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Mobility, Taxation and Welfare

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Abstract:
Income mobility is often thought to equalize permanent incomes and thereby to improve social welfare. The welfare analysis of mobility often fails, however, to account for the cost of the variability of periodic incomes around permanent incomes. This paper assesses the net welfare benefit of mobility by assuming both a social aversion to inequality in permanent incomes and an individual aversion to variability in periodic incomes. The paper further investigates the combined (and comparative) impact of mobility and of the tax system (another presumed income equalizer) on the dynamics of income across time and on the inequality of income across individuals. Using panel data, we find that Canada's tax system limits significantly the redistributive impact of mobility while also lowering considerably the cost of income variability. The permanent income equalizing effect of taxes can reach up to 23 percent of mean income at the higher values of inequality aversion that we use. Globally, the net social welfare effect of both mobility and taxation is (almost always) positive and substantial, often amounting to around 30 percent of mean income. For all choices of parameter values, the tax effect exceeds by far the net effect of mobility on inequality and social welfare.

Keywords: Mobility, social welfare, risk, income variability, inequality, permanent income

JEL Classification: D31, D63, H24

1 Introduction

The paper is concerned with the welfare impact of income dynamics across time and across individuals. Income mobility has at least two potential social welfare effects. The first effect is to make the distribution of permanent incomes potentially more equal than the distribution of periodic incomes (periodic incomes being cross-sectional incomes). This is usually seen to increase social welfare. Milton Friedman argued fifty years ago, for instance, that a society with a rigid income distribution where everyone remains in the same position year after year can almost certainly be declared "worst" than a mobile society with identical cross-sectional inequality — see page 12 for a full quote. The second effect is to generate variability at the individual level, because of the time variability of individual incomes that mobility induces. If individuals would prefer their incomes to be distributed as equally as possible across time (because they are risk averse), then this aspect of mobility will reduce social welfare. Gittleman and Joyce (1996) argue in this respect that mobility may make it difficult to retain one's position in the distribution, thus making mobility less desirable.

To address these questions, the paper first follows the spirit of "mobility as equalizer" introduced by Shorrocks (1978) and generalized by Maasoumi and Zandvakili (1999). Methodologically and conceptually, this is different from the use of mobility indices based on transition matrices — see Shorrocks (1976), Atkinson, Bourguignon, and Morrisson (1992), Dardanoni (1993), Gottschalk and Spolaore (2002a), and Klevmarken (2004) for more on this. Other approaches to measuring mobility are suggested and surveyed by Shorrocks (1993), Fields and Ok (1996), Fields and Ok (1999), Maasoumi (1998) and Fields (2010). In the context of mobility as equalizer of incomes, inequality in permanent incomes is compared to the average inequality of periodic incomes.[1] The lower the level of permanent income inequality, the higher is income mobility deemed to be.

This framework is enhanced to take into account the cost of the variability in the distribution of periodic incomes if individuals are averse to income variability across time. Aversion to temporal income variability is a natural assumption in economics. The effect of such an aversion has, however, curiously not featured prominently in the analysis of mobility — some of the rare exceptions include Gottschalk and Spolaore (2002b) and Salas and Rabadán (1998). Mobility is usually thought to be necessarily beneficial in social welfare terms; the income

[1] See, among many others, Chakravarty, Dutta, and Weymark (1985), Atkinson, Bourguignon, and Morrisson (1992), Jarvis and Jenkins (1998), Salas and Rabadan (1998), Trede (1998), Salas and Rabadán (1998), Benabou and Ok (2001), Creedy and Wilhelm (2002), and Beenstock (2004).

variability costs are typically not taken into account in the social assessment of the effects of mobility.

The combination of these two facets of mobility provides a unified framework to trade off the advantage of mobility as "equalizer" across individuals and the cost of mobility as "disequalizer" across time. Such a framework can also be useful for the purposes of evaluating the social welfare effects of tax systems. There is indeed an interesting analogy between the effects of mobility and taxation. In the words of Benabou and Ok (2001), "[j]ust like a tax scheme maps pre-tax incomes into post-tax incomes, a mobility process maps initial incomes into expected future incomes, or more generally into expected levels of intertemporal welfare. The extent to which the terminal distribution is equalized compared to the initial one is then precisely measured by the degree to which the mapping is progressive" (p.1). Equality of permanent incomes is increased by a progressive system; the time variability of periodic incomes around permanent incomes is decreased by tax progressivity. The precise quantitative welfare impact of progressivity nevertheless depends on the distribution of pre-tax incomes and the structure of the tax system. It also depends on the social evaluator's aversion to variability across time and to inequality across individuals.

Looking jointly at taxation and mobility helps assess both mobility's impact and the tax system's impact on intertemporal social welfare. How much is the usefulness of mobility as a longer-term equalizer diminished by the presence of a progressive tax system? How much is the cost of mobility as a temporal disequalizer reduced by a progressive tax system? Is the welfare benefit of tax progressivity reduced or increased by the presence of mobility? How do the inequality-reducing benefits and the variability-reducing benefits of tax progressivity compare? How do the welfare benefits of mobility and taxation compare? These are some of the original questions that this paper seeks to address, both through the provision of a measurement framework and empirically.

The main empirical results (obtained from recent Canadian panel data) are instructive. The cost of mobility in pre-tax income ranges from roughly 2 to 11 percent of pre-tax mean income for reasonable parameter values of aversion to variability. The tax system reduces considerably that variability cost, halving it for many parameter values. Mobility also contributes much less to the equalization of permanent post-tax incomes than of permanent pre-tax incomes. The permanent income equalizing effect of taxes can reach up to 23 percent of mean income at the higher values of inequality aversion that we use. The global result is that the net social welfare effect of both mobility and taxation is (almost always) positive and usually very significant, often amounting to around 30 percent of mean income.

The rest of the paper is organized as follows. Section 2 presents formally the measurement of individual and social welfare. Section 3 shows how the two aspects of mobility influence social welfare. It also derives measures of the impact of taxation on mobility, examines the effects of taxation on social welfare, and decomposes the overall effects of taxation and mobility on social welfare. Section 4 provides two methods for correcting statistical biases in the estimation of the welfare cost of the variability of periodic incomes. Section 5 applies the methodology to Canadian panel data. Section 6 concludes briefly. The Appendix presents proofs of some of the results and contains additional methodological precisions.

2 Measurement

Let the variables x and y stand for pre- and post-tax income, respectively, which we assume to be positive. The "tax system", which maps x into y, is a shorthand for the "net tax and transfer system". Transfers are also allowed, and the tax net of transfers can be negative (we then have a net positive transfer). Let $F_{x,y}(\cdot,\cdot)$ be the joint distribution function of pre- and post-tax incomes. The marginal distributions are denoted as $F_x(\cdot)$ and $F_y(\cdot)$ for x and y, respectively, and can be obtained as

$$F_x(v) = \int_{y=0}^{y=\infty} \int_{x=0}^{x=v} dF_{x,y}(x,y) \tag{1}$$

and similarly for $F_y(\cdot)$. To focus on the distributive effects of the tax system, we assume that the mean of x and y have been normalized to 100, so that $\int v dF_x(v) = \int v dF_y(v) = 100$. We further index individuals by their characteristics ω, of which the distribution function is $F_\omega(\cdot)$. $F_{s|\omega}(\cdot)$ is the distribution of income s (with s equal to x or to y) conditional on an individual having characteristics ω (we call this individual ω for short).

2.1 Individual welfare

A useful tool throughout the analysis will be that of "permanent income". For individual ω, it is given by

$$\overline{s}(\omega) = \int z dF_{s|\omega}(z) \tag{2}$$

with $s = x$ and $s = y$ for pre- and post-tax income respectively. Note here that we are not discounting future incomes and/or utilities. Discounting the future

could readily be done but at some expositional cost. Individuals are assumed to be averse to income variability over time. Their utility at period t, which we call periodic utility, is given by $U_\epsilon(s)$,

$$U_\epsilon(s) = \begin{cases} \frac{s^{1-\epsilon}}{1-\epsilon}, & \text{for} \quad \epsilon \neq 1, \\ \ln(s), & \text{for} \quad \epsilon = 1, \end{cases} \tag{3}$$

where $\epsilon \geq 0$ is a parameter of relative risk aversion. $U_\epsilon(s)$ is a standard utility function in the literature; although its constant level of inequality/risk aversion simplifies exposition, other choices of functional forms are possible in our paper's framework. Denote the inverse of the utility function $u = U_\epsilon(s)$ by

$$U_\epsilon^{-1}(u) = \begin{cases} (1-\epsilon)u^{\frac{1}{1-\epsilon}}, & \text{when} \quad \epsilon \neq 1, \\ \exp(u), & \text{when} \quad \epsilon = 1. \end{cases} \tag{4}$$

In the manner of Atkinson (1970) for the measurement of social welfare and inequality, let $\chi(\omega;\epsilon)$ be the pre-tax "equally distributed equivalent income" (EDEI) for individual ω. $\chi(\omega;\epsilon)$ is the value of pre-tax income that, if enjoyed by individual ω at each period of his lifetime, would yield him the same average utility over time as that generated by the distribution of his periodic incomes.[2] Using (3) and (4), $\chi(\omega;\epsilon)$ is thus given by

$$\chi(\omega;\epsilon) = U_\epsilon^{-1}\left(\int U_\epsilon(z)dF_{x|\omega}(z)\right). \tag{5}$$

For $\epsilon = 0$, $\chi(\omega;0)$ equals $\overline{x}(\omega)$. $\chi(\omega;\epsilon)$ is in general lower than $\overline{x}(\omega)$ because of ω's aversion to periodic income variability. The difference $\overline{x}(\omega) - \chi(\omega;\epsilon)$ can be interpreted as a risk premium that ω would be willing to pay to eliminate the variability in his periodic incomes.

Figure 1 helps understand this. Values of periodic utility $U_\epsilon(x)$ are shown at two different pre-tax incomes $x_1(\omega)$ and $x_2(\omega)$ for a single individual ω. Average pre-tax income across time is given by $\overline{x}(\omega)$. Average utility across time is $\int U_\epsilon(z)dF_{x|\omega}(z) = 0.5\left(U_\epsilon(x_1(\omega)) + U_\epsilon(x_2(\omega))\right) = U_\epsilon(\chi(\omega;\epsilon))$. $\chi(\omega;\epsilon)$ is thus the EDEI of individual ω. The difference between $\overline{x}(\omega)$ and $\chi(\omega;\epsilon)$ on the horizontal axis is the welfare cost of ω's income variability, measured in units of average income.

Define $\gamma(\omega;\epsilon)$ analogously as the post-tax EDEI for individual ω:

[2] Salas and Rabadan (1998) follow this approach to decompose overall intertemporal inequality into between- and within-household contributions.

$$\gamma(\omega;\epsilon) = U_{\epsilon}^{-1}\left(\int U_{\epsilon}(z)dF_{y|\omega}(z)\right). \tag{6}$$

Further, let $\xi(\omega;\epsilon)$ be the post-tax EDEI for individual ω, estimated by applying the variability in pre-tax incomes on permanent post-tax incomes, $\overline{y}(\omega)$:

$$\xi(\omega;\epsilon) = \overline{y}(\omega)\left(\frac{\chi(\omega;\epsilon)}{\overline{x}(\omega)}\right). \tag{7}$$

Seen differently, $\xi(\omega;\epsilon)$ gives the EDEI of the distribution of x scaled by $\overline{y}(\omega)/\overline{x}(\omega)$, that is, by forcing post-tax incomes y to display the same periodic inequality as pre-tax incomes x. The greater the progressivity of the tax system, the greater the gap between $\gamma(\omega;\epsilon)$ and $\xi(\omega;\epsilon)$.[3] For a proportional tax system, we have that $\xi(\omega;\epsilon) = \gamma(\omega;\epsilon)$.

2.2 Social welfare

We measure social welfare over the distribution of individual EDEI, that is, over the distribution of permanent incomes corrected for the cost of periodic income variability — we refer to this as "permanent welfare". Thus, we define $W_{\chi(\epsilon)}(\rho)$ as the EDEI of the distribution of pre-tax permanent welfare $\chi(\omega;\epsilon)$,

$$W_{\chi(\epsilon)}(\rho) = U_{\rho}^{-1}\left(\int U_{\rho}(\chi(\omega;\epsilon))dF_{\Omega}(\omega)\right). \tag{8}$$

$W_{\gamma(\epsilon)}(\rho)$ and $W_{\xi(\epsilon)}(\rho)$ are defined accordingly. ρ is the aversion to between-individual inequality. If $W_{\chi(\epsilon)}(\rho)$ were enjoyed by all, it would generate the same social welfare as that generated by the distribution of $\chi(\omega;\epsilon)$.

Let $W_{\overline{s}}(\rho)$ then be the EDEI of permanent incomes for the pre-tax ($\overline{s} = \overline{x}$) and post-tax ($\overline{s} = \overline{y}$) distributions

$$W_{\overline{s}}(\rho) = U_{\rho}^{-1}\left(\int U_{\rho}(\overline{s}(\omega))dF_{\Omega}(\omega)\right). \tag{9}$$

Let also $W_s(\rho)$ be the EDEI of periodic incomes, both for pre-tax ($s = x$) and post-tax ($s = y$) incomes:

$$W_s(\rho) = U_{\rho}^{-1}\left(\int U_{\rho}(z)dF_s(z)\right). \tag{10}$$

[3] See the Appendix for a proof of this statement.

$W_s(\rho)$ can be interpreted as social welfare imposing time anonymity on social evaluation. Time anonymity says that a social evaluator should show indifference regarding the dependence of temporal incomes: how periodic incomes are allocated intertemporally across individuals is not an input into social evaluation in the presence of time anonymity. The one-period temporal distributions of incomes are sufficient for social evaluation purposes; it is not necessary to know the joint distribution (and the dependence) of these temporal incomes for these purposes.

Note that if $\epsilon = \rho$, that is, if aversion to variability and inequality are the same, then $W_x(\rho) = W_{\chi(\epsilon)}(\rho)$ and $W_y(\rho) = W_{\gamma(\epsilon)}(\rho)$. The above notation concerning the distribution of individual and social welfare is summarized in Table 1.

3 The impact of mobility and taxation on social welfare

3.1 Impact of mobility on social welfare

A general belief is that mobility serves as an equalizer of permanent incomes. The stochastic nature of mobility is, however, also a source of periodic income variability. As a result, we can think of mobility as having two potential effects on social welfare:

1. it generates an uncertainty cost if individuals are averse to income variability across time;

2. it makes the distribution of permanent welfare more equal than the distribution of periodic welfare.

To quantify these two effects, define

$$M_x(\epsilon;\rho) = W_{\chi(\epsilon)}(\rho) - W_x(\rho) \tag{11}$$

as the net effect of mobility on pre-tax social welfare. The first term on the right-hand-side of (11) is welfare corrected for the cost of income variability and for the benefit of permanent welfare equalization. The second term is social welfare without such adjustments. This can be decomposed into two components:

$$
\begin{aligned}
M_x(\epsilon;\rho) \quad &\equiv W_{\chi(\epsilon)}(\rho) - W_x(\rho) \\
&= \underbrace{W_{\chi(\epsilon)}(\rho) - W_{\overline{x}}(\rho)}_{M_x^1(\epsilon;\rho)\leq 0} + \underbrace{W_{\overline{x}}(\rho) - W_x(\rho)}_{M_x^2(\rho)\geq 0}, \qquad (12)
\end{aligned}
$$

where

- $M_x^1(\epsilon;\rho)$ is the effect of periodic income variability, which is negative for $\epsilon > 0$,
- and $M_x^2(\rho)$ is the effect of the equalization of permanent pre-tax welfare, which is positive for $\rho > 0$.

The mobility effects on post-tax social welfare are obtained by replacing x, χ and $\overline{x}$ by y, γ and $\overline{y}$ in (11) and (12). Whenever $\epsilon = \rho$, that is, when across-time inequality aversion is exactly offset by across-individual inequality aversion, the negative variability effect is exactly counterbalanced by the positive effect of the equalization of permanent welfare, so that $M_x^1(\epsilon;\epsilon) = -M_x^2(\epsilon)$ and $M_x(\epsilon;\epsilon) = M_y(\epsilon;\epsilon) = 0$. This is because it then does not matter for social evaluation purposes whether the variability in the distribution of periodic incomes is variability across individuals or variability across time. The social welfare cost of both is the same. Any reduction in permanent welfare inequality induced by the effect of mobility is exactly canceled out from a social welfare perspective by the income variability that this introduces.[4]

When $\rho > \epsilon > 0$, we have that $M_x(\epsilon;\rho) > 0$ and the welfare effect of the equalization of permanent welfare dominates that of the cost of income variability. Mobility-accounting social welfare $W_{\chi(\epsilon)}(\rho)$ is thus larger than time-anonymous social welfare $W_x(\rho)$. An increase in periodic income variability (which decreases $W_x(\rho)$) can then yet improve social welfare $W_{\chi(\epsilon)}(\rho)$ if it induces a sufficient increase in permanent welfare equality.

A reverse reasoning applies when $0 < \rho < \epsilon$. We then have that $M_x(\epsilon;\rho) < 0$ and that the welfare cost of income variability dominates the beneficial effect that income mobility has on the equalization of permanent welfare. Mobility-accounting social welfare $W_{\chi(\epsilon)}(\rho)$ is then lower than time-anonymous social welfare $W_x(\rho)$. An increase in periodic income variability can still improve social

[4]The proof of this can be found in the Appendix.

welfare $W_{\chi(\epsilon)}(\rho)$, but it will then need to generate a sufficiently large fall in permanent welfare inequality.

To see this better, consider Figure 2. Vectors of two-period pre-tax ($\mathbf{x} = (x_1, x_2)$) and post-tax ($\mathbf{y} = (y_1, y_2)$) incomes are shown for two individuals, $\omega = a$ and $\omega = b$. Overall mean income is the same pre-tax and post-tax since $\overline{x}(a) + \overline{x}(b) = \overline{y}(a) + \overline{y}(b)$. There is no pre-tax income variability: $x_1(\omega) = x_2(\omega) = \overline{x}(\omega) = \chi(\omega; \epsilon)$ for both individuals. But there is pre-tax inequality in permanent welfare since $\chi(a; \epsilon) < \chi(b; \epsilon)$. Because $\overline{y}(a) = \overline{y}(b)$ and $\gamma(a; \epsilon) = \gamma(a; \epsilon)$, the tax system equalizes post-tax permanent welfare perfectly, but it also introduces temporal variability since we now have that $y_1(\omega) \neq y_2(\omega)$ and that $\overline{y}(\omega) > \gamma(\omega; \epsilon)$ for all $\epsilon > 0$. The question then is: does the tax system increase social welfare?

The answer depends on the social evaluator's comparative aversion to variability across time and to inequality across individuals. The pre-tax and post-tax distributions of periodic incomes are the same in Figure 2. Hence, a time anonymous social evaluation would judge the pre-tax and post-tax distributions as welfare equivalent. If time anonymity is removed, then the relative social evaluation of pre- and post-tax incomes will depend on the social evaluator's comparative aversion to income variability across time and to welfare inequality across individuals. Indifference towards variability across time will necessarily make the post-tax distribution better: $W_{\chi(\epsilon=0)}(\rho) < W_{\gamma(\epsilon=0)}(\rho) = \gamma(a; \epsilon = 0) = \gamma(b; \epsilon = 0)$ for all inequality aversion parameter values $\rho > 0$. Indifference towards inequality across individuals will conversely make the pre-tax distribution better.

For common values of ϵ and ρ, the pre-tax and post-tax distributions in Figure 2 have the same social welfare level, and that level is also the same as for time-anonymous social welfare. The social welfare benefit of the reduction in permanent welfare inequality that redistribution introduces is then exactly canceled out by the social welfare cost of greater income variability that redistribution introduces in Figure 2. A greater aversion to inequality makes the post-tax distribution preferable: $W_{\chi(\epsilon)}(\rho) < W_{\gamma(\epsilon)}(\rho)$ for any $\epsilon < \rho$. The converse is true for $\epsilon > \rho$: the pre-tax distribution is then better.

Figure 2 also shows the role of time anonymity in social evaluations. Say that an alternative post-tax distribution is given by $\mathbf{y}^*(\omega)$. Mean post-tax income is unchanged. Post-tax inequality is still nil but $\mathbf{y}^*$ now displays more temporal variability than $\mathbf{y}$. $\mathbf{y}^*$ is thus worse than $\mathbf{y}$. For a sufficiently large aversion to inequality across individuals, the distribution of $\mathbf{y}^*(\omega)$ will, however, be judged better than the distribution of pre-tax incomes. This is so even though the periodic pre-tax distributions of incomes are judged individually better (for both periods,

since they display less inequality) than the periodic post-tax distributions of incomes. Whether this worsening in the periodic income distributions is judged welfare improving depends not only on the aversion to inequality in permanent welfare, but also on the aversion to income variability across time. The greater the aversion to welfare inequality, the more likely will the distribution of $\mathbf{y}^*(\omega)$ be judged better than the distribution of $\mathbf{x}(\omega)$.

3.2 Marginal rates of substitution

We can further use Figure 2 to illustrate the marginal rates of substitution (MRS) of incomes across time and across individuals. MRS show by how much one income needs to be changed to keep constant social welfare W when another income changes. Consider an intra-individual-a change in incomes $y_1(a)$ and $y_2(a)$. The relevant MRS is given by:

$$\left.\frac{\partial y_2(a)}{\partial y_1(a)}\right|_{W \text{ constant}} = -\left(\frac{y_2(a)}{y_1(a)}\right)^{\epsilon}. \tag{13}$$

If, as in Figure 2, $y_2(a) > y_1(a)$, (13) says that we can sacrifice more than one dollar of $y_2(a)$ when $y_1(a)$ increases by one dollar and still maintain social welfare constant. The larger the ratio of $y_2(a)$ to $y_1(a)$, and the larger the value of ϵ, the greater in absolute value is the MRS.

We can also consider a common-time-period between-individual change in incomes involving variations in $x_2(b)$ and $y_2^*(a)$, say. This example is chosen because although $\mathbf{x}(b)$ can certainly be judged better than $\mathbf{y}^*(a)$, making a transfer from $x_2(b)$ to $y_2^*(a)$ increases income variability for both individuals. It also increases income inequality in period 2 (and leaves unchanged income inequality in period 1). It is thus conceivable that this transfer from a better-off to a lesser-off individual may decrease overall social welfare in the presence of aversion to income variability. The corresponding MRS is given by:

$$\left.\frac{\partial x_2(b)}{\partial y_2^*(a)}\right|_{W \text{ constant}} = -\underbrace{\left(\frac{x_2(b)}{y_2^*(a)}\right)^{\epsilon}}_{\text{income variability}} \cdot \underbrace{\left(\frac{\chi(b)}{\gamma^*(a)}\right)^{\rho-\epsilon}}_{\text{welfare inequality}}. \tag{14}$$

Equation (14) provides an explicit tradeoff between the cost of increasing income variability (and/or periodic income inequality), and the benefit of increasing welfare equality. Increasing $y_2^*(a)$ by one dollar and decreasing $x_2(b)$ by the same one

dollar increases income variability: the first term on the right-hand-side of (14) says that this tends to decrease social welfare (since $x_2(b) < y_2^*(a)$). Increasing $y_2^*(a)$ by one dollar and decreasing $x_2(b)$ by the same one dollar increases welfare equality; since $\chi(b) > \gamma^*(a)$, the first term on the right-hand-side of equation (14) also says that this tends to increase social welfare whenever $\rho - \epsilon > 0$. The net effect depends on the relative importance of the two terms. If $\rho = 0$, (14) yields

$$\left.\frac{\partial x_2(b)}{\partial y_2^*(a)}\right|_{W \text{ constant}} = -\left[\frac{x_2(b)}{\chi(b)} \Big/ \frac{y_2^*(a)}{\gamma^*(a)}\right]^{\epsilon} > -1. \tag{15}$$

The transfer from $x_2(b)$ to $y_2^*(a)$ then has a solely welfare decreasing effect of increasing income variability and periodic inequality. If $\rho = \epsilon$, (14) yields

$$\left.\frac{\partial x_2(b)}{\partial y_2^*(a)}\right|_{W \text{ constant}} = -\left(\frac{x_2(b)}{y_2^*(a)}\right)^{\epsilon} > -1, \tag{16}$$

and we are back to (13). The transfer from $x_2(b)$ to $y_2^*(a)$ again decreases welfare since it increases time-anonymous income inequality. For a sufficiently large value of $\rho - \epsilon$, however, the equality-enhancing effect can be judged to be sufficiently strong to offset the variability-increasing effect, so that time-non-anonymous social welfare increases. This is necessarily the case whenever $\rho > \epsilon$. Note that this condition is implied by the following view that a mobile society (with given cross-sectional inequality) should be deemed better than an immobile society:

> "Consider two societies that have the same distribution of annual income. In one there is great mobility and change so that the position of particular families in the income hierarchy varies widely from year to year. In the other, there is great rigidity so that each family stays in the same position year after year. Clearly, in any meaningful sense, the second would be the more unequal society." (Friedman 1962)

3.3 Impact of taxation on variability

The cost of variability in post-tax income, namely, $M_y^1(\epsilon;\rho)$, can also be decomposed as:

$$M_y^1(\epsilon,\rho) = \underbrace{W_{\gamma(\epsilon)}(\rho) - W_{\overline{y}}(\rho)} \tag{17}$$

(<0: cost of income variability in post-tax incomes)

$$= \underbrace{W_{\xi(\epsilon)}(\rho) - W_{\overline{y}}(\rho)} \tag{18}$$

(<0: cost of pre-tax income variability)

$$+ \underbrace{W_{\gamma(\epsilon)}(\rho) - W_{\xi(\epsilon)}(\rho).} \tag{19}$$

>0: (fall in cost of income variability due to tax system)

Expression (17) is the difference between post-tax social welfare and post-tax social welfare without income variability. It is thus the welfare cost of post-tax income variability. Expression (18) is the difference between post-tax social welfare with pre-tax income variability and post-tax social welfare without income variability. It is thus the welfare cost of pre-tax income variability, as measured on the distribution of post-tax incomes. Expression (19) is the difference between post-tax social welfare and post-tax social welfare with pre-tax income variability. This is thus the social welfare benefit of the reduction of income variability induced by the tax system. The sum of (18) and (19) is the cost of income variability in post-tax incomes. This also says that if the cost of income variability across time is reduced by a tax system, then the post-tax distribution will show a lower welfare cost of income variability than the pre-tax one.

3.4 Impact of taxation on the social welfare effect of mobility

As mentioned above, both aspects of mobility can be expected to be improved by a progressive tax system:

1. an equalizing tax system reduces the variability of periodic incomes around permanent incomes;

2. a redistributive tax system makes permanent post-tax welfare more equal than permanent pre-tax welfare.

Using (11), let the impact of the tax system on mobility be expressed as

$$\Delta M(\epsilon;\rho) \equiv M_y(\epsilon;\rho) - M_x(\epsilon;\rho) = \Delta M^1(\epsilon;\rho) + \Delta M^2(\rho), \tag{20}$$

where $\Delta M^1(\epsilon;\rho) = M_y^1(\epsilon;\rho) - M_x^1(\epsilon;\rho)$ and $\Delta M^2(\rho) = M_y^2(\rho) - M_x^2(\rho)$. This is the difference in the welfare impact of mobility before and after tax. $\Delta M^1(\epsilon;\rho)$ shows the welfare effect of taxation on income variability, and $\Delta M^2(\rho)$ shows the welfare effect of taxation on permanent welfare inequality. As argued above, a progressive tax system is expected to strengthen both of these aspects of mobility.

3.5 Overall effect of taxation on social welfare

Recall that $W_{\chi(\epsilon)}(\rho)$ is the EDEI of individual pre-tax welfare levels, that is, the EDEI of the distribution of pre-tax individual permanent incomes corrected for the cost of income variability. $W_{\gamma(\epsilon)}(\rho)$ is analogously defined as the EDEI of individual post-tax welfare levels. Let then $\Gamma(\epsilon;\rho) = W_{\gamma(\epsilon)}(\rho) - W_{\chi(\epsilon)}(\rho)$ be the total effect of taxation on such EDEI welfare. Note that $\Gamma(\epsilon = 0;\rho)$ is the welfare effect of taxation on the distribution of permanent incomes. We then have:

Proposition 1 *With degrees ϵ and ρ of aversion to income variability across time and to welfare inequality across individuals, respectively, the social welfare effect of the tax system on the distribution of individual welfare is given by*

$$\begin{aligned} \Gamma(\epsilon;\rho) &= (W_y(\rho) - W_x(\rho)) + \Delta M^1(\epsilon;\rho) + \Delta M^2(\rho) && (21) \\ &= \Gamma(0;\rho) + \Delta M^1(\epsilon;\rho). && (22) \end{aligned}$$

See appendix. ■

Let us consider the different components of (21) and (22). $W_y(\rho) - W_x(\rho)$ is the social welfare effect of the tax system on periodic incomes. This is the difference in anonymous social welfare; it fails to take into account both 1) the permanent welfare equalization effect and 2) and the income variability reduction effect of the tax system. This is corrected in (21) by $\Delta M^1(\epsilon;\rho)$ and $\Delta M^2(\rho)$. $\Delta M^1(\epsilon;\rho)$ is the effect of the tax system on income variability, and $\Delta M^2(\rho)$ is the effect of the tax system on permanent income inequality. By (21), the extent to which the tax system is judged welfare improving will then depend upon its ability 1) to equalize the distribution of periodic incomes; 2) to reduce the cost of income variability compared to the no-tax baseline; and 3) to reduce the cost of permanent welfare inequality, also relative to the no-tax scenario. Result (22) alternatively says that the net impact of the tax system on social welfare will depend on its ability to equalize permanent income, $\Gamma(0;\rho)$, and to reduce the pre-tax cost of welfare variability, $\Delta M^1(\epsilon;\rho)$.

3.6 Combined effect of mobility and taxation on social welfare

We can also think of the combined effect of mobility and taxation on social welfare. This is the social welfare difference between the distribution of periodic pre-tax incomes, F_x, and the distribution of post-tax incomes adjusted for the cost of variability. Let $\Lambda(\epsilon;\rho) = W_{\gamma(\epsilon)}(\rho) - W_x(\rho)$ be this combined effect of mobility and taxation on social welfare.

Corollary 2 *With degrees ϵ and ρ of aversion to income variability across time and to welfare inequality across individuals, respectively, the combined effect of mobility and the tax system on social welfare is given by*

$$\begin{aligned} \Lambda(\epsilon;\rho) &= M_x(\epsilon;\rho) + \Gamma(\epsilon;\rho) && (23) \\ &= M_x(\epsilon;\rho) + \Delta M^1(\epsilon;\rho) + \Gamma(0;\rho). && (24) \end{aligned}$$

See appendix. ■

For a progressive tax system, $\Gamma(\epsilon;\rho)$ is positive. However, $M_x(\epsilon;\rho)$ may be positive or negative according to whether the social welfare loss from pre-tax income variability is larger than the social welfare benefit of the equalization of permanent pre-tax welfare. For instance, as ϵ increases, the pre-tax EDEI $\chi(\omega;\epsilon)$ approaches the lowest pre-tax income individual ω can experience. This may yield welfare losses from mobility that are too large to be offset by the equalizing effects of the tax system.

For ease of reference, the notation on the above decompositions of the effects of mobility and taxation is summarized in Table 2.

4 Statistical procedures

We will need to use panel data below to estimate individual-level EDEIs and aggregate social welfare levels. Such panel data will, however, typically involve a relatively modest number of time periods (at least compared to the number of individuals observed). This can create biases between the expected value of sample estimates and the value of the true (unobserved) individual and social welfare levels. The effects of mobility and taxation on social welfare will also be biased since they are obtained as differences across such biased estimators.

We therefore introduce procedures that correct, at least partially, for these biases. We detail the nature of these corrections for estimating $\gamma(\omega;\epsilon)$; similar reasoning and corrections apply to $\chi(\omega;\epsilon)$ and $\xi(\omega;\epsilon)$.

4.1 Analytical bias corrections

Assume that, for each individual ω in our sample, a number of periodic income values are drawn randomly from an individual-specific distribution function $F_{y|\omega}(\cdot)$. Let $\gamma(\epsilon;\omega)$ be the true (as opposed to the estimated) EDEI of individual ω — see equation (6) for its formal definition using the true distribution function F. A natural estimator of $\gamma(\epsilon;\omega)$ is given by $\hat{\gamma}(\epsilon;\omega)$,

$$\hat{\gamma}(\omega;\epsilon) = U_{\epsilon}^{-1}\left(\int U_{\epsilon}(z)d\hat{F}_{y|\omega}(z)\right) = U_{\epsilon}^{-1}\left(t^{-1}\sum_{j=1}^{t} U_{\epsilon}\left(y_j(\omega)\right)\right), \qquad (25)$$

where t is the number of periodic observations drawn for individual ω (assumed to be the same for all individuals), $y_j(\omega)$ is income observed at time j for individual ω, and $\hat{F}_{y|\omega}(z)$ is the empirical (or sample) distribution of periodic incomes y for individual ω.

The estimator in (25) is, however, biased for a small number of time periods since $\gamma(\epsilon;\omega)$ is non linear in incomes y_j. To see this, define $\delta = 1-\epsilon$, $G(\delta;\omega) = \int z^{\delta} dF_{y|\omega}(z)$, and $\hat{G}(\delta;\omega) = \int z^{\delta} d\hat{F}_{y|\omega}(z)$. Using a Taylor expansion, we then have that

$$\begin{aligned} E\left[\hat{\gamma}(\delta;\omega)\right] = E\Big[&\gamma(\delta;\omega) + \delta^{-1}\gamma(\delta,\omega)^{(1-\delta)/\delta}\left[\hat{G}(\delta;\omega) - G(\delta,\omega)\right] \\ &-0.5\delta^{-2}(\delta-1)\gamma(\delta,\omega)^{(1-2\delta)/\delta}\left[\hat{G}(\delta;\omega) - G(\delta,\omega)\right]^2 + ...\Big]. \end{aligned} \qquad (26)$$

Since

$$E\left[\hat{G}(\delta;\omega) - G(\delta;\omega)\right] = 0,$$

and

$$E\left[\left(\hat{G}(\delta;\omega) - G(\delta;\omega)\right)^2\right] = t^{-1}\text{var}\left(y(\omega)\right),$$

we have (to leading order) that

$$\begin{aligned} E\left[\hat{\gamma}(\delta;\omega)\right] &\cong \gamma(\delta;\omega) - 0.5\delta^{-2}(\delta-1)\gamma(\delta;\omega)^{(1-2\delta)/\delta}t^{-1}\text{var}\left(y(\omega)\right) \\ &\geq \gamma(\delta;\omega). \end{aligned} \qquad (27)$$

This shows that $\hat{\gamma}(\delta;\omega)$ is biased upwards (as $\delta - 1 < 0$). A second-order correction for $\hat{\gamma}(\delta;\omega)$ is thus given by:

$$\widehat{\overline{\gamma}}(\delta;\omega) = \hat{\gamma}(\delta;\omega) + 0.5\delta^{-2}(\delta-1)\gamma(\delta;\omega)^{(1-2\delta)/\delta}t^{-1}\text{var}\left(y(\omega)\right). \qquad (28)$$

All of the elements in (28) can be estimated consistently.

Equations (26) to (28) can be similarly applied to the individuals' pre-tax incomes to provide second-order bias corrections for the natural estimator for $\chi(\delta;\omega)$. Given (7), this also provides second-order corrections to the natural estimator for $\xi(\omega;\epsilon)$. Applying bias corrections of order t^{-1} to estimators for $\gamma(\omega;\epsilon)$, $\chi(\delta;\omega)$ and $\xi(\omega;\epsilon)$ also provides bias corrections of the same order to estimators of $W_{\gamma(\epsilon)}(\rho)$, $W_{\chi(\epsilon)}(\rho)$ and $W_{\xi(\epsilon)}(\rho)$ since these estimators exhibit biases of the same order of magnitude.

4.2 Bootstrap bias corrections

An alternative approach to correcting for the biases found in and (27) is by estimating the biases that arise in numerical simulations of the periodic distributions of incomes. This can be done by bootstrapping the empirical distribution of each individual's periodic incomes. We can proceed as follows:

1. For each individual ω observed in the sample, we first compute a "plug-in" estimator using ω's original sample of periodic incomes; this is simply $\hat{\gamma}(\epsilon;\omega)$.

2. Then, again for each individual ω, we generate K samples, $k = 1, ..., K$, of periodic incomes, each sample being composed of t incomes drawn randomly (and with replacement) from the original observations of incomes for individual ω. We thus compute a new estimator $\gamma_k(\epsilon;\omega)$ for each k. K should be as large as is numerically sufficient and computationally reasonable.

3. Denoting by $\gamma^*(\epsilon;\omega)$ the mean of these K estimators $\gamma_k(\epsilon;\omega)$; that is, let $\gamma^*(\epsilon;\omega) = K^{-1}\sum_{k=1}^{K}\gamma_k(\epsilon;\omega)$. The bootstrap estimate of the bias is then given by the difference between $\gamma^*(\epsilon;\omega)$ and the plug-in estimator $\hat{\gamma}(\epsilon;\omega)$.

The $\gamma(\epsilon;\omega)$ for each of the individuals ω can then be corrected by the bootstrap-estimated biases, $\gamma^*(\epsilon;\omega) - \hat{\gamma}(\epsilon;\omega)$, leading to a bootstrap-corrected estimator given by

$$\begin{aligned}\widetilde{\gamma}(\epsilon;\omega) &= \hat{\gamma}(\epsilon;\omega) - (\gamma^*(\epsilon;\omega) - \hat{\gamma}(\epsilon;\omega)) \\ &= 2\hat{\gamma}(\epsilon;\omega) - \gamma^*(\epsilon;\omega).\end{aligned} \tag{29}$$

Bootstrap bias corrections can sometimes work better than asymptotic ones with small t since they are not restricted to the leading terms listed in (26). They come,

however, at a larger computational cost. A similar procedure to the above can be applied to compute $\widetilde{\chi}(\epsilon;\omega)$ and $\widetilde{\xi}(\epsilon;\omega)$.

5 Empirical application using Canadian data

We now turn to investigating the empirical social welfare effects of mobility and taxation. For this, we use panel data from the Canadian Survey of Labor and Income Dynamics (SLID). The panel runs from 1996 with 38567 observations to 2001 with only 31451 observations. Each household is observed 5.7 times on average, which is close to the six-year total length of the panel. More descriptive details on this panel can be found in Table 3.

Pre-tax-and-benefit income is called "market income" in the SLID. It includes wages and salaries, self-employment income, private pensions and investment income. Post-tax-and-post-benefit income is market income plus transfers minus taxes, and is referred to as "disposable income" in the SLID. Transfers include federal and provincial child and family allowances, old age security pensions and guaranteed income supplement, employment insurance benefits, social assistance benefits, and various tax credits. Taxes include both provincial and federal personal income taxes.

Note that consumption might be deemed to be a better indicator of living standards than income. Longitudinal and nationally representative consumption data are not available, however, in Canada (and are rare elsewhere too). The implicit assumption, therefore, is that all income variability is costly to the individual, even though the anticipation of income changes, borrowing and saving, and insurance will generally help smooth consumption in the presence of income variability. It is also the case that a panel longer than 6 years would probably be better than the one provided by SLID. Again, this is not available in Canada.

Finally, note that we do not allow for the effect of taxation on behavior, such as labor supply and savings behavior. It would be interesting to take into account such effects of taxation on intertemporal behavioral, but this is beyond the scope of the current paper.

To adjust for differences in household composition, we use the equivalence scale traditionally used by Statistics Canada, which assigns a weight of 1 to the household head, of 0.4 to each additional adult, and of 0.3 to each child less than 16 years old. We also normalize incomes such that the mean of pre- and post-tax incomes *per equivalent adult* equals 100 at each time period.

Asymptotic and bootstrap bias corrections reduce by roughly 2 to 5 percent

the estimates of the different indices of mobility and social welfare. Both turn out to yield almost identical estimates. For expositional simplicity, we thus use only the asymptotic correction below.

5.1 The welfare effect of mobility and taxation in Canada

The impact of mobility on social welfare is summarized in Table 4 for various values of ϵ and ρ. For ϵ between 0.3 and 0.9, the cost of variability in pre-tax income ($M_x^1(\epsilon;\rho)$) ranges from 2.09 to 10.84 percent of pre-tax mean income.[5] The tax system reduces considerably, however, the periodic variability of incomes; for instance, the variability cost ($M_y^1(\epsilon;\rho)$) is more than halved for $\epsilon = \rho \geq 0.6$. The variability cost of mobility is thus decreased significantly by the tax system.

Table 4 also shows the impact of mobility on the equalization of permanent pre-tax and post-tax welfare (M_x^2 and M_y^2). For $0.3 \leq \rho \leq 0.9$, the equalization benefit ranges from 1.8% to 10.84% of mean pre-tax incomes and from 1.02% to 3.77% of mean post-tax income. Thus, mobility enhances equality significantly in both distributions of income, but substantially more so in the absence of taxes and transfers. The net effect of mobility on social welfare is dominated by mobility's benefit for $\epsilon < \rho$ and by mobility's cost for $\epsilon > \rho$.

The effect of mobility and taxation on periodic income variability is shown in the two panels of Figure 3. The top panel deals with pre-tax incomes and the bottom panel, with post-tax incomes. The horizontal axes show the percentiles p of the population ordered by increasing levels of permanent incomes. The levels of permanent incomes that correspond to the different percentiles are shown on the left vertical axis (as a percentage of overall mean permanent incomes). The cost of income variability as a percentage of overall mean income is shown on the right vertical axis. This is the effect of periodic income variability on individual welfare: $\chi(p;\epsilon=0)-\chi(p;\epsilon)$ for the top panel of Figure 3 and $\gamma(p;\epsilon=0)-\gamma(p;\epsilon)$ for the bottom panel of the same figure, both relative to overall mean income.

Figure 3 shows both the welfare cost of income variability and the effect that taxation has on it. The top panel shows that the welfare cost of pre-tax income variability can be significant. For the lower values 0.3 and 0.6 of relative risk

[5] Jim Davies kindly suggested a useful way to interpret these numbers. Assume that there is only one individual with two possible income levels, y_l and y_h, each with a 50% probability of being realized. Then $M_y^1(\epsilon = 0.3;\rho) = 2.09$ is obtained with $y_l = 63.3$ and $y_h = 136.7$. If consumption smoothing decreased by 25% the implied variability (through borrowing/saving or some partial insurance schemes), then $M_y^1(\epsilon = 0.3;\rho)$ would fall to 1.16. This is approximately the level of the cost of mobility in post-tax income reported in Table 4.

aversion, this costs is between 2% and 5% of permanent pre-tax incomes. Said differently, individuals would be willing to pay a premium of the order of 2% to 5% of their permanent incomes to smooth their periodic incomes. For a larger value of 0.9 of relative risk aversion, that percentage is almost always above 5% at all percentiles, and can even exceed 10% for the top decile or permanent pre-tax incomes.

The bottom panel of Figure 3 indicates that the welfare cost of income variability is roughly reduced by half by the tax system, no matter what percentile is considered. For instance, at the 0.6 median value of the relative risk aversion parameter ϵ, the welfare cost of post-tax income variability is about 2% of permanent incomes — as oppose to 4%-5% for pre-tax income variability. This is a substantial welfare gain that is typically ignored in most welfare analyses of the tax and transfer system.

Let us turn consider the overall social welfare effects of the tax system. As Proposition 1 illustrates, these effects have two sources. Both of them can be understood from the two panels of Figure 4, which show pre-tax and post-tax permanent income levels (on the left vertical axes) for different percentiles of permanent incomes and the cost of income variability (on the right vertical axes) as a percentage of those permanent incomes. The top panel of Figure 4 does this for $\epsilon = 0.3$ and the second panel, for $\epsilon = 0.6$.

As (22) in Proposition 1 indicates, the first social welfare effect of the tax system comes from the tax equalization of permanent incomes. The top left panel of Figure 4 shows that for 95 percent of the population, pre-tax permanent income $\overline{x}(\omega)$ ranges from 0 percent to around 200 percent of mean pre-tax permanent incomes. The tax narrows considerably this range. The top and bottom panels indeed show that the tax system brings the lowest permanent post-tax incomes to around 40 percent of mean post-tax incomes; the tax system reduces correspondingly the top permanent income levels from 200% to 170% of the average. (The break-even point is approximately at the 55th percentile.) The quantitative welfare impact of this redistribution of permanent incomes is given in Table 5, which shows that $\Gamma(0;\rho)$ in (22) can reach up to 22.63 percent of mean income when $\rho = 0.9$.

The second social welfare effect of the tax system comes from the system's equalization of periodic incomes. This was already visible in Figure 3. It is also apparent in Figure 4, where the variability cost of income variability (as a proportion of percentile-specific permanent incomes) is reduced at all percentiles, and most strongly at the lower percentiles. Take $\epsilon = 0.3$ for instance. For the bottom 0.25 percentiles for instance, the variability of pre-tax income causes a cost of

between 5% and 25% of permanent incomes; that cost drops to less than 2% for post-tax incomes. Again, the welfare effect of the system's equalization of periodic incomes is strongest at the lowest percentiles; it is also about twice as large for $\epsilon = 0.6$ than for $\epsilon = 0.3$.

Quantitative estimates of the overall effects on social welfare of both mobility and taxation are further shown in Table 5. The effect of taxation on social welfare is given by $\Gamma(\epsilon;\rho)$; the net effect of mobility on pre-tax social welfare is given by $M_x(\epsilon;\rho)$; $\Lambda(\epsilon;\rho)$ is the combined welfare effect of taxation and mobility. Following Corollary 2, the redistributive benefit of the tax system may be enhanced, decreased, or even outdone by the effect of mobility on pre-tax incomes.

Table 5 shows that the net effect of both mobility and taxation on social welfare is usually non-negative. The only exception is given by $\rho = 0$ when $\epsilon > 0$. From Proposition 1 and Corollary 2, we can indeed show that $\Lambda(\epsilon;0) = M_y^1(\epsilon;0)$, meaning that the only effect that matters in the absence of an aversion to inequality in permanent welfare is that of post-tax income variability. Unless taxation eliminates income variability completely, the combined effect of taxation and mobility on social welfare then has to be strictly negative.

Otherwise, redistribution and mobility can have a considerable effect on social welfare. Pre-tax anonymous social welfare can be as low as 55% of mean income. ($W_x(\rho = 0.9)$). With $\epsilon = 0.6$ and $\rho = 0.9$, mobility increases that level of social welfare to 61% of mean income ($W_{\chi(\epsilon=0.6)}(\rho = 0.9)$). The effect of taxation moves that up to 87% of mean income ($W_{\gamma(\epsilon=0.6)}(\rho = 0.9)$). Had taxation not considerably decreased initial income variability, post-tax social welfare would have been 80.6% of mean income ($W_{\xi(\epsilon=0.6)}(\rho = 0.9)$). Hence, a considerable part of the welfare impact of taxation is through the equalization of permanent welfare.

Overall, the welfare impact of mobility and taxation can be very important. For $\epsilon = 0.6$ and $\rho = 0.9$, for instance, the combined impact is 31.25% of mean income, with 26.08 percentage points originating from the effect of taxation on social welfare. For all choices of parameter values, the tax effect exceeds by far the net effect of mobility on social welfare.

6 Conclusion

Social evaluations of welfare distributions are generally based on periodic, "time-anonymous", income distributions. It has long been recognized, however, that such evaluations can provide unsatisfactory accounts of welfare, especially

when individuals care about the inter-temporal allocation of their incomes and when societies have to trade off the advantage of mobility as a reducer of inequality in permanent incomes and the drawback of mobility as a source of variability of periodic incomes around permanent incomes.

This paper shows how the effect of mobility on social welfare can be decomposed into two components, one owing to "mobility as equalizer" (in line with most previous studies of mobility) and the other due to mobility as a source of income variability. Relative to time-anonymous social evaluations, the net impact of mobility is ambiguous and depends upon the comparative degree of aversion to income variability across time and to income inequality across individuals. It is also shown in this setting how redistributive tax and transfer policies may serve not only to increase the equality of the distribution of permanent incomes, but also to achieve a greater stability of individual incomes across time.

We use this framework to investigate both the combined and the comparative impacts of mobility and taxation on individual and social welfare in Canada. Redistribution and mobility (both usually argued to be important income equalizers) can have a considerable effect on social welfare, the precise effect depending on aversions to variability and inequality. For usual values of such aversion parameters, pre-tax anonymous social welfare can be as low as 55% of mean income; mobility increases that level of social welfare to 61% of mean income, and taxation moves it up to 87%. Mobility enhances equality significantly in both distributions of income, but substantially more so in the absence of taxes and transfers. The results also show that Canada's tax and transfer system enhances considerably the redistributive effects of mobility while also lowering the cost of income variability; for all choices of parameter values, the effect of taxation also exceeds by far the net effect of mobility on social welfare.

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7 Appendix: Proofs

Proof of Proposition 1.

Recall that $\Gamma(\epsilon;\rho) = W_{\gamma(\epsilon)}(\rho) - W_{\chi(\epsilon)}(\rho)$ stands for the effect of taxation on social welfare:

$$
\begin{aligned}
& W_{\gamma(\epsilon)}(\rho) - W_{\chi(\epsilon)}(\rho) && (30)\\
= \quad & \underbrace{W_{\gamma(\epsilon)}(\rho) - W_{\overline{y}}(\rho)} && (31)\\
& \quad (M_y^1(\epsilon;\rho)\text{: post-tax cost of mobility})\\
+ \quad & \underbrace{W_{\overline{y}}(\rho) - W_{\overline{x}}(\rho)} && (32)\\
& \quad (\Gamma(0;\rho)\text{: equalization of permanent incomes by the tax system})\\
- \quad & \underbrace{W_{\chi(\epsilon)}(\rho) - W_{\overline{x}}(\rho)} && (33)\\
& \quad (M_x^1(\epsilon;\rho)\text{: pre-tax cost of mobility.})
\end{aligned}
$$

Since $\chi(\omega;\epsilon=0) = \overline{x}(\omega)$ and $\gamma(\omega;\epsilon=0) = \overline{y}(\omega)$, $\Gamma(\epsilon;\rho)$ can be expressed as:

$$\Gamma(\epsilon;\rho) = \Delta M^1(\epsilon;\rho) + \Gamma(0;\rho). \tag{34}$$

Rearranging (32) by adding and subtracting $(W_y(\rho) - W_x(\rho))$, and using (12) and (20), Proposition 1 is obtained.

■

Proofs of two statements made on page 7.

For the first statement, which follows equation (7), note that there is a simple relationship between EDEI and permanent incomes,

$$
\begin{aligned}
\chi(\omega;\epsilon) &= \overline{x}(\omega)\left(1 - I_x(\omega;\epsilon)\right),\\
\gamma(\omega;\epsilon) &= \overline{y}(\omega)\left(1 - I_y(\omega;\epsilon)\right),
\end{aligned} \tag{35}
$$

where $I_t(\omega;\epsilon)$ is the cost of mobility (proportional to permanent income) for an individual ω. $I_s(\omega;\epsilon)$ takes the value of 0 when the s's are equally distributed over time, and is increasing in the time variability of individual incomes.

Thus, rearranging (7) using (35) yields

$$\xi(\omega;\epsilon) = \gamma(\omega;\epsilon)\frac{(1 - I_x(\omega;\epsilon))}{(1 - I_y(\omega;\epsilon))}. \tag{36}$$

It is well known that Atkinson's (1970) social welfare function is homothetic. For a proportional tax system, we therefore have that $I_x(\omega;\epsilon) = I_y(\omega;\epsilon)$ and this naturally leads to $\xi(\omega;\epsilon) = \gamma(\omega;\epsilon)$. By the Fellman-Jakobsson theorem (Jakobsson 1976 and Fellman 1976), the greater the progressivity of the tax system, the greater the fall in income variability across periods, and thus the greater the gap between $\gamma(\omega;\epsilon)$ and $\xi(\omega;\epsilon)$.

Turn now to the second statement, which follows equation (12). For $\epsilon = \rho$, equation (8) can be rewritten as

$$W_{\chi(\rho)}(\rho) = U_\rho^{-1}\left(\int U_\rho(\chi(\omega;\rho))dF_\Omega(\omega)\right). \tag{37}$$

Note from equation (5) that

$$\chi(\omega;\rho) = U_\rho^{-1}\left(\int U_\rho(z)dF_{y|\omega}(z)\right). \tag{38}$$

Using equation (38) to rearrange (37), the second statement is obtained for pre-tax incomes. The same procedure applies for $W_y(\rho) = W_{\gamma(\rho)}(\rho)$.

■

Table 1: Notation for individual and social welfare distributions

	Incomes	Distributions	Permanent income of individual ω	Individual EDEI	Social EDEI	Social EDEI of permanent incomes
Pre-tax	x	F_x	$\overline{x}(\omega)$	$\chi(\omega;\epsilon)$	$W_{\chi(\epsilon)}(\rho)$	$W_{\overline{x}}(\rho)$
Post-tax	y	F_y	$\overline{y}(\omega)$	$\gamma(\omega;\epsilon)$	$W_{\gamma(\epsilon)}(\rho)$	$W_{\overline{y}}(\rho)$
Post-tax with pre-tax variability	$\overline{y}(\omega)\frac{x(\omega)}{\overline{x}(\omega)}$		$\overline{y}(\omega)$	$\xi(\omega;\epsilon)$	$W_{\xi(\epsilon)}(\rho)$	$W_{\overline{y}}(\rho)$

Table 2: Notation for the decompositions of the effects of mobility and taxation

Incomes	Net effect of mobility on social EDEI	Effect of mobility on variability	Effect of mobility on inequality	Social EDEI with mobility	Anonymous social EDEI	Anonymous pre-tax and EDEI post-tax
(a) Pre-tax	M_x	M_x^1	M_x^2	$W_{\chi(\epsilon)}(\rho)$	$W_x(\rho)$	$W_x(\rho)$
(b) Post-tax	M_y	M_y^1	M_y^2	$W_{\gamma(\epsilon)}(\rho)$	$W_y(\rho)$	$W_{\gamma(\epsilon)}(\rho)$
(b) minus (a)	ΔM	ΔM^1	ΔM^2	$\Gamma(\epsilon;\rho)$	$W_y(\rho) - W_x(\rho)$	$\Lambda(\epsilon;\rho)$

Table 3: Descriptive statistics on SLID's panel data

	1996		1997		1998	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Household size	2.42	1.4	2.40	1.41	2.38	1.4
Equivalent adults	1.52	0.49	1.51	0.5	1.51	0.5
Per capita market income	15 846	16 590	16 575	18 415	17 568	19 792
Per capita net income	14 963	10 519	15 595	11 583	16 354	12 522
	1999		2000		2001	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Household size	2.36	1.39	2.35	1.37	2.34	1.36
Equivalent adults	1.50	0.49	1.50	0.49	1.50	0.49
Per capita market income	18 892	23 365	20 334	21 817	21 483	27 387
Per capita net income	17 415	14 229	18 628	13 927	20 144	17078

Table 4: The social welfare effect of mobility in pre- and post-tax incomes in Canada, 1996-2001 (mean incomes are normalized to 100)

		Pre-tax income			Post-tax income			Effect of taxation on mobility		
ϵ	ρ	$M_x^1(\epsilon;\rho)$	$M_x^2(\rho)$	$M_x(\epsilon;\rho)$	$M_y^1(\epsilon;\rho)$	$M_y^2(\rho)$	$M_y(\epsilon;\rho)$	$\Delta M^1(\epsilon;\rho)$	$\Delta M^2(\rho)$	$\Delta M(\epsilon;\rho)$
0	0.3	0	1.80	1.80	0	1.02	1.02	0	-0.78	-0.78
0	0.6	0	4.50	4.50	0	2.15	2.15	0	-2.35	-2.35
0	0.9	0	10.84	10.84	0	3.77	3.77	0	-7.07	-7.07
0.3	0	-2.09	0	-2.09	-1.26	0	-1.26	0.83	0	0.83
0.3	0,9	-2.30	10.84	8.54	-1.03	3.77	2.74	1.27	-7.07	-5.8
0.6	0.3	-4.17	1.80	-2.37	-2.25	1.02	-1.23	1.93	-0.78	1.15
0.6	0.6	-4.50	4.50	0	-2.15	2.15	0	2.36	-2.35	0
0.6	0.9	-5.67	10.84	5.17	-2.22	3.77	1.55	3.44	-7.07	-3.63
0.9	0.6	-7.89	4.50	-3.39	-3.53	2.15	-1.38	4.37	-2.35	2.01
0.9	0.9	-10.84	10.84	0	-3.77	3.77	0	7.07	-7.07	0

$M_s^1(\epsilon;\rho)$: effect of mobility on variability; $M_s^2(\rho)$: effect of mobility on inequality; $M_s(\epsilon;\rho)$: net effect of mobility on social welfare; Δ: effect of movement from pre-tax to post-tax distribution.

Table 5: The impact of taxation and mobility on social welfare Canada, 1996-2001 (mean incomes are normalized to 100)

		Levels of social welfare							Overall impact		
		Pre-tax income			Post-tax income				on social welfare		
ϵ	ρ	$W_x(\rho)$	$W_{\overline{x}}(\rho)$	$W_{\chi(\epsilon)}(\rho)$	$W_y(\rho)$	$W_{\overline{y}}(\rho)$	$W_{\gamma(\epsilon)}(\rho)$	$W_{\xi(\epsilon)}(\rho)$	$\Gamma(\epsilon;\rho)$	$M_x(\epsilon;\rho)$	$\Lambda(\epsilon;\rho)$
0	0	100	100	100	100	100	100	100	0	0	0
0	0.3	89.27	91.07	91.07	95.12	96.15	96.15	96.15	5.07	1.8	6.86
0	0.6	76.41	80.91	80.91	90.36	92.51	92.51	92.51	11.6	4.5	16.1
0	0.9	55.52	66.36	66.36	85.22	88.99	88.99	88.99	22.63	10.84	33.47
0.3	0	-	-	97.91	-	-	98.74	96.93	0.83	-2.09	-1.26
0.3	0.9	-	-	64.06	-	-	87.96	85.6	23.9	8.54	32.45
0.6	0.3	-	-	86.90	-	-	93.89	90.06	7	-2.37	4.63
0.6	0.6	-	-	76.41	-	-	90.36	85.67	13.95	0	13.95
0.6	0.9	-	-	60.69	-	-	86.77	80.6	26.08	5.17	31.25
0.9	0.6	-	-	73.02	-	-	88.98	80.77	15.96	-3.39	12.57
0.9	0.9	-	-	55.52	-	-	85.22	73.06	29.71	0	29.71

$W_s(\rho)$: time-anonymous social welfare; $W_{\overline{s}}(\rho)$: permanent income social welfare; $W_{\chi(\epsilon)}(\rho)$: pre-tax social welfare; $W_{\gamma(\epsilon)}(\rho)$: post-tax social welfare; $W_{\xi(\epsilon)}(\rho)$ post-tax social welfare with pre-tax income variability; $\Gamma(\epsilon;\rho)$: effect of taxation on social welfare; $M_x(\epsilon;\rho)$: net effect of mobility on pre-tax social welfare; $\Lambda(\epsilon;\rho)$: combined effect of taxation and mobility on social welfare

Figure 1: Welfare with temporal income variability

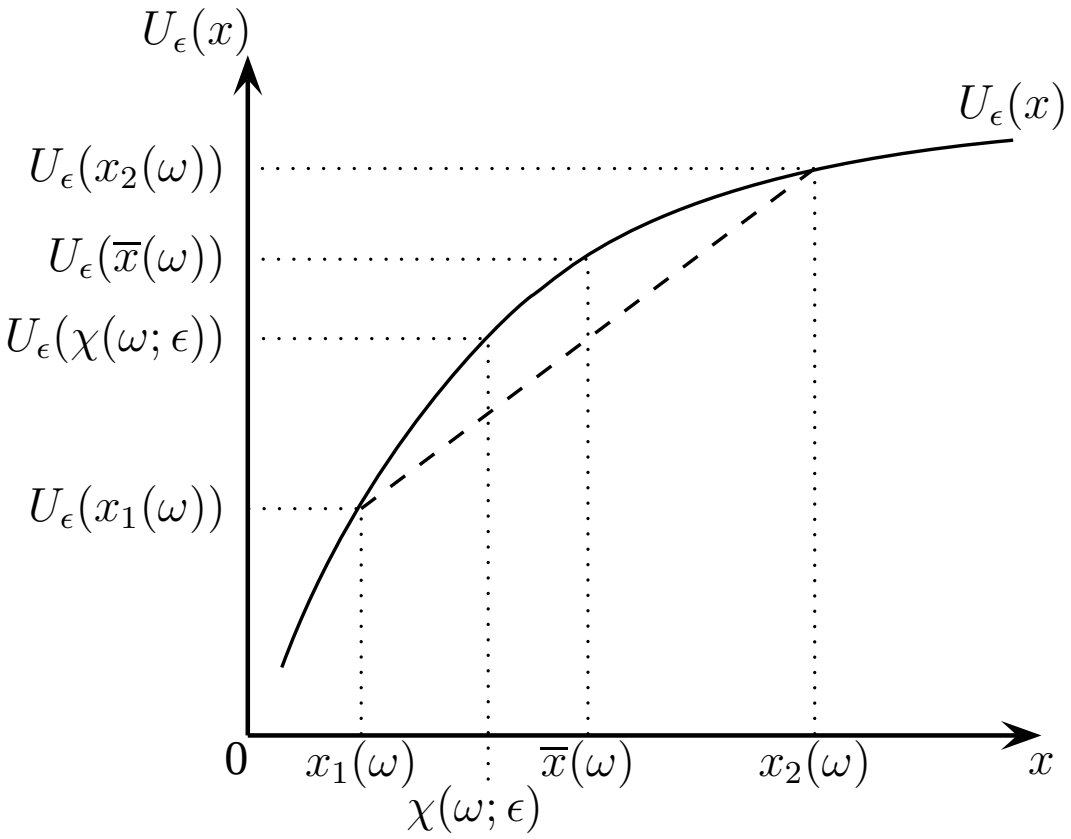

Figure 2: Effects of taxation and mobility on income variability across time and inequality across individuals

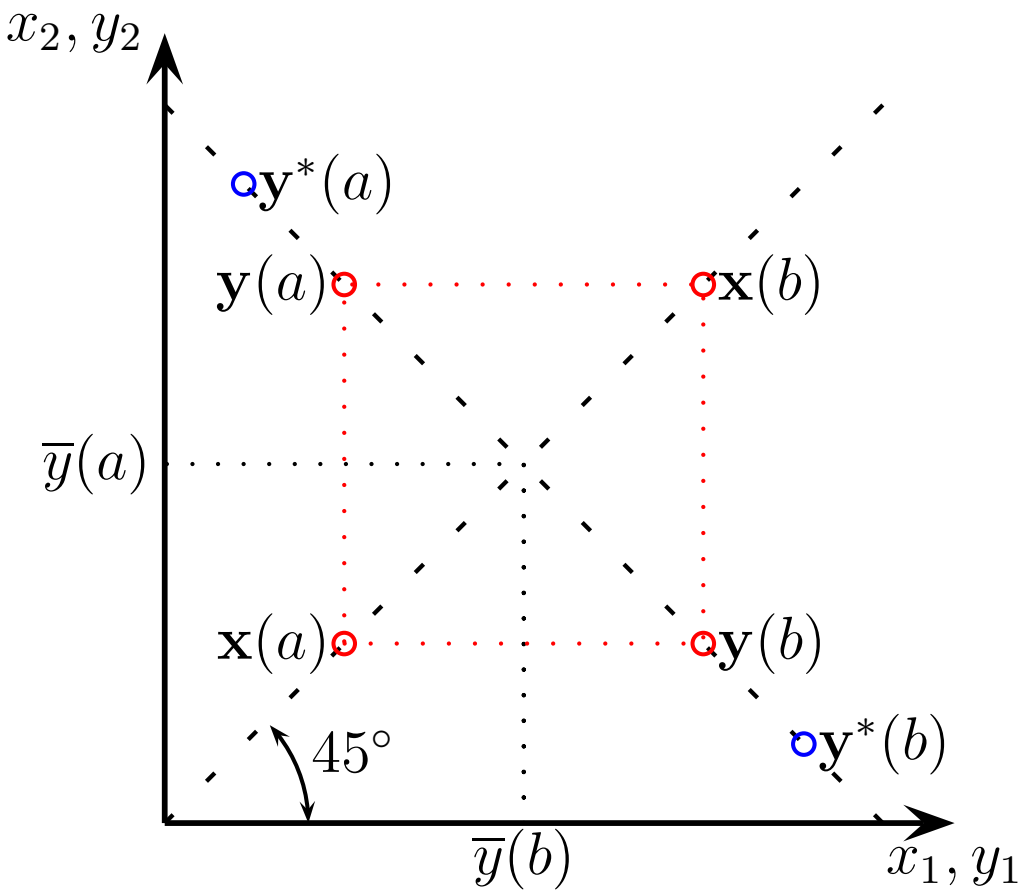

Figure 3: The cost of pre- and post-tax income variability according to different levels of aversion ϵ to inter-temporal variability, Canada 1996-2001

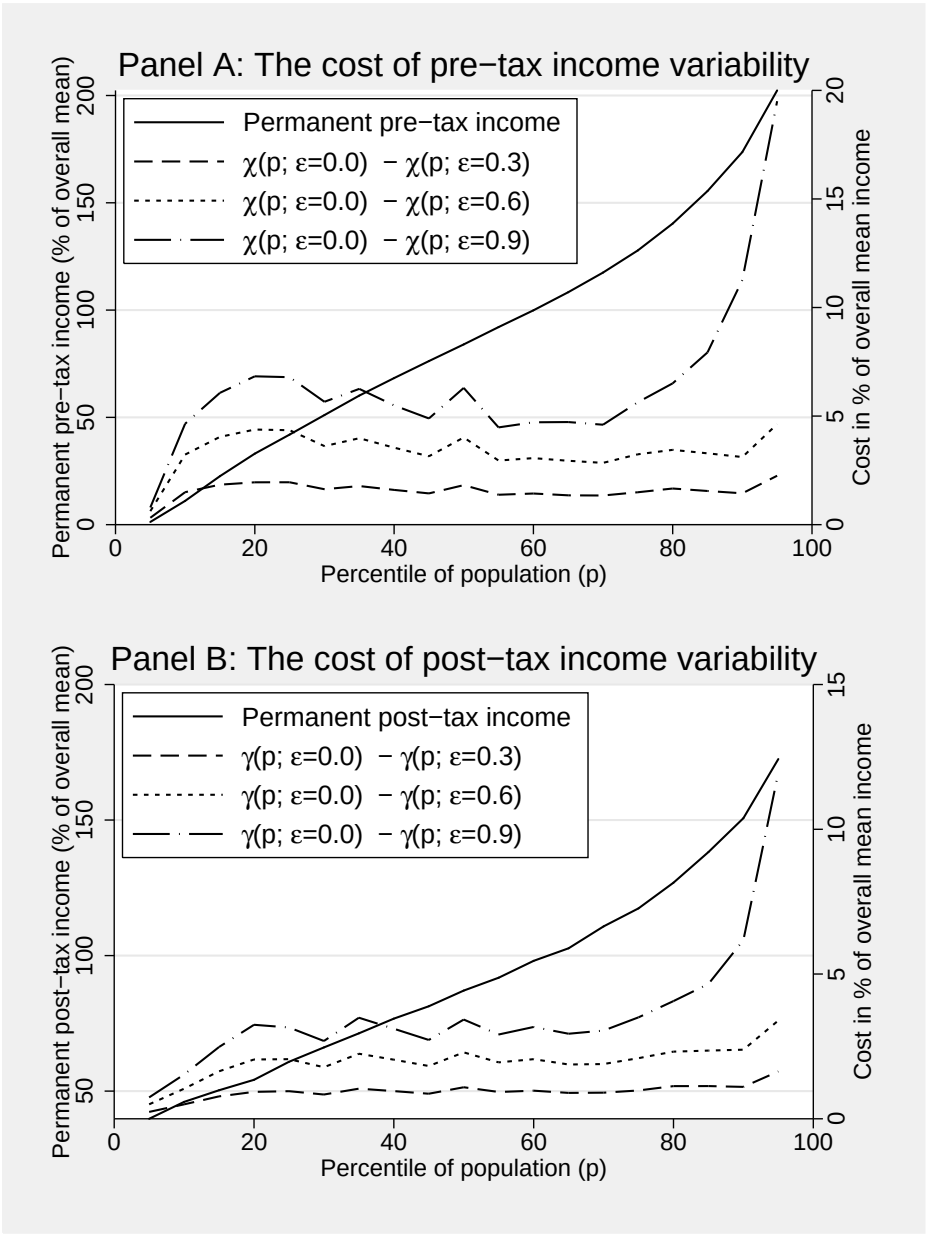

Figure 4: The impact of taxation on permanent income and on the cost of temporal variability, for two different levels of aversion ϵ to inter-temporal variability, Canada 1996-2001 (as a percentage of permanent income)