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Bargaining with Linked Disagreement Points

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Bargaining with Linked Disagreement Points^{*}

Justin Leroux[†], Walid Marrouch[‡]

Résumé / Abstract

Dans un contexte de négociations bilatérales, nous adressons la question de l'inter-connexion des sujets de négociation dans un modèle coopératif à deux-sujets-deux-agents. Les axiomes que nous proposons insistent sur le rôle des points de menace. Une famille de solutions ressort de l'analyse : les solutions monotones à taux nets identiques. Chacune de ces solutions préconise une issue Pareto efficace de sorte que les gains relatifs de chaque agent sont les mêmes pour les deux sujets de négociation. De plus, ces règles récompensent les agents pour des améliorations de leurs pouvoirs de négociation. Nous discutons nos résultats à la lumière des négociations de commerce international et environnementales, qui sont souvent amenées à la table de négociations de manière liée.

Mots clés : Négociations Multi-sujets, inter-connexion des sujets, solutions axiomatiques, point de Menace.

In the context of bilateral bargaining, we deal with issue linkage by developing a two-issue-two-players cooperative bargaining model. The axioms we propose focus on the role of the disagreement points. A family of bargaining rule stands out: the monotonic equal net ratio solutions. These solutions point to Pareto efficient outcomes such that the relative gains for players are equal across issues and reward the players for improving their bargaining power over each issue. We discuss our results in light of international trade and environmental negotiations, which are often put on the bargaining table in a linked fashion.

Keywords: Multi-issue bargaining, issue linkages, axiomatic solutions, disagreement point.

Codes JEL: C78, Q56.

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1. INTRODUCTION

Many bargaining situations involve multiple issues at once. For instance, international trade and environmental negotiations have often been put on the bargaining table in a linked fashion. From Kyoto in 1997 to Cartagena in 2003, international environmental agreements were negotiated with the lurking spectre of trade (dis)agreements like the WTO.

Multiple-issue bargaining is a complex process where negotiations often break down and disagreement ensues where a non-cooperative outcome prevails. The non-cooperative outcomes resulting from disagreement are typically Pareto-inferior. Keeping with our motivating example from international trade, when trade negotiations break down the result is a tariff war leading to substantial welfare losses.

In order to propose Pareto-improving recommendations, we model issue linkages using a cooperative bargaining model with two players and two issues. We build upon the Nash cooperative bargaining framework for a single issue and consider the linkage between issues by expliciting the relationships between disagreement points and possible Pareto-improving outcomes. Indeed, stylized facts suggest that countries' negotiation powers over each specific issue (trade or environment) play an important role in shaping the final outcome of international negotiations since they act as threat points. For instance, trade wars and trade negotiations in the pre-NAFTA context were driven by the parties' disagreement points (Harrison and Rutström, 1991). One can compute the non-cooperative Nash equilibrium of the trade protection game between the US and Europe and evaluate welfare relative to it. The Nash equilibrium can then serve as a "natural measure of nation's bargaining strength when entering into international trade negotiations, [where] this bargaining strength is based on relative gains and losses in a credible disagreement outcome, which [they] interpret as the disagreement outcome" (p. 421). This bargaining mechanism was also observed within the genetically modified organisms dispute in the years 2003-2006, which pitted the USA, Canada and Argentina on one side and the European Union on the other and was settled in favor of the former group, where negotiation power over trade favored the winners³.

The traditional single-issue Nash bargaining framework describes a feasible bargaining set X and a corresponding disagreement point d^X . To focus on the role of the latter, we normalize the set X to be a bargaining 'cake' of size E^X (See Figure 1). And, when considering simultaneous bargaining over two issues, X and Y , we augment the Nash bargaining framework by linking the two bargaining problems. This allows us to compare the relative bargaining power of the players over each issue. In this context, we propose axioms dealing with issue linkages in the presence of non-normalized disagreement points. Using these axioms we then propose a family of bargaining solutions, which we call *monotonic equal net ratio solutions*, that satisfy a number of intuitive properties. These solutions point to Pareto efficient outcomes such that the relative gains for players are equal across issues. Moreover, they are monotonic in the sense that they reward the players for improving their bargaining power over each issue.

³http://www.wto.org/english/tratop_e/dispu_e/cases_e/ds291_e.htm

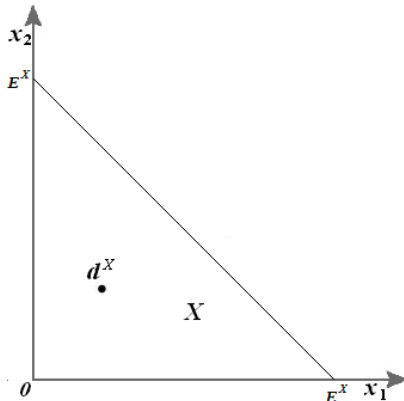


Figure 1. The feasible set X when normalized to unity

The paper unfolds as follows. Section 2 discusses the related literature and the theoretical contribution of our paper. Section 3 presents the two-issue bilateral bargaining model and the results. Section 4 concludes.

2. RELATED LITERATURE

The literature on bargaining is made up of two strands: one follows a non-cooperative approach à la Rubinstein (1982) and the other follows a cooperative or axiomatic approach à la Nash (1950). Our work belongs to the latter. This literature includes a number of attempts at modeling multiple-issue bargaining, which have mostly ignored the importance of changes in the disagreement points and the resulting spillovers between issues. An important general observation is that cooperative models have remained silent about the role played by disagreement points, which are typically normalized to zero. Meanwhile, stylized facts suggest that these points are pivotal in negotiations since they acts as threat points⁴. Moreover, the idea of concessions exchange that arises in non-cooperative models is also neglected in multi-issue cooperative models.

Most of the theoretical work on multiple-issue bargaining uses two-player models and generalizes existing solutions from single-issue bargaining. This is done by proposing new axioms that generalize or replace the classical ones found in the literature on single issue bargaining. When agents' preferences are represented by a utility function it is assumed that utilities are additive across issues. Ponsatí and Watson (1997) generalize the Nash solution and the symmetric utilitarian solution. Peters (1986), generalizes Kalai's (1977) extended family of proportional solutions and Harsanyi and Selten's (1972) extended family of non-symmetric Nash solutions. Another approach has been more recently proposed by Mármol and Ponsatí (2008) and uses maximin and leximin preferences when information about preference is limited or when those preferences do not admit a utility representation. This work follows Bossert *et al.* (1996) and Bossert and Peters (2001) by modeling the global bargaining problem as the Cartesian product of classical (single issue) bargaining problems.

Finally, under multiple issue bargaining three possible families of axioms exist. First, there are axioms that are related to changes in the bargaining set. These

⁴See Harrison and Rutström (1991).

appear in Peters (1985, 1986), and Ponsatí and Watson (1997) among others, where disagreement points are normalized to zero. Second, there are axioms related to changes in the population on which the literature has been mostly silent since bilateral bargaining is assumed.⁵ Finally, axioms related to changes in the disagreement points have so far not been considered under multiple issue bargaining. Here, we explore the relevance of these disagreement points, which constitutes a contribution of our model. In this regard, it should be noted that Thomson (1987, 1994) and Chun and Thomson (1990, 1992) introduce axioms related to the disagreement point but for single-issue bargaining only. More specifically, we propose a number of axioms related to issue linkages when the disagreement points are taken into consideration.

In this context, it is very important to draw the distinction between separate and linked Pareto efficiency. Classical axioms that are applied to single-issue problems are based on the idea of separate/local Pareto efficiency, where it is enough for the solution to be on the Pareto frontier of each set to be efficient⁶. In a more general context, Peters (1985) and Ponsatí and Watson (1997) discuss the idea of global efficiency in the context of multi-issue bargaining. They argue that efficiency demands that no possible gains from cooperation are lost, which means that each local solution must belong to Pareto frontier of the sum of the local sets. Given our normalization to linear bargaining frontiers to focus on the role of the disagreement points, any solution located on the Pareto frontiers of both sets, X and Y , maximizes the sum of players' utilities across issues and is thus Pareto efficient.

3. THE MODEL

Two agents, $i = 1, 2$, bargain simultaneously over two issues, X and Y . Successful bargaining on each issue consists in dividing total payoff, denoted E^X for issue X (resp. E^Y for issue Y), between the two agents. Failure to achieve an agreement in both issues results in agents falling back to their disagreement payoffs; we denote $d_i^X > 0$ (resp. d_i^Y) agent i 's payoff on issue X (resp. issue Y). We define $D^X = d_1^X + d_2^X \leq E^X$ and $D^Y = d_1^Y + d_2^Y \leq E^Y$, and denote $\Delta_\alpha = \{z \in \mathbb{R}_+^2 \mid 0 < z_1 + z_2 \leq \alpha\}$ for all $\alpha \in \mathbb{R}_+$. It will be useful to denote $d_i = (d_i^X, d_i^Y)$ and $D_i = d_i^X + d_i^Y$ agent i 's issue-wise and overall bargaining powers, respectively. The pair $(d; E) = (d^X, d^Y; E^X, E^Y)$ constitutes a *linked bargaining problem*. We denote by \mathcal{B} the class of linked bargaining problems.

A *linked bargaining solution* (or *solution*), $f : \mathcal{B} \rightarrow \mathbb{R}_+^2 \times \mathbb{R}_+^2$ maps each bargaining problem to a payoff vector, $f(d; E) = (x, y) \geq (d^X, d^Y)$ such that $x_1 + x_2 = E^X$ and $y_1 + y_2 = E^Y$. We denote by $S^X = \frac{d_2^X}{d_1^X}$ and $S^Y = \frac{d_2^Y}{d_1^Y}$ the agents' relative bargaining powers over issue X and issue Y , respectively. For instance, if S^X is very small (close to zero) and S^Y is larger, then player 1 has a strong advantage over issue X but player 2 has a better bargaining power over issue Y (See Figure 2). Lastly, we denote by $F_1(d; E) = x_1 + y_1$ and $F_2(d; E) = x_2 + y_2$ the overall payoffs of agent 1 and agent 2, respectively.

⁵See Thomson and Lensberg (1989) for single issue models with n-agents.

⁶This is the case when both issues are seen separately. The idea of global efficiency only makes sense when linkage is considered.

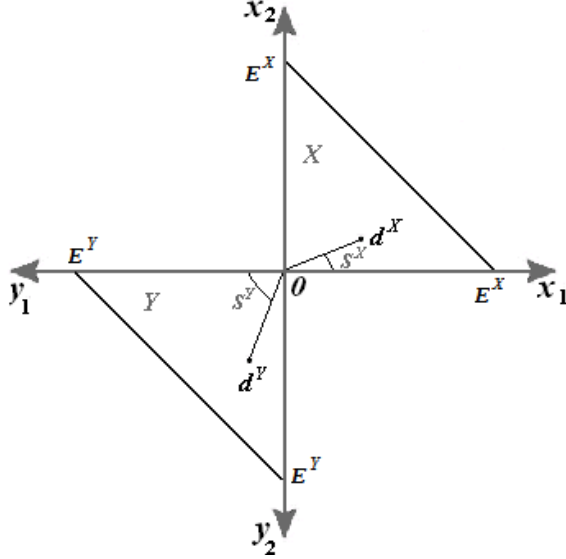


Figure 2. The two-issue bargaining problem

We introduce a number of properties that we deem desirable in a solution. The first axiom stipulates that if the relative bargaining power is the same across issues, the sharing rule should respect these relative strengths.

AXIOM 1. "Uniformity" $S^X = S^Y \implies \frac{x_2}{x_1} = \frac{y_2}{y_1} = S^X$.

Keeping with the spirit of impartiality, we argue that a solution should not behave differently across issues. More precisely once bargaining power has been taken into account, via the agents' issue-wise disagreement points, the solution treats both agents and issues symmetrically.

AXIOM 2. "Issue neutrality" $\frac{y_1 - d_1^Y}{x_1 - d_1^X} = \frac{y_2 - d_2^Y}{x_2 - d_2^X}$

Axiom 2 is an axiom of neutrality vis-a-vis the issues. For example, if $\frac{y_1 - d_1^Y}{x_1 - d_1^X} > \frac{y_2 - d_2^Y}{x_2 - d_2^X}$, the solution confers an *a priori* advantage to player 1 over player 2 in issue Y, which we view as undesirable. Therefore, this condition must hold at equality to ensure neutrality with respect to issues once bargaining powers are controlled for.

Next, we ask that a solution be consistent: achieving an agreement in several steps rather than in a single round should not affect the outcome.

AXIOM 3. "Composition" $f(d; E') = f(f(d; E); E')$ for any $E' \geq E$.

The next requirement is one of smoothness, which ensures that the solution be not wildly sensitive to changes in the bargaining powers:

AXIOM 4. "Smoothness" f is continuously differentiable in d .

We may now state our main result:

THEOREM 1. A solution satisfies axioms 1-4 if and only if:

$$a : \Delta_{E^X} \times \Delta_{E^Y} \rightarrow \mathbb{R}_+ \cup \{+\infty\}$$

$$(d^X, d^Y) \mapsto \frac{f_2^X - d_2^X}{f_1^X - d_1^X},$$

is a continuously differentiable function such that:

- i) $a(d^X, d^Y) = \frac{d_2^X}{d_1^X}$ if $\frac{d_2^X}{d_1^X} = \frac{d_2^Y}{d_1^Y}$, and
- ii) $a(d'^X, d'^Y) = a(d^X, d^Y)$ for all $(d'^X, d'^Y) \in (d^X, x) \times (d^Y, y)$.

Proof:

The reader can readily check sufficiency, but the proof of necessity, proviso ii), requires several steps. Let f be a bargaining solution satisfying axioms 1 through 4.

Claim 1: For all $d' = (d'^X, d'^Y) \in [d^X, x] \times [d^Y, y]$, the following holds:

- (a) $f(d'^X, d'^Y; E) = (x, y)$;
- (b) $f(d^X, d'^Y; E) = (x, y)$; and,
- (c) $f(d'; E) = (x, y)$.

Let $(d; E) \in \mathcal{B}$ and let $(d'^X, d'^Y) \in [d^X, x] \times [d^Y, y]$. We first prove point (a). By Composition, $y = f^Y(f(d; D'^X, E^Y); E) = f^Y(d; D'^X, E^Y)$ because the coordinates of the latter term already sum up to E^Y . By Issue Neutrality, $\overrightarrow{d^X f^X(d; D'^X, E^Y)}$ is colinear to $\overrightarrow{d^Y f^Y(d; D'^X, E^Y)}$ which, together with the fact that $\overrightarrow{f^Y(d; D'^X, E^Y)} = \underline{y}$ and the fact that $\overrightarrow{d^X x}$ and $\overrightarrow{d^Y y}$ are colinear, implies that $\overrightarrow{d^X f^X(d; D'^X, E^Y)}$ and $\overrightarrow{d^X x}$ are colinear. Lastly, the fact that the coordinates of $f^X(d; D'^X, E^Y)$ sum up to D'^X implies that $f^X(d; D'^X, E^Y) = d'^X$. Finally, by the Composition axiom, $f^X(f(d; D'^X, E^Y); E) = x$, yielding the result.

Note that, by assumption on f , $x \geq d'_X$ and $y' \geq d'_Y$. It follows that the rays (d^X, x) and (d^Y, y) are positively sloped, implying $a(d^X, d^Y) \in \mathbb{R}_+ \cup \{+\infty\}$. By Smoothness, a is continuously differentiable.

An analogous argument leads to $f(d^X, d'^Y; E^X) = (x, y)$. Finally applying (a) to the latter expression leads to $f(d'^X, d'^Y; E^X) = f(d^X, d'^Y; E^X) = (x, y)$, proving point (c).

Claim 2 For all $d'^X \in (d^X, x) \cap \Delta_{E^X}$ and all $d'^Y \in (d^Y, y) \cap \Delta_{E^Y}$, the following holds:

- (a) $f(d'^X, d'^Y; E) = (x, y)$;
- (b) $f(d^X, d'^Y; E) = (x, y)$; and
- (c) $f(d'; E) = (x, y)$.

We first prove statement (a). Let $(d; E) \in \mathcal{B}$. The line (d^X, x) divides Δ_{E^X} into two convex regions, $\Delta_{E^X}^+$ and $\Delta_{E^X}^-$ such that $\Delta_{E^X}^+ \cap \Delta_{E^X}^- = (d^X, x) \cap \Delta_{E^X}$. (See Figure 3)

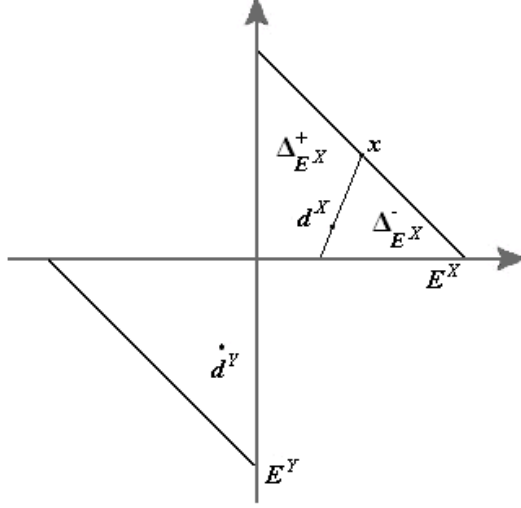


Figure 3.

Let $d^X \in (d^X, x) \cap \Delta_{E^X}$ and suppose $d^X \notin [d^X, x]$ (the case not covered by Claim 1). We shall show that $f(\cdot, d^Y; E)$ is stable on each of the subsets $\Delta_{E^X}^+$ and $\Delta_{E^X}^-$. Indeed, suppose there existed $\hat{d}^X \in \Delta_{E^X}^- \setminus \Delta_{E^X}^+$ such that $f(\hat{d}^X, d^Y; E) \in \Delta_{E^X}^+ \setminus \Delta_{E^X}^-$. For any $\lambda \in [0, 1]$ denote $d^{\lambda, X} = \lambda d^X + (1 - \lambda)\hat{d}^X$. By Continuity of f in d , $\lim_{\lambda \rightarrow 1} f^X(d^{\lambda, X}, d^Y; E) = x \in \Delta_{E^X}^-$. Yet, by Composition, it must be that $[d^{\lambda, X}, f^X(d^{\lambda, X}, d^Y; E)] \cap [d^X, x] = \emptyset$ for any $\lambda < 1$. Otherwise, there would exist some $\bar{d} \in [d^{\lambda, X}, f^X(d^{\lambda, X}, d^Y; E)] \cap [d^X, x]$, for which Claim 1 would imply $f(\bar{d}, d^Y, E) = x$ and, by Composition, we would have $f(\hat{d}^X, d^Y; E) = (f(\bar{d}, d^Y; E) = x$, contradicting the fact that $f(\hat{d}^X, d^Y; E) \in \Delta_{E^X}^+ \setminus \Delta_{E^X}^-$. Finally, because $[d^{\lambda, X}, f^X(d^{\lambda, X}, d^Y; E)] \cap [d^X, x] = \emptyset$ for any $\lambda < 1$, the convexity of $\Delta_{E^X}^-$ implies that $Cl\{f^X(d^{\lambda, X}, d^Y; E) | 0 \leq \lambda < 1\} \cap \{x\} = \emptyset$, where Cl is the closure operator, implying that $\lim_{\lambda \rightarrow 1} f^X(d^{\lambda, X}, d^Y; E) \neq x$, a contradiction.

Statement (b) is proved in a similar fashion as statement (a), and (c) is obtained by combining (a) and (b), as was done for Claim 1 ■

Theorem 1 provides the general structure of linked bargaining solutions satisfying the elementary axioms 1 through 4. In addition, one may find it desirable that the improvement of an agent's bargaining power in either issue should not hurt her overall payoff:

AXIOM 5. "Monotonicity" For all $(d; E) \in \mathcal{B}$,

$$\begin{cases} d'_i \geq d_i \\ d'_j = d_j \end{cases} \implies x'_i + y'_i \geq x_i + y_i$$

where $(x', y') = f(d'; E)$.

THEOREM 2. A solution satisfies axioms 1–5 if and only if:

$$\begin{aligned} \frac{\partial a}{\partial d_1^X} &\leq \frac{a}{F_1 - D_1} \\ \frac{\partial a}{\partial d_2^X} &\geq -\frac{1}{F_1 - D_1} \\ \frac{\partial a}{\partial d_1^Y} &\leq \frac{a}{F_1 - D_1} \\ \frac{\partial a}{\partial d_2^Y} &\geq -\frac{1}{F_1 - D_1} \end{aligned} ,$$

in addition to the conditions of Theorem 1.

Proof:

We show the first inequality. Let f satisfy axioms 1-5. Let $(d; E) \in \mathcal{B}$, and $\varepsilon > 0$ such that $(d_1^X + \varepsilon, d_2^X, d_1^Y, d_2^Y; E) \in \mathcal{B}$. Denote $a' = a(d_1^X + \varepsilon, d_2^X, d_1^Y, d_2^Y)$ and $f' = f(d_1^X + \varepsilon, d_2^X, d_1^Y, d_2^Y; E)$. By definition of $a(\cdot)$, $f_2'^X - d_2^X = a' \times (f_1'^X - d_1^X - \varepsilon)$ and $f_2''^Y - d_2^Y = a' \times (f_1'^Y - d_1^Y)$. Adding both equalities yields $F_2' - D_2 = a'(F_1' - D_1 - \varepsilon)$. The same operation applied to the original bargaining problem yields $F_2 - D_2 = a(F_1 - D_1)$. Subtracting the latter equality from the previous one yields $F_2' - F_2 = a(F_1 - D_1) - a'(F_1' - D_1 - \varepsilon)$. Using the fact that $F_1' + F_2' = F_1 + F_2 = E^X + E^Y$ leads to:

$$\begin{aligned} F_1' - F_1 &= a(F_1 - F_1') + (a - a')F_1' - (a - a')D_1 + a'\varepsilon \\ (1 + a')\frac{F_1' - F_1}{\varepsilon} &= a' + \frac{(a - a')}{\varepsilon}(F_1 - D_1). \end{aligned}$$

Taking the limit towards $\varepsilon = 0$ leads to:

$$(1 + a)\frac{\partial F_1}{\partial d_1^X} = a - \frac{\partial a}{\partial d_1^X}(F_1 - D_1).$$

It follows from this last expression that imposing monotonicity ($\frac{\partial F_1}{\partial d_1^X} \geq 0$) amounts to requiring $a - \frac{\partial a}{\partial d_1^X}(F_1 - D_1) \geq 0$, as was to be proven. The other inequalities are proven similarly. ■

Several solutions stand out among the ones satisfying axioms 1-5. For instance, any rule taking a convex combination of the relative bargaining powers in each issue, such that $a(d^X, d^Y) = \alpha S^X + (1 - \alpha)S^Y$ for $\alpha \in [0, 1]$, belongs to this class. We call this the class of *monotonic equal net ratio solutions* (See Figure. 4). This class consists of a continuum of solutions of which an extreme case stands out. The single-issue dictatorship solution requires bargaining gains be allocated according to the relative bargaining powers over issue X (S^X) only. In other words, the bargaining power S^Y over issue Y does not matter (See Figure. 5).

More refined solutions exist, where the weights of each issue may depend on the *absolute* bargaining powers of each agents, such as the natural one consisting in defining $a(d^X, d^Y) = \frac{D_2}{D_1}$, which amounts to defining the convex combination as $\alpha(d^X, d^Y) = \frac{d_1^X}{D_1}$. Graphically, this solution links both disagreement points d^X and d^Y , and locates the solution outcome on the Pareto frontier of each bargaining set (Figure 6). We call it the *balanced compromise solution* since it combines the bargaining powers over both issues: it takes the global bargaining power ratio between both players to determine the outcome.

It is noteworthy that the degrees of freedom granted by the class of monotonic equal net ratio solutions is "horizontal", in the sense that linkage is not a question of how strongly the two issues are linked, but a question of how much weight is given to the relative bargaining powers in each issue. In particular, a solution treating both issues separately would not belong to the class. This can be seen with the (single issue) Nash bargaining solution, for instance, which would correspond to $a \equiv 1$ at all profiles, thus violating Axiom 1. In other words, "no linkage" is not a special case of linkage.

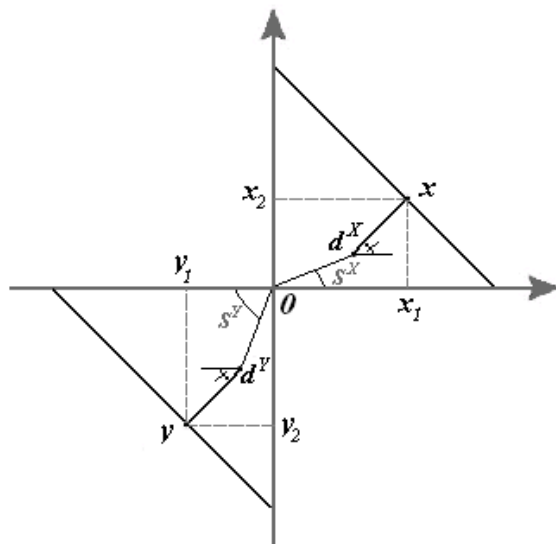


Figure 4. The monotonic equal net ratio solution where $\alpha = \frac{1}{2}$.

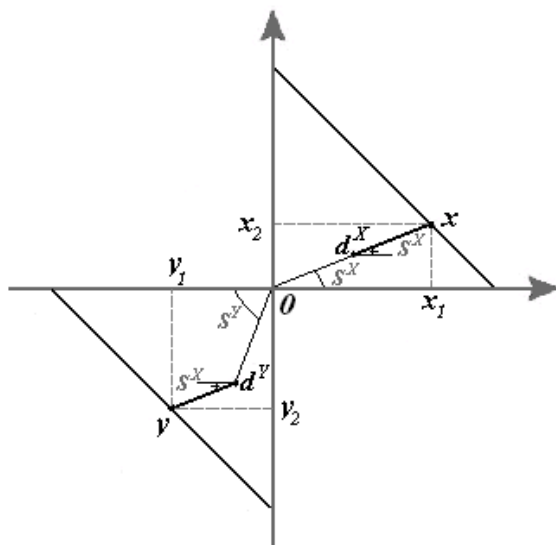


Figure 5. Single-issue dictatorship (issue X).

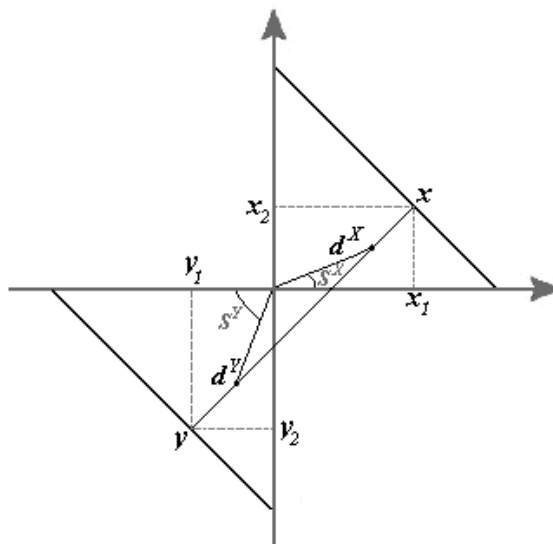


Figure 6. The balanced compromise solution.

4. CONCLUDING REMARKS

Stylized facts suggest that in international law, issues pertaining to commerce and environment are usually dealt with in a conflicting manner. This has been a trend since 1972 when the United Nations Environment Program (UNEP) was established. That year was the year of the United Nations' conference on the environment held in Stockholm, and is now seen as a turning point in international environmental awareness. The conflicting nature of international environmental law stems from the fact that trade and environmental concerns carry trade-offs. The GATT (WTO after 1995) is in general against unilateral discriminatory measures, as per Article XX. However, if these measures are required by an international environmental agreement (IEA) then the issue becomes more problematic because simultaneous negotiations are needed. Indeed, the class of *monotonic equal net ratio solutions*, which takes a convex combination of the relative bargaining powers in each issue, seem to reflect the way simultaneous bilateral bargaining over trade and environment has been taking place. In this example, if environmental measures are not in conflict with WTO's Article XX then a solution in the spirit of the *single-issue dictatorship* solution requires bargaining gains be allocated according to the relative bargaining powers over the trade issue only (See, e.g. the tuna case pitting Mexico versus USA, and the Shrimp case pitting the USA versus Malaysia, Philippines, and India). Otherwise, a convex combination of relative powers over both issues will determine the final outcome as was the case with the Genetically Modified Organisms (GMOs) conflict in 2003 between the USA, Canada, Argentina on one hand and the EU on the other. During this conflict, an IEA -Cartagena protocol on bio-safety- was used to challenge WTO rules, in other words a convex combination of trade and environment negotiation powers shaped the final solution of the conflict.

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