Centre Interuniversitaire sur le Risque, les Politiques Économiques et l'Emploi

Cahier de recherche/Working Paper **09-02**

Non-comparative versus Comparative Advertising as a Quality Signal

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Janvier/January 2009

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We thank Gerd Mühlheusser and Andras Niedermayer for helpful comments. The usual disclaimer applies.

Abstract:
Two firms produce a product with a horizontal and a vertical characteristic. We call the vertical characteristic quality. The difference in the quality levels determines how the firms share the market. Firms know the quality levels, consumers do not. Under non-comparative advertising a firm may signal its own quality. Under comparative advertising firms may signal the quality differential. In both scenarios the firms may attempt to mislead at a cost. If firms advertise, in both scenarios equilibria are revealing. Under comparative advertising the firms never advertise together which they may do under non-comparative advertising.

Keywords: Advertising, costly state falsification, signalling

JEL Classification: D82, K41, K42

1 Introduction

Comparative advertising was illegal in many European countries until the late 1990s. By contrast, in the US comparative advertising has been encouraged by the Federal Trade Commission since the 1970's. A 1997 EU directive (Directive 97/55/EC) changed the situation in Europe by legalizing comparative advertising subject to the restriction that it should not be misleading. European Competition Authorities now tend to agree with their American counterparts in that comparative advertising is an important tool in promoting competition. Comparative advertising increases consumers' information about alternative products. It allows consumers to evaluate the performance of particular products against other products, thus enabling more informed purchasing decisions.[1]

Despite its importance there has been little economic analysis on comparative advertising. We will review this literature at the end of the introduction. In this paper we address the following questions. Is the content of comparative advertisement more truthful than the content of non-comparative advertisements? Are comparative ads more informative for consumers than non-comparative ads? Is the amount of advertising higher or lower under comparative or non-comparative advertising? Can the two advertising regimes be compared using welfare criteria?

To answer these questions we consider a product with a horizontal and a vertical characteristic. We call the horizontal characteristic design and the vertical one quality. Consumers have different tastes concerning design but all consumers prefer higher to lower quality. Two firms produce opposite designs. To focus on advertising we consider a situation where prices cannot signal qualities. In equilibrium, the two firms charge identical prices which are low enough so that the market is always covered. The difference in the quality levels determines how the firms share the market. If both firms have

[1]See Barigozzi and Peitz (2006) for more details.

the same quality level, they share the market equally. If one firm has a higher quality level than its opponent, it has a higher market share. The quality differential defines a zero-sum game between the two firms.

Firms know the quality levels, consumers do not. Under non-comparative advertising a firm may send a message about its own quality but not about the competitor's one. Under comparative advertising firms may signal the quality differential. In both scenarios the firms attempt to mislead at a cost. The cost of misleading increases the further a firm moves away from the truth, i.e., the more a firm makes exaggerated claims. Moreover, in both scenarios firms may choose not to advertise.

In the non-comparative framework a firm advertises if its quality level is above a threshold. In the message it boasts about its quality level. Consumers rationally anticipate this boasting and infer the true quality level. Thus, if firms advertise, the equilibrium is revealing, yet firms attempt to mislead. Stated differently, because the marginal cost of slightly distorting the truth is negligible but the marginal return is not, consumers expect some boasting, leading firms to do so systematically. If the quality level is below the threshold, a firm does not advertise; the cost of sending the message is higher than the gain thereof. If both firms have low quality levels, both do not advertise and share the market equally. If one firm has a high quality level whereas the other one has a low one, the high quality firm advertises while the low quality one doesn't. The high quality firm has more customers than the low quality one. If both firms have high quality levels, both advertise. The last case may be highly inefficient: if both firms have the same high quality level, both advertise at a cost yet still share the market.

Under comparative advertising firms may send signals about the quality differential. An advertising firm wants to persuade consumers that the quality differential is in its favor. Again firms may distort the truth at a cost. If the quality differential is sufficiently small, both firms do not advertise. If the quality differential is, however, large, the high quality firm advertises while the low quality one doesn't. If a firm advertises, it attempts to boost the quality differential in its favor. Consumers account for this and infer the true value. In equilibrium the firms never advertise together.

In our equilibria if firms advertise, the signalling cost is minimal. This, in turn, implies that the range over which firms choose to advertise is maximal. Therefore, we have chosen in both scenarios out of the set of possible equilibria the least-cost maximum-advertising ones. Since both equilibria share these features, a comparison of their efficiency properties is reasonable.

Comparative advertising tends to perform better than non-comparative advertising in our set-up. Firms do not advertise if the quality differential is small and the information is of little value to consumers. If, however, the quality differential is large, the high quality firm advertises while the low quality one remains silent. There is no duplication of advertisement expenditures. By contrast, under non-comparative advertising firms advertise their high quality independently of their rival's quality level. Both firms may advertise even when the information is of little or no value to consumers.

Non-comparative advertising results in a two-sender simultaneous signalling game with a continuum of types where a type is given by the true quality. Comparative advertising yields a two-sender game with perfectly correlated types, i.e., the actions of both firms provide information about the same quantity. Since signals are non-stochastic, the true state is inferred in both contexts if firms advertise.

Let us now review the literature. The marketing literature has discussed comparative advertising quite extensively; see Grewal et al. (1997) for a survey. There is, however, little economics literature on comparative advertising. Aluf and Shy (2001) use a Hotelling-type model. Comparative advertising shifts transportation cost to the rival's product. The model does not deal with the informative role of advertising.

Anderson and Renault (2006) consider comparative advertising with respect to horizontal characteristics. If qualities are sufficiently different, the low quality firm will disclose horizontal attributes of both products. The main difference to our approach is that advertising is costless and firms can only communicate verifiable evidence but may attempt to milslead by disclosing only what they see fit.

Barigozzi, Garella, and Peitz (2007) consider an incumbent with known quality facing an entrant with unknown quality. The entrant can choose generic advertising which is standard money burning to signal quality. Moreover, the entrant can choose comparative advertising which involves a comparison of the two firms' qualities. If the entrant uses comparative advertising, the incumbent may sue hoping to obtain damages. If the entrant uses comparative rather than generic advertising, he signals to consumers that he has a strong case. It turns out that comparative advertising can signal quality in those cases where generic advertising cannot. An important difference to our model is that only the entrant can choose to advertise.

More generally, our analysis is related to the industrial organization literature on advertising as quality disclosure or quality signalling. Levin, Peck, and Ye (2009) analyze a duopoly where firms can disclose their true quality by presenting costly verifiable evidence in the form of, e.g., certification from a third party. In Daughety and Reinganum (2008), a monopolist may choose between costly disclosure or signalling quality through prices.

There is an important literature, going back to Milgrom and Roberts (1986), on quality signalling via prices or advertising as money burning. By and large, however, this literature has dealt with the case of a monopolist, i.e., it has considered a one-sender game. A recent exception is Daughety and Reinganum (2007) who consider signalling through prices in a duopoly. Two other exceptions, more closely related to the present analysis, are Hertzendorf and Overgaard (2001) and Fluet and Garella (2002). In these papers, as in the present one, the duopolists know each other's quality. In the resulting equilibria, signalling is either through prices alone or through the price-advertising mix. In the present paper, signalling through prices is not feasible. Moreover, we focus on the case where both firms may jointly signal about the same quantity, namely the quality differential.

The remainder of this paper is organized as follows. In the next section we describe the model. Section 3 analyzes non-comparative and section 4 comparative advertising. Section 5 compares welfare. Section 6 concludes.

2 The Model

Consider a product that has two characteristics. The first is horizontal; we will call it design. Design can take on two values $d \in \{-1, +1\}$. A computer may use the Mac OS X or the Microsoft Windows operating system; a vacation resort may be located in the mountains or at the seaside; a cigar may be produced in Cuba or the Dominican Republic. The second characteristic is vertical; we will refer to it as the product's quality. Let $q_d \in [0, 1]$ be the quality of design d. We will write q_m for the quality of $d = -1$ (m for minus) and q_p for the quality of $d = +1$ (p for plus).

Consumers have unit demand. They care about design and quality. A consumer's utility is given as

$$U = \theta d + 2q_d + 1,$$

where θ is uniform on $[-\hat{\theta}, \hat{\theta}], \hat{\theta} > 1$, or explicitly

$$U = \begin{cases} \theta + 2q_p + 1, & \text{if } d = 1; \\ -\theta + 2q_m + 1, & \text{if } d = -1. \end{cases}$$

The parameter θ measures the intensity with which a consumer cares about design. If θ is close to zero, design is not of great importance for the consumer and he cares more about quality. By contrast, if θ is close to $\hat{\theta}$ $(-\hat{\theta})$, the consumer is a $d = 1$ $(d = -1)$ aficionado for whom quality is of minor importance. The larger $\hat{\theta}$, the more the average consumer cares about design. θ is private information.[2]

[2] The average of $|\theta|$, i.e., the average preference intensity for one design or the other, is $\hat{\theta}/2$.

There are two firms: firm m offers design -1 and firm p design $+1$.[3] Both firms charge the price of 1 for their product. Below we provide a justification for why firms do not engage in price competition. When he purchases the product, a consumer's net utility is U minus the price; otherwise, it is zero. With unit prices, each consumer buys the product for all possible realizations of quality so that the market is covered. Which design a consumer chooses depends on his θ and the difference in quality levels: the consumer θ buys from firm p if $\theta + 2q_p \geq -\theta + 2q_m$ or $\theta \geq -(q_p - q_m) := -x$ where $x = q_p - q_m$ is the quality differential.

Production costs are normalized to zero so that each consumer generates a profit, net of advertising costs, of 1. Firm m's profit (market share) net of advertising cost is

$$\int_{-\hat{\theta}}^{-x} \frac{1}{2\hat{\theta}}\, d\theta = \frac{1}{2} - \frac{x}{2\hat{\theta}}.$$

Firm p's profit is

$$\int_{-x}^{\hat{\theta}} \frac{1}{2\hat{\theta}}\, d\theta = \frac{1}{2} + \frac{x}{2\hat{\theta}}.$$

If $q_m = q_p$, equivalently $x = 0$, both firms share the market; if $q_m < q_p$ or $x > 0$, firm p has more than half of the market; if $q_m > q_p$ or $x < 0$, firm m has more than half of the market. The quality difference defines a zero-sum game between the two firms. The marginal impact of x on profits is $+(-)1/2\hat{\theta}$: the less consumers care about design (the smaller $\hat{\theta}$), the higher the impact of the quality differential.

Consumers know the designs but do not observe the quality levels: the products are experience goods and consumers find out the actual quality only after they have purchased.[4] We assume that $\tilde{q}_m$ and $\tilde{q}_p$ are independent and uniform on $[0, 1]$. Both firms know the realizations q_m, q_p whereas consumers

[3] Abusing notation we will use $d \in \{-1, +1\}$ when we refer to design in the consumers' utility function and $d \in \{m, p\}$ as an index for firms and quality levels.

[4] As in Milgrom and Roberts (1986), quality may be interpreted as the probability that a consumer is fully satisfied with the product, which is non verifiable. Therefore, a warranty is not feasible.

do not. Without any additional information consumers expect $E(\tilde{q}_m) = E(\tilde{q}_p) = .5$ and the firms share the market. Unless $q_m = q_p$, this allocation is inefficient. If consumers learn, say, $x > 0$, then consumers with $\theta \in [-x, 0]$ buy from firm p rather than firm m. When they buy from m, their surplus is $\int_{-x}^{0}(-\theta + 2q_m)/2\hat{\theta}\, d\theta$; buying from p generates the surplus $\int_{-x}^{0}(\theta + 2(q_m + x)/2\hat{\theta}\, d\theta$. Becoming informed about x thus increases surplus by $x^2/2\hat{\theta}$. This expression also applies when $x < 0$.

Informing consumers about quality therefore not only redistributes profits, but typically also enhances efficiency. To put it differently, advertising quality in our set-up is on the one hand combative, acting to redistribute consumers among firms; on the other hand it is informative, increasing consumer surplus.[5]

To complete the set-up, we describe a simple framework where both products are priced at 1 in equilibrium. Suppose that, in addition to the quality conscious consumers just described, there is for each firm a mass M of quasi captive consumers who do not care about quality. The first (second) group consists of consumers with $U = 1$ if they purchase design m (p) and $U = \varepsilon < 1/2$ if they purchase design p (m). If one firm sets its price equal to 1, its rival can attract all the quality indifferent consumers only by setting a price at most equal to ε. The poacher's profit with respect to quality indifferent consumers amounts to $2\varepsilon M$, which is less than the profit of M earned by setting a price of 1 and selling only to its own quasi captive clientele. In the absence of quality conscious consumers, the Bertrand equilibrium prices are therefore equal 1.

Consider now the population of quality conscious consumers. Suppose for the time being that quality is observable before purchase. If the market consists only of quality conscious consumers, the equilibrium prices for m and p are $P_m = 2\hat{\theta} - 2x/3$ and $P_p = 2\hat{\theta} + 2x/3$. Prices are higher the more the average consumer cares about the horizontal characteristic, as this

[5] For a survey of the different views on advertising see, e.g., Bagwell (2007).

reduces the intensity of price competition. Prices also depend on the quality differential. Since $\hat{\theta} > 1$ and $x \in [-1, 1]$, both prices are strictly greater than unity.

Finally, let the actual market consist of both categories of consumers. Each product is sold at a uniform price, i.e., firms cannot discriminate between quality conscious and quality indifferent consumers. Depending on the relative weights of the two categories, equilibrium prices are then either $P_m = 2\hat{\theta} - 2x/3$ and $P_p = 2\hat{\theta} + 2x/3$ or $P_m = P_p = 1$. The latter occurs when the mass $2M$ of consumers with extreme brand loyalty is sufficiently large compared to that of quality conscious consumers. We assume this to be the case.[6] The resulting equilibrium prices also apply when quality is unobservable: a firm never has an incentive to set a price different from unity.[7]

In the sequel, we disregard prices and analyze how firms compete through advertising in order to increase their market share of quality conscious consumers. The timing is as follows. In stage 0, the firms learn their qualities and consumers learn their type. In stage 1, the firms simultaneously send messages about the qualities; this includes the possibility of saying nothing. In stage 2, consumers observe the messages, draw inferences, and make their purchase decisions.

3 Non-comparative advertising

In this section each firm may inform consumers about its own quality but not about the quality of its competitor.[8] It advertises its own quality by sending a message of the form $y_d \geq 0,\ d \in \{m, p\}$. It should be thought of as

[6] If the set of quality conscious has unit mass, the condition is $M > \hat{\theta} + 2/3 + 1/9\hat{\theta}$.

[7] Note that prices cannot signal quality because purchases are one-shot and unit costs are independent of quality; see, e.g., Daughety and Reinganum, 2007.

[8] In our set-up no, one, or both firms may advertise. In Barigozzi, Garella, amd Peitz (2007) the incumbent's quality is common knowledge and only the entrant may signal quality.

a story or argument rendering q_d plausible: the larger y_d, i.e., the "louder" the message, the more the firm claims about its quality.

The cost of advertising is related to actual quality q_d. We assume it to be of the form $C(y_d, q_d) = \gamma + .5(y_d - q_d)^2$. $\gamma \in (1/8\hat{\theta}, 1/4\hat{\theta})$ is a fixed cost of advertising.[9] The variable cost captures the idea that a distorting message is more costly than simply reporting the naked truth as it involves more fabrication. With the quadratic function the cost of misrepresenting the evidence increases at an increasing rate the more disconnected claims are from the truth: it becomes more difficult to produce the corresponding message, or advertising agencies charge more the more they embellish. When firms are restricted to advertise their own quality, they are obviously only interested in boasting, i.e., $y_d \geq q_d$.[10] Note that the marginal cost of sending the signal y_d is decreasing in the true quality so that the single-crossing property is satisfied. If firm d does not advertise, we will write $\emptyset_d$.

In the first stage of the game firms choose simultaneously whether or not to advertise. In the second stage consumers observe the firms' actions and form beliefs $E(x) = E(q_p|\cdot) - E(q_m|\cdot)$. Consumers then buy from the firm maximizing expected utility, i.e., consumers with $\theta < E(x)$ buy design m and the rest design p. If firm m doesn't advertise, its profit is $.5 - E(x)/2\hat{\theta}$; if it sends the message y_m profit is $.5 - E(x)/2\hat{\theta} - \gamma - .5(y_m - q_m)^2$. Firm p's profits are correspondingly $.5 + E(x)/2\hat{\theta}$ and $.5 + E(x)/2\hat{\theta} - \gamma - .5(y_p - q_p)^2$. Firms choose their advertising strategy so as to maximize expected profits. We focus on perfect Bayesian equilibria with minimum signalling costs.

The random variables $\tilde{q}_m$ and $\tilde{q}_m$ are independent meaning that the observation of the realization q_m provides no information about the realization of

[9] The upper bound on γ ensures that firms advertise at all, the lower bound guarantees that under comparative advertising firms never advertise together; see below.

[10] Alternatively, we could have specified the cost function as γ if $y_d < q_d$ and $\gamma + .5(y_d - q_d)^2$ otherwise. Since our choice of $C(\cdot)$ is everywhere differentiable, working with this specification simplifies the exposition. Moreover, in the next section firms send signals about the quality differential x; here they have an incentive to distort up- and downwards and we can continue to use $C(\cdot)$.

q_p and vice versa. We consider equilibria where this independence property carries over to the signals send by the firms. We assume that consumers' expectation $E(q_m)$ depends only on firm m's actions and is independent of what firm p does; $E(q_p)$ is a function of firm p's choices and is independent of what firm m does. This approach has the advantage of ruling out even implicit comparative advertising.[11]

Let us now derive the least cost signalling strategies $y(q_d),\ d \in \{m, p\}$. Suppose firms play a revealing strategy $\hat{y}(q_d)$ for some $q_d \geq q_0$ where q_0 is a threshold yet to be determined. Revealing means $E(q_d|\hat{y}(q_d)) = q_d$. Consider, say, firm p and denote by $\pi(y_p, q_p)$ its profit when it sends the message y_p and its actual quality is q_p. If firm p wants to mimic as q_p', its profit is

$$\begin{aligned}
\pi(\hat{y}(q_p'), q_p) &= \frac{1}{2} + \frac{1}{2\hat{\theta}}[q_p' - E(q_m|\cdot)] - \gamma - \frac{1}{2}(\hat{y}(q_p') - q_p)^2 \\
&= \frac{q_p'}{2\hat{\theta}} - \frac{1}{2}(\hat{y}(q_p') - q_p)^2 + \text{constant}
\end{aligned}$$

where the constant denotes the terms that do not depend on p's actions or type. Since the strategy is revealing, firm p's profit must be maximized at $q_p' = q_p$. This implies that for all q_p in the separating range

$$\left.\frac{\partial \pi}{\partial q_p'}\right|_{q_p'=q_p} = \frac{1}{2\hat{\theta}} - (\hat{y}(q_p) - q_p)\hat{y}'(q_p) = 0 \quad \text{and}$$

$$\left.\frac{\partial^2 \pi}{\partial q_p'^2}\right|_{q_p'=q_p} = -(\hat{y}(q_p) - q_p)\hat{y}''(q_p) - (\hat{y}'(q_p))^2 \leq 0.$$

If $\hat{y}(q_p)$ is revealing, it must solve the first-order and satisfy the second-order condition. It is easier to work with the inverse function $q_p = \varphi(y_p)$, noting that the inverse exists since $\hat{y}(q_p)$ must be strictly increasing. In terms of the inverse, the conditions are rewritten as

$$\varphi'(y_d) - 2\hat{\theta}\,(y_d - \varphi(y_d)) = 0 \quad \text{and} \tag{1}$$

[11] In effect our equilibrium strategies are the same as if each firm could only oberve its own quality.

$$\varphi''(y_d) \leq 2\hat{\theta}. \tag{2}$$

The general solution to the differential equation (1) is

$$\varphi(y_d) = Ke^{-2\hat{\theta}y_d} + y_d - \frac{1}{2\hat{\theta}} \tag{3}$$

where K is a constant. Condition (2) requires

$$Ke^{-2\hat{\theta}y_d} \leq \frac{1}{2\hat{\theta}}. \tag{4}$$

Signalling costs (excluding the fixed cost) are

$$\frac{1}{2}\left(y_d - \varphi(y_d)\right)^2 = \frac{1}{2}\left(\frac{1}{2\hat{\theta}} - Ke^{-2\hat{\theta}y_d}\right)^2.$$

Next we derive the threshold q_0 at which the firm starts to advertise. Let y_0 be the smallest message sent, corresponding to q_0. Signalling costs are minimized if $y_0 = q_0$. This implies

$$K = \frac{e^{2\hat{\theta}y_0}}{2\hat{\theta}} = \frac{e^{2\hat{\theta}q_0}}{2\hat{\theta}}.$$

Observe that (4) is then always satisfied given that $y_d \geq q_0$.

We can now rewrite (3) as

$$q_d = y_d - \frac{1 - e^{-2\hat{\theta}(y_d - q_0)}}{2\hat{\theta}}.$$

It is easily verified that y_d is strictly increasing in q_d, with $y_0 = q_0$; see the discussion below.

When the firm does not advertise, consumers' expectation about its quality is $E(q_d|\emptyset_d) = q_0/2$. The firm's profit is then

$$\pi(\emptyset_d) = \frac{(q_0/2)}{2\hat{\theta}} + \text{constant}.$$

The threshold is such that a firm with actual quality q_0 is just indifferent between advertising and not. If it does, its profit is

$$\pi(y_0) = \frac{q_0}{2\hat{\theta}} - \gamma + \text{constant}.$$

Indifference between advertising and not yields $q_0 = 4\hat{\theta}\gamma$. A firm with quality above this threshold prefers to advertise, the converse holds if quality is below the threshold.

Equilibrium strategies are, therefore, $\emptyset_d$ if $q_d < 4\hat{\theta}\gamma$ and $y_d \geq q_d$ satisfying

$$q_d = y_d - \frac{1 - e^{-2\hat{\theta}(y_d - 4\hat{\theta}\gamma)}}{2\hat{\theta}} \tag{5}$$

when $q_d \geq 4\hat{\theta}\gamma$.

Equilibrium beliefs are $E(q_d \mid \emptyset_d) = 2\hat{\theta}\gamma$ and

$$E(q_d \mid y_d) = y_d - \frac{1 - e^{-2\hat{\theta}(y_d - 4\hat{\theta}\gamma)}}{2\hat{\theta}}, \text{ for } y_d \geq 4\hat{\theta}\gamma.$$

Quality is perfectly revealed in this case.

Out-of-equilibrium beliefs need to be defined only for a message $y_d < 4\hat{\theta}\gamma$. One possibility is

$$E(q_d \mid y_d) \leq y_d, \text{ for } y_d < 4\hat{\theta}\gamma.$$

To sum up:

Proposition 1: *In the least-cost equilibrium if* $q_d < 4\hat{\theta}\gamma$*, firm d chooses* $\emptyset_d$*; consumers correctly expect* $E(q_d \mid \emptyset_d) = 2\hat{\theta}\gamma$*. If* $q_d \geq 4\hat{\theta}\gamma$*, firm d sends the message* $y_d \geq q_d$ *solving (5); consumers infer the true quality level* q_d*,* $d \in \{m, p\}$*.*

The outcome is depicted in Figure 1. If both firms have quality levels below $4\hat{\theta}\gamma$, both do not advertise. Consumers rationally expect quality $2\hat{\theta}\gamma$ of each firm. The more dispersed consumers (the higher $\hat{\theta}$) or the higher the fixed cost γ, the larger the non-advertising range. If one firm's quality level is below while the other firm's is above this threshold, the high quality one advertises while the low quality one doesn't. Consumers infer the quality of the advertising firm and expect $2\hat{\theta}\gamma$ of the non-advertising one. When both quality levels are above the threshold, both firms advertise. Consumers infer both quality levels and thus the true quality differential. The game has prisoners' dilemma features. If, say, $q_m = q_p > q_0$, both firms advertise yet share the market. They spend resources on boasted messages without raising consumer surplus.

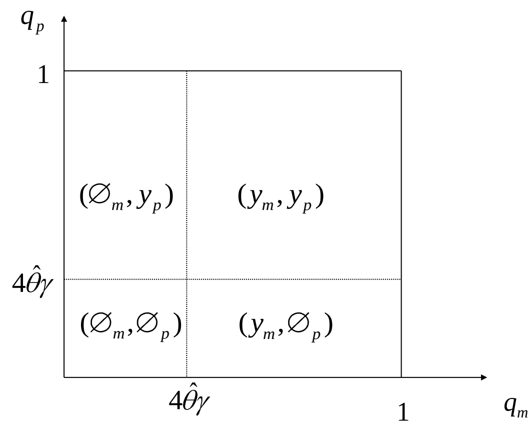

Figure 1: Non-comparative advertising

Let us now look at the message y_d in more detail. Solving (5) yields

$$y_d = q_d + \frac{1}{2\hat{\theta}}(1 + \text{ProductLog}(-e^{2\hat{\theta}(4\hat{\theta}\gamma - q_d)-1})$$

where the ProductLog is the inverse function of $f(w) = we^w$.[12] We have $y_d(q_0) = q_0$, i.e., at the threshold $q_0 = 4\hat{\theta}\gamma$ the firm signals the true quality at zero variable cost. For $q_d \in (q_0, 1]$, $y_d(q_d) \in (q_d, q_d + 1/2\hat{\theta})$. Firms boast quality; boasting increases with quality, yet at a decreasing rate. See Figure 2. Except for the threshold, if a firm advertises, it falsifies. For $y_d > y_d(q_0)$, $E(q_d|y_d)$ is strictly monotone in y_d: different q_d's give rise to different y_d's to which consumers react by computing the correct expectation. Given $E'(q_d|y_d) \neq 0$ and the marginal cost of lying is zero around the true value, it pays for firm d to falsify if it advertises.

[12]See, e.g., http://en.wikipedia.org/wiki/Lambert's_W_function for more on the ProductLog.

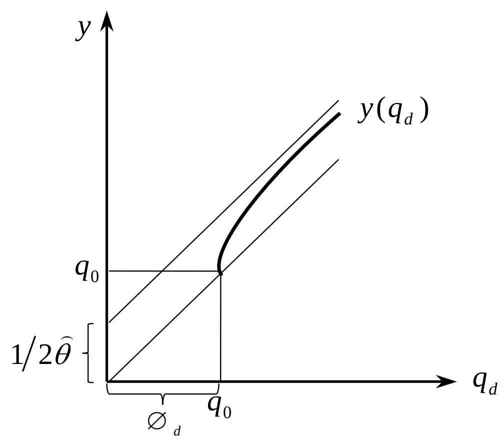

Figure 2: The least cost signalling strategy $y(q_d)$

Two remarks are in order. First, under a properness restriction on consumers' out-of-equilibrium beliefs an equilibrium cannot be totally unrevealing (Myerson (1978)). To see this, suppose on the contrary that consumers believe $E(q_d) = 1/2$ for $\emptyset_d$ and for all y_d. If firms actually choose not to advertise, consumers' beliefs are borne out in equilibrium. Now suppose for simplicity that, say, firm m has either quality 1/4 or 3/4. Sending signal $y_m = 3/4$ is a "big" mistake for the type 1/4-firm and a "small" mistake for the type 3/4-firm at the proposed equilibrium. If consumers believe that big mistakes are less likely than small ones, they should conclude upon observing $y_m = 3/4$ that firm m is of type 3/4.

Second, in our least cost signalling equilibrium the extent of boasting $(y_d - q_d)$, $d \in \{m, p\}$ is minimal. Since the extent of boasting is minimal given firms advertise, the range where firms do not advertise $[0, 4\hat{\theta}\gamma]$ is also minimal. Put differently, in our equilibrium falsification is minimal and, therefore, the range where firms advertise is maximal.

4 Comparative Advertising

Let us now consider comparative advertising. Firms send messages about the difference in quality levels $x = q_p - q_m$, the quantity consumers are ultimately interested in. Given $\tilde{q}_m$ and $\tilde{q}_p$ are independent and uniform on $[0,1]$, $\tilde{x}$ has density

$$f(\tilde{x}) = \begin{cases} 1+x, & \text{if } x \in [-1,0); \\ 1-x, & \text{if } x \in [0,1]. \end{cases}$$

If firm $d \in \{m,p\}$ advertises, it sends a message z_d. The cost of advertising is again $\gamma + .5(z_d - x)^2$ where x is the true difference in quality levels which is known to both firms. Now firm m has an incentive to distort x downwards while firm p wishes to boost upwards. If firm d does not advertise, we will write $\emptyset_d$. Only comparative advertising is allowed. We focus on symmetric equilibria.

Suppose firm p follows the strategy $\emptyset_p$ for $x < x_p^0$ and $z_p(x)$ otherwise; firm m's strategy is $\emptyset_m$ for $x > x_m^0$ and $z_m(x)$ otherwise. Let $z_d' > 0$ if d advertises alone and if both firms advertise together, $d \in \{m,p\}$.

Again we rule out totally unrevealing equilibria, i.e., $x_p^0 < 1$, or $x_m^0 > -1$, or both. There is thus some range where at least one firm sends a signal and x is revealed.

Next, we need some structure on out-of-equilibrium beliefs. We assume that at an out-of-equilibrium information set consumers believe that it was reached with the minimum number of deviations from the equilibrium strategies. A similar restriction on beliefs, termed the *minimality condition* has been used by Bagwell and Ramey (1991) or Emons and Fluet (2009).

First we show that under comparative advertising the firms never advertise together. The formal derivations of the following results are relegated to the Appendix.

Lemma 1: *In equilibrium the firms never advertise together, i.e.,* $x_m^0 < x_p^0$.

To show this result we assume that firms advertise together. Yet it always pays for a firm to deviate because this either changes the expected quality differential in its favor or signalling costs fall by more than revenues.

We can now state the least-cost signalling equilibrium. The equilibrium has the following structure:

(i) If $x \in (-2\hat{\theta}\gamma, 2\hat{\theta}\gamma)$, neither firm advertises and beliefs are $E(x \mid \emptyset_m, \emptyset_p) = 0$.

(ii) If $x \geq 2\hat{\theta}\gamma$, firm m plays $\emptyset_m$ and firm p sends the signal $z_p \geq x$ solving

$$x = z_p - \frac{1 - e^{-2\hat{\theta}(z_p - 2\hat{\theta}\gamma)}}{2\hat{\theta}}. \tag{6}$$

Beliefs are

$$E(x \mid \emptyset_m, z_p) = z_p - \frac{1 - e^{-2\hat{\theta}(z_p - 2\hat{\theta}\gamma)}}{2\hat{\theta}}.$$

(iii) If $x < -2\hat{\theta}\gamma$, firm p plays $\emptyset_p$ and firm m sends the signal $z_m \leq x$ solving

$$x = z_m + \frac{1 - e^{2\hat{\theta}(z_m + 2\hat{\theta}\gamma)}}{2\hat{\theta}}. \tag{7}$$

Beliefs are

$$E(x \mid z_m, \emptyset_p) = z_m + \frac{1 - e^{2\hat{\theta}(z_m + 2\hat{\theta}\gamma)}}{2\hat{\theta}}.$$

To sum up:

Proposition 2: *In the least-cost equilibrium if $x \in [-1, -2\hat{\theta}\gamma]$, firm m sends the message z_m solving (7) while firm p doesn't advertise. If $x \in (-2\hat{\theta}\gamma, 2\hat{\theta}\gamma)$, neither firm advertises. If $x \in [2\hat{\theta}\gamma, 1]$, firm p sends the message z_p solving (6) and firm m doesn't advertise. If one firm advertises, consumers infer the true quality differential; if both firms do not advertise, consumers rationally expect a quality differential of zero.*

The outcome is depicted in Figure 3. First note that unlike in the case of non-comparative advertising, the firms never advertise together. When $|x|$ and thus the informational value to consumers is small, the firms do not advertise. Only when $|x|$ is sufficiently large, the firm with the better quality advertises while the other firm remains silent.

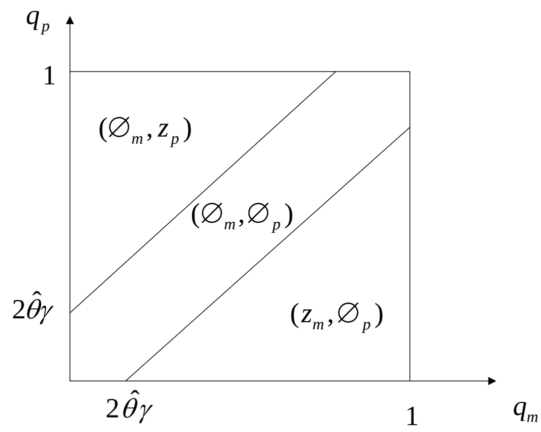

Figure 3: Comparative advertising

Here we have again that if a firm advertises, except at the threshold, it falsifies and the outcome if one firm advertises is the one with minimal falsification. The logic is the same as described in the previous section. See Figure 4.

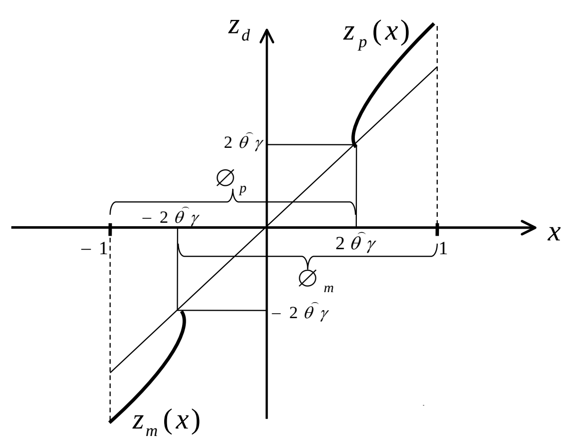

Figure 4: The least cost signalling strategies $z_m(x)$ and $z_p(x)$

5 Welfare

To compare the welfare properties of our least-cost equilibria under non-comparative and comparative advertising consider Figure 5.

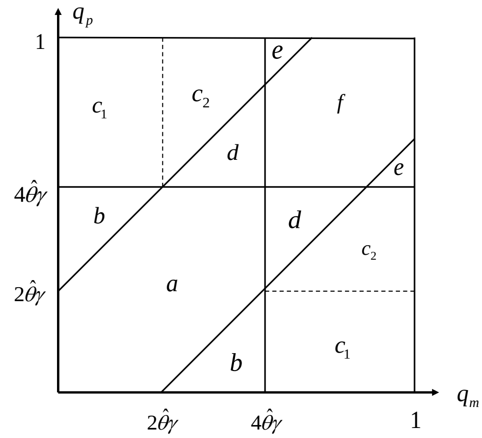

Figure 5: Welfare comparison

Under non-comparative advertising no firm advertises in the areas a and b; one firm advertises in the areas c_1, c_2, and d, providing imperfect information about the quality differential x; in the areas e and f both firms advertise and provide perfect information about x. Under comparative advertising no firm advertises in a, d, and f; one firm advertises in the area b, c_1, c_2, and e, providing perfect information about the quality differential.

Recall that welfare increases by $x^2/2\hat{\theta}$ when consumers learn x. If only one firm advertises and doesn't falsify, advertising costs γ. We may, therefore, say that advertising is efficient if the gain from informing consumers exceeds the cost, or formally, if $|x| \geq \sqrt{2\hat{\theta}\gamma}$. Note that $2\hat{\theta}\gamma < \sqrt{2\hat{\theta}\gamma} < 4\hat{\theta}\gamma$.

In areas a, d and f advertising is inefficient: providing perfect information about x is not worth the expenditure γ. In area a firms don't advertise under both regimes, thus welfare is the same. In areas d and f firms don't advertise under comparative advertising. Under non-comparative advertising one firm advertises in area d and both firms advertise in area f. Therefore, comparative advertising performs better than non-comparative advertising in areas d and f.

To compare welfare in the remaining areas we need the following

Lemma 2: *Variable signalling costs are higher for the non-comparative than for the comparative advertiser in c_2 and e and lower in c_1.*

In c_2 the non-comparative advertiser has the same fixed cost but a larger variable signalling cost than the comparative advertiser. Moreover, non-comparative advertising provides less information. Thus, information is better and signalling less costly under comparative advertising.

In e the information is the same under both regimes. However, each non-comparative advertiser has a higher signalling cost than the single comparative advertiser: the fixed cost is the same, and variable cost is higher. Furthermore, costs are duplicated under non-comparative advertising.

The welfare comparison is ambiguous in c_1. The non-comparative advertiser has a lower signalling cost than the comparative advertiser, but at the same time provides less information.

Finally, consider region b which can be partitioned in two subareas: one where $x \leq \sqrt{2\hat{\theta}\gamma}$ and one where $x > \sqrt{2\hat{\theta}\gamma}$. In the first subarea, signalling by the comparative advertiser is not worth its cost. Hence, non-comparative advertising does better because it does not signal at all. In the second subarea, the comparison is ambiguous as is in c_1.

To sum up: Under comparative advertising for $x \in (2\hat{\theta}\gamma, \sqrt{2\hat{\theta}\gamma})$ and $x \in (-\sqrt{2\hat{\theta}\gamma}, -2\hat{\theta}\gamma)$ firms advertise although it is inefficient to do so. Comparative advertising has the following virtues. Whenever firms do not advertise, their decision is efficient. Furthermore, firms never advertise together, i.e., there is no duplication of advertising expenditures. For non-comparative advertising the picture is less clear cut. When no or one firm advertises, the outcome may be efficient or not. When both firms advertise, the outcome is inefficient because signalling costs are duplicated.

Comparative advertising thus seems to do a better job than non-comparative advertising. Since under comparative advertising firms inform consumers directly about the quantity they are interested in and not just about one component thereof as under non-comparative advertising, this is after all not that surprising.

6 Conclusions

The purpose of this paper is to analyze non-comparative and comparative advertising in a framework where firms may signal their quality. In equilibrium, if firms advertise, they falsify at a cost. We consider the equilibria entailing minimal falsification which, in turn, implies that the range over which firms advertise is maximal. Comparative advertising performs better in our set-up than non-comparative advertising: firms do not advertise at all if the informational content is of little value to consumers; moreover, they never advertise together. By contrast, under non-comparative advertising a firm advertises if its quality level is above a threshold independently of the rival's quality. If both firms have high quality, both advertise leading to a duplication of falsification costs.

We have restricted attention to the two scenarios where firms either choose between no and non-comparative or no and comparative advertising. An interesting topic for future research is to give firms the choice between no, non-comparative, and comparative advertising. This extension is, however, a lot harder to analyze than the set-up at hand. The choice of the advertising format has also informational content, leading into the intricacies of multi-dimensional signalling.

Appendix

Proof of Lemma 1: Suppose on the contrary that there is some range $[x_p^0, x_m^0]$ where both firms advertise and the equilibrium is revealing. At, say, x_m^0 firm m signals $z_m(x_m^0)$, firm p sends $z_p(x_m^0)$, and $E(x|z_m(x_m^0), z_p(x_m^0)) = x_m^0$. See Figure 6.

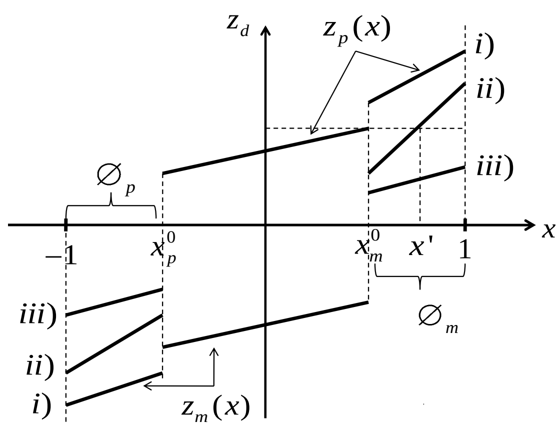

Figure 6: Signalling strategies $z_m(x)$ and $z_p(x)$ if $x_p^0 < x_m^0$

We have to distinguish three cases.

i) Let $z_p(x_m^0) \geq z_p(x_m^0 + \epsilon)$ with ϵ small. If firm m deviates to $\emptyset_m$ consumers observe $(\emptyset_m, z_p(x_m^0))$ which is off the equilibrium path. By the minimality condition consumers think at this out-of equilibrium node that either m deviated while p played his equilibrium strategy $z_p(x_m^0)$ and the underlying quality differential is x_m^0; or they think that p deviated while m played his equilibrium strategy $\emptyset_m$ and the underlying differential is $.5x_m^0 + .5$. Consumers assign equal probability to both possibilities. Hence, $E(x|(\emptyset_m, z_p(x_m^0)) = .75x_m^0 + .25$.

If m plays $z_m(x_m^0)$, his profit $\pi_m(z_m(x_m^0), z_p(x_m^0)) \leq .5 - x_m^0/2\hat{\theta} - \gamma$. If m plays $\emptyset_m$, his profit $\pi_m(\emptyset, z_p(x_m^0)) = .5 - (1/2\hat{\theta})(.75x_m^0/ + .25)$. m prefers $\emptyset_m$ to $z_m(x_m^0)$ if $x_m^0 > 1 - 8\hat{\theta}\gamma$. Likewise, p prefers $\emptyset_p$ to $z_p(x_p^0)$ if $x_p^0 < 8\hat{\theta}\gamma - 1$. Consequently, for $\gamma > 1/8\hat{\theta}$, $x_m^0 < x_p^0$.

ii) Let $z_p(x_m^0) < z_p(x_m^0 + \epsilon)$ with ϵ small and $z_p(1) \geq z_p(x_m^0)$. Now suppose p lowers the signal to $z_p(x_m^0) - \epsilon$ so that consumers observe $(z_m(x_m^0), z_p(x_m^0) - \epsilon)$ which is off the equilibrium path. Then they think with equal probability that either p deviated and $x = x_m^0$ or m deviated and $x = x' - \delta'$ or $x = x_m^0 - \delta$; let consumers assign probabilities $b > 0$ and $(1 - b)$ to the two possibilities. Hence, $E(x|((z_m(x_m^0), z_p(x_m^0) - \epsilon)) = 1/2 \cdot x_m^0 + 1/2 \cdot (b(x' - \delta') + (1 - b)(x_m^0 - \delta))$. For ϵ going to zero, so do δ and δ' so that $E(x|((z_m(x_m^0), z_p(x_m^0) - \epsilon)) > x_m^0$. p's revenue increases, hence he will deviate.

iii) Let $z_p(x_m^0) < z_p(x_m^0 + \epsilon)$ with ϵ small and $z_p(1) < z_p(x_m^0)$. Now m deviates to $\emptyset_m$ and the argument is along the same lines as in i). ■

Proof of Proposition 2. a) z_p as defined in (6) is a best response to $\emptyset_m$ when $x \geq 2\hat{\theta}\gamma$. The argument is similar the case of non-comparative advertising, except that the threshold is different. When firm p does not advertise, its profit is

$$\pi_p(\emptyset_m, \emptyset_p) = \frac{1}{2}.$$

It starts advertising at the threshold $x_p^0 = 2\hat{\theta}\gamma$, in which case it sends the message $z_p^0 = x_p^0$. Its profit is then

$$\pi_p(\emptyset_m, z_p^0) = \frac{1}{2} + \frac{x_p^0}{2\hat{\theta}} - \gamma.$$

Equating the two yields $x_p^0 = 2\hat{\theta}\gamma$. At $x = 2\hat{\theta}\gamma$, the firm is therefore indifferent between advertising and not. We assume that it does. At $x > 2\hat{\theta}\gamma$, it is easily seen that it is strictly better off by advertising.

b) $\emptyset_p$ is a best response to $\emptyset_m$ when $x \in (-2\hat{\theta}\gamma, 2\hat{\theta}\gamma)$. From the argument used to derive the threshold z_p^0, it is easily seen that profit is larger with $\emptyset_p$ than with some message $z_p \geq 2\hat{\theta}\gamma$. We therefore need only consider the case where the message sent is some $z_p < 2\hat{\theta}\gamma$. Applying the minimality condition, consumers infer that m played its equilibrium strategy while p deviated. They therefore infer that the true differential belongs to $(-2\hat{\theta}\gamma, 2\hat{\theta}\gamma)$, hence $E(x \mid \emptyset_m, z_p) < 2\hat{\theta}\gamma$. Firm p's profit is then

$$\pi_p(\emptyset_m, z_p) = \frac{1}{2} + \frac{E(x \mid z_p, \emptyset_m)}{2\hat{\theta}} - \gamma < \frac{1}{2},$$

i.e., its profit is smaller than the profit if it does not advertise.

c) $\emptyset_p$ is firm p's best response to z_m as defined as (7) when $x \leq -2\hat{\theta}\gamma$. Consider first the play of some $z_p < 2\hat{\theta}\gamma$. Applying the minimality condition, consumers infer that firm p deviated while m played its equilibrium strategy. Their beliefs are therefore determined by (7), i.e., firm p's deviation has no influence on beliefs. Hence firm m is better off with $\emptyset_p$ since it avoids the cost γ.

Consider now the play of some $z_p \geq 2\hat{\theta}\gamma$. Consumers do not know which firm has deviated but think that at most one did. Let them assign probability 1/2 to a deviation by firm m. Their beliefs are then

$$E(x \mid z_m, z_p) = \frac{1}{2}\left(z_m + \frac{1 - e^{2\hat{\theta}(z_m + 2\hat{\theta}\gamma)}}{2\hat{\theta}}\right) + \frac{1}{2}\left(z_p - \frac{1 - e^{-2\hat{\theta}(z_p - 2\hat{\theta}\gamma)}}{2\hat{\theta}}\right).$$

Because m is playing its equilibrium strategy,

$$E(x \mid \hat{z}_m(x), z_p) = \frac{1}{2}\left(z_p - \frac{1 - e^{-2\hat{\theta}(z_p - 2\hat{\theta}\gamma)}}{2\hat{\theta}}\right) + \frac{1}{2}x.$$

Firm p's profit is then

$$\pi_p(\hat{z}_m(x), z_p) = \frac{1}{2} + \frac{E(x \mid z_p, \hat{z}_m(x))}{2\hat{\theta}} - \gamma - \frac{1}{2}(z_p - x)^2.$$

If it plays the equilibrium strategy, its profit is

$$\pi_p(\hat{z}_m(x), \emptyset_p) = \frac{1}{2} + \frac{x}{2\hat{\theta}}.$$

We need to show that $\pi_p(\hat{z}_m(x), \emptyset_p) \geq \pi_p(\hat{z}_m(x), z_p)$ for all $x \leq -2\hat{\theta}\gamma$ and $z_p \geq 2\hat{\theta}\gamma$. Define

$$\begin{aligned}
\varphi(x, z_p) &= 2\hat{\theta}\left(\pi_p(\hat{z}_m(x), z_p) - \pi_p(\hat{z}_m(x), \emptyset_p)\right) \\
&= \frac{1}{2}\left(z_p - \frac{1 - e^{-2\hat{\theta}(z_p - 2\hat{\theta}\gamma)}}{2\hat{\theta}} - x\right) - 2\hat{\theta}\gamma - \hat{\theta}(z_p - x)^2.
\end{aligned}$$

We want to show that $\varphi(x, z_p) < 0$ when $x \leq -2\hat{\theta}\gamma$ and $z_p \geq 2\hat{\theta}\gamma$. Differentiating with respect to z_p yields

$$\varphi_2(x, z_p) = \frac{1}{2}\left(1 - e^{-2\hat{\theta}(z_p - 2\hat{\theta}\gamma)}\right) - 2\hat{\theta}(z_p - x) \quad \text{and}$$

$$\varphi_{22}(x, z_p) = 2\hat{\theta}\left(\frac{1}{2}e^{-2\hat{\theta}(z_p - 2\hat{\theta}\gamma)} - 1\right) < 0.$$

At $z_p = 2\hat{\theta}\gamma$, $\varphi_2(x, z_p) < 0$. Since $\varphi_{22} < 0$, this implies that p would never want to send a signal $z_p > 2\hat{\theta}\gamma$. Thus, it remains to evaluate the sign of

$$\varphi(x, 2\hat{\theta}\gamma) = \frac{1}{2}\left(2\hat{\theta}\gamma - x\right) - 2\hat{\theta}\gamma - \hat{\theta}\left(2\hat{\theta}\gamma - x\right)^2.$$

Observe that

$$\begin{aligned}
\varphi(-2\hat{\theta}\gamma, 2\hat{\theta}\gamma) &= -\widehat{\theta}\left(4\hat{\theta}\gamma\right)^2 < 0 \quad \text{and} \\
\varphi(-1, 2\hat{\theta}\gamma) &= \frac{1}{2}\left(2\hat{\theta}\gamma + 1\right) - 2\hat{\theta}\gamma - \hat{\theta}\left(2\hat{\theta}\gamma + 1\right)^2 < 0.
\end{aligned}$$

Furthermore

$$\varphi_1(x, 2\hat{\theta}\gamma) = -\frac{1}{2} + 2\hat{\theta}\left(2\hat{\theta}\gamma - x\right) > 0.$$

It follows that $\varphi(x, 2\hat{\theta}\gamma) < 0$ for all $x \leq -2\hat{\theta}\gamma$, which in turn implies $\varphi(x, z_p) < 0$ over the relevant domain.

Regarding the best responses of firm m, the same arguments can be made. The only difference is that m wants x to be perceived as small. ■

Proof of Lemma 2. Let $s_q = y_p - q_p$. s_q^2 is thus the variable signalling cost. Substituting in (5) yields

$$s_q(q_p) := \frac{1 - e^{-2\hat{\theta}(s_q + q_p - 4\hat{\theta}\gamma)}}{2\hat{\theta}}, \quad \text{for } q_p \geq 4\hat{\theta}\gamma.$$

Likewise, let $s_x = z_p - x$. Substituting in (6) yields

$$s_x(x) := \frac{1 - e^{-2\hat{\theta}(s_x + x - 2\hat{\theta}\gamma)}}{2\hat{\theta}}, \quad \text{for } x \geq 2\hat{\theta}\gamma.$$

Next we show that $s_x(t) > s_q(t)$ for $t \geq 2\hat{\theta}\gamma$. Let t and t' be such that $s_x(t) = s_q(t')$. Thus

$$\frac{1 - e^{-2\hat{\theta}(\hat{s}_q(t') + t' - 4\hat{\theta}\gamma)}}{2\hat{\theta}} = \frac{1 - e^{-2\hat{\theta}(\hat{s}_x(t) + x - 2\hat{\theta}\gamma)}}{2\hat{\theta}},$$

which yields

$$t' = t + 2\hat{\theta}\gamma.$$

Since $s_x(\cdot)$ and $s_q(\cdot)$ are increasing functions, it follows that for $t' > t + 2\hat{\theta}\gamma$, $\hat{s}_q(t') > \hat{s}_x(t)$. Conversely, $t' < t + 2\hat{\theta}\gamma$ implies $\hat{s}_q(t') < \hat{s}_x(t)$.

Consider now what happens in the areas c and e of Figure 5. For the comparative advertiser, $s_x = s_x(x)$. For the non comparative advertiser, $s_q = s_q(q_p)$. Since $q_p = x + q_m$, $s_q \geq s_x$ if $q_m \geq 2\hat{\theta}\gamma$, in which case variable signalling costs are at least as large for the non-comparative advertiser in the areas c_2 and e of Figure 5. Conversely, $s_q < s_x$ if $q_m < 2\hat{\theta}\gamma$. In c_1 signalling costs are lower with non- than with comparative advertising .

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