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Asset Value Constraints in Models of Incomplete Factor Taxation

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Abstract:
This paper clarifies the role of initial asset value constraints in Ramsey models of incomplete factor taxation. We show that the optimal long-run capital tax is zero in the long run if and only if there is no binding constraint on the initial capital tax rate. This finding contrasts with Armenter (2008) who argues that zero long-run capital taxes reappear in models of incomplete factor taxation as long as the government is barred from manipulating initial asset wealth. The reason for this difference is that the two constraints cannot both be binding at the same time. Hence, in Armenter's (2008) analysis, the initial asset value constraint is necessarily more restrictive than the constraint on the initial capital tax rate.

Keywords: Ramsey equilibrium, incomplete factor taxation

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1 Introduction

One of the most provocative results in the Ramsey taxation literature is that capital income should not be taxed in the long-run. This conclusion, first established by Chamley (1986) and Judd (1985), is surprisingly robust to a wide variety of assumptions about the economic environment.[1] As Correia (1996) and Jones et al. (1997) show, however, the Chamley-Judd result hinges on the condition that the government can freely tax every factor of production. If instead the tax system is incomplete, the long-run capital tax rate is generally different from zero.

In a recent paper, Armenter (2008) argues that the non-zero capital tax result of Correia (1996) and Jones et al. (1997) crucially depends on how fiscal policy is constrained at date $t = 0$. Armenter's starting point is the observation that the revenue flows from an untaxed factor contribute to the initial asset value of the economy. As long as marginal productivity of the untaxed factor is affected by the economy's capital stock, the long-run capital tax can thus be used to depress the initial asset value indirectly through its distorting role on capital accumulation. Under the standard assumption that the initial capital tax rate is restricted, this indirect effect on the initial asset value is what generates the non-zero capital tax result of Correia (1996) and Jones et al. (1997). If instead, the government is barred from manipulating initial asset wealth, the original Chamley-Judd result reappears. Based on this insight, Armenter (2008) concludes that the non-zero capital tax prescription in models of incomplete factor taxation should be considered with caution because a constraint on initial asset wealth is a priori no more restrictive than a constraint on the initial capital tax rate.

In this paper, we consider a more general definition of the Ramsey equilibrium that simultaneously imposes the restriction on the initial capital tax rate and the restriction on the initial asset value as inequality constraints (i.e., the initial capital tax rate cannot exceed some value $\hat{\theta}_0$ while initial asset wealth cannot be smaller than some value $\hat{A}_0$). We show how to implement this equilibrium and, in doing so, we are able to clarify the role of initial asset value constraints. These clarifications are important because the Ramsey taxation approach has recently been applied to richer environments with market imperfections that, from a mechanical point of view, contribute to initial asset wealth in very similar ways as in the setup considered in Armenter (2008).[2]

First, we show that capital taxes are zero in the long run if and only if the constraint on the initial capital tax rate is non-binding. Hence, Armenter (2008) is correct in arguing that the long-run capital tax rate in models of incomplete factor taxation crucially depends on how fiscal policy is constrained at date $t = 0$. But his reinstatement of the Chamley-Judd result has nothing to do

[1] See Atkeson, Chari, and Kehoe (1999) for a comprehensive review.

[2] Some examples of this "second generation" of Ramsey models are Guo and Lansing (1999), Domeij (2005), Arseneau and Chugh (2008), Aruoba and Chugh (2008), and Arseneau, Chugh, and Kurmann (2008).

per se with the assumption that tax rates have to imply some exogenously imposed initial asset value $\hat{A}_0$. Rather, the result obtains because there is no binding constraint on the initial capital tax rate.

Indeed, it is well known in the Ramsey literature that whenever possible, the government should raise all revenues through taxes on initial capital. Given the pre-determined nature of the initial capital stock, this is akin to a lump-sum tax. Consequently, all distortionary taxes are zero – including the long-run capital tax rate – and the economy attains its first best. This leads to the second clarification of the paper. We show that the initial capital tax constraint and the initial asset value constraint cannot both be binding at the same time. Hence, Armenter's (2008) constraint that the government is barred from manipulating initial asset wealth is necessarily more restrictive than the constraint on the initial capital tax rate. More generally, Armenter (2008) argues that a restriction on initial asset wealth is no more arbitrary than a constraint on the initial capital tax rate. This argument seems hard to follow. Restricting the initial capital tax rate can be justified on grounds of implementability constraints and political economy considerations.[3] The Ramsey problem then consists of setting equilibrium allocations such as to maximize welfare. Imposing instead an exogenous constraint on the initial asset value defeats the normative purpose of Ramsey taxation. It amounts to simply assuming that the equilibrium is suboptimal in much the same way as if we asserted that the government has to implement a pre-ordained present value of consumption to the private sector.

Both clarifications offered in this paper are the result of our general definition of the Ramsey equilibrium. By contrast, Armenter (2008) implements the Ramsey equilibrium under which the government is barred from manipulating initial asset wealth simply by treating initial asset wealth as a constant. This effectively assumes away the influence of both future capital stocks and the initial capital tax rate on initial asset wealth. While the resulting solution for the optimal long-run capital tax rate remains correct, it is for the wrong reasons. According to Armenter's (2008) implementation, it is because capital taxes do not affect initial asset wealth whereas in our analysis, it is because the effect of capital taxes on initial asset wealth is exactly offset by an inverse effect on the government's implementability constraint. It is therefore impossible to see from Armenter's (2008) analysis that the two constraints cannot both be binding at the same time and that in cases where the initial asset value constraint is binding, it is necessarily more restrictive than the constraint on the initial capital tax rate.

The remainder of the paper is organized as follows. Section 2 briefly describes Armenter's

[3] For example, if agents have the option to hold capital at some storage cost rather than renting it to firms, the initial tax rate is bounded for there to be an equilibrium (see Atkeson et al., 1999). Alternatively, consider a decentralized economy with financial frictions that constrain firms' financing. For sufficiently high initial tax rates, these constraints could be so important that firms can no longer invest and thus, the economy shuts down.

stylized economy that we reuse here one for one for the sake of comparison. Section 3 defines the Ramsey equilibrium and solves for the more general case that imposes both constraints simultaneously. Section 4 compares our results with Armenter (2008) and concludes.

2 The Economy

The economic environment and notation is the same as in Armenter (2008), so our description is kept brief. There are three agents in the economy: a representative firm, a representative household, and a government.

The representative firm purchases capital k_t, labor n_t and an unnamed factor z_t on competitive markets and uses them as inputs in a constant returns to scale production technology

$$y_t = f(k_t, n_t, z_t). \tag{1}$$

Output y_t is sold to households in a perfectly competitive market.

Households, in turn, discount the future at rate $\beta \in (0, 1)$, have preferences over consumption and leisure $u(c_t, 1 - n_t)$ and own all factors of production. Capital evolves according to

$$k_{t+1} = (1 - \delta)k_t + i_t$$

where i_t denotes investment and $0 < \delta < 1$ is the rate of depreciation. Labor is non-negative and the total time endowment per period is normalized to one. Leisure is thus $1 - n_t$. The factor z_t is not storable and is in fixed supply, i.e. $z_t = z \ \forall t$. Households own shares s_t of this factor, each of which represents a claim to a stream of dividends $\{d_v\}_{v=t}^{\infty}$. This claim is valued at ex-dividend price p_t.

The government, finally, has to finance an exogenous stream of government expenditures $\{g_t\}_{t=0}^{\infty}$ with flat-rate taxes τ_t and θ_t on labor income $w_t n_t$ and capital income $r_t k_t$, where w_t is the wage rate and r_t is the capital rental rate. Furthermore, the government can smooth tax revenues by issuing a one-period bond b_t^g at price q_t. However, the government cannot tax income from the unnamed factor z_t. The government's flow budget constraint is thus

$$g_t + b_t^g = \tau_t w_t n_t + \theta_t r_t k_t + q_t b_{t+1}^g \tag{2}$$

Given these definitions, the representative household's problem is

$$\max_{\{c_t, n_t, k_{t+1}, s_{t+1}, b_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t) \tag{3}$$

subject to the flow budget constraint

$$c_t + q_t b_{t+1} + k_{t+1} - (1 - \delta)k_t + p_t(s_{t+1} - s_t) \leq (1 - \tau_t) w_t n_t + (1 - \theta_t) r_t k_t + d_t s_t + b_t. \tag{4}$$

2.1 Competitive Equilibrium

The household's optimality conditions are

$$q_t = \frac{\beta u_{c,t+1}}{u_{c,t}} \tag{5}$$

$$1 = \frac{\beta u_{c,t+1}}{u_{c,t}} \left((1-\theta_{t+1})r_{t+1} + 1 - \delta\right) \tag{6}$$

$$-\frac{u_{n,t}}{u_{c,t}} = (1-\tau_t)w_t \tag{7}$$

$$p_t = \frac{\beta u_{c,t+1}}{u_{c,t}} \left(p_{t+1} + d_{t+1}\right). \tag{8}$$

The firm's optimality conditions with respect to factor inputs are

$$r_t = F_k(k_t, n_t, z_t) \tag{9}$$

$$w_t = F_n(k_t, n_t, z_t) \tag{10}$$

$$d_t = F_z(k_t, n_t, z_t). \tag{11}$$

The market-clearing conditions for bonds and claims to the untaxed factor are

$$b_t = b_t^g \tag{12}$$

$$s_t = 1. \tag{13}$$

Finally, the aggregate resource constraint is

$$c_t + k_{t+1} - (1-\delta)k_t + g_t \leq F(k_t, n_t, z_t). \tag{14}$$

Taken together, conditions (5)-(14) characterize the equilibrium quantities and prices $\{c_t, n_t, k_{t+1}, p_t, q_t, w_t, r_t, d_t, b_t, s_t\}_{t=0}^{\infty}$ for given $\{g_t, \tau_t, \theta_t\}_{t=0}^{\infty}$.

It is worth drawing attention to the fact that conditions (5), (6), and (8) together imply an arbitrage condition between the after-tax returns on holding a unit of physical capital and a claim to the untaxed factor, z_t, that must hold for all t,

$$\frac{1}{q_t} = \left((1-\theta_{t+1})r_{t+1} + 1 - \delta\right) = \frac{p_{t+1} + d_{t+1}}{p_t}. \tag{15}$$

This arbitrage condition links the capital tax rate directly to the equilibrium return on a claim to the untaxed factor. We can also express the date $t = 0$ asset price as a standard present-value condition. To see this, note that the period-zero pricing condition (8) can be expressed as

$$p_0 = \sum_{t=1}^{\infty} q^t d_t = \sum_{t=1}^{\infty} \beta^t \frac{u_{c,t}}{u_{c,0}} d_t = \sum_{t=1}^{\infty} \beta^t \frac{u_{c,t}}{u_{c,0}} F_z(k_t, n_t), \tag{16}$$

where $q^t \equiv q_1 ... q_t$.[4]

[4] To simplify notation, we write $F_z(k_t, n_t)$ instead of $F_z(k_t, n_t, z_t)$ from here on since $z_t = z$ in equilibrium.

3 Ramsey equilibrium

The Ramsey government's problem is to finance the exogenous stream of government expenditures by choosing the sequence of labor income and capital income tax rates $\{\tau_t, \theta_t\}_{t=0}^{\infty}$ that achieves the highest possible household welfare among the feasible competitive equilibrium allocations. As is customary in the Ramsey taxation literature, we assume full commitment on the part of the government.

The goal is to implement a general Ramsey equilibrium that imposes inequality constraints on both the initial capital tax rate and the initial asset value of the economy. We begin by setting up the core Ramsey problem with *neither* of these restrictions. We then add the two constraints and solve for the different possible solutions. For the sake of comparability with Armenter (2008) and the literature in general, we adopt the usual primal approach to solve the problem. It is important to point out, however, that all results go through for the dual approach initially employed by Chamley (1986).

3.1 The core Ramsey problem

The primal approach consists of using the private-sector equilibrium conditions to eliminate prices and tax rates from the Ramsey problem, and then suppose that the government directly chooses quantities among the feasible set of competitive equilibrium allocations. To do so, we construct the implementability constraint (IC) of the economy by substituting the different optimality conditions of the private equilibrium into the present-value version of the household budget constraint in (4). The details of this derivation can be found in Armenter (2008). Given the IC, the Ramsey problem can be formulated as follows.

Proposition 1. *A Ramsey equilibrium is an allocation $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$ and an initial capital income tax rate θ_0 that solves*

$$\max \sum_{t=0}^{\infty} \beta^t u\left(c_t, 1-n_t\right)$$

subject to

$$\sum_{t=0}^{\infty} \beta^t \left(u_{c,t}c_t + u_{n,t}n_t\right) \geq u_{c,0}\left[b_0 + (p_0+d_0) + \left((1-\theta_0)F_{k,0} + 1 - \delta\right)k_0\right] \equiv A_0 \tag{17}$$

and

$$F(k_t, n_t, z_t) \geq c_t + k_{t+1} - (1-\delta)k_t + g_t \tag{18}$$

at all dates $t \geq 0$, taking the initial values k_0, b_0 and the sequence $\{g_t\}_{t=0}^{\infty}$ as given.

Proof. See Armenter (2008). □

Two points are important to highlight about this description of the Ramsey equilibrium. First, our formulation of the IC in (17) directly incorporates the sequence of no-arbitrage conditions (15). Imposing such a sequence of constraints for investment in physical capital is part of the standard construction of the IC but, obviously, the same sequence of constraints can also be imposed for investment in shares of the untaxed factor z_t.

Second, the term A_0 in (17) is defined as initial asset wealth or the initial asset value of the economy. We emphasize that this asset value is *not* predetermined (as k_0 or b_0 are) but is actually a function of all allocations dated $t = 0...\infty$. This is because A_0 contains p_0, and in turn p_0 depends, as shown by (16), on the entire sequence of allocations dated $t = 0...\infty$. To emphasize this dependence of A_0 on allocations beyond date $t = 0$, we write from here on $A_0\left(\{c_t, n_t, k_t\}_{t=0}^{\infty}, \theta_0\right)$, or $A_0(.)$ for short. This point is crucial as it implies that $A_0(.)$ can never be taken as a constant when solving for the Ramsey equilibrium, even if $A_0(.)$ is constrained to satisfy a certain numerical value in equilibrium.[5]

3.2 Constraints on initial period taxes and initial asset wealth

The core Ramsey problem abstracts from restrictions on taxation except that the government's only tax instruments are flat-rate taxes on capital and labor income. We now impose two additional constraints:

1. **Initial capital tax constraint.** The initial capital tax rate is restricted to $\theta_0 \leq \hat{\theta}_0$, where $\hat{\theta}_0$ is an arbitrary constant.

2. **Initial asset value constraint.** The initial asset value is restricted to $A_0\left(\{c_t, n_t, k_t\}_{t=0}^{\infty}, \theta_0\right) \geq \hat{A}_0$, where $\hat{A}_0$ is an arbitrary constant.

The first constraint is standard in the Ramsey literature and is imposed, as we will revisit, to avoid that the government can finance the entire sequence of government expenditures via a lump-sum tax on initial capital. If one were to entertain only this first constraint, a Ramsey equilibrium similar to the ones in Correia (1996) and Jones et al. (1997) would emerge provided that $\hat{\theta}_0$ is sufficiently small. If instead one were to entertain only the second type of constraint — and in particular treat it as a strict equality constraint — Armenter's (2008) alternative Ramsey equilibrium concept in which the government is barred from manipulating initial asset wealth would emerge. Here, we impose both constraints simultaneously as inequality constraints.

[5] To elaborate a bit further on this, we note that Ljungqvist and Sargent (2004, p. 492) emphasize that in Ramsey models of *complete* factor taxation, the analogous "constant term A_0" is properly expressed as the function $A_0(c_0, n_0, \theta_0)$, hence it depends directly *only* on period-zero allocations. With *incomplete* factor taxation, however, because of the dependence of $A_0(.)$ on period $t > 0$ allocations through (16), as just described in the text, the function $A_0(.)$ depends on more than just period-zero allocations.

To solve for the thus constrained Ramsey problem, we simplify notation as in Armenter (2008) and define

$$
\begin{aligned}
V(c_t, n_t) &\equiv u_{c,t}c_t + u_{n,t}n_t \\
H(c_t, n_t, k_t) &\equiv u_{c,t}F_z(k_t, n_t).
\end{aligned}
$$

The Lagrangian for the government's optimization problem then becomes

$$
\begin{aligned}
L &= \sum_{t=0}^{\infty} \beta^t \left[u(c_t, 1-n_t) + \lambda_t(F(k_t, n_t) + (1-\delta)k_t - k_{t+1} - c_t - g_t)\right] \qquad (19) \\
&+ \phi \left\{ \sum_{t=0}^{\infty} \beta^t [V(c_t, n_t) - H(c_t, n_t, k_t)] - u_{c,0}\left[b_0 + ((1-\theta_0)F_{k,0} + 1 - \delta)\, k_0\right] \right\} \\
&+ \gamma \left\{ \hat{\theta}_0 - \theta_0 \right\} \\
&+ \mu \left\{ \sum_{t=0}^{\infty} \beta^t H(c_t, n_t, k_t) + u_{c,0}\left[b_0 + ((1-\theta_0)F_{k,0} + 1 - \delta)\, k_0\right] - \hat{A}_0 \right\},
\end{aligned}
$$

in which λ_t, ϕ, γ, and μ are the respective Lagrange multipliers. The first two lines of the Lagrangian capture the core Ramsey problem stated in Proposition 1. The third line is the constraint on the initial capital tax rate. The fourth line is the constraint on initial asset wealth. Since $\sum_{t=0}^{\infty} \beta^t H(c_t, n_t, k_t) = u_{c,0}(p_0 + d_0)$ (from the definitions above), this constraint could equivalently be expressed as $A_0\left(\{c_t, n_t, k_t\}_{t=0}^{\infty}, \theta_0\right) - \hat{A}_0 \geq 0$. The constraint on initial asset wealth is thus more complicated than the constraint on the initial capital tax rate because it does not apply to a single variable, but rather to the entire sequence of equilibrium allocations $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$ plus the initial capital tax rate θ_0.

3.3 Long-run capital taxation

With the setup of the problem clarified, we analyze the optimal long-run capital income tax. The relevant FOCs are the ones with respect to θ_0 and k_{t+1}

$$
\gamma = (\phi - \mu)u_{c,0}F_{k,0}k_0 \qquad (20)
$$

$$
\lambda_t = \beta[\lambda_{t+1}(F_{k,t+1} + 1 - \delta) + (\mu - \phi)H_{k,t+1}]. \qquad (21)
$$

Condition (20) states that the marginal benefit γ from increasing the initial capital tax rate θ_0 equals the net gain from reducing the initial asset value by $u_{c,0}F_{k,0}k_0$. On the one hand, the increase in θ_0 relaxes the IC because the government needs to raise less revenue through distortionary taxes, thus bringing the economy closer to its first-best solution. This increases the Ramsey planner's objective by $\phi u_{c,0}F_{k,0}k_0$. On the other hand, the increase in θ_0 decreases the initial asset value of the economy, which diminishes the Ramsey planner's objective by $\mu u_{c,0}F_{k,0}k_0$.

Condition (21), in turn, states that the Ramsey planner's marginal cost from investment today is equal to the discounted marginal gain from the increase in net capital return ($F_{k,t+1}+1-\delta$) plus the net gain from the change in asset wealth $(\mu-\phi)H_{k,t+1}$ that this investment entails tomorrow.

Under the usual assumption that the Ramsey allocation converges to a stationary equilibrium as $t\to\infty$, this second condition rewritten in steady state becomes

$$1=\beta\left[(F_k+1-\delta)+\frac{\mu-\phi}{\lambda}H_k\right]. \tag{22}$$

Comparing this condition to the steady state version of the competitive equilibrium condition for investment in (6),

$$1=\beta\left[(1-\theta_\infty)F_k+1-\delta\right], \tag{23}$$

it is clear that the long-run capital income tax rate θ_∞ is zero if either $H_k=0$ or $\mu=\phi$. Consider the first condition: $H_k=u_cF_{zk}=0$ if and only if the untaxed factor z and capital k are strictly separable inputs in the production function. This point is clear from Armenter (2008), Correia (1996), and Jones et al. (1997). So from here on we assume $H_k\neq 0$ to make the problem interesting, just as in the related literature.

Now consider the second condition: $\mu=\phi$ implies that the the marginal gain from relaxing the IC (i.e. decreasing the distortions from flat-rate taxation) equals the marginal loss from increasing the initial asset value constraint. Under this condition, $\theta_\infty=0$ even if $H_k\neq 0$. However, the following proposition shows that this condition is satisfied only for a very particular case.

Proposition 2. *Provided that $H_k\neq 0$, the optimal long-run capital income tax rate is zero $\theta_\infty=0$ if and only if $\{g_t\}_{t=0}^{\infty}$ and $\hat{\theta}_0$ are such that $\theta_0<\hat{\theta}_0$ in the Ramsey equilibrium.*

Proof. Consider two different Ramsey equilibria: one in which the constraint on the initial capital tax binds and one in which it does not. If the constraint on the initial capital tax binds (i.e. $\theta_0=\hat{\theta}_0$), then $\gamma>0$ by Kuhn-Tucker.[6] Since $u_{c,0}F_{k,0}k_0$ in (20) is strictly positive, it therefore has to be that $\mu\neq\phi$. But then, comparison of (22) and (23) implies $\theta_\infty\neq 0$. If instead the constraint on the initial capital tax does not bind (i.e. $\theta_0<\hat{\theta}_0$), then $\gamma=0$ and by (20), we have $\mu=\phi$, which in turn implies $\theta_\infty=0$.[7] This proves that θ_∞ can only be zero if and only if $\{g_t\}_{t=0}^{\infty}$ and $\hat{\theta}_0$ are such that $\theta_0<\hat{\theta}_0$ in the Ramsey equilibrium.

[6]More precisely, the Kuhn-Tucker theorem states that at an optimum, we have $\gamma(\theta_0-\hat{\theta}_0)=0$. If $\theta_0=\hat{\theta}_0$, then $\gamma\geq 0$. But we can discard the case $\gamma=0$ because it only applies if the problem is subject to another constraint that needs to be relaxed simultaneously with $\theta_0=\hat{\theta}_0$ in order to increase the objective. In our problem, there is no such additional constraint; i.e. allowing the government to shift from distortionary taxes to lump-sum taxes on initial capital income is always welfare-improving. Note also that the constraint qualification necessary for the Kuhn-Tucker theorem to hold is always satisfied for our problem.

[7]Alternatively, we can show that for preferences consistent with balanced growth of the form

$$u(c,1-n)=[c(1-n)^{-\gamma}]^{1-\sigma}/(1-\sigma),$$

□

At first sight, Proposition 2 may seem evident since in the standard incomplete factor taxation problem without inital asset value constraints, it is well known that $\theta_\infty = 0$ if and only if there is no binding constraint on the initial capital tax rate. In this case, the Ramsey planner finances all of $\{g_t\}_{t=0}^{\infty}$ with non-distortionary taxes on initial capital income $\theta_0 F_{kn0} k_0$ and the economy attains the first best equilibrium. In other words, $\theta_\infty = 0$ if and only if $\hat{\theta}_0$ is such that $\theta_0^{FB} \leq \hat{\theta}_0$ where θ_0^{FB} is the initial capital tax rate associated with the first best equilibrium. The implications of Proposition 2 for our generalized case with initial asset value constraints are more subtle, however, as the following proposition shows.

Proposition 3. *The initial capital tax constraint* $\theta_0 \leq \hat{\theta}_0$ *and the initial asset value constraint* $A_0(.) \geq \hat{A}_0$ *are mutually exclusive; i.e. except for non-generic parameter values, the two constraints cannot bind simultaneously in a Ramsey equilibrium. If the initial asset value constraint binds (i.e.* $A_0(.) = \hat{A}_0$*), then* $\theta_\infty = 0$ *even though the Ramsey equilibrium is generally not at its first best.*

Proof. For the first part of the proposition, suppose we are in a Ramsey equilibrium where the initial asset value constraint binds; i.e. $A_0 = \hat{A}_0$. By the above definition of $A_0(.)$, this implies the following value for the initial capital tax rate

$$\theta_0 = 1 - \left(\frac{\frac{\hat{A}_0 - \sum_{t=0}^{\infty} \beta^t H(c_t, n_t, k_t)}{u_{c,0}} - b_0}{k_0} - (1-\delta) \right) / F_{k0},$$

where the sequence $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$ implements the Ramsey equilibrium. Except for non-generic parameter values, this value of θ_0 is necessarily higher or lower than the imposed upper bound $\hat{\theta}_0$. If $\theta_0 > \hat{\theta}_0$ then the supposed Ramsey equilibrium cannot exist. If $\theta_0 < \hat{\theta}_0$ then the initial capital tax constraint does not bind. An analogous argument proves that if the initial capital tax constraint binds; i.e. $\theta_0(.) = \hat{\theta}_0$, then the initial asset value A_0 associated with this Ramsey equilibrium is necessarily higher or lower than the imposed lower bound $\hat{A}_0$.

For the second part of the proposition, if we are in a Ramsey equilibrium where $A_0 = \hat{A}_0$, then we necessarily have that $\theta_0 < \hat{\theta}_0$ as just demonstrated. But by Proposition 2, this implies $\theta_\infty = 0$. Furthermore, by the same arguments than used above, the initial capital tax rate θ_0 implied by this Ramsey equilibrium is either higher or lower than θ_0^{FB}, the intial capital tax rate associated with

the condition $\phi = \mu$ (i.e. $\theta_\infty = 0$) implies that it is optimal for the Ramsey planner to set all distortionary taxes to zero and finance government expenditures exclusively with lump-sum taxes on initial capital income. Of course, this is only possible if there is no binding constraint on θ_0, which is just another way of stating that $\gamma = 0$. The strategy of this alternative proof is similar to a strategy used by Atkeson et al. (1999) and is available from the authors upon request.

the first best. Hence, the Ramsey planner must resort to some distortionary taxes (while keeping $\theta_\infty = 0$) to attain the equilibrium where $A_0 = \hat{A}_0$, which is necessarily suboptimal.[8]

□

To summarize, Proposition 3 reveals two of the main implications of our generalized treatment of Ramsey equilibria with inequality constraints on both the initial capital tax and initial asset wealth. First, for sufficiently large values of $\hat{A}_0$ such that the initial asset value constraint binds, we have $\theta_\infty = 0$ even though the resulting Ramsey equilibrium is not at its first best. Second, in an equilibrium where $A_0 = \hat{A}_0$, it necessarily has to be the case that the the initial asset value constraint is more restrictive – from a welfare point of view – than the initial capital tax constraint. This is important for our discussion of Armenter (2008) in what follows.

4 Discussion of Armenter (2008) and conclusion

The preceeding analysis allows us to bring about several clarifications of Armenter's (2008) recent note on incomplete factor taxation. Using the model described in Section 2, Armenter (2008) first illustrates that the non-zero capital tax result of Correia (1996) and Jones et al. (1997) stems from the possibility of taxing initial asset wealth by distorting capital accumulaton in the long-run. Armenter (2008) then considers an alternative equilibrium without any constraint on initial capital taxes but where the government is restricted from manipulating initial asset wealth (i.e. the initial asset value constraint binds with equality). In this case, the Chamley-Judd result of zero long-run capital taxation reappears. Armenter (2008) thus concludes that the non-zero capital tax prescription in models of incomplete factor taxation should be considered with caution because *"...a constraint on the initial asset value is not necessarily more restrictive than a constraint on the initial capital tax rate"* (page 2276).

We certainly agree with Armenter's (2008) first point that optimal long-run capital taxes in models of incomplete factor taxation are non-zero because such a tax can affect initial asset wealth. This insight is particularly useful for recent applications of the Ramsey taxation approach to models with market imperfections (see references in the introduction). These imperfections give rise to rents that contribute to the initial asset wealth of the economy much in the same way than dividends from the untaxed factor z_t do in the present model.

[8] An alternative proof of this last part goes as follows. Consider a Ramsey equilibrium where the initial capital tax rate binds; i.e. $\theta_0 = \hat{\theta}_0$. Now, suppose this constraint is gradually relaxed. The Ramsey planner would obviously take advantage of this loosening of constraints by increasing θ_0 so as to decrease distortionary taxes. The associated change of allocations decreases the initial asset value A_0. As the Ramsey planner continues this substitution of distortionary taxes with θ_0, the economy either attains its first best or A_0 hits the imposed lower bound $\hat{A}_0$. In the latter case, it necessarily has to be true that $\theta_0 < \theta_0^{FB}$.

However, we find less interest in Armenter's (2008) reinstatement of the Chamley-Judd result when initial asset wealth is restricted to a fixed constant. First, our analysis shows that the optimal long-run capital income tax rate in models of incomplete factor taxation is zero in only two cases: either capital and the untaxed factor are strictly separable in the production function (i.e. $H_k = 0$); or the Ramsey planner has unconstrained access to lump-sum taxes on initial capital income (i.e. the constraint $\theta_0 \leq \hat{\theta}_0$ does not bind). Hence, Armenter's (2008) reinstatement of the Chamley-Judd result has nothing do per se with the assumption that tax rates have to imply some exogenously imposed initial asset wealth $\hat{A}_0$. Rather, the result obtains because there is no binding constraint on the initial capital tax rate.

Second and as shown by Proposition 3, imposing $A_0(.) = \hat{A}_0$ implies that the initial asset value constraint is necessarily more restrictive than any (inequality) constraint on the initial capital tax rate because the two constraints cannot both be binding. This contrasts with Armenter's (2008) conclusion. More generally, $A_0(.) = \hat{A}_0$ restricts the economy from attaining its first best not because of a restriction on the available tax instruments (i.e. a binding limit to lump-sum taxation as is assumed in the standard Ramsey equilibrium) but simply because of a restriction that the equilibrium be suboptimal. Or put another way, the equality constraint $A_0(.) = \hat{A}_0$ makes θ_0 endogenous, but endogenous in a non-optimizing way. A Ramsey government that were to *optimally choose* the period-zero capital tax rate would with probability one *not* choose that particular tax rate. This seems to defeat the normative purpose of Ramsey taxation models and is more or less on the same level than, say, an ad-hoc constraint that consumption be no larger than a certain value even if that is suboptimal. By contrast and whatever other criticisms one may levy against the standard Ramsey equilibrium concept, the typical *exogeneity* assumption regarding the initial capital tax rate can at least be justified on grounds of implementability constraints (see footnote 3 in introduction).

Neither of the two clarifications made in this paper come out of Armenter's (2008) analysis. The reason is that Armenter (2008) implements the Ramsey equilibrium under which the government is barred from manipulating initial asset wealth by simply combining the IC with $A_0(.) = \hat{A}_0$. The IC thus becomes $\sum_{t=0}^{\infty} \beta^t V(c_t, n_t) \geq \hat{A}_0$, which effectively assumes away the influence of future capital stocks or the initial capital tax rate on initial asset wealth. This simplification is correct under the assumption that the Ramsey planner is free to choose θ_0 such that $A_0(.) = \hat{A}_0$ is always satisfied. The problem is that this assumption is only made implicitly. As a result, the optimality conditions resulting from Armenter's (2008) implementation are equivalent to the ones obtained under the assumption of $H_k = 0$. But this means that the Chamley-Judd result would obtain even if there was a binding constraint on θ_0, which is clearly incorrect. By the same token, Armenter's (2008) implementation does not reveal that the initial asset value constraint cannot be binding simultaneously with the initial capital tax constraint. It is therefore impossible to see that when the initial asset value constraint is binding, it is necessarily more restrictive than a constraint on the initial capital tax rate.

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