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# Relative Consumption and Resource Extraction

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# Titre Relative Consumption and Resource Extraction

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**Résumé / *Abstract***

On présente un modèle d'exploitation d'une ressource naturelle dans lequel les ménages accordent une importance à la consommation relative, soit la différence entre leur consommation et celle de leur groupe de référence. On identifie deux dimensions de distorsion. Premièrement, il y a la distorsion dans le choix du niveau d'effort. Deuxièmement, il y a la distorsion dans le choix entre la consommation présente et la consommation dans le futur. En général, les ménages ont tendance à exploiter les ressources naturelles de facon excessive. Par conséquent, les stocks de ressources à l'état stationnaire sont plus petits que ceux qu'aurait choisis un planificateur central. On propose une règle de taxation qui assure les résultats optimaux.

**Mots clés** : consommation relative, revenue relative, l'hypothèse de la revenue permanente.

*This paper presents a simple model of resource extraction where preferences are household's preferences depend on relative consumption levels. We identify two dimensions along which consumption externalities distort the efficient extraction of resources: (i) the static trade-off between consumption and effort, and (ii) the dynamic trade-off between current and future consumption. In general, households over-exploit the natural resource stocks, resulting in steady state stocks lower than the efficient stocks of resources that would be chosen by a benevolent central planner. We propose a tax mechanism to induce the first best outcome.*

**Keywords:** *relative consumption; relative income hypothesis; permanent income hypothesis.*

**Codes JEL** : D62, Q20, Q50

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# 1 Introduction

The assumption that preferences are independent across households is standard in the economic literature, although it is not particularly appealing. Indeed, social scientists and philosophers have long stressed the relevance of status seeking as being an important characteristic of human behavior (see Aristotle (1941, Rhetoric, Book II, Chapter 10), Kant (1960, Chapter 6), Rawls (1971, Sections 80-82), Schoeck (1966)). In our discipline, the idea that the overall level of satisfaction derived from a given level of consumption depends not only on the consumption level itself but also on how it compares to the consumption of other members of society, is not new. Though origins of this proposition can be traced as far back as Smith (1759) and Veblen (1899), it was not until the work of Duesenberry (1949) and Pollak (1976) that the idea was subjected to systematic analysis. The subsequent literature has often referred to this type of interdependence as "catching up with the Joneses" as in Abel (1990), "keeping up with the Joneses" as in Gali (1994), "status" as in Fisher and Hof (2000), "jealousy" as in Dupor and Liu (2003), or "envy" as in Eaton and Eswaran (2003).

There is a growing body of empirical evidence that confirms the importance of preference interdependence. Clark and Oswald (1996), using a sample of 5,000 British workers, find that workers' reported satisfaction levels are inversely related to their comparison wage rates, supporting the hypothesis of positional externalities. Neumark and Postlewaite (1998) propose a model of relative income to rationalize the striking rise in the employment of married women in the U.S. during the past century. Using a sample of married sisters, they find that married women are 16 to 25 percent more likely to work outside the home if their sisters' husbands earn more than their own husbands. Luttmer (2005) matches individual-level panel data on well-being from the U.S. National Survey of Families and Households to census data on local average earnings. After controlling for income and other own characteristics, he finds that local average earnings have a significantly negative effect on self-reported happiness[1].

On the other hand, there is a vast literature on the over-exploitation of natural resources, in which economists and other scientists have traditionally focused in the "common property" characteristics of many resources. Gordon (1954) presents a lucid treatment of the economics of common property resources. Hardin (1968) conveys the "tragedy of the commons" to the scientific community. Smith (1968) focuses on the steady-state inefficiency while Plourde (1971), Brown (1974) and Smith (1975) explicitly consider models that exhibit transitional dynamics. Brown (1974) points out that a harvest tax, which must change over time as the stock level evolves, should be introduced to correct for congestion externalities. Smith (1975) reviews the debate on the cause of the extinction of many animal species in prehistoric time, and assesses the role of "over-hunting" by primitive human societies. Kremer and Morcom (2000) analyze multiplicity of equilibria in common property resources. Considering the environment as an international common property, Withagen and van der Ploeg (1991), Dockner and Long (1993), Copeland and Taylor (1995), de Zeeuw and Mäler (1998) show that the environment is over-exploited and analyze the role of coordination and governmental regulation.

In this paper we connect these two streams of literature: envy and over-exploitation of natural resources. Our goal is to explore the effects of relative consumption concerns on the process of resource extraction. Complementing the traditional concerns of over-exploitation that results from the common property feature of many natural resource stocks, our results highlight an additional source of inefficiency that leads to over-exploitation: envy. We present a standard model of resource extraction where preferences are

[1] Beyond these studies, status concerns have been introduced to account for observed departures from the neoclassical paradigm in the asset pricing literature (Abel (1990), Gali (1994) and Campbell and Cochrane (1999)), the literature on labor market outcomes (Akerlof and Yellen (1990), the consumption literature (van de Stadt et al. (1985), Kapteyn et al. (1997), Alvarez-Cuadrado and Sutthiphisal (2006)), the experimental literature (Solnick and Hemenway (1998), Johansson-Stenman et al. (2002) and Alpizar et al. (2005)) and the real business cycle literature (Ravn et al. (2006)).

defined over the individual's consumption level, her effort and the comparison of her consumption with that of other members of the community. We identify two dimensions along which consumption externalities distort the efficient extraction of resources. First, when effort is costly, envy distorts the marginal rate of substitution between consumption and effort. We call this the *static/steady-state distortion*. Since status-seeking individuals overvalue consumption, their willingness to exert effort in order to achieve additional consumption is higher than the efficient level. As a consequence of this they over-exploit the resource, resulting in a steady-state stock that is lower than the efficient stock of resources chosen by a central planner. Second, even when effort is costless, consumption externalities might distort the willingness to shift consumption through time, resulting in an inefficient path of extraction. We call this the *dynamic distortion*. We explore the conditions under which these two distortions arise and we show that there exists an optimal tax scheme which induces the competitive agents to replicate the choices of the planner. The tax rate is positive and, in general, time-varying. We calibrate our model under widely used functional forms and find that, under consumption externalities, the competitive steady-state stock of resources is less than two thirds of the efficient stock. Moreover the welfare costs associated with this over-exploitation are very large, close to one third of the laissez-faire steady-state level of consumption. Finally we revisit two important topics in the natural resource literature: amenities and extinction. The intuition we developed with our general model carries through in these special cases: over-exploitation arises even when the natural resource generates a variety of amenity services, and the possibility of extinction increases with consumption externalities.

Our work is related to Fischer and Hof (2000) and Liu and Turnovsky (2005). These authors explore the effects of relative consumption on the rate of capital accumulation and growth. They show that, when labor is endogenous, the concerns for relative consumption lead to the **over-accumulation** of capital. In contrast, in our context, consumption externalities lead to the

**under-accumulation** of the stock of natural resources. Our welfare results are closely related to the recent literature that explores the determinants of (self-reported) well-being such as Frank (1985), Easterlin (1995), Frey and Stutzer (2002) and Layard (2005). This literature highlights the importance of interpersonal comparisons as a key determinant of self-reported happiness.

Finally, in order to focus on the role of relative consumption, we decided to abstract from the common property problem[2]. All the results we present are derived under the assumption that the stocks of resources are privately owned. This assumption allows us to clearly identify the specific distortions associated with consumption externalities abstracting from other external effects, such as the ones caused by congestion or common property.

The paper is organized as follows. Section 2 sets out the basic model, compares the decentralized and centrally planned solutions, explores the distortions associated with envy and characterizes the optimal fiscal policy. Section 3 quantifies the consequences of comparative consumption on the equilibrium allocation of resources. Section 4 presents additional results under consumption externalities: amenities and extinction. The conclusions are summarized in Section 5, while the Appendix provides some technical details.

# 2 Consumption externalities and resource extraction

## 2.1 Economic environment

Consider an economy populated by $N$ identical infinitely-lived individuals. Each individual owns a resource stock, $S$, and has access to the following harvesting function,

$$y = F(S, L) \tag{1}$$

that satisfies $F_L > 0$, $F_S > 0$ and $F(0, L) = F(S, 0) = 0$, where $L$ is the representative individual's harvesting effort. The change in the stock at any

[2] See Long and Wang (2007) for a model of exploitation of a common property resource under envy.

point in time is the difference between the natural growth rate of the resource, $G(S)$, and the amount harvested:

$$\dot{S} = G(S) - y \tag{2}$$

where $G(.)$ is a strictly concave function, with $G(0) = 0$ and $G'(0) > 0$. Assume $G(S)$ reaches a global maximum at $S_M > 0$. We call $S_M$ the maximum-sustainable-yield stock level. The quantity harvested is consumed and therefore denoting by $c$ the consumption of the representative individual, $c = y$.

Let $C$ denote the average per capita consumption in our economy, $C = \sum_{i=1}^{N} c_i/N$. Following Abel (1990) and Carroll et al. (1997), we assume that the utility function of our representative individual depends not only on her own level of consumption and effort, but also on the average consumption level in the economy: $U(c, C, L)$. This specification captures the intuition that lies behind the growing body of empirical evidence that places interpersonal comparisons as a key determinant of individual well-being. We denote the marginal utility of own consumption, average consumption and effort by $U_1$, $U_2$ and $U_L$, respectively. The level of utility achieved by our representative individual is increasing in her own consumption but at a decreasing rate, $U_1 > 0$ and $U_{11} < 0$, and decreasing in effort, $U_L < 0$. In addition we assume that the utility function is jointly concave on individual consumption and effort with $U_{1L} \leq 0$, so the marginal utility of consumption decreases with effort. The crucial aspect of our preference specification concerns the externality imposed by average consumption on the well-being of the individual agent. In the terminology of Dupor and Liu (2003) our agents are jealous, i.e., $U_2 < 0$. Furthermore we impose the following restrictions on the consumption externality for symmetric increases in individual and average consumption, $U_1 + U_2 > 0$, $U_{11} + U_{12} < 0$ and $U_{L1} + U_{L2} \leq 0$. These inequalities guarantee that along a symmetric equilibrium, where $c = C$, direct effects (i.e., effects through the individual's own consumption), always dominate indirect effects (i.e., effects through average consumption).

Finally, it is convenient to invert the harvesting function to get the "effort requirement" function

$$L = L(c, S) \tag{3}$$

with $L_c > 0$ and $L_S < 0$. Our specification assumes that there are $N$ identical stocks of resources that are privately owned and each individual internalizes the effects of her harvesting choices on the evolution of her stock. Our goal is to explore the effects of consumption externalities on the rate of depletion of the stock of resources and therefore we choose to ignore the additional external effect that would be introduced under the assumption of common property of resources, which has been widely explored.

## 2.2 Model solution: decentralized versus centralized exploitation

Each individual chooses consumption and effort in order to maximize the present value of her intertemporal utility, where $\rho$ is the discount rate, given by

$$\Omega = \int_0^\infty U(c, C, L) e^{-\rho t} dt \tag{4}$$

subject to her resource constraint, (2), the effort requirement function, (3) and $S(0) = S_0$. In a decentralized solution each individual ignores the effects of her own consumption choices on average consumption and therefore takes the path of $C$ as given. Denoting by $\psi^d$ the private shadow value of the resource, the optimality conditions associated with this program, where the superscript $d$ denotes decentralized choices, are

$$U_1\left(c^d, C^d, L\left(c^d, S^d\right)\right) + U_L\left(c^d, C^d, L\left(c^d, S^d\right)\right) L_c\left(c^d, S^d\right) = \psi^d \tag{5}$$

$$\rho - \frac{\dot{\psi}^d}{\psi^d} = G'(S^d) + \frac{U_L\left(c^d, C^d, L\left(c^d, S^d\right)\right) L_S\left(c^d, S^d\right)}{U_1 + U_L L_c} \tag{6}$$

together with (2) and the transversality condition, $\lim\limits_{t\to\infty} \psi^d S^d e^{-\rho t} = 0$. Equation (5) equates at the margin the utility of a unit of consumption, net of the effort cost required to extract it, to the private shadow value of the resource. Equation (6) is the standard intertemporal allocation condition that requires the equalization of the rate of return of consumption and the rate of return of the unextracted resource, which consists of two terms: the marginal reproduction rate of the resource and the lower effort cost required to extract it when it is marginally more abundant.

In contrast to the optimization problem of private agents, the central planner acknowledges that each individual's consumption choice creates distortions through its effects on average consumption. Therefore he perceives the following utility specification for the representative individual, $U(c, c, L)$. The planner chooses the levels of consumption and effort to maximize the present value of the flow of utility subject to (2), (3) and $S(0) = S_0$. The first order conditions for this program, where the superscript $p$ denotes the planner's choices, are

$$U_1\left(c^p, c^p, L\left(c^p, S^p\right)\right) + U_2\left(c^p, c^p, L\left(c^p, S^p\right)\right) \\ + U_L\left(c^p, c^p, L\left(c^p, S^p\right)\right) L_c\left(c^p, S^p\right) = \psi^p \quad (7)$$

$$\rho - \frac{\dot{\psi}^p}{\psi^p} = G'(S^p) + \frac{U_L\left(c^p, c^p, L\left(c^p, S^p\right)\right) L_S\left(c^p, S^p\right)}{U_1 + U_2 + U_L L_c} \quad (8)$$

together with (2) and the transversality condition, $\lim_{t\to\infty} \psi^p S^p e^{-\rho t} = 0$. The difference between the two solutions arises because the planner internalizes the negative impact of average consumption on individual welfare by adjusting the marginal utility of private consumption to take into account its marginal social cost. In a general set-up, with endogenous consumption and effort choices, consumption externalities can introduce distortions along two margins; the trade-off between consumption and effort at any given time, the *static distortion*, and the trade-off between consumption at different points in time, the *dynamic distortion*. We will explore the effects of both distortions along a symmetric equilibrium where $c = C$.[3]

[3] The optimality conditions presented in this section are necessary for an interior optimum path. Under well-behaved preferences and reproduction functions, these conditions are also sufficient. In our analysis we assume that our necessary conditions are also sufficient.

## 2.3 The steady-state distortion

Since status-seeking individuals overvalue consumption, their willingness to exert effort in order to achieve additional consumption is higher than the efficient level. As a result, for any given stock of the natural resources competitive agents choose levels of consumption and effort above the efficient levels chosen by the central planner. First, comparing (6) and (8), it is clear that the steady-state level of the stock of resources, and therefore steady-state consumption, achieved in the decentralized solution coincides with the efficient solution if and only if either effort is costless (i.e., $U_L = 0$) or the stock does not enter the harvest function (i.e., $L_S = 0$). Second, assuming that the resource stock enters the harvesting function, under costly effort, we compare the decentralized steady state with the efficient steady state using linear approximations of the first, given by (5), (6) and (2) evaluated at $\left(c^d_\infty, S^d_\infty, \psi^d_\infty\right)$, around the second, given by (7), (8) and (2) evaluated at $(c^p_\infty, S^p_\infty, \psi^p_\infty)$, where the subscript $\infty$ denotes steady-state values. Our results, proved in the Appendix, conclude that the laissez-faire steady-state stock of resources, $S^d_\infty$, is lower than the efficient level, $S^p_\infty$. It follows from the steady-state versions of (2) and (6) that deviations in the stock of resources imply deviations in consumption and effort. Our findings are summarized in the following proposition:

**Proposition 1**

(i) *In a decentralized economy where (a) either effort is costless, or (b) the harvest function is independent of the stock, the steady-state stock of resources and consumption are efficient.*

(ii) *In a decentralized economy where effort is endogenous* ($U_L < 0$) *and the effort requirement function is decreasing in the stock* ($L_S < 0$), *the steady-state stock of resources is* ***lower*** *than the efficient stock chosen by a central planner. If* $G'(S^d_\infty) < 0$ *(i.e.,* $S^d_\infty$ *is greater than the maximum sustainable yield stock), the laissez-faire outcome is associated with* ***over-consumption*** *in the steady state and an inefficiently high level of effort. If* $G'(S^d_\infty) > 0$ *the laissez-faire outcome is associated with an* ***inefficiently low level of***

***steady-state consumption*** *and its effects on steady-state effort is ambiguous.*

**Proof:** See the Appendix.

Intuitively, relative consumption concerns trigger a process of excessive extraction of the natural resource. If at the steady state, the stock of resources is relatively abundant, i.e., its reproduction rate is locally decreasing in the stock, over-consumption can be maintained but only at the expense of an inefficiently high level of effort. On the other hand, if the steady-state stock of resources is low, i.e., its reproduction rate is locally increasing in the stock, then the laissez-faire solution provides a permanently lower level of consumption relative to the efficient solution.

**Example 1a:**

Assume

$$U(c,C,L) = \frac{c^{1-\sigma}}{L^{\gamma}}\left(\frac{c}{C}\right)^{\alpha} \text{ and } L(c,S) = \frac{c}{S^{\mu}} \text{ with } \gamma \geq 0 \text{ and } 1 \geq \mu \geq 0$$

where

$$0 \leq \alpha < \sigma < 1 \text{ and } 1 - \sigma - \gamma > 0$$

Here $\alpha$ is an indicator of status-consciousness.

Then the planner's choices satisfy the condition

$$\mu\gamma\frac{\dot{S}^p}{S^p} - (\sigma+\gamma)\frac{\dot{c}^p}{c^p} = \rho - G'(S^p) - \left(\frac{\mu\gamma}{1-\sigma-\gamma}\right)\frac{c^p}{S^p}$$

while the (symmetric) laissez-faire outcome satisfies the condition

$$\mu\gamma\frac{\dot{S}}{S} - (\sigma+\gamma-\alpha)\frac{\dot{c}}{c} = \rho - G'(S^d) - \left(\frac{\mu\gamma}{1-\sigma-\gamma+\alpha}\right)\frac{c}{S}$$

Thus, the steady state under the social planner satisfies

$$\rho - G'(S^p_{\infty}) = \left(\frac{\mu\gamma}{1-\sigma-\gamma}\right)\frac{G(S^p_{\infty})}{S^p_{\infty}}$$

while the steady state under laissez-faire satisfies

$$\rho - G'(S^d_{\infty}) = \left(\frac{\mu\gamma}{1-\sigma-\gamma+\alpha}\right)\frac{G(S^d_{\infty})}{S^d_{\infty}}$$

Thus, if $\mu > 0$ and $\gamma > 0$, we can see that $S_\infty^d < S_\infty^p$ given that $\alpha > 0$. The greater is $\alpha$, the smaller is the steady state stock under laissez-faire.

If we specifiy, in addition, that

$$G(S) = AS^\theta - \delta S \text{ where } A > 0,\ 0 < \theta < 1 \text{ and } \delta > 0$$

then the maximum-sustainable-yield stock level is

$$S_M = \left(\frac{\theta A}{\delta}\right)^{1/(1-\theta)}$$

and one can verify that the steady-state stock under laissez-faire exceeds $S_M$ if and only if

$$\delta\mu\gamma(1-\theta) > \rho\theta(1-\sigma-\gamma+\alpha).$$

## 2.4 The dynamic distortion

The dynamic distortion arises when concerns for relative consumption cause a deviation of the private willingness to shift consumption through time from the efficient rate of change of consumption chosen by a central planner. Differentiating (5) and combining the result with (6), along a symmetric equilibrium where $c = C$, we obtain the following system of differential equations for the decentralized solution,

$$\frac{\dot{c}^d\left[U_{11}+U_{12}+U_{1L}L_c+\left(U_{L1}+U_{L2}+U_{LL}L_c\right)L_c+U_LL_{cc}\right]}{U_1+U_LL_c}$$

$$\frac{+\dot{S}^d\left[U_{1L}L_S+U_{LL}L_SL_c+U_LL_{cS}\right]}{U_1+U_LL_c} = \rho - G'(S^d) - \frac{U_LL_S}{U_1+U_LL_c} \tag{9}$$

$$\dot{S}^d = G(S^d) - c^d \tag{10}$$

Proceeding similarly with (7) and (8) we obtain the following pair of differential equations for the efficient solution

$$\frac{\dot{c}^p\left[U_{11}+2U_{12}+U_{1L}L_c+U_{2L}L_c+U_{22}+\left(U_{L1}+U_{L2}+U_{LL}L_c\right)L_c+U_LL_{cc}\right]}{U_1+U_2+U_LL_c}+$$

$$\frac{\dot{S}^p\left[U_{1L}L_S+U_{2L}L_S+U_{LL}L_SL_c+U_LL_{cS}\right]}{U_1+U_2+U_LL_c} = \rho - G'(S^p) - \frac{U_LL_S}{U_1+U_2+U_LL_c} \tag{11}$$

$$\dot{S}^p = G(S^p) - c^p \tag{12}$$

By definition, the dynamic distortion is limited to the transitional path and therefore it is better illustrated using a simpler variant of the model where relative consumption does not introduce steady-state distortions. From Proposition 1, both steady states coincide if $L(c, S) = L(c)$, and for the sake of exposition we shall make this assumption throughout Section 2.4. Then the transitional path of the competitive solution is efficient if and only if

$$\begin{aligned} & \frac{U_{11} + U_{12} + U_{1L}L_c + (U_{L1} + U_{L2} + U_{LL}L_c)\, L_c + U_L L_{cc}}{U_1 + U_L L_c} \qquad (13) \\ = \;& \frac{U_{11} + 2U_{12} + U_{1L}L_c + U_{2L}L_c + U_{22} + (U_{L1} + U_{L2} + U_{LL}L_c)\, L_c + U_L L_{cc}}{U_1 + U_2 + U_L L_c} \end{aligned}$$

At this stage is convenient to define the following function

$$V(c, C) \equiv U(c, C, L(c)) \tag{14}$$

**Proposition 2:** *There are no dynamic distortions if and only if the function $V(c, C)$ displays "scale-independent" marginal rate of substitution along the 45 degree line ($c = C$ line), i.e. iff*

$$\frac{V_C(x, x)}{V_c(x, x)} = k$$

*where $k$ is a constant (independent of $x$).*

**Proof:** Using (14) we can express condition (13) as follows

$$\frac{V_{11} + V_{12}}{V_1} = \frac{V_{11} + V_{12} + (V_{21} + V_{22})}{V_1 + V_2}$$

where all derivatives are evaluated at $(c, C) = (x, x)$, for any $x > 0$. This equality holds if and only if

$$(V_1 + V_2)(V_{11} + V_{12}) = (V_1)(V_{11} + V_{12}) + (V_1)(V_{21} + V_{22})$$

i.e.,

$$-\left[\frac{V_{22}}{V_1} - \frac{V_2 V_{12}}{(V_1)^2}\right] = \frac{V_{21}}{V_1} - \frac{V_2 V_{11}}{(V_1)^2}$$

which is equivalent to

$$-\frac{\partial}{\partial C}\left[\frac{V_2}{V_1}\right] = \frac{\partial}{\partial c}\left[\frac{V_2}{V_1}\right]$$

i.e.,

$$\frac{d}{dx}\left[\frac{V_1(x,x)}{V_2(x,x)}\right] = 0$$

Under exogenous effort, this condition reduces to the result presented by Fisher and Hof (2000) and Turnovsky and Liu (2004) in the context of a growing economy[4].

**Example 1b**

$$U(c,C,L) = \frac{1}{1-\theta}\left[\left(\frac{c}{C}\right)^{\beta} cL^{-\mu}\right]^{1-\theta} \text{ with } \beta > 0, \mu > 0$$

where

$$L = c^{\gamma} \text{ with } \gamma > 0$$

Then, assuming $1 > \mu\gamma$ and $0 < (1+\beta-\mu\gamma)(1-\theta) < 1$,

$$V(c,C) = \frac{1}{1-\theta}\left[\left(\frac{1}{C}\right)^{\beta} c^{\beta+1-\mu\gamma}\right]^{1-\theta}$$

This function is concave and homogeneous of degree $(1-\mu\gamma)(1-\theta)$ in $(c, C)$. Thus the marginal rate of substitution is constant along any ray through the origin, i.e., any line $C = bc$ where $b > 0$. Thus, using Proposition 2, there are no dynamic distortions.

**Example 2**

$$U(c,C,L) = \left[\left(\frac{c^{\beta+1}}{C^{\beta}}\right)^{-1} + \left(\frac{1}{L}\right)^{-1}\right]^{-1} \text{ with } \beta > 0$$

and

$$L = c^{\gamma} \text{ with } \gamma > 0$$

[4] We refer our readers to Turnovsky and Liu (2004) for an extensive analysis of the effects of the dynamic distortion. Under costless effort their results are readily applicable to our context.

Then

$$V(c,C)=\left[\left(\frac{c^{\beta+1}}{C^{\beta}}\right)^{-1}+(c)^{\gamma}\right]^{-1}$$

$V(c,C)$ is not homogeneous in $(c,C)$, and the marginal rate of substitution is not constant along the 45 degree line $c=C$, so consumption externalities distort the transition.

## 2.5 Optimal fiscal policy

In the presence of consumption externalities the decentralized allocation of resources is not Pareto efficient. We now show that the government can restore efficiency by means of corrective taxation. Consider the decentralized economy described in sub-section 2.2, with a government that imposes a time varying tax, $\tau(t)$, on resource extraction. The government is assumed to run a balanced budget, returning at each instant in time the amount raised through taxes as a lump sum transfer, $T(t)$. Under this government tax and transfer program, the individual, taking $T$ as given, perceives the following relationship between her level of extraction and her consumption

$$c=(1-\tau)y+T$$

i.e. she takes it that

$$dc/dy=1-\tau$$

The consumer chooses the time path $y^r$, where the superscript $r$ denotes variables under the scenario with government tax or regulations, to maximize

$$\Omega=\int_0^\infty U((1-\tau)y^r+T,C,L(y^r,S^r))e^{-\rho t}dt$$

subject to

$$\dot{S}^r=G(S^r)-y^r \tag{15}$$

The necessary conditions for the representative consumer are

$$(1-\tau)U_1+U_LL_y=\psi^r \tag{16}$$

$$\frac{\dot{\psi}^r}{\psi^r} = (\rho - G'(S^r)) - \frac{U_L L_S}{(1-\tau)U_1 + U_L L_y} \tag{17}$$

Differentiate (16) with respect to time

$$(1-\tau)\left[\frac{d}{dt}(U_1)\right] - (U_1)\left[\frac{d}{dt}\tau\right] + \left[\frac{d}{dt}(U_L L_y)\right] = \dot{\psi}^r \tag{18}$$

Combining equations (18) and (17), we get

$$\frac{(1-\tau)\left[\frac{d}{dt}(U_1)\right] - (U_1)\left[\frac{d}{dt}\tau\right] + \left[\frac{d}{dt}(U_L L_y)\right]}{(1-\tau)U_1 + U_L L_y} = (\rho - G'(S^r)) - \frac{U_L L_S}{(1-\tau)U_1 + U_L L_y} \tag{19}$$

Suppose the government set $\tau(t)$ as follows:

$$\tau(t)U_1(c^r, c^r, L(c^r, S^r)) = -U_2(c^r, c^r, L(c^r, S^r)) \tag{20}$$

then, differentiating both sides of (20) with respect to $t$

$$U_1\left[\frac{d}{dt}\tau\right] + \tau(t)\left[\frac{d}{dt}(U_1)\right] = -\left[\frac{d}{dt}(U_2)\right] \tag{21}$$

Substituting (20) and (21) into (19), we obtain the following differential equation that together with (15) describes the dynamics of the competitive solution under our tax scheme:

$$\frac{\left[\frac{d}{dt}(U_1)\right] + \left[\frac{d}{dt}(U_2)\right] + \left[\frac{d}{dt}(U_L L_y)\right]}{U_1 + U_2 + U_L L_y} = (\rho - G'(S^r)) - \frac{U_L L_S}{U_1 + U_2 + U_L L_y}$$

This equation coincides with (11) and therefore achieves the social optimum. Our results are summarized in the following proposition.

**Proposition 3:** *Along a symmetric path, $c = C$, the efficient equilibrium can be decentralized by setting a tax on resource extraction at each point in time equal to*

$$\tau = -\frac{U_2\left(c^r, c^r, L\left(c^r, S^r\right)\right)}{U_1\left(c^r, c^r, L\left(c^r, S^r\right)\right)}$$

In general $\tau$ will be time-varying along the transitional path converging to a positive constant at the steady state.

# 3 Calibrating the effects of the externality

Consider an economy populated by a representative individual endowed with the following iso-elastic utility function:

$$U(c, C, L) = \frac{1}{1-\theta}\left[c\left(\frac{c}{C}\right)^{\beta} L^{-\mu}\right]^{1-\theta}$$

where $\theta > 0$, $\theta \neq 1$, $\beta \geqslant 0$, $\theta + \beta(\theta - 1) > 0$, $\mu > 0$ and $\mu(\theta - 1) - 1 > 0$ . Multiplicative relative consumption has been widely used in the asset pricing literature, Abel (1990) and Gali (1994), growth literature, Alvarez-Cuadrado et al. (2004), and experimental literature, Solnick and Hemenway (1998). In the competitive solution the individual takes $C$ as given and chooses $c$ to maximize

$$\int_0^{\infty} e^{-\rho t}\left\{\frac{1}{1-\theta}\left[c\left(\frac{c}{C}\right)^{\beta}[L(c,S)]^{-\mu}\right]^{1-\theta}\right\} dt$$

subject to (2) and $S(0) = S_0$. Evaluating the optimality conditions along a symmetric equilibrium path where $C = c$, we obtain

$$(1+\beta)c^{-\theta}L^{-\mu(1-\theta)} - \mu c^{1-\theta}L^{-\mu(1+\theta)-1}L_c - \psi = 0 \tag{22}$$

$$\dot{\psi} = \psi\left[\rho - G'(S)\right] + \mu c^{1-\theta}L^{-\mu(1+\theta)-1}L_S \tag{23}$$

Consider the following harvesting and natural resource reproduction functions[5],

$$c = F(S, L) = S^{\alpha}L^{\gamma} \tag{24}$$

$$G(S) = S^{\lambda} \tag{25}$$

with $(1-\gamma) \leq \alpha \leq 1$, $(1-\alpha) \leq \gamma \leq 1$ and $0 < \lambda < 1$. Our harvesting function nests the popular Schaefer harvesting function, where $\gamma = \alpha = 1$ (Schaefer (1957), Brander and Taylor (1998)) and the constant returns to

[5] Our numerical results are based on a reproduction function strictly increasing in the resource stock. Although this specification is more restrictive than our previous analytical results, we believe that in the resource extraction context the increasing range of this function is the empirically relevant one.

scale Cobb-Douglas function, where $\alpha+\gamma=1$ (Brown (1974), Smith (1975)) as special cases. Inverting (24) the necessary conditions become[6]

$$S^{\frac{\mu(1-\theta)\alpha}{\gamma}} c^{\frac{-(\mu(1-\theta)+\theta\gamma)}{\gamma}} \left[(1+\beta)-\frac{\mu}{\gamma}\right]-\psi=0 \tag{26}$$

$$\dot{\psi}=\psi\left[\rho-\lambda S^{\lambda-1}\right]-\mu S^{\frac{\mu(1-\theta)\alpha}{\gamma}-1} c^{\frac{-(\mu(1-\theta)+\theta\gamma)}{\gamma}+1} \tag{27}$$

Combining (26), (27), (25) and (2) we get a system of two differential equations that together with the initial stock of resources and the transversality condition fully describes the dynamic behavior of our model. In the unique steady state, the stock of natural resources satisfies

$$\rho-\lambda\left(S_{\infty}^{d}\right)^{\lambda-1}=\frac{\alpha\mu}{\gamma(1+\beta)-\mu}\left(S_{\infty}^{d}\right)^{\lambda-1} \tag{28}$$

It is straightforward to show that the steady-state stock of natural resources, $S_{\infty}^{d}$, is decreasing in the importance of relative consumption captured by $\beta$. Furthermore if effort is costless (i.e., $\mu=0$) the steady-state level of the natural resource is not affected by relative consumption concerns. Finally it is worth noticing that the efficient steady state satisfies (28) with $\beta=0$ and therefore in line with our analytical results, $S_{\infty}^{d}<S_{\infty}^{p}$ and $C_{\infty}^{d}<C_{\infty}^{p}$.

We calibrate our model to illustrate the quantitative effects of relative consumption in the steady-state stock of resources, consumption, effort and welfare. Our measure of the welfare cost of the distortion is standard: we denote by $\varphi$ the percentage increase in individual (and average) steady-state consumption that an agent living in the competitive world must receive in order to enjoy the same welfare level as that of an agent living in the steady state of the planned economy[7]. In our benchmark calibration, we set our

[6] In order to ensure that the planner's optimization problem is strictly concave in $c$ we impose the restriction $\mu/\gamma<1$.

[7] The steady state level of welfare achieved by the planned economy is given by $\Omega^{p}\left(c_{\infty}^{p}, L_{\infty}^{p}\right)=\int_{0}^{\infty} e^{-\rho t}\left\{\frac{1}{1-\theta}\left[c_{\infty}^{p}\left(L_{\infty}^{p}\right)^{-\mu}\right]^{1-\theta}\right\} dt$, the corresponding measure for the decentralized economy is given by $\Omega^{m}\left(c_{\infty}^{m}, L_{\infty}^{m}\right)=\int_{0}^{\infty} e^{-\rho t}\left\{\frac{1}{1-\theta}\left[c_{\infty}^{m}\left(L_{\infty}^{m}\right)^{-\mu}\right]^{1-\theta}\right\} dt$. We define our welfare cost as the value of $\varphi$ that satisfies $\Omega^{p}\left(c_{\infty}^{p}, L_{\infty}^{p}\right)=\Omega^{m}\left((1+\varphi) c_{\infty}^{p}, L_{\infty}^{m}\right)$.

extraction parameters $\alpha = \gamma = 0.5$, the stock elasticity of the reproduction function $\lambda = .5$, the rate of time preference $\rho = 0.02$ and the parameter that governs the disutility of effort $\mu = 0.2$. Since we restrict our analysis to steady-state outcomes our results are independent of the value of $\theta$.Direct evidence on the importance of relative consumption, captured by $\beta$, is sparse. The literature on the equity premium puzzle suggests that only relative consumption matters; see Abel (1990), Gali (1994) and Campbell and Cochrane (1999). Easterlin (1995) and Frey and Stutzer (2002) evaluate the time series and cross-sectional properties of several measures of self-reported happiness. Their findings are consistent with preference specifications that again place most of the weight on relative consumption. Alpizar et al. (2005) conduct several experiments to assess the importance of relative consumption. In the case of cars and housing their median estimate for the weight of relative consumption lies between 0.5 and 0.75. Alvarez-Cuadrado and Sutthiphisal (2007), using individual consumption data, estimates a weight of relative consumption close to one third. In view of these estimates, we choose a conservative value, $\beta = 1$ , for the benchmark calibration and conduct extensive sensitivity analysis based on the range of reported estimates.

As reported in Table 1, at the steady state of our benchmark calibration (in bold numerals), the competitive stock of resources is less than two thirds of the efficient stock. As a result, there is a shortfall in consumption equal to one fourth of the efficient level. Since under our benchmark parameter values steady-state effort is not distorted, this lower level of consumption is associated with a welfare cost, expressed in units of permanent consumption, equal to one third of the competitive level of consumption. Table 1 presents some robustness checks for our benchmark results. As we increase the weight of relative consumption (i.e., $\beta/(1+\beta)$) from zero to more than nine tenths, the competitive stock of resources falls to slightly more than one third of the

Admittedly, $\varphi$ overstates the true welfare cost since it ignores the welfare effects along the transition. Nontheless given the size of the welfare costs associated with the steady state distortion we believe our measure is a good proxy for the overall welfare costs of the distortion from any set of initial conditions. See Alvarez-Cuadrado (2007) for an analysis of the welfare costs along the transitional path in the context of a growing economy.

efficient stock. The diminishing returns in the resource reproduction function limit the drop in consumption that remains close to two thirds of the efficient level, but despite of this the associated welfare costs, as a result of diminishing marginal utility, almost double relative to our benchmark calibration. Since relative consumption concerns affect the steady-state allocation of resources through the trade-off between consumption and effort, the distortion is very sensitive to changes in $\mu$. As effort becomes more costly, $\mu = 0.4$, the planner reduces both consumption and effort to reach a steady state with a relatively high stock of resources. Since competitive agents overvalue consumption they exert an inefficiently high level of effort that eventually runs the stock of resources down to only one tenth of the efficient level. The associated welfare costs are huge, equivalent to a consumption loss three times as large as the steady state laissez faire level. As we change de elasticity of the extraction function to the resource stock, $\alpha$, a similar effect arises but now it even distorts the steady state allocation of effort. When effort is a major ingredient in the extraction process, $\alpha = 0.3$, the planner chooses relatively high levels of effort, even higher than the market in steady state, and the distortion only reduces the laissez faire stock of resources by one fifth with an associated welfare cost slightly above one tenth. On the other hand as the weight of effort on extraction decreases, the market reduces the amount of effort exerted at a slower rate than the efficient pace. As a result, when $\alpha = 0.7$, competitive agents exert almost three times more effort than the the efficient level and reduce the efficient stock of resources by four fifths, again the welfare losses associated with this process of over-extraction are tremendous.

The steady-state results for the Schaefer extraction function are presented in Table 2. Under our benchmark calibration, the competitive stock of resources is only two thirds of the efficient stock, consumption only four fifths and the laissez-faire level of effort exceeds the efficient level by approximately one quarter. This combination of lower consumption and higher effort is associated with a welfare cost slightly below one third of the laissez-faire level of consumption. As in the constant returns to scale case, increases in the weight of relative consumption, $\beta$, or increases on the disutility of effort, $\mu$, exacerbate the effects of the distortion and its welfare costs.

**Table 1.**Steady-state distortion for different parameter configurations.
Constant returns to scale extraction technology

| | $\alpha = 0.5;$ | | $\mu = 0.2$ | | |
|---|---|---|---|---|---|
| | $\beta = 0$ | $\beta = 0.5$ | $\beta = 1$ | $\beta = 5$ | $\beta = 10$ |
| Resource Stock | 0% | -33% | **-44%** | -59% | -61% |
| Consumption | 0% | -18% | **-25%** | -36% | -38% |
| Effort | 0% | 0% | **0%** | 0% | 0% |
| Welfare | 0% | 22% | **33%** | 56% | 61% |

| | | $\alpha = 0.5;$ | $\beta = 1$ | |
|---|---|---|---|---|
| | $\mu = 0$ | $\mu = 0.1$ | $\mu = 0.3$ | $\mu = 0.4$ |
| Resource Stock | 0% | -21% | -67% | -89% |
| Consumption | 0% | -11% | -43% | -67% |
| Effort | 0% | 0% | 0% | 0% |
| Welfare | 0% | 13% | 75% | 200% |

| | | $\mu = 0.2;$ | $\beta = 1$ | |
|---|---|---|---|---|
| | $\alpha = 0.3$ | $\alpha = 0.4$ | $\alpha = 0.6$ | $\alpha = 0.7$ |
| Resource Stock | -21% | -31% | -60% | -80% |
| Consumption | -11% | -17% | -36% | -55% |
| Effort | -7% | -6% | 25% | 192% |
| Welfare | 11% | 19% | 64% | 177% |

**Table 2.**Steady-state distortion for different parameter configurations.
Schaefer extraction technology

| | | | $\mu = 0.2$ | | |
|---|---|---|---|---|---|
| | $\beta = 0$ | $\beta = 0.5$ | $\beta = 1$ | $\beta = 5$ | $\beta = 10$ |
| Resource Stock | 0% | -24% | **-34%** | -49% | -52% |
| Consumption | 0% | -13% | **-19%** | -29% | -36% |
| Effort | 0% | 15% | **23%** | 40% | 45% |
| Welfare | 0% | 18% | **28%** | 50% | 56% |

| | $\beta = 1$ | | | |
|---|---|---|---|---|
| | $\mu = 0$ | $\mu = 0.1$ | $\mu = 0.3$ | $\mu = 0.4$ |
| Resource Stock | 0% | -18% | -47% | -59% |
| Consumption | 0% | -10% | -27% | -36% |
| Effort | 0% | 11% | 37% | 56% |
| Welfare | 0% | 12% | 51% | 86% |

# 4 Additional results with consumption externalities

## 4.1 A model with amenity values

In addition to being consumption goods or production inputs, some natural resources generate a variety of amenity services that include, for instance, the recreational and aesthetic values associated with a well-preserved environment[8]. Following Krautkraemer (1985) we assume that the owner of a resource stock not only earns income from extraction, but also enjoys other amenities from the preservation of the stock. To capture this idea, we adopt the following preference specification for our representative individual,

$$U = \left[\alpha\left(\left(\frac{c}{C}\right)^{\beta} c\right)^{\omega} + (1-\alpha)S^{\omega}\right]^{\mu/\omega} = \left[\alpha C^{-\beta\omega} c^{(1+\beta)\omega} + (1-\alpha)S^{\omega}\right]^{\mu/\omega} \tag{29}$$

where

$$-\infty < \omega < 1,\ 0 < \alpha < 1 \text{ and } 0 < \mu \leq 1$$

Here, in order to focus on the role of amenities, we assume that extraction does not require effort:

$$\dot{S} = G(S) - c$$

Define

$$Z = \alpha C^{-\beta\omega} c^{(1+\beta)\omega} + (1-\alpha)S^{\omega}$$

[8] The experimental literature on relative consumption highlights important differences in the degree of comparison among different consumption goods. In general, easily observable goods or services, such as housing or cars, are subject to stronger externalities than unobservable ones, such as medical insurance or leisure. In line with this evidence, we make the extreme assumption that interpersonal comparisons do not involve comparing the amenity services provided by the stock of natural resources.

In a decentralized setting the representative individual chooses the path of $c$ to maximize the intertemporal value of (29) subject to (2) and $S(0) = S_0$. The necessary conditions for this program are

$$\frac{\mu}{\omega} Z^{(\mu/\omega)-1}(1+\beta)\alpha C^{-\beta\omega}\omega c^{(1+\beta)\omega-1} - \psi = 0 \tag{30}$$

$$\dot{\psi} = \psi(\rho - G') - \frac{\mu}{\omega} Z^{(\mu/\omega)-1}(1-\alpha)\omega S^{\omega-1} \tag{31}$$

together with (2) and the transversality condition, $\lim_{t\to\infty} \psi S e^{-\rho t} = 0$. Differentiating (30) with respect to time and combining the result with (30) and (31), we obtain the following differential equation

$$\frac{\dot{\psi}}{\psi} = (\rho - G'(S)) - \frac{(1-\alpha)S^{\omega-1}}{(1+\beta)\alpha C^{-\beta\omega} c^{(1+\beta)\omega-1}}$$

A symmetric steady state is reached when $C = c = G(S_\infty)$ and $\dot{\psi} = 0$ and therefore the steady-state stock of resources satisfies

$$\rho - G'(S_\infty) = \frac{1-\alpha}{(1+\beta)\alpha} \left[\frac{G(S_\infty)}{S_\infty}\right]^{1-\omega} \tag{32}$$

It is straightforward to show that the steady-state stock of natural resources, $S_\infty$, is unique. Furthermore, increases in the importance of relative consumption, captured by $\beta$, decrease the steady-state stock of resources. Intuititively increases in the importance of relative consumption increase the willingness of the competitive agent to increase extraction at the cost of lowering the amenity services provided by the unexploited resource. This process of over-extraction is associated with a lower steady-state stock of the natural resource. Finally, it is worth noticing that the efficient stock of resources is given by (32) with $\beta = 0$ and therefore is always larger than the laissez-faire solution.

## 4.2 A model with extinction of privately-owned resource stocks

So far we have assumed that all consumption goods come from resource extraction. It follows that, if the marginal utility of consumption, evaluated at $c = 0$, is infinite, then given our assumption that each agent has perfect property rights over her own resource stock it is not possible that the market outcome, with agents maximizing over an infinite horizon, will result in the extinction of the resource.[9]

We now show that if the marginal utility of consumption from harvested resources, evaluated at $c = 0$, is finite, then under certain parameter values, the social planner would want to maintain a positive stock level, but the market outcome results in extinction, even though each individual has full control of her own resource stock.

Assume the utility function is a function of a basic good $x$, and a resource good (e.g. timber furniture), the consumption of which is denoted by $c$. We posit in the case where envy applies only to the consumption of the resource good. We specify the following utility function:

$$U(x, c, C, L) = v(x) + \ln\left[1 + c^{\beta+1}C^{-\beta}L^{-\mu}\right]$$

where $v(x)$ is a concave and increasing function, and

$$0 < \beta < \mu < 1, \; 0 < 1 + \beta - \mu < 1$$

The harvesting function is $y = LS$, and as before the resource havested is consumed, $c = y$. Then

$$L = cS^{-1}$$

Substitution yields

$$U = v(x) + \ln\left[1 + c^{\beta+1}C^{-\beta}c^{-\mu}S^{\mu}\right] \tag{33}$$

Assume that at each point of time, each agent is endowed with a fixed flow $\overline{x} > 0$ of a non-storable basic good, so her consumption of this good is equal to $\overline{x}$.

[9] Of course with common property resources, extinction is a definite possibility even if the marginal utility of consumption at zero consumption is infinite. For a model with extinction under the regime of common-property resources, see Dutta and Rowat (2006).

Assume $G(S)$ is strictly concave, with $G(0) = 0 = G(\overline{S})$ for some $\overline{S} > 0$. Then $G(S)/S$ is a decreasing function, and

$$\lim_{S \to 0} \frac{G(S)}{S} = G'(0) \equiv a > 0$$

We shall prove the following:

**Proposition 4:** *Assume that the parameter values satisfy the following inequalities*

$$\frac{(1+\beta-\mu)}{\mu}(\rho - G'(0)) \geq G'(0) > \frac{1-\mu}{\mu}(\rho - G'(0)) > 0 \tag{34}$$

*Then under laissez-faire, there exists no positive steady-state stock, while the social planner's problem has a unique positive steady-state stock level.*

**Proof:**

(i) Market outcome (extinction):

In a decentralized setting the representative individual chooses the path of $c$, taking $C$ as given, to maximize the intertemporal value of (33) subject to (2) and $S(0) = S_0$. The necessary conditions for this program are[10]

$$\frac{C^{-\beta} S^{\mu} (1+\beta-\mu) c^{\beta-\mu}}{1 + c^{\beta+1-\mu} C^{-\beta} S^{\mu}} - \psi = 0$$

$$\dot{\psi} = \psi(\rho - G') - \frac{\mu c^{\beta+1-\mu} C^{-\beta} S^{\mu-1}}{[1 + c^{\beta+1-\mu} C^{-\beta} S^{\mu}]}$$

Along a symmetric equilibrium, $c/C = 1$. Furthermore, let

$$z(t) \equiv \frac{c(t)}{S(t)}$$

Then we have the following system

$$\frac{(1+\beta-\mu)\,[z(t)]^{-\mu}}{1 + S(t)\,[z(t)]^{1-\mu}} - \psi(t) = 0$$

$$\dot{\psi}(t) = \psi(t)(\rho - G') - \frac{\mu\,[z(t)]^{1-\mu}}{1 + S(t)\,[z(t)]^{1-\mu}}$$

[10] The second order condition is satisfied because $\mu - \beta > 0$.

$$\dot{S}(t) = G(S(t)) - z(t)S(t)$$

Assume $\rho > G'(0)$ . Then it is easy to verify that the following triple is a steady state, where the steady-state stock is zero,

$$(S^d_\infty, \psi^d_\infty, z^d_\infty) = \left(0, (1+\beta-\mu)\left[\frac{(1+\beta-\mu)(\rho-a)}{\mu}\right]^{-\mu}, \frac{(1+\beta-\mu)(\rho-a)}{\mu}\right)$$

Furthermore, let us show there are no steady states with a positive stock. Suppose there was one, denoted by $\widehat{S} > 0$. Then

$$\widehat{z} = \frac{G(\widehat{S})}{\widehat{S}}$$

$$\frac{(1+\beta-\mu)\widehat{z}^{-\mu}}{1+\widehat{S}\widehat{z}^{1-\mu}}(\rho - G'(\widehat{S})) = \frac{\mu(\widehat{z})^{1-\mu}}{\left[1+\widehat{S}\widehat{z}^{1-\mu}\right]}$$

i.e.

$$\frac{(1+\beta-\mu)}{\mu}(\rho - G'(\widehat{S})) = \frac{G(\widehat{S})}{\widehat{S}} \tag{35}$$

But, from assumption (34) and the strict concavity of $G(S)$, for all $S > 0$, the following inequalities hold:

$$\frac{(1+\beta-\mu)}{\mu}(\rho - G'(S)) > \frac{(1+\beta-\mu)}{\mu}(\rho - G'(0)) \geq G'(0) > \frac{G(S)}{S}$$

It follows that there is no $\widehat{S} > 0$ that satisfies condition (35).

(ii) Social planner (non-extinction):

The social planner internalizes the negative impact of individual consumption, $c$, on average consumption, $C$, and therefore the optimality conditions for his program, where $z = c/S$, are

$$\frac{(1-\mu)z^{-\mu}}{1+Sz^{1-\mu}} = \psi$$

$$\dot{\psi} = \psi(\rho - G') - \frac{\mu z^{1-\mu}}{1+Sz^{1-\mu}}$$

Then the steady state stock $S^p_\infty$ must satisfy

$$\frac{1-\mu}{\mu}(\rho - G'(S)) = \frac{G(S)}{S} \tag{36}$$

The right-hand side of (36) starts, when $S = 0$, at $G'(0)$ and falls as $S$ rises, and eventually becomes negative. The left-hand side starts at $(\rho - G'(0))(1 - \mu)/(\mu)$ and rises as $S$ rises. It follows that a unique steady state $S^p_\infty \in \left(0, \overline{S}\right)$ exists if $(\rho - G'(0))(1-\mu)/\mu < G'(0)$.

# 5 Conclusions

The negative welfare consequences of competitive consumption have been long noted by social and natural scientists. In the words of the evolutionary biologist Richard Dawkins (1986, p.184):

> Why, for instance, are trees in the forest so tall? The short answer is that all the other trees are tall, so no one tree can afford not to be. It would be overshadowed if it did... But if only they were all shorter, if only there could be some sort of trade-union agreement to lower the recognized height of the canopy in forests, all the trees would benefit. They would be competing with each other in the canopy for exactly the same sun light, but they would all have "paid" much smaller growing costs to get into the canopy.

Only recently has the economic profession begun to pay closer attention to the welfare consequences of consumption externalities. In this paper we have presented a simple model of resource extraction where preferences are defined over the individual's consumption level, her effort and the comparison of her consumption with that of other members of the community. Our specification captures the intuition that lies behind the growing body of empirical evidence that places interpersonal comparisons as a key determinant of well-being. We find that envious individuals ignore the negative effects that their extraction choices impose on the welfare of their neighbors; as a result they over-exploit the natural resource, resulting in an inefficiently low steady-state stock.

We have identified two dimensions along which consumption externalities distort the efficient extraction of resources. In the case where effort is endogenous, envy distorts the marginal rate of substitution between consumption and effort, the *static/steady state distortion.* Even when effort is costless, consumption externalities might distort the willingness to shift consumption through time and therefore the path of extraction displays a *dynamic distortion.* These distortions provide a new rationale for the increasing concerns about over-exploitation of resources, possible extinction, and the general deterioration of the environment caused by human activities. Our results highlight an important scope for government intervention even in the absence of the externalities associated with common property arrangements. In a world where agents envy the consumption of their neighbors, an appropriately chosen harvesting tax must be imposed to induce the preservation of the natural resource and improve welfare.

Our results can be extended along several dimensions. In future work, we aim to fully explore the transitional dynamics of our model. A similar exercise could be conducted under the more plausible scenario of common-property resources. We anticipate that the negative effects associated with consumption externalities will exacerbate the problems caused by the "tragedy of commons".

## APPENDIX

## Proof of Proposition 1

The symmetric competitive solution satisfies the following conditions

$$U_1\left(c,c,L\left(c,S\right)\right)+U_L L_c-\psi=0$$

$$\dot{\psi}=\psi\left[\rho-G'(S)\right]-U_L\left(c,c,L\left(c,S\right)\right)L_S\left(c,S\right)$$

$$\dot{S}=G(S)-c$$

Therefore the steady-state values of the symmetric competitive solution, denoted by $\left(c_\infty^d,\psi_\infty^d,S_\infty^d\right)$, satisfy the following system of equations

$$U_1\left(c,c,L\left(c,S\right)\right)+U_L L_c-\psi=0$$

$$\psi\left[\rho-G'(S)\right]-U_L\left(c,c,L\left(c,S\right)\right)L_S\left(c,S\right)=0$$

$$G(S)-c=0$$

The social planner's solution satisfies the following conditions

$$U_1\left(c,c,L\left(c,S\right)\right)+U_2\left(c,c,L\left(c,S\right)\right)+U_L L_c-\psi=0$$

$$\dot{\psi}=\psi\left[\rho-G'(S)\right]-U_L\left(c,c,L\left(c,S\right)\right)L_S\left(c,S\right)$$

$$\dot{S}=G(S)-c$$

The steady-state values of the social planner's solution, denoted by $(c_\infty^p,\psi_\infty^p,S_\infty^p)$, satisfy the following system of equations

$$U_1\left(c,c,L\left(c,S\right)\right)+U_2\left(c,c,L\left(c,S\right)\right)+U_L L_c-\psi=0$$

$$\psi\left[\rho-G'(S)\right]-U_L\left(c,c,L\left(c,S\right)\right)L_S\left(c,S\right)=0$$

$$G(S)-c=0$$

We assume that the planner's steady state has the usual saddlepoint property. We wish to compare $(c_\infty^p,S_\infty^p,\psi_\infty^p)$ with $\left(c_\infty^d,S_\infty^d,\psi_\infty^d\right)$.

**Proof of Part (i):** Assume either $U_L=0$ or $L_S=0$. Then the steady-state stock satisfies $G'(S)=\rho$ under the social planner, and also under the laissez-faire regime. Since $G(S)$ is strictly concave, this condition implies $S^p_\infty = S^d_\infty$. This in turn implies $c^p_\infty = c^d_\infty$.

**Proof of Part (ii):** Assume that $U_L > 0$ and $L_S < 0$. Then, since the shadow price $\psi$ is positive, at the steady state $\rho - G'(S) = U_L L_S > 0$. We now show that this implies $S^d_\infty < S^p_\infty$.

For this purpose, we define the following vector-valued function

$$\mathbf{J}(c, S, \psi, \phi) \equiv \begin{bmatrix} J^{(1)}(c, \psi, S, \phi) \\ J^{(2)}(c, \psi, S, \phi) \\ J^{(3)}(c, \psi, S, \phi) \end{bmatrix}$$

where

$$J^{(1)}(c, \psi, S, \phi) \equiv U_1\left(c, c, L\left(c, S\right)\right) + (1-\phi)U_2\left(c, c, L\left(c, S\right)\right) + U_L L_c - \psi$$

$$J^{(2)}(c, \psi, S, \phi) \equiv \psi\left[\rho - G'(S)\right] - U_L\left(c, c, L\left(c, S\right)\right) L_S\left(c, S\right)$$

$$J^{(3)}(c, \psi, S, \phi) \equiv G(S) - c$$

By construction, when we set $\phi = 0$, we obtain the social planner's system of equations, i.e.,

$$\mathbf{J}(c^p_\infty, \psi^p_\infty, S^p_\infty, 0) = \mathbf{0}$$

Similarly, setting $\phi = 1$, we obtain the market outcome

$$\mathbf{J}\left(c^d_\infty, \psi^d_\infty, S^d_\infty, 1\right) = \mathbf{0}$$

Applying Taylor expansion around the point $(c^p_\infty, \psi^p_\infty, S^p_\infty, 0)$, we get

$$\mathbf{0} = \mathbf{J}(c^p_\infty, \psi^p_\infty, S^p_\infty, 0) - \mathbf{J}\left(c^d_\infty, \psi^d_\infty, S^d_\infty, 1\right) = \begin{bmatrix} J^{(1)}_c & J^{(1)}_\psi & J^{(1)}_S & J^{(1)}_\phi \\ J^{(2)}_c & J^{(2)}_\psi & J^{(2)}_S & J^{(2)}_\phi \\ J^{(3)}_c & J^{(3)}_\psi & J^{(3)}_S & J^{(3)}_\phi \end{bmatrix} \begin{bmatrix} c^p_\infty - c^d_\infty \\ \psi^p_\infty - \psi^d_\infty \\ S^p_\infty - S^d_\infty \\ 0 - 1 \end{bmatrix}$$

where we have ignored the higher order terms. Now, since $J^{(1)}_\phi(c^p_\infty, S^p_\infty, \psi^p_\infty, 0) = -U_2\left(c^p_\infty, c^p_\infty, L\left(c^p_\infty, S^p_\infty\right)\right) \equiv -U^p_2$, $J^{(3)}_c = -1$, $J^{(3)}_\psi = 0$, $J^{(2)}_\psi = \rho - G'(S^p_\infty) > 0$, and $J^{(2)}_\phi = 0 = J^{(3)}_\phi$, we get

$$\begin{bmatrix} J^{(1)}_c & J^{(1)}_\psi & J^{(1)}_S \\ J^{(2)}_c & \rho - G'(S^p_\infty) & J^{(2)}_S \\ -1 & 0 & G'(S^p_\infty) \end{bmatrix} \begin{bmatrix} c^p_\infty - c^d_\infty \\ \psi^p_\infty - \psi^d_\infty \\ S^p_\infty - S^d_\infty \end{bmatrix} = \begin{bmatrix} -U^p_2 \\ 0 \\ 0 \end{bmatrix}$$

Let $D$ denote the determinant of the matrix on the left-hand side. It can be shown (see Lemma 1 below) that $D > 0$. Then, using Cramer's Rule,

$$S_\infty^p - S_\infty^d = \frac{-\left(U_2^p\right)\left(\rho - G'(S_\infty^p)\right)}{D} > 0$$

This shows that the market outcome results in a lower steady-state stock level as compared with the outcome under the social planner. Similarly

$$c_\infty^p - c_\infty^d = \frac{-\left(U_2^p\right)\left(\rho - G'(S_\infty^p)\right)G'(S_\infty^p)}{D}$$

This expression is positive iff $G'(S_\infty^p) > 0$.

**Lemma 1:** Consider any autonomous optimal control problem with one state variable, $c$, and one control variable, $S$. Assume the Hamiltonian is concave in $(c, S)$, and strictly concave in $c$, and the existence of a steady state that displays saddlepoint stability. Let $D$ be the determinant of the $3 \times 3$ matrix

$$\begin{bmatrix} H_{cc} & H_{c\psi} & H_{cS} \\ -H_{Sc} & \rho - H_{S\psi} & -H_{SS} \\ H_{\psi c} & H_{\psi\psi} & H_{\psi S} \end{bmatrix}$$

evaluated at that steady state. Then $D > 0$.

**Proof:** (Not intended for publication).

Let us define

$$v(c, S) \equiv U(c, c, L(c, S))$$

Consider the problem of the planner. He chooses $c$ to maximize

$$\int_0^\infty e^{-\rho t} v(c, S) dt$$

subject to

$$\dot{S} = F(c, S)$$

where $F(c, S) = G(S) - c$ or any other general function that is concave in $(c, S)$.

The Hamiltonian is

$$H(c, \psi, S) = v(c, S) + \psi F(c, S)$$

We get a system of 3 equations

$$H_c(c,\psi,S) = v_c + \psi F_c = 0$$

$$\dot{\psi} = \rho\psi - H_S(c,\psi,S)$$

$$\dot{S} = H_\psi(c,\psi,S)$$

At the steady state we have

$$H_c(c,\psi,S) = 0$$

$$\rho\psi - H_S(c,\psi,S) = 0$$

$$H_\psi(c,\psi,S) = 0$$

Consider the maximized Hamiltonian

$$\overline{H}(\psi,S) = \max_c H(c,\psi,S)$$

Then we get a system of two equations

$$\dot{\psi} = \rho\psi - \overline{H}_S(\psi,S)$$

$$\dot{S} = \overline{H}_\psi(\psi,S)$$

Let $(\psi_\infty, S_\infty)$ be a steady state of this system. Linearizing around $(\psi_\infty, S_\infty)$ we get

$$\begin{bmatrix} \dot{\psi} \\ \dot{S} \end{bmatrix} = \begin{bmatrix} \rho - \overline{H}_{S\psi} & -\overline{H}_{SS} \\ \overline{H}_{\psi\psi} & \overline{H}_{\psi S} \end{bmatrix} \begin{bmatrix} \psi - \psi_\infty \\ S - S_\infty \end{bmatrix}$$

The determinant is

$$\Delta = \left(\rho - \overline{H}_{S\psi}\right)\overline{H}_{\psi S} + \overline{H}_{\psi\psi}\overline{H}_{SS}$$

Assume saddlepoint stability, so that one root is positive and one root is negative. Then $\Delta < 0$ because $\Delta$ is equal to the product of the two roots.

From the envelope theorem

$$\overline{H}_S(\psi,S) = H_S(c,\psi,S)$$

$$\overline{H}_{\psi}(\psi, S) = H_{\psi}(c, \psi, S)$$

Thus

$$\overline{H}_{\psi\psi} = H_{\psi\psi} + H_{\psi c}\frac{\partial c}{\partial \psi} = H_{\psi\psi} - H_{\psi c}\frac{H_{c\psi}}{H_{cc}}$$

$$\overline{H}_{SS}(\psi, S) = H_{SS}(c, \psi, S) + H_{Sc}(c, \psi, S)\frac{\partial c}{\partial S} = H_{SS} - H_{Sc}\frac{H_{cS}}{H_{cc}}$$

$$\overline{H}_{S\psi} = H_{S\psi} + H_{Sc}\frac{\partial c}{\partial \psi} = H_{S\psi} - H_{Sc}\frac{H_{c\psi}}{H_{cc}}$$

Now consider the matrix

$$\begin{bmatrix} H_{cc} & H_{c\psi} & H_{cS} \\ -H_{Sc} & \rho - H_{S\psi} & -H_{SS} \\ H_{\psi c} & H_{\psi\psi} & H_{\psi S} \end{bmatrix}$$

Let $D$ be the determinant of this matrix. Evaluating this determinant by expanding along the second row, it is easy to verify that

$$H_{cc}D = (H_{cc})^2\,\Delta$$

It follows that $D > 0$. This concludes the proof of the Lemma.

**REFERENCES:**


Abel, A. (1990), "Asset Prices Under Habit Formation and Catching Up With the Joneses," *American Economic Review,* 80, 38–42.

Akerlof, G. and J. Yellen (1990), "The Fair Wage-Effort Hypothesis and Unemployment," *Quarterly Journal of Economics,* 105, 2, 255-283

Alpizar, F., F. Carlsson, and O. Johansson-Stenman (2005), "How Much do We Care about Absolute versus Relative Income and Consumption?" *Journal of Economic Behavior and Organization,* 56, 405-421.

Alvarez-Cuadrado, F., (2007), "Envy, leisure and restrictions on working hours," *Canadian Journal of Economics* (forthcoming).

Alvarez-Cuadrado, F., G. Monteiro, and S.J. Turnovsky (2004), "Habit Formation, Catching up with the Joneses, and Economic Growth," *Journal of Economic Growth,* 9, 47-80.

Alvarez-Cuadrado, F. and D. Sutthiphisal (2007), "Envy and Consumption Behavior: A panel data approach," McGill University Working Paper.

Aristotle (1941), *The Basic Works of Aristotle,* R. McKeon (ed.), New York: Random House.

Brander, James A. and M. Scott Taylor (1998), "The Simple Economics of Easter Island: A Ricardo-Malthus Model", *American Economic Review,* Vol. 88, pp. 119-138.

Brown, G. (1974), "An Optimal Program for Managing Common Property Resources with Congestion Externalities," *Journal of Political Economy,* Vol. 82(1), pp. 163-173.

Campbell, J.Y. and J.H. Cochrane (1999), "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy,* Vol. 107, 205–251.

Carroll, C.D., J.R. Overland, and D.N. Weil (1997), "Comparison Utility in a Growth Model", *Journal of Economic Growth* 2, 339–367.

Clark, A.E. and A.J. Oswald (1996), "Satisfaction and Comparison Income," *Journal of Public Economics,* 61, 359-381.

Copeland, B. and M. S. Taylor (1995), "Trade and Transboundary Pollution," *American Economic Review*, 716-737.

Dawkins, R. (1986), *The Blind Watchmaker*, Oxford University Press, Oxford, UK.

de Zeeuw, A. and K.-G. Mäler (1998), "The Acid Rain Differential Game," *Environmental and Resource Economics,* Vol. 12 (2), 167-184.

Dockner, E. and N.V. Long (1993), "International Pollution Control: Cooperative versus Noncooperative Strategies," *Journal of Environmental Economics and Management*, 24(3), 13-29.

Dupor, B. and W-F. Liu (2003), "Jealousy and equilibrium overconsumption," *American Economic Review*, 93, 423-428

Duesenberry, J.S. (1949), *Income, Saving and the Theory of Consumer Behavior,* Harvard University Press, Cambridge, Mass.

Dutta, J. and C. Rowat (2006), "The Road to Extinction: Commons with Capital Markets," Manuscript, Department of Economics, University of Birmingham.

Easterlin, R.A. (1995), "Will Raising the Incomes of All Increase the Happiness of All?" *Journal of Economic Behavior and Organization*, 27, 35-47

Eaton, B.C., and M. Eswaran (2003), "The Evolution of Preferences and Competition: a Rationalization of Veblen's Theory of Invidious Consumption," *Canadian Journal of Economics,* 36, 832-59.

Fisher, W. and F. Hof (2000), "Relative Consumption and Endogenous Labour Supply in the Ramsey Model: Do Status-Conscious People Work Too Much?" Institute for Advance Studies, Vienna.

Frank, R.H. (1985), *Choosing the Right Pond,* New York and Oxford: Oxford University Press.

Frey, B.S. and A. Stutzer (2002), "What Can Economists Learn from Happiness Research?" *Journal of Economic Literature*, 40, 402-435.

Gali, J. (1994). "Keeping Up with the Joneses: Consumption Externalities, Portfolio Choice, and Asset Prices," *Journal of Money, Credit, and*

*Banking* 26, 1–8.

Gordon, H. Scott (1954), "Economic Theory of Common Property Resources," *Journal of Political Economy*, Vol. 62(2), 124-142

Hardin, G. (1968), "The Tragedy of the Common," *Science,* Vol. 162, pp. 1243-1248.

Johansson-Stenman, O., Carlsson, F., and D. Daruvala (2002), "Measuring Future Grandparents' Preferences for Equality and Relative Standing," *Economic Journal* 112, 362-383

Kant, I. (1960), *Education*, translated by Annette Churton, Ann Arbor: University of Michigan Press.

Kapteyn, A., S. van de Geer, H. van de Stadt, and T. Wansbeek (1997), "Interdependent Preferences: An Econometric Analysis," *Journal of Applied Econometrics*, 12, 6, 665-686.

Krautkraemer, J.A., 1985, "Optimal Growth, Resource Amenities and Preservation of Natural Environments," *Review of Economic Studies,*Vol 85(1), pp. 153-170

Kremer, M. and C. Morcom, 2000, "Elephants," *American Economic Review,* Vol. 90 (1), 212-234.

Layard, R. (2005), *Happiness: Lessons from a New Science*, New York: Penguin Press.

Liu, W.-F. and S.J. Turnovsky (2005), "Consumption Externalities, Production Externalities, and Long-run Macroeconomic Efficiency," *Journal of Public Economics*, 89, 1097-1129

Long, N.V. and S. Wang (2007), "Resource Grabbing by Status-Conscious Agents", Paper presented at the 2007 CREE conference, Ottawa, October 2007.

Luttmer, E. (2005), "Neighbors as Negatives: Relative Earnings and Well-Being," *Quarterly Journal of Economics,* 120(3), 963-1002.

Neumark, D. and A. Postlewaite (1998), "Relative Income Concerns and the Rise in Married Women's Employment," *Journal of Public Economics,* 70. 157-183

Plourde, G. C. (1971), "Exploitation of Common Property Renewable Natural Resources," *Economic Inquiry,*Vol. 9, 256-61.

Pollak, R.A. (1976), "Interdependent Preferences," *American Economic Review*, 66(3), 309-320

Ravn, M., Schmitt-Grohe, S. and M. Uribe (2006), "Deep Habits," *Review of Economic Studies,* 73, 195-218

Rawls, J. (1971), *A Theory of Justice,* Cambridge, MA: Belknap Press.

Schoeck, H. (1966), *Envy: A Theory of Social Behavior,* Glenny and Ross (trans.), New York: Harcourt, Brace.

Schaefer, M. B. (1957), "Some Considerations of Population Dynamics and Economics in Relation to the Management of Marine Fisheries," *The Fisheries Research Board of Canada*, Vol. 14, 669-81.

Smith, A. (1759), *The Theory of Moral Sentiments*, Oxford: Clarendon Press.

Smith, V. L., (1968), "Economics of Production from Natural Resources," *American Economic Review*, Vol. 58, pp 409-31

Smith, V. L., 1975, "The Primitive Hunter Culture, Pleistocene Extinction and the Rise of Agriculture," *Journal of Political Economy*, Vol. 83, pp 727-755.

Solnick, S. and D. Hemenway (1998), "Is More Always Better? A Survey on Positional Concerns," *Journal of Economic Behavior and Organization* 37, 373-383

van der Ploeg, F. and C. A. Withagen (1991), "Pollution Control and the Ramsey Problem," *Environmental and Resource Economics*, Vol. 1, 215-230.

van de Stadt, H., A. Kapteyn, and S. van de Geer (1985), "The Relativity of Utility: Evidence from Panel Data," *Review of Economics and Statistics* 67, 179–187.

Veblen, T.B. (1899), *The Theory of the Leisure Class: An Economic Study of Institutions,* Modern Library, New York.