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Résumé / *Abstract*

L'efficacité des accords économiques constitue le centre d'intérêt de la recherche en économie, mais on peut se demander si les améliorations observées en matière d'efficacité au sein des économies développées ont été accompagnées d'une vulnérabilité accrue de la production aux événements catastrophiques. Pour répondre à cette question de façon utile, nous avons besoin d'outils pouvant être mis en œuvre dans le but de mesurer la vulnérabilité de la production aux perturbations ou aux bouleversements.
La présente étude tente de mettre en place des mesures rudimentaires de cette vulnérabilité potentielle accrue au sein des systèmes économiques. Pour ce faire, nous nous appuyons sur les concepts issus de la théorie de l'information, sur la conception rigoureuse de systèmes et, indirectement, sur des statistiques robustes pour mesurer la vigueur de la production et pour appliquer cette mesure à certains cas intéressants.

Mots clés : Entropie, redondance, conception rigoureuse

While the efficiency of economic arrangements is the primary focus of economic research, we may ask whether the efficiency improvements that have been experienced in the developed economies have been accompanied by increased vulnerability of output to catastrophic events. In order to address this question usefully, we need some implementable measures of vulnerability of output to disruptive events or large shocks. This study attempts to provide some rudimentary measures of this potential increased vulnerability in economic systems. To do so we draw on concepts from information theory, robust system design and, indirectly, robust statistics to define a measure of the robustness of production, and apply the measure to some cases of interest.

Keywords*: Entropy, redundancy, robust design.*

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1. Introduction

The efficiency of economic arrangements is the primary focus of economic research. This focus properly reflects the potential for improvements in the efficiency of production and the gains that have in fact been realized, over long historical periods, in the productivity and aggregate output of the economy.

Nevertheless, it is reasonable to ask whether the productivity, or efficiency, improvements that have been experienced in the developed economies have been accompanied by increased vulnerability of output to catastrophic events. Consider for example a manufacturing facility with a marginal cost of production which is always declining in the quantity produced, that is, a natural monopoly. If the costs of distribution are small, the most efficient arrangement will entail concentration of all production at a single location. As long as the probability of a disruption at this location is sufficiently small, this efficient arrangement is also optimal in a broader sense. By contrast, if there is a substantial probability of disruption at any given location, this efficient arrangement may be less robust to catastrophe than production at multiple points.

To take another example, consider the water supplies of a set of scattered houses. If each householder has his or her own supply of water, each must maintain a separate system of well, pump, etc. If the householders form a village and centralize water supply, the single system potentially reduces expense, effort in maintenance and so on very substantially: that is, there is a potential increase in efficiency. However, the single system now makes this set of householders more vulnerable to serious disruption; equipment failure or contamination eliminate water supply for all, whereas with separate systems neighbours could supply each other in the event of a failure of one of the individual systems. Increased efficiency under some typical circumstances may have a cost in increased vulnerability.

The study of the vulnerability of output to disruptive events, or conversely the robustness of economic arrangements, has a parallel in the statistical literature; the most efficient estimator of a population parameter is not in general the most robust, and *vice versa*. It may be sensible to sacrifice some degree of efficiency in ideal cases in return for a more robust estimator which will perform better under some adverse circumstances; it is possible that the same may be true of economic arrangements.

This study attempts to provide some rudimentary measures of this potential increased vulnerability in economic systems. To do so we draw on concepts from information theory, robust system design and, indirectly, robust statistics to define a measure of the robustness of production, and apply the measure to some cases of interest. The next section ...

2. Concepts and definitions

There are substantial literatures in engineering and biology on system robustness, redundancy or degeneracy; see for example Sussman (2007), Tononi et al. (1999), Edelman and Gally (2001).

When a destructive event arises in part of a system, there are a number of features of the system which will tend to reduce the impact of the event. The impact will tend to be lower where:

- another part of the system can directly replace the output of the part affected by the destructive event

- the destructive effect can be localized to the part of the system where the event arose (error propagation is limited)
- another part of the system can be adapted to replace all or part of the lost output
- the affected part of the system can be repaired quickly

In order to provide some corresponding operational measures we will begin with some formal definitions of terms. The definition of robustness that we use is based on concepts that date at least to Box and Andersen (1955) and Tukey (1960); the definitions of redundancy and adaptability are based on those given by Tononi et al. (1999), who use the term 'degeneracy' rather than 'adaptability'. Entropy is used as a measure of information content by Shannon (1948).

Definition 2.1 Robustness. One system is more robust than another if it performs relatively well when conditions differ from the ideal.

Definition 2.2 Redundancy. A system contains redundant features if some elements duplicate the function of other structurally identical elements.

Definition 2.3 Adaptability (degeneracy). A system contains adaptable features if the functions of some elements of the system can be performed by other structurally different elements.

Note that in a system containing redundancies, some elements can be made inoperative without negatively affecting performance of the system; in a system with adaptable features, adaptation to inoperative elements may entail some sacrifice in performance.

Definition 2.4 Entropy. Let $i = 1, 2, \ldots N$ index elements of a set Ω_N which can be aggregated to a total value $\omega = \sum_{i=1}^{N} \omega_i$. Then the entropy of the set Ω_N is

$$\rho(\Omega_N) = -\sum_{i \in \Omega_N} \frac{\omega_i}{\omega} ln \left(\frac{\omega_i}{\omega} \right).$$

A set with high entropy is one for which the aggregated value is highly dispersed among the individual elements.

3. Measures of system robustness

We now need to integrate these various features of a robust system into a measure suitable for application to a substantial aggregate economic system, at the level of a city or a larger economic unit.

Different possible measures will indicate robustness to different types of possible disruption. In the present study we will attempt to measure robustness to a single event at the system's most vulnerable point; we might think of this as a measure of the most damage that can be done by a planned attack on one element of the system. Of course, the economy of a city can be completely destroyed by a sufficiently large destructive event, as has often happened during wars; in measuring robustness we attempt to characterize performance of a system in response to relatively small deviations from ideal conditions. However, an ideal measure would indicate a continuum of degrees of vulnerability to events of differing magnitude.

In discussing these measures, we need to define what we mean by a source of supply.

Definition 3.0 Source of supply.

3.1 Properties of measures

We will only consider measures that fulfil a few simple conditions. Let M be any measure of system robustness which is consistent with the properties given below.

i (strict monotonicity) If an additional source of supply x_{N+1}, is added to the system, then M increases.

ii If additional supply in quantity Z is added to the system, (a) as one source, $x_{N+1} = Z$, or (b) as multiple sources $\sum_{i=1}^{k} x_{N+i} = Z$, then M increases more in case (b) than in (a).

iii If there is only one independent source (if all output can be lost through destruction at one point), then $M \equiv 0$.

3.2 Measures

Let $x_i, i = 1, \ldots, N$ index sources as defined above with $X = \sum_{i=1}^{N} x_i$, and let r_i be the proportion of total *required supply* R which remains if source i is destroyed. Next compute the order statistics of the $\{r_i\}$, that is, the re-ordered values $\{r_{(i)}\}$ such that $r_{(1)} \leq r_{(2)} \leq \ldots \leq r_{(N)}$; $r_{(1)}$ therefore corresponds with the largest independent source of supply.

Definition 3.1 Measure 1. $m_1 = \sum_{i=1}^{n} r_{(j)}$.

Measure 1 has a number of advantages, as we will note below, but imposes the strong informational requirement that we be able to characterize the loss of required capacity arising from each source of supply.

Another measure which is commonly used in analogous contexts is the simple entropy of sources of supply.

Definition 3.2 Measure 2 (entropy of sources). $m_2 = \sum_{i=1}^{n} \left(\frac{x_i}{X}\right) \ln \left(\frac{x_i}{X}\right)$.

Although m_2 measures dispersion, and is therefore related to the limitation of error propagation, it takes no account of any excess capacity, or redundancy, in the system. Measure 1 does so; for example, consider a system in which there is sufficient redundancy that any two single sources could be destroyed while leaving sufficient supply to cover the requirement. Then $\ell_1 = \ell_2 = 0$ and the first two entries in m_1 both take the maximum value, 1. This system therefore will have a higher value of measure 1 than an otherwise identical system, with identical total output, but in which the requirement is higher and matches the total output exactly. By contrast, the entropy measure m_2 depends only on the outputs of individual sources and the total output, and so will be the same in the two systems: whether total output exceeds the requirement does not enter the calculation.

A measure such as m_2 which takes no account of redundancy is of some value, but is not fully satisfactory for our purposes. An alternative is to supplement the simple entropy measure m_2 with a non-negative component that measures redundancy. We do this with measure 3; if there is no redundancy, measures 3 and 2 are equal.

To make this definition we take the order statistics of proportion of required supply remaining, $\{r_{(i)}\}$, defined above. Let c be the largest index such that $r_{(c)} \geq 1$: that is, full capacity remains if ordered sources 1 through $c-1$ are destroyed, but not if the $c-$th most important is also destroyed. The additional component of the measure takes the entropy of the sources through c inclusive. If there is no redundancy in the system, or if any excess capacity will be lost with loss of the most important source of supply, then $m_3 = m_2$.

Definition 3.3 Measure 3 (combined entropy). $m_3 = m_2 + \sum_{i=1}^{c} \left(\frac{x_{(i)}}{X}\right) \ln \left(\frac{x_{(i)}}{X}\right)$.

Note that measure 3 requires less information than measure 1, in that we do not actually need the set of values $\{r_{(i)}\}$: we only need the point c at which $r_{(c)}$ falls below 1. Apart from this, the definition refers to observed supplies and total supply.

4. Examples

TABLE 4.1
Values of measures $m_1 - m_3$ in example cases

	Measure:				
	m_1	$m_2(= m_3)$	m_1	m_2	m_3
	$X = R$	$X = R$	$X = 1.5R$	$X = 1.5R$	$X = 1.5R$
Case:					
A	0	0	0	0	0
A'	0	0	0	0	0
B	1.5	1.386	2.25	1.386	2.079
C	0.5	0.693	0.5	0.693	0.693
C'	0.4	0.673	0.4	0.673	0.673
D	0.5	0.693	0.75	0.693	0.693

5. Measures specific to urban economies

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