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Trade and the External Wealth of Nations

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Résumé :

Dans la plupart des MEGC de commerce international, le solde du compte courant est exogène, conformément au point de vue largement répandu que la politique commerciale peut influencer les flux d'échanges, mais que le compte courant est contraint par le compte capital, qui lui est symétrique, sur lequel la politique commerciale a peu d'effet. Le modèle *MIRAGE-D* a été développé pour rendre explicites les flux internationaux de capital qui doivent avoir lieu pour équilibrer les implications sur le compte courant des flux d'échanges simulés, et pour compiler les conséquences cumulatives de tels flux de capital sur la position extérieure nette des pays. Dans *MIRAGE-D*, les soldes de compte courant et leur pendant, les soldes de compte capital, sont endogènes, suivant un modèle de gestion de portefeuille à trois niveaux, adapté de Decaluwé et Souissi (1994; Souissi, 1994; Souissi et Decaluwé, 1997), qui représente le comportement d'allocation de la richesse des agents-pays. L'allocation du capital entre pays et industries est déterminée par un mécanisme d'équilibrage de l'offre et de la demande d'investissement. L'offre d'investissement est la demande de nouveaux titres de propriété de capital physique qui découle du processus d'allocation de la richesse, tandis que la demande d'investissement est une fonction à élasticité constante du q de Tobin, à la Jung-Thorbecke (2001). Pour fins d'illustration, un scénario de simulation a été résolu en parallèle avec *MIRAGE-D* et avec la version standard de *MIRAGE*. À part la position extérieure nette des pays, que la version standard ne calcule pas, les autres résultats, bien qu'ils ne soient pas identiques, montrent des différences modérées, qui s'expliquent pleinement par les aspects financiers et découlent de la nécessaire cohérence entre ces aspects financiers et le reste du modèle.

Mots-clés : Modèles d'équilibre général concurrentiel, Position extérieure nette, Actifs financiers, Commerce international

Classification JEL : C68, D58, F17, F37, G11, G15

Abstract :

Most CGE trade models fix current account balances exogenously, in accordance with the widely accepted view that trade policy may influence trade flows, but that current accounts are constrained by symmetric capital account balances, on which trade policy has little effect. The *MIRAGE-D* model was developed to make explicit the international capital flows which must take place to balance the current account implications of the simulated trade flows, and to compute the cumulative consequences of such capital flows on the international investment positions (IIP) of countries. In *MIRAGE-D*, current account balances and their capital account counterparts are endogenous, following a three-tier portfolio management model, adapted from Decaluwé and Souissi (1994; Souissi, 1994; Souissi and Decaluwé, 1997), which represents country-agent wealth allocation behavior. The allocation of capital among countries and industries is determined by an investment supply and demand equilibrating mechanism. Investment supply is the demand for new physical capital ownership titles resulting from the wealth allocation process, while investment demand is a constant elasticity function of Tobin's q in the Jung-Thorbecke (2001) style. An illustrative simulation scenario was run with both *MIRAGE-D* and the standard version of *MIRAGE*. Apart from the IIP of countries, which the standard version does not produce, other simulation results, although not identical, show moderate differences, which are fully explained by the financial aspects, and arise from the consistency required between such financial aspects and the rest of the model.

Key words : CGE models, International Investment Position (IIP), Financial assets, International trade

JEL Codes : C68, D58, F17, F37, G11, G15

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Introduction

« Global imbalances » is shortcut for the United States current account deficit and its counterparts, East Asian surpluses and petrodollar recycling. For several years, growing global imbalances have claimed the attention of, among others, the International Monetary Fund (IMF). In its April 2007 *World Economic Outlook*, the Fund observed that « The persistence of global imbalances brings with it an important financial stability issue — the problem of sustaining the financing flows needed to support the imbalances. The April 2007 World Economic Outlook projects that imbalances are unlikely to fall much over the short term, and thus continued large cross-border net capital flows will be needed to finance current accounts at close to their present levels. This is clearly the case for the United States, which had an estimated current account deficit of \$848 billion, or 6.4 percent of GDP, in 2006. The rising dependence on fixed-income inflows to finance the U.S. current account deficit suggests that capital flows may have become more sensitive both to changes in world interest rate differentials and to expected exchange rate shifts » (IMF, 2007-04, p. 15).

Confronted with the same facts, the World Trade Organization (WTO) 2007 *World Trade Report* stated that « In part, larger current account imbalances reflect the impact of greater capital and financial market integration. A current account deficit reflects dissaving by domestic residents, an excess of absorption over income. The fact that it is occurring reflects a willingness by foreigners to finance that excess absorption by accumulating future claims on the earnings of domestic residents. It is important to emphasize that sustained imbalances are primarily a macroeconomic phenomenon and they have little to do with trade policy » (WTO 2007, p. 25-26). In other words, trade policy may influence trade flows, but current accounts are constrained by symmetric capital account balances, or imbalances, on which trade policy has little effect².

² In a speech given in 2005, Ben Bernanke goes so far as to consider that « one leading explanation for rising global imbalances traces it to an excess of savings in emerging markets (specifically East Asia) and the attractiveness of the United States as an investment destination, the depth and sophistication of US financial markets and the role of the dollar as leading international reserve currency » (Bernanke, 2005). Bernanke's position is in agreement with that implicit in many IMF analyses; for example, « Capital inflows not only reflect relatively low domestic saving and weak foreign domestic demand but also, more importantly, they are a consequence of global investors seeking the best risk-adjusted returns and diversification opportunities » (IMF 2006-09, p.34).

This view seems to be shared by the modeling community. WTO staff members Piermartini and Teh (2005) review several models that were used to perform simulations of the Uruguay and Doha rounds of multilateral trade negotiations. In their introductory presentation of CGE trade models, Piermartini and Teh discuss the issue of substitutability between imports and domestic products (the Armington hypothesis), its implications for terms of trade and, ultimately, for welfare gains expected from liberalization. But there is not a word on the implications of trade on current account balances. Even under « Model closure », no mention is made of hypotheses concerning current account balances.

In fact, most CGE trade models fix current account balances exogenously, in accordance with the WTO view. The initiators of the GTAP model, Hertel and Tsigas (1997, p.29), write : « The exogeneity of the current account balance embodies the notion that this balance is a macroeconomic, rather than microeconomic, phenomenon: to a great extent, the causality in [the savings-investment] identity [...] runs from the left side [savings minus investment] to the right side [current account surplus] ». Alternatively, « We exogenize the three key macroeconomic ratios: government spending, net national saving, and the trade balance, all relative to net national income » (Hertel et al., 2008, p. 6). Similarly, in the Harrison-Rutherford-Tarr model, « The current account balances the value of exports and imports taking into account exogenously-fixed capital inflows » (Harrison et al. 2002, p. 6; also see Rutherford and Tarr 2002). The same is true of van der Mensbrugge's LINKAGE model : « The value of foreign saving [...] is equal to some exogenous level multiplied by a world price. The world price is a price index of OECD manufactured exports and thus each unit of exogenous foreign saving is essentially equated to the purchase of an average unit of OECD manufactured exports » (van der Mensbrugge 2005, p. 33). Finally, in the Michigan Model of World Production and Trade, income « is determined by the sum of payments to factors plus net borrowing in the base period data set. This is equivalent to holding the change in the trade balance equal to zero. The trade balance is simply the difference between the value of exports and imports » (Equation derivations, p. 10³).

Some versions of the GTAP model, however, endogenize the current account balance : « Once the left-hand side of (2.15) [savings minus investment] is permitted to adjust, a mechanism is needed to ensure that the global demand for savings equals the global demand for investment in the postsolution equilibrium. The easiest way to do so is through the use of a "global bank" to assemble savings and disburse investment. [...] The global bank in the GTAP model uses receipts from the sale of a homogeneous savings commodity to the individual regional households in order to purchase (at price PSAVE) shares in a portfolio of regional investment goods. The size of this portfolio adjusts to

³ <http://www.fordschool.umich.edu/rsie/model/DERIVE.pdf>

accommodate changes in global savings. Therefore, the global closure in this model is neoclassical. However, on a regional basis, some adjustment in the mix of investment is permitted, thereby adding another dimension to the determination of investment in the model » (Hertel and Tsigas, 1997, p. 29).

This paper pursues in the same direction. It presents a CGE modeling methodology that makes it possible to explore *What if* scenarios of the evolution of trade and international finance. This is achieved by including financial assets in the model, together with a portfolio allocation mechanism, so that international saving flows, and hence current account balances, are endogenous.

The proposed approach is illustrated using a modified version of the *MIRAGE* model (Bchir *et al.*, 2002), nicknamed *MIRAGE-D* (D for « debt »), modified to track the evolution of international investment positions (IIP's; Lane and Milesi-Ferretti, 2006). *MIRAGE* is a multi-sector, multi-country CGE model of international trade developed at the *Centre d'Études Prospectives et d'Informations Internationales* (CEPII, Paris). The model is used extensively to simulate the consequences of various WTO and other international trade policy arrangements. Among other features, *MIRAGE* includes a sophisticated foreign direct investment (FDI) mechanism, closely related to the gravity model, well-known in economic geography (for details, see Lemelin and Decaluwé, 2007, p.40-48). The original *MIRAGE*, however, like other CGE trade models, does not track the evolution of countries' external debts.

It should be pointed out that, in spite of the fact that it is based on the fully operational *MIRAGE* model and uses real data, *MIRAGE-D* should be considered as a prototype, or as a work-in-progress.

The rest of the paper is organized as follows. Section 1 gives an overview of the model. Section 2 presents the portfolio allocation model. Section 3 discusses the investment allocation mechanism. Section 4 describes the implementation of the model. In section 5, simulation results illustrate the functioning of the model. Concluding remarks complete the article.

1. Overview of the model

Each country (or group of countries, depending on the aggregation chosen) in *MIRAGE* is modeled as a single agent. In the *MIRAGE-D* version, every country-agent owns a portfolio of assets which constitutes its net wealth. There are two types of wealth : financial wealth, and physical assets. The latter are ownership titles to productive capital or, equivalently, claims on the flow of income generated by the capital. The financial component of the portfolio is made up of assets and liabilities (debt). The asset-liability structure of the financial portfolio is endogenous, and it is possible for a country-agent to have negative net financial assets (liabilities in excess of assets). The possibility of borrowing is limited, however, by the willingness of other country-agents to lend, which reflects their own portfolio choices, and by the competition from other borrowing countries.

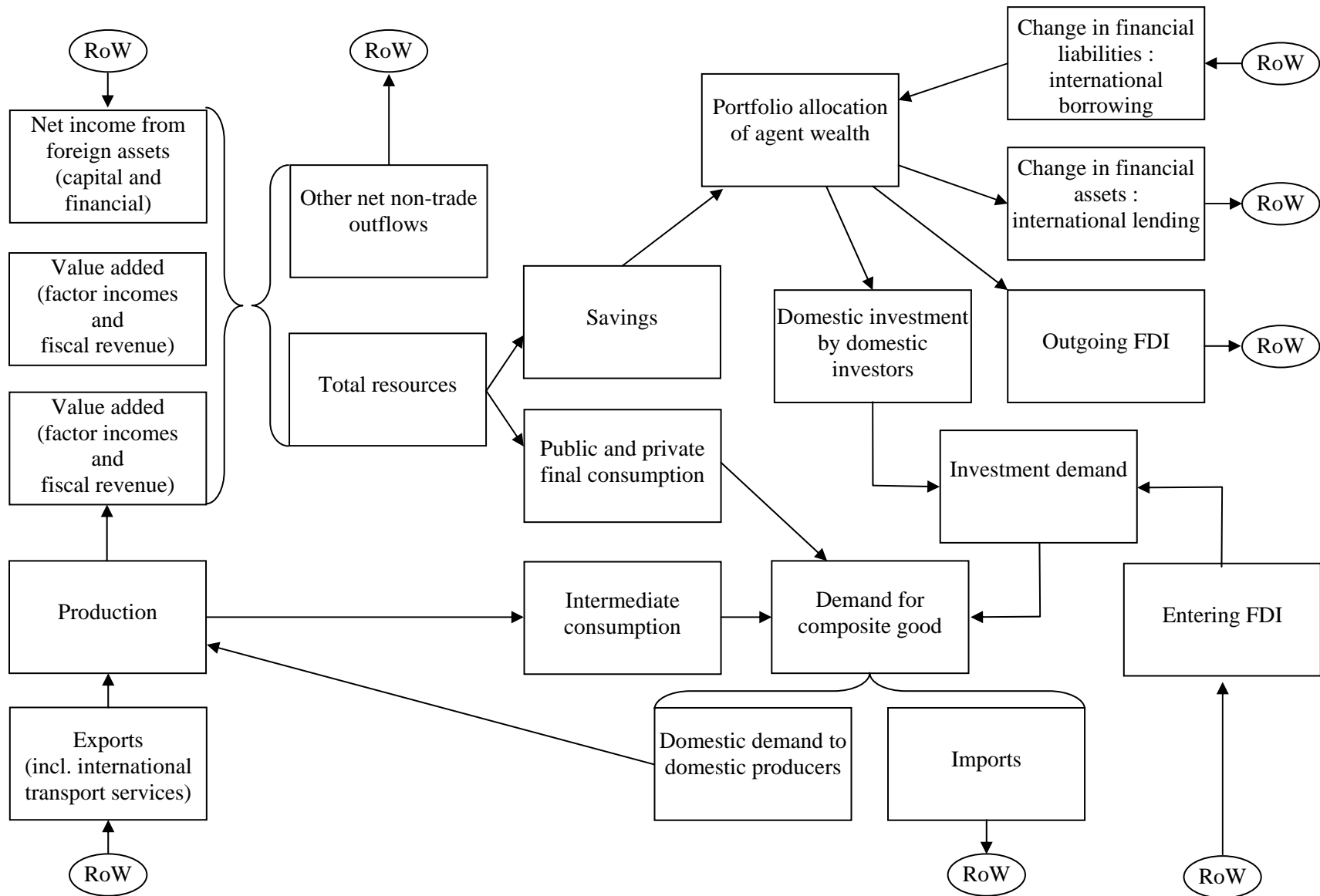
Figure 1 represents the structure of *MIRAGE-D* relative to a single country. In each period, factor incomes and fiscal revenue combine with income from assets owned abroad. After subtracting « Other net non-trade outflows »⁴, the remainder is the amount of resources allocated by the country-agent, in fixed shares, between consumption and savings. Savings, together with net assets inherited from the preceding period constitute the country-agent's net wealth. The portfolio allocation mechanism determines domestic and foreign direct investment, and changes in international financial assets and liabilities. The rest of the model follows the *MIRAGE* model structure and is relatively standard (for details, see Bchir et al., 2002).

Figure 2 presents the portfolio allocation mechanism. For reasons to be explained below, the value to be allocated is equal to net wealth (savings and wealth inherited from the preceding period), plus a credit margin. Each period, all of the portfolio is reallocated, in several stages :

- first, the portfolio is allocated between financial wealth and physical assets (capital ownership titles);
- the second stage on the physical assets side is the allocation of capital among countries and industries;
- on the financial side, the asset-liability structure of financial wealth is determined : a given increase (decrease) of the stock of financial wealth can be achieved as a combination of an increase (decrease) in assets and a decrease (increase) in liabilities;

⁴ These consist of flows not accounted for in the model : (net labor income paid to foreigners) + (net current transfers to foreign countries) + (net capital transfers to foreign countries) + (net acquisitions of non-produced nonfinancial assets). The definitions of these concepts can be found in the International Monetary Fund's *Balance of Payments Manual*, Fifth edition (1993). In *MIRAGE-D*, these flows are assumed to grow proportionately to world GDP.

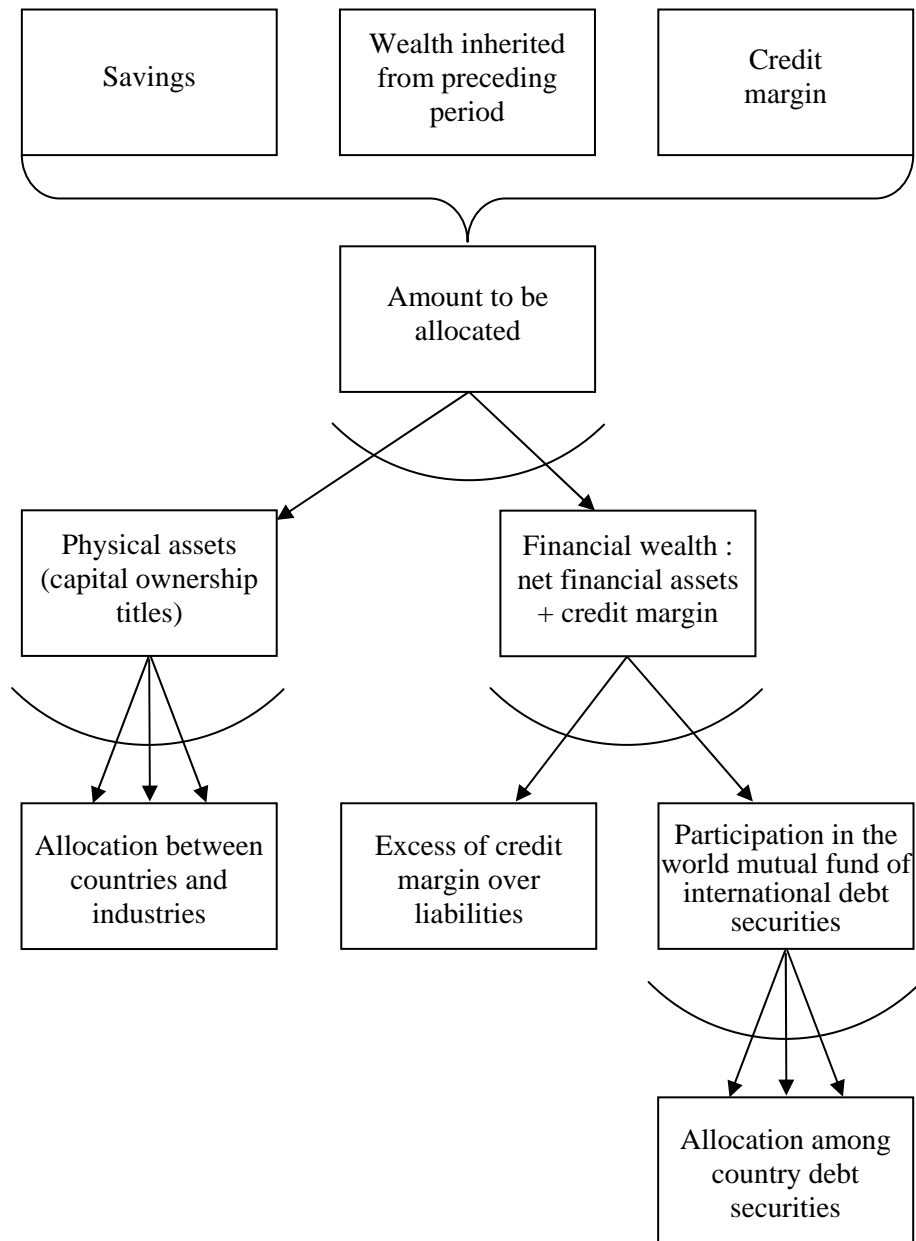
Figure 1 : Structure of the MIRAGE-D model



- there is a third stage on the financial side, where country-agent financial assets are pooled, and then distributed among country international debt securities⁵.

At every stage, portfolio allocation follows the Decaluwé-Souissi approach (1994; Souissi, 1994; Souissi and Decaluwé, 1997), described later.

Figure 2 : Portfolio allocation in the MIRAGE-D model



⁵ This is reminiscent of Hertel and Tsigas' (1997) global bank.

2. Portfolio management

This section first gives a general presentation of the Decaluwé-Souissi portfolio allocation model. Then it describes successively how the Decaluwé-Souissi model is applied to the different stages in the *MIRAGE-D* portfolio allocation mechanism, except for the allocation of physical capital between countries and industries, which is discussed in the following section.

2.1 GENERAL PRESENTATION OF THE DECALUWÉ-SOUISSI PORTFOLIO ALLOCATION MODEL

We agree with Souissi's (1994) criticism of the Rosensweig-Taylor (1990) model. In the latter, *income flows* from different assets are viewed as imperfect substitutes : a more appealing assumption, proposed by Souissi and Decaluwé, is that the assets themselves, rather than the income they generate, are not perfect substitutes (for a survey of portfolio management models in CGE models, and a further discussion of the Rosensweig-Taylor model, see Lemelin and Decaluwé, 2007, p.118-124). We begin with a presentation of the Decaluwé-Souissi portfolio allocation model in its general form.

The portfolio manager can acquire a unit of asset i in period t , for a price equal to q_i , and this will yield an investment income of $r_i q_i$, resulting in a capitalized value of $\xi_i = (1+r_i) q_i$.⁶

In each period, every portfolio manager maximizes the capitalized value of his/her wealth

$$\text{MAX}_{a_i} VC = \sum_i \xi_i a_i, \text{ where } \xi_i = (1+r_i)q_i \quad [001]$$

subject to diversification constraint⁷

$$W = A_w \left[\sum_i \alpha_i a_i^\beta \right]^{\frac{1}{\beta}} \quad [002]$$

with elasticity of transformation

$$\tau = \frac{1}{1-\beta} \quad (\beta > 1) \quad [003]$$

⁶ In *MIRAGE-D*, it is assumed that interest payments begin in the same period in which acquisition takes place. This is consistent with the *MIRAGE* time-structure, where current period investment is instantly productive. Also note that q_i here bears no relationship whatsoever with Tobin's « q », discussed later on.

⁷ In his statement of the model, Souissi describes that constraint as the wealth constraint. But A_w in [002] is not a parameter : it is a variable whose value depends on wealth accounting identity [004]. That is why we prefer to call [002] a diversification constraint. That aspect of the Decaluwé-Souissi model is discussed in more detail in Lemelin (2007b), especially in Appendix 2.

Clearly, the form of the utility function implies that all of the portfolio be reallocated in every period. It follows that wealth W consists of the value of assets owned at the end of period $t-1$, plus current savings.

Given the wealth accounting identity

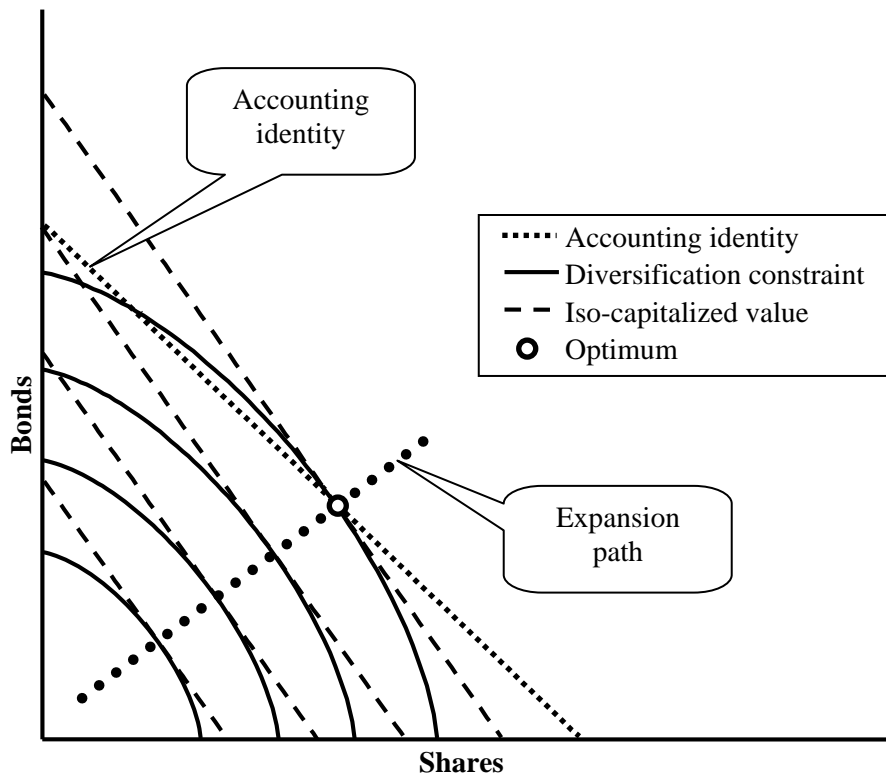
$$\sum_i q_i a_i = W \tag{004}$$

demand functions follow :

$$q_i a_i = W \frac{\alpha_i^\tau q_i \xi_i^{-\tau}}{\sum_j \alpha_j^\tau q_j \xi_j^{-\tau}} \tag{005}$$

The Decaluwé-Souissi portfolio allocation model is illustrated in the following diagram for the case of two assets, labeled « Bonds » and « Shares ».

Figure 3 – Portfolio allocation



Portfolio allocation equilibrium is located at the intersection of the expansion path and the wealth accounting identity constraint. The expansion path consists of the set of optimal asset combinations, for given return rates and different levels of wealth; for any optimal asset combination, the marginal rate of

transformation of the diversification constraint is equal to the slope of the iso-capitalized value line (whose equation is given by [001], with a constant value for VC).

We make a slight modification to the Decaluwé-Souissi portfolio allocation model : in our application, the diversification constraint is stated in terms of the *value* of different assets in the portfolio.

$$W = A_w \left[\sum_i \alpha_i (q_i a_i)^\beta \right]^{\frac{1}{\beta}} \quad [006]$$

The whole model can then be reformulated in terms of asset values. Let

$$b_i = q_i a_i \quad [007]$$

The Decaluwé-Souissi objective function can be written

$$\underset{b_i}{MAX} VC = \sum_i (1 + r_i) b_i \quad [008]$$

subject to

$$W = A_w \left[\sum_i \alpha_i b_i^\beta \right]^{\frac{1}{\beta}} \quad [009]$$

and the wealth accounting identity

$$\sum_i b_i = W \quad [010]$$

This leads to asset demand functions of the form

$$b_i = W \frac{\alpha_i^\tau (1 + r_i)^{-\tau}}{\sum_j \alpha_j^\tau (1 + r_j)^{-\tau}} \quad [011]$$

Asset demand is homogenous in wealth, and increasing in rate of return r_i . Portfolio proportions are given

by the ratio $\frac{b_i}{b_j} = \frac{\alpha_i^\tau (1 + r_i)^{-\tau}}{\alpha_j^\tau (1 + r_j)^{-\tau}}$: in order to persuade the portfolio manager to increase the proportion of

the i asset in the portfolio, the rate of return of the i asset must increase relative to others.

2.2 ALLOCATION OF WEALTH BETWEEN FINANCIAL WEALTH AND PHYSICAL ASSETS (CAPITAL)

The amount to be allocated by each country consists of net wealth inherited from the past (net value of international financial assets – assets, minus liabilities, and domestic and foreign physical assets – capital ownership titles), plus current savings, and, for reasons to be explained below, a credit margin. All of the

portfolio is reallocated each period, in several stages. The first stage allocates the portfolio between financial wealth and physical assets (capital), following the Decaluwé-Souissi model, reformulated in terms of the *value* of net financial wealth and physical assets. The corresponding return rates are weighted averages of the return rates of component assets of each form of wealth.

2.3 COUNTRY CREDIT MARGINS AND THE STRUCTURE OF COUNTRIES' EXTERNAL FINANCIAL WEALTH

Some countries have negative net financial wealth (they are net international debtors). This can happen even if a country has positive savings : for example, if investment expenditures, both domestic and abroad, have been in excess of savings, without incoming FDI being sufficient to compensate. But the Decaluwé-Souissi model as presented above cannot represent the allocation of a negative amount of net financial wealth.

Moreover, a country's net financial position (assets, minus liabilities) is obviously far more volatile than the underlying stocks of assets and liabilities, making a net position variable often unstable, and therefore difficult to model. So it would seem desirable to model assets and liabilities as distinct variables. But, once again, how can the Decaluwé-Souissi model accommodate negative asset values (liabilities)? The problem is illustrated in Figure 4 below, which is similar to Figure 3, but for the presence of liabilities.

The credit margin is a device to adapt the Decaluwé-Souissi model to handle liabilities. Negative liability variables are converted to positive variables by a simple shift of origin : rather than choosing the positive amount of assets and the negative amount of liabilities, subject to net financial wealth, the country portfolio manager chooses the positive amount of assets, and the – also positive – amount of his/her unused credit margin, subject to a constraint on the positive total of net financial-wealth-cum-credit-margin. The latter is defined as assets, plus the difference between the maximum amount the country is capable of borrowing, and the actual amount of its liabilities. This is illustrated in Figure 5 below.

Figure 4 – Asset-liability structure

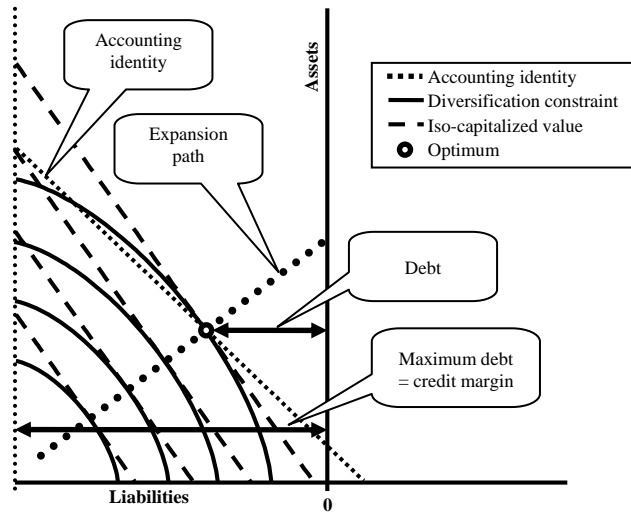
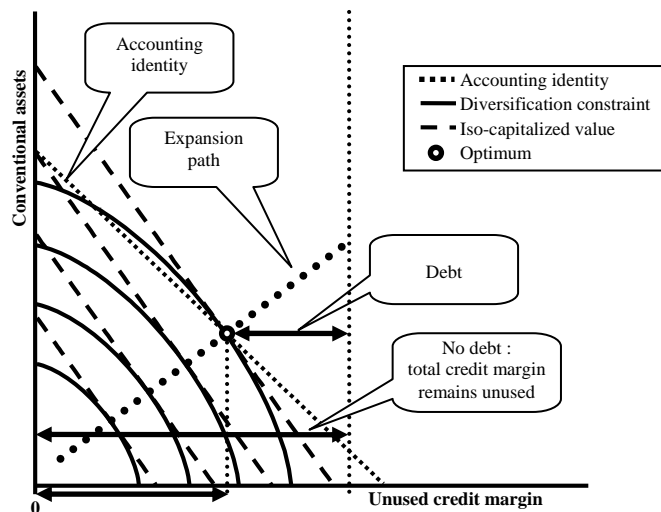


Figure 5 – Unused credit margin as an asset



So the financial wealth of country-agents is defined in *MIRAGE-D* as assets, minus liabilities, plus a credit margin. Financial wealth is allocated between (1) a composite asset, and (2) the surplus of the credit margin over liabilities, i.e. remaining borrowing capacity : debt reduction increases the maximum amount of new loans that could be contracted, and further borrowing reduces it. The rate of return on the composite asset is an aggregate of the interest rates on country debt securities (see below), while the rate of return on debt reduction is the opportunity cost of debt, i.e. the interest rate on a country's own debt. The asset-liability structure of each country's external financial wealth is determined by applying a Decaluwé-Souissi portfolio allocation model.

The credit margin has been arbitrarily set in the first period to equal the sum of assets and liabilities (in other words, each country is allowed to increase its debt by the amount of its assets). The credit margin is then assumed to grow in proportion to the world GDP. In spite of its simplicity, we believe that this formulation is not totally out of line with the reality of international financial markets : countries do have a total borrowing capacity, which usually exceeds their current level of debt. It is nonetheless recognized that, contrary to our specification, real total borrowing capacity is a fuzzy number, not an exact value. Moreover, the level of credit margins influences the values of the calibrated portfolio parameters, and consequently agents' behavior in the model. Therefore, setting credit margins at arbitrary levels as is done here can be acceptable only in the context of a prototype model : in the future, therefore, efforts should be dedicated to a more careful determination of these credit margins.

2.4 COUNTRY INTERNATIONAL DEBT SECURITIES

For simplicity, each country issues a single international debt security. Its total external debt is renegotiated in every period⁸. Individual country financial assets are pooled in what could be called a world mutual fund of international debt securities. Such a pooling mechanism, which is obviously a radical simplification of world financial markets, was made necessary for lack of credible complete data on bilateral debt. Moreover, it implies that the composite asset owned by each country includes portions of its own international debt, an incongruity which this author was unable to resolve. However, since the single agent representing each country in the model is really an abstraction from the large number of actual agents whose aggregate behavior it is meant to represent, perhaps the incongruity is not so great.

The world fund is allocated among individual country securities following, once again, a Decaluwé-Souissi portfolio model. So the interest rate on each country's security adjusts to clear the market : *ceteris paribus*, an increase (decrease) in the interest rate paid on a country's debt security is an incentive for the world debt portfolio manager to acquire more (less) of it.

The rate of return on the composite financial asset must be the same for all countries, since they all hold stakes in the same mutual fund. In reality, however, the observed ratio of foreign financial investment income to the value of foreign assets varies from country to country. So, in order for *MIRAGE-D* to reproduce the base-year data, the income of each country from its holdings of international debt securities

⁸ Redemption does not follow the term structures of bond issues, and past interest rates have no bearing on current transactions. This is completely different from what is proposed in Lemelin (2007b).

is modeled as a linear function of its assets, with the slope (marginal return) for all countries equal to the current equilibrium rate of interest, but with the intercept calibrated, specific to each country⁹.

3. Investment allocation mechanism

In *MIRAGE*, the owners of inherited physical capital are forced to keep it, except for the fraction that evaporates with depreciation. That restriction was maintained in *MIRAGE-D*. It could be seen as a way to characterize the difference between FDI and portfolio investments. This implies, however, that physical assets are not fully reallocated in each period among countries and industries : the portfolio allocation mechanism applies only to new physical assets, i.e. investments. In future developments, it would seem possible to do away with that restriction, and allow zero-base allocation of the total stock of physical assets, through buying and selling of industry and country-specific capital ownership titles, provided they are traded at their revised value (see below), and under the condition that markets clear. But for now, the process of allocating capital between countries and industries is limited to allocating investments.

Once wealth has been allocated between physical capital and financial wealth, the modeler has several options regarding the allocation of capital among countries and industries (regardless of whether the above restriction is enforced or not). This section first discusses the different modeling options, of which the one chosen for *MIRAGE-D* is to allocate investments through a supply and demand interaction mechanism combining industry investment demand together with a Decaluwé-Souissi allocation of *new* capital (investment supply). Next, the investment demand model is developed. Then, the supply and demand market clearing equilibrium is discussed. Finally, it is shown how the valuation of capital inherited from the preceding period is revised to be consistent with investment financing.

3.1 MODELING OPTIONS

One possible model is *MIRAGE*'s gravity-type distribution of investments. In accordance with *MIRAGE*'s restriction that the owners of inherited physical capital are forced to keep it, the gravity model allocates investment expenditures according to the rates of return of competing uses, net of taxes and depreciation, simultaneously determining domestic and foreign direct investment, by destination country and industry. In that case, the rate of return on the composite physical asset is an aggregate of country-specific industry rates of return.

⁹ In the model, interests paid by each country on its foreign debt are formally modeled in the same way. But in the current version of *MIRAGE-D*, all the intercepts of the interest payment functions are zero.

A second option is to replace the *MIRAGE* gravity model with a Decaluwé-Souissi allocation of *new* capital. As before, the rate of return on the composite physical asset is an aggregate of country-specific industry rates of return.

A third option is to introduce investment *demand* into the model. Investment demand models distribute investment among industries, given the total amount of investment expenditures. Investment-savings equilibrium is guaranteed by the interest rate, which plays the part of a discount rate and enters the determination of the user cost of capital : since the rate of interest is the same for all industries and countries, it follows that the rate of return for all capital owners is uniform across industries and countries, and is therefore equal to the rate of return on the composite physical asset.

A fourth option is a supply and demand interaction mechanism which combines investment demand together with, either the *MIRAGE* gravity model, or a Decaluwé-Souissi allocation of *new* capital. In this case, the rate of return to capital owners may vary between industries and countries. This is the option chosen for *MIRAGE-D* : the supply of new capital to countries and to industries within countries is determined by the interaction of a Decaluwé-Souissi portfolio allocation model of *new* capital with investment demand.

In what follows, we first describe investment demand. Then, under the assumption that investment is financed by issuing new shares (capital ownership titles), we explain how the demand for new shares follows the portfolio allocation model, and how the rate of return on new shares adjusts to equilibrate supply and demand. Finally, we show how old shares are revalued so that the market value of capital inherited from the preceding period is consistent with the investment financing mechanism.

3.2 INVESTMENT DEMAND

We shall now describe investment demand as a function of Tobin's « q », the ratio of the market value of capital to its replacement cost. The market value of capital is the present value of the income stream it is expected to generate. It is shown that, with myopic expectations regarding the future rental rate of capital, the ratio of market value to replacement cost is the ratio of the rental rate to the user cost of capital. These ratios are also equal to the ratio of the gross rate of return on capital, before depreciation, but net of taxes on capital income ($\rho_{i,s,t}$), to the rate of return required for the present value of the income stream generated by an investment to be equal to its cost (break-even rate of return $\zeta_{i,s,t}$).

The capital accumulation rule in *MIRAGE* is

$$K_{i,r,s,t} = (1 - \delta_{i,s})K_{i,r,s,t-1} + INV_{i,r,s,t} \quad [012]$$

where

$K_{i,r,s,t}$ is the quantity of capital owned by country-agent r in industry i of country s in period t ;¹⁰

$\delta_{i,s}$ is the rate of depreciation of capital in industry i of country s ;

$INV_{i,r,s,t}$ is the volume of investment by country-agent r in industry i of country s in period t .

Of course, the total stock of capital in industry i of country s is the sum over country-agent owners r :

$\sum_r K_{i,r,s,t}$. The total volume of investments in country s is the sum over industries of destination i and

country-agent investors r : $\sum_i \sum_r INV_{i,r,s,t}$. Total investment in country s is produced as a CES composite

of goods and services, at a cost $PK_{s,t}$ which reflects the prices of inputs; $PK_{s,t}$ is the replacement cost of capital in period t , and it is assumed to be the same for all industries i in a given country s .

Note that the time subscript of $INV_{i,r,s,t}$ in [012] is t , not $t-1$, since current investments add to productive capital in the current period : in *MIRAGE*, new investment is instantaneously productive¹¹. Our investment demand equation must take into account the time structure of *MIRAGE*.

Now let

$R_{i,s,t}$ be the rental rate of capital in industry i of country s in period t , net of capital income taxes;

$\delta_{i,s}$ be the rate of depreciation of capital in industry i of country s ;

$\phi_{i,s,t}$ be the market discount rate applied in period t .

It should be emphasized that $\phi_{i,s,t}$ is specific to each industry in each country : a different market discount rate is applied to every industry-country pair. Such variations could be interpreted as the market's valuation of industry and country risks.

Then, with myopic expectations regarding the future rental rate of capital, the present value $PV_{i,s,t}$ of the income stream generated by one unit of capital, beginning in current period t at $R_{i,s,t}$, and declining thereafter at a rate of $\delta_{i,s}$ per period, is equal to

¹⁰ The *MIRAGE* convention is that the first country index (in this case, r) is the country of origin, the exporting country, the investor country, or the owner country; the second index (s here) is the country of destination, the importing country, the country in which the investment takes place, or in which the capital is located.

¹¹ According to the *MIRAGE* model builders, this was adopted to eliminate the instability that could arise from investment overshooting.

$$PV_{i,s,t} = \sum_{\theta=0}^{\infty} \left(\frac{1 - \delta_{i,s}}{1 + \phi_{i,s,t}} \right)^{\theta} R_{i,s,t} = \frac{1 + \phi_{i,s,t}}{\phi_{i,s,t} + \delta_{i,s}} R_{i,s,t} \quad [013]$$

Present value $PV_{i,s,t}$ is therefore the market value of one unit of capital. Let

$$\zeta_{i,s,t} = \frac{(\phi_{i,s,t} + \delta_{i,s})}{(1 + \phi_{i,s,t})} = \frac{1}{\sum_{\theta=0}^{\infty} \left(\frac{1 - \delta_{i,s}}{1 + \phi_{i,s,t}} \right)^{\theta}} \quad [014]$$

and the market value of [013] can be written as

$$PV_{i,s,t} = \sum_{\theta=0}^{\infty} \left(\frac{1 - \delta_{i,s}}{1 + \phi_{i,s,t}} \right)^{\theta} R_{i,s,t} = \frac{1}{\zeta_{i,s,t}} R_{i,s,t} \quad [015]$$

Now let $\rho_{i,s,t}$ be the gross rate of return on capital in industry i of country s , before depreciation, but net of taxes on capital income :

$$\rho_{i,s,t} = \frac{R_{i,s,t}}{PK_{s,t}} \quad [016]$$

Then [015] implies that, under myopic expectations, for the present value of the income stream generated to be equal to the cost of investment, the rate of return $\rho_{i,s,t}$ must be equal to $\zeta_{i,s,t}$. So $\zeta_{i,s,t}$ is the rate of return required for an investment to break even. The break-even rate of return $\zeta_{i,s,t}$ takes a form that is slightly different from the usual one, which would be $\zeta_{i,s,t} = \phi_{i,s,t} + \delta_{i,s,t}$. The reason is that, in this model, compared to standard formulations, returns are paid, so to speak, one period in advance. Indeed, with investment being instantly productive, new capital begins to generate income in the very period in which investment takes place, so the whole stream of income is shifted one period back, and its present value is enhanced by a factor of $1 + \phi_{i,s,t}$; consequently, the required rate of return is reduced in the same proportion.

The user cost of capital in industry i of country s for period t is

$$U_{i,s,t} = \zeta_{i,s,t} PK_{s,t} = \frac{(\phi_{i,s,t} + \delta_{i,s})}{(1 + \phi_{i,s,t})} PK_{s,t} \quad [017]$$

which differs from the usual formulation in that it is divided by $(1 + \phi_{i,s,t})$ as pointed out above.

Then

$$\frac{(1 + \phi_{i,s,t})R_{i,s,t}}{(\phi_{i,s,t} + \delta_{i,s})PK_{s,t}} = \frac{R_{i,s,t}}{\zeta_{i,s,t}PK_{s,t}} = \frac{R_{i,s,t}}{U_{i,s,t}} \quad [018]$$

is the ratio of the market value [015] to the replacement cost $PK_{s,t}$ of a unit of capital, and it can be interpreted as Tobin's « q ». Investment demand is now specified following Jung and Thorbecke (2001)¹² as a constant elasticity increasing function of Tobin's « q »¹³ :

$$\frac{Id_{i,s,t}}{KD_{i,s,t}} = \gamma_{i,s} \left(\frac{R_{i,s,t}}{U_{i,s,t}} \right)^{el_Id_{i,s}} \quad [019]$$

where

$Id_{i,s,t}$ is investment demand by industry i of country s in period t ;

$KD_{i,s,t}$ is the capital stock of industry i in country s in period t ¹⁴;

$el_Id_{i,s}$ is the demand elasticity of investment demand of industry i in country s ;

$\gamma_{i,s}$ is a calibrated parameter.

From [016] and [014], investment demand [019] can be written alternatively as

$$\frac{Id_{i,s,t}}{KD_{i,s,t}} = \gamma_{i,s} \left(\frac{R_{i,s,t}}{U_{i,s,t}} \right)^{el_ind_{i,s}} = \gamma_{i,s} \left(\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} \right)^{el_ind_{i,s}} \quad [020]$$

The present value of the stream of income expected from new investment in industry i of country s is larger or smaller than its cost (Tobin's « q » is above or below 1), according to whether the actual rate of return $\rho_{i,s,t}$ is more or less than the break-even rate $\zeta_{i,s,t}$.

¹² Our specification differs from Jung-Thorbecke in accordance with the *MIRAGE* time structure as characterized in [012], and also in that depreciation is taken into account in user cost of capital [017].

¹³ It is acknowledged that this specification is at variance with Tobin's theory. Indeed, according to the « q » theory, q equals 1 in equilibrium. Other investment demand models, most notably the Bourguignon *et al.* (1989) theoretical formulation, conform more closely to Tobin's q theory. But that specification implies a degree of investment demand elasticity that is simply too high for the model to be stable. For that reason, Bourguignon *et al.* themselves have given it up for an *ad hoc* formulation. We have found the same difficulties when we tried to implement the theoretical form in the small scale *EXTER-Debt* model (Lemelin, 2007b). Even the Jung-Thorbecke specification is susceptible to instability if the elasticity value is set too high. See Lemelin and Decaluwé (2007) for a more detailed discussion of this point.

¹⁴ $KD_{i,s,t}$ is the sum of $K_{i,r,s,t}$ over country-agent capital owners r .

3.3 INVESTMENT SUPPLY AND DEMAND EQUILIBRIUM

The portfolio allocation mechanism determines the amount to be invested by each country-agent r in each industry i of each country s , according to the return rate on new shares (capital ownership titles). This section exposes the relationship between the investor's rate of return on new shares and the internal rate of return of investments, thereby establishing the equilibrating mechanism between the demand for new shares, and investment demand and the value of new shares issued.

The Decaluwé-Souissi demand for physical assets (capital ownership titles) in industry i of country s by country-agents r is given by :

$$PK_{s,t} INV_{i,r,s,t} = DelK_{r,t} \frac{(\alpha_{i,r,s}^K)^{\tau_K} (1 + r_{i,s,t}^K)^{-\tau_K}}{\sum_z \sum_j (\alpha_{j,r,z}^K)^{\tau_K} (1 + r_{j,z,t}^K)^{-\tau_K}} \quad [021]$$

parallel to [011], where

$DelK_{r,t}$ is total investment expenditure (acquisition of *new* capital ownership titles) by country-agent r in period t ;

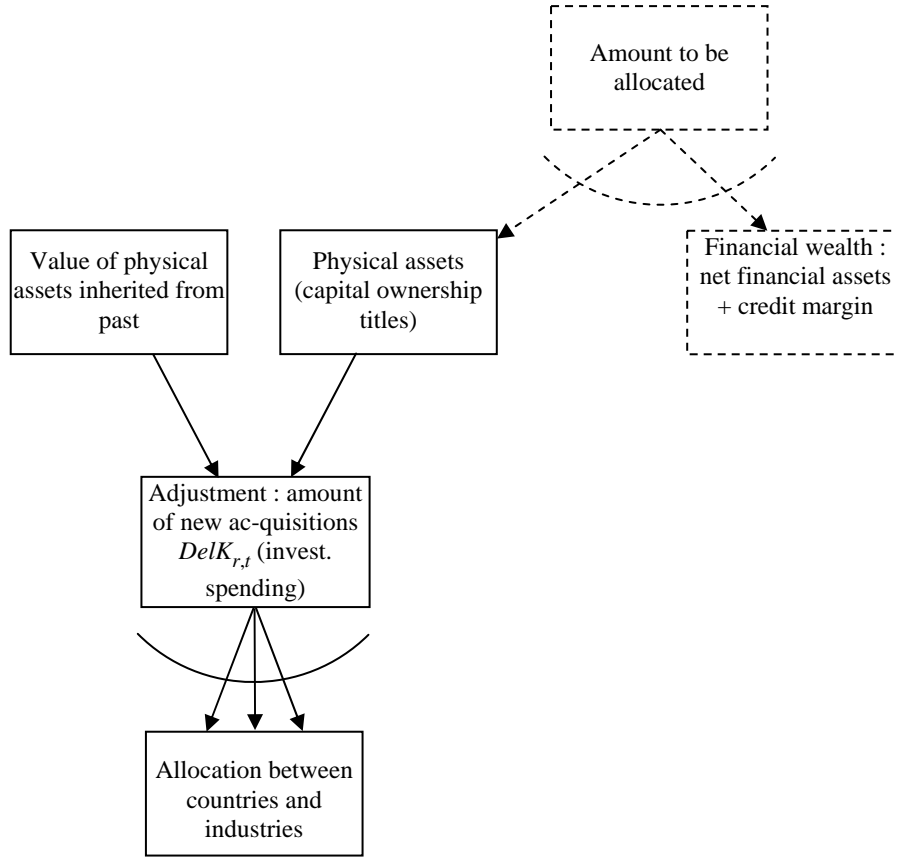
$r_{i,s,t}^K$ is the return rate on *new* capital ownership titles in industry i of country s ;

$\alpha_{i,r,s}^K$ is a calibrated parameter;

τ_K is the elasticity of transformation of the physical asset diversification constraint.

Total investment expenditure is determined taking into account that the process of allocating capital between countries and industries is limited to allocating investments, as illustrated in Figure 6 below, which completes Figure 2.

Figure 6 – Physical assets



The relevant rate for the portfolio manager is the rate of return *net of depreciation*, $r_{i,s,t}^K$, which is defined implicitly by

$$\sum_{\theta=0}^{\infty} \frac{1}{(1 + \phi_{i,s,t})^{\theta}} r_{i,s,t}^K = 1 \quad [022]$$

Since

$$\sum_{\theta=0}^{\infty} \frac{1}{(1 + \phi_{i,s,t})^{\theta}} = \frac{1 + \phi_{i,s,t}}{\phi_{i,s,t}} \quad [023]$$

we substitute in [022] :

$$\sum_{\theta=0}^{\infty} \frac{1}{(1 + \phi_{i,s,t})^{\theta}} r_{i,s,t}^K = \frac{1 + \phi_{i,s,t}}{\phi_{i,s,t}} r_{i,s,t}^K = 1 \quad [024]$$

and there results

$$r_{i,s,t}^K = \frac{\phi_{i,s,t}}{1 + \phi_{i,s,t}} \quad [025]$$

reflecting once again the discrepancy resulting from the time structure of *MIRAGE*, between the market rate of return $r_{i,s,t}^K$ which guides the portfolio manager in his/her choices, and the market discount rate $\phi_{i,s,t}$ (in other words, if the sum on the left-hand side of [022] were to begin at $\theta = 1$, rather than $\theta = 0$, we would have $r_{i,s,t}^K = \phi_{i,s,t}$).

Equilibrium between investment demand and investment spending imposes the following constraint :

$$PK_{s,t} Id_{i,s,t} = PK_{s,t} \sum_r INV_{i,r,s,t} \quad [026]$$

where the price of capital $PK_{s,t}$ appears on both sides of the equation to emphasize that investor portfolio decisions are in terms of amounts.

The market equilibrating mechanism thus rests upon the relationship between $r_{i,s,t}^K$, the return rate on *new* capital ownership titles (henceforth new shares), and the break-even rate of return $\zeta_{i,s,t}$ and the user cost of capital which enters the investment demand equation.

Indeed, combining [014] and [025], we obtain

$$r_{i,s,t}^K \frac{1 + \phi_{i,s,t}}{\phi_{i,s,t}} = \zeta_{i,s,t} \frac{(1 + \phi_{i,s,t})}{(\phi_{i,s,t} + \delta_{i,s})} = 1 \quad [027]$$

which implies

$$\zeta_{i,s,t} = r_{i,s,t}^K + (1 - r_{i,s,t}^K) \delta_{i,s} \quad [028]$$

Substituting into [017] results in

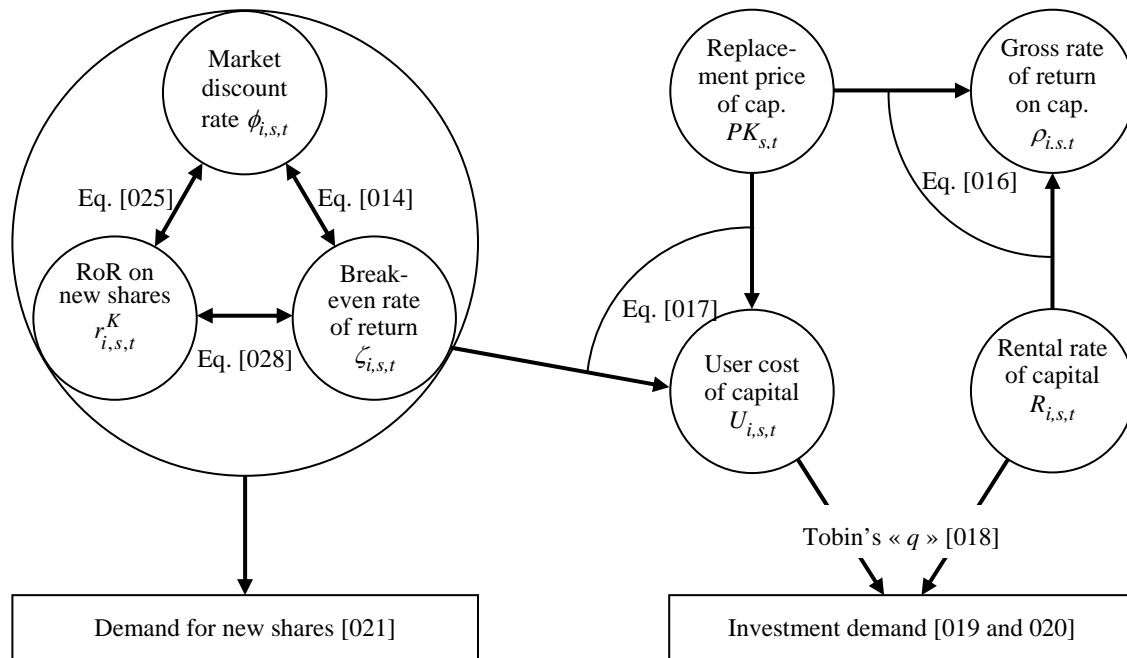
$$U_{i,s,t} = \zeta_{i,s,t} PK_{s,t} = [r_{i,s,t}^K + (1 - r_{i,s,t}^K) \delta_{i,s}] PK_{s,t} \quad [029]$$

The key role played by market rate of return $r_{i,s,t}^K$ in the savings-investment equilibrating mechanism is now clear. Through user cost of capital $U_{i,s,t}$, any rise in $r_{i,s,t}^K$ dampens investment demand [019]. A rise

in $r_{i,s,t}^K$ also increases the fraction of all country-agent portfolios dedicated to new shares in industry i of country s (demand equation [021]).

The equilibrating mechanism is illustrated in Figure 7. Given depreciation rate $\delta_{i,s}$, the market discount rate $\phi_{i,s,t}$, the rate of return on new shares $r_{i,s,t}^K$, and the break-even rate of return $\zeta_{i,s,t}$ are identically linked together. This triad and the replacement price of capital $PK_{i,s}$ jointly determine user cost of capital $U_{i,s,t}$, which in turn, combines with rental rate of capital $R_{i,s,t}$ to form Tobin's « q ». The latter drives investment demand and the supply of new shares. On the other hand, the demand for new shares by each country-agent portfolio manager depends positively on the rate of return on new shares $r_{i,s,t}^K$ (or, equivalently, on the break-even rate of return $\zeta_{i,s,t}$ or the market discount rate $\phi_{i,s,t}$). The triad of variables adjusts so as to equilibrate the supply and demand of new shares of every industry in every country.

Figure 7 : Investment allocation



3.4 NEW AND OLD SHARES AND THE STOCK MARKET VALUATION OF CAPITAL

But what about the owners of old shares, who hold titles to the capital inherited from the past (after depreciation)? Do they receive the same rate of return as the buyers of new shares? Yes, but the return is

computed on a value that is adjusted in accordance with the way investment is funded. Here we take a closer look at investment financing and the market valuation of old and new capital.

Buyers of new shares in industry i of country s will demand the market rate of return $r_{i,s,t}^K$. As we have shown, a constant income stream of $r_{i,s,t}^K$, beginning in the current period is equivalent to an income stream of $\zeta_{i,s,t}$ in the current period, declining thereafter at the rate of $\delta_{i,s}$. So, for every dollar invested, new shareholders demand a current return of $\zeta_{i,s,t}$. Specifically, they will demand a number of shares that will entitle them to a fraction of the income generated by the industry that will be sufficient for them to realize the yield they expect. So, to raise new funds to finance investments in the amount of $PK_{s,t} Id_{i,s,t}$ the number of new shares issued $\Delta N_{i,s,t}$ will have to be such that

$$\frac{\Delta N_{i,s,t}}{N_{i,s,t} + \Delta N_{i,s,t}} R_{i,s,t} KD_{i,s,t} = \zeta_{i,s,t} PK_{s,t} Id_{i,s,t} \quad [030]$$

where $N_{i,s,t}$ is the number of shares outstanding at the beginning of the period.

Equation [030] reflects the view that, in accordance with the *MIRAGE* time structure, part of the income generated in the current period goes to the new shareholders. So the left-hand side of [030] is the amount of income that will be paid to holders of the new shares in the current period, according to their fraction of ownership. The right-hand side is the income they must receive for the present value of expected incomes to be equal to the cost of the investment under myopic expectations. In other words, the number of shares issued must be such that the fraction of expected income attributed to the holders of new shares will produce the income they demand.

Substitute

$$\rho_{i,s,t} = \frac{R_{i,s,t}}{PK_{s,t}} \quad [016]$$

and [030] becomes

$$\frac{\Delta N_{i,s,t}}{N_{i,s,t} + \Delta N_{i,s,t}} \rho_{i,s,t} KD_{i,s,t} = \zeta_{i,s,t} Id_{i,s,t} \quad [031]$$

$$\frac{\Delta N_{i,s,t}}{N_{i,s,t} + \Delta N_{i,s,t}} = \frac{\zeta_{i,s,t}}{\rho_{i,s,t}} \frac{Id_{i,s,t}}{KD_{i,s,t}} \quad [032]$$

So the equity owned by new investors is more or less than their contribution to the industry's current capital stock, according to whether $\zeta_{i,s,t}$ is greater or smaller than $\rho_{i,s,t}$: if $\zeta_{i,s,t}$ is greater than $\rho_{i,s,t}$, new investment is financed by equity dilution; in the opposite case, there is equity concentration. Now, since the ratio of $\rho_{i,s,t}$ over $\zeta_{i,s,t}$ is Tobin's « q », [032] can be interpreted as follows :

$$\begin{aligned} \text{Share of equity} & \quad \text{Number of} & \quad \text{Investment} \\ \text{owned by new} & = \frac{\text{new shares issued}}{\text{Total number}} & = \frac{\text{(new capital)}}{\text{Total}} \text{ if Tobin's « } q \text{ »} = 1; \\ \text{investors} & \quad \text{of shares} & \quad \text{capital stock} \\ \\ \text{Share of equity} & \quad \text{Number of} & \quad \text{Investment} \\ \text{owned by new} & = \frac{\text{new shares issued}}{\text{Total number}} & > \text{ or } < \frac{\text{(new capital)}}{\text{Total}} \text{ if Tobin's « } q \text{ »} < \text{ or } > 1. \\ \text{investors} & \quad \text{of shares} & \quad \text{capital stock} \end{aligned}$$

Since a number $\Delta N_{i,s,t}$ of new shares have been issued for a total amount of $PK_{s,t} Id_{i,s,t}$, their unit price is

$$\frac{PK_{s,t} Id_{i,s,t}}{\Delta N_{i,s,t}}$$

As for the holders of shares outstanding at the beginning of the period, it follows from [030] that what they must give up to the new shareholders is $\zeta_{i,s,t} PK_{s,t} Id_{i,s,t} = U_{i,s,t} Id_{i,s,t}$ which is the user cost of the new capital, as it should. So they expect to receive a current period income of

$$\rho_{i,s,t} PK_{s,t} KD_{i,s,t} - \zeta_{i,s,t} PK_{s,t} Id_{i,s,t} = PK_{s,t} (\rho_{i,s,t} KD_{i,s,t} - \zeta_{i,s,t} Id_{i,s,t}) \quad [033]$$

Given that

$$KD_{i,s,t} = (1 - \delta_{i,s}) KD_{i,s,t-1} + Id_{i,s,t} \quad [034]$$

we have

$$\begin{aligned} \rho_{i,s,t} PK_{s,t} KD_{i,s,t} - \zeta_{i,s,t} PK_{s,t} Id_{i,s,t} \\ = PK_{s,t} \left\{ \rho_{i,s,t} \left[(1 - \delta_{i,s}) KD_{i,s,t-1} + Id_{i,s,t} \right] - \zeta_{i,s,t} Id_{i,s,t} \right\} \end{aligned} \quad [035]$$

$$\begin{aligned} \rho_{i,s,t} PK_{s,t} KD_{i,s,t} - \zeta_{i,s,t} PK_{s,t} Id_{i,s,t} \\ = PK_{s,t} \left[\rho_{i,s,t} (1 - \delta_{i,s}) KD_{i,s,t-1} + (\rho_{i,s,t} - \zeta_{i,s,t}) Id_{i,s,t} \right] \end{aligned} \quad [036]$$

In the following periods, that stream of income declines with depreciation. The present value of the stream of income that old shareholders expect to receive, assuming myopic expectations, is equal to :

$$\sum_{\theta=0}^{\infty} \left(\frac{1-\delta_{i,s}}{1+\phi_{i,s,t}} \right)^{\theta} PK_{s,t} \left[\rho_{i,s,t} (1-\delta_{i,s}) KD_{i,s,t-1} + (\rho_{i,s,t} - \zeta_{i,s,t}) Id_{i,s,t} \right]$$

$$= \frac{1}{\zeta_{i,s,t}} PK_{s,t} \left[\rho_{i,s,t} (1-\delta_{i,s}) KD_{i,s,t-1} + (\rho_{i,s,t} - \zeta_{i,s,t}) Id_{i,s,t} \right] \quad [037]$$

$$= PK_{s,t} \left[\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} (1-\delta_{i,s}) KD_{i,s,t-1} + \left(\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} - 1 \right) Id_{i,s,t} \right] \quad [038]$$

$$= PK_{s,t} \left[\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} KD_{i,s,t} - Id_{i,s,t} \right] \quad [039]$$

The unit price of old shares is their value, divided by their number :

$$\frac{PK_{s,t}}{N_{i,s,t}} \left[\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} KD_{i,s,t} - Id_{i,s,t} \right]$$

It is demonstrated in the following box that the price of old shares is identical to the price of new ones. So share pricing is perfectly compatible with the fact that new and old shareholders are the same country-agents.

Demonstration of the equality of prices between old and new shares :

From [032], the number of new shares is

$$\Delta N_{i,s,t} = \frac{\zeta_{i,s,t}}{\rho_{i,s,t}} \frac{Id_{i,s,t}}{KD_{i,s,t}} (N_{i,s,t} + \Delta N_{i,s,t}) \quad [040]$$

$$\left(1 - \frac{\zeta_{i,s,t}}{\rho_{i,s,t}} \frac{Id_{i,s,t}}{KD_{i,s,t}} \right) \Delta N_{i,s,t} = \frac{\zeta_{i,s,t}}{\rho_{i,s,t}} \frac{Id_{i,s,t}}{KD_{i,s,t}} N_{i,s,t} \quad [041]$$

$$\Delta N_{i,s,t} = \frac{\frac{\zeta_{i,s,t}}{\rho_{i,s,t}} \frac{Id_{i,s,t}}{KD_{i,s,t}}}{\left(1 - \frac{\zeta_{i,s,t}}{\rho_{i,s,t}} \frac{Id_{i,s,t}}{KD_{i,s,t}}\right)} N_{i,s,t} \quad [042]$$

$$\Delta N_{i,s,t} = \frac{1}{\left(\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} \frac{KD_{i,s,t}}{Id_{i,s,t}} - 1\right)} N_{i,s,t} \quad [043]$$

It follows that the price of new shares can be written as

$$\frac{PK_{s,t} Id_{i,s,t}}{\Delta N_{i,s,t}} = \frac{PK_{s,t} Id_{i,s,t}}{N_{i,s,t}} \left(\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} \frac{KD_{i,s,t}}{Id_{i,s,t}} - 1 \right) \quad [044]$$

$$\frac{PK_{s,t} Id_{i,s,t}}{\Delta N_{i,s,t}} = \frac{PK_{s,t}}{N_{i,s,t}} \left(\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} KD_{i,s,t} - Id_{i,s,t} \right) \quad [045]$$

which is precisely the price of old shares.

Q.E.D.

Since the price of old and new shares is the same, the total stock market value of industry i in period t is simply

$$\frac{PK_{s,t} Id_{i,s,t}}{\Delta N_{i,s,t}} \Delta N_{i,s,t} + \frac{PK_{s,t}}{N_{i,s,t}} \left(\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} KD_{i,s,t} - Id_{i,s,t} \right) N_{i,s,t} = \frac{\rho_{i,s,t}}{\zeta_{i,s,t}} PK_{s,t} KD_{i,s,t} \quad [046]$$

So the current stock market value of an industry is equal to the replacement cost of its capital, multiplied by Tobin's q .

The second term on the left-hand side of [046] is the current stock market value of shares issued in the past. This can be rewritten as in the right hand side of [038] :

$$PK_{s,t} \left(\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} KD_{i,s,t} - Id_{i,s,t} \right) = PK_{s,t} \left[\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} (1 - \delta_{i,s}) KD_{i,s,t-1} + \left(\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} - 1 \right) Id_{i,s,t} \right] \quad [047]$$

If Tobin's q is 1, the stock market value of old shares that enters the country-agent's wealth is equal to the replacement value of the capital inherited from the preceding period after depreciation. Otherwise, it is greater or less, according to whether Tobin's q is greater or less than 1.

4. Implementation of *MIRAGE-D*

The *MIRAGE-D* GAMS code was developed on the basis of the *MIRAGE* GAMS code. The model was tested on an aggregate data set. The moderate size of result files allowed a detailed examination and facilitated diagnostics during model development. This was made possible by *MIRAGE*'s sophisticated flexibility, which makes it possible to apply the same GAMS program to variously aggregated data bases.

However, the level of aggregation is not so coarse as to render results uninteresting. In the version of *MIRAGE-D* presented here, there are 14 countries or groups of countries :

- Africa South of the Sahara (AfriSS)
- China (incl. Hong Kong) (ChinaHK)
- European Union Fifteen (EU15)¹⁵
- Rest of the EU (before 2007) (EUplus)
- India
- Japan
- Middle-East and North Africa (ME&NA)
- Latin American developing countries (LAmDev)
- Asia-Pacific developing countries (AsPaDev)
- Rest of Latin America (RoLAm)
- Rest of Asia (RoAsia)
- Rest of the world (RoW)
- Transition countries (Transit)
- United States of America (USA)

There are 5 categories of goods and services and industries :

- Agriculture (Agri)
- Other primary products (OthPrim)
- Manufacturing (Manuf)
- Services, except transportation (Serv)
- Transportation services (Transp)

Simulations run over a fifteen-year period, from 2001 to 2015. The reference (BAU) scenario is generated by a version of the original *MIRAGE*, adapted to comparisons with *MIRAGE-D*. In the reference scenario, real GDP is exogenous, fixed according to World Bank country real GDP forecasts, which foresee a 54% expansion of the world economy between 2001 and 2015, and endogenous total factor productivity adjusts. In simulations and in *MIRAGE-D*, total factor productivity is fixed at its BAU levels, and GDP is endogenous.

5. Simulation results

The *MIRAGE-D* model was developed to make explicit the international capital flows which must take place to balance the current account implications of the simulated trade flows, and to compute the cumulative consequences of such capital flows on the international investment positions (IIP) of

¹⁵ Member countries in 1995.

countries. An illustrative simulation scenario was run with both *MIRAGE-D* and the standard version of *MIRAGE*. Our expectation is that the results will not be identical, but neither will they be wildly different. It would be worrisome if the *MIRAGE-D* results were to be wildly different from those of the standard version, whose credibility is well established. On the other hand, if the results were identical, why should we bother with the complications of *MIRAGE-D*? And of course, the differences should be explicable by the financial factors taken into account in *MIRAGE-D*.

5.1 *MIRAGE-D* RESULTS

Before examining the differences between the *MIRAGE* reference scenario and the *MIRAGE-D* simulation, let us cast an eye on the *MIRAGE-D* results. Table 1 presents the evolution of real GDP by country or country group.

Table 1
Evolution of real GDP by country or group of countries, 2001-2015
(2001=100)

	AfriSS	ChinaHK	EU15	EUplus	India	Japan	ME&NA	LAmDev	AsPaDev	RoLAm	RoAsia	RoW	Transit	USA	Total
2001	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
2002	106	109	101	102	105	100	103	99	106	99	107	104	107	101	101
2003	110	119	102	106	115	103	109	99	113	101	110	106	116	104	104
2004	114	130	104	111	121	108	114	104	121	106	117	110	126	108	108
2005	119	140	106	115	129	110	119	108	128	110	123	114	134	111	112
2006	124	150	109	120	137	113	124	111	136	114	129	117	140	114	116
2007	130	160	112	125	146	115	129	114	143	118	136	121	146	118	120
2008	135	171	115	130	154	117	135	117	151	122	142	125	151	121	124
2009	140	183	118	134	163	119	141	120	158	126	149	129	156	125	128
2010	145	194	121	138	173	121	146	124	166	130	156	133	160	129	132
2011	150	207	124	142	183	124	152	127	174	134	164	137	164	133	136
2012	156	220	127	146	193	126	159	131	183	138	171	142	169	137	141
2013	162	234	130	150	204	128	165	135	192	143	179	146	173	141	145
2014	167	249	133	155	215	130	172	138	201	147	187	150	178	146	150
2015	173	264	136	160	227	132	179	142	210	152	196	155	183	150	154
Rank	7	1	13	8	2	14	6	12	3	10	4	9	5	11	

Of course, the rapid growth of China stands out, but also that of India, Asia-Pacific developing countries and the rest of Asia. The more mature economies of the EU Fifteen and Japan grow more slowly than the rest, while the USA maintains its pace¹⁶. That unequal growth results in a corresponding evolution of world GDP shares, as shown in the following table.

¹⁶ This scenario was developed early in 2006, before the recent events that shook the world economy.

Table 2
Evolution of country or group of countries shares in world real GDP, 2001-2015
 (%)

	AfriSS	ChinaHK	EU15	EUplus	India	Japan	ME&NA	LAmDev	AsPaDev	RoLAm	RoAsia	RoW	Transit	USA	Total
2001	1.03	4.36	24.50	1.12	1.55	13.34	2.84	1.63	2.08	4.60	2.85	5.33	1.60	33.17	100.00
2002	1.07	4.67	24.30	1.12	1.60	13.09	2.87	1.59	2.17	4.48	2.99	5.45	1.68	32.91	100.00
2003	1.08	4.97	23.77	1.13	1.70	13.10	2.96	1.55	2.25	4.45	3.00	5.41	1.77	32.85	100.00
2004	1.08	5.21	23.31	1.14	1.72	13.18	2.96	1.57	2.30	4.48	3.06	5.38	1.84	32.78	100.00
2005	1.09	5.42	23.08	1.14	1.77	13.04	2.99	1.56	2.36	4.49	3.10	5.37	1.89	32.71	100.00
2006	1.10	5.62	22.96	1.16	1.82	12.87	3.02	1.55	2.41	4.50	3.16	5.36	1.92	32.55	100.00
2007	1.11	5.81	22.83	1.16	1.88	12.71	3.05	1.54	2.47	4.51	3.21	5.36	1.93	32.42	100.00
2008	1.12	6.01	22.68	1.17	1.93	12.55	3.08	1.54	2.51	4.52	3.27	5.36	1.94	32.33	100.00
2009	1.12	6.21	22.53	1.17	1.97	12.39	3.11	1.53	2.56	4.52	3.32	5.36	1.94	32.28	100.00
2010	1.13	6.40	22.36	1.16	2.02	12.23	3.14	1.53	2.60	4.52	3.37	5.36	1.93	32.25	100.00
2011	1.13	6.60	22.20	1.16	2.07	12.06	3.17	1.52	2.65	4.52	3.42	5.35	1.92	32.23	100.00
2012	1.14	6.81	22.03	1.16	2.12	11.90	3.19	1.52	2.69	4.52	3.46	5.35	1.91	32.21	100.00
2013	1.14	7.01	21.87	1.16	2.17	11.73	3.22	1.51	2.73	4.51	3.51	5.34	1.90	32.20	100.00
2014	1.15	7.23	21.70	1.15	2.22	11.56	3.25	1.50	2.77	4.51	3.56	5.33	1.89	32.18	100.00
2015	1.15	7.44	21.53	1.15	2.27	11.40	3.27	1.50	2.81	4.50	3.60	5.33	1.88	32.16	100.00

But the interest of *MIRAGE-D* lies elsewhere, in its taking into account the international investment position (IIP) of countries. A country's IIP is defined as the discrepancy between its assets and liabilities *vis-à-vis* the rest of the world, excluding direct investments, but including exchange reserves other than gold (see formal accounting definition in the appendix). Figures 8a and 8b show the evolution of net IIP's. The first figure clearly displays the plummeting net IIP of the United States. By 2015, the combined net international financial assets of China, Japan and the rest of Asia amount to more than 90% of US net negative IIP. Figure 8b is nothing but an enlargement of the central part of Figure 8a. The trajectories of the US and Japan are off the chart, while those of China and the rest of Asia shoot through the roof. The enlargement makes it easier to follow the trajectories of other countries and regions.

Figure 8a
Evolution of net international investment positions, excluding FDI
 (tens of billions of 2001 US dollars)

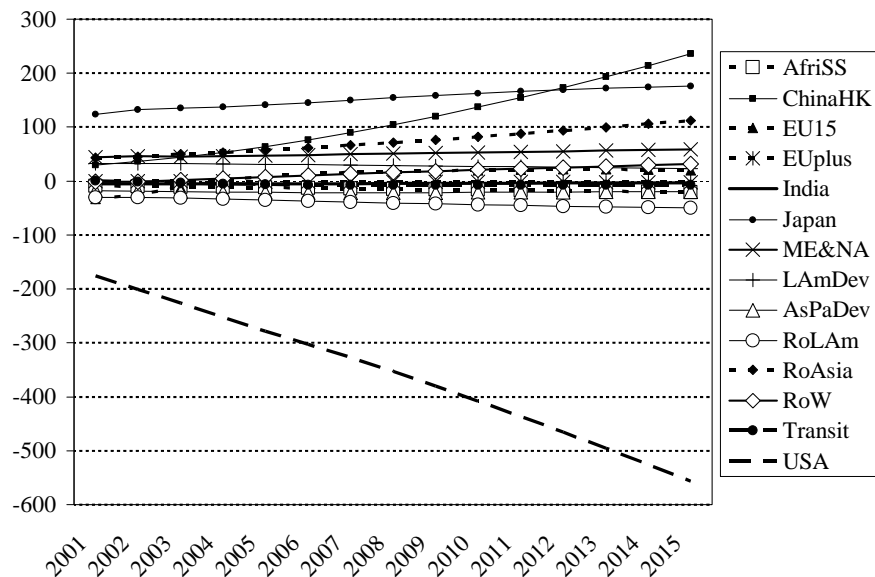


Figure 8b (enlargement of central part of Figure 8a)
Evolution of net international investment positions, excluding FDI
(tens of billions of 2001 US dollars)

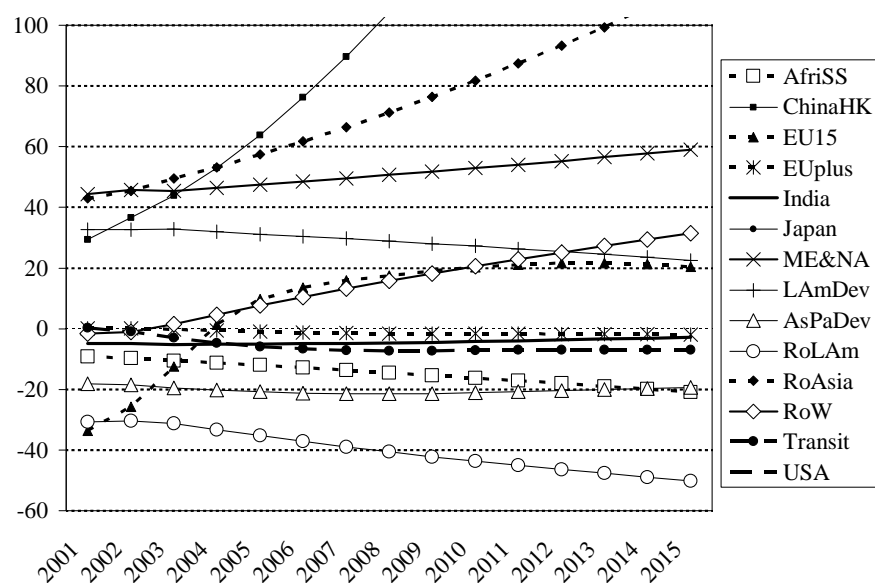


Table 3 examines the ratio of net IIP over GDP. The most spectacular evolutions are those of China, which jumps from 22% to 73%, and of the United States, whose negative position worsens from 17% to 40% of its GDP. As for Africa South of the Sahara, whose financial indebtedness in 2001 was 28% of its GDP, its position deteriorates to 38% in 2015.

Table 3
Ratio of net international investment position over GDP, 2001-2015
(%)

	AfriSS	ChinaHK	EU15	EUplus	India	Japan	ME&NA	LAmDev	AsPaDev	RoLAm	RoAsia	RoW	Transit	USA
2001	-28.4	22.3	-4.3	0.3	-10.3	29.7	50.3	62.5	-28.5	-20.8	48.7	-0.9	0.5	-17.4
2002	-28.2	25.4	-3.2	0.1	-9.5	31.5	50.8	64.5	-27.2	-21.2	48.1	-0.6	-1.7	-20.5
2003	-29.3	27.8	-1.5	-0.8	-9.4	30.8	46.8	64.4	-26.8	-21.0	51.0	0.8	-5.1	-22.5
2004	-30.1	31.0	0.2	-1.6	-8.8	29.7	46.7	58.9	-26.3	-21.3	51.5	2.4	-7.6	-24.2
2005	-30.9	35.0	1.1	-2.2	-8.2	29.9	45.8	56.0	-25.7	-21.9	53.3	3.9	-9.0	-26.0
2006	-31.8	39.3	1.6	-2.9	-7.6	30.4	45.1	53.4	-25.0	-22.4	54.8	5.2	-9.7	-27.6
2007	-32.7	43.5	1.8	-3.4	-7.0	30.7	44.5	50.8	-24.0	-22.9	56.3	6.3	-10.0	-29.1
2008	-33.6	47.7	1.9	-3.6	-6.4	31.0	43.9	48.3	-23.0	-23.2	57.8	7.3	-10.0	-30.6
2009	-34.4	51.8	2.0	-3.6	-5.8	31.2	43.2	45.7	-21.9	-23.4	59.3	8.2	-9.7	-32.1
2010	-35.1	55.8	2.1	-3.5	-5.2	31.3	42.6	43.1	-20.7	-23.6	60.8	9.0	-9.3	-33.5
2011	-35.8	59.6	2.1	-3.4	-4.6	31.3	42.0	40.6	-19.5	-23.6	62.3	9.7	-8.8	-34.9
2012	-36.5	63.2	2.1	-3.4	-4.0	31.3	41.4	38.2	-18.3	-23.7	63.7	10.3	-8.5	-36.2
2013	-37.2	66.6	2.1	-3.4	-3.5	31.2	40.9	35.9	-17.3	-23.7	65.1	10.8	-8.2	-37.4
2014	-37.8	69.8	2.0	-3.5	-3.0	31.0	40.4	33.5	-16.2	-23.6	66.5	11.3	-7.9	-38.6
2015	-38.4	72.9	1.9	-3.5	-2.6	30.8	39.8	31.3	-15.2	-23.5	67.9	11.8	-7.8	-39.7

Changes in the net IIP is not without consequences on the balance of payments flows. Table 4 shows the evolution of net portfolio investment income resulting from the IIP, in proportion to GDP. The case of China calls for attention. It may seem surprising that, except in the first periods, China, whose IIP is heftily positive and growing, pays more portfolio investment income than it receives. But, according to

the financial data used to calibrate the model (see appendix), the return rate paid by China on its international financial liabilities is by far the highest of all (around 12%). Now, China's rapid growth requires important investments, part of which are financed indirectly by borrowing, and that pushes the rate paid on liabilities even higher, while the return received on assets remains that of the world debt portfolio. The result is a negative net income which, by 2015, amounts to 3.5% of GDP, double the corresponding ratio for the USA, whose GDP, however, is projected by then to more than quadruple China's.

It is reasonable to think such a scenario will not materialize. Indeed, portfolio manager behavior in *MIRAGE-D* is governed by parameters that have been calibrated on 2001 financial data. It is to be expected that there will be structural changes (changes in parameter values) whereby China will be less and less disadvantaged in her international financial transactions.

Table 4
Ratio of net international payments of portfolio investment income over GDP, 2001-2015
(%)

	AfriSS	ChinaHK	EU15	EUplus	India	Japan	ME&NA	LAmDev	AsPaDev	RoLAm	RoAsia	RoW	Transit	USA
2001	0.56	-0.44	0.09	-1.95	0.29	-0.90	-1.65	-0.12	1.38	1.43	1.00	0.81	1.87	-0.06
2002	0.68	-0.26	-0.15	-1.95	0.27	-0.99	-1.70	-0.20	1.31	1.40	0.91	0.73	1.92	0.16
2003	0.82	0.01	-0.44	-1.85	0.30	-0.96	-1.53	-0.29	1.28	1.34	0.76	0.59	2.06	0.36
2004	0.99	0.16	-0.71	-1.77	0.29	-0.97	-1.62	-0.37	1.28	1.36	0.55	0.42	2.16	0.60
2005	1.13	0.31	-0.94	-1.70	0.28	-1.02	-1.65	-0.45	1.25	1.38	0.35	0.26	2.22	0.81
2006	1.26	0.49	-1.12	-1.63	0.26	-1.07	-1.67	-0.50	1.20	1.39	0.20	0.11	2.22	0.98
2007	1.38	0.69	-1.28	-1.58	0.23	-1.10	-1.68	-0.55	1.12	1.38	0.05	-0.03	2.19	1.14
2008	1.47	0.94	-1.44	-1.57	0.21	-1.13	-1.68	-0.59	1.02	1.35	-0.08	-0.16	2.13	1.28
2009	1.54	1.23	-1.60	-1.57	0.18	-1.15	-1.66	-0.62	0.91	1.31	-0.19	-0.29	2.05	1.40
2010	1.60	1.55	-1.76	-1.58	0.15	-1.16	-1.64	-0.65	0.80	1.26	-0.29	-0.40	1.96	1.50
2011	1.64	1.91	-1.91	-1.59	0.12	-1.16	-1.62	-0.69	0.68	1.21	-0.37	-0.51	1.87	1.59
2012	1.67	2.29	-2.06	-1.60	0.09	-1.15	-1.59	-0.72	0.56	1.14	-0.45	-0.61	1.78	1.65
2013	1.68	2.70	-2.20	-1.61	0.06	-1.14	-1.56	-0.76	0.45	1.07	-0.50	-0.71	1.70	1.70
2014	1.69	3.11	-2.34	-1.62	0.04	-1.13	-1.53	-0.80	0.34	0.99	-0.55	-0.80	1.62	1.73
2015	1.69	3.52	-2.48	-1.62	0.02	-1.11	-1.50	-0.85	0.24	0.92	-0.59	-0.89	1.55	1.75

5.2 COMPARISON

How different are *MIRAGE-D* results when compared to *MIRAGE*? In order to make such comparisons, we have grafted the financial asset module of *MIRAGE-D* to *MIRAGE*, while keeping the *MIRAGE* specification of consumption and investment, so that the results of this modified version of *MIRAGE* are the same as those of the standard *MIRAGE* with respect to all the variables that exist in the original *MIRAGE*.

Table 5 compares the evolution of real GDP's. It can be seen that in *MIRAGE-D*, countries which have recently joined the EU, Latin American countries and the USA grow less than in *MIRAGE*. This can be interpreted as the dampening effect of negative or deteriorating IIP's, particularly in the case of the USA.

Table 5
Evolution of real GDP, by country or group of countries, 2001-2015
Ratio MIRAGE-D / MIRAGE (%)

	AfriSS	ChinaHK	EU15	EUplus	India	Japan	ME&NA	LAmDev	AsPaDev	RoLAm	RoAsia	RoW	Transit	USA	Total
2001	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
2002	103	102	100	100	101	100	100	99	102	100	101	101	102	99	99
2003	104	102	100	100	101	101	102	98	103	100	101	101	103	99	99
2004	104	103	100	99	101	101	101	99	104	100	102	101	104	98	99
2005	105	103	100	99	102	101	102	98	104	100	102	101	103	98	100
2006	105	103	100	99	102	101	102	98	105	100	102	102	103	98	100
2007	106	104	101	98	102	101	102	97	105	100	102	102	103	98	100
2008	106	104	101	98	102	102	102	97	105	100	102	102	102	97	100
2009	107	104	101	98	102	102	102	96	106	99	102	102	102	97	100
2010	107	104	101	98	102	102	102	96	106	99	103	102	102	97	100
2011	107	104	101	97	103	102	102	95	106	98	103	103	102	97	100
2012	107	104	102	97	103	102	102	95	106	98	103	103	102	97	100
2013	107	104	102	97	103	103	102	94	106	97	103	103	101	97	100
2014	107	104	102	97	103	103	101	94	106	97	103	103	101	97	100
2015	107	104	102	97	103	103	101	93	106	96	103	103	101	97	100
Rank	1	3	8	11	5	6	10	14	2	13	7	4	9	12	

The composition of GDP is different also. Depending on country and year, real investments in *MIRAGE* are greater or less than in *MIRAGE-D*, as seen in Table 6. Notice that the four regions with slower growth in Table 5 are also those with less investment in Table 6.

Table 6
Ratio of real investment : MIRAGE-D / MIRAGE, 2001-2015
(%)

	AfriSS	ChinaHK	EU15	EUplus	India	Japan	ME&NA	LAmDev	AsPaDev	RoLAm	RoAsia	RoW	Transit	USA	Total
2001	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
2002	130	111	100	96	105	102	104	90	117	99	113	112	121	90	100
2003	130	113	98	98	109	107	112	89	121	101	106	107	123	90	100
2004	126	112	98	96	105	109	105	95	117	103	109	106	118	90	100
2005	126	111	102	95	106	107	105	92	116	101	107	107	111	89	101
2006	125	110	104	95	106	107	105	90	114	100	107	107	106	89	101
2007	123	111	105	94	106	107	104	89	113	97	107	108	104	89	101
2008	122	111	105	93	106	107	104	88	111	96	107	108	102	90	101
2009	120	111	106	92	106	108	103	87	110	94	106	109	100	90	102
2010	119	111	106	92	106	108	103	86	110	92	106	109	100	90	102
2011	118	111	106	92	107	109	102	85	109	91	106	109	100	90	102
2012	116	111	107	93	107	109	102	85	109	89	106	109	100	90	102
2013	115	111	107	93	107	110	102	84	109	88	106	110	101	90	102
2014	114	111	107	93	107	110	101	83	109	87	106	110	101	90	102
2015	114	110	108	93	108	110	101	82	109	86	106	110	101	90	102
Rank	1	3	6	11	7	2	10	14	5	13	8	4	9	12	

Grosso modo, relative to *MIRAGE*, *MIRAGE-D* directs more investment to Africa South of the Sahara and Asia-Pacific, and less towards the Americas. Of course, that implies, in the end, a different geographic distribution of the stock of capital. This is shown in Table 7, together with changes in the distribution of capital among industries.

Table 7
Capital stock, by country or group of countries
and by industry, 2015
MIRAGE-D / MIRAGE Ratio
 (%)

	Agri	OthPrim	Manuf	Serv	Transp	Total
AfriSS	116	126	108	112	115	113
ChinaHK	90	110	106	112	124	109
EU15	101	96	107	103	100	103
EUplus	88	88	92	98	94	95
India	88	104	103	111	101	106
Japan	99	93	109	105	104	106
ME&NA	95	105	97	103	106	102
LAmDev	88	91	92	90	86	90
AsPaDev	103	115	108	108	108	108
RoLAm	93	94	94	95	95	95
RoAsia	98	102	106	105	111	105
RoW	98	103	112	105	106	106
Transit	99	112	100	102	103	103
USA	77	78	89	95	89	93
Total	93	102	101	101	103	101

It is interesting to note that changes are almost bi-proportional, that is, the columns of Table 7 are more or less proportional to the column *Total*, and lines, to the line *Total*¹⁷. Changes in the geographical and industrial distribution of productive capacity reflects upon value added by region and industry (Table 8) : the correlation coefficient between the values of Tables 7 and 8 is 94%.

Table 8
Real value added, by country or group of countries
and by industry, 2015
MIRAGE-D / MIRAGE Ratio
 (%)

	Agri	OthPrim	Manuf	Serv	Transp	Total
AfriSS	104	112	103	105	105	105
ChinaHK	99	102	103	104	109	103
EU15	101	97	102	101	100	101
EUplus	99	96	96	99	98	98
India	99	101	102	104	102	102
Japan	101	95	103	102	100	102
ME&NA	99	102	98	101	103	101
LAmDev	98	96	95	95	95	95
AsPaDev	102	108	104	105	104	104
RoLAm	99	97	96	98	97	97
RoAsia	101	101	104	101	104	102
RoW	100	100	104	102	101	102
Transit	100	105	100	101	101	101
USA	93	90	96	98	98	97
Total	99	100	100	100	100	100

¹⁷ The weak Khi-square statistic, approximately 14.4, confirms that impression.

Finally, such differences have implications for international trade. But the relation between capital stock and value added, on one hand, and net exports on the other is not very clear. Although correlation between the numbers in Table 9 and Tables 7 and 8 is positive (26% and 34% respectively), it is unexpectedly weak. Also noteworthy is that the volume of trade increases in *MIRAGE-D* relative to *MIRAGE*, except for agricultural products (see the last line of Table 9).

Table 9
Variation of net exports, 2015
by country or group of countries and by industry
MIRAGE-D - MIRAGE
 (% of the volume of exports in MIRAGE)

	Agri	OthPrim	Manuf	Serv	Transp
AfriSS	-0.0295	1.3035	-0.1724	-0.1928	-0.1214
ChinaHK	0.4692	-0.1357	0.4638	0.4959	2.2045
EU15	0.2094	-1.5672	-0.0744	-2.2593	-2.4313
EUplus	0.2901	0.0086	-0.0412	0.1906	-0.0416
India	-0.3564	-0.1397	-0.0169	0.2143	-0.0798
Japan	0.0129	-0.8839	-0.1978	-0.5094	-0.6237
ME&NA	0.2573	1.5742	-0.1456	0.2474	0.3831
LAmDev	0.8723	-0.0202	0.0941	0.1243	-0.0465
AsPaDev	-0.5219	0.6622	-0.0559	-0.2910	-0.3428
RoLAm	1.0976	0.0332	-0.0863	0.2635	-0.1088
RoAsia	0.2074	-0.3736	0.3372	0.0525	0.3587
RoW	-0.4254	-0.4974	-0.0864	-0.7457	-0.6474
Transit	0.2013	1.4955	-0.0696	-0.0989	-0.1254
USA	-2.2842	-1.4594	0.0514	2.5089	1.6224
Volume of trade (MIRAGE-D / MIRAGE, %)	99	104	101	102	101

Concluding remarks

Most CGE trade models fix current account balances exogenously, in accordance with the widely accepted view that trade policy may influence trade flows, but that current accounts are constrained by symmetric capital account balances, on which trade policy has little effect. The *MIRAGE-D* model was developed to make explicit the international capital flows which must take place to balance the current account implications of the simulated trade flows, and to compute the cumulative consequences of such capital flows on the international investment positions (IIP) of countries. In *MIRAGE-D*, current account balances and their capital account counterparts are endogenous, following a three-tier portfolio management model, adapted from Decaluwé and Souissi (1994; Souissi, 1994; Souissi and Decaluwé, 1997), which represents country-agent wealth allocation behavior. In the first stage, wealth is distributed between physical assets (capital) and international financial assets. In the second stage, on the financial side, the asset-liability structure of external financial wealth is determined. Next, international financial assets are pooled and allocated among country debt securities. On the physical assets side, the allocation of capital among countries and industries is determined by an investment supply and demand

equilibrating mechanism. Investment supply is the demand for new physical capital ownership titles resulting from the wealth allocation process, while the investment demand equation is a constant elasticity function of Tobin's q in the Jung-Thorbecke (2001) style.

An illustrative simulation scenario was run with both *MIRAGE-D* and the standard version of *MIRAGE*. Apart from *MIRAGE-D*'s results on the international investment positions of countries, which the standard version does not produce, other simulation results, although they are not identical, show moderate differences. In our view, this is an indication that the proposed extension of *MIRAGE* does not compromise the credibility of the model. Moreover, those differences that exist are fully explained by the financial aspects, and arise from the consistency required between such financial aspects and the rest of the model. Most notably, in the scenario run for illustration purposes, the deepening negative US IIP is accommodated by corresponding surpluses, most notably in China, Japan and the rest of Asia. This in itself is rather interesting, given the spectacular size of the negative US position projected in 2015. In *MIRAGE-D*, however, consistent with the deteriorating US IIP, GDP growth in the US is dampened relative to *MIRAGE*, thus highlighting the potential interest of endogenizing the current account balance¹⁸.

It is acknowledged that several aspects of *MIRAGE-D* are in need of improvement. First, its credibility would be enhanced if portfolio diversification constraint elasticities – arbitrary in the current version – were given an empirical basis. Simulations with different sets of elasticity values were run to test model robustness, and results showed that, indeed, the choice of elasticities does matter. High elasticity values generate results that look somewhat more chaotic. It is nonetheless reassuring to see that, except for financial variables and the evolution of IIP's, other results are not too sensitive to portfolio elasticity values (except for China; see Lemelin, 2007a).

One might also question the stability of the portfolio management model parameters over a fifteen year period, and point to the many volatile factors omitted from the model that intervene in financial markets. It should be emphasized however that the objective here is to show that, even assuming stable parameters, the exogenous current account balance hypotheses may not be compatible with rational international financial behavior. Moreover, computing the evolution of « global imbalances » consistent with trade scenarios is a way of displaying potential sources of instability that could make these scenarios improbable.

¹⁸ Strictly speaking, it is not the current account balance which is exogenous in *MIRAGE*, but the sum of the trade surplus and net income from capital owned abroad, minus net foreign direct investments.

Next, it is not entirely satisfactory that the composite international financial asset of each country consists of shares in a world mutual fund that includes liabilities of that same country¹⁹. The model could be improved in that respect if it were possible to construct bilateral (origin-destination) international financial data. Similarly, when several countries are aggregated into a single country-group, it would be preferable to distinguish investments by investors from the same country, and investments from one country to other countries within the same group of countries²⁰. Finally, it would be desirable (and, I conjecture, relatively easy) to relax the constraint which forces the owners of physical assets to keep the capital inherited from the past, in order to extend the allocation procedure to *all* physical assets.

So there are several desirable improvements that could be applied to *MIRAGE-D*. But it does run and produce economically sensible results. Comparisons with the original *MIRAGE* (without financial assets) have shown that the evolution of international investment positions matters. This is sufficient motivation to pursue its development.

¹⁹ Maybe that is not totally unlikely, however, since the single agent representing each country in the model is really an abstraction from the large number of actual agents whose aggregate behavior it is meant to represent.

²⁰ Thanks to David Laborde for having drawn my attention to that point.

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APPENDIX : DATA SOURCES

Most of the data used to calibrate *MIRAGE-D* are the same that underlie *MIRAGE*, and are described in Appendix 1 of Bchir *et al.* (2002). This includes data on foreign direct investments, of which an updated version was provided to the author by David Laborde of the CEPII. Other data sources are described in what follows.

Data on country international investment positions (IIP)

Under the title « The External Wealth of Nations, Mark II » (EWN-II), Lane and Milesi-Ferretti (2006) have published downloadable international financial data. The data file relates to individual country IIP's for the years 1970-2004, and includes the following variables :

- Portfolio equity assets
- Portfolio equity liabilities
- Foreign direct investment (FDI) assets
- Foreign direct investment (FDI) liabilities
- Debt assets (portfolio debt + other investment)
- Debt liabilities (portfolio debt + other investment)
- financial derivatives (assets)
- financial derivatives (liabilities)
- Total reserves minus gold
- Net external position

In order to be usable in *MIRAGE-D*, the Lane and Milesi-Ferretti data needed to be balanced. It had been known for some time that, because of data inconsistencies, the world total of current account surpluses and deficits is not zero, but negative. Lane and Milesi-Ferretti showed that the same occurs with the sum of international assets and liabilities. That problem had to be solved in order for the data to be usable for calibration purposes.

Also, EWN-II data on foreign direct investments are different from the *MIRAGE* data mentioned above, which have been elaborated from other sources. And EWN-II data contain no bilateral (origin-destination, by industry) information, while *MIRAGE* data do. For that reason, and for *MIRAGE-D* to rest as much as possible on the same data base as *MIRAGE* for comparison purposes, the EWN-II FDI information was discarded. The remainder of EWN-II data, once aggregated into 14 regions, were adjusted using cross-entropy minimization (MinXEnt), as generalized by Junius and Oosterhaven (2003) to accommodate negative data.

After data adjustment, the base-year value of countries' international financial assets is calibrated in *MIRAGE-D* as the sum of :

Portfolio equity assets
Debt assets (portfolio debt + other investment)
Financial derivatives (assets)
Total reserves minus gold

The base-year value of international financial liabilities is the sum of :

Portfolio equity liabilities
Debt liabilities (portfolio debt + other investment)
Financial derivatives (liabilities)

Balance of payments data

The International Monetary Fund's *Balance of Payments Statistics Yearbook* (BOPSY) provides information on :

01- 1. Current account

02- 1.A&B Goods and services

03- 1.C. Income

04- 1.C.1. Compensation of employees including border, seasonal, and other workers

05- 1.C.2. Investment income

06- 1.C.2.1 Direct investment

07- 1.C.2.2 Portfolio investment

08- 1.C.2.3 Other investment

09- 1.D. Current transfers

10- 1.D.1. General government

11- 1.D.2. Other sectors

12- 1.D.2.1 Workers' remittances

13- 1.D.2.2 Other transfers

14- 2. Capital and Financial Acct

15- 2.E. Capital Account

16- 2.E.1. Capital transfers

17- 2.E.2. Acquisitions/disposal of non-produced nonfinancial assets

18- 2.F. Financial Account

19- 2.F.1. Direct investment

20- 2.F.1.1 Abroad

21- 2.F.1.2 In reporting economy

22- 2.F.2. Portfolio investment

23- 2.F.2.1 Assets

24- 2.F.2.2 Liabilities

25- 2.F.3. Financial derivatives (net)

26- 2.F.4. Other investment

27- 2.F.4.1 Assets

28- 2.F.4.2 Liabilities

29- 2.F.5. Official reserve assets

30- 3. Net errors and omissions

For each variable, the BOPSY gives debit, credit, and net amounts. Most of the balance of payments data for individual countries are balanced, thanks to the item « Net errors and omissions ». However, after aggregating into 14 regions, we noticed that, for certain regions, some variables are subtotals whose value

does not equal the sum of their components. When the components played no part in the adjustment process described below, it didn't matter, and we chose to keep the subtotal figure. In the opposite case, a correction was applied, based on the assumption that subtotals are more reliable than their components, by modifying the component with the highest value.

Next, the BOPSY data was replaced with the corresponding *MIRAGE* data-base values for :

02- 1.A&B Goods and services

06- 1.C.2.1 Direct investment

19- 2.F.1. Direct investment

The rest of the data were adjusted using the Junius-Oosterhaven (2003) generalized cross-entropy minimization method. The adjusted data obeys the world zero-sum constraint of debits and credits for each variable, as well as balance of payments equilibrium for each region, without a « Net errors and omissions » term, while being consistent with previously calibrated *MIRAGE* values.

LIST OF EQUATIONS

$$MAX_{a_i} VC = \sum_i \xi_i a_i, \text{ where } \xi_i = (1 + r_i)q_i \quad [001]$$

$$W = A_w \left[\sum_i \alpha_i a_i^\beta \right]^\frac{1}{\beta} \quad [002]$$

$$\tau = \frac{1}{1 - \beta} \quad (\beta > 1) \quad [003]$$

$$\sum_i q_i a_i = W \quad [004]$$

$$q_i a_i = W \frac{\alpha_i^\tau q_i \xi_i^{-\tau}}{\sum_j \alpha_j^\tau q_j \xi_j^{-\tau}} \quad [005]$$

$$W = A_w \left[\sum_i \alpha_i (q_i a_i)^\beta \right]^\frac{1}{\beta} \quad [006]$$

$$b_i = q_i a_i \quad [007]$$

$$MAX_{b_i} VC = \sum_i (1 + r_i) b_i \quad [008]$$

$$W = A_w \left[\sum_i \alpha_i b_i^\beta \right]^\frac{1}{\beta} \quad [009]$$

$$\sum_i b_i = W \quad [010]$$

$$b_i = W \frac{\alpha_i^\tau (1 + r_i)^{-\tau}}{\sum_j \alpha_j^\tau (1 + r_j)^{-\tau}} \quad [011]$$

$$K_{i,r,s,t} = (1 - \delta_{i,s}) K_{i,r,s,t-1} + INV_{i,r,s,t} \quad [012]$$

$$PV_{i,s,t} = \sum_{\theta=0}^{\infty} \left(\frac{1 - \delta_{i,s}}{1 + \phi_{i,s,t}} \right)^\theta R_{i,s,t} = \frac{1 + \phi_{i,s,t}}{\phi_{i,s,t} + \delta_{i,s}} R_{i,s,t} \quad [013]$$

$$\zeta_{i,s,t} = \frac{(\phi_{i,s,t} + \delta_{i,s})}{(1 + \phi_{i,s,t})} = \frac{1}{\sum_{\theta=0}^{\infty} \left(\frac{1 - \delta_{i,s}}{1 + \phi_{i,s,t}} \right)^{\theta}} \quad [014]$$

$$PV_{i,s,t} = \sum_{\theta=0}^{\infty} \left(\frac{1 - \delta_{i,s}}{1 + \phi_{i,s,t}} \right)^{\theta} R_{i,s,t} = \frac{1}{\zeta_{i,s,t}} R_{i,s,t} \quad [015]$$

$$\rho_{i,s,t} = \frac{R_{i,s,t}}{PK_{s,t}} \quad [016]$$

$$U_{i,s,t} = \zeta_{i,s,t} PK_{s,t} = \frac{(\phi_{i,s,t} + \delta_{i,s})}{(1 + \phi_{i,s,t})} PK_{s,t} \quad [017]$$

$$\frac{(1 + \phi_{i,s,t}) R_{i,s,t}}{(\phi_{i,s,t} + \delta_{i,s}) PK_{s,t}} = \frac{R_{i,s,t}}{\zeta_{i,s,t} PK_{s,t}} = \frac{R_{i,s,t}}{U_{i,s,t}} \quad [018]$$

$$\frac{Id_{i,s,t}}{KD_{i,s,t}} = \gamma_{i,s} \left(\frac{R_{i,s,t}}{U_{i,s,t}} \right)^{el_Id_{i,s}} \quad [019]$$

$$\frac{Id_{i,s,t}}{KD_{i,s,t}} = \gamma_{i,s} \left(\frac{R_{i,s,t}}{U_{i,s,t}} \right)^{el_ind_{i,s}} = \gamma_{i,s} \left(\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} \right)^{el_ind_{i,s}} \quad [020]$$

$$PK_{s,t} INV_{i,r,s,t} = DelK_{r,t} \frac{(\alpha_{i,r,s}^K)^{\tau_K} (1 + r_{i,s,t}^K)^{-\tau_K}}{\sum_z \sum_j (\alpha_{j,r,z}^K)^{\tau_K} (1 + r_{j,z,t}^K)^{-\tau_K}} \quad [021]$$

$$\sum_{\theta=0}^{\infty} \frac{1}{(1 + \phi_{i,s,t})^{\theta}} r_{i,s,t}^K = 1 \quad [022]$$

$$\sum_{\theta=0}^{\infty} \frac{1}{(1 + \phi_{i,s,t})^{\theta}} = \frac{1 + \phi_{i,s,t}}{\phi_{i,s,t}} \quad [023]$$

$$\sum_{\theta=0}^{\infty} \frac{1}{(1 + \phi_{i,s,t})^{\theta}} r_{i,s,t}^K = \frac{1 + \phi_{i,s,t}}{\phi_{i,s,t}} r_{i,s,t}^K = 1 \quad [024]$$

$$r_{i,s,t}^K = \frac{\phi_{i,s,t}}{1 + \phi_{i,s,t}} \quad [025]$$

$$PK_{s,t} Id_{i,s,t} = PK_{s,t} \sum_r INV_{i,r,s,t} \quad [026]$$

$$r_{i,s,t}^K \frac{1 + \phi_{i,s,t}}{\phi_{i,s,t}} = \zeta_{i,s,t} \frac{(1 + \phi_{i,s,t})}{(\phi_{i,s,t} + \delta_{i,s})} = 1 \quad [027]$$

$$\zeta_{i,s,t} = r_{i,s,t}^K + (1 - r_{i,s,t}^K) \delta_{i,s} \quad [028]$$

$$U_{i,s,t} = \zeta_{i,s,t} PK_{s,t} = [r_{i,s,t}^K + (1 - r_{i,s,t}^K) \delta_{i,s}] PK_{s,t} \quad [029]$$

$$\frac{\Delta N_{i,s,t}}{N_{i,s,t} + \Delta N_{i,s,t}} R_{i,s,t} KD_{i,s,t} = \zeta_{i,s,t} PK_{s,t} Id_{i,s,t} \quad [030]$$

$$\frac{\Delta N_{i,s,t}}{N_{i,s,t} + \Delta N_{i,s,t}} \rho_{i,s,t} KD_{i,s,t} = \zeta_{i,s,t} Id_{i,s,t} \quad [031]$$

$$\frac{\Delta N_{i,s,t}}{N_{i,s,t} + \Delta N_{i,s,t}} = \frac{\zeta_{i,s,t} Id_{i,s,t}}{\rho_{i,s,t} KD_{i,s,t}} \quad [032]$$

$$\rho_{i,s,t} PK_{s,t} KD_{i,s,t} - \zeta_{i,s,t} PK_{s,t} Id_{i,s,t} = PK_{s,t} (\rho_{i,s,t} KD_{i,s,t} - \zeta_{i,s,t} Id_{i,s,t}) \quad [033]$$

$$KD_{i,s,t} = (1 - \delta_{i,s}) KD_{i,s,t-1} + Id_{i,s,t} \quad [034]$$

$$\begin{aligned} \rho_{i,s,t} PK_{s,t} KD_{i,s,t} - \zeta_{i,s,t} PK_{s,t} Id_{i,s,t} \\ = PK_{s,t} \left\{ \rho_{i,s,t} \left[(1 - \delta_{i,s}) KD_{i,s,t-1} + Id_{i,s,t} \right] - \zeta_{i,s,t} Id_{i,s,t} \right\} \end{aligned} \quad [035]$$

$$\begin{aligned} \rho_{i,s,t} PK_{s,t} KD_{i,s,t} - \zeta_{i,s,t} PK_{s,t} Id_{i,s,t} \\ = PK_{s,t} \left[\rho_{i,s,t} (1 - \delta_{i,s}) KD_{i,s,t-1} + (\rho_{i,s,t} - \zeta_{i,s,t}) Id_{i,s,t} \right] \end{aligned} \quad [036]$$

$$\sum_{\theta=0}^{\infty} \left(\frac{1 - \delta_{i,s}}{1 + \phi_{i,s,t}} \right)^{\theta} PK_{s,t} \left[\rho_{i,s,t} (1 - \delta_{i,s}) KD_{i,s,t-1} + (\rho_{i,s,t} - \zeta_{i,s,t}) Id_{i,s,t} \right]$$

$$= \frac{1}{\zeta_{i,s,t}} PK_{s,t} \left[\rho_{i,s,t} (1 - \delta_{i,s}) KD_{i,s,t-1} + (\rho_{i,s,t} - \zeta_{i,s,t}) Id_{i,s,t} \right] \quad [037]$$

$$= PK_{s,t} \left[\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} (1 - \delta_{i,s}) KD_{i,s,t-1} + \left(\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} - 1 \right) Id_{i,s,t} \right] \quad [038]$$

$$= PK_{s,t} \left[\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} KD_{i,s,t} - Id_{i,s,t} \right] \quad [039]$$

$$\Delta N_{i,s,t} = \frac{\zeta_{i,s,t}}{\rho_{i,s,t}} \frac{Id_{i,s,t}}{KD_{i,s,t}} (N_{i,s,t} + \Delta N_{i,s,t}) \quad [040]$$

$$\left(1 - \frac{\zeta_{i,s,t}}{\rho_{i,s,t}} \frac{Id_{i,s,t}}{KD_{i,s,t}} \right) \Delta N_{i,s,t} = \frac{\zeta_{i,s,t}}{\rho_{i,s,t}} \frac{Id_{i,s,t}}{KD_{i,s,t}} N_{i,s,t} \quad [041]$$

$$\Delta N_{i,s,t} = \frac{\frac{\zeta_{i,s,t}}{\rho_{i,s,t}} \frac{Id_{i,s,t}}{KD_{i,s,t}}}{\left(1 - \frac{\zeta_{i,s,t}}{\rho_{i,s,t}} \frac{Id_{i,s,t}}{KD_{i,s,t}} \right)} N_{i,s,t} \quad [042]$$

$$\Delta N_{i,s,t} = \frac{1}{\left(\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} \frac{KD_{i,s,t}}{Id_{i,s,t}} - 1 \right)} N_{i,s,t} \quad [043]$$

$$\frac{PK_{s,t} Id_{i,s,t}}{\Delta N_{i,s,t}} = \frac{PK_{s,t} Id_{i,s,t}}{N_{i,s,t}} \left(\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} \frac{KD_{i,s,t}}{Id_{i,s,t}} - 1 \right) \quad [044]$$

$$\frac{PK_{s,t} Id_{i,s,t}}{\Delta N_{i,s,t}} = \frac{PK_{s,t}}{N_{i,s,t}} \left(\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} KD_{i,s,t} - Id_{i,s,t} \right) \quad [045]$$

$$\frac{PK_{s,t} Id_{i,s,t}}{\Delta N_{i,s,t}} \Delta N_{i,s,t} + \frac{PK_{s,t}}{N_{i,s,t}} \left(\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} KD_{i,s,t} - Id_{i,s,t} \right) N_{i,s,t} = \frac{\rho_{i,s,t}}{\zeta_{i,s,t}} PK_{s,t} KD_{i,s,t} \quad [046]$$

$$PK_{s,t} \left(\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} KD_{i,s,t} - Id_{i,s,t} \right) = PK_{s,t} \left[\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} (1 - \delta_{i,s}) KD_{i,s,t-1} + \left(\frac{\rho_{i,s,t}}{\zeta_{i,s,t}} - 1 \right) Id_{i,s,t} \right] \quad [047]$$