting our focus on the two main opposite groups, situated

[1] Among the other dimensions of polarization that have intersected researches, one can cite the ethnic and religious aspects. For instance, see Reynal-Querol (2002) and Horowitz (1985).

in general at the bottom and top of the distribution of income, we redefine our polarization concept as bipolarisation[2].

The theoretical frameworks for inequality and poverty were extensively and widely developed in the last few decades compared to that of polarization. Furthermore, there is an evident link between polarization and some other negative aspects of the distribution. For instance, the severe poverty, the disappearing of middle class or the higher level of between-group inequality are certainly related to the polarization phenomenon. While inequality measurements are conceived to assess the expected divergence or disparity between incomes, polarization measurements are sensitive to the level of identification of individuals by their income levels. For a given population group, which is predetermined by an income range, the higher the population size of this group, the more the group is identified. Thus, polarization measurements are sensitive to the concentration of individuals around their expected income. This gives additional information that inequality and poverty measurements fail to catch.

During the last two decades, there has been a growing interest to develop a theoretical framework to assess accurately polarization through income levels. Pioneered by Esteban and Ray (1991) and Esteban and Ray (1994), Foster and Wolfson (1992) and Wolfson (1994), they have proposed some measurements of polarization in the distribution of incomes. An important and recent contribution was the work of Duclos, Esteban, and Ray (2004) (DER). The DER index is distinguished by its axiomatic approach and it obeys to a set of desirable properties for polarization measurements. Compared to Esteban and Ray (1994) polarization index (ER), the latter:

- is defined in continues form in income;
- does not require to fix arbitrarily given income ranges to identify groups;
- is normalized by population size and can be normalized by income scale.

Obviously, the normalizations by income scale and population size are fundamental to enable the comparison between distributions for any distributive phenomenon, like poverty, inequality or polarization.

The main part of this paper is devoted to decompose polarization measurements by population groups or income sources. Showing how the different population groups or income sources contribute to polarization index can help policymakers to implement policies to fight the drawbacks underlining the development

[2]Some polarization measurements are based on this bipolarisation concept. See Wolfson (1994).

process. Moreover, we investigate the empirical link between polarization and inequality and highlight the difference between them. To this end, we propose a pure polarization index to extract the contribution of the more identified groups.

The rest of the paper is organized as follows. In section 2, we present the DER polarization index and we show how polarization is different from inequality by deriving the pure popularization index. In section 3, we propose an analytical decomposition of the DER index by population groups. The proposed decomposition by income sources is presented in section 4. We illustrate, in section 5, the proposed developments and we conclude in section 6.

2 Polarization, concept and measurements

The popular polarization measurements are based on two important ingredients: the *alienation* and the *identification*, denoted by A and I respectively. The greater the population concentration of a given group within a small range of income, the higher the identification the group members feel. On the other hand, the greater the disparity of income between two individuals, the higher the alienation they feel. When two groups are well identified and their average incomes are more distant, the society may be frustrated with the apparent gap. Thus, in contrast with inequality, polarization indices are characterized by their local sensitivity to the interaction between the alienation and identification, which forms the so called *antagonism*. In general, one can divide the polarization axioms or the desirable properties into two subsets. The first subset is common for the majority of distributive indices, such as:

- A1: *The Anonymity*: Distributive indices do not depend on the individual characteristics except its income.
- A2: *Population-principle*: Distributive indices are invariant with the increase of population size by replicating it.
- A3: *Free scale*: Distributive indices are invariant with income scale.

The second subset is more specific to polarization indices. Let the local unimodal distribution be a part of the global distribution with different averages of income:

- A4: Collapsing the global distribution around its mean does not increase polarization (*Alienation sensitivity*).

A5: Collapsing the local unimodal distribution around its mean does not decrease polarization (*Identification sensitivity*).

A6: Increasing the distance, by the same level, between incomes which form the local distribution and average income does not decrease polarization (*Interaction sensitivity*).

2.1 The DER polarization index

The Duclos, Esteban, and Ray (2004) polarization index have a functional form derived exclusively with an axiomatic approach and that is close to the functional form of the Gini index to capture the alienation part. Precisely, the DER index can be written as follows:

$$P = \int \int f(x)^{1+\alpha} f(y)|x - y|dydx, \tag{1}$$

where $f(.)$ is the density function and the parameter α is a normative parameter that expresses the sensitivity of the index to the local identification[3]. This parameter must be bounded and is between 0.25 and 1. This interval is mainly derived to respect the IA interaction structure and to have an optimal tradeoff for the sensitivity of this index between the alienation and identification components[4]. Under the simplest algebric form, one can say that the contribution of the group, composed from individuals with income x, is the product of its identification component $i(x) = f(x)^\alpha$, multiplied by its alienation one $a(x) = \int f(y)|x - y|dy$.

The alienation component, which expresses simply the expected absolute distance between income x and the other incomes, can be decomposed into two more comprehensible components. Indeed, one can check easily that:

$$|y - x| = (y - x)_+ + (x - y)_+, \tag{2}$$

where $(\varepsilon)_+ = \varepsilon$ if $\varepsilon > 0$ and zero otherwise. According to Runciman (1966), the magnitude of relative deprivation is the difference between the desired situation and the actual situation of a person[5]. We define the relative deprivation of household with income x compared to that with income y as follows:

[3]The local identification at income x depends on the proportion of individuals that have the level of this income. The higher this proportion is higher, the greater is the identification component $f(x)^\alpha$.

[4]Ideally, the functional form must ensure that the index will not be biased or determined mainly by one of the two IA components.

[5]See also Yitzhaki (1979) and Hey and Lambert (1980).

$$\tau(x, y) = (y - x)_+ = \begin{cases} y - x & if \quad x < y \\ 0 & otherwise. \end{cases} \tag{3}$$

Hence, the expected deprivation of individual with income x is equal to:

$$\delta(x) = \int \tau(x, y) f(y) dy. \tag{4}$$

Similarly, the expected surplus of individual with income x is equal to:

$$\sigma(x) = \int \tau(y, x) f(y) dy. \tag{5}$$

Thus, one can decompose the local alienation into expected deprivation plus expected surplus components.

$$a(x) = \delta(x) + \sigma(x). \tag{6}$$

By replacing (6) in equation (1), we find that:

$$\begin{aligned} P &= \int f(x)^{1+\alpha} a(x) dx & (7) \\ &= \int f(x)^{1+\alpha} \left[\delta(x) + \sigma(x)\right] dx & (8) \\ &= D + S, & (9) \end{aligned}$$

where $D = \int f(x)^{1+\alpha} \left[\delta(x)\right] dx$ is the deprivation component and the complement part S is the surplus. When the distribution is symmetric or when the parameter α equals zero, these two components are equal. In general, with the usual asymmetric distribution of incomes, one can expect that $D > S$ [6]. The normalized DER index can be written as follows:

$$P = \int f(\dot{x})^{1+\alpha} \left[\delta(\dot{x}) + \sigma(\dot{x})\right] d\dot{x}, \tag{10}$$

where $\dot{x} = x/\mu$. As indicated by the Duclos, Esteban, and Ray (2004), to obtain the free scale index, one has to divide the absolute polarization index by $\mu.\mu^{-\alpha}$. Indeed, the part $\mu^{-\alpha}$ comes from the relative change between $f(x)^\alpha$ and $f(\dot{x})^\alpha$ considering $f(\dot{x})^\alpha = f(x)^\alpha/\mu^\alpha$. We note again that for the normalized DER, one can divide the latter by 2 to set its value between zero and one.

[6] Esteban and Ray (1994) and Duclos, Esteban, and Ray (2004) indicated the asymmetric importance of alienation to assess polarization. This can be explained by the level of the social ethical weight, which decreases with the increase in income.

2.2 The DER polarization index and inequality

Recall that the Gini index represents a particular case of the DER, i.e., when the parameter $\alpha = 0$. Using equation (10), one can express the inequality with the expected deprivation and surplus as follows[7]:

$$A = \int f(\dot{x}) \left(\delta(\dot{x}) + \sigma(\dot{x})\right) d\dot{x} \tag{11}$$

It is easy to check that the expected deprivation equals the expected surplus at the level of population ($E[\delta(x)] = E[\sigma(x)]$). When the parameter α is greater than zero, we weight the expected local deprivation and surplus by the local identification component ($f(x)^{\alpha}$). Except for the case where the distribution is symmetric, with this process, we give more weight to the poor individuals, which are in general more identified. In addition to the parameter α, the divergence between the DER index and inequality, measured by the Gini index, depends on the level of asymmetry of the distribution. The other popular tool to represent and to study inequality is the Lorenz curve. When one notes the usual Lorenz curve by $L(p)$ and the quantile by $Q(p)$, the component $a(Q(p))$ can be written as follows:

$$a(Q(p)) = \mu \left[2(1 - L(p)) + p\varsigma(p) - (1-p)\varsigma(p) - 1\right], \tag{12}$$

where $\varsigma(p) = Q(p)/\mu$[8]. Based on this, the DER index can be written as follows:

$$P \;=\; 2\int f(L'(p))^{\alpha} \left[(1 - L(p)) - 0.5(1 + L'(p)) - pL'(p))\right] dp \tag{13}$$

When the parameter $\alpha = 0$, it is straightforward to find the usual definition of the Gini index with the Lorenz curve[9]. With the usual asymmetric density functions, the surplus component -$S = \int_0^1 f(\varsigma(p))^{\alpha} \left[2p\varsigma(p) - \varsigma(p) - 1)\right] dp$- diverges from the deprivation one. This divergence is caused, inter alia, by the difference in levels of identification at the different segments of percentiles. The lower the tangency of the Lorenz curve $L'(p)$ is (this occurs in general in the lower tail of

[7]See Sen (1973), Yitzhaki (1979) and Araar and Duclos (2003).

[8]The deprivation for an individual with income $Q(p)$ is $\delta(Q(p)) = \mu - GL(p) - Q(p)(1-p)$ and surplus is $\sigma(Q(p)) = \mu - GL(p) + pQ(p) - \mu$, where $GL(p)$ is simply the generalized Lorenz curve.

[9]Note that $\int(2p\varsigma(p) - \varsigma(p) - 1)dp = 1 - \int 2L(p)dp$, or simply equals to the Gini index. This result can be derived using the integration by part, i.e. $V = p$ and $U' = \varsigma(p)$. Recall again that the DER index is twice the Gini when $\alpha = 0$.

the distribution), the higher is the local identification and the more skewed the distribution is[10].

Even if the difference between inequality and polarization was already well discussed and illustrated in the literature, some empirical studies were devoted to check for the empirical correlation between them[11]. Results of these studies confirm such correlation when some polarization indices are used. The question that can now be raised is: how one can explain this correlation? Recall that most of polarization indices are sensitive to alienation, as is the case for inequality indices, and this even if the identification part is ignored. The magnitude of divergence between the trend of polarization and that of inequality depends on the importance of change in the identification component. To show this in clear way, one can recall the other form of the DER index as proposed by Duclos, Esteban, and Ray (2004).

$$P = AI(\alpha)[1+\rho], \tag{14}$$

where $I(\alpha) = \int f(y)^{1+\alpha}dy$ is the average identification component and the parameter ρ is the normalized covariance between alienation and identification, which equals to:

$$\rho = \frac{\int (a(x) - E[a])\,(f(x)^{\alpha} - E[f^{\alpha}])\,f(x)dx}{AI}. \tag{15}$$

If the significant change in inequality occurs with a neglected effect on local identification components, this decomposition shows why inequality and polarization can follows the same path[12]. However, this finding cannot confirm the neglected importance of identification to the polarization index. By their functional form, polarization indices may be more sensitive to alienation than to identification components.

By *pure polarization* concept, we refer to polarization of the more identified groups. Assume that $\bar{f}$ is the identification threshold, which is simply the minimum level of identification to separate between the more identified groups and the others. Under this assumption, the new polarization index is defined as follows:

$$P^* = \int f^*(x)^{1+\alpha}a(x)dx \tag{16}$$

where $f^*(x) = f(x)$ if $f(x) > \bar{f}$ and zero otherwise.

[10]Note that when $L'(p)$ is low, the increase in the ranked incomes is small and this implies a higher level of $f(L'(p) = Q(p)/\mu)$ and high level of identification.

[11]See Ravallion and Chen (1997) and Zhang and Kanbur (2001).

[12]In general, an increase in inequality involves a positive change of the parameter ρ and a neglected change in average identification component.

Theorem 1 *For any given distribution, there is a unique equivalent distribution, which satisfies simultaneously the following constraints:*

A: Have the same level of inequality, measured by the Gini index;

B: Have the same average income;

C: Have a uniform distribution and incomes can be defined by: $y_i = a + bi$, where y_i is the i_{th} ranked income, in ascending order.

Proof. Easily, one has to resolve a system of two equations with two parameters. The first equation refers to the equality between averages of income, i.e. $a + b\int ydy - \mu = 0$. The second refers to the equality between the Gini's social welfare, such that: $\int(a+by)\upsilon(y)dy - \xi_A = 0$, where $\upsilon(y)$ is a function of social ethical weights and $\xi_A = \int y\upsilon(y)dy$. ■

This equivalent distribution can be perceived as a counterfactual distribution, with the minimum level of identified groups with the prevailed level of inequality. One can check easily that for the uniform distribution, the link between the Gini and DER index is as follows:

$$P^u = (E[f(.)]\mu)^\alpha A. \tag{17}$$

The density $f(.)$ is the same for all incomes and is equal to the inverse of population size when the distance between two successive incomes, for the uniform distribution, is scaled to one. This level of density may be the standard for threshold identification, up to which the apparent groups are detected, as indicated previously. If one assumes that the uniform distribution is the initial one, the pure polarization can be perceived as the impact of the transformation of this uniform distribution to the more polarized. If one focuses simply on this side of polarization, he has to add the following axiom, which pure polarization measurements must fulfill.

A7: *Concentration focusing axiom*: The pure polarization measurements are insensitive to the distribution of the less identified groups, i.e. $(f(x) < \bar{f})$.

3 Decomposing the DER index by population groups

Before going into the development of the decomposition of the DER index by group components, one has to keep in mind the difference between the main objectives of this decomposition. Indeed, questions of interest can be addressed as follows:

1. How population groups contribute to total polarization?
2. How population groups can explain polarization?
3. What are the main masses -modals of the distribution- that attract each population group?

For instance, for the first question, if we assume that all groups have identical distributions of income, but differ by their population size, the relative contribution of each group must depend simply on its population share (See the illustration of this case in figure (1)) . For the second question, if we assume that there is no income overlap, the retained groups represent well the different masses of the distribution (See the illustration of this case in figure (2))[13]. In this case, by their average incomes and population shares, groups can reproduce the distribution of income. For the third question, assume that groups are formed according to household head education attainment. The two main masses that can attract mainly the illiterate group will be those situated at the bottom of the distribution (for instance, they can be the two first modals of distribution, presented in figure (3)).

In chronic way, the decomposition of the Gini index by population groups was pioneered by Bhattacharaya and Mahalanobis (1967). There was a growing interest to develop well founded methods for decomposition of this inequality index by sub-population groups [14]. Assume that the population is composed of the G exclusive population groups (rural vs urban households or group households according to household head education attainment, etc.). In general, when we suppose that the between-group inequality represents the inequality when each household has the average income of its group, the algebraic decomposition of the Gini index takes the following form[15]:

$$A = \sum_{g} \phi_g \psi_g A_g + \bar{A} + R, \tag{18}$$

where ϕ_g and ψ_g are respectively the population and income shares of group g. The component $\bar{A}$ is the between group inequality and equals simply to the Gini index when each individual have the average income of its group. One can recall

[13]Gradin (2000) used the generalized ER index (1999) and has proposed an indicator to assess how the formed population groups reproduce polarization in distribution of incomes.

[14]See Pyatt (1976), Lambert and Aronson (1993) and Araar (2006).

[15]See also the interpretation of Lambert and Aronson (1993).

here that if group incomes do not overlap, the residual part of this decomposition (R) vanishes. The lower the residual of the relative contribution is, the more significant the explanation of the formed population to polarization. In this sense, the decomposition of the Gini index may be useful to shed light on this aspect and one can use the indicator R/A to assess the explanatory power of the formed groups to polarization.

Now we turn to the decomposition of the DER index by population groups. If one notes the density function for group g by f_g and based on equation (8), the contribution of individual(s) with income x to the DER index is as follows:

$$c(x) \quad = \quad \frac{a(x)f(x)^{1+\alpha}}{\mu^{1-\alpha}}. \tag{19}$$

The alienation component $a(x)$ for the individual with income x belonging to group g can be decomposed as follows:

$$a(x) \quad = \phi_g a_g(x) + \tilde{a}_g(x), \tag{20}$$

where $a_g(x)$ is the alienation for the individual at the level of its group g and $\tilde{a}_g(x)$ the alienation component at population level when the within-group alienation is ignored. Let $\pi_g(x)$ denotes the local proportion of individuals belonging to group g and having income x. Let again $c_g(x)$ denotes the local contribution of group g with income x to the DER polarization index[16]:

$$c_g(x) \quad = \quad \pi_g(x)f(x)^\alpha \frac{f(x)a(x)}{\mu^{1-\alpha}} \tag{21}$$

$$= \quad \frac{\mu_g^{1-\alpha}}{\mu^{1-\alpha}} \left[\frac{\pi_g(x)\phi_g a_g(x)f(x)^{1+\alpha}}{\mu_g^{1-\alpha}} \right] + \frac{\pi_g(x)\tilde{a}(x)f(x)^{1+\alpha}}{\mu^{1-\alpha}} \tag{22}$$

$$= \quad \phi_g^\alpha \psi_g^{1-\alpha} \left[\frac{\pi_g(x)a_g(x)f(x)^{1+\alpha}}{\mu_g^{1-\alpha}} \right] + \frac{\pi_g(x)\tilde{a}(x)f(x)^{1+\alpha}}{\mu^{1-\alpha}}. \tag{23}$$

Based on these local group contributions, one can write the DER index as follows:

$$P = \sum_g \int c_g(x)dx. \tag{24}$$

[16]With this decomposition, the local contribution of the group to the local identification is attributed linearly, i.e. $i_g(x) = \pi_g(x)f(x)^\alpha$. One can propose more refined forms of attributions, but one must maintain the equality: $\sum i_g(x) = i(x)$. With the use of household surveys, the impact of such approximation is insignificant, since it is rare to find the same level of incomes for two samples of households.

Hence, the DER index can be decomposed as follows:

$$P = \underbrace{\sum_g \phi_g^{1+\alpha}\psi_g^{1-\alpha} R_g P_g}_{W} + \underbrace{\tilde{P}}_{B}, \quad (25)$$

where

$$R_g = \frac{\int a_g(x)\pi_g(x)f(x)^{1+\alpha}dx}{\phi_g \int a_g(x)f_g(x)^{1+\alpha}dx}, \quad (26)$$

and $\tilde{P}$ is the DER polarization index when the within-group polarization or inequality is ignored. If group incomes do not overlap, ($\pi_g(x) = 1 \; \forall x$), the component R_g equals simply to 1. In general, in addition to the population share, the component R_g depends again on the correlation between the density function of the group and that of the population. One can check easily that for the special case, when $\alpha = 0$, this decomposition is similar to that of the Gini index[17]. Let $\bar{P}$ denote the between-group polarization component, which is simply the polarization index when we suppose that the within-group polarization is suppressed. This is equivalent to assuming that each individual have the average income of its group. The functional form of $\bar{P}$ is practically similar to that proposed by Esteban and Ray (1994) and can be defined as follows:

$$\bar{P} = \sum_g \phi_g^{1+\alpha} a(\dot{\mu}_g), \quad (27)$$

where $\dot{\mu}_g = \frac{\mu_g}{\mu}$ and $a(\dot{\mu}_g) = \sum_h \phi_h |\dot{\mu}_g - \dot{\mu}_h|$. Based on the analytical approach for the decomposition of the Gini index, suppressing the alienation at group levels makes the between-group component lower than the Gini index. However, with polarization indices, this process has two opposite effects. It is true that the alienation is reduced, but the local identification increases in general with this process and it can happen that $\bar{P} > P$. When we attribute the between-group polarization to that implied by the interpersonal alienation between groups, the component B equals the between-group polarization component, as shown in equation (25) [18]. The indicator $(1 - W/P)$ shows how much groups are locally polarized. Perfect identification of groups and lower local polarization coincide, in general, with the

[17]Note that when the parameter $\alpha = 0$, we have $R_g = 1$. In addition, the residue can be attributed to the between-group component when the latter is based on interpersonal comparison between groups.

[18]See Dagum (1997) and Araar (2006) for the case of the decomposition of the Gini index.

higher relative contribution of the between-group component to the polarization index. The indicator B/W can be used to show how much the formed groups polarize the distribution[19]. Now we turn to the third main objective of decomposition by group population, which is to identify the main masses that attract each group. Starting from equations (8) and (24), one can write the following:

$$P = \sum_g \frac{1}{\mu^{1-\alpha}} \int \left([\delta(x) + \sigma(x)] f(x)^{\alpha} \right) \pi_g(x) f(x) d(x) dx \quad (28)$$

$$= \sum_g D_g + S_g. \quad (29)$$

If group g is composed of a significant part of poor individuals, the ratio $D_g/S_g > 0$ will be relatively higher than that of the other groups. In such a case, the group is more attracted by masses situates in the lower tail of the distribution. The conclusion is the inverse for the wealthy groups.

4 Decomposition of the DER index by income components

The decomposition of polarization indices by income sources allows having a clear idea on how each source contributes to the total polarization. First, suppose that the sum of K income sources equals the total income and the amount of source k, noted by s_k, is non negative. To develop the intuition of such decomposition and to impose some desired rules, one can argue the following:

1. If distributions of income sources are similar, the relative contribution of each source must simply be equal to its income share.
2. Assume that there are only two similar income sources. Increasing polarization in a given source, by a series of polarization transformations, must imply a higher contribution of this source to total polarization.

For the decomposition of the Gini index, a straightforward method was proposed by Rao (1969). With Rao's method, the contribution of each income source equals

[19] Note that the idea of using the ratio between the between-group and the within-group components of inequality to catch the importance of polarization of groups was already proposed by Zhang and Kanbur (2001). Precisely, they use the generalized entropy index of inequality to estimate the within and between-group inequality components.

to the product between its income share and its coefficient of concentration. Starting from equation (6), one can write the following:

$$\begin{aligned} a(x) &= \int [\tau(x,y) + \tau(y,x)] f(y)dy \\ &= \int \left[\tau(\sum_k s_k(x), \sum_k s_k(y)) + \tau(\sum_k s_k(y), \sum_k s_k(x)) \right] f(y)dy \\ &= \sum_k \int [(s_k(x) - s_k(y))I(x < y) + (s_k(y) - s_k(x))I(y < x)] f(y)dy \\ &= \sum_k a_k(x). \end{aligned} \tag{30}$$

The term $a_k(x)$ gives the level of alienation in income source k for the individual with income x, but conditioned by the validity of deprivation using total incomes $I(x < y)$ and the validity of surplus $I(y < x)$. By replacing this finding in equation (7), one can find that:

$$P = \frac{1}{\mu^{1-\alpha}} \int f(x)^{1+\alpha} a(x)dx \tag{31}$$

$$= \sum_k \psi_k \frac{\int f(x)^{1+\alpha} a_k(x)dx}{\psi_k^\alpha \mu_k^{\alpha-1}} \tag{32}$$

$$= \sum_k \psi_k CP_k \tag{33}$$

where CP_k is the pseudo-polarization index of income source k. It is similar to the concentration index except for its sensitivity to income identification component, $f(x)^\alpha$. One can remark that if the parameter α equals zero, this decomposition is the same as that of Rao[20]. This analytical development shows that the contribution of each income source mainly depends on its income share and on its pseudo polarization index. The more equally distributed is the income source, the lower is its pseudo polarization index. Furthermore, for a given level of the parameter α, one can check for the progressivity of income source under polarization context. First, let the usual transformation of incomes to quantiles be $F^{-1}(x) = Q(p)$, where $F()$ is simply the cumulative density function. One can compare between

[20] With term ψ_k^α in the denominator, the pseudo-polarization index is free scale and easily comparable with the DER polarization index. This enables us to judge the direction of the contribution simply by checking the sign of $(CP_k - P)$.

the Cumulative-Polarization Curve -$CC(p)$- and the Concentration-Polarization curve -$CC_k(p)$- to check for the progressivity of income sources[21]:

$$CC(p) = \frac{1}{\mu^{1-\alpha}} \int_0^p f(Q(p))^\alpha a(Q(p))dp, \tag{34}$$

$$CC_k(p) = \frac{1}{\psi_k^\alpha \mu_k^{\alpha-1}} \int_0^p f(Q(p))^\alpha a_k(Q(p))dp. \tag{35}$$

For the analytical issue, it will be useful to assess the impact of the marginal increase in income source k on the polarization. First, assume that $\lambda_k - 1 \geq 0$, denotes the proportion of increase in income source k, i.e. $x(\lambda_k) = (x - s_k) + \lambda_k s_k$. The impact of this increase on the DER index is as follows[22]:

$$\left.\frac{\partial P(\lambda_k)}{\partial \lambda_k}\right|_{\lambda_k=1} = \frac{\int \left([\alpha f(x)^{\alpha-1} g(x) s_k(x) - (1-\alpha)\psi_k f(x)^\alpha] a(x) + f(x)^\alpha a_k(x)\right) f(x)dx}{\mu^{1-\alpha}}, \tag{36}$$

where $g(x)$ denotes the first derivative of the density function at x. If income source is simply a constant, the impact can be written as follows:

$$\frac{\partial P(c)}{\partial c} = (1 + c/\mu)^{\alpha-1} P \tag{37}$$

5 Illustration Using Chinese and Nigerian Data

To illustrate how the evolution of polarization can differ from that of inequality, we illustrate this using a panel survey from rural China. This panel survey was conducted during China's economic transition and covers the period 1986-2002. Recent descriptive research emphasizes the marked decline in the incidence of poverty over the twenty-five years of economic reform in China accompanied by an increase in inequality[23]. The data comes from annual household surveys conducted by the Survey Department of the Research Center on Rural Economy

[21]Here, one must recall that the cumulative polarization curve was proposed by El Lahga (2005).

[22]See Appendix 1 for the prove.

[23]For instance, see Benjamin, Brandt, Giles, and Wang (2005) and Khan (2004).

(RCRE) in Beijing. We use household level surveys from 82 villages in nine provinces (Anhui, Gansu, Guangdong, Henan, Hunan, Jiangsu, Jilin, Shanxi, and Sichuan) where households were surveyed annually from 1986 through 2002, with gaps in 1992 and 1994 when funding difficulties prevented survey activities.[24] In each province, counties in the upper, middle and lower income terciles were selected, from which a village was then randomly chosen. Depending on village size, between 40 and 120 households were randomly surveyed in each village. The panel component of the household survey (from panel villages) includes 3983 households per year from 1986 to 2002. Our measure of consumption includes nondurable goods expenditure per capita. All income and expenditure are deflated to 1990 prices using the NBS rural consumer price index for each province.

One additional advantage of using panel data is to prevent our estimates from unnecessary noise which may occur as a result of the changes in the structure of the sample. This allows us to focus more on the nature of evolvement of the different distributive phenomena.

In figure (4), we show the trend of inequality and polarization, measured respectively by the Gini and DER indices, when the parameter of identification $\alpha = 0.5$. As noted in earlier work and confirmed with this figure, inequality and polarization seems to be highly correlated. As reported previously, this correlation may be implied by the biased weight effect of alienation on polarization indices. Figure (5) shows the trends of alienation -Gini index- and average identification components. It is clear that population groups became less identified with income distribution in China during the last years. To show how polarization can evolve differently from inequality, we present in figure (6) the proposed pure polarization index. Starting form results reported in this figure, it seems that inequality and polarization have evolved differently. More important, the pure part of polarization has decreased in average during this studied period. This result is not surprising since poverty in China has significantly decreased during this period and this must lighten the level of polarization.

Now, to illustrate the proposed methods for the decomposition of the polarization index, we use a recent and rich nationally representative survey of Nigerian households (NLSS). This was carried out between September 2003 and August

[24]These 82 villages are a subsample of the 110 villages originally surveyed in 1986 in which survey administrators successfully followed a significant share of households through 2002. The complete RCRE survey covers over 22,000 households in 300 villages in 31 provinces and administrative regions. RCRE's complete national survey is 31 percent of the annual size of the NBS rural household survey. By agreement, we have obtained access to data from nine provinces, or roughly one-third of the RCRE survey.

2004 by Nigeria's National Bureau of Statistics. Nigeria is the most populous country in Africa. In addition, it is one of the very few countries in Africa where income is surveyed, and this is of course important in motivating our choice of this particular country. Out of the 22,200 households that were randomly selected in the NLSS, we remove those that did not report their income components. Therefore, 17764 observations were used for this application. We use *per capita* total household income as a measure of living standards. Household observations are weighted by household size and sampling weights.

In table (1), we perform the decomposition of total polarization (DER with $\alpha = 0.75$) by population groups. The exclusive population groups are formed according to the six main geopolitical zones of Nigeria[25]. First, one can recall that many studies have found that the northern zones are relatively poorer than the southern zones. Through the reported results in table (1), one can note that even if groups differ by their population size or income share, their level of polarization is relatively higher and the northern zones has practically the same level of polarization. The north-west region has the highest level of the ratio deficit/surplus and contributes the most to the intra-group component. The contribution of the between-group polarization is higher and this seems to confirm that Nigerian population is spatially polarized in wealth. One can report again the pure between-group polarization, $\bar{P}$ equals to 0.063. Its lowest level indicates the non neglected impact of reducing the within-group disparities on inequality, poverty and polarization[26].

To show how each income source contribute to the total polarization, we use six main income sources. As shown in table (2), the components *Employment income* and *Non farm business income* contribute more to total polarization, and this, for two levels of parameter of identification (α=0.25 and $\alpha = 0.75$). The pseudo-polarization index for the *Agricultural income* component is the lowest. Indeed, this phenomenon of lower disparity in agricultural incomes is confirmed by many empirical studies concerning developing countries.

6 Conclusion

Inequality and polarization phenomena continue to receive a growing interest from researchers and policymakers. There is a consensus on the importance of maintaining an adequate level of equity in the distribution of wealth in developing

[25]These geopolitical zones cut across the three main agro-ecological zones of the country.

[26]See Araar and Awoyemi (2006) and Araar and Duclos (2007).

and developed countries. Indeed, for developing countries, stakeholders advocate the reduction of the alarming disparities to enhance the impact of economic growth on poverty. For developed countries, there is a consensus on the necessity of ensuring a minimum level of equity to combat the social exclusion of a large part of the population or the disappearance of the middle class.

The theoretical framework for inequality measurement or its decomposition by the distributive components was widely and extensively developed during the last century. The proposed developments for polarization are recent and seem to be less extended in comparison to those for inequality and poverty. In general, polarization measurements of income are characterized by their sensitivities to the concentration of population groups meaning income levels. Assessing the level of polarization is not sufficient to propose adequate policies to fight this distributive phenomenon. In this paper we provide some theoretical developments to decompose the DER polarization index by population groups or income sources. In our knowledge, there is no other work devoted to the decomposition of polarization indices by distributive components. For policy purpose, these decompositions may help to target the apparent poor groups of population or to propose efficient redistributive corrections. Also, in this paper we focus on the empirical and analytical links between inequality and polarization and we propose the pure polarization index with the new axiom concerning the focus of pure polarization measurements on the apparent or identified groups.

Using the Chinese panel data, which cover the rural area between the years 1986 to 2002, we have shown a close link between trends of inequality and polarization. More importantly, we have shown that the pure polarization component has evolved differently with inequality. To illustrate the proposed theoretical framework for the decomposition of polarization by population groups or income sources, we have used the Nigerian household survey for the year of 2004. Results from these decompositions confirm the spatial polarization of the Nigerian society. Starting from results of decomposition by income sources, we conclude that components *Employment income* and *Non farm business income* contribute more to total polarization in Nigeria.

Note finally that the proposed theoretical developments are made to contribute to enrich the existing distributive tools, which are usually used to help policy-makers in shaping policies designed to fight simultaneously the different negative aspects of distribution. In addition, analytical developments may contribute to shed light on the different aspects that surround the polarization concept and to attract the interest of researchers to continue to investigate and study the polarization phenomenon.

Figure 1: Global and local symmetric unimodal distribution

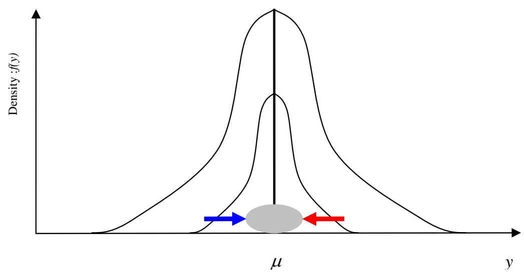

Figure 2: Pure bimodal distribution

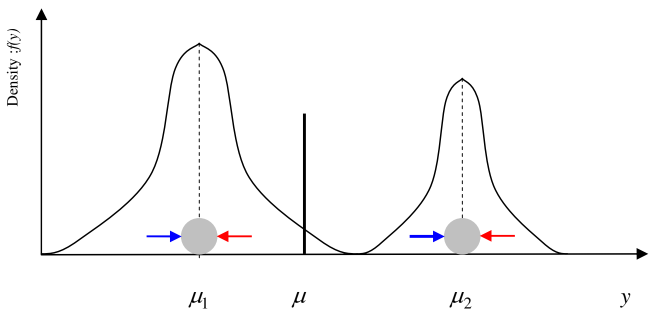

Figure 3: Global multi-modal distribution

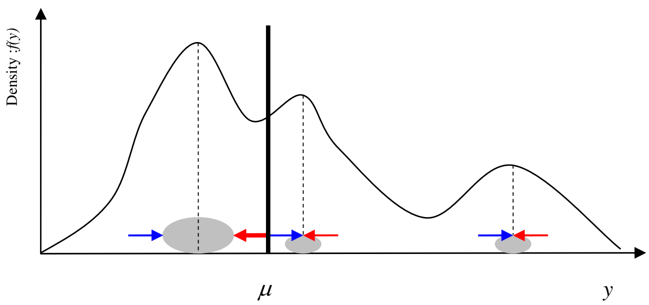

Figure 4: Inequality and polarization trend in China
(Rural China: 1986-2002)

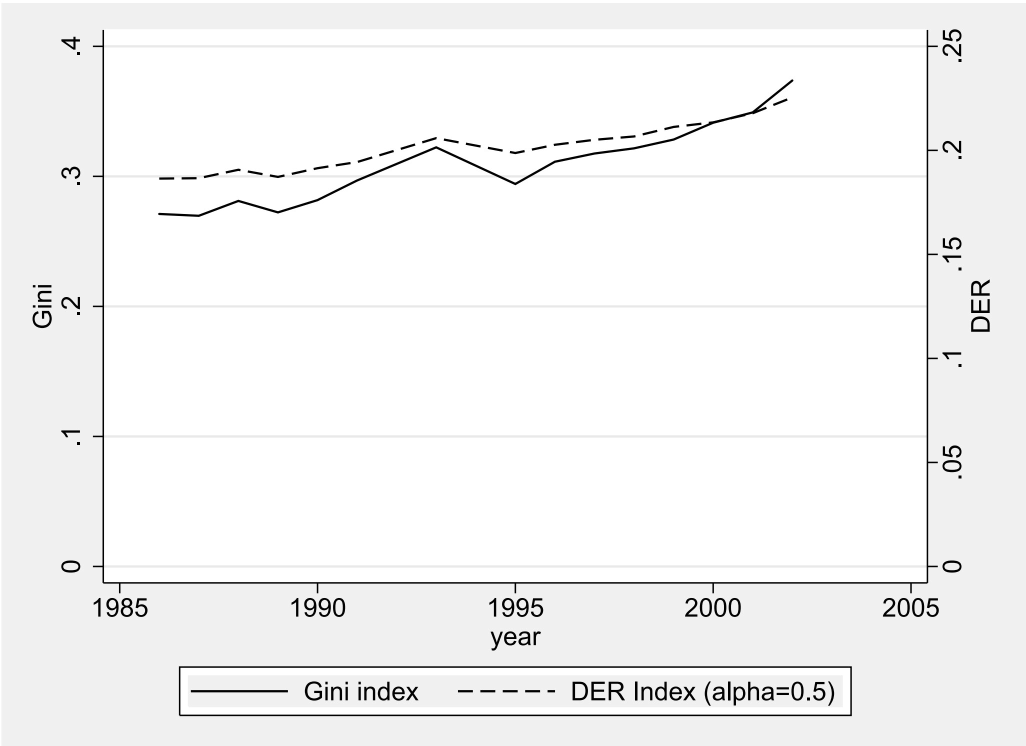

Figure 5: The trend of identification and alienation components
(Rural China: 1986-2002)

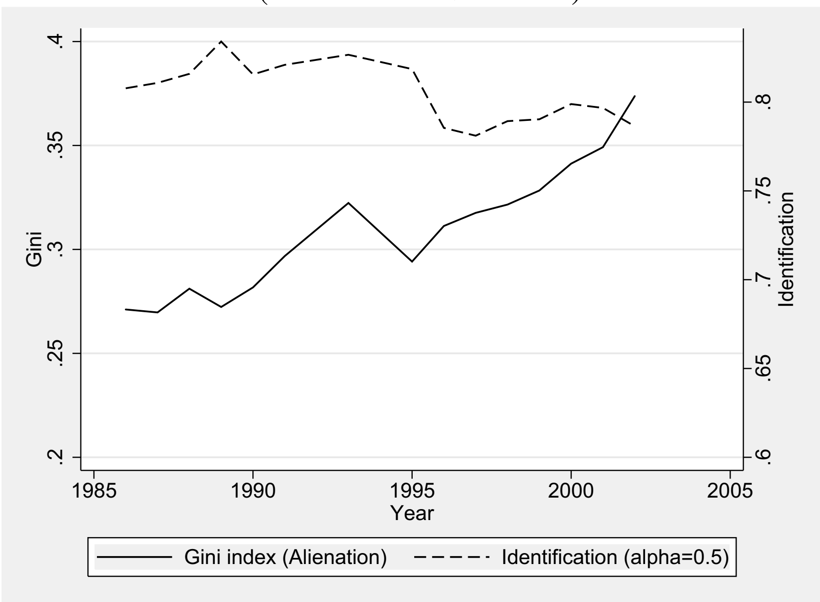

Figure 6: Inequality and pure polarization index
(Rural China: 1986-2002)

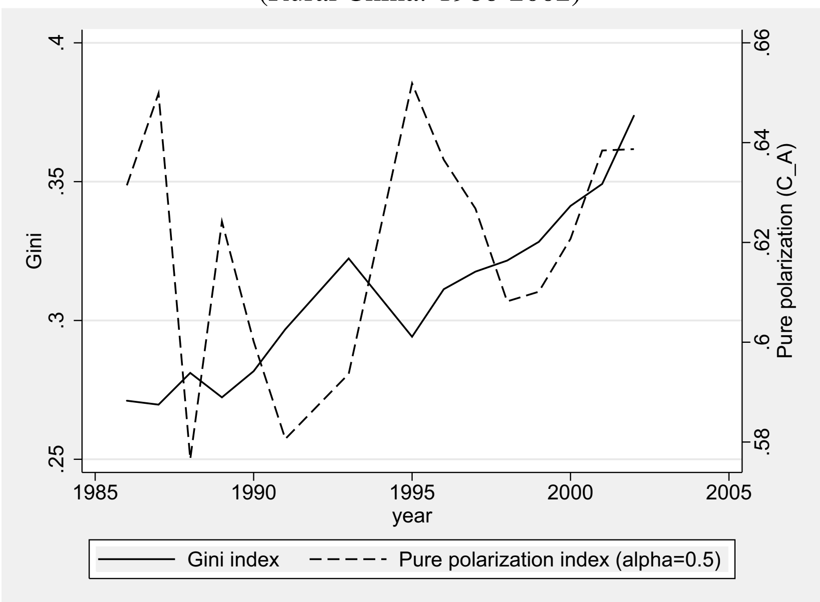

Table 1: Decomposition of the DER polarization index according to the Nigeria's geopolitical zones ($\alpha = 0.75$)

Group g	ϕ_g	ψ_g	P_g	R_g	D	S	D/S	AC^{*}	RC^{**}
North-central	0.139	0.105	0.304	0.910	0.041	0.006	6.637	0.005	0.016
North-east	0.135	0.094	0.309	0.884	0.041	0.006	7.386	0.005	0.015
North-west	0.263	0.157	0.287	0.858	0.082	0.011	7.531	0.015	0.048
South-east	0.119	0.117	0.335	0.953	0.032	0.006	5.625	0.004	0.015
South-south	0.150	0.193	0.300	1.075	0.033	0.009	3.658	0.008	0.025
South-west	0.194	0.333	0.243	1.153	0.028	0.014	1.980	0.012	0.039
Within-group	—	—	—-	—-	—	—	—	0.049	0.158
Between-group***	—	—	—-	—-	—	—	—	0.260	0.842
Nigeria	1.000	1.000	—-	—-	0.257	0.051	5.039	0.309	1.000

* AC : The absolute contribution.

** RC : The relative contribution.

*** The pure between-group polarization $\bar{P}(\alpha = 0.75)$ is equals to 0.063.

Table 2: Decomposition of polarization index according to income sources (Nigeria 2004)

Parameter α		$\alpha = 0.25$			$\alpha = 0.75$		
Component k	ψ_k	CP_k	AC_k^{*}	RC_k^{**}	CP_k	AC_k^{*}	RC_k^{**}
Employment income	0.353	0.497	0.175	0.406	0.337	0.119	0.386
Agricultural income	0.200	0.268	0.054	0.124	0.226	0.045	0.146
Fish-processing income	0.024	0.406	0.010	0.022	0.304	0.007	0.023
Non-farm business income	0.344	0.468	0.161	0.373	0.330	0.114	0.368
Remittances received	0.025	0.407	0.010	0.023	0.302	0.007	0.024
All other income	0.055	0.404	0.022	0.051	0.294	0.016	0.052
Total	1.000	—	0.432	1.000	—	0.309	1.000

* AC_k : The absolute contribution of income component k.

** RC_k : The relative contribution of income component k.

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Appendix 1 Marginal increase in income source component and the DER index

Recall that with the scheme of proportional increase in income source k, the income $x(\lambda_k)$ can be defined as follows:

$$x(\lambda_k) = x + (\lambda_k - 1)s_k(x). \tag{A.1}$$

The average of $x(\lambda_k)$ is give in its turn by

$$\mu(\lambda_k) = \mu(1 + (\lambda_k - 1)\psi_k). \tag{A.2}$$

In discrete form, the DER index being defined as:

$$P(\lambda_k) = \frac{\sum_i f(x_i(\lambda_k))^\alpha a_i(\lambda_k)}{N\mu(\lambda_k)^{1-\alpha}} \tag{A.3}$$

Where N denotes the population size. Using the *chain rule* method of derivative, we have inter alia, $\frac{\partial f(x_i(\lambda_k))^\alpha}{\partial \lambda_k} = \alpha f(x_i)^{\alpha-1}g(x_i)s_k(x_i)$, where $g(x_i)$ denotes the first derivative of density function with respect to x_i. Further, when incomes are ranked in ascending order, one can find that $\frac{\partial a_i(\lambda_k)}{\partial \lambda_k} = a_{i,k}$, where $a_{i,k}$ have the same definition as a_i except the change of arguments, i.e. x_i by $s_k(x_i)$. If we note the numerator and the denominator of the last equation by A and B respectively, we have then:

$$\left.\frac{\partial P(\lambda_k)}{\partial \lambda_k}\right|_{\lambda_k=1} = \frac{A'}{B} - \frac{B'A}{B^2} \tag{A.4}$$

where

$$\frac{A'}{B} = \alpha\frac{\sum_i f(x_i)^{\alpha-1}g(x_i)s_k(x_i)a_i}{N\mu^{1-\alpha}} + \frac{\sum_i f(x_i)^\alpha a_{i,k}}{N\mu^{1-\alpha}} \tag{A.5}$$

$$= \psi_k(Z_k + CP_k) \tag{A.6}$$

Values of density function and its derivative can be estimated using the Gaussian kernel method. For the other side of equation, we have:

$$\frac{B'A}{B^2} = (1-\alpha)\psi_k\frac{\sum_i f(x_i)^\alpha a_i}{N\mu^{1-\alpha}} \tag{A.7}$$

$$= (1-\alpha)\psi_k P \tag{A.8}$$

Hence, one can write the impact as follows:

$$\left.\frac{\partial P(\lambda_k)}{\partial \lambda_k}\right|_{\lambda_k=1} = \frac{\sum_i [\alpha f(x_i)^{\alpha-1} g(x_i) s_k(x_i) - (1-\alpha)\psi_k f(x_i)^{\alpha}] a_i + f(x_i)^{\alpha} a_{i,k}}{N\mu^{1-\alpha}} \quad \text{(A.9)}$$

The impact in continues form is then:

$$\left.\frac{\partial P(\lambda_k)}{\partial \lambda_k}\right|_{\lambda_k=1} = \frac{\int \left([\alpha f(x)^{\alpha-1} g(x) s_k(x) - (1-\alpha)\psi_k f(x)^{\alpha}] a(x) + f(x)^{\alpha} a_k(x)\right) f(x) dx}{\mu^{1-\alpha}}, \quad \text{(A.10)}$$

and in more compact format:

$$\left.\frac{\partial P(\lambda_k)}{\partial \lambda_k}\right|_{\lambda_k=1} = \psi_k \left[Z_k + CP_k - (1-\alpha)P\right] \quad \text{(A.11)}$$

Obviously, one can check easily that when $\alpha = 0$, the derived impact is similar to that with the Gini index, i.e. $\psi_k(C_k - A)$ where C_k is the concentration index of income source k.