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A Simple Model of Performance-enhancing Goals^{*}

Daniel Léonard[†], Ngo Van Long[‡], Antoine Soubeyran[§]

Résumé / Abstract

Nous montrons que les objectifs choisis influencent le niveau d'effort et qu'un objectif bien choisi peut améliorer les performances. Nous déduisons une relation 'en forme de cloche' entre l'objectif et l'effort. Nous généralisons le modèle en introduisant une seconde période et démontrons que l'objectif choisi pour la première période influence le moral dans la deuxième période, et de là l'effort, l'objectif choisi et la performance en deuxième période. Les choix optimaux des objectifs sont caractérisés sous différentes hypothèses à propos des relations entre celui qui choisit l'objectif (l'entraîneur) et celui qui tente de l'atteindre (l'athlète).

Mots clés : effort, moral, performance, succès

We show that goal setting influences effort level, and that an appropriately set goal can enhance performance. We derive an inverted U-shaped relationship between the goal and the effort level. We then extend the model to a two-period framework, and demonstrate that the goal level set for period 1, together with success or failure, can affect the self-confidence level in period two, which in turn influences period-two optimal goal, effort, and performance. The optimal choice of period-one goal to maximize period-two payoff is derived, under alternative assumptions about the relationship between the goal setter and the (potential) goal achiever.

Keywords: confidence, effort, performance, success

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1 Introduction

Ability is necessary to success but, as it is a given, not much of interest can be said about it, except how to discover it. Attitude, training, confidence and strategy on the other hand, are policy variables and therefore worthy of study by the social scientist.

Business success is rarely the achievement of a single individual (these days are long gone). More often there is a complex structure where individuals have somewhat different objectives and different responsibilities towards one another. We do have CEO's who make decisions, but also boards of directors who influence the background and limits of these decisions. We have best-selling authors with a long time suffering editor on their back, giving them unwelcome but useful advice. We have athletes and coaches, as in the parable of this paper. The tasks of strategy formulation and the execution of decisions are often separate.

Is this two-tier structure just a historical oddity or one of the many imperfections of markets? If so, it is strange that it is so prevalent in organizations.

Our contention here is that it is not fortuitous and we attempt to provide a framework in which its usefulness emerges naturally.

For the purpose of this paper we simplify the above complex notions of execution and strategy to performance and goal-setting .

Goal setting has been known to be a crucial element in achieving success, be it in sports, education, academia, or in the business world. Psychologists, researchers in sports science and management science have emphasized the importance of appropriate goal setting. (See Locke and Latham (1990a, 1990b), Hardy et al. (1986), Bell (1983), Botterill (1983), Cury and Sarrazin (1993), among others.) It is generally recognized that, subject to goals being realistic, performance increases as the goals become more difficult. This is known as "the hard goal effect" . (For empirical evidence in sports, see Beggs,

1990, Cury and Sarrazin, 1993).

Clearly goals should not be too low, but there cannot be a monotone relationship between goal and performance; the world is not that easy. Therefore there must be an inverted U-shaped relationship between goal and performance. The first objective of this paper is to provide a framework in which this relationship emerges.

Psychologists have explained the upward-sloping portion of the goal-performance curve by using the expectancy-valence theory of motivation (Vrom (1964), Mitchell (1974), Arnold (1981), Levy-Leboyer (2003)). Valence is the affective evaluation of the outcome (for example, a promotion) and its by-products (for example, stress and fatigue). Expectancy is a person's assessment of his likelihood of achieving the set goal. While these two concepts are akin to economists' concepts of "utility" and "subjective probability", there are different nuances. For example, expectancy involves self-image, self-confidence (or self-doubt), which is partly a reflection of one's perception of the confidence others have in one's abilities and judgements. Motivation is not simply a product of valence and expectancy. A goal that one fixes for oneself may not have the same effect on motivation as a goal imposed by one's superior (Erez et al. (1983), Hollenbeck et al. (1989)¹). On the other hand, a "hard goal" assigned by your superior may indicate the latter has confidence in you, which in turn reinforces your attitude and stimulates your motivation. The more motivated a person is in performing a task, the more intense will be his efforts, which influences the probability of success. The importance of self-confidence is well acknowledged in the psychology literature (Bandura, 1977).

The U-shaped relationship that we derive bears some similarity to the Yerkes-Dobson law in the psychology literature (Petri, 1986, Yerkes and

¹They showed that participation in decision making stimulates motivation. On the other hand, a series of studies by Latham et al. (1979, 1992) showed that once goals are in fact accepted, then it made no difference how they were chosen.

Dobson, 1980). As Kaufman (1999, p. 137) put it, “this law states that the relationship between arousal and performance resembles an inverted U or bell-shaped curve.(...) Although controversy continues among psychologists over the correct specification, domain, and theoretical explanation of the law,(...) the relationship (...) has now been documented in a sufficiently large number of studies with human beings that (it) is one of the few in psychology to be called a ‘law’ (giving it roughly equal status to the ‘law of demand’ in economics).” In our formulation, we depict the goal level on the horizontal axis, and the expected performance level on the vertical axis. To the extent that a higher goal is correlated with a higher emotional intensity, our U shaped curve, derived from optimization behavior, is compatible with the “arousal-performance” relationship.

Our simple model can apply to many real-world situations, for both humans and animals. Dawkins(1976) reports that insects such as crickets seem to have evolving level of self-confidence. "Crickets have a general memory of what happens in past fights. A cricket which has recently won a large number of fights become more hawkish. A cricket which has recently had a losing streak becomes more dovish. This was neatly shown by R. D. Alexander. He used a model cricket to beat up real crickets. After this treatment the real crickets became more likely to lose fights against other real crickets. Each cricket can be thought of as constantly updating his own estimate of his fight ability, relative to that of an average individual in his population."(Dawkins, 1976, pp 88-89). Of course, success in a trivial task is no success at all. That is why the goal level should be appropriately set to maximize self-confidence and enhance future performance.

Our first task, in the next section, is to present a simple model that takes on board the interaction between goals, effort and performance and show that the empirically true U-shaped relationship emerges from it. We begin by analyzing the different performance outcomes, depending on who sets the

goals. Within this framework we then examine the incidence of asymmetric information. Finally, in order to more fully appreciate the strategic aspects of goal-setting, we study a two-period model where the all important self-confidence level evolves as a consequence of performance, but also of future goals.

2 A Simple One-Period Model

An individual's objective probability of achieving a given target such as an increased volume of sales, a higher examination score, may depend on a number of factors. Among these are (i) the individual's innate ability, (ii) the reward received if the target is met, (iii) the time available, (iv) the level of effort and (v) the level of self-confidence.

Let us consider at first the simple case of an individual for whom a goal is already set. For example, parents set a school performance target (such as exam scores) for their children, a swimmer sets for himself a target of swimming across a river in A seconds, a high school student sets the goal of being successful in an entrance examination to a prestigious university². There are many such instances in business and in sport (which is a business). The goal metric A can be smooth or abrupt. What is it worth being second in developing a product? Not much. What is it worth being a silver medalist in the Olympics? A lot. What about the loser in the World Cup soccer final? On the other hand, the exact time in which a 400 m hurdle or a marathon are run do matter a lot. We attempt to capture these facets of the outcomes of competition in the following way. In our simple model we use a metric A to measure the value of the goal but we also characterize the outcome as

²In Japan, admission to prestigious universities is based on highly competitive entrance examinations. One cannot sit for more than two exams in any given year. We thank Koji Shimomura for providing the information. The choice of A would correspond to the choice of the targeted university, as universities have different entrance standards.

success or failure. The probability of achieving a goal depends on the effort level, denoted by E , on the goal level A , as well as other factors such as on innate ability or self-confidence. Inspired by Tullock’s hypothesis about probability of success in a n -person contest for rents³, we assume

$$p = P(E, A, \lambda) = \lambda \left[\frac{E}{E + A} \right] = \lambda \left[1 - \frac{A}{E + A} \right] = \lambda \left[\frac{1}{1 + \frac{A}{E}} \right] \quad (1)$$

where $\lambda \in (0, 1]$ represents his *self-confidence*. Here A is non-negative real number that can be interpreted as the goal level. E is also a non-negative real number⁴.

From (1), for any given effort level, the higher is the target A , the lower is the probability of success. Similarly an increase in the effort level E , given A , will increase the probability of success.

2.1 The Athlete’s Choice

In what follows, we consider a “representative scenario”, where the individual is an athlete who may set his own goal or may be managed by a coach.

For any goal A , the athlete chooses his effort level $E \geq 0$. His objective

³See Tullock (1980), where the probability of success of agent i is

$$p_i = \frac{E_i}{E_i + \sum_{j \neq i} E_j}$$

Therefore, just a competitors’ efforts lower the probability of success in his formulation, in ours it is the set goal that plays that role. Instead of competing against other players, the athlete plays against "nature", although the state of nature, A , has been artificially set.

See also Hillman and Riley (1989) for a detailed treatment of the case of contests among heterogeneous agents.

⁴Here λ is assumed to be an objective parameter. It is different from another factor that is amply discussed in the psychology literature, namely that some individuals tend to overestimate their abilities and rate their own probabilities for favorable future events as above the true average (see Alloy and Abrahamson, 1979, Taylor and Brown, 1988, Weinstein, 1980), but some individuals suffer from self-doubt.

function is to maximize his expected utility net of effort cost:

$$\max_{E \geq 0} W(E) \equiv P(E, A, \lambda)u(A) - bE$$

where $b > 0$ is his cost per unit of effort E , and $u(A) \geq 0$ is his evaluation of the “prize” A , where $u(0) = 0$ and $u'(A) > 0$.

This formulation implies that either (i) the individual gets zero utility if he fails to achieve the set goal A (regardless of how “close” the actual performance is to the goal), or (ii) if the individual fails, he does not know how close he was to the goal (e.g., in some entrance examinations, or job interviews, one knows if one fails, but one is not told how badly one fails.) On the other hand the measurability of A accounts for the measurability of many sporting records and economic objectives.

We assume for simplicity that

$$u(A) = A^\beta \text{ where } 0 < \beta < 1$$

Then the athlete’s expected utility (net of effort cost) is

$$W(A, E; \lambda) = \lambda \left[\frac{E}{E + A} \right] A^\beta - bE \quad (2)$$

The first order condition is

$$\frac{\partial W}{\partial E} = \frac{\lambda A^{\beta+1}}{(E + A)^2} - b \leq 0 \quad (= 0 \text{ if } E > 0)$$

The second order condition is satisfied, because W is concave in E for all $E \geq 0$.

The first order condition implies

$$E^*(A; \lambda) = \begin{cases} \left(\sqrt{\frac{\lambda}{b}} \right) A^{(\beta+1)/2} - A \geq 0 & \text{if } A \leq \bar{A}(\lambda) \equiv \left(\frac{\lambda}{b} \right)^{1/(1-\beta)} \\ 0, & \text{if } A \geq \bar{A}(\lambda) \end{cases} \quad (3)$$

Clearly, $E^*(0, \lambda) = 0 = E^*(\bar{A}(\lambda), \lambda)$. The function $E^*(A, \lambda)$ as given by (3) has positive derivative at $A = 0$ and negative derivative at $A = \bar{A}(\lambda)$.

Thus it has the inverted U-shape property if the equation $\partial E^*/\partial A = 0$ has a unique solution, which is in fact the case (See remark (i) below). One interpretation is that, as the coach sets higher goals, the athlete is motivated, then discouragement sets in, and finally if the coach sets a goal larger than \bar{A} , the athlete quits.

Remarks:

(i) Effort E increases with A (the level of difficulty of the goal) for small A , and decreases with A for large A . Effort is maximized at $A = \tilde{A}(\lambda)$ where

$$\tilde{A}(\lambda) \equiv \left[\frac{\beta + 1}{2} \right]^{2/(1-\beta)} \bar{A}(\lambda) \quad (4)$$

We will refer to $\tilde{A}(\lambda)$ as the “**effort-maximizing goal**”.

(NOTE: the ratio $\tilde{A}(\lambda)/\bar{A}(\lambda)$ is increasing in $\beta \in [0, 1)$. It is 1/4 if $\beta = 0$, and tends to 1 as $\beta \rightarrow 1$.)

(ii) Given A , an increase in self-confidence will increase the effort level, hence the probability of success, after the athlete has chosen the effort accordingly.

(iii) Since the level of effort chosen by the athlete is a function of goal A , we can define the endogenous probability of success, given the goal A , as follows:

$$p(A, \lambda) \equiv P(E^*(A, \lambda), A, \lambda)$$

Using (1) and (3) we get:

$$p = \lambda \left[1 - \frac{A}{E^* + A} \right] = \lambda \left[1 - \frac{A}{\left(\sqrt{\frac{\lambda}{b}} \right) A^{(\beta+1)/2}} \right]$$

$$p(A, \lambda) = \lambda - \sqrt{b\lambda} (A^{(1-\beta)/2}) \quad \text{for } A \leq \bar{A}(\lambda) \quad (5)$$

or

$$p(A, \lambda) = \lambda \left[1 - \left(\frac{A}{\bar{A}(\lambda)} \right)^{(1-\beta)/2} \right] \quad \text{for } A \leq \bar{A}(\lambda) \quad (6)$$

Clearly, $p(\bar{A}(\lambda), \lambda) = 0$, $p(0, \lambda) = \lambda < 1$, and for all $A \in (0, \bar{A}(\lambda))$, $p(A, \lambda)$ is a **decreasing** function of A

$$\frac{\partial p}{\partial A} = -\lambda \left(\frac{1-\beta}{2} \right) \left(\frac{1}{\bar{A}(\lambda)} \right)^{(1-\beta)/2} A^{-(\beta+1)/2} < 0$$

Thus, the higher is the goal, the lower is the probability of achieving the target, again, once the effort level has been chosen.

(iv) Finally we can define the athlete's net utility as a function of A and λ , once effort has been chosen, as

$$V(A, \lambda) \equiv W(A, E^*(A, \lambda), \lambda)$$

Using (2), (3) and (4) we obtain, after simplification

$$V(A, \lambda) = \begin{cases} \left[\sqrt{b}A^{1/2} - \sqrt{\lambda}A^{\beta/2} \right]^2 & \text{if } A \leq \bar{A}(\lambda) \equiv \left(\frac{\lambda}{b}\right)^{1/(1-\beta)} \\ 0 & \text{if } A \geq \bar{A}(\lambda) \end{cases}. \quad (7)$$

Note that $V(0, \lambda) = 0 = V(\bar{A}(\lambda), \lambda)$.

2.2 Alternative Strategies

We now consider the two scenarios for goal-setting. Either the athlete acts independently and sets his own goal to maximize his net utility⁵ $V(A, \lambda)$ or the coach sets the goal in order to maximize expected performance

$$\pi(A, \lambda) = p(A, \lambda)A$$

The latter seems a reasonable objective criterion as the coach is not directly concerned by the athlete's effort and $u(A)$ reflects the athlete's personal assessment of the goal.

$$\pi(A; \lambda) = \begin{cases} \lambda \left[1 - \left(\frac{A}{\bar{A}(\lambda)} \right)^{(1-\beta)/2} \right] A & \text{if } A \leq \bar{A}(\lambda) \equiv \left(\frac{\lambda}{b}\right)^{1/(1-\beta)} \\ 0 & \text{if } A \geq \bar{A}(\lambda) \end{cases}. \quad (8)$$

⁵Another interpretation of this criterion is that the coach has the objective of maximizing the athlete's net utility.

This function is strictly concave in A for $A \in [0, \bar{A}(\lambda)]$. (Clearly the coach will not set $A \geq \bar{A}(\lambda)$, as it would result in the athlete quitting.)

The first-order condition for maximizing expected performance is

$$\lambda - \lambda \left(\frac{3 - \beta}{2} \right) \left(\frac{A}{\bar{A}} \right)^{(1-\beta)/2} = 0$$

Solving, we obtain the performance-maximizing goal

$$A^{**}(\lambda) = \left(\frac{2}{3 - \beta} \right)^{2/(1-\beta)} \bar{A}(\lambda) \quad (9)$$

This is less than $\bar{A}(\lambda)$ as expected.

Substituting (9) into (8) we obtain the maximum value for this problem:

$$\begin{aligned} \pi^{**}(\lambda) &= \pi(A^{**}(\lambda), \lambda) \\ \pi^{**}(\lambda) &= \left(\frac{1 - \beta}{3 - \beta} \right) \left(\frac{2}{3 - \beta} \right)^{2/(1-\beta)} \lambda \bar{A}(\lambda) \end{aligned} \quad (10)$$

or

$$\pi^{**}(\lambda) = \left(\frac{1 - \beta}{3 - \beta} \right) \left(\frac{2}{3 - \beta} \right)^{2/(1-\beta)} b^{1/(\beta-1)} \lambda^{(2-\beta)/(1-\beta)} \quad (11)$$

It is easy to see that $\pi^{**}(\lambda)$ is an increasing and convex function of λ , and a decreasing function of b . Note that the probability of success is now $p^{**}(\lambda) = p(A^{**}(\lambda), \lambda)$, hence using (6) and (9)

$$p^{**}(\lambda) = \lambda \left(\frac{1 - \beta}{3 - \beta} \right) < \lambda/3 \quad (12)$$

The fact that $\pi^{**}(\lambda)$ is increasing and convex indicates that a coach would be very keen indeed to work with more self-confident athletes.

We now turn to the characterization of outcomes when the athlete is self-managed and sets his own goal. His objective is naturally to maximize net

utility $V(A, \lambda)$. Clearly he won't choose a goal higher than or equal to $\bar{A}(\lambda)$. The first-order condition to maximize (7) reduces to

$$\sqrt{b}A^{-1/2} - \beta\sqrt{\lambda}A^{(\beta/2)-1} = 0$$

and the net utility maximizing goal is

$$A^*(\lambda) = \beta^{2/(1-\beta)} \left(\frac{\lambda}{b}\right)^{1/(1-\beta)} = \beta^{2/(1-\beta)}\bar{A}(\lambda) \quad (13)$$

This is also less than $\bar{A}(\lambda)$, as expected.

Substituting (13) into (7) we obtain the maximum value function for this problem

$$\begin{aligned} V^*(\lambda) &= V(A^*(\lambda), \lambda) \\ V^*(\lambda) &= \left(\frac{\beta}{\sqrt{b}}\right)^{2\beta/(1-\beta)} (\beta - 1)^2 \lambda^{1/(1-\beta)} = \beta^{2\beta/(1-\beta)} (\beta - 1)^2 \bar{A}(\lambda) \end{aligned} \quad (14)$$

Note that the probability of success is now

$$p^*(\lambda) = p(A^*(\lambda), \lambda) = \lambda [1 - \beta^\beta (1 - \beta)^{1-\beta}] \quad (15)$$

It is interesting to note that this probability is lowest when $\beta = 0.5$. Mild-mannered (when it comes to risk) athletes do worst. Cautious ones and dare-devils do better.

We now proceed to rank the various outcomes in terms of the goals that are set.

Proposition 1: *The net utility maximizing goal is the lowest, while the performance-maximizing goal is the highest; they bracket the effort-maximizing goal. The corresponding probabilities of success have the opposite ranking. These differences are exacerbated by the “aversion to risk” of the athlete -small β .*

To show this, all we need is to compare the coefficients of $\bar{A}(\lambda)$ in $A^*(\lambda)$ and $A^{**}(\lambda)$ with that of $\tilde{A}(\lambda)$. From (4) and (13),

$$\left(\frac{\beta+1}{2}\right)^{2/(1-\beta)} > \beta^{2/(1-\beta)} \text{ since } \beta < 1$$

From (4) and (9),

$$\left(\frac{\beta+1}{2}\right)^{2/(1-\beta)} < \left(\frac{2}{3-\beta}\right)^{2/(1-\beta)}$$

since $(\beta+1)(\beta+3) < 4$ or $-(\beta-1)^2 < 0$. Note that differences are sharpest when β is small and vanish when β approaches 1. Clearly

$$0 < A^*(\lambda) < \tilde{A}(\lambda) < A^{**}(\lambda) < \bar{A}(\lambda) \quad (16)$$

Since the derivative of (5) is negative, the endogenous probability of success is a decreasing function of A , the opposite ranking applies to them.

$$p^*(\lambda) > \tilde{p}(\lambda) > p^{**}(\lambda) > 0 \quad (17)$$

Corollary: *It follows that when left to his own devices the athlete will set a low goal below that at which effort is maximized. When the coach is in charge the goal is set much higher; the effort expended by the athlete is also higher. Indeed it is close to the maximum effort $E^*(\tilde{A}(\lambda), \lambda)$.*

We can calculate the effort expended by the athlete. Substituting (9) into (3) gives

$$E^*(A^{**}(\lambda), \lambda) = \bar{A}(\lambda) \left[\frac{1-\beta}{3-\beta} \right] \left[\frac{2}{3-\beta} \right]^{(1+\beta)/(1-\beta)} \quad (18)$$

Substituting (13) into (3) gives

$$E^*(A^*(\lambda), \lambda) = \bar{A}(\lambda) [1-\beta] \beta^{(1+\beta)/(1-\beta)} \quad (19)$$

It is easy to show that $E^*(A^{**}(\lambda), \lambda) > E^*(A^*(\lambda), \lambda)$, the difference being very large for small β and vanishing when β approaches 1 -when effort also approaches zero.

The maximum level of effort chosen by the athlete can be expressed in terms of λ only by using (3) and (4)

$$\tilde{E} = \bar{A}(\lambda) \left(\frac{1 - \beta}{2} \right) \left(\frac{\beta + 1}{2} \right)^{(1+\beta)/(1-\beta)}.$$

Numerical calculations (see Diagram 1) show that for most parameter ranges E^{**} and \tilde{E} are extremely close; thus the coach sets a goal that yields near maximum effort. E^* and \tilde{E} are usually very far apart. The exception is when $\lambda/b > 1$ and $\bar{A} = (\lambda/b)^{1/(1-\beta)}$ gets very large when β approaches 1, drowning the difference exhibited by (17).

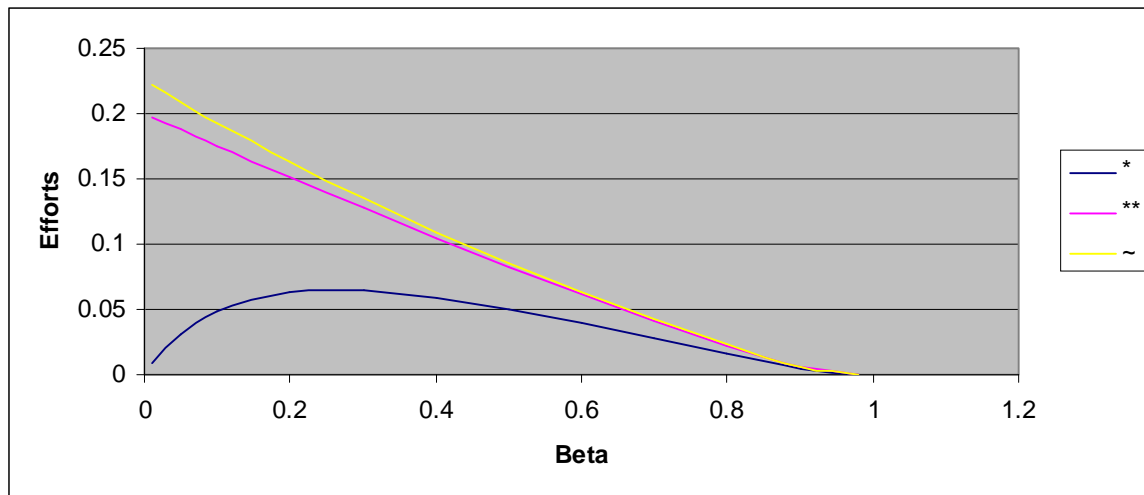


Diagram 1

Various Efforts over the Range of β

We have been able to establish an immutable ranking among the various goals, depending on which criterion is maximized. One of the feature of that ranking is that the magnitudes of the various ratios depend solely on β , the athlete’s “risk aversion.” The parameter b , the cost of effort, does not influence these magnitudes, though it does influence the value of \bar{A} , lowering \bar{A} and all other goals as it increases. For illustrative purposes we have calculated the goal ratios for a few values of β . (See Table 1).

TABLE 1

β	A^*/\bar{A}	\tilde{A}/\bar{A}	A^{**}/\bar{A}
0.1	0.0059	0.2648	0.4379
0.2	0.0178	0.2788	0.4312
0.3	0.0320	0.2920	0.4242
0.4	0.0471	0.3045	0.4170
0.5	0.0625	0.3164	0.4096
0.6	0.0777	0.3276	0.4018
0.7	0.0927	0.3384	0.3938
0.8	0.1073	0.3468	0.3855
0.9	0.1215	0.3584	0.3768

It is clear that an athlete would set for himself goals considerably below those a coach would set. It is also worth noting that the coach sets goals above the effort-maximizing level because he wants to maximize expected payoffs. He acts as if he were in some sense more of a gambler than the athlete and the difference is more marked, the more risk averse (low β) the athlete is.

Furthermore, in this section, our formulation of the probability of success (equation (1)) has been shown to be consistent with the empirical observations and the psychology literature in that it results in an inverted U-shaped relationship between goal and effort, and other facts, as indicated in the Remarks (i)-(iv) in subsection 2.1. We now proceed to build on this formulation.

3 The Consequence of Asymmetric Information

So far we have assumed that both coach and athlete know all the parameters of the problem. While it is perhaps reasonable to assume the athlete knows how confident he feels, it is less so for the coach. Suppose therefore that the coach cannot observe λ . Nonetheless he has great expertise in assessing the morale of athletes in his charge. The way we model this is to have the coach view the level of confidence as a random variable Λ and assume he

knows the probability distribution from which it is drawn. Therefore he has general information but not specific information. His objective must now be to maximize $\mathcal{E}(\pi(A, \Lambda))$ where \mathcal{E} is the expected value operator. Once the coach has set a goal A , the athlete will choose $E^*(A, \lambda)$ but the coach cannot predict accurately what it will be and views it as $E^*(A, \Lambda)$, a random variable. The coach will thus choose A to maximize, from (8),

$$\mathcal{E}(\pi(A, \Lambda)) = A\mathcal{E}(\Lambda) - \mathcal{E}(\Lambda^{1/2})\sqrt{b}A^{(3-\beta)/2} \quad (20)$$

Hence he will choose

$$A_{\mathcal{E}}^{**} = \left[\frac{\mathcal{E}(\Lambda)}{\mathcal{E}(\Lambda^{1/2})} \right]^{2/(1-\beta)} \left(\frac{2}{3-\beta} \right)^{2/(1-\beta)} \left(\frac{1}{\sqrt{b}} \right)^{2/(1-\beta)} \quad (21)$$

instead of

$$A^{**}(\lambda) = \left(\frac{2}{3-\beta} \right)^{2/(1-\beta)} \left(\sqrt{\frac{\lambda}{b}} \right)^{2/(1-\beta)}$$

as in the case of complete information. Therefore $\mathcal{E}(\Lambda)/\mathcal{E}(\Lambda^{1/2})$ replaces $\sqrt{\lambda}$. Clearly the ratio of these goals depends on the true λ value. However we can make a meaningful comparison by supposing that the coach's expectations are rational and suppose that the true λ value is $\mathcal{E}(\Lambda)$, hence compare $A_{\mathcal{E}}^{**}$ with $A^{**}(\mathcal{E}(\Lambda))$.

Proposition 2: *The coach always sets a higher goal when he cannot observe λ , compared to what he would have set, had he know the true value -and supposing the true value is the mean of the distribution. Incomplete information makes the coach bolder, not more prudent.*

To see this, simply note that $\mathcal{E}(\Lambda^{1/2}) < (\mathcal{E}(\Lambda))^{1/2}$ because the square root is a strictly concave function, so Jensen's inequality yields the inequality. Therefore

$$\frac{\mathcal{E}(\Lambda)}{\mathcal{E}(\Lambda^{1/2})} > [\mathcal{E}(\Lambda)]^{1/2}.$$

Proposition 3: *The larger the ratio of variance of $\Lambda^{1/2}$ over the square of its expected value, the larger the discrepancy between $A_{\mathcal{E}}^{**}$ and $A^{**}(\mathcal{E}(\Lambda))$.*

Proof: Make a change of variable $X = \Lambda^{1/2}$. We must compare $\mathcal{E}(X^2)/\mathcal{E}(X)$ with $[\mathcal{E}(X^2)]^{1/2}$. We know $\mathcal{E}(X^2)/\mathcal{E}(X) > [\mathcal{E}(X^2)]^{1/2}$, but by how much? Recall that

$$\mathcal{E}(X^2) = \text{var}(X) + [\mathcal{E}(X)]^2$$

Hence

$$\frac{\text{var}(X) + [\mathcal{E}(X)]^2}{\mathcal{E}(X)} > \sqrt{\text{var}(X) + [\mathcal{E}(X)]^2}$$

which yields

$$\sqrt{\text{var}(X) + [\mathcal{E}(X)]^2} > \mathcal{E}(X)$$

and

$$\sqrt{\frac{\text{var}(X)}{[\mathcal{E}(X)]^2} + 1} > 1$$

Example: Suppose Λ is uniformly distributed on $[\bar{\lambda} - \theta, \bar{\lambda} + \theta]$ where $0 < \bar{\lambda} - \theta < \bar{\lambda} + \theta < 1$. The density is $1/(2\theta)$ and the mean is $\bar{\lambda}$.

$$\mathcal{E}(\Lambda^{1/2}) = \frac{(\bar{\lambda} + \theta)^{3/2} - (\bar{\lambda} - \theta)^{3/2}}{3\theta}$$

In our calculations of goals, $3\theta \bar{\lambda} / [(\bar{\lambda} + \theta)^{3/2} - (\bar{\lambda} - \theta)^{3/2}]$ replaces $\sqrt{\bar{\lambda}}$. It is easy to show that the first expression tend to $\bar{\lambda}$ when $\theta \rightarrow 0$. Plotting these expressions for several λ values show that the larger θ , hence the more dispersion and the more incomplete the coach's information, the larger is the discrepancy and the higher he sets the goal. Therefore the less precisely informed the coach is, the higher he sets the goal. Note that this results in a lower effort by the athlete, since all these goals are above $\tilde{A}(\lambda)$.

These results do not augur well for unknown hopefuls, as they will be treated more harshly than athletes with whom the coach is acquainted. Competitive sports – and the entertainment industry, and high-flying marketing

and business ventures- are thus seen as careers where a positive introduction is most helpful. This seems to be borne out by casual observation.

4 Sequential goals set by the coach

In sports and in many other forms of endeavour, it is often the case that a series of sequential goals are set at the beginning, even though goals may be revised upon receiving new information. In any case, when agents know there is a sequence of contests, it makes sense for them to set their goals strategically. In this section, we model optimal goal setting by the coach; in the next section we will analyze goal setting by the athlete. The level of self-confidence will be affected (positively) by success or (negatively) by failure in the preceding period.

Suppose there are two periods. In period 1, the self-confidence level is λ_1 (which is given by history). Assume that λ_2 is a function of several factors: the actual outcome of the result in period 1 (e.g. success or failure or luck), the initial self-confidence level λ_1 , and the size of the goal in period 1.

We suppose that the athlete acts myopically in each period: given the goal A_t sets for period t , he simply chooses $E^*(A_t)$ as in the single period problem. The coach on the other hand thinks strategically and chooses the sequence (A_1, A_2) to maximize his objective, which we take to be $p_2 A_2$. In other words, the coach is interested in the ultimate performance of his athlete; earlier contests are viewed as try-outs. (In a non-sports application of the model, the try-outs could be a series of marketing campaigns with the goal being the ultimate market share.)

The dynamics of the problem are encapsulated in the level of self confidence the player has in the second period (the first period's λ_1 is given). We take the view that the athlete, whether he succeeds or fails to attain the goal, will have a higher level of confidence if he has done well in other dimensions

of the first period contest; thus we introduce a “luck” variable in period 1, Y_1 . This is observable before the coach sets the goal in period 2, but not in period 1. This Y_1 is not related to “success” or “failure” (because the probability of success or failure depends on E_1 and A_1 , while Y_1 is random and is assumed to be entirely independent of the values of A and E). Moreover, having aimed high will boost the athlete’s self-confidence, everything else equal.

Subscripts indicate the period.

As before we have

$$p_t = \lambda_t \left[\frac{E_t}{E_t + A_t} \right] \quad (22)$$

At the beginning of period 1, the athlete’s level of confidence is λ_1 , determined by his history. At the beginning of period 2, his confidence level λ_2 will also be known. However, at the beginning of period 1, it is a random variable, denoted by Λ_2 . It will turn out that we need to maximize various powers of Λ_2 . So we posit

$$\mathcal{E}\Lambda_2^\omega = \left[(1 - p_1)\lambda_1^{(1+\gamma)\omega} + p_1\lambda_1^{(1-\alpha)\omega} \right] \left(\frac{BA_1}{A_1} \right)^\omega \mathcal{E}h^\omega(Y_1) \quad (23)$$

where $0 < \alpha < 1$, $\gamma > 1$, $\omega > 0$ and we assume that $h(\cdot)$ is positively valued,

strictly increasing and bounded, Y_1 is independently distributed from success and failure, is symmetrically distributed around its mean, and $\mathcal{E}(Y_1) = 0$, $h(0) = 1$ and $\mathcal{E}[(h(Y_1)^\omega)] = 1$. B is a scaling parameter.

The random term $h(Y_1)$ accounts for the fact that the actual (irrespective of his preparation, just due to luck) performance of the athlete is unknown at the beginning of period 1 but will be known at the beginning of period 2. The term inside the square brackets in equation (23) represents the expectation of how the athlete's initial self-confidence and his preparation for the period 1's contest interact to mould his confidence level in the next period. Recall that $\lambda_1 < 1$. Therefore if the probability of failure $(1 - p_1)$ is large, this will impact negatively on the future λ_2 (since $\lambda_1^{1+\gamma} < \lambda_1$ for $\lambda_1 < 1$). Conversely, if the probability of success p_1 is large, the impact will be positive, as $\lambda_1^{1-\alpha} > \lambda_1$ for $\lambda_1 < 1$. How sharp or mild are these effects depends on the values of α and γ . As an indication of the strength of these effects, note the following calculations:

$$(0.5)^{0.1} = 0.93, (0.5)^{0.9} = 0.53, (0.5)^{1.2} = 0.43, (0.5)^4 = 0.06,$$

One of many candidates for $h(Y_1)$ could be

$$h(Y_1) = \begin{cases} 1 - \theta (e^{-Y_1} - 1) & \text{for } Y_1 \geq 0 \\ 1 + \theta (e^{Y_1} - 1) & \text{for } Y_1 \leq 0 \end{cases} . \quad (24)$$

with $Y_1 \sim N(0, 1)$. Then $h(Y_1)$ is bounded, $1 - \theta \leq h(Y_1) \leq 1 + \theta$, and $\mathcal{E}[h^\omega(Y_1)] = 1$, by symmetry. A large θ gives a large weight to "luck".

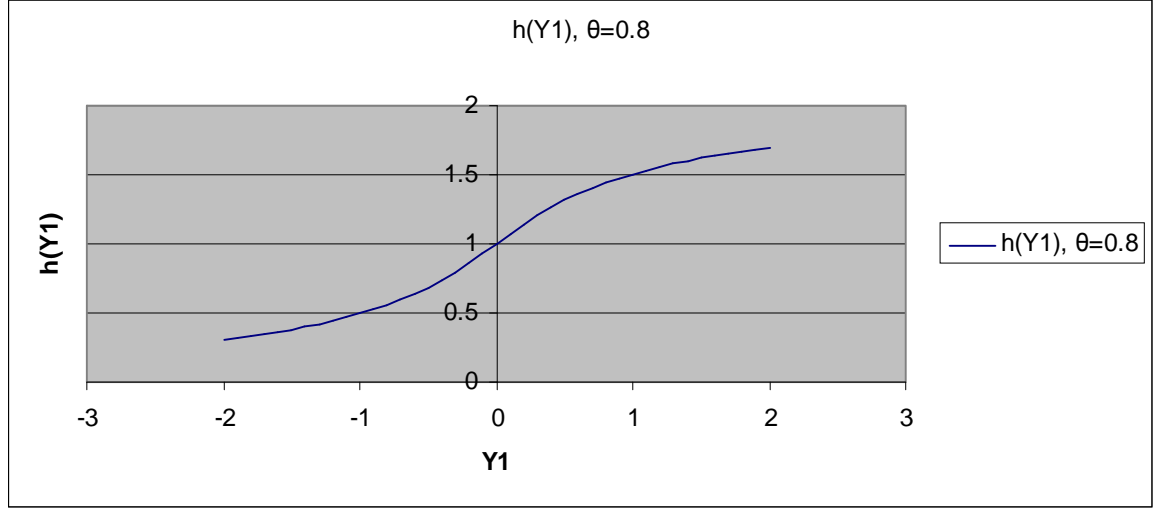


Diagram 2

Finally the term BA_1/\bar{A}_1 is a scaling factor reflecting the influence of the size of the set goal. The probability of success will boost self-confidence more, the higher the goal. Conversely the probability of failure will lessen self-confidence less sharply if the goal that was set was higher. Failure to reach a modest goal is more damaging to morale than failure to reach a very high goal. Success in reaching a high goal is more morale-boosting than success in reaching a low goal.

Equation (24) reflects the ex-ante distribution of Λ_2 . However, ex-post, it will be known whether the athlete has succeeded or not, and the realization of Y_1 , say y_1 , will also have been observed. Consequently the ex-post value of λ_2 will exhibit a dichotomy.

$$\left. \begin{aligned} \text{(Success 1)} \quad \lambda_2^S(\lambda_1) &= \lambda_1^{1-\alpha} \left(\frac{BA_1}{\bar{A}_1(\lambda_1)} \right) h(y_1) \\ \text{(Failure 1)} \quad \lambda_2^F(\lambda_1) &= \lambda_1^{1+\gamma} \left(\frac{BA_1}{\bar{A}_1(\lambda_1)} \right) h(y_1) \end{aligned} \right\} \quad (25)$$

Throughout, the athlete always chooses effort as in the single-period game, once the goal is set.

$$E_t^*(A_t, \lambda_t) = \begin{cases} \left(\sqrt{\frac{\lambda_t}{b}}\right) A_t^{(1+\beta)/2} - A_t & \text{if } A_t \leq \bar{A}_t(\lambda_t) \equiv \left(\frac{\lambda_t}{b}\right)^{1/(1-\beta)} \\ 0 & \text{if } A_t \geq \bar{A}_t(\lambda_t) \end{cases}. \quad (26)$$

NOTE: we can also express E_t^* as

$$E_t^*(A_t, \lambda_t) = A_t \min \left\{ \left[\left(\frac{\bar{A}_t(\lambda_t)}{A_t} \right)^{(1-\beta)/2} - 1 \right], 0 \right\}$$

In fact we will restrict the coach's choice to $A_t \leq \bar{A}_t$. We stated earlier that if the coach were to set $A_t \geq \bar{A}_t$, the athlete would quit.

4.1 The Coach's Strategy

The coach solves the game recursively, beginning with the last stage of the game when he chooses A_2 to maximize $p_2 A_2$. Note that at that time Y_1 has been realized and has taken on a value, say y_1 , and success or failure has been recorded; consequently Λ_2 has taken a value λ_2 . The coach's value of the game is $p_2(A_2^{**})A_2^{**}$ which is increasing in λ_2 . This is because, by an argument parallel to that used to establish equation (11),

$$p_2(A_2^{**})A_2^{**} = \left(\frac{2}{3-\beta}\right)^{2/(1-\beta)} b^{1/(\beta-1)} \left(\frac{1-\beta}{3-\beta}\right) \lambda_2^{(2-\beta)/(1-\beta)} \quad (27)$$

Note that while it was obvious that, everything else equal (in particular E_2 and A_2), a higher λ_2 was preferable (partial dependency), we have shown that this is so when the second period choices of both the athlete and the coach are optimally chosen (total dependency).

For given λ_1 and A_1 , the expected value of Λ_2^ω (where here $\omega = (2 - \beta)/(1 - \beta)$, the exponent of λ_2 in equation (27)) is, from (23)

$$\mathcal{E}(\Lambda_2^\omega) = \left(\frac{B}{A_1}\right)^\omega \left\{ \left[\lambda_1^{(1-\alpha)\omega} - \lambda_1^{(1+\gamma)\omega} \right] p_1 A_1^\omega + \lambda_1^{(1+\gamma)\omega} A_1^\omega \right\} \quad (28)$$

since

$$\mathcal{E}(h^\omega(Y_1)) = 1$$

Assuming that an interior solution prevails for the athlete, i.e. $E_1^*(A_1, \lambda_1) = \left(\sqrt{\lambda_1/b}\right) A_1^{(1+\beta)/2} - A_1$, we can use (5) to obtain $\mathcal{E}(\Lambda_2^\omega)$ in terms of λ_1 and A_1

$$\mathcal{E}(\Lambda_2^\omega) = \left(\frac{B}{A_1}\right)^\omega A_1^\omega \left[\lambda_1^{1+\omega(1-\alpha)} - \lambda_1^{\omega(1+\gamma)} (\lambda_1 - 1) \right] - \left(\frac{B}{A_1}\right)^\omega \left(\lambda_1^{\omega(1-\alpha)} - \lambda_1^{\omega(1+\gamma)} \right) \sqrt{b\lambda_1} \left(A_1^{(2\omega+1-\beta)/2} \right) \quad (29)$$

In period 1, the coach chooses A_1 to maximize $\mathcal{E}(\Lambda_2^\omega)$. The first-order condition is

$$\omega \left[\lambda_1^{1+\omega(1-\alpha)} - (\lambda_1 - 1) \lambda_1^{\omega(1+\gamma)} \right] = \left(\frac{2\omega + 1 - \beta}{2} \right) \left[\lambda_1^{\omega(1-\alpha)} - \lambda_1^{\omega(1+\gamma)} \right] \left(\sqrt{b\lambda_1} \right) A_1^{(1-\beta)/2}$$

This gives the coach's optimal choice of goal for period 1 for his two-period optimization problem:

$$A_1^{00}(\lambda_1) = \left[\frac{\lambda_1^{\omega(1-\alpha)} + (\lambda_1^{-1} - 1) \lambda_1^{\omega(1+\gamma)}}{\lambda_1^{\omega(1-\alpha)} - \lambda_1^{\omega(1+\gamma)}} \right]^{2/(1-\beta)} \left(\frac{2\omega}{2\omega + 1 - \beta} \right)^{2/(1-\beta)} \left(\frac{\lambda_1}{b} \right)^{1/(1-\beta)}$$

Thus

$$A_1^{00}(\lambda_1) = \left[\frac{1 + (\lambda_1^{-1} - 1) \lambda_1^{\omega(\alpha+\gamma)}}{1 - \lambda_1^{\omega(\alpha+\gamma)}} \right]^{2/(1-\beta)} \left(\frac{2\omega}{2\omega + 1 - \beta} \right)^{2/(1-\beta)} \left(\frac{\lambda_1}{b} \right)^{1/(1-\beta)} \quad (30)$$

or

$$A_1^{00}(\lambda_1) = \left[1 + \frac{1}{\lambda_1(\lambda_1^{-\omega(\alpha+\gamma)} - 1)} \right]^{2/(1-\beta)} \left(\frac{2\omega}{2\omega + 1 - \beta} \right)^{2/(1-\beta)} \left(\frac{\lambda_1}{b} \right)^{1/(1-\beta)} \quad (31)$$

where $\omega = (2 - \beta)/(1 - \beta) > 1$.

Compare this with the coach's optimal goal in the single-period problem

$$A_1^{**}(\lambda_1) = \left(\frac{2}{3 - \beta} \right)^{2/(1-\beta)} \left(\frac{\lambda_1}{b} \right)^{1/(1-\beta)}$$

$$\frac{A_1^{00}(\lambda_1)}{A_1^{**}(\lambda_1)} = \left[\left(1 + \frac{1}{\lambda_1(\lambda_1^{-\omega(\alpha+\gamma)} - 1)} \right) \left(\frac{\omega(3-\beta)}{2\omega+1-\beta} \right) \right]^{2/(1-\beta)}$$

which simplifies to

$$\frac{A_1^{00}(\lambda_1)}{A_1^{**}(\lambda_1)} = \left[\left(1 + \frac{1}{\lambda_1(\lambda_1^{-\omega(\alpha+\gamma)} - 1)} \right) \left(1 + \frac{1}{3-\beta+\frac{2}{1-\beta}} \right) \right]^{2/(1-\beta)}$$

Therefore, for all values of the parameters,

$$A_1^{00}(\lambda_1) > A_1^{**}(\lambda_1)$$

The coach sets a higher first-period goal A_1 in the two-period problem because it has a positive effect on the expected second period self-confidence of the athlete although it will produce a smaller effort in period 1.

Simulations (Table 2) shows that this goals ratio is fairly stable for most λ_1 values. It is between 1.5 and 2 for most parameter values. However it always rises abruptly when λ_1 gets very high. A coach will push a self-confident athlete quite hard, but will in any case push an athlete whom he coaches for two periods harder than one he guides for one period only.

TABLE 2: The coach's goals ratio

	example 1	example 2	example 3
	$\alpha = 0.8, \beta = 0.5$	$\alpha = 0.5, \beta = 0.5$	$\alpha = 0.8, \beta = 0.2$
	$\gamma = 10.5, \omega^{**} = 3$	$\gamma = 6.5, \omega^{**} = 3$	$\gamma = 9.5, \omega^{**} = 2.25$
λ_1	A_1^{00}/A_1^{**}	A_1^{00}/A_1^{**}	A_1^{00}/A_1^{**}
0.1	1.772522	1.772522	1.540500
0.2	1.772522	1.772522	1.540500
0.3	1.772522	1.772522	1.540500
0.4	1.772522	1.772522	1.540500
0.5	1.772522	1.772529	1.540501
0.6	1.772522	1.772781	1.540546
0.7	1.772579	1.778189	1.541915
0.8	1.777124	1.856477	1.568132
0.9	2.011571	2.957267	1.981268

Let us use the notation

$$s^{00} \equiv \left[1 + \frac{1}{\lambda_1(\lambda_1^{-\omega(\alpha+\gamma)} - 1)} \right]^{2/(1-\beta)} \left(\frac{2\omega}{2\omega + 1 - \beta} \right)^{2/(1-\beta)}$$

so that

$$A_1^{00}(\lambda_1) = s^{00} \bar{A}_1(\lambda_1) \quad (32)$$

Remarks:

1. The expression for s^{00} is not smaller than 1 for all assignments of α, β, γ and λ_1 . A problem may arise if for instance λ_1 is already large. However a very large γ can always keep A_1^{00} below \bar{A}_1 . The reason for wanting to keep A_1^{00} below \bar{A}_1 is that if A_1^{00} exceeds \bar{A}_1 , the athlete would choose $E^* = 0$ (or quit) and he would fail with probability 1. His $\mathcal{E}\Lambda_2^\omega$ would then become $\mathcal{E}[h^\omega(Y_1)] \lambda_1^{\omega(1+\gamma)} (A_1/\bar{A}_1)$ which would mean that the coach would perversely rely on only one of the mechanisms described by Λ_2 , namely the effect of the size of the goal. We wish to rule out such an outcome and assume $s^{00} < 1$. (In practice this means that some high λ_1 values are out of range.) (See column 2 of Table 3 for illustrations.)

2. The expression for $\mathcal{E}(\Lambda_2^\omega)$ in (23) has three factors. The first and the third are bounded by 1 but we have chosen not to bound the second as three factors smaller than 1 would tend to favour a decrease in self-confidence, which is unwarranted. The suggestion we made for the form of $h(\cdot)$ shows that by varying θ , λ_2 can be scaled. In any case, the upper bound of 1 on λ_2 is not essential. As we show below (equation (46)), all we need is that $p_2 = \lambda_2(1-\beta) \leq 1$. Therefore in all our simulations we have set $B = 1/(1-\beta)$. It is still not possible to maintain $s^{00} < 1$ for all λ_1 values and the highest admissible λ_1 value is usually around 0.9.

We now calculate the realized value λ_2 . Two separate events have occurred: (i) the athlete has either succeeded or failed, and (ii) the random variable Y_1 has taken on a value y_1 . The effect of the second event is straightforward; $\mathcal{E}(h(Y_1))$ is replaced by $h(y_1)$. As for the first event, recall that in the $\mathcal{E}(\Lambda_2^\omega)$ expression we had two non-linear functions of λ_1 weighted by the probability of success and failure, namely $(1-p_1)\lambda_1^{\omega(1+\gamma)} + p_1\lambda_1^{\omega(1-\alpha)}$. Now that either success or failure is known to have occurred we need two separate formulas as we did in equation (25). If success (respectively, failure) has occurred the ex-post probability of success (respectively, failure) is simply 1. Thus we have

$$\begin{aligned} \text{(Success)} \quad \lambda_2^S(\lambda_1) &= Bh(y_1)\lambda_1^{1-\alpha} \left(\frac{A_1^{00}}{A_1} \right) \\ &= Bh(y_1)\lambda_1^{1-\alpha} \left(\frac{2\omega}{2\omega + 1 - \beta} \right)^{2/(1-\beta)} \left[1 + \frac{1}{\lambda_1(\lambda_1^{-\omega(\alpha+\gamma)} - 1)} \right]^{2/(1-\beta)} \end{aligned} \quad (33)$$

$$\begin{aligned} \text{(Failure)} \quad \lambda_2^F(\lambda_1) &= Bh(y_1)\lambda_1^{1+\gamma} \left(\frac{A_1^{00}}{A_1} \right) \\ &= Bh(y_1)\lambda_1^{1+\gamma} \left(\frac{2\omega}{2\omega + 1 - \beta} \right)^{2/(1-\beta)} \left[1 + \frac{1}{\lambda_1(\lambda_1^{-\omega(\alpha+\gamma)} - 1)} \right]^{2/(1-\beta)} \end{aligned} \quad (34)$$

where $\omega = (2 - \beta)/(1 - \beta) > 1$. Clearly $\lambda_2^S(\lambda_1) > \lambda_2^F(\lambda_1)$.

Corresponding to these two λ_2 values, the coach sets a goal, as in equation (9),

$$A_2^{**}(\lambda_2) = \left(\frac{2}{3-\beta}\right)^{2/(1-\beta)} \bar{A}_2 = \left(\frac{2}{3-\beta}\right)^{2/(1-\beta)} \left(\frac{\lambda_2}{b}\right)^{1/(1-\beta)}$$

which may take on two values

either (Success) $A_2^{**S}(\lambda_2) =$

$$\left(\frac{2}{3-\beta}\right)^{2/(1-\beta)} \left[\frac{Bh(y_1)\lambda_1^{1-\alpha}}{b}\right]^{1/(1-\beta)} \left(\frac{2\omega}{2\omega+1-\beta}\right)^{2/(1-\beta)^2} \left[1 + \frac{1}{\lambda_1(\lambda_1^{-\omega(\alpha+\gamma)} - 1)}\right]^{2/(1-\beta)^2} \quad (35)$$

or (Failure) $A_2^{**F}(\lambda_2) =$

$$\left(\frac{2}{3-\beta}\right)^{2/(1-\beta)} \left[\frac{Bh(y_1)\lambda_1^{1+\gamma}}{b}\right]^{1/(1-\beta)} \left(\frac{2\omega}{2\omega+1-\beta}\right)^{2/(1-\beta)^2} \left[1 + \frac{1}{\lambda_1(\lambda_1^{-\omega(\alpha+\gamma)} - 1)}\right]^{2/(1-\beta)^2} \quad (36)$$

Again $A_2^{**S} > A_2^{**F}$.

We can also calculate the the second period effort

$$E_2^*(A_2, \lambda_2) = \left(\sqrt{\frac{\lambda_2}{b}}\right) (A_2^{**})^{(1+\beta)/2} - A_2^{**}$$

Note that, again,

$$A_2^{**} = \left(\frac{2}{3-\beta}\right)^{2/(1-\beta)} \bar{A}_2$$

but \bar{A}_2 is now endogenous.

$$\begin{aligned} \left(\sqrt{\frac{\lambda_2}{b}}\right) &= (\bar{A}_2)^{(1-\beta)/2} \\ E_2^* &= (\bar{A}_2)^{(1-\beta)/2} \left(\frac{2}{3-\beta}\right)^{(1+\beta)/(1-\beta)} (\bar{A}_2)^{(1+\beta)/2} - \left(\frac{2}{3-\beta}\right)^{2/(1-\beta)} \bar{A}_2 \\ E_2^* &= \bar{A}_2 \left(\frac{2}{3-\beta}\right)^{(1+\beta)/(1-\beta)} \left[\frac{1-\beta}{3-\beta}\right] \end{aligned} \quad (37)$$

Thus the athlete who was successful in period 1 tries harder in period 2. However the ratio

$$\frac{A_2^{**}}{E_2^*} = \frac{2}{1 - \beta}$$

is the same in both cases. The second period probability of success is

$$p_2 = \lambda_2 \left[\frac{E_2^*}{E_2^* + A_2^{**}} \right] = \lambda_2 \left[\frac{1}{1 + \frac{A_2^{**}}{E_2^*}} \right] = \frac{\lambda_2(1 - \beta)}{3 - \beta} < \frac{\lambda_2}{3} \quad (38)$$

Therefore the admissible upper-bound of λ_2 is 3 here.

The ratio of second period expected payoff in the event of first-period success to the payoff in the event of first-period failure is

$$\frac{p_2(A_2^{**S})A_2^{**S}}{p_2(A_2^{**F})A_2^{**F}} = \frac{\lambda_2^S A_2^{**S}}{\lambda_2^F A_2^{**F}} = \left(\frac{\lambda_2^S}{\lambda_2^F} \right) \left(\frac{\lambda_2^S}{\lambda_2^F} \right)^{1/(1-\beta)} = (\lambda_1)^{-(\alpha+\gamma)\left[\frac{2-\beta}{1-\beta}\right]}$$

This power is negative and (since $\alpha + \gamma > 1$) greater than 2 in absolute value. The values of this ratio are calculated below for different values of λ_1 for illustration (where $(\alpha + \gamma)(2 - \alpha)/(1 - \beta) = 3$):

$$(0.1)^{-3} = 1000, (0.5)^{-3} = 8, (0.9)^{-3} = 1.37$$

Therefore success in period 1 can increase the expected payoff of athletes manyfold, particularly for those who start with a low self-esteem. Note also that goal and effort increase in the same ratio, therefore successful athletes will try much harder in period 2. Success in period 1 will lead to higher self-confidence and a higher goal set by the coach, as well as a higher effort by the athlete. To understand this last point we must remember that a higher self-confidence level (λ_2) increases the range ($\bar{A}_2(\lambda_2)$) over which effort (E_2) is determined.

The probability of success in period 2 is also affected by the realization of the random variable Y_1 . “Luck” will increase self-confidence, and the

probability of success in period 2, by the factor $h(y_1) > 0$. This will also lead a coach to set a higher goal and the athlete to try harder. However the “luck” effect will be proportionately the same, whether the athlete meets with success or failure in period 1.

4.2 Simulations

Although our primary purpose in this paper is to analyze the qualitative differences between one-period and two-period outcomes and contrast outcomes when either the coach or the athlete is in charge of strategy, it is of interest to give a quantitative account of these differences.

Surprisingly this is relatively easy to do objectively in the context of our model. If we take for granted equations (1), (2) and (23), that is, the laws governing the probability of success, the athlete’s utility and the laws governing the revision of self-confidence, there are few parameters for which we do not know the range. One of them is b , the cost of effort, that influences the size of \bar{A} -but not comparisons between levels. In our simulations we have opted for a neutral $b = 1$. As we indicated in Section 4.1, we choose $B = 1/(1 - \beta)$ as the common scaling factor to insure that $p_2 = \lambda_2(1 - \beta)$ remains below 1. The terms s^0 and s^{00} must also be monitored so that they remain below 1 -see Remark 1 that follows equation (32). Unfortunately this is not possible for all λ_1 values on $[0, 1]$ and we sometimes have to exclude λ_1 values near 0.9 from our simulations. (Inadmissible values are indicated in boldface in the tables.)

TABLE 3(a) : $\alpha = 0.8, \beta = 0.5, \gamma = 10.5, \omega^{**} = 3$

λ_1	s^{00}	λ_2^S	λ_2^F	A_2^S	A_2^F	E_2^S	E_2^F	p_2^S	p_2^F
0.1	0.73	0.916	$5E - 12$	0.3438	$8.9E - 24$	0.0860	$2.2E - 24$	0.18324	$9.2E - 13$
0.2	0.73	1.052	$1E - 8$	0.4537	$7.2E - 17$	0.1134	$1.8E - 17$	0.21048	$2.7E - 09$
0.3	0.73	1.141	$1E - 6$	0.5335	$8.1E - 13$	0.1334	$2E - 13$	0.22826	$2.8E - 07$
0.4	0.73	1.209	$4E - 5$	0.5986	$6.1E - 10$	0.1497	$1.5E - 10$	0.24178	$7.7E - 06$
0.5	0.73	1.264	0.0005	0.6545	$1E - 7$	0.1636	$2.6E - 08$	0.25282	0.00010
0.6	0.73	1.311	0.0041	0.7040	$6.8E - 6$	0.1760	$1.7E - 06$	0.26221	0.00082
0.7	0.73	1.352	0.0240	0.7488	0.00024	0.1872	$5.9E - 05$	0.27042	0.00480
0.8	0.73	1.392	0.1119	0.7940	0.00512	0.1985	0.00128	0.27846	0.02237
0.9	0.82	1.614	0.4906	1.1064	0.09858	0.2666	0.02464	0.32270	0.09812

TABLE 3(b) : $\alpha = 0.5, \beta = 0.5, \gamma = 6.5, \omega^{**} = 3$

λ_1	s^{00}	λ_2^S	λ_2^F	A_2^S	A_2^F	E_2^S	E_2^F	p_2^S	p_2^F
0.1	0.73	0.459	$5E - 8$	0.0864	$8.6E - 16$	0.0216	$2.2E - 16$	0.09184	$9.2E - 09$
0.2	0.73	0.649	$8E - 6$	0.1727	$2.8E - 11$	0.0432	$7.1E - 12$	0.12988	$1.7E - 06$
0.3	0.73	0.795	0.0002	0.2591	$1.2E - 08$	0.0648	$3.1E - 09$	0.15906	$3.5E - 05$
0.4	0.73	0.918	0.0015	0.3454	$9.3E - 07$	0.0864	$2.3E - 07$	0.18367	0.00030
0.5	0.73	1.027	0.0080	0.4318	$2.6E - 05$	0.1080	$6.6E - 06$	0.20535	0.00160
0.6	0.73	1.125	0.0315	0.5183	0.00041	0.1296	0.00010	0.22498	0.00630
0.7	0.73	1.219	0.1004	0.6084	0.00413	0.1521	0.00103	0.24375	0.02007
0.8	0.77	1.360	0.2853	0.7579	0.03333	0.1895	0.00833	0.27205	0.05705
0.9	1.21	2.298	1.0993	2.1635	0.49495	0.5409	0.12374	0.45965	0.21985

TABLE 3(c) : $\alpha = 0.8, \beta = 0.2, \gamma = 9.5, \omega^{**} = 2.25$

λ_1	s^{00}	λ_2^S	λ_2^F	A_2^S	A_2^F	E_2^S	E_2^F	p_2^S	p_2^F
0.1	0.66	0.524	$3E - 11$	0.1922	$2.6E - 14$	0.0769	$1E - 14$	0.14969	$7.5E - 12$
0.2	0.66	0.602	$4E - 08$	0.2286	$2.3E - 10$	0.0914	$9.2E - 11$	0.17195	$1.1E - 08$
0.3	0.66	0.653	$3E - 06$	0.2529	$4.7E - 08$	0.1012	$1.9E - 08$	0.18647	$7.7E - 07$
0.4	0.66	0.691	$6E - 05$	0.2718	$2E - 06$	0.1087	$9.2E - 07$	0.19751	$1.6E - 05$
0.5	0.66	0.723	0.0006	0.2874	$3.8E - 05$	0.1150	$1.5E - 05$	0.20653	0.00016
0.6	0.66	0.750	0.0039	0.3008	0.00042	0.1203	0.00017	0.21420	0.00111
0.7	0.66	0.774	0.0196	0.3130	0.00317	0.1252	0.00127	0.22111	0.00561
0.8	0.68	0.808	0.0812	0.0812	0.01868	0.1322	0.00747	0.23095	0.02319
0.9	0.85	1.046	0.3532	0.3532	0.11743	0.1824	0.04697	0.29875	0.10093

TABLE 3(d) : $\alpha = 0.8, \beta = 0.2, \gamma = 5, \omega^{**} = 2.25$

λ_1	s^{00}	λ_2^S	λ_2^F	A_2^S	A_2^F	E_2^S	E_2^F	p_2^S	p_2^F
0.1	0.66	0.524	$3E - 05$	0.1922	$8.1E - 07$	0.0769	$3.2E - 07$	0.14969	$7.5E - 06$
0.2	0.66	0.602	0.0006	0.2286	$4E - 05$	0.0914	$1.6E - 05$	0.17195	0.00017
0.3	0.66	0.653	0.0037	0.2530	0.00039	0.1012	0.00016	0.18648	0.00105
0.4	0.66	0.692	0.0135	0.2721	0.00198	0.1088	0.00079	0.19769	0.00384
0.5	0.67	0.727	0.0369	0.2896	0.00698	0.1158	0.00279	0.20779	0.01055
0.6	0.68	0.772	0.0859	0.3122	0.02005	0.1249	0.00802	0.22067	0.02454
0.7	0.74	0.867	0.1870	0.3607	0.05303	0.1443	0.02121	0.24766	0.05343
0.8	0.97	1.159	0.4439	0.5184	0.15622	0.2073	0.06249	0.33105	0.12682
0.9	2.24	2.746	1.7457	1.5244	0.86526	0.6097	0.34610	0.78462	0.49877

In Table 3, the values of $\lambda_2, A_2, E_2,$ and p_2 are calculated in the event of success or failure, respectively, and for a range of λ_1 values. Only in two instances does a λ_1 value (0.9) yields inadmissible values. In all other cases we obtain admissible values. Among these, success boosts self-confidence and failure lowers it.

The effect of failure can be shattering -particularly for athletes who start with a low self-confidence. It leads to a much lower self-esteem, lower goals, lower effort and a reduced probability of achieving these modest goals. A high γ exacerbates this phenomenon, as expected. A competitive world is a harsh environment. It would be interesting to investigate whether a multi-period model would reveal less abrupt shifts in outcomes.

5 Sequential goals set by the athlete

In the simple one-period model, the outcomes in terms of goal setting, effort and probability of success differed markedly depending on whether the coach or the athlete set the goal. One might expect this result to hold at each period but, as shown below, we find that there is a striking similarity of behavior in period 1.

5.1 The Athlete's Strategy

The dynamics of self-confidence (the law that determines Λ_2) remain unchanged. The objective of the athlete is to maximize net utility⁶ in period 2, not expected payoff as for the coach. The athlete also solves the game recursively, choosing $E_2^*(A_2, \lambda_2)$ and then chooses A_2 to maximize $V(A_2, \lambda_2)$. We know the solution to that problem.

In period two, the athlete will select

$$A_2^*(\lambda_2) = \beta^{2/(1-\beta)} \bar{A}_2 = \beta^{2/(1-\beta)} \left(\frac{\lambda_2}{b} \right)^{1/(1-\beta)}$$

which yields, with (7)

$$\begin{aligned} V_2^*(\lambda_2) &\equiv V(A_2^*, \lambda_2) = \left[\sqrt{\lambda_2} (\bar{A}_2)^{\frac{\beta}{2}} (\beta)^{\frac{\beta}{1-\beta}} - \sqrt{b} \sqrt{\bar{A}_2} (\beta)^{\frac{1}{1-\beta}} \right]^2 \\ &= \left[\sqrt{\lambda_2} (\lambda_2)^{\frac{\beta}{2(1-\beta)}} (b)^{\frac{-\beta}{2(1-\beta)}} (\beta)^{\frac{\beta}{1-\beta}} - \sqrt{b} (b)^{\frac{-1}{2(1-\beta)}} (\lambda_2)^{\frac{1}{2(1-\beta)}} (\beta)^{\frac{1}{1-\beta}} \right]^2 \\ &= (\lambda_2)^{\frac{1}{1-\beta}} \left(\frac{1}{\sqrt{b}} \right)^{\frac{2\beta}{1-\beta}} (\beta)^{\frac{2\beta}{1-\beta}} [1 - \beta]^2 \end{aligned} \quad (39)$$

It follows that the second period expected utility of the athlete is an increasing function of λ_2^ω , where $\omega = 1/(1 - \beta)$, which is a smaller ω than when the coach sets the goal (when we had $\omega = (2 - \beta)/(1 - \beta) = 1 + [1/(1 - \beta)]$).

The recursive method of solution proceeds as in the preceding section. The athlete must treat Λ_2 as a random variable in period 1 -as it is unknown- and maximize $\mathcal{E}(\Lambda_2^\omega)$. This is exactly the same problem faced by the coach in section 3 with a smaller ω .

The choice of A_1 , now called $A_1^0(\lambda_1)$, is given by (31) with $\omega = 1/(1 - \beta)$

$$A_1^0(\lambda_1) = \left[1 + \frac{1}{\lambda_1 (\lambda_1^{-\omega(\alpha+\gamma)} - 1)} \right]^{2/(1-\beta)} \left(\frac{2\omega}{2\omega + 1 - \beta} \right)^{2/(1-\beta)} \left(\frac{\lambda_1}{b} \right)^{1/(1-\beta)} \quad (40)$$

⁶Please refer to the first footnote of Section 2.2.

$$\equiv s^0 \bar{A}_1(\lambda_1)$$

In order to compare the goals set by the coach and the athlete respectively, we must compare the values of s^{00} and s^0 . Simulations (see column 2 of Tables 3 and 4) show that the athlete usually sets a lower goal, sometimes as low as 3/4 of that set by the coach, but for other parameter values the goals are very close. It is also noteworthy that, for some λ_1 values, the athlete sometimes sets a slightly higher goal than the coach. For instance for many parameter values the ratio of the athlete's goal over the coach's vary from 95% to 98% as λ_1 goes from 0.1 to 0.8. But when λ_1 approaches 0.9 the ratio moves to 110%.

Therefore the very different maximands used by the two agents often result in rather similar outcomes *for period 1*. The reason for this is that they both want the athlete to be well prepared for the period two contest, both maximizing $\mathcal{E}(\Lambda_2^\omega)$, but with different ω values. The similarity ends there, though, as we now show.

The values of $\lambda_2^S(\lambda_1)$ and $\lambda_2^F(\lambda_1)$ are exactly as in (33) and (34) but with $\omega = 1/(1 - \beta)$ now. This modifies the last two terms only, the product of which, for many parameter values, and λ_1 not too high, is not much affected by the change in the ω value. (See Tables 3 and 4.)

Thus the self-confidence of the athlete often (but not always) remains roughly the same as when the coach chooses the goal.

Corresponding to these two λ_2 values, the athlete sets a goal (as in equation (13))

$$A_2^*(\lambda_2) = \beta^{2/(1-\beta)} \left(\frac{\lambda_2}{b} \right)^{1/(1-\beta)} = \beta^{2/(1-\beta)} \bar{A}_2(\lambda_2) \quad (41)$$

which may take on two values

$$\text{(Success 1) } A_2^{*S} = \beta^{2/(1-\beta)} \left[\frac{Bh(y_1)\lambda_1^{1-\alpha}}{b} \right]^{1/(1-\beta)} \left[\frac{2}{2 + (1 - \beta)^2} \right]^{2/(1-\beta)^2} Z \quad (42)$$

$$\text{(Failure 1) } A_2^{*S} = \beta^{2/(1-\beta)} \left[\frac{Bh(y_1)\lambda_1^{1+\gamma}}{b} \right]^{1/(1-\beta)} \left[\frac{2}{2 + (1-\beta)^2} \right]^{2/(1-\beta)^2} Z \quad (43)$$

where

$$Z \equiv \left[1 + \frac{1}{\lambda_1(\lambda_1^{-\omega(\alpha+\gamma)/(1-\beta)} - 1)} \right]^{2/(1-\beta)^2} \quad (44)$$

The A_2^* values are smaller than the corresponding A_2^{**} values and the differences come mainly from the first terms, $(2/(3-\beta))^{2/(1-\beta)}$ and $\beta^{2/(1-\beta)}$, respectively, the former being at least three times larger (for high β) and sometimes more than 50 times larger (for low β). Therefore in the final period the athlete and the coach set vastly different goals and effort levels are consequently very different.

We can calculate the effort exerted by the athlete using (3) and (41),

$$E_2^* = (1-\beta)\beta^{(1+\beta)/(1-\beta)}\bar{A}_2(\lambda_2) \quad (45)$$

and the ratio

$$\frac{A_2^*(\lambda_2)}{E_2^*(\lambda_2)} = \frac{\beta}{1-\beta}.$$

Therefore the probability of success is now

$$p_2 = \lambda_2(1-\beta) \quad (46)$$

Comparing (46) and (38) and recalling our earlier observation that in many cases the λ_2^* and λ_2^{**} values are not very different (λ_2^* being the smaller), we can conclude that the probability of success is usually much higher when the athlete sets the goals. (Often roughly twice as high.) Of course he sets very much smaller goals.

5.2 Simulations

The calibration is exactly the same as that described in Section 4.2. See Table 4.

Once again, all values are admissible, save some when $\lambda_1 = 0.9$. The observations we made when the coach set the goals are also true here: failure is devastating. The first-period goal set by the athlete is often 15% to 20% lower than the one set by the coach, but rises sharply as λ_1 reaches its upper range and sometimes exceeds that set by the coach. Nonetheless, even in these instances the athlete always sets a much lower goal in period two.

Our earlier observation is confirmed. The probability of success is roughly twice as high when the athlete sets the goal; but the goals themselves are much smaller. A self-managed athlete may feel comfortable but is very much an under-achiever.

TABLE 4(a) : $\alpha = 0.8, \beta = 0.5, \gamma = 10.5, \omega^* = 2$

λ_1	s^0	λ_2^S	λ_2^F	A_2^S	A_2^F	E_2^S	E_2^F	p_2^S	p_2^F
0.1	0.624	0.788	4E - 12	0.0338	1E - 24	0.03879	9.7E - 25	0.3939	2E - 12
0.2	0.624	0.905	1E - 08	0.0512	8E - 18	0.05118	8.2E - 18	0.4525	6E - 09
0.3	0.624	0.981	1E - 06	0.0602	9E - 14	0.06020	9.2E - 14	0.4907	6E - 07
0.4	0.624	1.040	3E - 05	0.0675	7E - 11	0.06754	6.5E - 11	0.5198	2E - 05
0.5	0.624	1.087	4E - 04	0.0738	1E - 08	0.07384	1.2E - 08	0.5435	0.0002
0.6	0.624	1.127	0.0040	0.0794	8E - 07	0.07944	7.7E - 07	0.5637	0.0018
0.7	0.625	1.165	0.0210	0.0848	3E - 05	0.08479	2.7E - 05	0.5824	0.0103
0.8	0.645	1.233	0.0990	0.0951	0.0006	0.09507	0.00061	0.6167	0.0495
0.9	0.959	1.877	0.5710	0.2203	0.0204	0.22026	0.02036	0.9386	0.2854

TABLE 4(b) : $\alpha = 0.5, \beta = 0.5, \gamma = 6.5, \omega^* = 2$

λ_1	s^0	λ_2^S	λ_2^F	A_2^S	A_2^F	E_2^S	E_2^F	p_2^S	p_2^F
0.1	0.624	0.395	4E - 08	0.0097	1E - 16	0.00974	9.7E - 17	0.1974	2E - 08
0.2	0.624	0.558	7E - 06	0.0195	3E - 12	0.01949	3.2E - 12	0.2792	4E - 06
0.3	0.624	0.684	1E - 04	0.0292	1E - 09	0.02923	1.4E - 09	0.3419	7E - 05
0.4	0.624	0.790	0.001	0.0390	1E - 07	0.03898	1E - 07	0.3948	0.0006
0.5	0.625	0.883	0.007	0.0488	3E - 06	0.04877	3E - 06	0.4417	0.0035
0.6	0.628	0.972	0.027	0.0591	3E - 05	0.05908	4.6E - 05	0.4861	0.0136
0.7	0.649	1.086	0.089	0.0737	0.0005	0.07371	0.00050	0.5430	0.0447
0.8	0.781	1.397	0.293	0.1219	0.0054	0.12192	0.00536	0.6983	0.1465
0.9	1.951	3.702	1.771	0.8564	0.1959	0.85643	0.19596	1.8509	0.8853

TABLE 4(c) : $\alpha = 0.8, \beta = 0.2, \gamma = 9.5, \omega^* = 1.25$

λ_1	s^0	λ_2^S	λ_2^F	A_2^S	A_2^F	E_2^S	E_2^F	p_2^S	p_2^F
0.1	0.5	0.394	2E - 11	0.0056	7E - 16	0.02233	3E - 15	0.3152	2E - 11
0.2	0.5	0.453	3E - 08	0.0066	7E - 12	0.02656	2.7E - 11	0.3621	2E - 08
0.3	0.5	0.491	2E - 06	0.0073	1E - 09	0.02939	5.4E - 09	0.3926	2E - 06
0.4	0.5	0.520	4E - 05	0.0079	6E - 08	0.03159	2.4E - 07	0.4159	3E - 05
0.5	0.5	0.544	4E - 04	0.0084	1E - 06	0.03343	4.4E - 06	0.4352	0.0003
0.6	0.502	0.567	0.003	0.0088	1E - 05	0.03521	4.8E - 05	0.4536	0.0024
0.7	0.518	0.603	0.015	0.0095	1E - 04	0.03801	0.00039	0.4823	0.0122
0.8	0.598	0.715	0.072	0.0118	0.0007	0.04707	0.00266	0.5723	0.0575
0.9	1.129	1.381	0.467	0.0268	0.0069	0.10715	0.02760	1.1051	0.3733

TABLE 4(d) : $\alpha = 0.8, \beta = 0.2, \gamma = 5, \omega^* = 1.25$

λ_1	s^0	λ_2^S	λ_2^F	A_2^S	A_2^F	E_2^S	E_2^F	p_2^S	p_2^F
0.1	0.5	0.394	$6E - 07$	0.0056	$3E - 10$	0.02233	$1.3E - 09$	0.3152	$5E - 07$
0.2	0.5	0.453	$4E - 05$	0.0066	$6E - 08$	0.02656	$2.3E - 07$	0.3621	$3E - 05$
0.3	0.5	0.491	$5E - 04$	0.0074	$1E - 06$	0.02944	$4.8E - 06$	0.3932	0.0004
0.4	0.504	0.524	0.003	0.0080	$1E - 05$	0.03191	$4.2E - 05$	0.4193	0.0021
0.5	0.516	0.562	0.010	0.0087	$6E - 05$	0.03480	0.00023	0.4494	0.0081
0.6	0.554	0.625	0.032	0.0099	0.0002	0.03976	0.00098	0.5	0.0258
0.7	0.658	0.766	0.097	0.0128	0.0010	0.05121	0.00386	0.6125	0.0774
0.8	0.98	1.171	0.321	0.0218	0.0043	0.08719	0.01729	0.9370	0.2568
0.9	2.718	3.326	1.805	0.0804	0.0374	0.32145	0.14975	2.6612	1.4444

TABLE 5
Comparison of First Period Goals by Coach and Athlete

Ex a		Ex a	Ex b	Ex b	Ex c		Ex c	Ex d	Ex d
λ_1	s^{00}	s^0	s^{00}	s^0	λ_1	s^{00}	s^0	s^{00}	s^0
0.1	0.73	0.62	0.73	0.62	0.1	0.66	0.50	0.66	0.50
0.2	0.73	0.62	0.73	0.62	0.2	0.66	0.50	0.66	0.50
0.3	0.73	0.62	0.73	0.62	0.3	0.66	0.50	0.66	0.50
0.4	0.73	0.62	0.73	0.62	0.4	0.66	0.50	0.66	0.51
0.5	0.73	0.62	0.73	0.63	0.5	0.66	0.50	0.67	0.52
0.6	0.73	0.62	0.73	0.63	0.6	0.66	0.50	0.68	0.55
0.7	0.73	0.63	0.73	0.65	0.7	0.66	0.52	0.74	0.66
0.8	0.73	0.64	0.77	0.78	0.8	0.68	0.60	0.97	0.98
0.9	0.82	0.96	1.21	1.95	0.9	0.85	1.13	2.24	2.72

6 Concluding Remarks

Psychologists and management consultants have long been aware that goal-setting can be an important determinant of success, be it in business ventures or in sport. Here we have constructed a simple model that generates the inverted U-shaped relationship between goal and effort. Our focus, though, is primarily on the objective of the agent setting the goals to which the athlete will respond. We find that a coach with a criterion of expected pay-off will set a much higher goal than the utility-maximizing athlete, although the latter will have a higher probability of reaching his modest goal. The athlete on his own also tries much less hard. Here is perhaps a model-generated occurrence that is familiar to academics and businessmen. An author needs the discipline of an editor and a manager needs the guidance of a board.

When the assumption of complete information is put aside, we find that

a coach will set higher goals, to be realized with a smaller probability, the less information he has. A converse proposition is that, in business, getting to know your “team” will lead to more realistic goal setting and a higher probability of success.

Finally we expand our model to two-periods, with the objectives still related to the ultimate period performance. We devise a realistic “confidence modification” mechanism which depends on performance and goals. We show that a self-managing athlete will behave not too differently from one who has a coach, in period 1. (He generally sets a lower goal, still.) The consequences of failure are quite dramatic with lower self-esteem, lower goals, lower effort and lower probability of success. A more ambitious multi-period model might be able to uncover a pattern in a tree diagram of successes and failures.

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