



Centre Interuniversitaire sur le Risque,
les Politiques Économiques et l'Emploi

Cahier de recherche/Working Paper **09-30**

A Dynamic Model of Risk-Shifting Incentives with Convertible Debt

Pascal François

Georges Hübner

Nicolas Papageorgiou

Août/August 2009

François: HEC Montréal and CIRPÉE. Corresponding author : HEC Montréal, Department of Finance, 3000 Côte-Ste-Catherine, Montréal, Canada H3T 2A7

pascal.francois@hec.ca

Hübner: HEC-University of Liège and Maastricht University

Papageorgiou: HEC Montréal and CIRPÉE

We thank Murray Carlson as well as seminar participants at ESSEC, HEC Lausanne, Solvay Business School, and the University of Guelph for helpful comments. Financial supports from FQRSC and IFM² (François and Papageorgiou) and Deloitte Luxembourg (Hübner) are gratefully acknowledged. All remaining errors are ours.

Abstract:

In a one-period setting Green (1984) demonstrates that convertible debt perfectly mitigates the asset substitution problem by curbing shareholders' incentive to increase risk. This is because claimholders design the capital structure precisely when the risk-shifting opportunity is available. In practice, firms do not alter their capital structure over the life of the convertible debt. Hence, when the risk-shifting opportunity arises, convertible debt design may no longer match with firm asset value to mitigate the asset substitution problem. This leaves room for a strategic non-cooperative game between shareholders and convertible debtholders. We show that two risk-shifting scenarios arise as attainable Nash equilibria. *Pure asset substitution* occurs when, despite convertible debtholders not exercising their conversion option, shareholders still find it profitable to shift risk. *Strategic conversion* occurs when, despite convertible debtholders giving up the conversion option value, they are better off receiving their share of the wealth expropriation from straight debtholders. We use contingent claims analysis and the Black and Scholes (1973) setup to characterize the equilibria. Even when initial convertibles debt is endogenously designed so as to minimize the likelihood of risk-shifting equilibria, we show that asset substitution cannot be completely eliminated. Our overall conclusion is that – in contrast to agency theory's claim – convertible debt is an imperfect instrument for mitigating shareholders' incentive to increase risk.

Keywords: Convertible debt, risk-shifting, non-cooperative game

JEL Classification: C72, G32

1 Introduction

This paper addresses the strategic dimension surrounding the exercise of the conversion option that is embedded in convertible debt contracts. Among the possible rationales for issuing convertible debt is the mitigation of the asset substitution problem put forward by Jensen and Meckling (1976).¹ Due to the limited liability principle, equity payoff is a convex function of firm value that provides shareholders with an incentive to increase risk (see Galai and Masulis 1976). By selecting riskier projects in a way that is not anticipated by creditors, shareholders can transfer wealth to their own benefit. Eisdorfer (2008) provides recent evidence of asset substitution.

In this paper, we argue that some issues regarding the use of convertible debt as a risk mitigating financial instrument are still left unresolved. Green (1984) and MacMinn (1992) show that the risk-shifting problem can be fully eliminated when the investment opportunity and the convertible debt issue are *contemporaneous*.² Their one-period setting ensures the existence of a convertible debt contract that perfectly mitigates shareholders' incentive to increase risk. However, if shareholders have the opportunity to benefit from shifting risk, it is unclear why they would, at the same time, agree on designing a convertible debt contract in the capital structure. Rather, the agency theory literature views convertible debt as a *pre-commitment* device for curbing shareholders' incentive to increase risk. That is, at some point in time, claimholders agree on a capital structure that reflects their commitment to avoid asset substitution over a certain time horizon.

¹Other rationales for issuing convertible debt include: heterogeneous risk assessment (Brennan and Schwartz 1988), backdoor equity financing (Stein 1992), or financing sequential investments (Mayers 1998).

²Lewis, Rogalski and Seward (1998, 1999) and Elliott, Koëter-Kant and Warr (2004) provide empirical evidence of firms using convertibles to reduce asset substitution.

Yet, when the risk-shifting opportunity arises, they may not be able to reach a similar agreement to redesign capital structure. In other words, they will have to deal with the risk-shifting opportunity according to contract terms that were previously negotiated.³ As a matter of fact, Chang, Chen and Liu (2004) provide evidence that firms issue convertible debt to finance future growth opportunities and that they do not alter their capital structure over the life of the convertible debt.

We therefore extend Green's analysis by allowing the risk-shifting opportunity to occur *after* the capital structure is designed, leaving room for a strategic game to be played between shareholders and convertible debtholders when the opportunity to shift risk arises.⁴ Despite the initial capital structure, there are two reasons why asset substitution might still take place. First, shareholders may find that the convertible no longer mitigates their incentive to increase risk. Hence, whatever the convertible debtholders' action, shareholders will seize the opportunity to shift risk. We refer to such a situation as pure asset substitution. Second, convertible debtholders can act strategically when considering the decision to convert. Specifically, they trade off the suboptimal exercise of the conversion option with the benefits of aligning their interests with those of initial shareholders, i.e. restoring the incentive to take risk and transferring wealth at the expense of straight debtholders. We refer to such a situation as strategic conversion.

We analyze the decisions to shift risk and to convert as a non-cooperative game

³Mayers (1998) further argues that convertible debt financing is motivated by a desire to minimize security issue costs. That is, convertible debt is designed precisely to avoid having to redesign capital structure in the future.

⁴Another direction, not pursued here, is to allow shareholders to adjust asset volatility (see Hennessy and Tserlukevich 2004). However, evidence that corporate investment is lumpy (see Doms and Dunne, 1998, or Nilsen and Schiantarelli, 2003) suggests that in practice, shareholders may not have the option to continuously select asset volatility.

between shareholders and convertible debtholders.⁵ When the game is simultaneous, we show that pure asset substitution arises as an attainable Nash equilibrium. When the game is sequential, we further show that strategic conversion arises as an additional attainable Nash equilibrium. Strategic conversion is further favored by a contractual means that shareholders may have at their disposal, namely the payment of a special dividend. We then assess the ex ante likelihood of asset substitution by assuming that convertible debt is initially designed so as to minimize the probability of risk-shifting. Although our setup provides the greatest chances for convertible debt to play its role as a risk-mitigating instrument, there is still room for asset substitution. Our overall conclusion is that convertible debt is an imperfect instrument for mitigating shareholders' incentive to increase risk. The agency theoretic rationale for issuing convertible debt may therefore be overstated.

The rest of the paper is organized as follows. Section 2 reviews the assumptions of the model. Section 3 derives the Nash equilibria for both the simultaneous and the sequential games. Section 4 carries out a sensitivity analysis to investigate how equilibria are affected ex post by the risk-shifting magnitude, financial leverage and asset value. In section 5, the design of convertible debt is endogenously determined so as to minimize the likelihood of risk-shifting equilibria. This makes it possible to assess, ex ante, the risk-mitigating performance of convertible debt. Section 6 shows how this performance can be further weakened by a special dividend. Section 7 concludes.

⁵The strategic exercise of other options is examined in the literature (see e.g. Grenadier 1996, for the timing of real estate development, or Kulatilaka and Perotti 1998, for investment with strategic competition). In Bühler and Koziol (2002), a gradual conversion of convertible debt allows for strategic interplay between claimholders yet, their analysis does not address the asset substitution problem.

2 Assumptions

Our analysis is based on the following assumptions.

The economy The market for assets is complete and arbitrage-free. Trading takes place in continuous time.

The firm's assets The value of the firm's assets is a stochastic process $(V_t)_{t \geq 0}$ defined on the filtered probability space $(\Omega, \mathcal{F}, P, \{\mathcal{F}_t\})$, and is independent of capital structure choices. In the absence of arbitrage, prices of traded assets discounted at the risk-free rate are martingales under the equivalent risk-neutral measure Q .

The firm's capital structure The firm is financed with equity, senior straight discount debt and junior convertible discount debt.⁶ Without any loss in generality, we assume that the firm does not pay any regular dividend. This implies that conversion policies are solely driven by strategic considerations, and not by the excess yield on equity.⁷ The number of outstanding shares is N . Straight debt promises a face value of F and matures at date T . Convertible debt promises a face value of M , matures at T and can be converted anytime up to T into n newly issued shares. We denote by q the conversion ratio where $q = n/(n + N)$. All agents are supposed to act as a single class of claimholders.⁸

⁶Spatt and Sterbenz (1993) and Chesney and Gibson (2001) introduce a distinction between convertibles and warrants. They show that convertibles, being "bundled" securities, are more effective at mitigating the asset substitution problem.

⁷Asquith and Mullins (1991) provide evidence that voluntary conversion is related to the differential between the coupon and the dividends on the converted stock.

⁸Gradual conversion of convertible debt is examined by Emanuel (1983), Constantinides (1984), Constantinides and Rosenthal (1984), and Bühler and Koziol (2002).

The risk-shifting opportunity The asset substitution problem consists of the opportunity to shift risk before debt matures. Specifically, we assume that the firm will be presented with the option to increase the risk of assets.⁹ This increase in risk reflects for instance a technology shock, a change in regulation, or the entry of a new competitor that the firm will face. The timing of the risk-shifting opportunity is random.

Claimholders' behavior All claimholders share the same information. They observe asset value dynamics. They agree on the characteristics of the risk-shifting opportunity. It is assumed that the asset value process is observable to all claimholders. However, it is not verifiable. Therefore claimholders cannot write a contract contingent on every sample path of asset value.¹⁰ A second-best alternative is the convertible debt contract, which induces local concavity in equity payoff, thereby mitigating shareholders' incentive to increase risk. Claimholders also agree on designing the capital structure that minimizes the probability of asset substitution. We therefore examine the setup under which convertible debt has the greatest chances to fulfill its role as a risk-mitigating instrument.

The timing of events The risk-shifting opportunity can randomly occur at any time θ between the initial contracting date and debt maturity. At date θ , convertible debtholders decide on the conversion of their claim and shareholders choose the level of asset risk. Claimholders can act either simultaneously or sequentially. The timing of

⁹In the spirit of Jensen and Meckling (1976), Gavish and Kalay (1983), and Green and Talmor (1986), we focus on the strict asset substitution problem. That is, shareholders may manipulate firm risk without altering the value of firm assets. This induces that, as in the model of Green (1984), the martingale pricing technique still applies with the same numeraire.

¹⁰Specifically, the future level of asset risk cannot be contracted upon. If it could, asset substitution would not be an issue.

events is shown in Figure 1.

The major difference from Green's setup is that the design of convertible debt may not be synchronous with the possible shift in risk. The one-period setup in Green (1984) is equivalent to assuming that claimholders can always redesign contracts every time a risk-shifting opportunity arises. However, it is not clear why shareholders would accept such a renegotiation if they can actually benefit from shifting risk. We therefore rely on the more realistic assumption that capital structure is designed *before* the risk-shifting opportunity occurs so that all claimholders agree on their best commitment to avoid asset substitution over a certain time horizon. Our standpoint is consistent with Ju and Ouyang (2006) who show that the asset substitution problem becomes more severe as the firm cannot issue debt on a repetitive basis. As a matter of fact, straight and convertible bonds are mostly issued as long-term contracts.¹¹

3 Game theory analysis

Whenever the risk-shifting opportunity occurs, shareholders will decide whether or not to seize this opportunity. Similarly, convertible debtholders will reconsider the exercise of their conversion option. Table 1 indicates the four possible values of equity and convertible debt at time θ whether asset substitution takes place or not, and whether convertible debtholders exercise their conversion option at date θ or postpone their decision until T .

We denote by S_t and C_t the time- t values of equity and convertible debt, respectively.

¹¹Krishnaswami and Yaman (2008) report a median maturity of about 20 years for convertibles and 10 years for straight debt in their sample of 660 straight and 1,862 convertible bonds issued between 1983 and 1998. Korkeamaki and Moore (2004) report an average maturity of 17 years for their sample of convertible bonds issued between 1980 and 1996.

Superscripts a and \bar{a} refer to the occurrence and non-occurrence of asset substitution, respectively. Similarly, superscripts e and \bar{e} refer to the occurrence and non-occurrence of early conversion, respectively.

Since the decision of shareholders to substitute assets and that of convertible debtholders to convert early are both discretionary, Table 1 can be viewed as the representation of the non-cooperative game between these two claimants.¹² In his setup, Green (1984) only considers two of the four cells of Table 1: either the convertible debt design precludes asset substitution until time T (south-east cell), or this design does not make the shareholders' payoff concave enough to offset the gains from shift in risk (north-east cell).

Lemma 1 establishes three inequalities with which shareholders and convertible debtholders are confronted, irrespective of any strategic consideration.

Lemma 1 *For any time $\theta < T$, the following inequalities prevail*

$$S_{\theta}^{a,e} > S_{\theta}^{\bar{a},e}, \quad C_{\theta}^{a,\bar{e}} > C_{\theta}^{a,e}, \quad C_{\theta}^{\bar{a},\bar{e}} > C_{\theta}^{\bar{a},e}.$$

The first inequality is a consequence of asset substitution: provided that convertibles are converted, only straight debt remains in the firm's liabilities. Hence, equity payoff is convex again and shareholders are better off increasing asset risk. Second and third inequalities follow from Merton's (1973) non-early exercise property for American call options on non-dividend paying asset. Ingersoll (1977) applies this property to convertibles and shows that it is not optimal to exercise a convertible before maturity.

We call *Status Quo* the Nash equilibrium where: (i) shareholders are better off not substituting assets at date θ and (ii) convertible debtholders do not exercise their con-

¹²Note that straight debtholders do not interfere in the game and, because firm asset value is invariant to asset substitution, the value of their claim can be retrieved from the other two securities values.

version option before T . It yields the pair $\{S_{\theta}^{\bar{a},\bar{e}}, C_{\theta}^{\bar{a},\bar{e}}\}$ and rules out any wealth transfer at the expense of straight and convertible debtholders.

The situation depicted in the north-west cell of the table is of special interest. On the one hand, convertible debtholders depart from Ingersoll's (1977) optimal exercise rule and are better off converting early. On the other hand, shareholders choose to shift risk despite the presence of convertible debt in the capital structure. This contingent decision that yields the pair $\{S_{\theta}^{a,e}, C_{\theta}^{a,e}\}$ is referred to as the *Strategic Conversion* Nash equilibrium.

The other possible Nash equilibrium is the *Pure Asset Substitution* that yields the pair $\{S_{\theta}^{a,\bar{e}}, C_{\theta}^{a,\bar{e}}\}$. Note that early conversion restores the convexity of shareholders' payoff. Consequently, asset substitution implies that $S_{\theta}^{a,e} > S_{\theta}^{\bar{a},e}$, and the pair $\{S_{\theta}^{\bar{a},e}, C_{\theta}^{\bar{a},e}\}$ is dominated by *Strategic Conversion*. In other words, the south-west cell cannot support a Nash equilibrium.

To solve the non-cooperative game depicted in Table 1, one must specify the timing of actions. The game can be played either sequentially – and a first mover must be defined – or simultaneously. Typically, the following factors will determine how the game is played: the observability of the risk-shifting opportunity, the duration of the opportunity window, and the reversibility of the decision to increase risk.

Admittedly, the combined effect of these factors will generate case-by-case outcomes. Nevertheless, the incentive for shareholders to move first is all the stronger since:

- they observe the risk-shifting opportunity before all other claimholders. Shareholders can then announce that they have increased asset risk¹³ and let convertible

¹³The limiting case is where the risk-shifting opportunity is observed *solely* by shareholders and information available to convertible debtholders only originates from shareholders' announcements.

debtholders decide whether to convert or not;

- the risk-shifting opportunity window is short.¹⁴ Shareholders who run the company must hurry to seize the opportunity and cannot wait for convertible debtholders to move first;
- the decision to seize the risk-shifting opportunity is reversible. Convertible debtholders will be reluctant to make their move – which is contractually irreversible – if they fear shareholders may retract.

The following proposition characterizes the equilibria for all possible game sequences (all proofs are gathered in the appendix).

Proposition 2 (A). *In the sequential game in which convertible debtholders move first, the sub-game perfect Nash equilibria are:*

- *Status Quo* iff $S_{\theta}^{\bar{a},\bar{e}} \geq S_{\theta}^{a,\bar{e}}$ and $C_{\theta}^{\bar{a},\bar{e}} \geq C_{\theta}^{a,e}$,
- *Strategic Conversion* iff $S_{\theta}^{\bar{a},\bar{e}} \geq S_{\theta}^{a,\bar{e}}$ and $C_{\theta}^{a,e} > C_{\theta}^{\bar{a},\bar{e}}$,
- *Pure Asset Substitution* iff $S_{\theta}^{a,\bar{e}} > S_{\theta}^{\bar{a},\bar{e}}$.

(B). In the sequential game in which shareholders move first and in the simultaneous game, the sub-game perfect Nash equilibria are:

- *Status Quo* iff $S_{\theta}^{\bar{a},\bar{e}} \geq S_{\theta}^{a,\bar{e}}$,
- *Pure Asset Substitution* iff $S_{\theta}^{a,\bar{e}} > S_{\theta}^{\bar{a},\bar{e}}$.

Proposition 2 challenges the risk-mitigating effect of convertible debt. It provides the necessary and sufficient conditions for equilibria other than Status Quo to prevail. Independent of the game sequence, the dynamic nature of the game leaves room for

¹⁴The risk-shifting opportunity window can be short because of technology constraints, regulatory changes or competitors' pressure.

Pure Asset Substitution. This equilibrium arises when shareholders find it profitable to increase asset risk irrespective of the convertible debt design. In the sequential game in which convertible debtholders move first, Proposition 2 shows that a third equilibrium can arise. Convertible debtholders choose to convert, which leaves the capital structure with equity and straight debt only. Hence, the shareholders' payoff becomes convex, which restores their incentive for asset substitution.

Note that the condition ensuring Pure Asset Substitution is the same for all game sequences. Some cases that lead to Status Quo in game sequence (B) will lead to Strategic Conversion in game sequence (A).

The next subsection introduces the contingent claims framework that is needed to assess the likelihood of equilibria.

4 Contingent claims framework

Analyzing corporate securities as contingent claims on the firm's assets allows us to analytically characterize the inequalities given in Proposition 2. In the standard contingent claims approach, the payoffs of corporate securities are expressed in terms of options written on the value of assets, and the level of asset risk is adequately measured by asset return volatility. Let σ denote the initial level of asset return volatility. The risk-shifting opportunity consists in increasing asset return volatility to the level $\alpha > \sigma$.

Let $c(v, \tau, k, s)$ denote the value of the call option written on v , with time to maturity τ , strike price k , and asset return volatility s . The value of equity upon the risk-shifting opportunity is given by

$$S_{\theta}^{\bar{\alpha}, \bar{\sigma}} = c(V_{\theta}, T - \theta, F + M, \sigma) - q \cdot c(V_{\theta}, T - \theta, F + M/q, \sigma), \quad (1)$$

if shareholders do not seize the opportunity to shift risk and there is no early conversion, and by

$$S_{\theta}^{a,\bar{e}} = c(V_{\theta}, T - \theta, F + M, \alpha) - q \cdot c(V_{\theta}, T - \theta, F + M/q, \alpha), \quad (2)$$

if they do (and still no early conversion).

In the last two equations, the first call option reflects shareholders' limited liability on their total obligations $F + M$. The second call option captures the equity dilution if conversion occurs upon debt maturity (that is, when $q(V_T - F) > M$ or, equivalently $V_T > F + M/q$).

The value of convertible debt upon the risk-shifting opportunity is given by

$$C_{\theta}^{\bar{a},\bar{e}} = c(V_{\theta}, T - \theta, F, \sigma) - c(V_{\theta}, T - \theta, F + M, \sigma) + q \cdot c(V_{\theta}, T - \theta, F + M/q, \sigma), \quad (3)$$

if shareholders do not seize the opportunity to shift risk and there is no early conversion, and by

$$C_{\theta}^{a,e} = q \cdot c(V_{\theta}, T - \theta, F, \alpha), \quad (4)$$

if they do and there is early conversion.

In equation (3), the first two call options reflect a debt claim that is junior to straight debt. The third call option represents the value of the option to convert upon maturity. In equation (4), debt is converted early so convertible debtholders hold a fraction q of equity, which turns out to be a call option on the firm's assets with shifted volatility α and debt nominal F .

Contracts are designed at the initial date, but asset value will fluctuate between the initial date and the occurrence of the risk-shifting opportunity. Hence, the equilibrium that will prevail at date θ will depend on the value of assets at that date. Using the call option analogies from equations (1)–(4), we can further characterize the equilibria

identified in Proposition 2. Specifically, combining equations (1) and (2), Appendix A shows that the equation $S_{\theta}^{\bar{a},\bar{e}} = S_{\theta}^{a,\bar{e}}$ has only one root in V . By contrast, combining equations (3) and (4), the equation $C_{\theta}^{\bar{a},\bar{e}} = C_{\theta}^{a,e}$ has either zero or two roots in V . We therefore obtain the following proposition.

Proposition 3 *The occurrence of Status Quo, Strategic Conversion, or Pure Asset Substitution is determined by the value of assets upon θ in the following way:*

- If the equation $C_{\theta}^{\bar{a},\bar{e}} = C_{\theta}^{a,e}$ has no root in V

$$\text{Status Quo} \iff \text{not attainable}$$

$$\text{Strategic Conversion} \iff V_{\theta} \geq V_{\theta}^S$$

$$\text{Pure Asset Substitution} \iff V_{\theta} < V_{\theta}^S$$

- If the equation $C_{\theta}^{\bar{a},\bar{e}} = C_{\theta}^{a,e}$ has two roots in V

$$\text{Status Quo} \iff V_{\theta} \in [\min(V_{\theta}^{C+}, V_{\theta}^S), V_{\theta}^{C+}]$$

$$\text{Strategic Conversion} \iff V_{\theta} \in [\max(V_{\theta}^S, V_{\theta}^{C+}), \infty[$$

$$\text{Pure Asset Substitution} \iff V_{\theta} < V_{\theta}^S$$

where V_{θ}^S is the unique root of $S_{\theta}^{\bar{a},\bar{e}} = S_{\theta}^{a,\bar{e}}$, and V_{θ}^{C+} is the highest root, when it exists, of $C_{\theta}^{\bar{a},\bar{e}} = C_{\theta}^{a,e}$.

We see from Proposition 3 that Status Quo can only occur if $V_{\theta}^{C+} > V_{\theta}^S$.

Finally, the contingent claims analysis allows us to formulate the next proposition.

Proposition 4 *Strategic Conversion makes both shareholders and convertible debtholders better off than in Status Quo.*

Strategic Conversion involves a wealth transfer at the expense of straight debtholders (and that is why it will only occur if there is straight debt in the capital structure). Proposition 4 further shows that this wealth transfer is at the *sole* expense of straight debtholders. This implies that shareholders have the incentive to make the equilibrium shift from Status Quo to Strategic Conversion. They can actually do so by increasing the moneyness of the conversion option. One way is to pay a dividend upon the risk-shifting opportunity to induce early conversion. We will explore this possibility in section 7.

5 Ex post risk-shifting incentives in a diffusion setting

Proposition 3 indicates that, for a given convertible debt contract designed at the initial date, the dynamics of asset value will determine which equilibrium prevails when the opportunity to shift risk occurs. For the remainder of our analysis, we position ourselves in the Black and Scholes (1973) setting. The stochastic process $(V_t)_{t \geq 0}$ follows a geometric Brownian motion under P

$$\frac{dV_t}{V_t} = \mu dt + \sigma dW_t,$$

where $(W_t)_{t \geq 0}$ is a standard Brownian motion.

Figure 2 represents a Black-Scholes illustration of our model. It shows, for different values of assets, the difference $\Delta S = S_{\theta}^{\bar{a}, \bar{e}} - S_{\theta}^{a, \bar{e}}$. That is, what shareholders gain (or lose if negative) from not substituting assets conditional on no early exercise from convertible debtholders. Figure 2 also shows the difference $\Delta C = C_{\theta}^{\bar{a}, \bar{e}} - C_{\theta}^{a, e}$. That is, what convertible debtholders gain (or lose if negative) from a state with no asset substitution and no early exercise to a state of asset substitution and early exercise. From Proposition 2, we know that the signs of ΔS and ΔC determine the equilibrium.

The top figures (2a and 2b) represent cases where the three equilibria are attainable. By contrast, the bottom figures (2c and 2d) show cases where Status Quo no longer exists either because $V_{\theta}^{C^+} < V_{\theta}^S$ (figure 2c) or because the equation $C_{\theta}^{\bar{a},\bar{e}} = C_{\theta}^{a,e}$ has no root in V (figure 2d).

Figure 2a represents a situation where Status Quo is a dominant equilibrium. Initial capital structure is: straight debt nominal $F = 40$, convertible debt nominal $M = 20$, conversion ratio $q = 0.4$, maturity of both debts $T = 10$. The initial value of assets is $V = 100$. Five years later, shareholders are presented with the opportunity to increase risk from $\sigma = 0.2$ to $\alpha = 0.3$. Our model shows that the convertible debt design effectively curbs shareholders' incentive to increase risk (i.e. Status Quo prevails) if asset value lies between 104 and 372. If asset value is below 104, then Pure Asset Substitution occurs. Indeed, for low asset values, the conversion option is out-of-the-money, which has two consequences. First, convertible debtholders have no incentive to exercise their option. Second, the convertible debt contract is no longer suited to preventing shareholders from shifting risk. If asset value exceeds 372, then Strategic Conversion occurs. Indeed, for very high asset values, the conversion option is deep in-the-money. Convertible debtholders find that the benefit of getting their share of the assets with shifted volatility more than offsets the loss incurred from prematurely exercising their American call option on equity.

Figure 2b depicts a similar situation, but the firm is now more highly levered ($F = 60$ and $M = 40$). Higher leverage leaves more room for Pure Asset Substitution (now occurring for asset values below 170 after five years). Furthermore, the conversion option is more in-the-money ($q = 0.6$) and the risk-shifting opportunity is more tempting ($\alpha = 0.35$) leaving more room for Strategic Conversion (now occurring for asset values above 219 after five years). Therefore, the Status Quo equilibrium shrinks from both

sides.

In Figure 2c, firm leverage is the same as in figure 2a yet, convertible debtholders have a lower weight in the initial capital structure ($M = 10$ and $F = 50$). In addition, their conversion option is further out-of-the-money ($q = 0.2$). Consequently, they can no longer play their risk-mitigating role when the risk-shifting opportunity arises five years later. Status Quo is not attainable and the prevailing equilibrium is Pure Asset Substitution if asset value is below 132 after five years, or Strategic Conversion otherwise.

Figure 2d is another example where Status Quo is unattainable. This is mostly driven by the fact that the risk-shifting magnitude is high ($\alpha = 0.4$) and the conversion option is in-the-money ($q = 0.4$). The equilibrium is Pure Asset Substitution if asset value is below 130 after five years or Strategic Conversion otherwise.

Figure 3 shows the prevailing equilibria as a function of asset value and also as a function of shifted volatility for given initial straight and convertible debt contracts. Figure 3a stands as a base case with straight debt nominal $F = 40$, convertible debt nominal $M = 20$, conversion ratio $q = 0.4$, initial maturity of both debts $T = 10$ and where the risk-shifting opportunity arises at date $\theta = 5$. When asset value is low, any opportunity to increase risk will be seized by shareholders irrespective of the convertible debtholders' move, leading to Pure Asset Substitution. As asset value increases, the convertible debt curbs shareholders' incentive to shift risk and shareholders therefore require a higher shifted volatility to induce Pure Asset Substitution. Even though the level of shifted volatility is not high enough to lead to Pure Asset Substitution, it may still be high enough for convertible debtholders to convert early and induce Strategic Conversion. In all other cases where asset value is relatively high and shifted volatility is relatively low, Status Quo prevails.

Figures 3b, 3c and 3d represent a sensitivity analysis with respect to a single parameter from the base case. In Figure 3b, straight debt nominal is $F = 50$ and Pure Asset Substitution is more prevalent. In Figure 3c, convertible debt nominal is $M = 30$. Convertible debtholders have more power to curb shareholders' incentive to increase risk – making Status Quo more prevalent. At the same time, convertible debtholders have more incentive to keep their debt claim – reducing the scope for Strategic Conversion. In Figure 3d, the conversion ratio is $q = 0.5$, which basically leaves the area for Pure Asset Substitution unchanged. However, Strategic Conversion prevails for lower values of shifted volatility.

6 Risk-shifting and initial contract design

So far, we have conducted our analysis of equilibria using exogenously specified convertible debt contracts. No specific effort was made as such to structure a convertible debt contract that would best cope with shareholders' incentive to increase risk. This could underestimate the effectiveness of convertible debt as a risk-mitigating instrument. In this section, we endogenize the design of convertible debt and determine the initial contract terms (M, q) that maximize the probability of Status Quo.

To this end, we must make an assumption as to the timing of the risk-shifting opportunity. Specifically, all claimholders agree that the timing of the risk-shifting opportunity is characterized by the first jump time of a standard Poisson process with constant intensity λ . Thus, the cumulative distribution function of θ is given by

$$P(x < \theta) = e^{-\lambda x}, x \in \mathbb{R}^+.$$

Parameter λ can be interpreted as the instantaneous probability of the risk-shifting op-

portunity. For simplicity, risk-shifting magnitude is kept constant at α .

Under our assumption on the dynamics of asset value, the probability that the time- t asset value V_t lies within the interval $[x_{\min}, x_{\max}]$ is given by

$$\pi(V, x_{\min}, x_{\max}, t) = \Phi\left(\frac{\ln \frac{x_{\max}}{V} - \left(\mu - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}\right) - \Phi\left(\frac{\ln \frac{x_{\min}}{V} - \left(\mu - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}\right),$$

where V is the initial asset value and $\Phi(\cdot)$ stands for the standard cumulative normal distribution function.

Using Proposition 3, the probability of Status Quo is the probability that asset value upon θ lies within the interval $[V_\theta^S, V_\theta^{C+}]$, conditional on there being a risk-shifting opportunity before debt contracts mature, that is

$$\int_0^T \lambda e^{-\lambda\theta} \cdot \pi(V_\theta, V_\theta^S, V_\theta^{C+}, \theta) d\theta.$$

The initial convertible debt contract (M^*, q^*) is found numerically by maximizing the above expression.

Figure 4 plots the probability of Status Quo as a function of M and q . The optimal contract for the case depicted by Figure 4 is $M^* = 29.9$ and $q^* = 0.50$. Numerical results regarding equilibrium probabilities are reported in Table 2. We rely on a base case and perform a sensitivity analysis by changing one parameter value at a time. To simplify the interpretation, we report the equilibrium probabilities conditional on the occurrence of a risk-shifting opportunity before debt maturity, that is, all probabilities are divided by $1 - e^{-\lambda T}$ so that they sum up to unity.

The difference between the probability of Status Quo and 1 measures how imperfectly convertible debt mitigates shareholders' incentive to increase risk. In our simulations, conditional probabilities of Status Quo range from 45.7% to 78.1%, indicating that our

dynamic model leaves significant room for risk-shifting. Note that Strategic Conversion remains a marginal equilibrium (with conditional probability never above 5%). risk-shifting mostly occurs through Pure Asset Substitution – an equilibrium which, in some cases, is even more likely than Status Quo. The likelihood of risk-shifting increases for higher straight nominal debt or shifted volatility. A higher straight nominal debt raises the equity payoff convexity, inducing Pure Asset Substitution. A higher shifted volatility provides more benefits for shareholders, whether they substitute assets on their own initiative or with the cooperation of convertible debtholders. The likelihood of risk-shifting also increases for higher debt maturity or a higher probability of risk-shifting opportunity.

As far as the optimal convertible debt contract is concerned, a striking result is that the optimal conversion ratio is almost insensitive to all parameters except for the risk-shifting magnitude (α). We also note that the likelihood of risk-shifting opportunity (λ) has very little effect on the optimal convertible debt nominal. As a consequence, if claimholders' beliefs are the same for the magnitude of the risk-shifting opportunity and only differ with regard to its likelihood, they will agree on the same design of convertible debt.

7 The case for a special dividend

As mentioned in our comment of Proposition 4, shareholders can make the equilibrium shift from Status Quo to Strategic Conversion by paying a special dividend.¹⁵ Let δ

¹⁵The following analysis is however subject to two caveats. First, special dividends must be announced prior to actual distribution. This delay may jeopardize the implementation of the strategic dividend policy as the opportunity to shift assets may have vanished before the payment date. Second, as initially put forward by Smith and Warner (1979), typical

denote the special dividend that shareholders will distribute just after θ . We assume that δ is financed by a volatility-invariant asset sale (typically retained earnings to be reinvested in projects with same level of risk). We therefore preclude any security issuing to finance a special dividend.

The conditions given in Proposition 2, which characterize the equilibria, remain the same except that equity and convertible debt values must now account for the payment of the special dividend. Specifically

$$\begin{aligned}
S_{\theta}^{\bar{a},\bar{e}} &= c(V_{\theta} - \delta, T - \theta, F + M, \sigma) - q \cdot c(V_{\theta} - \delta, T - \theta, F + M/q, \sigma), \\
S_{\theta}^{a,\bar{e}} &= c(V_{\theta} - \delta, T - \theta, F + M, \alpha) - q \cdot c(V_{\theta} - \delta, T - \theta, F + M/q, \alpha), \\
C_{\theta}^{\bar{a},\bar{e}} &= c(V_{\theta} - \delta, T - \theta, F, \sigma) - c(V_{\theta} - \delta, T - \theta, F + M, \sigma) \\
&\quad + q \cdot c(V_{\theta} - \delta, T - \theta, F + M/q, \sigma), \\
C_{\theta}^{a,e} &= q \cdot c(V_{\theta} - \delta, T - \theta, F, \alpha) + q \cdot \delta.
\end{aligned}$$

Note that the payment of a special dividend has two effects. First, since asset value is reduced by the amount δ , the moneyness of all call options decreases. Second, the second term in $C_{\theta}^{a,e}$ reflects the fact that if convertible debtholders choose to convert early, they will capture a fraction of the dividend.

Figures 5a and 5b plot the frontiers $\Delta\tilde{S} := S_{\theta}^{\bar{a},\bar{e}} - S_{\theta}^{a,\bar{e}}$ and $\Delta\tilde{C} := C_{\theta}^{\bar{a},\bar{e}} - C_{\theta}^{a,e}$ and the corresponding equilibria. The special dividend payment makes Strategic Conversion a more likely outcome at the expense of Status Quo – leaving the likelihood of Pure Asset Substitution mostly unaffected.

covenants restrict the scope for special dividends. In their sample of over 14,000 loans issued between 1993 and 2001, Bradley and Roberts (2004) find that more than 85% of issues have restrictions on dividend payments.

8 Conclusion

The rationale advocated by agency theory for issuing convertible debt is to mitigate shareholders' incentive to increase risk. Admittedly, in a one-period setting where both the design of financial claims and the opportunity to shift risk occur simultaneously, convertible debt can efficiently solve the asset substitution problem.

In this paper, we allow for the opportunity to shift risk to occur at some random time after the capital structure is designed. If this opportunity arises prior to debt maturity, the initial contracts may no longer be suited to curbing the incentive to increase risk. This leaves room for shareholders and convertible debtholders to act strategically. The outcome of this non-cooperative game is that, in many cases, shareholders do seize the opportunity to shift risk, and therefore, convertible debt appears to be an imperfect risk-mitigating instrument. In the most favorable case where convertible debt is initially designed so as to minimize the risk-shifting probability, the likelihood for asset substitution is still non-negligible. The risk-mitigating effect of convertible debt is further weakened when contractual provisions allow shareholders to pay a special dividend.

As an extension for future research, one might want to examine the case when convertible debt is also callable by shareholders. When the opportunity to shift risk arises, shareholders must decide upon the exercise of the call option to force conversion. Once shareholders have decided to force conversion, convertible debtholders know for sure that the opportunity to shift risk will be seized, irrespective of their behavior. Therefore, this is no longer a Nash game. Shareholders will assess their optimal decision in view of their own payoff. We should therefore expect that the call feature will further weaken the risk-mitigating effect of convertible debt.

Appendix

Proof of Proposition 2

(A). When convertible debtholders move first, two sub-game equilibria must be solved, depending on their initial move. Suppose convertible debtholders opt for conversion at date θ ; they know that shareholders are better off substituting assets (since by Lemma 1 $S_\theta^{a,e} > S_\theta^{\bar{a},e}$). Therefore, their payoff will be $C_\theta^{a,e}$. This is to be compared with their payoff in the case of no early conversion:

- If $S_\theta^{a,\bar{e}} > S_\theta^{\bar{a},\bar{e}}$, their payoff will be $C_\theta^{a,\bar{e}}$. From Lemma 1, $C_\theta^{a,\bar{e}} > C_\theta^{a,e}$, hence the sub-game perfect Nash equilibrium is given by the pair $\{S_\theta^{a,\bar{e}}, C_\theta^{a,\bar{e}}\}$, i.e. Pure Asset Substitution obtained.
- If $S_\theta^{a,\bar{e}} \geq S_\theta^{\bar{a},\bar{e}}$, their payoff will be $C_\theta^{\bar{a},\bar{e}}$. Two cases must be distinguished:
 - If $C_\theta^{\bar{a},\bar{e}} \geq C_\theta^{a,e}$, the sub-game perfect Nash equilibrium is given by the pair $\{S_\theta^{\bar{a},\bar{e}}, C_\theta^{\bar{a},\bar{e}}\}$, i.e. Status Quo obtained,
 - If $C_\theta^{a,e} > C_\theta^{\bar{a},\bar{e}}$, the sub-game perfect Nash equilibrium is given by the pair $\{S_\theta^{a,e}, C_\theta^{a,e}\}$, i.e. Strategic Conversion obtained.

(B). In the simultaneous game, one of these three sets of conditions ensures the existence of the Status Quo:

$$(a) \left\{ \begin{array}{l} C_\theta^{\bar{a},\bar{e}} > C_\theta^{\bar{a},e} \\ S_\theta^{\bar{a},\bar{e}} > S_\theta^{a,\bar{e}} \end{array} \right. \quad (b) \left\{ \begin{array}{l} C_\theta^{\bar{a},\bar{e}} > C_\theta^{\bar{a},e} \\ S_\theta^{\bar{a},\bar{e}} = S_\theta^{a,\bar{e}} \\ C_\theta^{a,e} > C_\theta^{a,\bar{e}} \end{array} \right. \quad (c) \left\{ \begin{array}{l} C_\theta^{\bar{a},\bar{e}} = C_\theta^{\bar{a},e} \\ S_\theta^{\bar{a},\bar{e}} > S_\theta^{a,\bar{e}} \\ C_\theta^{a,\bar{e}} > C_\theta^{a,e} \end{array} \right. .$$

Sets (b) and (c) are immediately ruled out since, from Lemma 1, $C_\theta^{a,e} < C_\theta^{a,\bar{e}}$ and $C_\theta^{\bar{a},e} < C_\theta^{\bar{a},\bar{e}}$. Note that these two inequalities rule out Strategic Conversion and pure early

conversion (south-west cell of Table 1), respectively. Furthermore, for set (a) to ensure uniqueness of the Status Quo, we must have either $C_\theta^{a,\bar{e}} > C_\theta^{a,e}$ or $S_\theta^{\bar{a},\bar{e}} > S_\theta^{a,e}$. Since the first inequality always holds from Lemma 1, uniqueness follows.

If shareholders move first, the strategic motive for early conversion is eliminated. As Strategic Conversion is inaccessible, this type of sequential game produces exactly the same outcomes as the simultaneous game. \square

Proof of Proposition 3

One needs to study the sign of $S_\theta^{\bar{a},\bar{e}} - S_\theta^{a,\bar{e}}$ and $C_\theta^{\bar{a},\bar{e}} - C_\theta^{a,e}$ as a function of V .

The function $S_\theta^{\bar{a},\bar{e}} - S_\theta^{a,\bar{e}}$ is a linear combination of calls involving two different strikes: $F + M$ and $F + M/q$. Therefore, the behavior of the function $S_\theta^{\bar{a},\bar{e}} - S_\theta^{a,\bar{e}}$ with respect to V must be examined in three different regions at most.

For $V \ll F + M$, the options with the highest strike price are the deepest out of the money. Thus,

$$S_\theta^{\bar{a},\bar{e}} - S_\theta^{a,\bar{e}} \approx c(V_\theta, T - \theta, F + M, \sigma) - c(V_\theta, T - \theta, F + M, \alpha),$$

which decreases with V (since the call with volatility α gains moneyness more quickly).

Consequently,

$$\lim_{V_\theta \rightarrow 0^+} S_\theta^{\bar{a},\bar{e}} - S_\theta^{a,\bar{e}} = \lim_{V_\theta \rightarrow 0^+} c(V_\theta, T - \theta, F + M, \sigma) - c(V_\theta, T - \theta, F + M, \alpha) = 0^-.$$

For $V \gg F + M/q$, the options with the lowest strike price are the deepest out of the money. Thus,

$$S_\theta^{\bar{a},\bar{e}} - S_\theta^{a,\bar{e}} \approx q \cdot c(V_\theta, T - \theta, F + M/q, \alpha) - q \cdot c(V_\theta, T - \theta, F + M/q, \sigma),$$

which decreases with V (since the call with volatility σ loses moneyness more quickly). As V goes to infinity, all the calls converge to their intrinsic values. Yet, the calls with volatility $\alpha > \sigma$ converge more slowly. Thus,

$$\lim_{V \rightarrow \infty} S_{\theta}^{\bar{a}, \bar{e}} - S_{\theta}^{a, e} = 0^+.$$

Combining all of these results, we obtain the following table for $S_{\theta}^{\bar{a}, \bar{e}} - S_{\theta}^{a, e}$

Domain	0	$0 < V \ll F + M$	$F + M/q \ll V$	$+\infty$
$S_{\theta}^{\bar{a}, \bar{e}} - S_{\theta}^{a, e}$	0^-	decreasing	decreasing	0^+

Therefore, there is a unique root for equation $S_{\theta}^{\bar{a}, \bar{e}} - S_{\theta}^{a, e} = 0$, denoted by V_{θ}^S .

The function $C_{\theta}^{\bar{a}, \bar{e}} - C_{\theta}^{a, e}$ is a linear combination of calls involving three different strikes: F , $F + M$ and $F + M/q$. Therefore, the behavior of the function $C_{\theta}^{\bar{a}, \bar{e}} - C_{\theta}^{a, e}$ with respect to V must be examined in four different regions at most.

For $V \ll F$, the options with the highest strike price are the deepest out of the money. Thus,

$$C_{\theta}^{\bar{a}, \bar{e}} - C_{\theta}^{a, e} \approx c(V_{\theta}, T - \theta, F, \sigma) - q \cdot c(V_{\theta}, T - \theta, F, \alpha).$$

The increasing or decreasing behavior of $C_{\theta}^{\bar{a}, \bar{e}} - C_{\theta}^{a, e}$ as a function of V then depends on the net effect of the conversion ratio q and the shifted volatility α . The conversion ratio effect is linear in call value ($q \cdot c(V_{\theta}, T - \theta, F, \alpha)$) whereas the shifted volatility effect is convex in call value (with slope higher than 1). Consequently, the net effect is such that $C_{\theta}^{\bar{a}, \bar{e}} - C_{\theta}^{a, e}$ decreases with V (since the call with volatility α gains moneyness more quickly) except for the extreme case where q is very close to 1 and α is very close to σ . Since this extreme case is economically meaningless (convertible debtholders have the option to own almost the entire firm and risk-shifting opportunity is almost insignificant), we will discard it throughout the analysis.

Furthermore,

$$\lim_{V_\theta \rightarrow 0^+} C_\theta^{\bar{a}, \bar{e}} - C_\theta^{a, e} = \lim_{V_\theta \rightarrow 0^+} c(V_\theta, T - \theta, F, \sigma) - q \cdot c(V_\theta, T - \theta, F, \alpha) = 0^-.$$

For $V \gg F + M/q$, the options with the lowest strike price are the deepest out of the money. Thus,

$$C_\theta^{\bar{a}, \bar{e}} - C_\theta^{a, e} \approx q \cdot c(V_\theta, T - \theta, F + M/q, \sigma),$$

which increases with V .

Furthermore, as V goes to infinity, all the calls converge to their intrinsic values. Yet, the calls with volatility $\alpha > \sigma$ converge more slowly

$$\lim_{V_\theta \rightarrow \infty} C_\theta^{\bar{a}, \bar{e}} - C_\theta^{a, e} = 0^-.$$

Combining all of these results, we obtain the following table sign for $C_\theta^{\bar{a}, \bar{e}} - C_\theta^{a, e}$

Domain	0	$0 < V \ll F$	$F + M/q \ll V$	$+\infty$
$C_\theta^{\bar{a}, \bar{e}} - C_\theta^{a, e}$	0^-	decreasing	increasing	0^-

The function $C_\theta^{\bar{a}, \bar{e}} - C_\theta^{a, e}$ can exhibit two more behaviors with respect to V because of the intermediate strike $(F + M)$. Thus, there is either zero or two roots for equation $C_\theta^{\bar{a}, \bar{e}} - C_\theta^{a, e} = 0$.

Note that if there are two roots, then the lowest one $V_\theta^{C^-}$ is lower than $F + M$. Hence, we obtain from the analysis of $S_\theta^{\bar{a}, \bar{e}} - S_\theta^{a, \bar{e}}$ that

$$V_\theta^{C^-} < V_\theta^S,$$

which concludes the proof of Proposition 3. \square

Proof of Proposition 4

From Proposition 2, the condition $C_\theta^{a,e} - C_\theta^{\bar{a},\bar{e}} > 0$ is necessary for Strategic Conversion to hold. Hence, Strategic Conversion is chosen only if, as a result, convertible debtholders are better off. Furthermore, we show below that there is no wealth expropriation at the expense of shareholders. Indeed, first note that under Strategic Conversion, equity value is given by

$$S_\theta^{a,e} = (1 - q) \cdot c(V_\theta, T - \theta, F, \alpha).$$

Hence, using equation (1), the wealth transfer to shareholders is given by

$$\begin{aligned} S_\theta^{a,e} - S_\theta^{\bar{a},\bar{e}} &= (1 - q) \cdot c(V_\theta, T - \theta, F, \alpha) \\ &\quad - c(V_\theta, T - \theta, F + M, \sigma) + q \cdot c(V_\theta, T - \theta, F + M/q, \sigma) \\ &\geq (1 - q) \cdot c(V_\theta, T - \theta, F, \sigma) \\ &\quad - c(V_\theta, T - \theta, F + M, \sigma) + q \cdot c(V_\theta, T - \theta, F + M/q, \sigma) \\ &\geq 0 \end{aligned}$$

where the first inequality stems from the fact that $\alpha > \sigma$, and the second inequality stems from the convexity of call option prices with respect to the strike price. Thus, Strategic Conversion is chosen only if, as a result, both shareholders and convertible debtholders are better off. The wealth expropriation is entirely made at the expense of straight debtholders. \square

References

- [1] Asquith, P. and Mullins, D., 1991, Convertible debt, corporate call policy and voluntary conversion, *Journal of Finance* 46, 1273–1289.
- [2] Black, F. and Scholes, M., 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637–654.
- [3] Bradley, M. and Roberts, M. 2004, The structure and pricing of corporate debt covenants, Unpublished working paper, Duke University.
- [4] Brennan, M. and Schwartz, E., 1988, The case for convertibles, *Journal of Applied Corporate Finance* 1, 55–64.
- [5] Bühler, W. and Koziol, C., 2002, Valuation of convertible bonds with sequential conversion, *Schmalenbach Business Review* 54, 302–334.
- [6] Chang, S.-C., Chen, S.-S. and Liu, Y., 2004, Why firms use convertibles: A further test of the sequential-financing hypothesis, *Journal of Banking and Finance* 28, 1163–1183.
- [7] Chesney, M. and Gibson, R., 2001, Reducing asset substitution with warrant and convertible debt issues, *Journal of Derivatives* 9, 39–52.
- [8] Constantinides, G., 1984, Warrant exercise and bond conversion in competitive markets, *Journal of Financial Economics* 13, 371–397.
- [9] Constantinides, G. and Rosenthal, R., 1984, Strategic analysis of the competitive exercise of certain financial options, *Journal of Economic Theory* 32, 128–138.

- [10] Doms, M., and Dunne, T., 1998, Capital adjustment patterns in manufacturing plants, *Review of Economic Dynamics* 1, 409–429.
- [11] Eisdorfer, A., 2008, Empirical evidence of risk-shifting in financially distressed firms, *Journal of Finance* 63, 609–637.
- [12] Elliott, W., Koëter-Kant, J. and Warr, R., 2004, Valuation errors in equity and the motives for issuing convertible debt, Unpublished working paper, Vrije Universiteit Amsterdam.
- [13] Emanuel, D., 1983, Warrant valuation and exercise strategy, *Journal of Financial Economics* 12, 211–235.
- [14] Galai, D. and Masulis, R., 1976, The option pricing model and the risk factor of stock, *Journal of Financial Economics* 3, 53–81.
- [15] Gavish, B. and Kalay, A., 1983, On the asset substitution problem, *Journal of Financial and Quantitative Analysis* 18, 21–30.
- [16] Green, R., 1984, Investment incentives, debt, and warrants, *Journal of Financial Economics* 13, 115–136.
- [17] Green, R. and Talmor, E., 1986, Asset substitution and the agency costs of debt financing, *Journal of Banking and Finance* 10, 391–399.
- [18] Grenadier, S., 1996, The strategic exercise of options: Development cascades and overbuilding in real estate markets, *Journal of Finance* 51, 1653–1679.
- [19] Hennessy, C. and Tserlukevich, Y., 2004, Dynamic hedging incentives, debt, and warrants, Unpublished working paper, University of California at Berkeley.

- [20] Ingersoll, J., 1977, An examination of corporate call policies on convertible securities, *Journal of Finance* 32, 463-478.
- [21] Jensen, M. and Meckling, W., 1976, Theory of the firm: Managerial behavior, agency costs, and capital structure, *Journal of Financial Economics* 3, 305–360.
- [22] Ju, N. and Ou-Yang, H., 2006, Asset substitution and underinvestment: A dynamic view, Unpublished working paper, Duke University.
- [23] Korkeamaki, T. P. and Moore, W. T., 2004, Convertible bond design and capital investment: The role of call provisions, *Journal of Finance* 54, 391–405.
- [24] Krishnaswami, S. and Yaman, D., 2008, The role of convertible bonds in alleviating contracting costs, *Quarterly Review of Economics and Finance*, forthcoming.
- [25] Kulatilaka, N. and Perotti, E., 1998, Strategic growth options, *Management Science* 44, 1021–1031.
- [26] Lewis, C., Rogalski, R. and Seward, J., 1998, Agency problems, information asymmetries, and convertible debt security design, *Journal of Financial Intermediation* 7, 32–59.
- [27] Lewis, C., Rogalski, R. and Seward, J., 1999, Is convertible debt a substitute for straight debt or for common equity? *Financial Management* 28, 5–27.
- [28] MacMinn, R., 1992, On the risk-shifting problem and convertible bonds, Unpublished working paper, Illinois State University.
- [29] Mayers, D., 1998, Why firms issue convertible bonds: The matching of financial and real investment options, *Journal of Financial Economics* 47, 83–102.

- [30] Nilsen, O.A. and Schiantarelli, F., 2003, Zeros and lumps in investment: Empirical evidence on irreversibilities and nonconvexities, *Review of Economic Statistics* 85, 1021–1037.
- [31] Smith, C. and Warner, J., 1979, On financial contracting, *Journal of Financial Economics* 7, 117–161.
- [32] Spatt, C. and Sterbenz, F., 1993, Incentive conflicts, bundling claims, and the interaction among financial claimants, *Journal of Finance* 48, 513–528.
- [33] Stein, J., 1992, Convertible bonds as backdoor equity financing, *Journal of Financial Economics* 32, 3–21.

Tables

**Table 1: Sets of security values
at the time of risk-shifting opportunity.**

	Early conversion	No early conversion
Asset substitution	$\{S_{\theta}^{a,e}, C_{\theta}^{a,e}\}$	$\{S_{\theta}^{a,\bar{e}}, C_{\theta}^{a,\bar{e}}\}$
No asset substitution	$\{S_{\theta}^{\bar{a},e}, C_{\theta}^{\bar{a},e}\}$	$\{S_{\theta}^{\bar{a},\bar{e}}, C_{\theta}^{\bar{a},\bar{e}}\}$

Table 1 indicates the four possible values at the time of risk-shifting opportunity (θ) of equity (S) and convertible debt (C) whether asset substitution takes place or not, and whether convertible debtholders exercise their conversion option at date θ (early conversion) or at debt maturity T (no early conversion). Superscripts (a, \bar{a}) and (e, \bar{e}) account for the asset vs. no asset substitution cases and the early vs. no early conversion cases, respectively.

Table 2: Equilibrium probabilities.

	Optimal contract (M^*, q^*)	Conditional probability (%)			Probability of risk-shifting opportunity (%)
		SQ	PAS	SC	
Base case	(29.9, 0.50)	59.1	37.3	3.6	39.35
$F = 30$	(24.9, 0.50)	79.9	15.1	5.0	39.35
$F = 50$	(33.0, 0.50)	45.7	51.4	2.9	39.35
$\alpha = 0.25$	(28.5, 0.65)	69.8	25.9	4.3	39.35
$\alpha = 0.35$	(27.4, 0.33)	49.3	47.4	3.3	39.35
$\lambda = 0.02$	(29.9, 0.50)	60.8	35.0	4.2	18.13
$\lambda = 0.1$	(29.7, 0.50)	56.0	41.1	2.9	63.21
$T = 5$	(19.9, 0.50)	78.1	17.3	4.6	22.12
$T = 15$	(38.9, 0.50)	53.5	43.0	3.5	52.76

Base case parameters are as follows. Initial asset value is $V = 100$, growth rate of asset value is $\mu = 0.1$, asset volatility is $\sigma = 0.2$. Straight debt contract design is $F = 40$, and $T = 10$. Risk-free rate is $r = 0.04$. Shifted volatility is $\alpha = 0.3$. Instantaneous probability of the risk-shifting opportunity is $\lambda = 0.05$. In subsequent lines, one parameter value is changed above and below its base case setting. Optimal convertible debt contract is the couple (M^*, q^*) that maximizes the initial probability of Status Quo and is found numerically. Conditional probabilities are unconditional probabilities divided by the probability of risk-shifting opportunity $(1 - \exp(-\lambda T))$.

Figures

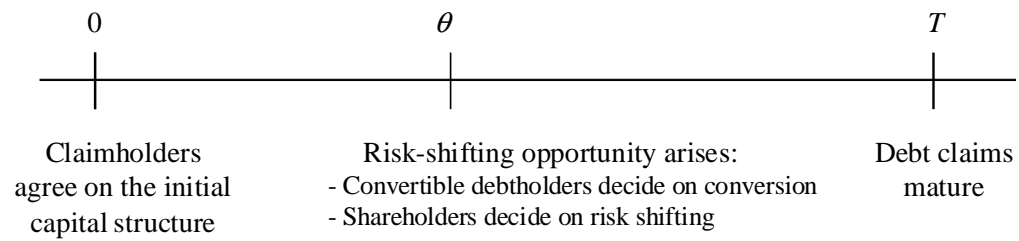


Figure 1: Timing of events.

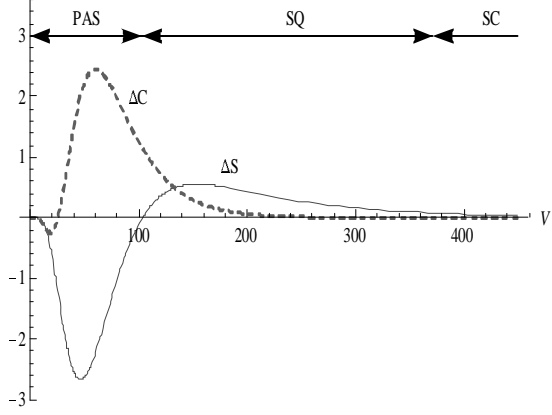


Figure 2a

($F = 40, M = 20, q = 0.4, \alpha = 0.3$)

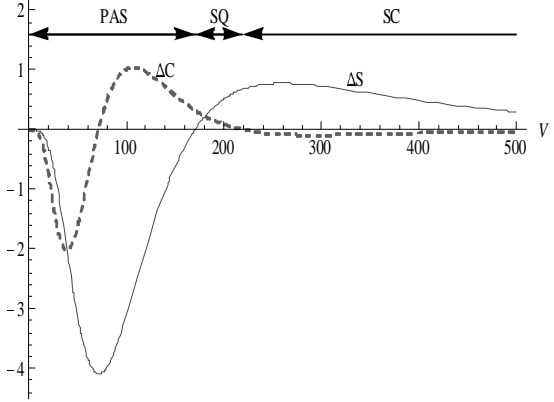


Figure 2b

($F = 60, M = 40, q = 0.6, \alpha = 0.35$)

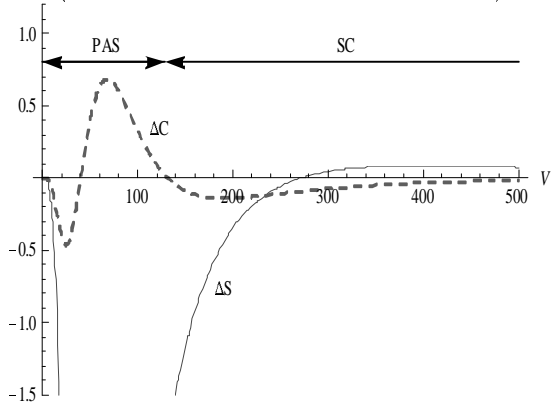


Figure 2c

($F = 50, M = 10, q = 0.2, \alpha = 0.4$)

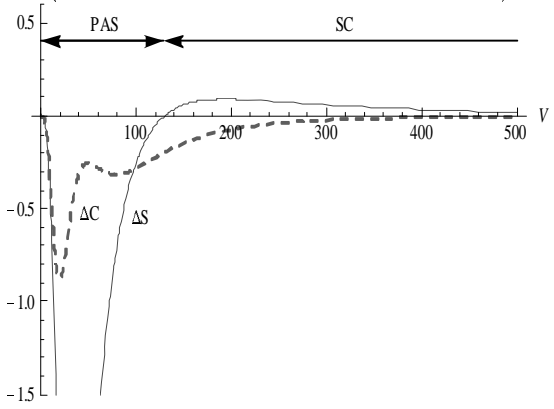


Figure 2d

($F = 30, M = 10, q = 0.4, \alpha = 0.4$)

Figure 2: Equilibria as a function of asset value for various capital structures and risk-shifting opportunity.

Figure 2 plots the value of $S_{\theta}^{\bar{a},\bar{e}} - S_{\theta}^{a,\bar{e}}$ (solid line denoted ΔS) and $C_{\theta}^{\bar{a},\bar{e}} - C_{\theta}^{a,e}$ (dashed line denoted ΔC) as a function of asset value. Common parameters across the four figures are: remaining debt maturity at the time of risk-shifting opportunity $T - \theta = 5$, risk-free rate $r = 0.04$, and initial volatility $\sigma = 0.2$. Other parameters (straight debt face value F , convertible debt face value M , conversion ratio q , and shifted volatility α) are reported in the figure's caption. Call option values are obtained under the Black-Scholes model.

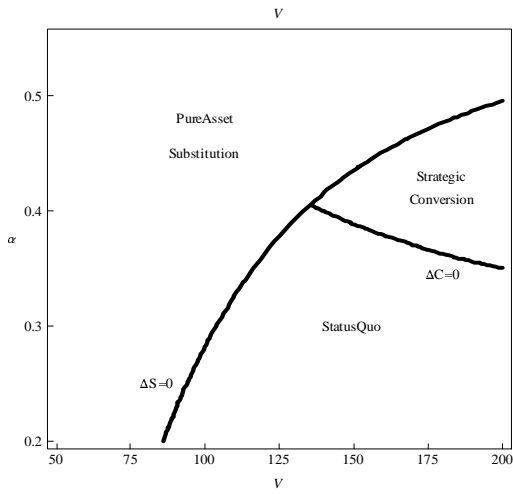


Figure 3a
 $(F = 40, M = 20, q = 0.4)$

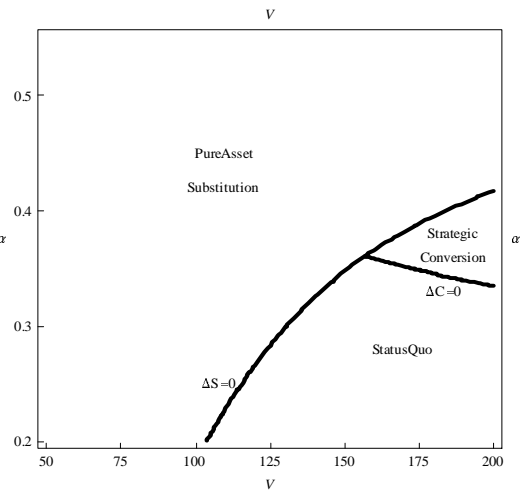


Figure 3b
 $(F = 50, M = 20, q = 0.4)$

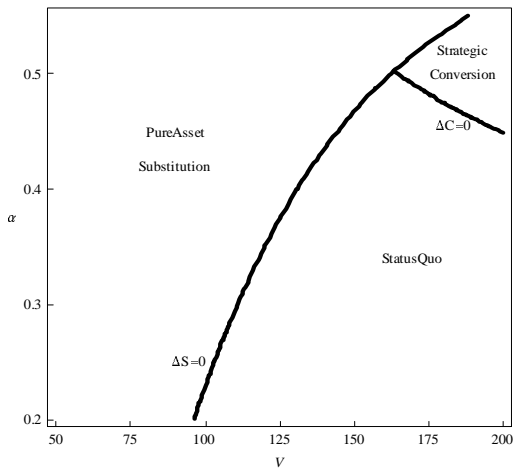


Figure 3c
 $(F = 40, M = 30, q = 0.4)$

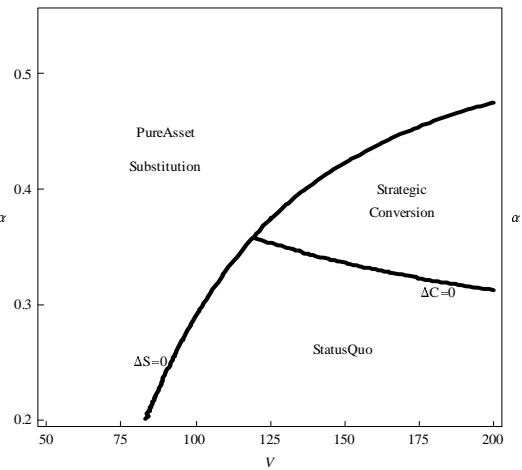


Figure 3d
 $(F = 40, M = 20, q = 0.5)$

Figure 3: Equilibria for different levels of asset value and shifted volatility.

Figures 3a, 3b, 3c and 3d plot the frontiers separating Status Quo from Pure Asset Substitution ($\Delta S = 0$) and Status Quo from Strategic Conversion ($\Delta C = 0$). Base case parameters are: remaining debt maturity at the time of risk-shifting opportunity $T - \theta = 5$, risk-free rate $r = 0.04$, and initial volatility $\sigma = 0.2$. On Figure 3a, straight debt face value is $F = 40$, convertible debt face value is $M = 20$, and conversion ratio is $q = 0.4$. In Figure 3b, straight debt face value is modified to $F = 50$. In Figure 3c, convertible debt face value is modified to $M = 30$. In Figure 3d, conversion ratio is modified to $q = 0.5$. Call option values are obtained under the Black-Scholes model.

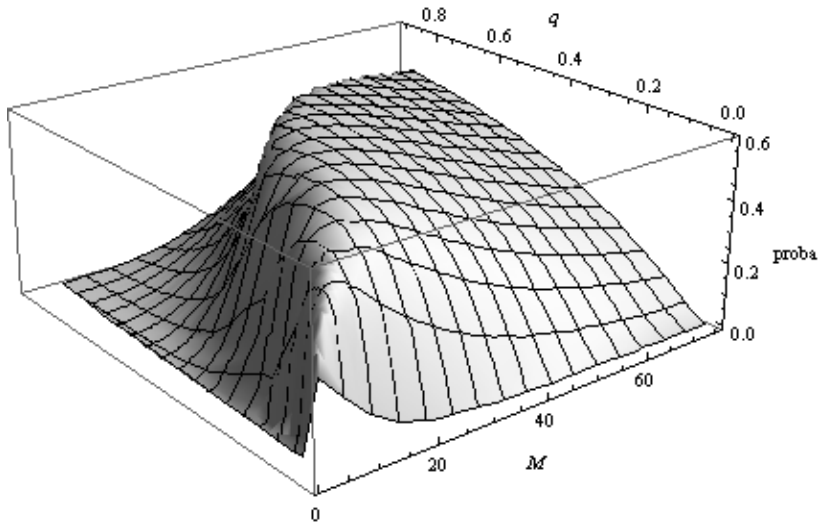


Figure 4: Probability of Status Quo.

In the space (M, q) of convertible debt design, Figure 4 plots the probability of Status Quo. Base case parameters are: initial value of assets $V = 100$, debt maturity $T = 10$, straight debt nominal $F = 40$, risk-free rate $r = 0.04$, growth rate of assets $\mu = 0.1$, initial volatility $\sigma = 0.2$, shifted volatility $\alpha = 0.3$, and instantaneous probability of the risk-shifting opportunity $\lambda = 0.05$. Call option values are obtained under the Black-Scholes model

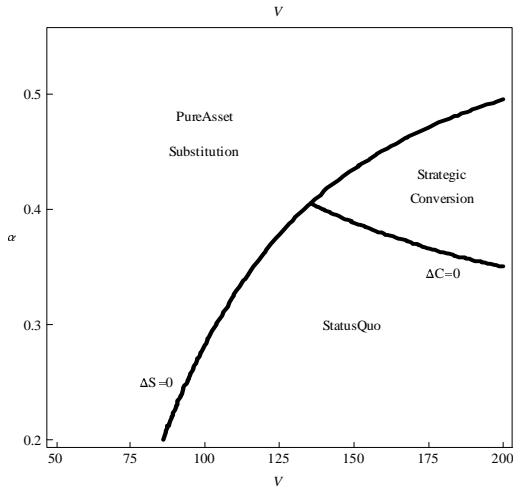


Figure 5a: Without dividend

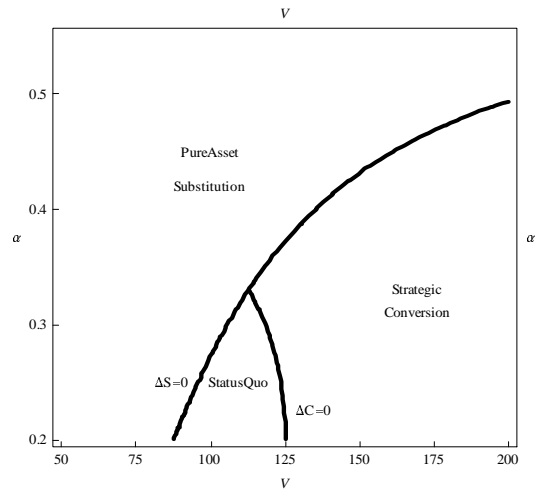


Figure 5b: With dividend

Figures 5a and 5b: Special dividend and equilibria.

Figures 5a and 5b plot the frontiers separating Status Quo from Pure Asset Substitution ($\Delta S = 0$) and Status Quo from Strategic Conversion ($\Delta C = 0$). Base case parameters are: remaining debt maturity at the time of risk-shifting opportunity $T - \theta = 5$, straight debt face value $F = 40$, risk-free rate $r = 0.04$, initial volatility $\sigma = 0.2$, convertible debt face value $M = 20$, and conversion ratio $q = 0.4$. In Figure 5a, there is no dividend. In Figure 5b, there is a special dividend of $\delta = 2$. Call option values are obtained under the Black-Scholes model.