The Reaction of Stock Returns to News about Fundamentals

Tolga Cenesizoglu

Septembre/September 2010

Cenesizoglu: Department of Finance, HEC Montréal and CIRPÉE
tolga.cenesizoglu@hec.ca
Please check http://neumann.hec.ca/pages/tolga.cenesizoglu/ for updates.

Abstract:
This paper analyzes the reaction of stock returns to news about the state of the economy. We develop a general equilibrium asset pricing model where the investor learns about the growth rate of the economy through two sources of information, dividend realizations and regularly scheduled announcements about the state of the economy. We distinguish between dividend news and the unexpected part of the external signal and characterize the reaction of stock returns to news from these two sources of information. We show that the reaction to these news variables can be quite different under different assumptions about their precisions in different states. Our model is able to account for several empirical facts about the reaction of stock returns to news, such as time-varying and state-dependent reaction, asymmetric reaction to extreme news and stronger reaction to more precise signals.

1 Introduction

There is no question that the reaction of stock prices to news is of central importance to financial decision making. The main problem in analyzing the reaction of stock prices to news is that it is relatively difficult to observe and distinguish the arrival of additional information. Furthermore, it is also relatively difficult to accurately measure the information content of an announcement. Scheduled news such as releases of macroeconomic data provides a good starting point. First of all, the timing of these news is generally exogenously determined and known in advance by financial market participants. Secondly, it is relatively easy to quantify investors' expectations about scheduled announcements by employing either model- or survey-based measures. Hence, it is not surprising to find a large literature on the reaction of stock prices to macroeconomic announcements.

The literature mostly focuses on the reaction of an aggregate index rather than individual stock prices to macroeconomic news to establish the link between the stock market and fundamentals. In face of new information about the state of the economy, investors update their beliefs about fundamentals such as the growth rate of the economy and interest rates, which in turn affects the dynamics of stock returns. This fact that new information about macroeconomic variables affects returns on an aggregate index is well documented in the finance literature.[1]

In contrast to the vast empirical evidence on the reaction of the stock market to news about macroe-conomic variables, there still remain unanswered questions about the theoretical link between news about the state of the economy and the reaction of stock prices. A formal model is crucial not only for analyzing the theoretical link but also for constructing reasonable proxies for investors' expectations and uncertainty about an announcement. Instead of the current practice of using either ad hoc forecasting models or sur-veys, a formal model provides guidelines on how to construct such proxies for market expectations about announcements.

The contribution of this paper is to the theoretical literature on the link between news about the state of the economy and the reaction of stock returns. Kim and Verrecchia (1991) develop a three-period partial equilibrium model to analyze the market reaction to anticipated announcements. They conclude that a price change reflects the change in investors' expectations due to the arrival of new information, whereas volume

---

[1] A partial list of studies analyzing the reaction of stock prices to macroeconomic announcements includes McQueen and Roley (1993), Thorbecke (1997), Balduzzi, Elton, and Green (2001), Flannery and Protopapadakis (2002), Bomfim (2003), Guo (2004), Andersen, Bollerslev, Diebold, and Vega (2007), Bernanke and Kuttner (2005), Boyd, Hu, and Jagannathan (2005) and Gilbert (2009). Most of these studies focus on the reaction of an aggregate market index rather than individual stocks or portfolios with different characteristics with the exceptions of Thorbecke (1997), Guo (2004) and Bernanke and Kuttner (2005) who analyze the reaction to unanticipated changes in the target rate. Bernanke and Kuttner (2005) analyze the reaction of industry portfolios whereas Guo (2004) and Thorbecke (1997) analyze the reaction of portfolios formed on size.

arises due to information asymmetries. David (1997) develops a general equilibrium model where the investor learns about the unobserved profitability switches between different industries in the economy. He shows that this model with regime switches is able to generate three stylized facts of stock market returns: negative skewness, excess kurtosis, and predictive asymmetry. Veronesi (1999) analyzes the reaction of the aggregate stock market to news about the growth rate of dividends and finds that stock prices overreact to bad news when the growth rate of dividends is high and underreact to good news when it is low. In a similar framework to ours, Veronesi (2000) analyzes the relation between stock returns and the quality of information and finds that higher quality information leads to an increase in the risk premium.

In this paper, we analyze the reaction of stock returns to news about the state of the economy in a Lucas-type pure exchange economy with a representative agent (Lucas (1978)). Specifically, we first develop a general equilibrium asset pricing model where the investor learns about the growth rate of the economy through two sources of information, dividend realizations and regularly scheduled announcements about the state of the economy. In between announcement periods, the investor updates his beliefs about the current state of the economy through dividend realizations. On an announcement period, not only the growth rate of the economy can possibly change to a new level, but also the investor receives an external signal about the state of the economy in addition to the dividend realization. The two main features that distinguish our model from those in the literature are the assumptions on the volatility of the dividend growth process or the external signal in different states and on the number of periods between announcements. The volatility of the dividend growth process or the external signal affects how the investor updates his beliefs about the state variable whereas the number of periods between announcements affects the dividend growth process itself. These assumptions have important implications for the reaction of stock returns to news variables from these two sources of information. In this framework, we characterize the reaction of stock returns to news when there are only two possible states of the economy. Our results provide theoretical support for the recent empirical findings on the asymmetric reaction of stock returns to macroeconomic news (Flannery and Protopapadakis (2002), Boyd, Hu, and Jagannathan (2005), Andersen, Bollerslev, Diebold, and Vega (2007)), the timing effect of announcements (Andersen, Bollerslev, Diebold, and Vega (2003)) and the effect of the external signal's precision on the reaction (Gilbert (2009)).

The implications of our model for the reaction of stock returns to dividend news can be summarized as follows: First of all, our model is able to generate time-varying and state-dependent reaction of stock returns to dividend news. Both the sign and the magnitude of the reaction depend on the observed dividend news and the reaction to the same dividend news might be completely different depending on the investor's prior beliefs and the number of periods until the next announcement. Under the assumption that the dividend growth process has different volatilities in different states, the reaction of stock returns to large dividend

news is asymmetric in the sense that stock returns react positively to negative dividend news and negatively to positive dividend news.

Dividend news affects unexpected stock returns through two different channels. The first channel is its direct effect on the investor's consumption since, in equilibrium, the investor's consumption is the realized dividend. The second channel is its indirect effect through the investor's beliefs about the state variable. It is the relation between the direct and indirect effects that determines the sign and the magnitude of the reaction to dividend news. The direct effect is symmetric and always equal to one resulting in a one percent increase (decrease) in unexpected stock returns following a dividend realization that is one percent higher (lower) than expected. The indirect effect, on the other hand, can be symmetric or asymmetric in the sense that the indirect effect of a higher than expected dividend realization on stock returns can be negative. If the indirect effect has the opposite sign of the direct effect and dominates it, then the overall reaction to dividend news would be asymmetric.

To gain more intuition on the possible asymmetric reaction of stock returns to dividend news, consider the case where the dividend growth process has only two possible states, high growth state with low volatility and low growth state with high volatility. The indirect effect depends on the investor's risk aversion as well as the change in the investor's beliefs due to dividend news. For any news variable that increases the probability of the high growth state, the indirect effect is negative if and only if the investor is more risk averse than log utility. This is due to the fact that the positive impact of a higher than expected dividend growth rate on stock returns is dominated by the negative impact of a higher than expected discount rate in equilibrium if the investor is more risk averse than log utility. However, whether a news variable increases or decreases the probability of the high growth state depends on the investor's prior beliefs as well as the probability ratio of observing the realized dividend news under the low and the high growth state. Under the assumption that the volatility of the dividend growth process is higher in the low growth state, the probability of observing a large positive dividend news is higher under the low growth state than in the high growth state and the investor decreases the probability of the high growth state following a large positive dividend news. The probability of observing a large negative dividend news is still higher in the low growth state than in the high growth state and the investor continues to decrease the probability of the high growth state following a large negative dividend news. Hence, the reaction of stock returns to large negative dividend news would be positive since the positive indirect effect would dominate the negative direct effect.

On the other hand, the reaction of stock returns to dividend news can be guaranteed to be symmetric if the dividend growth process has the same volatility in both states. This is mainly due to the fact that a positive dividend news always increases the probability of the high growth state when the dividend growth process has the same volatility in both states. Furthermore, our calibration results show that the reaction of

stock returns is generally smaller in magnitude under this assumption.

The reaction of returns to dividend news also depends on the number of periods until the next announcement. We show that the dividend news released earlier in between announcement periods has a different impact on stock returns than the same dividend news released later, everything else equal. Under the assumption that the reaction is symmetric, dividend news released later would have a stronger impact on stock returns than those released earlier if and only if the investor is more risk averse than log utility. The intuition behind this result depends once again on the relation between the direct and the indirect effects of dividend news on stock returns. Independent of the investor's risk aversion, the indirect effect decreases in magnitude as the next announcement approaches given that everything else remains the same. The investor puts more weight on dividend news observed earlier in between announcements when updating his beliefs since he knows that the dividend growth process will be in the same state until the next announcement. However, whether stock returns react more strongly to dividend news released earlier depends on whether the direct effect has the same sign as the indirect effect. Under the assumptions that the investor is more risk averse than log utility and the reaction to dividend news is symmetric, the direct effect dominates the indirect effect which decreases as the next announcement approaches. In this case, the overall reaction increases as the next announcement approaches since the direct effect has the opposite sign of the indirect effect. Under the assumption that the investor is less risk averse than log utility, the direct and the indirect effects have the same sign. Thus, the overall reaction decreases as the next announcement approaches.

The implications of our model for the reaction of stock returns to the external signal can be summarized as follows: The reaction to the unexpected part of the external signal is relatively different than the reaction to dividend news as the only channel through which the external signal affects returns on the risky asset is through the investor's beliefs. Hence, the reaction to the external signal is similar to the indirect effect of dividend news. Under the assumption that the external signal has the same volatility in both states, the reaction to the unexpected part of the external signal is always asymmetric if and only if the investor is more risk averse than log utility. The intuition from the indirect effect of dividend news applies here as well. In this case, a positive news observed from the external signal increases the probability that the investor assigns to the high growth state. However, the negative effect of a higher than expected discount rate dominates the positive effect of a higher than expected growth rate if the investor is more risk averse than log utility. Hence, stock returns react negatively to positive news from the external signal. On the other hand, a large positive news decreases the probability that the investor assigns to the high growth state when the external signal is more volatile in the low growth state. Thus, stock returns react positively to large positive dividend news when the investor is more risk averse than log utility.

Finally, we show that stock returns react more strongly to a perfect external signal than an imperfect

external signal if the precision of the imperfect external signal is low enough. The intuition behind this result depends on the effect of the investor's prior beliefs on the reaction of stock returns to an imperfect external signal. When the external signal is perfect, the reaction to the unexpected part of the external signal does not depend on the investor's prior beliefs. However, this is not true when the external signal is imperfect. An imperfect signal with high enough precision might have a stronger impact on stock returns than a perfect signal if the investor's uncertainty prior to observing news variables is high enough. However, the reaction to a perfect signal would be stronger if the precision of the imperfect external signal is low enough.

The rest of the paper is organized as follows: Section 2 introduces the setup and the assumptions of our model and presents analytical solutions for the return on the risky asset. Section 3.1 characterizes the reaction of stock returns to dividend news. Section 3.2 analyzes the reaction of stock returns to the unexpected part of the external signal. Section 4 summarizes our findings and gives direction for future research.

2 The Model

In this section, we discuss the main assumptions as well as the information structure of our model. We consider a pure exchange economy (Lucas (1978)) in discrete time where a representative investor infers about the true state of the dividend growth rate through dividend realizations and external public signals that reveal additional information about the growth rate of dividends. The preferences of the representative investor in this economy are represented by a constant relative risk aversion utility over consumption,

$$U(C_t) = \begin{cases} \frac{C_t^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ \log(C_t) & \text{if } \gamma = 1 \end{cases} \tag{1}$$

where C_t denotes the investor's consumption in period t and γ is his coefficient of relative risk aversion. The investor's opportunity set consists of a risky asset whose supply is fixed and normalized to 1 and a riskless asset whose risk-free rate of return is r_t^f. Dividends of the risky asset at time t, D_t, grow according to the following process

$$\Delta d_t = \mu_{d,S_n} + \sigma_{d,S_n} \varepsilon_{d,t} \quad \text{for } T_{n-1} < t \leq T_n \tag{2}$$

where $d_t = \log(D_t)$ is the log-dividend at time t, Δ denotes the first difference operator (i.e. $\Delta d_t = d_t - d_{t-1}$) and $\varepsilon_{d,t}$ is an independently and identically distributed Gaussian random variable with zero mean and unit variance. S_n represents the true state of the dividend growth rate for the time period between T_{n-1} and T_n. We assume that the investor infers about the state of the dividend growth rate not only

through dividend realizations but also through an imperfect external signal, x_n, that is observed only on announcement periods,

$$x_n = \mu_{x,S_n} + \sigma_{x,S_n}\varepsilon_{x,n} \tag{3}$$

where $\varepsilon_{x,n}$ is an independently and identically distributed Gaussian random variable with zero mean and unit variance and is independent of ε_{d,T_n}.

We assume that the state variable S_n can take on a finite number of values, i.e. $S_n \in \{1, 2, \ldots, N\}$. We further assume without loss of generality that $\mu_{d,1} > \mu_{d,2} > \ldots > \mu_{d,N}$ and $\mu_{x,1} > \mu_{x,2} > \ldots > \mu_{x,N}$. We do not impose any specific restrictions on the volatilities of the dividend growth process or the external signals. The state variable, S_n, follows a first-order N-state Markov chain where the transition probabilities are given by an $N \times N$ matrix $\mathbf{Q}$

$$\{\Pr(S_n = j | S_{n-1} = i)\} = \{q_{ij}\} = \mathbf{Q} \tag{4}$$

We assume that the state of the dividend growth process can take on a new value according to the transition probability matrix only on announcement periods. In other words, S_n is realized on announcement period T_{n-1} and is the state of the dividend growth process until the next announcement period T_n when the investor observes the external signal x_n that reveals additional information about S_n. Hence, we use the index n to track the state variable and t to track the realizations of the dividend growth process. For analytical tractability, we consider regularly scheduled announcements every T periods, i.e. $T_n - T_{n-1} = T$ for $n = 1, 2, \ldots$ and $T_0 = 0$.

One should also note that T_n is not only the announcement period of the external signal, x_n, but it is also the period where the dividend growth process possibly switches to a new state. The assumption that the dividend growth process might switch to a new regime only on announcement days might be considered unrealistic. There are several reasons why we make this assumption in our model. First of all, this assumption allows us to solve our model analytically and derive implications without resorting to numerical solutions. Secondly, this assumption makes sense from the point of view of an econometrician who is trying to identify turning points for the economy. Whether it is industrial production, employment numbers or GDP, the data that the econometrician needs is not available every period and is released regularly only on certain days. Although the economy might have changed to a new regime in between the announcement days, the only point in time that he would be able to identify this change is when new data becomes available on announcement days.

In this setup, the investor never observes the true state of the dividend growth process. However, he infers about the state variable in between announcement periods by observing dividend realizations every period. On announcement periods, he not only receives an additional imperfect signal about the state variable but also knows that the state of the dividend growth process might change. The main advantage of our model

is that not only is it analytically tractable and suitable for the question posed in this paper but it is also realistic. A special case of our model where the representative investor observes daily or weekly dividend realizations and quarterly public announcements would closely replicate the structure of information flow in financial markets. They infer about it through many signals such earnings and dividends realizations of individual companies during the quarter. At the end of the quarter (with a one month delay), investors receive additional signals such as the unemployment rate or the growth rate of GDP and update their beliefs.

Our model nests many other preceding models in the literature as special cases. The closest model to ours is that of Veronesi (2000) where he analyzes the effect of information quality on stock returns and equity premium. One can obtain the model of Veronesi (2000) as a special case of our model by assuming that the external signal is observed every period rather than on pre-specified announcement periods (i.e. by setting $T = 1$ and $\mu_{d,S_n} = \mu_{x,S_n}$ for all S_n in our model). This extension of the Veronesi's model is important as our model is better suited to analyze the reaction of stock returns to public announcements about the state of the economy and it replicates the structure of information flow in financial markets more closely. If we assume that there are only two possible states of the dividend growth process ($N = 2$), that the external signal is observed every period ($T = 1$) and that the dividend growth process and the external signal have the same mean for all states ($\mu_{d,S_n} = \mu_{x,S_n}$ for all S_n), then we obtain the model of Veronesi (1999) when the external signal does not reveal any information ($\sigma_{x,S_n} = \infty$ for $n = 1, 2$) and the model of Cecchetti, Lam, and Mark (1990) when the external signal reveals the true state ($\sigma_{x,S_n} = 0$ for $n = 1, 2$). Although our model builds on other models in the literature, the questions we address and the implications we derive are the contributions of this paper. As we discuss in further detail in Section 2.3, one of the main distinguishing features of our model is that the dividend growth process is allowed to switch to a new state only on announcement periods. This assumption has important implications for return dynamics and the reaction of stock returns to dividend news.

2.1 Investor's Belief

A model where investors update their beliefs in face of information is a natural choice for the question analyzed in this paper. These types of models with learning are known to generate dynamics such as time-varying volatility and expected returns where the investor's beliefs play a central role. In this section, we analyze how the investor's beliefs evolve over time as new information about the state of the dividend growth process arrives.

The investor's beliefs about the current state of the dividend growth process depends on his current information set. As he receives additional information about the current state of the dividend growth process through dividend realizations and announcements, he updates his prior beliefs according to the Bayes' rule.

Specifically, let $\tilde{\pi}_{jt}$ denote the probability that he assigns to state j before observing the information revealed at time t (i.e. the dividend realization and possibly the external signal if t is an announcement period). Similarly, let π_{jt} denote the probability that he assigns to state j after observing the information revealed at time t. The information set of the investor, $\mathcal{F}_t$, includes past dividend realizations and external signals and whether time t is an announcement period. Assuming that the investor has prior beliefs about the initial state of the dividend growth process at time 0 before observing any dividend realizations or announcements (π_{j0} for $j = 1, 2, \ldots, N$), the following lemma characterizes the investor's beliefs about the state variable:

Lemma 1.

$$\pi_{j,t} = \begin{cases} \dfrac{\frac{1}{\sigma_{d,j}}\phi(\frac{\Delta d_t - \mu_{d,j}}{\sigma_{d,j}})\tilde{\pi}_{j,t}}{\sum_{i=1}^{N}\frac{1}{\sigma_{d,i}}\phi(\frac{\Delta d_t - \mu_{d,i}}{\sigma_{d,i}})\tilde{\pi}_{i,t}} & \text{if} \quad T_{n-1} \leq t < T_n \\[4ex] \dfrac{\frac{1}{\sigma_{d,j}\sigma_{x,j}}\phi(\frac{\Delta d_t - \mu_{d,j}}{\sigma_{d,j}})\phi(\frac{x_n - \mu_{x,j}}{\sigma_{x,j}})\tilde{\pi}_{j,t}}{\sum_{i=1}^{N}\frac{1}{\sigma_{d,i}\sigma_{x,i}}\phi(\frac{\Delta d_t - \mu_{d,i}}{\sigma_{d,i}})\phi(\frac{x_n - \mu_{x,i}}{\sigma_{x,i}})\tilde{\pi}_{i,t}} & \text{if} \quad t = T_n \end{cases} \qquad \text{for } n = 1, 2, \ldots \qquad (5)$$

and

$$\tilde{\pi}_{j,t} = \begin{cases} \sum_{i=1}^{N} \pi_{i,t-1}q_{ij} & \text{if} \quad t = T_{n-1} + 1 \\[2ex] \pi_{j,t-1} & \text{if} \quad T_{n-1} + 1 < t \leq T_n \end{cases} \qquad \text{for } n = 1, 2, \ldots \qquad (6)$$

where $\phi(\cdot)$ is the standard normal density function.

Proof. All proofs are in the appendix. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

The investor's beliefs need to be analyzed in three different cases. On the period after the $(n-1)^{th}$ announcement, $T_{n-1} + 1$, before observing the dividend realization, the investor knows that the dividend growth process might have switched to a new state according to the transition probability matrix. Hence, his prior beliefs before observing the dividend realization at time $T_{n-1} + 1$ ($\tilde{\pi}_{j,T_{n-1}+1}$) is a function of his beliefs about the past state variable S_{n-1} and the transition probability matrix. In between announcement periods T_{n-1} and T_n, the investor observes dividend realizations and updates his beliefs according to the Bayes' rule based on the law of motion of the dividend growth process. Finally, on the announcement period T_n, the investor receives an additional signal x_n about the current state and updates his beliefs according to the Bayes' rule using the information embedded in the dividend realization as well as the external signal.

The probability that the investor assigns to different states characterizes not only the investor's fluctuating expectations but also his uncertainty about the state of the dividend growth process. As we discuss in the next section, it is the investor's fluctuating expectations that generates dynamics in prices and returns as in David (1997).

9

2.2 Equilibrium Asset Prices and Returns

The equilibrium price process of the risky asset can be derived analytically from the investor's utility maximization problem. In this section, we derive closed-form expressions for the price of the risky asset on announcement and non-announcement periods. We then derive general expressions for unexpected log returns.

The investor chooses the fraction of wealth invested in the risky asset, α_t, and consumption, C_t, in order to solve the following maximization problem:

$$\max_{C_t, \alpha_t} E_t[\sum_{\tau=0}^{\infty} \beta^{\tau} U(C_{t+\tau})] \tag{7}$$

subject to the budget constraint:

$$W_{t+1} = \left(W_t - C_t\right)\left(\alpha_t\left(\frac{P_{t+1} + D_{t+1} - P_t}{P_t}\right) + (1 - \alpha_t)r_{t+1}^f\right) \tag{8}$$

where W_t denotes the investor's wealth at time t and P_t denotes the price of the risky asset. β is the investor's time impatience parameter and $E_t[\cdot]$ denotes expectation conditional on the available information at time t, $\mathcal{F}_t$. The Euler equation for the maximization problem is given by

$$P_t = \beta E_t\left[\frac{U'(C_{t+1})}{U'(C_t)}(P_{t+1} + D_{t+1})\right] \tag{9}$$

An equilibrium is defined by a vector process $(C_t, \alpha_t, P_t, r_t^f)$ such that the Euler equation in (9) holds and markets clear, i.e. $\alpha_t = 1$ and $C_t = D_t$.

In order to solve for the price of the risky asset, we first need to express the price-dividend ratio on announcement periods as a function of the true state variable. We then derive the closed-form expression for the price-dividend ratio of the risky asset as a function of the investor's beliefs about the state variable. Our solution approach is closest to those of Veronesi (2000) and Cecchetti, Lam, and Mark (1990). The following lemma characterizes the price-dividend ratio on announcement periods as a function of the true state variable assuming that the transversality condition for our model holds.[2]

Lemma 2. *Let* λ_j *denote the price-dividend ratio of the risky asset on an announcement period if the true state of the dividend growth process since the last announcement period is j. Then, λ_j can be expressed as:*

$$\lambda_j = [(\mathbf{I} - \mathbf{HQ})^{-1}\mathbf{QG}]_j \geq 0 \quad for\ j = 1, 2, \ldots, N \tag{10}$$

where the operator $[\cdot]_j$ refers to the jth element of a vector. $\mathbf{I}$ is the $N \times N$ identity matrix and $\mathbf{Q}$ is the transition probability matrix defined in Equation (4). $\mathbf{G}$ is a $N \times 1$ vector whose i^{th} element is βe^{a_i} and $\mathbf{H}$

---

[2]The transversality condition for our model can be expressed as $\lim_{\tau \to \infty} E_t\left[\beta^{\tau}\left(\frac{D_{t+\tau}}{D_t}\right)^{-\gamma} P_{t+\tau}\right] = 0$. A necessary and sufficient condition for the transversality condition to hold is $\beta e^{a_i} < 1$ for $i = 1, 2, \ldots, N$ where a_i is defined in Lemma 2.

is a $N \times N$ diagonal matrix whose i^{th} diagonal element is $\sum_{k=1}^{N} \beta e^{a_k} q_{ik}$ where $a_i = (1 - \gamma)\mu_{d,i} + (1 - \gamma)^2 \sigma_{d,i}^2 / 2$.

Proof. All proofs are in the appendix. □

The lemma suggests the price-dividend ratio would take one of the N possible values if the investor observes the true state of the dividend growth process. A direct implication of the above lemma is that the price-dividend ratio on the current announcement period would take one of the N possible values if the signal is perfect and reveals the true state of the dividend growth process since the last announcement period. However, since we assume that the signal is imperfect and does not reveal the true state variable, the price-dividend ratio on any announcement period is a weighted average of the N possible values given by λ_j in Lemma 2 where the weights are the investor's beliefs about the state variable. The following proposition characterizes the price of the risky asset on announcement and non-announcement periods as well as the unexpected return on the risky asset.

Proposition 1. *The price of the risky asset at time t is given by:*

$$P_t = \sum_{j=1}^{N} k_{j\tau} \pi_{jt} D_t \qquad \text{for } T_{n-1} < t \le T_n \text{ and } n = 1, 2, \ldots \tag{11}$$

where $\tau = T_n - t$ is the number of days until the next announcement and $k_{j\tau}$ is a time varying positive function:

$$k_{j\tau} = \frac{(\beta e^{a_j})^{\tau+1} - 1}{\beta e^{a_j} - 1} - 1 + (\beta e^{a_j})^{\tau} \lambda_j \qquad \text{for } \tau = 0, 1, 2, \ldots, T - 1 \tag{12}$$

Let r_t denote the log return on the risky asset at time t (i.e. $r_t = \log(\frac{P_t + D_t}{P_{t-1}})$), then the unexpected log return on the risky asset at time t can be approximated by:

$$r_t^* = r_t - E_{t-1}[r_t] \approx \frac{1}{1 + \overline{\lambda}} \sum_{j=1}^{N} k_{j,\tau} (\pi_{jt} - \tilde{\pi}_{j,t}) + \Delta d_t - \sum_{j=1}^{N} \mu_{d,j} \tilde{\pi}_{j,t} \tag{13}$$

where $\overline{\lambda}$ is the long-term average price-dividend ratio: $\overline{\lambda} = E[P_t/D_t] = (1/T) \sum_{j=1}^{N} \Omega_j \sum_{\tau=0}^{T-1} k_{j\tau}$ and $[\Omega_1, \Omega_2, \ldots, \Omega_N]$ is the stationary distribution vector of the transition probability matrix $\mathbf{Q}$.

Proof. All proofs are in the appendix. □

The price-dividend ratio of the risky asset in between announcements fluctuates with respect to changes in the investor's beliefs about the current state of the dividend growth process as he receives additional information through dividend realizations. The price-dividend ratio changes also deterministically as a function of time until the next announcement. In an extreme case where the investor knows the true state until the next announcement, the price-dividend ratio monotonically approaches to one of the N values

described in Lemma 2. The unexpected returns on the risky asset depend on the current dividend realization as well as the investor's current and prior beliefs about the state variable. Hence, unexpected returns will not only react to dividend realizations but also to changes in the investor's beliefs about the state variable. To gain further intuition about the dynamics of unexpected returns, the following corollary presents the unexpected returns as a function of the investor's beliefs and the dividend realizations when there are only two possible states of the dividend growth process.

Corollary 1. *Assume that there are only two possible states of the dividend growth process, i.e. $N = 2$, then the unexpected return on the risky asset at time t such that $T_{n-1} < t \leq T_n$ and $n = 1, 2, \ldots$ is given by:*

$$r_t^* \approx \frac{1}{1 + \bar{\lambda}}(k_{1,\tau} - k_{2,\tau})(\pi_{1,t} - \tilde{\pi}_{1,t}) + \Delta d_t - (\mu_{d,1}\tilde{\pi}_{1,t} + \mu_{d,2}(1 - \tilde{\pi}_{1,t})) \tag{14}$$

Unexpected returns on the risky asset is a function of two factors. The first is the dividend news defined as the difference between the realized dividend growth rate, Δd_t, and the expected dividend growth rate, $\mu_{d,1}\tilde{\pi}_{1,t} + \mu_{d,2}(1 - \tilde{\pi}_{1,t})$. The second is the change in the investor's beliefs due to additional information received at time t. The second factor depends on the dividend news on non-announcement days and also on the unexpected part of the external signal on announcement days. As we discuss in section 3, the distinction between news observed from dividend realizations and those observed from external signals is important. Dividend news affects the returns through two channels, its direct effect through investor's consumption and its indirect effect through investor's beliefs, whereas the effect of external news on returns is only through its effect on the investor's beliefs. For simplicity and analytical tractability, we choose to focus on the case where there are only two possible states for the rest of the paper.

2.3 Calibration

Our model builds on existing models in the literature. As mentioned above, the main distinguishing feature of our model is due to the assumption underlying the regime switching process. The dividend growth process is allowed to switch to a new state every T periods on announcement days rather than every period. This assumption has important implications for return dynamics as well as for the reaction of stock returns to dividend news. Another assumption that changes the implications of our model is the volatility of the dividend growth process or the external signal in different states. The reaction of unexpected stock returns to news variables changes dramatically whether the dividend growth process or the external signal have the same volatility in different states or not. To gain some intuition on the effect of these two assumptions on the dynamics of the investor's beliefs and returns, we present simulated paths for the investor's beliefs and returns under different assumptions. As mentioned before, we restrict our attention to a model where there are only two possible states of the dividend growth process.

We first calibrate our model by estimating Markov regime switching models of Hamilton (1989) for the growth rate of real GDP. For several reasons, we choose to calibrate our model to the growth rate of real GDP rather than the growth rate of real corporate dividends. First of all, in a pure exchange general equilibrium asset pricing model like ours, dividends, consumption and output are identical. Secondly, evidence of regime-switching type behavior is more pronounced in real GDP than real corporate dividends. Finally, we also calibrated our model to real corporate dividends. Our results do not change qualitatively since the important factor is the difference between the volatilities of the growth process in different states and the growth rate continues to have a higher volatility in the low growth state whether we use real GDP or real corporate dividends.

Specifically, we estimate two Markov regime switching models with two states for the log growth rate of quarterly real GDP between 1950 and 2008.[3] The two empirical models differ from each other only in terms of the specification for the standard deviation. In the first model, we allow both the mean and the standard deviation of the process to be state dependent whereas in the second model, we restrict the standard deviation to be identical across states.[4] Table 1 presents the estimated parameters from these two empirical models.

We assume that there are 63 trading days in a quarter and use the corresponding daily values of parameter estimates for quarterly data from Table 1. We use the following transformations to convert parameter estimates based on quarterly data to their corresponding daily values

$$
\begin{aligned}
\mu_i^{\text{daily}} &= \mu_i^{\text{quarterly}}/63; \text{ for } i = 1, 2 \\
\sigma_i^{\text{daily}} &= \sigma_i^{\text{quarterly}}/\sqrt{63}; \text{ for } i = 1, 2
\end{aligned}
$$

We consider four special cases of the dividend growth process in Equation (2). In the first model, we assume that the dividend growth process can switch to a new state only every 63 periods and that the standard deviation of the dividend growth process is higher in the low growth state. The second model is similar to the first model except that the dividend growth process can switch to a new state every period. The third and fourth models are similar to the first and the second model, respectively, except that we restrict the dividend growth process to have the same standard deviation in both states.

- Model 1: $T = 63$ and $\sigma_{d,1} < \sigma_{d,2}$;

- Model 2: $T = 1$ and $\sigma_{d,1} < \sigma_{d,2}$;

---

[3]The data is available to us from the Federal Reserve of Bank of St. Louis.

[4]The estimated empirical models are $\Delta \ln(GDP_t) = \mu_{S_t} + \sigma_{S_t}\varepsilon_t$ and $\Delta \ln(GDP_t) = \mu_{S_t} + \sigma\varepsilon_t$ where S_t is a two-state Markov chain and ε_t is an independently and identically distributed Gaussian random variable with zero mean and unit variance. GDP_t is the real GDP in quarter t. Both models are estimated via maximum likelihood.

- Model 3: $T = 63$ and $\sigma_{d,1} = \sigma_{d,2}$;

- Model 4: $T = 1$ and $\sigma_{d,1} = \sigma_{d,2}$.

In all models, for simplicity, we assume that the mean and the standard deviation of the external signal in both states are identical to those of the dividend growth process in the corresponding state. This assumption is not crucial and our results carry through even when we assume that the external signal has different mean and standard deviation than those of the dividend growth process. Furthermore, we choose to use the corresponding daily parameter values from empirical model 1 for all models except for the standard deviation. For models 1 and 2, the daily standard deviations are based on the estimated parameter values from empirical model 1. For models 3 and 4, the daily standard deviation in both states is based on the standard deviation estimate from empirical model 2. This allows us to focus on the effect of the standard deviation on the dynamics of our model without having to consider the effect of other model parameters. Table 2 presents the daily parameter values used for our simulation results and for figures in the rest of the paper.

To simulate from our models, we first simulate the paths of the state variable (S_n) and the dividend shocks (ε_t). Although we use different sets of random variables for the state variable and the dividend shocks, they are the same across all models in order to facilitate the comparison between models. The investor's beliefs about the state variable ($\pi_{1,t}$ and $\pi_{2,t}$) do not depend on the investor's risk aversion or his time impatience parameter. However, in order to simulate daily returns, we assume that the investor has a coefficient of risk aversion of 1.5 and a daily time impatience parameter of 0.9998 corresponding to an annual value of 0.9508.

Figure 1 presents the simulated path of the probability that the investor assigns to the high growth state ($\pi_{1,t}$). This figure allows us to compare graphically the implications of different assumptions on the dynamics of the investor's beliefs and returns which we analyze in further detail in the rest of the paper. First of all, comparing models 1 and 3 to models 2 and 4, respectively, one can easily see that the investor's beliefs are less persistent under the assumptions of models 2 and 4. This is not surprising since the state variable can possibly switch to a new regime every period and the investor observes an external signal about the state variable every period under the assumptions of models 2 and 4. Comparing models 1 and 2 to models 3 and 4, respectively, one can analyze how the assumption on the volatility of the dividend growth process in different states affects the dynamics of the investor's beliefs. The investor is more pessimistic under the models 1 and 2 in the sense that he assigns, on average, a higher probability to the low growth state compared to models 3 and 4. As we discuss below in further detail, this is a direct consequence of the assumption that the dividend growth process is more volatile in the low growth state. The probability of observing large positive or negative news is generally higher in the low growth state under the assumption

that the volatility of the dividend is higher in the low growth state.

Figure 2 presents the simulated paths of unexpected returns under the assumptions of these four models. Comparing models 1 and 2 to models 3 and 4, respectively, one can easily see that returns are more volatile under the assumptions of models 1 and 2. This is not surprising since the average volatility of the dividend growth process is larger under models 1 and 2 than under models 3 and 4. More interestingly, returns exhibit a clearer pattern of volatility clustering under models 1 and 3 than under models 2 and 4. This is mainly due to the higher persistence of the investor's beliefs under the assumptions of models 1 and 3.

3   The Reaction of Stock Returns to News

In this section, we derive analytical expressions for the reaction of unexpected stock returns to additional information. We distinguish between dividend news and the unexpected part of the external signal as these two news variables affect stock returns differently. Dividend realizations affect not only the investor's beliefs about the state variable but also his consumption whereas the external signal only affects his beliefs. We denote dividend news by $u_{d,t}$ defined as the unexpected dividend realization at time t:

$$u_{d,t} = \Delta d_t - \tilde{\mu}_{d,t} \tag{15}$$

where $\tilde{\mu}_{d,t} = \sum_{j=1}^{2} \mu_{d,j} \tilde{\pi}_{j,t}$ is the expected growth rate of dividends based on the investor's beliefs prior to observing the dividend realization at time t. Similarly, news observed from the nth announcement released at time T_n is defined as the unexpected part of the external signal:

$$u_{x,T_n} = x_n - \tilde{\mu}_{x,T_n} \tag{16}$$

where $\tilde{\mu}_{x,T_n} = \sum_{j=1}^{2} \mu_{x,j} \tilde{\pi}_{j,T_n}$ is the expected part of the external signal based on the investor's beliefs prior to observing the dividend realization and the external signal at time T_n. We should note that, by definition, news due to announcements (u_{x,T_n}) is observed only on announcement days every T periods whereas news due to dividend realizations ($u_{d,t}$) is observed every period. Hence, the stock price reacts to dividend realizations on non-announcement days and reacts to both dividend realizations and external signals on announcement days.

3.1   The Reaction of Stock Returns to Dividend News

In this section, we focus on the reaction of stock returns to dividend news under different assumptions. We first analyze the reaction when the volatility of the dividend growth process is different in different states. We then turn our attention to the case where the dividend growth process is equally volatile in both states. The implications of our model are quite different under these different sets of assumptions. The following

proposition characterizes the reaction of stock returns to dividend news when the dividend growth process has different volatilities in different states:

Proposition 2. *The reaction of stock returns to dividend news on*

(1) non-announcement days:

$$\partial r_t^*/\partial u_{d,t} = 1 + f_1(u_{d,t}, \tilde{\pi}_{1,t})\left(\frac{k_{2,\tau} - k_{1,\tau}}{1+\overline{\lambda}}\right)\left(\frac{\sigma_{d,2}^2 - \sigma_{d,1}^2}{\sigma_{d,1}^2 \sigma_{d,2}^2}\right)\left(u_{d,t} - \frac{(\mu_{d,1} - \mu_{d,2})\tilde{\sigma}_{d,t}^2}{\sigma_{d,2}^2 - \sigma_{d,1}^2}\right) \quad (17)$$

for $T_{n-1} < t < T_n$ and $n = 1, 2, \ldots$.

(2) on announcement days:

$$\partial r_{T_n}^*/\partial u_{d,T_n} = 1 + f_2(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n})\left(\frac{\lambda_2 - \lambda_1}{1+\overline{\lambda}}\right)\left(\frac{\sigma_{d,2}^2 - \sigma_{d,1}^2}{\sigma_{d,1}^2 \sigma_{d,2}^2}\right)\left(u_{d,T_n} - \frac{(\mu_{d,1} - \mu_{d,2})\tilde{\sigma}_{d,T_n}^2}{\sigma_{d,2}^2 - \sigma_{d,1}^2}\right) \quad (18)$$

for $n = 1, 2, \ldots$.

The functions $f_1 : \mathbb{R} \times [0,1] \to [0, 0.25]$ and $f_2 : \mathbb{R}^2 \times [0,1] \to [0, 0.25]$ are real-valued positive functions bounded above by 0.25 and are defined in the appendix.

Proof. All proofs are in the appendix. □

Proposition 2 shows that the reaction of stock returns to dividend news on non-announcement days depends on the investor's beliefs prior to observing current dividend news and the number of periods until the next announcement among other factors. This suggests that our model is able to generate time- and state-dependent reaction of stock returns to dividend news. In other words, same dividend news might have different effects on stock returns depending on the investor's beliefs about the current state as well as the number of periods until the next announcement day where the dividend growth process might change to a new regime. On announcement days, stock returns do not only react to dividend news but also to the unexpected part of the external signal. The reaction to dividend news on announcement days is similar to the reaction on non-announcement days and it depends on the investor's beliefs prior to observing the external signal among other factors.[5] It is also easy to see from Proposition 2 that the sign and the magnitude of the reaction to news also depends on the magnitude of the news variable itself. This suggests that stock returns react differently to a positive news than a negative one of the same magnitude.

---

[5]The reaction to dividend news on announcement days is similar to the reaction of stock returns to dividend news in Veronesi (1999) where the dividend growth process might switch to a new regime every period and the investor does not receive an external signal. However, our model has implications for the reaction of stock returns to dividend news not only on announcement days but also on non-announcement days. The fact that the dividend growth process does not switch to a new regime in between announcement days is a distinguishing feature of our model compared to that of Veronesi (1999).

In the rest of this section, we analyze the implications of our model for the reaction of stock returns to dividend news. We first focus on the case where the volatility of the dividend growth process is different in different states and show that the reaction of stock returns to extreme dividend news is asymmetric in the sense that returns react positively (negatively) to large negative (positive) dividend news. We then turn our attention to the case where the dividend growth process is equally volatile in both states and show that the reaction can be guaranteed to be symmetric under this assumption. Finally, we analyze the effect of the number of periods between announcement days on the reaction of stock returns to dividend news. The following proposition characterizes the reaction of stock returns to dividend news when the volatility of the dividend growth process is different in different states.

Proposition 3. *(a) Assume that the investor is more risk averse than a log-utility investor, i.e. $\gamma > 1$, and the dividend growth process is more volatile in the low growth state, i.e. $\sigma_{d,2} > \sigma_{d,1}$, or equivalently assume that the investor is less risk averse than a log-utility investor, i.e. $\gamma < 1$, and the dividend growth process is more volatile in the high growth state, i.e. $\sigma_{d,2} < \sigma_{d,1}$; then stock returns react positively to large negative dividend news.*

(b) Assume that the investor is more risk averse than a log-utility investor, i.e. $\gamma > 1$, and the dividend growth process is less volatile in the low growth state, i.e. $\sigma_{d,2} < \sigma_{d,1}$, or equivalently assume that the investor is less risk averse than a log-utility investor, i.e. $\gamma < 1$, and the dividend growth process is more volatile in the high growth state, i.e. $\sigma_{d,2} > \sigma_{d,1}$; then stock returns react negatively to large positive dividend news.

(c) Assume that the investor is a log-utility investor, i.e. $\gamma = 1$, then the reaction of unexpected returns to dividend news is always symmetric and a one percent higher (lower) than expected dividend realization will result in a one percent increase (decrease) in unexpected returns independent of the investor's prior beliefs.

Proposition 3 formalizes the asymmetric reaction of stock returns to dividend news. The intuition behind this result follows from the direct and the indirect effects of dividend news. There are two channels through which dividend news affects the unexpected returns on the risky asset. The first channel is its direct effect on the investor's consumption through the dividend realization. The effect of dividend news through the first channel is always equal to one, the first term in Equations (17) and (18). The second channel is its indirect effect through the investor's beliefs, the second term in Equations (17) and (18). It is the relation between these two effects that determines the sign and the magnitude of the overall reaction. If the indirect effect is symmetric, then the overall reaction of unexpected returns would be symmetric given that the direct

effect is always symmetric. In other words, stock returns would react positively to positive dividend news and negatively to negative dividend news. However, if the indirect effect is asymmetric, then the sign of the overall reaction depends on the relation between these two effects. More specifically, if the magnitude of the indirect effect in absolute value is greater than one, then the overall reaction to dividend news would be asymmetric with returns reacting positively to negative dividend news and vice versa.

The sign and the magnitude of the indirect effect depends on the investor's coefficient of risk aversion as well as the change in his beliefs following a news variable. We first focus on how the sign of the indirect effect depends on the investor's coefficient of risk aversion. For any news variable that increases the probability of the high growth state, the indirect effect is negative if and only if the investor is more risk averse than log utility. The intuition follows from the opposing effects of the dividend growth rate (the income effect) and the discount rate (the substitution effect) in equilibrium. Following a news variable that increases the probability of the high growth state, the investor believes that both the dividend growth rate and the discount rate are higher than previously expected. A higher than expected dividend growth rate has a positive effect on returns whereas the opposite holds true for a higher than expected discount rate. Which of these two effects dominates in equilibrium depends on the investor's risk aversion. If the investor is more risk averse than a log-utility investor, i.e. $\gamma > 1$, then the negative impact of a higher than expected discount rate dominates the positive effect of a higher than expected dividend growth rate. Thus, the indirect effect would be negative for any news variable that increases the probability of the high growth state.

We now turn our attention to the other factor that affects the sign and the magnitude of the indirect effect, namely the change in the investor's beliefs. As we discuss below, a positive (negative) dividend news always increases (decreases) the probability of the high growth state under the assumption that the dividend growth process has the same volatility in both states. However, this does not hold true under the assumption that the dividend growth process has different volatilities in different states. Whether a dividend news decreases or increases the probability of the high growth state depends on the investor's beliefs prior to observing dividend news and the probability ratio of observing the realized dividend news under the low and the high growth states. Specifically, consider the case where the dividend growth process is more volatile in the low growth state than in the high growth state, i.e. $\sigma_{d,1} < \sigma_{d,2}$. Under this assumption, the probability of observing a large positive dividend news is higher under the low growth state than in the high growth state. Hence, the investor decreases the probability of the high growth state following a large positive dividend news. On the other hand, the probability of observing a large negative dividend news is still higher in the low growth state than in the high growth state. Hence, the investor continues to decrease the probability of the high growth state following a large negative dividend news. To gain more intuition, Figure 3 plots the change in the investor's beliefs about the high growth state ($\pi_{1,t} - \tilde{\pi}_{1,t}$) as a function of dividend news ($u_{d,t}$)

on non-announcement days using daily calibrated parameters for model 1.

Having discussed the two factors that determine the indirect effect, we now focus on the overall reaction of stock returns to dividend news. Consider the case where the investor is more risk averse than log utility and the dividend growth process is more volatile in the low growth state (i.e., the first case in part (a) of Proposition 3). Under these assumptions, the indirect effect is positive for any news variable that decreases the probability of the high growth state.

Any positive dividend news that decreases the probability of the high growth state will have a positive impact on returns through its indirect effect. The overall reaction to positive dividend news will always be positive since the direct effect of any positive dividend news is also positive. On the other hand, any negative dividend news that decreases the probability of the high growth state will also have a positive indirect effect on returns in contrast to the direct effect which is negative for negative dividend news. Hence, the overall reaction to large negative dividend news will be positive when the positive impact of the indirect effect dominates the negative impact of the direct effect. Under the assumptions of part (a) of Proposition 3, the overall reaction to large negative dividend news is asymmetric. The intuition for the reaction of stock returns under the different assumptions discussed in Proposition 3 is similar and depends on the sign of the indirect effect.

To gain more intuition on the possible asymmetric reaction of stock returns to dividend news, Figure 4 plots the reaction of unexpected stock returns to dividend news ($\partial r_t^* / \partial u_{d,t} \times u_{d,t}$) as a function of dividend news ($u_{d,t}$) for the calibrated daily parameter values of model 1 and assuming that there are 31 periods until the next announcement.[6] Figure 4 confirms our theoretical findings graphically. Returns react positively to large negative dividend news depending on the investor's prior beliefs. Several other interesting facts emerge from Figure 4. First of all, as mentioned above, the magnitude of the reaction depends on the investor's beliefs prior to observing dividend news. The higher the probability that the investor assigns to the high growth state prior to observing dividend news, the stronger the asymmetric reaction to large negative dividend news. Secondly, the direct effect of dividend news dominates the indirect effect for very large dividend news and the reaction just becomes the direct effect of dividend news (the 45 degree line) since the maximum change in the investor's beliefs is bounded above by 1. A dividend news of the magnitude 0.01 as presented in Figure 4 is quite rare to observe. However, one can easily see from Figure 4 that the returns continue to react asymmetrically to negative dividend news relatively smaller in magnitude.

Proposition 2 also shows that the magnitude of the reaction to dividend news is a function of the difference between the volatility of the dividend growth process in different states. The impact of the indirect effect on stock returns becomes more pronounced when the difference between the volatility of the dividend

---

[6]The figure looks qualitatively similar to Figure 4 when we consider other values for the number of periods until the next announcement. We choose to present the results for 31 days since it is the mid point of a quarter with 63 trading days.

growth process in different states is larger. The opposite holds true if the difference is smaller. Under the assumption of equal volatility in both states, although the magnitude of the reaction continues to depend on the dividend news itself, the reaction of stock returns to dividend news can be guaranteed to be symmetric. The following proposition presents the reaction of stock returns to dividend news under the assumption that both dividend growth process and the external signal are equally volatile in both states.

Proposition 4. *Assume that the dividend growth process is equally volatile in both states, i.e. $\sigma_{d,1} = \sigma_{d,2}$, then the reaction of stock returns to dividend news on*

(1) non-announcement days:

$$\partial r_t^* / \partial u_{d,t} = 1 + f_3(u_{d,t}, \tilde{\pi}_{1,t}) \left(\frac{\mu_{d,1} - \mu_{d,2}}{\sigma_{d,1}^2} \right) \left(\frac{k_{1,\tau} - k_{2,\tau}}{1 + \bar{\lambda}} \right) \tag{19}$$

for $T_{n-1} < t < T_n$ and $n = 1, 2, \ldots$.

(2) announcement days:

$$\partial r_{T_n}^* / \partial u_{d,T_n} = 1 + f_4(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n}) \left(\frac{\mu_{d,1} - \mu_{d,2}}{\sigma_{d,1}^2} \right) \left(\frac{\lambda_1 - \lambda_2}{1 + \bar{\lambda}} \right) \tag{20}$$

for $n = 1, 2, \ldots$.

The functions $f_3 : \mathbb{R} \times [0, 1] \rightarrow [0, 0.25]$ and $f_4 : \mathbb{R}^2 \times [0, 1] \rightarrow [0, 0.25]$ are real-valued positive functions bounded above by 0.25 and are defined in the appendix.

Proof. All proofs are in the appendix. □

Proposition 4 shows that the sign of the indirect effect does not depend on the dividend news itself although the magnitude continues to do so under the assumption that dividend growth process is equally volatile in both states. Furthermore, Proposition 4 shows that the sign of the overall effect can be characterized independent of the dividend news or the external signal since f_3 and f_4 are positive-valued functions bounded above by 0.25. For example, if the investor is less risk averse than log utility, then the indirect effect is always positive resulting in a symmetric overall reaction of stock returns to dividend news. The following proposition characterizes the reaction of unexpected returns on the risky asset to dividend news under the assumption that the volatility of the dividend growth process is the same in both states:

Proposition 5. *(a) Assume that the investor is more risk averse than a log-utility investor, i.e. $\gamma > 1$ and that the dividend growth process has the same volatility in both states, i.e. $\sigma_{d,1} = \sigma_{d,2}$, then the reaction of unexpected returns to dividend news is symmetric independent of the investor's prior beliefs, the dividend news and the external signal if $(k_{1,\tau} - k_{2,\tau})/(1 + \bar{\lambda}) > -4\sigma_{d,1}^2/(\mu_{d,1} - \mu_{d,2})$ for all $\tau = 0, 1, 2, \ldots, T - 1$.*

Otherwise, the reaction can be symmetric or asymmetric depending on the news variable and the investor's beliefs as well as the number of periods until the next announcement.

(b) Assume that the investor is less risk averse than a log-utility investor, i.e. $\gamma < 1$, and the dividend growth process has the same volatility in both states, i.e. $\sigma_{d,1} = \sigma_{d,2}$, then the reaction of unexpected returns to dividend news is symmetric independent of the investor's prior beliefs, the dividend news and the external signal.

(c) Assume that the investor is a log-utility investor, i.e. $\gamma = 1$, then the reaction of unexpected returns to dividend news is always symmetric and a one percent higher (lower) than expected dividend realization will result in a one percent increase (decrease) in unexpected returns independent of the investor's prior beliefs.

Proof. All proofs are in the appendix. □

Part (a) of Proposition 5 presents the region of model parameters for which the reaction to dividend news would be unambiguously symmetric. In other words, if the model parameters are such that the condition in Part (a) holds, then stock returns will react positively to positive dividend news and negatively to negative dividend news. Otherwise, the reaction to dividend news can be symmetric or asymmetric depending on the investor's prior beliefs, the dividend news and the external signal. Part (b) of Proposition 5 shows that the reaction to dividend news is always symmetric if the investor is less risk averse than log utility. This is due to the fact that the indirect effect of dividend news is always positive under this assumption. Part (c) shows that the indirect effect of dividend news is always equal to zero and the overall reaction would be always equal to one when the investor has a log utility function. Under this assumption, the dividend news does not have any effect on stock returns through the investor's beliefs.

The intuition behind these results is similar to the intuition behind Proposition 3. The overall reaction depends on the relation between the direct and the indirect effects of dividend news on stock returns. The direct effect is always equal to one and the indirect effect is determined by the investor's coefficient of risk aversion as well as the change in the investor's beliefs. As in Proposition 3, the indirect effect is negative for any news variable that increases the probability of the high growth state if and only if the investor is more risk averse than log utility. The main difference is the effect of dividend news on the investor's beliefs. Differently from a model where the dividend growth process has different volatilities in different states, the change in the investor's beliefs is an increasing function of dividend news under the assumption of equal volatility. In other words, the investor assigns a higher (lower) probability to the high growth state following a positive (negative) dividend news. This is due to the fact that the probability of observing large positive

(negative) dividend news is greater under the high (low) growth state since the volatility of the dividend growth process is identical in both states.

Using daily calibrated parameters for model 3, Figure 5 plots the change in the investor's beliefs about the high growth state $(\pi_{1,t} - \tilde{\pi}_{1,t})$ as a function of dividend news $(u_{d,t})$ and Figure 6 plots the overall reaction of stock returns to dividend news $(\partial r_t^* / \partial u_{d,t} \times u_{d,t})$ as a function of dividend news $(u_{d,t})$ assuming that there are 31 days until the next announcement. Comparing Figures 5 and 6 to Figures 3 and 4, respectively, reveals the impact of the assumption on the volatility of dividend growth process in different states. Although the overall reaction of stock returns are similar for extremely small dividend news (in the order of magnitude 0.0001) under these two different assumptions about the volatility of the dividend growth process, the striking difference becomes more pronounced for large dividend news. The first difference is the sign of the reaction to large negative dividend news. As mentioned above, the reaction is asymmetric if the volatility of the dividend growth process is different in different states whereas the reaction is generally symmetric if the volatility of the dividend growth process is the same in different states. The second difference is the magnitude of the reaction to dividend news. For very small and very large dividend news, the reaction has similar magnitudes whether the dividend growth process has same or different volatilities in different states. This is mainly due to the fact that the direct effect which is the same under both models dominates the indirect effect for very small and large dividend news. The main difference is for dividend news between these two extremes. The reaction is much stronger when the dividend growth process has different volatilities in different states. This is due to the fact that the indirect effect is much more important than the direct effect for large dividend news.

Before proceeding to the effect of the number of periods until the next announcement on the reaction of stock returns to dividend news, a note is in order for the sufficient condition in part (a) of Proposition 5. This condition is satisfied for a wide range of model parameters and guarantees that the direct effect dominates the indirect effect. However, it is not satisfied for all choices of model parameters. When it is not satisfied, the overall reaction of stock returns to dividend news might be symmetric or asymmetric depending on the investor's prior beliefs and the magnitude of the dividend news among other factors. This condition also changes as a function of the number of days until the next announcement. Hence, it might hold for certain periods and might be violated in others. In other words, everything else equal, stock returns might react positively or negatively to the same dividend news depending on the number of days until the next announcement under certain model parameters.

We now turn our attention to the effect of the number of days until the next announcement on the reaction of stock returns to dividend news. Under the assumption that the dividend growth process has the same volatility in both states, the following proposition characterizes the magnitude of the reaction of

unexpected returns to dividend news as a function of time until the next announcement.

Proposition 6. *For periods t_1 and t_2 such that $T_{n-1} < t_1, t_2 < T_n$ and $\tilde{\pi}_{1,t_1} = \tilde{\pi}_{1,t_2}$,*

(a) Assume that the investor is more risk averse than a log-utility investor, i.e. $\gamma > 1$, and the condition in part (a) of Proposition 5 is satisfied so that the overall reaction is symmetric. Then, the reaction of unexpected returns to a dividend news in period t_1 is smaller in absolute value than the reaction to the same dividend news in period t_2 if and only if $t_2 > t_1$. Mathematically, $|\partial r^*_{t_1}/\partial u_{d,t_1}| < |\partial r^*_{t_2}/\partial u_{d,t_2}|$ for $u_{d,t_1} = u_{d,t_2}$ if and only if $t_2 > t_1$.

(b) Assume that the investor is less risk averse than a log-utility investor, i.e. $\gamma < 1$. Then, the reaction of unexpected returns to a dividend news in period t_1 is greater in absolute value than the reaction to the same dividend news in period t_2 if and only if $t_2 > t_1$. Mathematically, $|\partial r^*_{t_1}/\partial u_{d,t_1}| > |\partial r^*_{t_2}/\partial u_{d,t_2}|$ for $u_{d,t_1} = u_{d,t_2}$ if and only if $t_2 > t_1$.

(c) Assume that the investor is a log-utility investor, i.e. $\gamma = 1$, then the reaction of unexpected returns to dividend news does not depend on the number of periods until the next announcement.

Proof. All proofs are in the appendix. ◻

Part (a) of Proposition 6 shows that dividend news observed later in between announcements have a stronger effect on stock returns than dividend news observed earlier when the investor is more risk averse than log utility. Part (b) shows that the opposite holds true when the investor is less risk averse than log utility. In other words, the reaction of stock returns is stronger to dividend news released earlier. Finally, part (c) shows that the reaction to dividend news is constant in between announcement days if the investor has a log-utility.

The intuition behind these results also follows from the relation between the direct and the indirect effects of dividend news on stock returns. First of all, it is the indirect effect that has a time-varying impact on stock returns. Independent of the investor's coefficient of risk aversion and the dividend news, the absolute value of the indirect effect decreases as the next announcement day approaches. This is due to the fact that the indirect effect is a function of the difference between the price-dividend ratios ($k_{1,\tau} - k_{2,\tau}$) in different states and this difference between the price dividend ratios is at its largest on periods following an announcement since the investor knows that the dividend growth process will stay in the current state until the next announcement. As discussed above, the overall reaction depends on whether the direct and the indirect effects have the same sign or not. Consider the case where the investor is more risk averse than log utility and the reaction is symmetric. In this case, the direct effect has the opposite sign of the indirect effect

and dominates the indirect effect so that the overall reaction is symmetric. Given that the negative indirect effect becomes smaller in magnitude as the next announcement approaches, the overall reaction increases monotonically everything else equal. When the investor is less risk averse than log utility, the direct and indirect effects have the same sign. Thus, the magnitude of the overall reaction of stock returns to dividend news decreases as the next announcement approaches. Using parameters of model 3, Figure 7 presents the reaction of stock returns to dividend news as a function of time since the last announcement under the assumptions of part (a) of Proposition 6. Panels (a) and (b) present the reaction to negative dividend news and Panels (c) and (d) present the reaction to positive dividends. As shown in part (a) of Proposition 6, the magnitude of the reaction to dividend news increases monotonically as the next announcement approaches.

Several notes are in order for the reaction of stock returns to dividend news as a function of the number of periods until the next announcement. First of all, when the reaction to dividend news is not guaranteed to be symmetric, the same dividend news might have a positive effect on stock returns in one period and a negative effect in another period everything else equal. This makes it more complicated to characterize the magnitude of the reaction to dividend news as a function of the number periods until the next announcement. Hence, we decided to focus on the case when the reaction is symmetric. Secondly, the implications of our model discussed in Proposition 6 continue to hold even if we assume that there is no external signal observed on announcement days. Finally, if we assume that the dividend growth process can possibly switch to a new regime every period, the reaction of stock returns to dividend news will not obviously exhibit the time-varying behavior presented in Proposition 6.

3.2 The Reaction of Stock Returns to the External Signal

We now turn our attention to the reaction of stock returns to the unexpected part of the external signal. There are several differences between the dividend news and the external signal. First of all, the external signal is observed only on announcement periods whereas the dividend news is observed every period. Secondly, the dividend growth process might possibly switch to a new state on announcement days following the external signal. Finally, in contrast to dividend news which affects not only the investor's beliefs but also his consumption, the external signal only affects his beliefs about the state variable. The following proposition characterizes the reaction of unexpected stock returns to the unexpected part of the external signal.

Proposition 7. *The reaction of stock returns to unexpected part of the external signal*

(1) if the dividend growth process has different volatilities in different states, i.e. $\sigma_{d,1} \neq \sigma_{d,2}$

$$\partial r^*_{T_n}/\partial u_{x,T_n} = f_2(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n})\left(\frac{\lambda_2 - \lambda_1}{1 + \overline{\lambda}}\right)\left(\frac{\sigma^2_{x,2} - \sigma^2_{x,1}}{\sigma^2_{x,1}\sigma^2_{x,2}}\right)\left(u_{x,T_n} - \frac{(\mu_{x,1} - \mu_{x,2})\tilde{\sigma}^2_{x,T_n}}{\sigma^2_{x,2} - \sigma^2_{x,1}}\right) \quad (21)$$

(2) if the dividend growth process has the same volatility in different states, i.e. $\sigma_{d,1} = \sigma_{d,2}$

$$\partial r^*_{T_n}/\partial u_{x,T_n} = f_4(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n}) \left(\frac{\lambda_2 - \lambda_1}{1 + \overline{\lambda}} \right) \left(\frac{\sigma^2_{x,2} - \sigma^2_{x,1}}{\sigma^2_{x,1}\sigma^2_{x,2}} \right) \left(u_{x,T_n} - \frac{(\mu_{x,1} - \mu_{x,2})\tilde{\sigma}^2_{x,T_n}}{\sigma^2_{x,2} - \sigma^2_{x,1}} \right) \quad (22)$$

for $n = 1, 2, \ldots$ and $\tilde{\sigma}^2_{x,T_n} = \sigma^2_{x,1}\tilde{\pi}_{1,T_n} + \sigma^2_{x,2}(1 - \tilde{\pi}_{1,T_n})$.

The functions $f_2, f_4 : \mathbb{R}^2 \times [0,1] \to [0, 0.25]$ are real-valued positive functions bounded above by 0.25 and are defined in the appendix.

Proof. All proofs are in the appendix. □

As mentioned above, the external signal affects unexpected returns only through its effect on the investor's beliefs. The reaction of stock returns to the unexpected part of the external signal is similar to the indirect effect of dividend news on stock returns. Hence, the sign of the reaction to the unexpected part of the external signal can be unambiguously characterized as in the following proposition.

Proposition 8. *(a) Assume that the external signal has a higher volatility in the low growth state of the dividend growth process, i.e. $\sigma_{x,2} > \sigma_{x,1}$, and the investor is more (less) risk averse than a log-utility investor; then the reaction of unexpected returns to the unexpected part of the external signal is symmetric for any news variables u_{x,T_n} greater (smaller) than $\overline{u}_{x,T_n}$ where $\overline{u}_{x,T_n} = \frac{(\mu_{x,1} - \mu_{x,2})\tilde{\sigma}^2_{x,T_n}}{\sigma^2_{x,2} - \sigma^2_{x,1}}$.*

(b) Assume that the external signal has a higher volatility in the low growth state of the dividend growth process, i.e. $\sigma_{x,2} < \sigma_{x,1}$, and the investor is more (less) risk averse than a log-utility investor; then the reaction of unexpected returns to the unexpected part of the external signal is symmetric for any news variables u_{x,T_n} smaller (greater) than $\overline{u}_{x,T_n}$.

(c) Assume that the external signal has the same volatility in both states, i.e. $\sigma_{x,2} = \sigma_{x,1}$; then the reaction of unexpected returns to the unexpected part of the external signal is asymmetric independent of the investor's beliefs and the external signal if and only if the investor is more risk-averse than log utility, i.e. $\gamma > 1$.

(d) Assume that the representative agent is a log-utility investor, i.e. $\gamma = 1$, then unexpected returns do not react to the external signal independent of the investor's prior beliefs.

Parts (a) and (b) of Proposition 8 show that the sign of the reaction to the unexpected part of the external signal can be characterized as a function of the news variable itself. Part (c) shows that the sign of the reaction does not depend on the magnitude of the news variable when the external signal is equally volatile

in both states. Finally, part (d) shows that the external signal does not have any effect on returns when the investor has a log-utility.

As mentioned above, the effect of the unexpected part of the external signal on stock returns is similar to the indirect effect of dividend news and the intuition for the indirect effect applies to the external signal as well. Any external signal that increases the probability of high growth state will have a negative impact on stock returns if the investor is more risk averse than log utility. This is again due to the fact that the negative impact of a higher than expected discount rate dominates the positive effect of a higher than expected dividend growth rate when the investor is more risk averse than log utility. However, whether the reaction to the external signal is symmetric or not depends on whether an external signal increases or decreases the probability of the high growth state. Similar to the dividend news, any positive (negative) news from an external signal increases (decreases) the probability of high growth state if the external signal is equally volatile in both states. However, this does not hold true under the assumption that the external signal has different volatilities in different states.

Consider the case in part (a) when the external signal is more volatile in the low growth state, i.e. $\sigma_{x,2} > \sigma_{x,1}$ and the investor is more risk averse than log utility. We should first note that the $\overline{u}_{x,T_n}$ is positive under these assumptions and the fact that $\mu_{x,1} > \mu_{x,2}$. Any negative news observed from the external signal decreases the probability of the high growth state since the probability of observing negative news is higher under the low growth state than under the high growth state. Hence, the reaction of stock returns to any negative news observed from the external signal is positive under these assumptions. On the other hand, any positive news increases the probability of the high growth state if and only if the magnitude of the news variable is smaller than $\overline{u}_{x,T_n}$. This is due to the fact that the probability of observing a news variable greater than $\overline{u}_{x,T_n}$ is larger under the low growth state than under the high growth state. Hence, the reaction of stock returns is positive to any news variable greater than $\overline{u}_{x,T_n}$. In other words, the reaction to the unexpected part of the external signal is generally positive (except for news variable between 0 and $\overline{u}_{x,T_n}$) under the assumption that $\sigma_{x,2} > \sigma_{x,1}$ and $\gamma > 1$. Figure 8 presents the reaction of unexpected returns to news from the external signal under the assumptions of model 1. We also assume that the mean and standard deviation of the external signal are identical to those of the dividend growth process, respectively and that the dividend news on this announcement day is zero, i.e. $u_{d,T_n} = 0$.

As shown in Proposition 7, the magnitude of the reaction to the unexpected part of the external signal depends on the volatility of the external signal. The following proposition characterizes the reaction of returns to the unexpected part of the external signal as a function of the external signal's volatility.

Proposition 9. *The reaction of returns to the unexpected part of a perfect external signal ($\sigma_{x,1} = \sigma_{x,2} = 0$) is greater in absolute value than the reaction of returns to the unexpected part of an imperfect signal if*

the volatility of the imperfect external signal is equal in both states and large enough, i.e. $\sigma_{x,1} = \sigma_{x,2} > (\mu_{x,1} - \mu_{x,2})/2.$

Proposition 9 shows that the stock returns would react more strongly to a perfect signal than an imperfect signal if the volatility of the imperfect signal is high enough. In other words, stock returns would not always react more strongly to more precise external signals. The intuition behind this result is as follows: The reaction of stock returns to the unexpected part of the external signal does not depend on the investor's beliefs prior to observing the external signal and the dividend news when the external signal is perfect. This follows from the fact that all uncertainty about the state variable is resolved on the announcement day when the external signal is perfect. On the other hand, when the external signal is not perfect, the reaction to the unexpected part still depends on the investor's beliefs prior to the announcement. Hence, an imperfect signal with high enough precision might have a stronger effect on stock returns than a perfect signal if the investor's uncertainty prior to observing the signal (defined as $\pi_{1,T_n-1}(1 - \pi_{1,T_n-1})$) is high enough. However, even if the investor's uncertainty is at its highest at 0.5 prior to the announcement, the reaction to a perfect signal would be stronger than the reaction to an imperfect signal if the precision of the imperfect signal is low enough.

This implication of our model can also be interpreted in another fashion. The reaction of stock returns to an imperfect signal reveals some information about the precision of the external signal. Although the precision of an external signal is not generally observed, one would be able to back it out from the reaction of stock returns calibrating the values of other model parameters. This implication of our model is also related to the findings in Gilbert (2009) where he analyzes the predictability of revisions to macroeconomic variables based on the reaction of a broad-based index to the initial surprise. He finds that the announcement day reaction of the S&P 500 returns is able to predict future revisions to macroeconomic variables. Although there are no revisions to the external signal in our model, our model predicts that announcement day return controlling for the investor's uncertainty prior to the announcement will contain important information about the precision of the external signal which is related to future revisions to the external signal.

4   Conclusion

This paper analyzes how dynamics of stock returns are affected by news about the state of the economy in a Lucas-type model where investors never observe the true growth rate of the economy but rather infer about it through two different sources of information. In between announcement periods, investors observe dividend realizations and update their beliefs about the current state of the economy. On announcement periods, investors receive an additional external signal about the state of the economy. In this framework, we characterize the reaction of stock returns to dividend news and the unexpected part of the external signal.

We show that our model is able to account for many of the recent empirical findings on the reaction of stock returns to news. First of all, our model is able to generate time-varying and state-dependent reaction of stock returns to dividend news. Secondly, we show that the reactions of stock returns to dividend news and the external signal are quite different as dividend news affects not only the investor's beliefs about the state of the economy but also his current consumption whereas external signals only affect his beliefs. Under certain assumptions, stock returns react asymmetrically to both dividend news and the unexpected part of the external signal when these two signals have different volatilities in different states. The reaction can be guaranteed to be symmetric under the assumption that these two signals have the same volatility in both states. We also show that stock returns react differently to dividend news released earlier in between announcement since the investor knows that the dividend growth process will be in the same state until the next announcement. Finally, we show that the reaction to a perfect external signal can be guaranteed to be stronger than the reaction to an imperfect signal if the precision of the imperfect external signal is low enough.

Although our model is realistic, analytically tractable and most importantly suitable for the question addressed in this paper, it has its shortcomings like any other model. To obtain analytical solutions we assume that the investor has a power utility. In this framework, the investor's relative degree of risk aversion and his intertemporal elasticity of substitution is closely linked making it impossible to distinguish their effects on returns. One can extend the model where the investor has an Epstein-Zin type utility (Epstein and Zin (1989)) at the cost of losing analytical solutions. However, it is still possible to obtain analytical solutions for several extensions of our model. For example, one can think of modeling consumption and dividend growth processes separately as in Cecchetti, Lam, and Mark (1993) to analyze the reaction of stock returns in a partial equilibrium framework. Another possible extension of our model where analytical solutions might be still feasible is to model dividends and the price of the consumption good separately as in David and Veronesi (2004). In this framework, one can consider analyzing the effect of releases about interest rates.

References

Andersen, Torben, Tim Bollerslev, Francis Diebold, and Clara Vega, 2003, Micro effects of macro announcements: Real-time price discovery in foreign exchange markets, *American Economic Review* 93, 38–61.

Andersen, Torben G., Tim Bollerslev, Francis X. Diebold, and Clara Vega, 2007, Real-time price discovery in global stock, bond and foreign exchange markets, *Journal of International Economics* 73, 251–277.

Balduzzi, Pierluigi, Edwin J. Elton, and T. Clifton Green, 2001, Economic news and bond prices: Evidence from the U.S. Treasury market, *Journal of Financial and Quantitative Analysis* 36, 523–543.

Bernanke, Ben S., and Kenneth N. Kuttner, 2005, What explains the stock market's reaction to Federal Reserve policy?, *Journal of Finance* 60, 1221–1257.

Bomfim, Antulio N., 2003, Pre-announcement effects, news effects, and volatility: Monetary policy and the stock market, *Journal of Banking and Finance* 27, 133–151.

Boyd, John H., Jian Hu, and Ravi Jagannathan, 2005, The stock market's reaction to unemployment news: Why bad news is usually good for stocks, *Journal of Finance* 60, 649–672.

Cecchetti, Stephen G., Poksang Lam, and Nelson C. Mark, 1990, Mean reversion in equilibrium asset prices, *American Economic Review* 80, 398–418.

——— , 1993, The equity premium and the risk-free rate: Matching the moments, *Journal of Monetary Economics* 31, 21–45.

David, Alexander, 1997, Fluctuating confidence in stock markets: Implications for returns and volatility, *Journal of Financial and Quantitative Analysis* 32, 427–462.

——— , and Pietro Veronesi, 2004, Inflation and earnings uncertainty and volatility forecasts, University of Chicago, GSB Working Paper.

Debreu, Gerard, and I. N. Herstein, 1953, Nonnegative square matrices, *Econometrica* 21, 597–607.

Epstein, Larry, and Stanley Zin, 1989, substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937:968.

Flannery, Mark J., and Aris A. Protopapadakis, 2002, Macroeconomic factors do influence aggregate stock returns, *Review of Financial Studies* 15, 751–782.

Gilbert, Thomas, 2009, Information aggregation around macroeconomic announcements: The link between revisions and stock returns, Michael G. Foster School of Business, University of Washington, Working Paper.

Guo, Hui, 2004, Stock prices, firm size, and changes in the federal funds rate target, *The Quarterly Review of Economics and Finance* 44, 487–507.

Hamilton, James D., 1989, A new approach to the economic analysis of nonstationary time series and the business cycle, *Econometrica* 57, 357–384.

Kim, Oliver, and Robert E. Verrecchia, 1991, Market reaction to anticipated announcements, *Journal of Financial Economics* 30, 273–309.

Lucas, Robert E., 1978, Asset prices in an exchange economy, *Econometrica* 46, 1429–1445.

McQueen, Grant, and V. Vance Roley, 1993, Stock prices, news, and business conditions, *Review of Financial Studies* 6, 683–707.

Thorbecke, Willem, 1997, On stock market returns and monetary policy, *Journal of Finance* 52, 635–654.

Veronesi, Pietro, 1999, Stock market overreaction to bad news in good times: A rational expectations equilibrium model, *Review of Financial Studies* 12, 975–1007.

——— , 2000, How does information quality affect stock returns?, *Journal of Finance* 55, 807–837.

A Proofs

Proof of Lemma 1. Investor's beliefs need to be characterized for three different time periods. Case 1 is the investor's beliefs on the period after the $(n-1)^{th}$ announcement, $t = T_{n-1} + 1$, Case 2 is the investor's beliefs in between announcement days, $T_{n-1} < t < T_n$, and finally Case 3 is the investor's beliefs on the current announcement period, $t = T_n$. We analyze these three cases separately.

Case 1. ($t = T_{n-1} + 1$): On the announcement day, the investor knows that the dividend growth process might have possibly switched to a new state. Hence, his prior beliefs about state j before observing the dividend realization at time $T_{n-1} + 1$ is a weighted function of the transition probabilities into state j where the weights are his beliefs about the state variable of the dividend growth process on the announcement day T_{n-1}. His beliefs about state j prior to observing the dividend realization is given by $\pi_{j,t} = \sum_{i=1}^{N} \pi_{i,T_{n-1}} q_{ij}$. Given his prior beliefs, the investor updates his beliefs about the state variable after observing the dividend news as in Case 2. Case 2. ($T_{n-1} + 1 < t < T_n$): In between announcement days, the only source of information about the state variable is the dividend realization. The investor updates his beliefs from the previous period according to the Bayes' rule based on the observed dividend realization. Recall that the probability of being in state j, $\pi_{j,t} = \Pr(S_n = j | \mathcal{F}_t)$.

$$
\begin{aligned}
\pi_{j,t} &= \Pr(S_n = j | \Delta d_t, \mathcal{F}_{t-1}) & (23) \\
&= \frac{\Pr(\Delta d_t | S_n = j, \mathcal{F}_{t-1}) \Pr(S_n = j | \mathcal{F}_{t-1})}{\Pr(\Delta d_t | \mathcal{F}_{t-1})} & (24) \\
&= \frac{\Pr(\Delta d_t | S_n = j, \mathcal{F}_{t-1}) \Pr(S_n = j | \mathcal{F}_{t-1})}{\sum_{i=1}^{N} \Pr(\Delta d_t | S_n = i, \mathcal{F}_{t-1}) \Pr(S_n = i | \mathcal{F}_{t-1})} & (25) \\
&= \frac{\frac{1}{\sigma_{d,j}} \phi\left(\frac{\Delta d_t - \mu_{d,j}}{\sigma_{d,j}}\right) \tilde{\pi}_{j,t}}{\sum_{i=1}^{N} \frac{1}{\sigma_{d,i}} \phi\left(\frac{\Delta d_t - \mu_{d,i}}{\sigma_{d,i}}\right) \tilde{\pi}_{i,t}} & (26)
\end{aligned}
$$

where $\phi(\cdot)$ is the standard normal density function. Equations (23) follows from the definition of the information set, $\mathcal{F}_t$, which includes all information and the current dividend realization. Equation (24) and (25) follow from Bayes' rule and law of total probability, respectively.[7] Note that, by definition, $\tilde{\pi}_{i,t} = \Pr(S_n = i | \mathcal{F}_{t-1})$ in between announcement periods. Equation (26) follows from the law of motion for the dividend growth process in Equation (2).

Case 3. ($t = T_n$): On the announcement day, T_n, there are two sources of information about the state variable, the dividend realization and the external signal. The investor updates his beliefs from the previous

---

[7] Recall that Bayes' rule is $\Pr(A|B,C) = \frac{\Pr(B|A,C)\Pr(A|C)}{\Pr(A|C)}$.

period according to the Bayes' rule based on the observed dividend realization and the external.

$$\pi_{j,T_n} = \Pr(S_n = j | \Delta d_{T_n}, x_n, \mathcal{F}_{T_n-1}) \tag{27}$$

$$= \frac{\Pr(\Delta d_{T_n} | S_n = j, \mathcal{F}_{T_n-1}) \Pr(x_n | S_n = j, \mathcal{F}_{T_n-1}) \Pr(S_n = j | \mathcal{F}_{T_n-1})}{\Pr(\Delta d_t, x_n | \mathcal{F}_{T_n-1})} \tag{28}$$

$$= \frac{\Pr(\Delta d_{T_n} | S_n = j, \mathcal{F}_{T_n-1}) \Pr(x_n | S_n = j, \mathcal{F}_{T_n-1}) \Pr(S_n = j | \mathcal{F}_{T_n-1})}{\sum_{i=1}^{N} \Pr(\Delta d_{T_n} | S_n = i, \mathcal{F}_{T_n-1}) \Pr(x_n | S_n = i, \mathcal{F}_{T_n-1}) \Pr(S_n = i | \mathcal{F}_{T_n-1})} \tag{29}$$

$$= \frac{\frac{1}{\sigma_{d,j}\sigma_{x,j}}\phi\left(\frac{\Delta d_{T_n}-\mu_{d,j}}{\sigma_{d,j}}\right)\phi\left(\frac{x_n-\mu_{x,j}}{\sigma_{x,j}}\right)\tilde{\pi}_{j,T_n}}{\sum_{i=1}^{N} \frac{1}{\sigma_{d,i}\sigma_{x,i}}\phi\left(\frac{\Delta d_{T_n}-\mu_{d,i}}{\sigma_{d,i}}\right)\phi\left(\frac{x_n-\mu_{x,i}}{\sigma_{x,i}}\right)\tilde{\pi}_{i,T_n}} \tag{30}$$

The proof of Case 3 is similar to that of Case 2. Equation (27) follows from the definition of the information set on the announcement day T_n, $\mathcal{F}_{T_n}$, which includes all past information, the current dividend realization and the external signal. Equations (28) and (29) follow from the independence of Δd_{T_n} and x_n conditional on the current state variable. Equation (30) follows from the law of motion for dividend growth in Equation (2) and the law of motion for the external signal in Equation (3).

$\square$

Proof of Lemma 2. By recursive substitution of future prices into Euler equation in (9), the price of the risky asset can be expressed as a discounted sum of expected future dividends where the discount factor is the intertemporal marginal rate of substitution:

$$P_t = E_t\left[\sum_{\tau=1}^{\infty} \beta^{\tau} \frac{U'(C_{t+\tau})}{U'(C_t)} D_{t+\tau}\right] \tag{31}$$

Imposing the equilibrium condition, $C_t = D_t$, substituting the functional form for the utility function and rearranging the terms, the price-dividend ratio at time t can be expressed as follows:

$$\frac{P_t}{D_t} = E_t\left[\sum_{\tau=1}^{\infty} \beta^{\tau} \left(\frac{D_{t+\tau}}{D_t}\right)^{1-\gamma}\right] \tag{32}$$

The infinite sum in Equation (32) can be expressed as a sum of two terms, sum of discounted future dividends until the upcoming announcement day and sum of discounted future dividends after the upcoming announcement day. The price-dividend ratio can be expressed as follows:

$$\frac{P_t}{D_t} = \sum_{\tau=1}^{T_n-t} \beta^{\tau} E_t\left[\left(\frac{D_{t+\tau}}{D_t}\right)^{1-\gamma}\right] + \beta^{T_n-t} E_t\left[\left(\frac{D_{T_n}}{D_t}\right)^{1-\gamma} \frac{P_{T_n}}{D_{T_n}}\right] \tag{33}$$

Conditioning on the current state, the following holds:

$$\frac{P_t}{D_t} = \sum_{i=1}^{N}\sum_{\tau=1}^{T_n-t} \beta^{\tau} E_t\left[\left(\frac{D_{t+\tau}}{D_t}\right)^{1-\gamma} \Big| S_n = i\right]\pi_{it}$$

$$+ \sum_{i=1}^{N} \beta^{T_n-t} E_t\left[\left(\frac{D_{T_n}}{D_t}\right)^{1-\gamma} \Big| S_n = i\right] E_t\left[\frac{P_{T_n}}{D_{T_n}} \Big| S_n = i\right]\pi_{it} \tag{34}$$

where Equation (34) follows from law of total probability and conditional independence of $\frac{D_{T_n}}{D_t}$ and $\frac{P_{T_n}}{D_{T_n}}$ when the conditioning information is the current state variable. Note that for t such that $T_{n-1} \leq t \leq T_n$ and $1 \leq \tau \leq T_n - t$, we have

$$E_t\left[\left(\frac{D_{t+\tau}}{D_t}\right)^{1-\gamma}\bigg| S_n = i\right] = E_t[\exp((1-\gamma)\mu_{d,i}\tau + (1-\gamma)\sigma_{d,i}\sum_{l=1}^{\tau}\varepsilon_{t+l})] \tag{35}$$

$$= \exp((1-\gamma)\mu_{d,i} + (1-\gamma)^2\sigma_{d,i}^2/2)^\tau \tag{36}$$

$$\equiv (e^{a_i})^\tau \tag{37}$$

where $a_i \equiv (1-\gamma)\mu_{d,i} + (1-\gamma)^2\sigma_{d,i}^2/2$. Equation (35) follows from the law of motion for the dividend growth rate. Equation (36) follows from the formula for the expectation of a lognormal variable where the mean and variance of the normal variable are $(1-\gamma)\mu_{d,i}\tau$ and $(1-\gamma)^2\sigma_{d,i}^2\tau$, respectively. The price-dividend ratio can be expressed as:

$$\frac{P_t}{D_t} = \sum_{i=1}^{N}\sum_{\tau=1}^{T_n-t}(\beta e^{a_i})^\tau \pi_{it} + \sum_{i=1}^{N}(\beta e^{a_i})^{T_n-t}E_t\left[\frac{P_{T_n}}{D_{T_n}}\bigg| S_n = i\right]\pi_{it}$$

$$= \sum_{i=1}^{N}\left(\frac{(\beta e^{a_i})^{T_n-t+1}-1}{\beta e^{a_i}-1}-1\right)\pi_{it} + \sum_{i=1}^{N}(\beta e^{a_i})^{T_n-t}E_t\left[\frac{P_{T_n}}{D_{T_n}}\bigg| S_n = i\right]\pi_{it} \tag{38}$$

The price-dividend ratio on the previous announcement day T_{n-1} can be expressed as follows by setting $t = T_{n-1}$:

$$\frac{P_{T_{n-1}}}{D_{T_{n-1}}} = \sum_{i=1}^{N}\left(\sum_{j=1}^{N}\left(\frac{(\beta e^{a_j})^{T+1}-1}{\beta e^{a_j}-1}-1\right)q_{i,j} + \sum_{j=1}^{N}(\beta e^{a_j})^T E_t\left[\frac{P_{T_n}}{D_{T_n}}\bigg| S_n = j\right]q_{i,j}\right)\pi_{i,T_{n-1}} \tag{39}$$

where $q_{i,j}$ is the ij^{th} element of the transition probability matrix $\mathbf{Q}$.

In order to solve the difference equation in (39), we conjecture a solution for the price-dividend ratio on announcement periods of the following form:

$$\frac{P_{T_n}}{D_{T_n}} = \lambda_i \text{ for } n = 1, 2, \ldots \text{ and } i = 1, 2, \ldots, N \tag{40}$$

Plugging in the conjecture in Equation (40), we obtain the following system of N linear equations in N variables, $(\lambda_1, \ldots, \lambda_N)$:

$$\lambda_i = \sum_{j=1}^{N}\left(\frac{(\beta e^{a_j})^{T+1}-1}{\beta e^{a_j}-1}-1\right)q_{ij} + \left(\sum_{j=1}^{N}(\beta e^{a_j})^T q_{ij}\right)\left(\sum_{j=1}^{N}\lambda_j q_{ij}\right) \tag{41}$$

for $i = 1, 2, \ldots, N$. To reduce notation, we define a $N \times 1$ vector, $\mathbf{G}$, whose j^{th} element, g_j, is given by $g_j = \frac{(\beta e^{a_j})^{T+1}-1}{\beta e^{a_j}-1}-1$ and a $N \times N$ diagonal matrix, $\mathbf{H}$, whose i^{th} diagonal element, h_i, is given by $h_i = \sum_{j=1}^{N}(\beta e^{a_j})^T q_{ij}$. The system of equations in (41) can be expressed as follows:

$$\lambda = \mathbf{QG} + \mathbf{HQ}\lambda \tag{42}$$

33

Solving for the vector λ, we obtain the price-dividend ratio on announcement days in Lemma 2.

First note that elements of $\mathbf{HQ}$ and $\mathbf{QG}$ are non-negative. To prove that elements of λ are non-negative, it suffices to show that the elements of $(\mathbf{I} - \mathbf{HQ})^{-1}$ are non-negative. According to Theorem III* of Debreu and Herstein (1953), the elements of $(\mathbf{I} - \mathbf{HQ})^{-1}$ are non-negative if and only if the maximal non-negative characteristic root of $\mathbf{HQ}$ is less than 1. Let p denote the maximal nonnegative characteristic root of $\mathbf{HQ}$. According to the Perron-Frobenius theorem for non-negative matrices, we know that $\min_i \sum_{j=1}^{N}[\mathbf{HQ}]_{i,j} \leq p \leq \max_i \sum_{j=1}^{N}[\mathbf{HQ}]_{i,j}$ where $[\cdot]_{i,j}$ refers to the ij^{th} element of the matrix. We know that $\sum_{j=1}^{N}[\mathbf{HQ}]_{i,j} = h_i < 1$ and thus $p < 1$. This completes the proof.

$\square$

Proof of Proposition 1. Proof of Proposition follows from Equation (38). Note that $E_t[\frac{P_{T_n}}{D_{T_n}}|S_n = j] = \lambda_j$ from the result in Lemma 2. Plugging in, we obtain Equation (11) for the price-dividend ratio on non-announcement days.

Using a first-order Taylor expansion of the log function around the long term average of the dividend price ratio, log returns on the risky asset can be expressed as follows:

$$
\begin{aligned}
r_t &= \log(1 + P_t/D_t) - \log(P_{t-1}/D_{t-1}) + \Delta d_t \\
&\approx \log(1 + \overline{\lambda}) + \frac{1}{1 + \overline{\lambda}}(P_t/D_t - \overline{\lambda}) - \log(\overline{\lambda}) - \frac{1}{\overline{\lambda}}(P_{t-1}/D_{t-1} - \overline{\lambda}) + \Delta d_t \quad (43)
\end{aligned}
$$

Using the above approximation, the conditional expectation of log returns based on the information set at time $t - 1$ can be written as follows:

$$
E_{t-1}[r_t] = \log(1 + \overline{\lambda}) + \frac{1}{1 + \overline{\lambda}}(\sum_{j=1}^{N} k_{j,\tau}\pi_{j,t-1} - \overline{\lambda}) - \log(\overline{\lambda}) - \frac{1}{\overline{\lambda}}(P_{t-1}/D_{t-1} - \overline{\lambda}) + \sum_{j=1}^{N} \mu_{d,j}\pi_{j,t-1} \quad (44)
$$

The unexpected log return on the risky asset in Equation (13) can be obtained as the difference between Equations (43) and (44). The long term average of the dividend price ratio is the unconditional expectation of the price-dividend ratio as defined in Proposition 1. $\square$

Proof of Corollary 1. Equation (14) can be directly obtained from Equation (13) by setting $N = 2$. $\square$

Proof of Proposition 2. Recall that $\pi_{1,t}$ denotes the probability that the investor assigns to the high growth state when there are only two possible states of the dividend growth process. For a non-announcement period t such that $T_{n-1} < t < T_n$, $\pi_{1,t}$ can be expressed as follows:

$$
\pi_{1,t} = \left[1 + \frac{1 - \tilde{\pi}_{1,t}}{\tilde{\pi}_{1,t}} \exp\left(-\frac{(\mu_{d,1} - \mu_{d,2})^2}{2(\sigma_{d,2}^2 - \sigma_{d,1}^2)}\right) \exp\left(\frac{\sigma_{d,2}^2 - \sigma_{d,1}^2}{2\sigma_{d,1}^2\sigma_{d,2}^2}\left(u_{d,t} - \frac{(\mu_{d,1} - \mu_{d,2})\tilde{\sigma}_{d,t}^2}{\sigma_{d,2}^2 - \sigma_{d,1}^2}\right)^2\right)\right]^{-1}
$$

and for an announcement period, T_n, π_{1,T_n} can be expressed as follows:

$$
\begin{aligned}
\pi_{1,T_n} &= \Bigg[1 + \frac{1 - \tilde{\pi}_{1,T_n}}{\tilde{\pi}_{1,T_n}} \exp\left(-\frac{(\mu_{d,1} - \mu_{d,2})^2}{2(\sigma_{d,2}^2 - \sigma_{d,1}^2)}\right) \exp\left(-\frac{(\mu_{x,1} - \mu_{x,2})^2}{2(\sigma_{x,2}^2 - \sigma_{x,1}^2)}\right) \\
&\quad \exp\left(\frac{\sigma_{d,2}^2 - \sigma_{d,1}^2}{2\sigma_{d,1}^2 \sigma_{d,2}^2}\left(u_{d,T_n} - \frac{(\mu_{d,1} - \mu_{d,2})\tilde{\sigma}_{d,T_n}^2}{\sigma_{d,2}^2 - \sigma_{d,1}^2}\right)^2\right) \\
&\quad \exp\left(\frac{\sigma_{x,2}^2 - \sigma_{x,1}^2}{2\sigma_{x,1}^2 \sigma_{x,2}^2}\left(u_{x,T_n} - \frac{(\mu_{x,1} - \mu_{x,2})\tilde{\sigma}_{x,T_n}^2}{\sigma_{x,2}^2 - \sigma_{x,1}^2}\right)^2\right)\Bigg]^{-1}
\end{aligned}
\tag{45}
$$

Then the derivative of $\pi_{1,t}$ on a non-announcement period t such that $T_{n-1} < t < T_n$ is given by

$$
\partial \pi_{1,t} / \partial u_{d,t} = f_1(u_{d,t}, \tilde{\pi}_{1,t})\left(\frac{\sigma_{d,2}^2 - \sigma_{d,1}^2}{\sigma_{d,1}^2 \sigma_{d,2}^2}\right)\left(u_{d,t} - \frac{(\mu_{d,1} - \mu_{d,2})\tilde{\sigma}_{d,t}^2}{\sigma_{d,2}^2 - \sigma_{d,1}^2}\right)
$$

where

$$
f_1(u_{d,t}, \tilde{\pi}_{1,t}) = \kappa_1(u_{d,t}, \tilde{\pi}_{1,t}) / (1 + \kappa_1(u_{d,t}, \tilde{\pi}_{1,t}))^2
$$

and

$$
\kappa_1(u_{d,t}, \tilde{\pi}_{1,t}) = \frac{1 - \tilde{\pi}_{1,t}}{\tilde{\pi}_{1,t}} \exp\left(-\frac{(\mu_{d,1} - \mu_{d,2})^2}{2(\sigma_{d,2}^2 - \sigma_{d,1}^2)}\right) \exp\left(\frac{\sigma_{d,2}^2 - \sigma_{d,1}^2}{2\sigma_{d,1}^2 \sigma_{d,2}^2}\left(u_{d,t} - \frac{(\mu_{d,1} - \mu_{d,2})\tilde{\sigma}_{d,t}^2}{\sigma_{d,2}^2 - \sigma_{d,1}^2}\right)^2\right)
$$

On the other hand, the derivative of π_{1,T_n} with respect to $\partial u_{d,T_n}$ on an announcement period T_n is given by

$$
\partial \pi_{1,T_n} / \partial u_{d,T_n} = f_2(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n})\left(\frac{\sigma_{d,2}^2 - \sigma_{d,1}^2}{\sigma_{d,1}^2 \sigma_{d,2}^2}\right)\left(u_{d,T_n} - \frac{(\mu_{d,1} - \mu_{d,2})\tilde{\sigma}_{d,T_n}^2}{\sigma_{d,2}^2 - \sigma_{d,1}^2}\right)
$$

where

$$
f_2(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n}) = \kappa_2(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n}) / (1 + \kappa_2(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n}))^2
$$

and

$$
\begin{aligned}
\kappa_2(u_{d,T_n}, u_{x,T_n}, \pi_{1,T_n}) &= \frac{1 - \tilde{\pi}_{1,T_n}}{\tilde{\pi}_{1,T_n}} \exp\left(-\frac{(\mu_{d,1} - \mu_{d,2})^2}{2(\sigma_{d,2}^2 - \sigma_{d,1}^2)}\right) \exp\left(-\frac{(\mu_{x,1} - \mu_{x,2})^2}{2(\sigma_{x,2}^2 - \sigma_{x,1}^2)}\right) \\
&\quad \exp\left(\frac{\sigma_{d,2}^2 - \sigma_{d,1}^2}{2\sigma_{d,1}^2 \sigma_{d,2}^2}\left(u_{d,T_n} - \frac{(\mu_{d,1} - \mu_{d,2})\tilde{\sigma}_{d,T_n}^2}{\sigma_{d,2}^2 - \sigma_{d,1}^2}\right)^2\right) \\
&\quad \exp\left(\frac{\sigma_{x,2}^2 - \sigma_{x,1}^2}{2\sigma_{x,1}^2 \sigma_{x,2}^2}\left(u_{x,T_n} - \frac{(\mu_{x,1} - \mu_{x,2})\tilde{\sigma}_{x,T_n}^2}{\sigma_{x,2}^2 - \sigma_{x,1}^2}\right)^2\right)
\end{aligned}
$$

It is easy to see that both f_1 and f_2 are positive functions ad bounded above by 0.25. The derivative of unexpected returns with respect to dividend news, $\partial r_t^* / \partial u_{d,t}$, is given by

$$
\partial r_t^* / \partial u_{d,t} = 1 + \frac{(k_{1,\tau} - k_{2,\tau})}{1 + \overline{\lambda}} \partial \pi_{1,t} / \partial u_{d,t}
\tag{46}
$$

where $\tau = T_n - t$. Equations (17) and (18) of Proposition 2 can be obtained by plugging in the appropriate derivative of $\pi_{1,t}$ with respect to dividend news Equation (46). $\square$

Proof of Proposition 3. To prove Proposition 3, we start by showing that λ_j is a non-increasing function of μ_i for $i, j = 1, 2, \ldots, N$ if and only if the investor is more risk averse than a log utility investor. In other words, $\partial \lambda_j / \partial \mu_i$ is non-positive for $i, j = 1, 2, \ldots, N$ if and only if $\gamma > 1$. To show this, first note that $\partial a_j / \partial \mu_i$ is zero for $i \neq j$ and negative for $i = j$ if and only if $\gamma > 1$. This implies that the diagonal elements of $\partial \mathbf{H} / \partial \mu_i$ and all elements of $\partial \mathbf{G} / \partial \mu_i$ are negative if and only if $\gamma > 1$. This in turn implies

$$
\begin{aligned}
\frac{\partial \lambda}{\partial \mu_i} &= \frac{\partial (\mathbf{I} - \mathbf{HQ})^{-1} \mathbf{QG}}{\partial \mu_i} \\
&= \frac{\partial (\mathbf{I} - \mathbf{HQ})^{-1}}{\partial \mu_i} \mathbf{QG} + (\mathbf{I} - \mathbf{HQ})^{-1} \mathbf{Q} \frac{\partial \mathbf{G}}{\partial \mu_i} \\
&= -(\mathbf{I} - \mathbf{HQ})^{-1} \frac{\partial (\mathbf{I} - \mathbf{HQ})}{\partial \mu_i} (\mathbf{I} - \mathbf{HQ})^{-1} \mathbf{QG} + (\mathbf{I} - \mathbf{HQ})^{-1} \mathbf{Q} \frac{\partial \mathbf{G}}{\partial \mu_i} \\
&= (\mathbf{I} - \mathbf{HQ})^{-1} \left[\frac{\partial \mathbf{H}}{\partial \mu_i} \mathbf{Q} \lambda + \mathbf{Q} \frac{\partial \mathbf{G}}{\partial \mu_i} \right]
\end{aligned}
$$

is non-positive if and only if $\gamma > 1$ since we know that elements of $(\mathbf{I} - \mathbf{HQ})^{-1}$ are nonnegative. This implies that $\lambda_1 \leq \lambda_2$ and $k_{1,\tau} \leq k_{2,\tau}$ for $\tau = 0, 1, \ldots, T - 1$ if and only if $\gamma > 1$.

Proofs of (a) and (b). Here, we only prove the first case in Part (a) of Proposition 3, the proofs of other cases in Parts (a) and (b) are similar, hence omitted. Under the assumptions of the first case in part (a), it is easy to see that

$$
f_1(u_{d,t}, \tilde{\pi}_{1,t}) \left(\frac{k_{2,\tau} - k_{1,\tau}}{1 + \overline{\lambda}} \right) \left(\frac{\sigma_{d,2}^2 - \sigma_{d,1}^2}{\sigma_{d,1}^2 \sigma_{d,2}^2} \right) > 0
$$

$$
f_2(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n}) \left(\frac{\lambda_2 - \lambda_1}{1 + \overline{\lambda}} \right) \left(\frac{\sigma_{d,2}^2 - \sigma_{d,1}^2}{\sigma_{d,1}^2 \sigma_{d,2}^2} \right) > 0
$$

since f_1 and f_2 are positive valued functions and $k_{2,\tau} > k_{1,\tau}$ for $\tau = 0, 1, \ldots, T - 1$ when $\gamma > 1$. Hence, the sign of the indirect effect depends on the sign of $\left(u_{d,t} - \frac{(\mu_{d,1} - \mu_{d,2}) \tilde{\sigma}_{d,t}^2}{\sigma_{d,2}^2 - \sigma_{d,1}^2} \right)$. For large dividend news, the indirect is negative and dominates the direct effect which is always equal to one. Hence, $\partial r_t^* / \partial u_{d,t}$ is negative for large dividend news which implies the stock return reacts positively to large negative dividend news.

Proof of (c) When $\gamma = 1$, $k_{2,\tau} = k_{1,\tau}$ for $\tau = 0, 1, \ldots, T - 1$. Then, it is easy to see that the indirect effect is equal to zero. Hence, $\partial r_t^* / \partial u_{d,t} = 1$. $\qquad\square$

Proof of Proposition 4. Recall that $\pi_{1,t}$ denotes the probability that the investor assigns to the high growth state when there are only two possible states of the dividend growth process. For a non-announcement period t such that $T_{n-1} < t < T_n$, $\pi_{1,t}$ can be expressed as follows under the assumption that $\sigma_{d,1} = \sigma_{d,2}$:

$$
\pi_{1,t} = \left[1 + \frac{1 - \tilde{\pi}_{1,t}}{\tilde{\pi}_{1,t}} \exp\left(\frac{(\mu_{d,1} - \mu_{d,2})^2 (1 - 2\tilde{\pi}_{1,t})}{2\sigma_{d,1}^2} \right) \exp\left(\frac{\mu_{d,2} - \mu_{d,1}}{\sigma_{d,1}^2} u_{d,t} \right) \right]^{-1}
$$

and for an announcement period, T_n, π_{1,T_n} can be expressed as follows:

$$\pi_{1,T_n} = \left[1 + \frac{1 - \tilde{\pi}_{1,T_n}}{\tilde{\pi}_{1,T_n}} \exp\left(\frac{(\mu_{d,1} - \mu_{d,2})^2 (1 - 2\tilde{\pi}_{1,t})}{2\sigma_{d,1}^2} \right) \exp\left(-\frac{(\mu_{x,1} - \mu_{x,2})^2}{2(\sigma_{x,2}^2 - \sigma_{x,1}^2)} \right) \right.$$
$$\left. \exp\left(\frac{\mu_{d,2} - \mu_{d,1}}{\sigma_{d,1}^2} u_{d,T_n} \right) \exp\left(\frac{\sigma_{x,2}^2 - \sigma_{x,1}^2}{2\sigma_{x,1}^2 \sigma_{x,2}^2} \left(u_{x,T_n} - \frac{(\mu_{x,1} - \mu_{x,2})\tilde{\sigma}_{x,T_n}^2}{\sigma_{x,2}^2 - \sigma_{x,1}^2} \right)^2 \right) \right]^{-1} \quad (47)$$

Then the derivative of $\pi_{1,t}$ on a non-announcement period t such that $T_{n-1} < t < T_n$ is given by

$$\partial \pi_{1,t} / \partial u_{d,t} = f_3(u_{d,t}, \tilde{\pi}_{1,t}) \left(\frac{\mu_{d,1} - \mu_{d,2}}{\sigma_{d,1}^2} \right) \left(\frac{k_{1,\tau} - k_{2,\tau}}{1 + \bar{\lambda}} \right)$$

where

$$f_3(u_{d,t}, \tilde{\pi}_{1,t}) = \kappa_3(u_{d,t}, \tilde{\pi}_{1,t}) / (1 + \kappa_3(u_{d,t}, \tilde{\pi}_{1,t}))^2$$

and

$$\kappa_3(u_{d,t}, \tilde{\pi}_{1,t}) = \frac{1 - \tilde{\pi}_{1,t}}{\tilde{\pi}_{1,t}} \exp\left(\frac{(\mu_{d,1} - \mu_{d,2})^2 (1 - 2\tilde{\pi}_{1,t})}{2\sigma_{d,1}^2} \right) \exp\left(\frac{\mu_{d,2} - \mu_{d,1}}{\sigma_{d,1}^2} u_{d,t} \right)$$

On the other hand, the derivative of π_{1,T_n} with respect to $\partial u_{d,T_n}$ on an announcement period T_n is given by

$$\partial \pi_{1,T_n} / \partial u_{d,T_n} = f_4(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n}) \left(\frac{\mu_{d,1} - \mu_{d,2}}{\sigma_{d,1}^2} \right) \left(\frac{\lambda_1 - \lambda_2}{1 + \bar{\lambda}} \right)$$

where

$$f_4(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n}) = \kappa_4(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n}) / (1 + \kappa_4(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n}))^2$$

and

$$\kappa_4(u_{d,T_n}, u_{x,T_n}, \pi_{1,T_n}) = \frac{1 - \tilde{\pi}_{1,T_n}}{\tilde{\pi}_{1,T_n}} \exp\left(\frac{(\mu_{d,1} - \mu_{d,2})^2 (1 - 2\tilde{\pi}_{1,t})}{2\sigma_{d,1}^2} \right) \exp\left(-\frac{(\mu_{x,1} - \mu_{x,2})^2}{2(\sigma_{x,2}^2 - \sigma_{x,1}^2)} \right)$$
$$\exp\left(\frac{\mu_{d,2} - \mu_{d,1}}{\sigma_{d,1}^2} u_{d,T_n} \right) \exp\left(\frac{\sigma_{x,2}^2 - \sigma_{x,1}^2}{2\sigma_{x,1}^2 \sigma_{x,2}^2} \left(u_{x,T_n} - \frac{(\mu_{x,1} - \mu_{x,2})\tilde{\sigma}_{x,T_n}^2}{\sigma_{x,2}^2 - \sigma_{x,1}^2} \right)^2 \right)$$

It is easy to see that both f_3 and f_4 are positive functions ad bounded above by 0.25.

The derivative of unexpected returns with respect to dividend news, $\partial r_t^* / \partial u_{d,t}$, is given by

$$\partial r_t^* / \partial u_{d,t} = 1 + \frac{(k_{1,\tau} - k_{2,\tau})}{1 + \bar{\lambda}} \partial \pi_{1,t} / \partial u_{d,t} \quad (48)$$

where $\tau = T_n - t$. Equations (19) and (20) of Proposition 4 can be obtained by plugging in the appropriate derivative of $\pi_{1,t}$ with respect to dividend news Equation (48). $\qquad \square$

Proof of Proposition 5. The proof of Proposition 5 is similar to the proof of Proposition 3. First recall that $k_{1,\tau} < k_{2,\tau}$ for $\tau = 0, 1, \ldots, T-1$ if and only if $\gamma > 1$ and f_3 and f_4 are positive valued functions bounded above by 0.25. *Proof of (a)* When $\gamma > 1$, the indirect effect has the opposite sign as the direct effect. If the indirect effect dominates the direct effect, then the overall reaction of stock returns to dividend news is

asymmetric ($\partial r_t^*/\partial u_{d,t} < 0$). If the opposite holds, then the overall reaction of stock returns to dividend news is symmetric ($\partial r_t^*/\partial u_{d,t} > 0$). A necessary and sufficient condition for $\partial r_t^*/\partial u_{d,t} > 0$ is:

$$
\frac{k_{1,\tau} - k_{2,\tau}}{1 + \bar{\lambda}} > -\frac{1}{f_3(u_{d,t}, \tilde{\pi}_{1,t})}\left(\frac{\sigma_{d,1}^2}{\mu_{d,1} - \mu_{d,2}}\right) \qquad \text{on non-announcement days}
$$

$$
\frac{\lambda_1 - \lambda_2}{1 + \bar{\lambda}} > -\frac{1}{f_4(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n})}\left(\frac{\sigma_{d,1}^2}{\mu_{d,1} - \mu_{d,2}}\right) \qquad \text{on announcement days}
$$

(49)

Given that both f_3 and f_4 are bound above by 0.25, a sufficient condition for $\partial r_t^*/\partial u_{d,t} > 0$ is given by the condition in part (a).

Proof of (b) When $\gamma < 1$, the indirect effect always has a positive sign. Thus, the sign of the overall reaction is also positive resulting in a symmetric reaction to dividend news.

Proof of (c) When $\gamma = 1$, $k_{2,\tau} = k_{1,\tau}$ for $\tau = 0, 1, \ldots, T - 1$. Then, it is easy to see that the indirect effect is equal to zero. Hence, $\partial r_t^*/\partial u_{d,t} = 1$. $\qquad\qquad\square$

Proof of Proposition 6. To prove Proposition 6, we start by showing that $k_{1,\tau} < k_{1,\tau-1}$ and $k_{2,\tau} > k_{2,\tau-1}$ if and only if $\gamma > 1$. Define $N \times 1$ vector $\Delta \mathbf{K}_\tau$ whose i^{th} element is given by $k_{i,\tau-1} - k_{i,\tau}$ for $\tau = 1, 2, \ldots, T - 1$ and define $N \times N$ diagonal matrix $\mathbf{Z}$ whose i^{th} diagonal element is $z_i = \beta \exp(a_i)$. One can show that $\Delta \mathbf{K}_\tau$ can be expressed as follows:

$$
\Delta \mathbf{K}_\tau = (\mathbf{I} - \mathbf{A})\mathbf{A}^\tau(\mathbf{I} - \mathbf{HQ})^{-1}[\mathbf{Q}(\mathbf{I} - \mathbf{A}^T) - (\mathbf{I} - \mathbf{HQ})]\mathbf{A}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{1}
$$

where $\mathbf{1}$ is a $N \times 1$ vector of ones and $(\mathbf{I} - \mathbf{A})\mathbf{A}^\tau$ is a diagonal matrix whose elements are positive. For $N = 2$, one can show that

$$
(\mathbf{I} - \mathbf{HQ})^{-1}[\mathbf{Q}(\mathbf{I} - \mathbf{A}^T) - (\mathbf{I} - \mathbf{HQ})]\mathbf{A}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{1} =
$$
$$
\frac{1}{c}\left(\frac{z_1}{1 - z_1} - \frac{z_2}{1 - z_2}\right)\left(\begin{array}{c} (1 + (z_2^T - z_1^T)(1 - [\mathbf{Q}]_{1,1} - [\mathbf{Q}]_{2,2})[\mathbf{Q}]_{2,2})(1 - z_2^T)([\mathbf{Q}]_{1,1} - 1) \\ (1 + (z_1^T - z_2^T)(1 - [\mathbf{Q}]_{1,1} - [\mathbf{Q}]_{2,2})[\mathbf{Q}]_{1,1})(1 - z_1^T)(1 - [\mathbf{Q}]_{2,2}) \end{array}\right)
$$

where c is the determinant of $(\mathbf{I} - \mathbf{HQ})$, hence positive. $[\mathbf{Q}]_{i,j}$ denotes the ij^{th} element of the transition probability matrix, $\mathbf{Q}$. Using the above equation and the fact that $(\mathbf{I} - \mathbf{A})\mathbf{A}^\tau$ is a diagonal matrix whose elements are positive, one can show that the first element of $\Delta \mathbf{K}_\tau$ is positive and the second element is negative if and only if $\gamma > 1$. Hence, the difference between $k_{1,\tau}$ and $k_{2,\tau}$ in absolute value gets smaller as τ approaches 0 if $\gamma \neq 1$.

Proof of (a) First note that the indirect effect has the opposite sign of the direct effect under the assumptions of part (a). However, the direct effect dominates the indirect effect since the reaction is assumed to be symmetric.

(1) For positive dividend news, the direct effect is positive and dominates the negative indirect effect. The indirect effect gets smaller in magnitude as the next announcement approaches due to the above result. Hence, the overall reaction is positive and get larger in magnitude.

(2) For negative dividend news, the direct effect is negative and dominates the positive indirect effect. The indirect effect gets smaller in magnitude as the next announcement approaches due to the above result. Hence, the overall reaction is negative and get smaller in magnitude.

Proof of (b) First note that the indirect effect has the same sign as the direct effect. The indirect effect gets smaller in magnitude as the next announcement approaches due to the above result. Hence, the overall reaction gets smaller in magnitude.

Proof of (c) When $\gamma = 1$, the indirect effect is zero and hence, the overall reaction is just the direct effect which does not depend on the number of periods until the next announcement. $\square$

Proof of Proposition 7. Taking the partial derivative of Equations (45) and (47) with respect to u_{x,T_n}, we obtain the reaction of stock returns to unexpected part of the external signal in Proposition 7. $\square$

Proof of Proposition 8. Proof of (a) and (b) Here, we prove only the first condition in part (a). The proofs of other conditions are similar, hence, omitted. Consider the case where $\gamma > 1$ and $\sigma_{x,2} > \sigma_{x,1}$. Under the assumptions, the sign of $\partial r^*_{T_n}/\partial u_{x,T_n}$ depends only on the sign of $u_{x,T_n} - \frac{(\mu_{x,1}-\mu_{x,2})\tilde{\sigma}^2_{x,T_n}}{\sigma^2_{x,2}-\sigma^2_{x,1}}$ since f_2 and f_4 are positive functions and $\lambda_2 > \lambda_1$. Note that $\frac{(\mu_{x,1}-\mu_{x,2})\tilde{\sigma}^2_{x,T_n}}{\sigma^2_{x,2}-\sigma^2_{x,1}} > 0$. Hence, $\partial r^*_{T_n}/\partial u_{x,T_n} > 0$ if and only if $u_{x,T_n} > \frac{(\mu_{x,1}-\mu_{x,2})\tilde{\sigma}^2_{x,T_n}}{\sigma^2_{x,2}-\sigma^2_{x,1}}$.

Proof of (c) When $\sigma_{x,1} = \sigma_{x,2}$,

$$\partial r^*_{T_n}/\partial u_{x,T_n} = \begin{cases} f_5(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n})\left(\frac{\mu_{x,1}-\mu_{x,2}}{\sigma^2_{x,1}}\right)\left(\frac{\lambda_1-\lambda_2}{1+\lambda}\right) & \text{if } \sigma_{d,1} \neq \sigma_{d,2} \\ f_6(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n})\left(\frac{\mu_{x,1}-\mu_{x,2}}{\sigma^2_{x,1}}\right)\left(\frac{\lambda_1-\lambda_2}{1+\lambda}\right) & \text{if } \sigma_{d,1} = \sigma_{d,2} \end{cases}$$

where f_5 and $f_6 : \mathbb{R}^2 \times [0,1] \to [0, 0.25]$ are real-valued positive functions bounded above by 0.25 similar to functions f_1, f_2, f_3 and f_4.

$$f_5(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n}) = \kappa_5(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n})/(1 + \kappa_5(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n}))^2$$

$$f_6(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n}) = \kappa_6(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n})/(1 + \kappa_6(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n}))^2$$

and

$$\kappa_5(u_{d,T_n}, u_{x,T_n}, \pi_{1,T_n}) = \frac{1 - \tilde{\pi}_{1,T_n}}{\tilde{\pi}_{1,T_n}} \exp\left(-\frac{(\mu_{d,1} - \mu_{d,2})^2}{2(\sigma_{d,2}^2 - \sigma_{d,1}^2)}\right) \exp\left(\frac{(\mu_{x,1} - \mu_{x,2})^2(1 - 2\tilde{\pi}_{1,T_n})}{2\sigma_{x,1}^2}\right)$$

$$\exp\left(\frac{\sigma_{d,2}^2 - \sigma_{d,1}^2}{2\sigma_{d,1}^2\sigma_{d,2}^2}\left(u_{d,T_n} - \frac{(\mu_{d,1} - \mu_{d,2})\tilde{\sigma}_{d,T_n}^2}{\sigma_{d,2}^2 - \sigma_{d,1}^2}\right)^2\right) \exp\left(\frac{\mu_{x,2} - \mu_{x,1}}{\sigma_{x,1}^2}u_{x,T_n}\right)$$

$$\kappa_6(u_{d,T_n}, u_{x,T_n}, \pi_{1,T_n}) = \frac{1 - \tilde{\pi}_{1,T_n}}{\tilde{\pi}_{1,T_n}} \exp\left(\frac{(\mu_{d,1} - \mu_{d,2})^2(1 - 2\tilde{\pi}_{1,T_n})}{2\sigma_{d,1}^2}\right) \exp\left(\frac{(\mu_{x,1} - \mu_{x,2})^2(1 - 2\tilde{\pi}_{1,T_n})}{2\sigma_{x,1}^2}\right)$$

$$\exp\left(\frac{\mu_{d,2} - \mu_{d,1}}{\sigma_{d,1}^2}u_{d,T_n}\right) \exp\left(\frac{\mu_{x,2} - \mu_{x,1}}{\sigma_{x,1}^2}u_{x,T_n}\right)$$

In this case, the reaction of stock returns to the unexpected part of the external signal is similar to the indirect effect of dividend news on stock return when $\sigma_{d,1} = \sigma_{d,2}$. Hence, the sign of the reaction depends on $\lambda_1 - \lambda_2$ which is negative if and only if $\gamma > 1$.

Proof of (d) When $\gamma = 1$, $\partial r_{T_n}^*/\partial u_{x,T_n} = 0$. Hence, stock returns do not react to the external signal.

$\square$

Proof of Proposition 9. When the external signal reveals the true state of the dividend growth process since the last announcement, unexpected return on the risky asset can be expressed as:

$$r_{T_n}^* = \frac{\lambda_1 - \lambda_2}{1 + \overline{\lambda}} \frac{u_{x,T_n}}{\mu_{x,1} - \mu_{x,2}} + u_{d,T_n}$$

and the reaction to the unexpected part of the external signal can be expressed as:

$$\partial r_{T_n}^*/\partial u_{x,T_n} = \frac{\lambda_1 - \lambda_2}{(1 + \overline{\lambda})(\mu_{x,1} - \mu_{x,2})}$$

Hence,

$$\left|\frac{\partial r_{T_n}^*}{\partial u_{x,T_n}}|_{\sigma_{x,1} = \sigma_{x,2} = 0}\right| > \left|\frac{\partial r_{T_n}^*}{\partial u_{x,T_n}}|_{\sigma_{x,1} = \sigma_{x,2} > 0}\right|$$

if

$$\frac{\sigma_{x,1}^2}{(\mu_{x,1} - \mu_{x,2})^2} > \begin{cases} f_5(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n}) & \text{if } \sigma_{d,1} \neq \sigma_{d,2} \\ f_6(u_{d,T_n}, u_{x,T_n}, \tilde{\pi}_{1,T_n}) & \text{if } \sigma_{d,1} = \sigma_{d,2} \end{cases}$$

Given that both f_5 and f_6 are bounded above by 0.25, a sufficient condition for the above inequalities to hold is $\sigma_{x,1} = \sigma_{x,2} > (\mu_{x,1} - \mu_{x,2})/2$.

$\square$

Table 1: Estimation Results from Markov Regime Switching Models for the Growth Rate of Real GDP

Parameters	Empirical Model 1	Empirical Model 2
μ_1	1.125384%***	1.122117%***
μ_2	-0.068479%	-0.125108%
σ_1	0.803234%***	0.826359%***
σ_2	0.924049%***	-
$q_{1,1}$	0.927855***	0.927869***
$q_{2,2}$	0.783603***	0.767423***

Note: The table presents the parameter estimates from Markov regime switching models for the log growth rate of quarterly real GDP between 1950 and 2008. Empirical Model 1 allows both the mean and standard deviation to be state-dependent ($\Delta \ln(GDP_t) = \mu_{S_t} + \sigma_{S_t}\varepsilon_t$). Empirical Model 2 restricts the standard deviation to be identical in both states ($\Delta \ln(GDP_t) = \mu_{S_t} + \sigma\varepsilon_t$). μ_1 and μ_2 are the quarterly growth rates of real GDP in states 1 and 2, respectively. σ_1 and σ_2 are the quarterly standard deviations of real GDP growth rates in states 1 and 2, respectively. $q_{1,1}$ and $q_{2,2}$ are the probabilities of staying in the same state for states 1 and 2, respectively. ***, **, * denote significance at 1% level, 5% and 10% levels, respectively.

Table 2: Daily Parameter Values

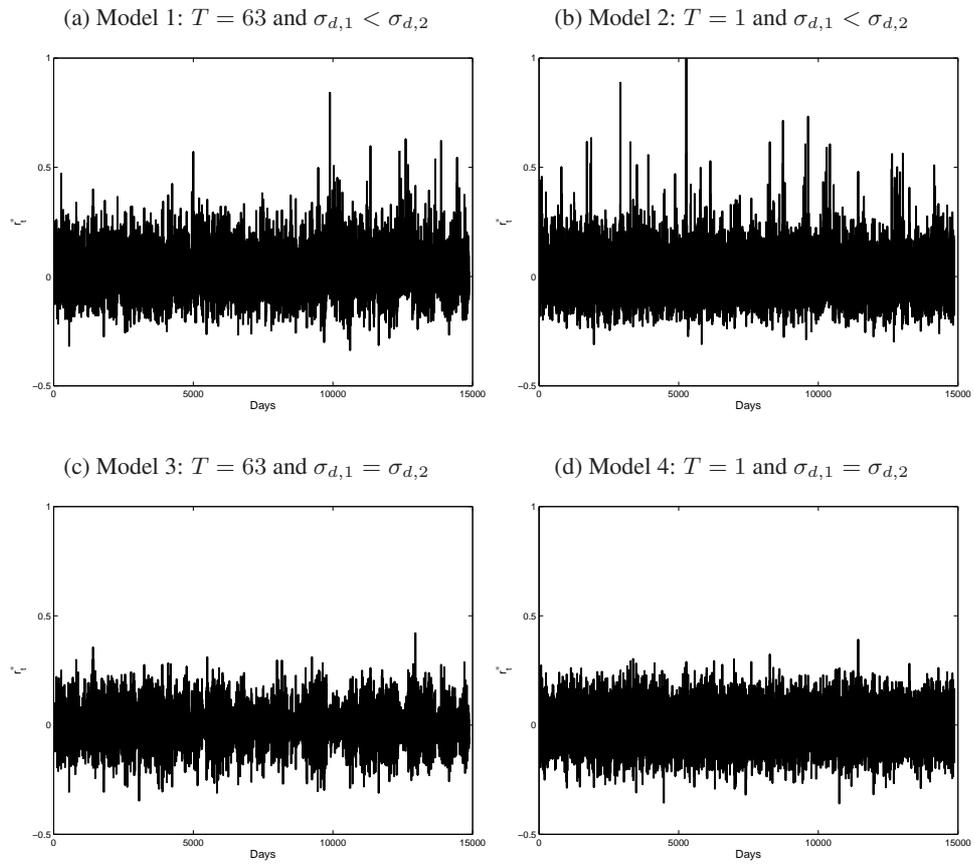| Parameters	Model 1	Model 2	Model 3	Model 4
$\mu_{d,1}$	0.017863%	0.017863%	0.017863%	0.017863%
$\mu_{d,2}$	-0.001087%	-0.001087%	-0.001087%	-0.001087%
$\sigma_{d,1}$	0.101198%	0.101198%	0.104111%	0.104111%
$\sigma_{d,2}$	0.116419%	0.116419%	0.104111%	0.104111%
$\mu_{x,1}$	0.017863%	0.017863%	0.017863%	0.017863%
$\mu_{x,2}$	-0.001087%	-0.001087%	-0.001087%	-0.001087%
$\sigma_{x,1}$	0.101198%	0.101198%	0.104111%	0.104111%
$\sigma_{x,2}$	0.116419%	0.116419%	0.104111%	0.104111%
$q_{1,1}$	0.927855	0.927855	0.927855	0.927855
$q_{2,2}$	0.783603	0.783603	0.783603	0.783603
T	63	1	63	1

Note: The table presents the calibrated daily parameter values for different models. $\mu_{d,1}$ and $\mu_{d,2}$ are the means of the daily dividend growth rate in states 1 and 2, respectively. Similarly, $\sigma_{d,1}$ and $\sigma_{d,2}$ are the standard deviations of the daily dividend growth rate in states 1 and 2, respectively. $\mu_{x,1}$, $\mu_{x,2}$, $\sigma_{x,1}$ and $\sigma_{x,2}$ are defined similarly for the external signal. $q_{1,1}$ and $q_{2,2}$ are the transition probabilities for states 1 and 2, respectively. T is the number of periods between announcement days.

Figure 1: Simulated Probability of the High Growth State ($\pi_{1,t}$)

(a) Model 1: $T = 63$ and $\sigma_{d,1} < \sigma_{d,2}$

(b) Model 2: $T = 1$ and $\sigma_{d,1} < \sigma_{d,2}$

(c) Model 3: $T = 63$ and $\sigma_{d,1} = \sigma_{d,2}$

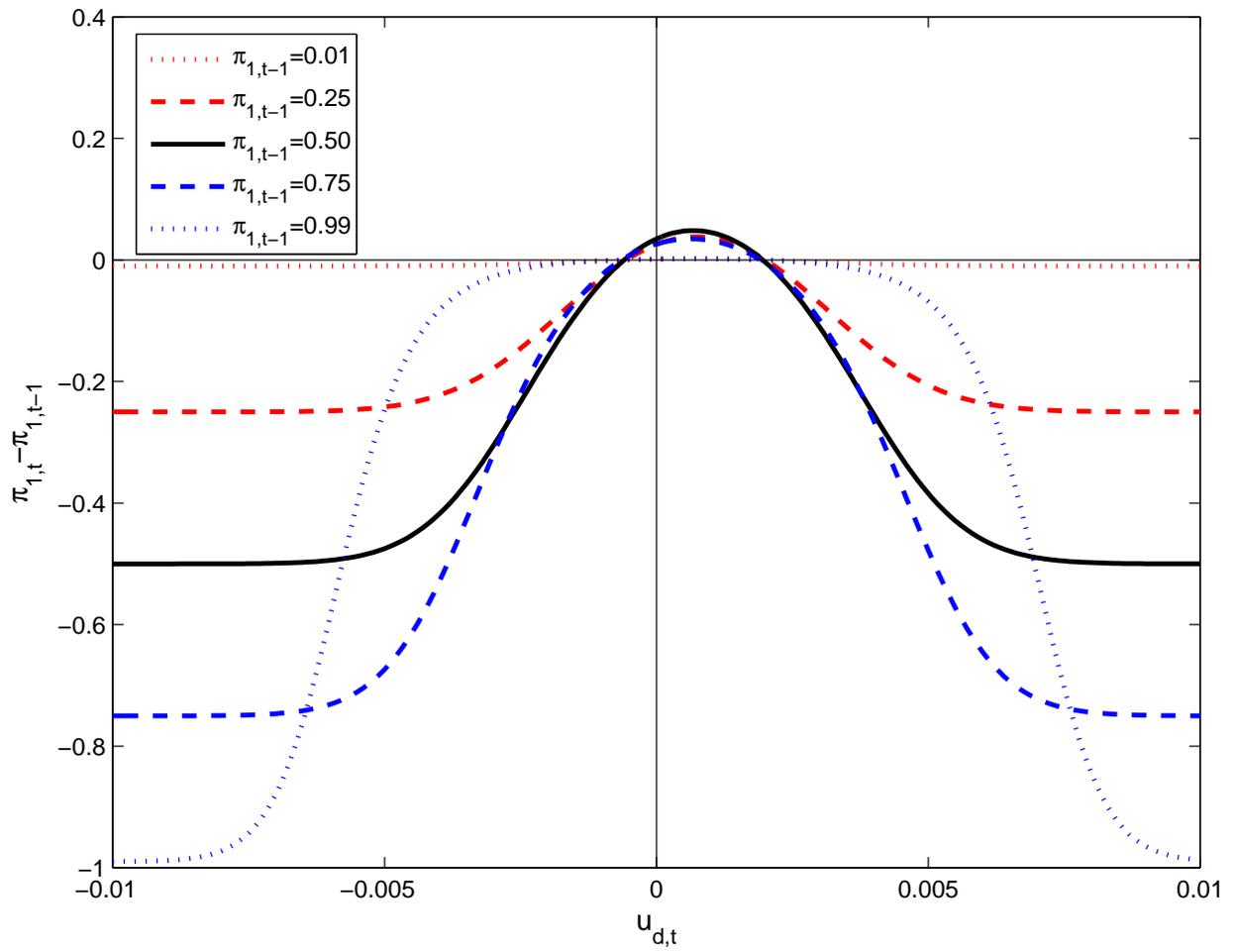(d) Model 4: $T = 1$ and $\sigma_{d,1} = \sigma_{d,2}$
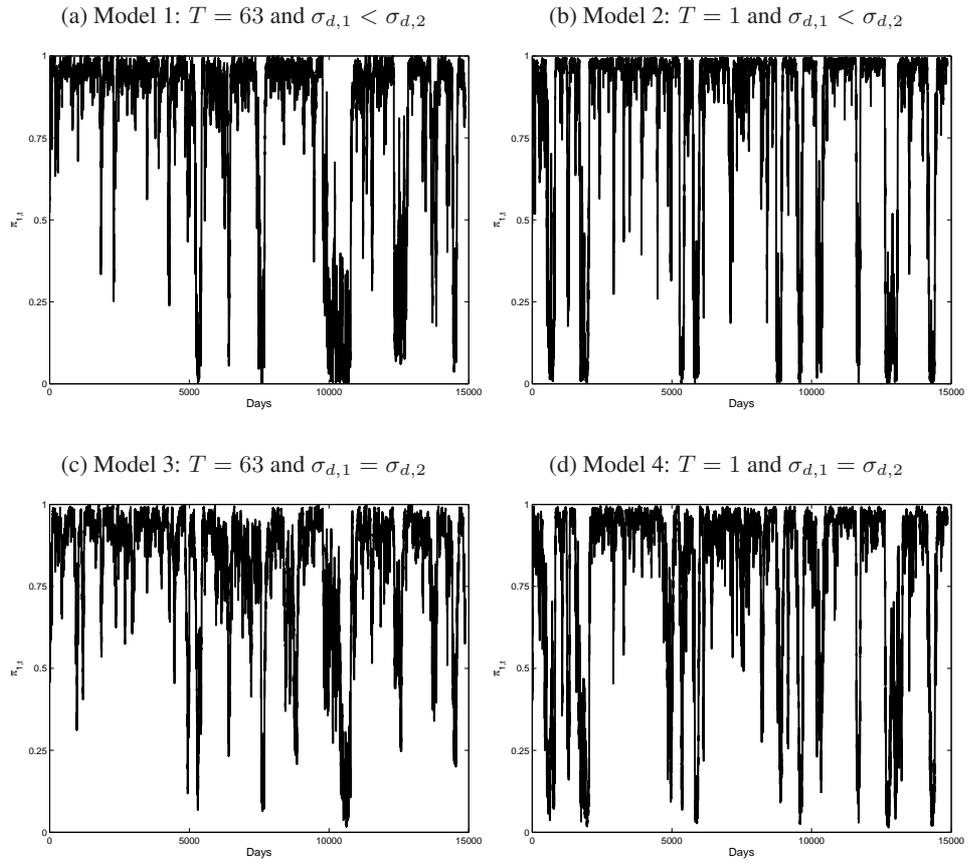
Note: This figure presents the daily evolution of the investor's beliefs about the high growth state of the dividend growth process.

Figure 2: Simulated Returns (r_t^*)

(a) Model 1: $T = 63$ and $\sigma_{d,1} < \sigma_{d,2}$

(b) Model 2: $T = 1$ and $\sigma_{d,1} < \sigma_{d,2}$

(c) Model 3: $T = 63$ and $\sigma_{d,1} = \sigma_{d,2}$

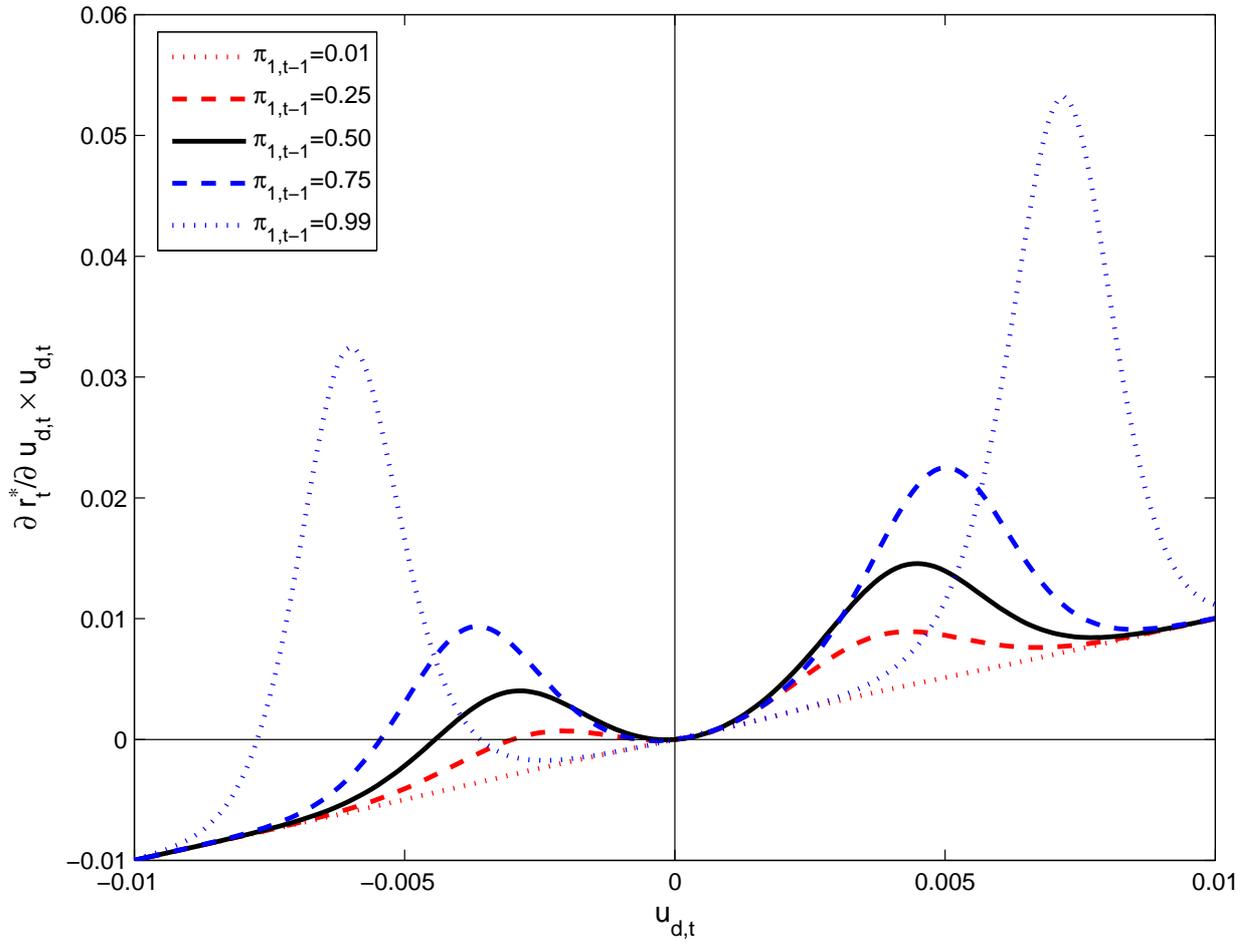(d) Model 4: $T = 1$ and $\sigma_{d,1} = \sigma_{d,2}$

Note: This figure presents the daily simulated returns.

Figure 3: The Effect of Dividend News on the Investor's Beliefs about the High Growth State ($\sigma_{d,1} < \sigma_{d,2}$)
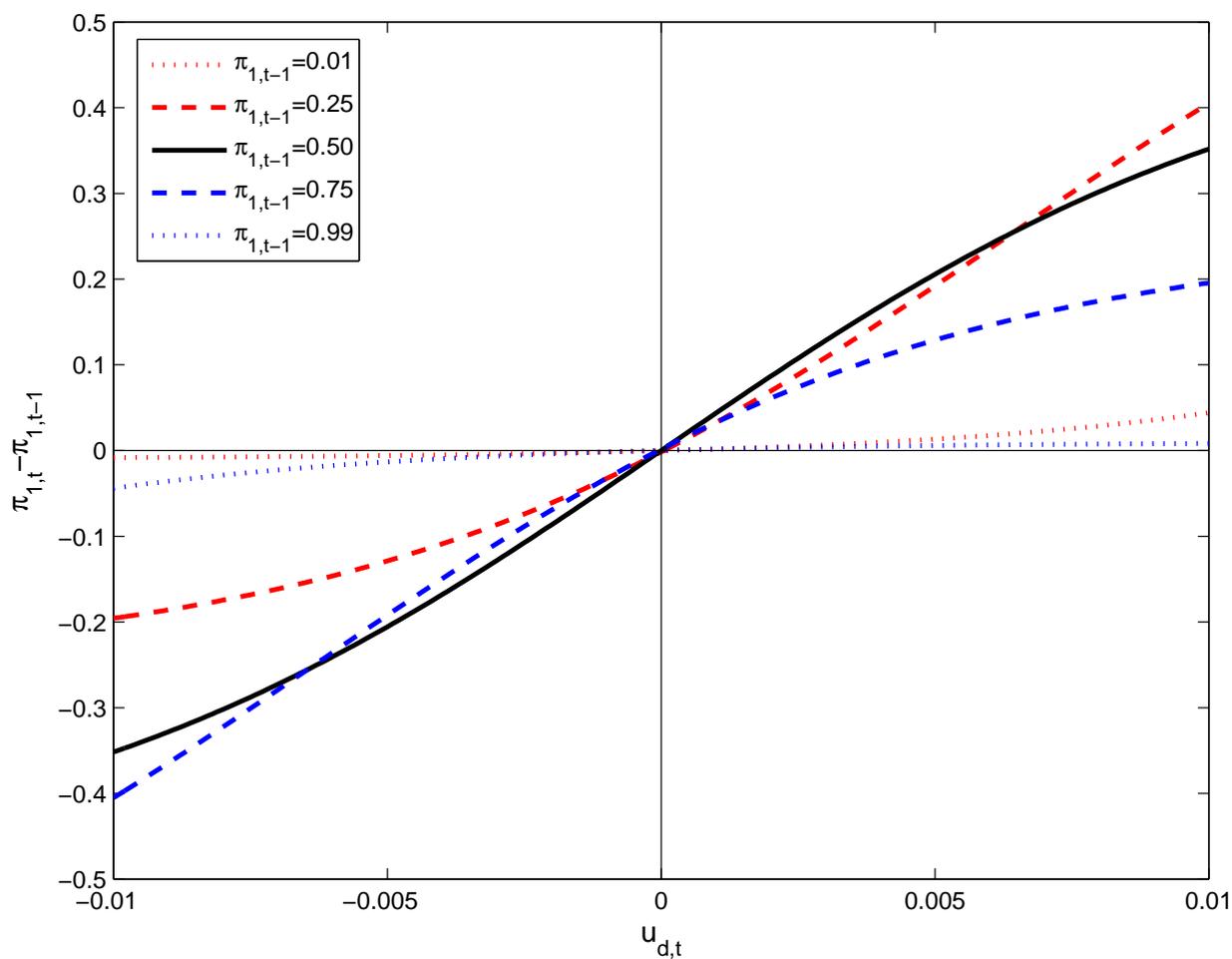

Note: This figure presents the effect of dividend news on the investor's beliefs about the high growth for different values of the investor's beliefs prior to observing the dividend news. The figure is generated using the calibrated daily parameter values for model 1 in Table 2.

Figure 4: The Reaction of Unexpected Returns to Dividend News ($\sigma_{d,1} < \sigma_{d,2}$)
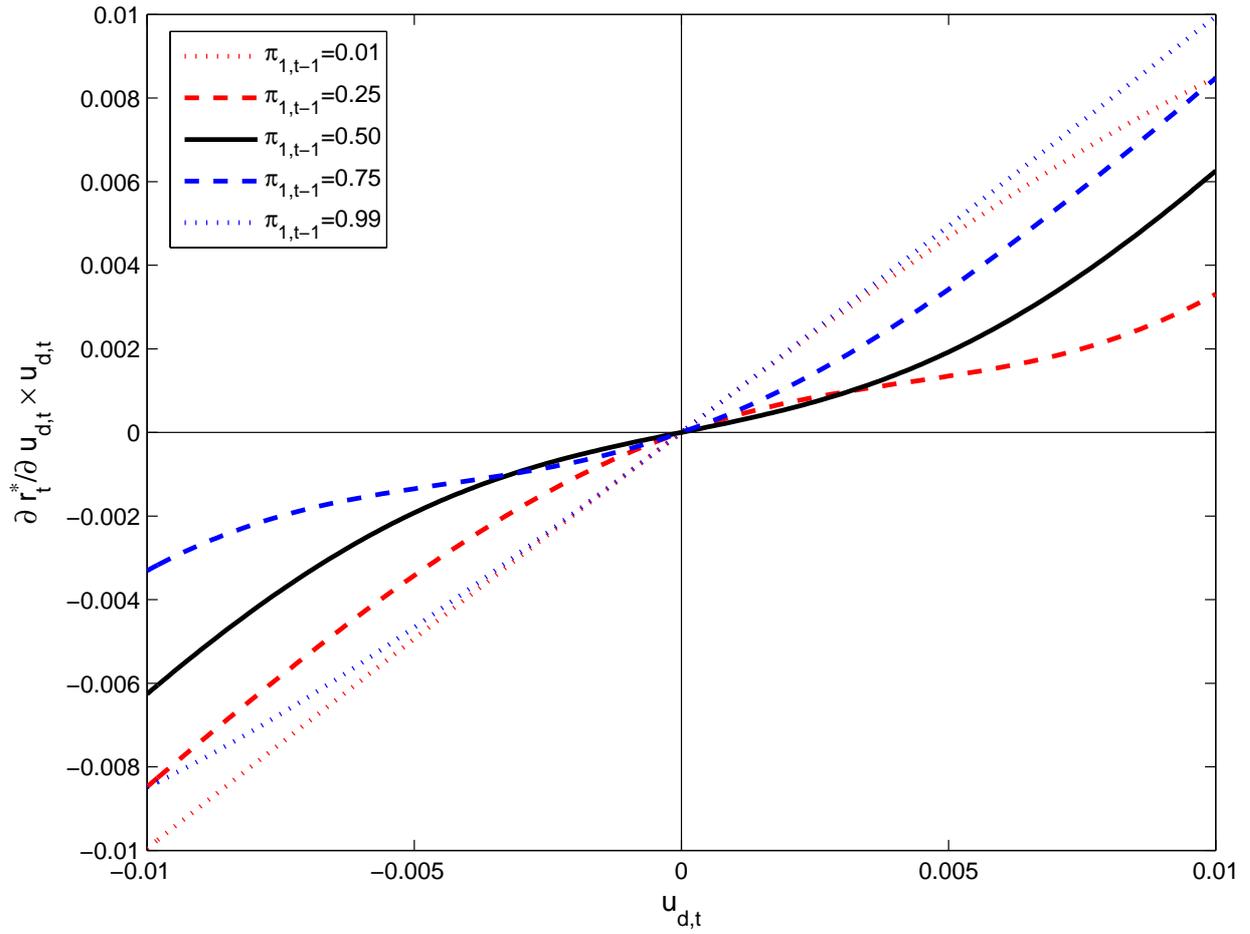

Note: This figure presents the reaction of unexpected returns as a function of dividend news for different values of the investor's beliefs about the high growth state prior to observing the dividend news. The figure is generated using the calibrated daily parameter values for model 1 in Table 2 and assuming that there are 31 periods until the next announcement.

Figure 5: The Effect of Dividend News on the Investor's Beliefs about the High Growth State ($\sigma_{d,1} = \sigma_{d,2}$)
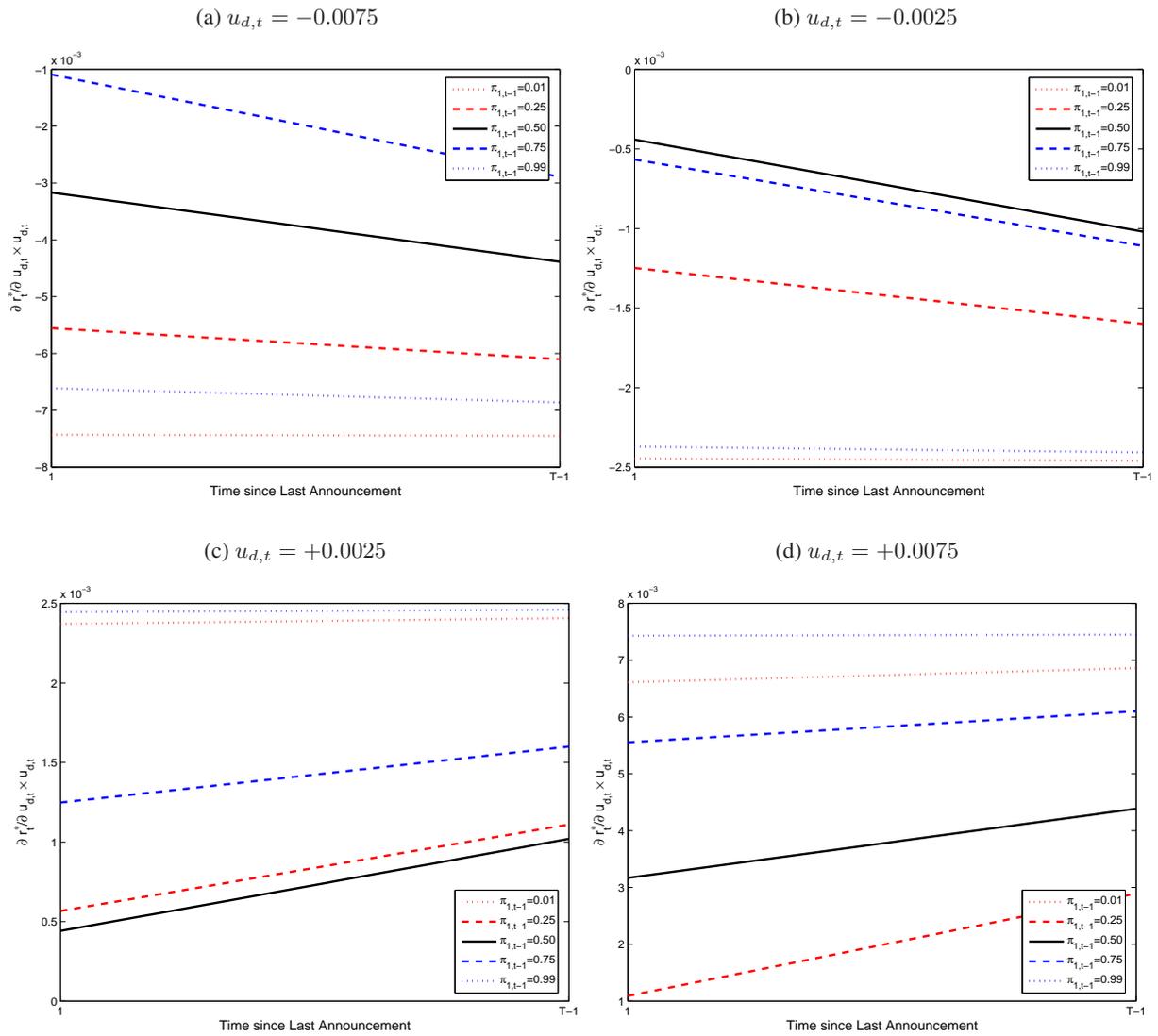

Note: This figure presents the reaction of unexpected returns as a function of dividend news for different values of the investor's beliefs about the high growth state prior to observing the dividend news. The figure is generated using the calibrated daily parameter values for model 3 in Table 2.

Figure 6: The Reaction of Unexpected Returns to Dividend News ($\sigma_{d,1} = \sigma_{d,2}$)
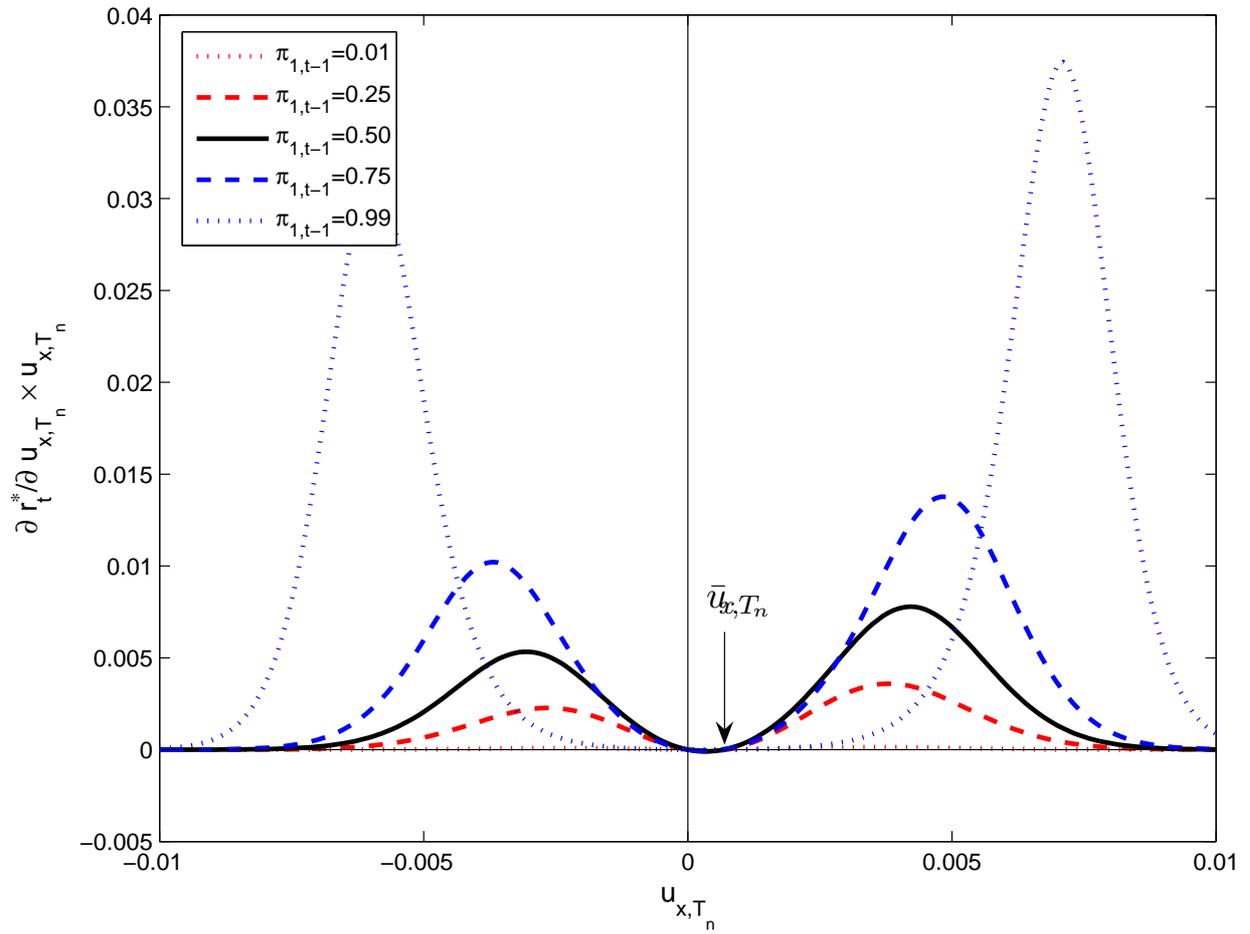

Note: This figure presents the reaction of unexpected returns as a function of dividend news for different values of the investor's beliefs about the high growth state prior to observing the dividend news. The figure is generated using the calibrated daily parameter values for model 3 in Table 2 and assuming that there are 31 periods until the next announcement.

Figure 7: The Reaction of Stock Returns to Dividend News as a Function of the Number of Periods until the Next Announcement

(a) $u_{d,t} = -0.0075$

(b) $u_{d,t} = -0.0025$

(c) $u_{d,t} = +0.0025$

(d) $u_{d,t} = +0.0075$

Note: This figure presents the reaction of unexpected returns to dividend news as a function of the number of periods until the next announcement for different values of dividend news and the investor's beliefs prior to observing dividend news. The figure is generated using the calibrated daily parameter values for model 3 in Table 2.

Figure 8: The Reaction of Stock Returns to the Unexpected Part of the External Signal

Note: This figure presents the reaction of unexpected returns to the unexpected part of the external signal as a function of the news variable itself for different values of the investor's beliefs prior to observing dividend news. The dividend news is assumed to be zero. The figure is generated using the calibrated daily parameter values for model 1 in Table 2.