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Trade, Competition, and Efficiency

(revised version)

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Abstract:
We present a general equilibrium model of monopolistic competition featuring pro-competitive effects and a competitive limit, and investigate the impact of trade on welfare and efficiency. Contrary to the constant elasticity case, in which all gains from trade are due to product diversity, our model allows for a welfare decomposition between gains from product diversity and gains from pro-competitive effects. We show that the market outcome is not efficient because too many firms operate at an inefficiently small scale by charging too high markups. We further illustrate that trade raises efficiency by narrowing the gap between the equilibrium utility and the optimal utility. As the population gets arbitrarily large in the integrated economy, the equilibrium utility converges to the optimal utility because of the competitive limit. We finally extend the variable elasticity model to a multi-sector setting, and show that intersectoral distortions are eliminated in the limit. The multi-sector model allows us to illustrate some new aspects arising from intersectoral and intrasectoral allocations, namely that trade leads to structural convergence, rather than sectoral specialization, and that trade induces domestic exit in the nontraded sector.

Keywords: Pro-competitive effects, competitive limit, excess entry, trade and efficiency, monopolistic competition

JEL Classification: D43, D51, F12

1 Introduction

Few trade theorists would disagree with the statement that product diversity, scale economies, and pro-competitive effects are central to any discussion about gains from trade and efficiency under monopolistic competition.[1] Yet, it is fair to say that these questions have not been fully and jointly explored within a simple and solvable general equilibrium model.[2] This is largely due to the fact that the workhorse model, namely the constant elasticity of substitution (henceforth, CES) framework, displays two peculiar features. First, it does not allow for pro-competitive effects so that "there is no effect of trade on the scale of production, and the gains from trade come solely through increased product diversity" (Krugman, 1980, p.953). Second, the equilibrium in the CES model is usually constrained (second-best) optimal, i.e., the market provides the socially desirable number of varieties at an efficient scale (Dixit and Stiglitz, 1977). Consequently, trade is not efficiency enhancing in the CES model because it does not correct the only existing market failure, pricing above marginal cost.

In order to more fully explore gains from trade and efficiency under monopolistic competition, we must depart from the standard CES model. Doing so, however, has long been difficult since the variable elasticity of substitution (henceforth, VES) model in Krugman (1979) has "not proved tractable, and from Dixit and Norman (1980) and Krugman (1980) onwards, most writers have used the CES specification [...] with its unsatisfactory implications that firm size is fixed by tastes and technology, and all adjustments in industry size (due to changes in trade policy, for example) come about through changes in the number of firms" (Neary, 2004, p.177). Building on the new VES specification by Behrens and Murata (2007), which satisfies the properties of the utility function in Krugman (1979), we present a simple general equilibrium model of international trade featuring pro-competitive effects (i.e., profit-maximizing prices are decreasing in the mass of competing firms) and a competitive limit (i.e., profit-maximizing prices converge to marginal costs when the mass of competing firms becomes arbitrarily large). Within this framework, where varieties, markups, firm-level scale economies are endogenous, we investigate the impact of trade on welfare and efficiency.

Our results can be summarized as follows. First, unlike in the standard CES model, the market outcome is not efficient because too many firms operate at an inefficiently small scale due to the negative externality each firm imposes on the others through markups. Note that in a more general CES model presented in Benassy (1996), where market power and taste for

[1] Dixit (2004, p.128) summarizes the gains from trade under monopolistic competition as follows: (i) availability of greater variety; (ii) better exploitation of economies of scale; and (iii) greater degree of competition, driving prices closer to marginal costs. More recently, the World Trade Organization (2008, pp.48-50) provides a similar classification: (i) gains from increased variety; (ii) gains from increased competition; and (iii) gains from increased economies of scale.

[2] For instance, the World Trade Organization (2008, p.48) states that "(a)s far as the gains from intra-industry trade are concerned, most studies have focused on either one of the variety, scale or pro-competitive (price) effects of trade opening".

variety are disentangled, the equilibrium mass of firms can also be larger (or smaller) than the optimal one. However, that inefficiency is not due to pro-competitive effects as markups are constant because of the CES specification. Accordingly, entry restriction (or promotion), if any, would not affect price-cost margins in Benassy (1996), whereas it would in our model since markups depend on the mass of firms competing in the market.

Second, due to pro-competitive effects, autarky markups are no longer the same across countries of different sizes. It is therefore not obvious that free trade leads to the equalization of price-cost margins and to product and factor price equalization when country sizes differ, labor markets are segmented, and products are differentiated. For instance, the seminal paper by Krugman (1979, p.476) states, after pointing out that "countries have identical tastes and technologies", that "(s)ymmetry will ensure that wage rates in the two countries will be equal and that the price of any good produced in either country will be the same", even in the presence of country-size asymmetry. As we are not aware of any formal proof of this assertion, we provide one in this paper.[3]

Third, contrary to the CES case, in which all gains from trade are due to greater product diversity, our model allows for a welfare decomposition between gains from product diversity and gains from pro-competitive effects. It is worth emphasizing that such a welfare decomposition is necessary for understanding each channel through which gains from trade materialize. Recently, Feenstra and Weinstein (2010) compared the estimated gains from trade in a VES model based on Feenstra (2003) with those in the CES model by Broda and Weinstein (2006). Interestingly, although the overall gains are roughly the same between the two specifications, the underlying mechanism is quite different: the CES model ascribes all gains to new import varieties, whereas in the VES model increased product diversity explains only two-thirds of the overall gains with the remaining one-third being driven by pro-competitive effects. Ignoring endogenous markups may thus overstate gains from new import varieties.

Fourth, we illustrate that trade raises efficiency by narrowing the gap between the equilibrium utility and the optimal utility. In our model, product diversity is greater in equilibrium than in optimum. The associated equilibrium gains approach zero as the population gets arbitrarily large in the integrated economy. By contrast, while markups are too high, the associated equilibrium losses also vanish in the limit as prices converge to marginal costs. Hence, we obtain the overall efficiency result.

Our approach is closely related to that of Feenstra (2003) in that both the mass of varieties and markups are made endogenous without relying on an additively separable numeraire good. However, there are several important differences. For instance, to solve for prices, Feenstra (2003) uses an approximation that applies to the case where markups are sufficiently small. In our framework, exact prices are obtained. Furthermore, in Feenstra (2003) there is no closed

[3]The absence of a formal proof in Krugman (1979) is not an exception. On the contrary, most monopolistic competition models of trade assume, rather than prove, that product price equalization holds under free trade. See Helpman (1981) for another representative example.

form solution for the direct utility function, although it is homothetic. By contrast, we use a class of non-homothetic preferences that admits a workable direct utility function. Finally, our model is tractable enough for obtaining several analytical results and for incorporating labor market clearing and zero profit conditions that must be satisfied in general equilibrium.[4] Hence, our approach is complementary to that of Feenstra (2003).

Finally, we extend our framework to include two monopolistically competitive sectors. Extending Krugman (1979) to a multi-sector setting has been difficult since preferences over varieties in each sector are non-homothetic. As is well known and as pointed out by Dixit and Stiglitz (1977, p.302) in the context of monopolistic competition, under such non-homothetic preferences, two-stage budgeting is not applicable.[5] Our specification, however, allows for closed form solutions for all equilibrium expressions, even in the two-sector case. This enables us to explore some new aspects arising from intersectoral and intrasectoral allocations.

The main contribution of the two-sector analysis is threefold. First, despite intersectoral heterogeneity, we can establish the efficiency result, namely that intersectoral distortions are eliminated as the population gets arbitrarily large in the integrated economy, while losses from intrasectoral distortions vanish in the limit as in the single-sector case. Second, when both sectors are freely traded, we can establish structural convergence, i.e., countries having different sectoral compositions under autarky converge to the same industry structure under free trade. This is in sharp contrast to the prediction of Ricardian and Heckscher-Ohlin models that trade causes sectoral specialization. Furthermore, unlike the new trade theory that emphasizes intra-industry trade between similar countries, with similarity giving rise to more trade, we show that countries become more similar due to trade, thus suggesting circular causation between similarity and intra-industry trade. Finally, when either of the two sectors is nontraded, we can show that trade induces domestic exit, or a variety loss in the nontraded sector.[6] Unlike in the single-sector case with a traded good, such a variety loss in the nontraded sector is not compensated by import varieties. Given that the observed share of nontraded goods is not negligible, this suggests an important welfare implication: monopolistic competition models that abstract from nontraded varieties may overestimate gains from trade.[7]

The remainder of the paper is organized as follows. Section 2 develops a single-sector model, and Section 3 focuses on the autarky case. Section 4 analyzes the trade equilibrium,

[4]See Behrens *et al.* (2009) for an example using such general equilibrium conditions when estimating a gravity equation.

[5]See Epifani and Gancia (2011) for a recent multi-sector analysis on the class of utility functions to which two-stage budgeting is applicable.

[6]This is reminiscent of what Feenstra and Weinstein (2010) call the "domestic exit effect" in the traded sector, i.e., a decrease in the mass of varieties produced in each country that is illustrated in Krugman (1979) and Feenstra (2004, Ch.5). One notable difference is that, in our extended model with a nontraded good, trade induces domestic exit in the *nontraded* sector.

[7]Dotsey and Duarte (2008), for instance, state that consumption of nontraded goods accounts for about 40 percent of GDP in the United States.

decomposes the gains from trade, and shows that trade enhances efficiency. Section 5 extends the single-sector model to a multi-sector setting. Section 6 concludes.

2 Model

We first analyze the single-sector case. Consider a world with two countries, labeled r and s. Variables associated with each country will be subscripted accordingly. There is a mass L_r of workers/consumers in country r, and each worker supplies inelastically one unit of labor. Thus, L_r also stands for the total amount of labor available in country r. We assume that labor is internationally immobile and that it is the only factor of production.

2.1 Preferences

There is a single monopolistically competitive industry producing a horizontally differentiated consumption good with a continuum of varieties. Let Ω_r (resp., Ω_s) be the set of varieties produced in country r (resp., s), of measure n_r (resp., n_s). Hence, $N \equiv n_r + n_s$ stands for the endogenously determined mass of available varieties in the global economy. International markets are assumed to be integrated, so that each firm in each country sets a unique free-on-board price for consumers in both countries. We assume that preferences are additively separable over varieties as in Krugman (1979), and that the sub-utility functions are of the 'constant absolute risk aversion' (CARA) type as in Behrens and Murata (2007). A representative consumer in country r solves the following utility maximization problem:

$$\begin{aligned} \max_{q_{rr}(i),\, q_{sr}(j)} \quad U_r \equiv \int_{\Omega_r} \left[1 - \mathrm{e}^{-\alpha q_{rr}(i)}\right] \mathrm{d}i + \int_{\Omega_s} \left[1 - \mathrm{e}^{-\alpha q_{sr}(j)}\right] \mathrm{d}j \\ \text{s.t.} \quad \int_{\Omega_r} p_r(i) q_{rr}(i) \mathrm{d}i + \int_{\Omega_s} p_s(j) q_{sr}(j) \mathrm{d}j = E_r, \end{aligned} \tag{1}$$

where $\alpha > 0$ is a utility parameter; E_r stands for the expenditure; $p_r(i)$ denotes the price of variety i, produced in country r; and $q_{sr}(j)$ stands for the per-capita consumption of variety j, produced in country s and sold in country r.

The demand functions for country-r consumers are given by (see Appendix A.1):

$$q_{rr}(i) = -\frac{1}{\alpha} \ln p_r(i) + \frac{E_r}{P} + \frac{1}{\alpha}\frac{H}{P} \tag{2}$$

$$q_{sr}(j) = -\frac{1}{\alpha} \ln p_s(j) + \frac{E_r}{P} + \frac{1}{\alpha}\frac{H}{P}, \tag{3}$$

where $P \equiv \int_{\Omega_r} p_r(i)\mathrm{d}i + \int_{\Omega_s} p_s(j)\mathrm{d}j$ and $H \equiv \int_{\Omega_r} p_r(i) \ln p_r(i)\mathrm{d}i + \int_{\Omega_s} p_s(j) \ln p_s(j)\mathrm{d}j$ are the sum of prices and a measure of price dispersion, respectively. Mirror expressions hold for country-s consumers. Because of the continuum assumption firms are negligible, and thus take P and H as given. The own-price derivatives of the demand functions are then as follows:

$$\frac{\partial q_{rr}(i)}{\partial p_r(i)} = -\frac{1}{\alpha p_r(i)} \qquad \frac{\partial q_{sr}(j)}{\partial p_s(j)} = -\frac{1}{\alpha p_s(j)}, \tag{4}$$

which yields the variable demand elasticities $\varepsilon_{rr}(i) = [\alpha q_{rr}(i)]^{-1}$ and $\varepsilon_{sr}(j) = [\alpha q_{sr}(j)]^{-1}$.[8] Mirror expressions hold again for country-s consumers.

2.2 Technology

All firms have access to the same increasing returns to scale technology. To produce $Q(i)$ units of any variety requires $l(i) = cQ(i) + F$ units of labor, where F is the fixed and c is the marginal labor requirement. We assume that firms can costlessly differentiate their products and that there are no scope economies. Thus, there is a one-to-one correspondence between firms and varieties, so that the mass of varieties N also stands for the mass of firms operating in the global economy. There is free entry and exit in each country, which implies that n_r and n_s are endogenously determined by the zero profit conditions. Consequently, the expenditure E_r equals the wage w_r. Under integrated markets, the profit of firm $i \in \Omega_r$ is then as follows:

$$\Pi_r(i) = [p_r(i) - cw_r]\, Q_r(i) - Fw_r, \tag{5}$$

where $Q_r(i) \equiv L_r q_{rr}(i) + L_s q_{rs}(i)$ stands for its total output.

2.3 Equilibrium

Country-r (resp., country-s) firms maximize their profit (5) with respect to $p_r(i)$ (resp., $p_s(j)$), taking the vectors (n_r, n_s) and (w_r, w_s) of firm distribution and wages as given.[9] This yields the following first-order conditions:

$$\frac{\partial \Pi_r(i)}{\partial p_r(i)} = Q_r(i) + [p_r(i) - cw_r]\left[L_r \frac{\partial q_{rr}(i)}{\partial p_r(i)} + L_s \frac{\partial q_{rs}(i)}{\partial p_r(i)}\right] = 0, \tag{6}$$

$$\frac{\partial \Pi_s(j)}{\partial p_s(j)} = Q_s(j) + [p_s(j) - cw_s]\left[L_s \frac{\partial q_{ss}(j)}{\partial p_s(j)} + L_r \frac{\partial q_{sr}(j)}{\partial p_s(j)}\right] = 0. \tag{7}$$

We define a *price equilibrium* as a distribution of prices satisfying (6) and (7) for all $i \in \Omega_r$ and $j \in \Omega_s$. We will discuss its existence, uniqueness, and some other properties in the following sections.[10] An *equilibrium* is a price equilibrium and vectors (n_r, n_s) and (w_r, w_s) of firm distribution and wages such that national labor markets clear, trade is balanced, and firms

[8] Our utility function U_r thus has the same properties as those in Krugman (1979) because it is additively separable across varieties, the sub-utility is increasing and concave in q, and the elasticity of demand ε decreases with q.

[9] It is well known that price and quantity competition yield the same outcome in monopolistic competition models with a continuum of firms (Vives, 1999, p.168). We thus focus on prices as the only choice variable.

[10] As shown by Roberts and Sonnenschein (1977), the existence of (price) equilibria is usually problematic in monopolistic competition models, since firms' reaction functions may be badly behaved. Because our model relies on a continuum of firms, which are individually negligible, we do not face this problem. In a similar spirit, Neary (2003) uses a general equilibrium model of oligopolistic competition with a continuum of sectors, in which firms are 'large' in their own markets but 'negligible' in the whole economy. This also allows to restore equilibrium since firms cannot directly influence aggregates of the whole economy.

earn zero profits. Formally, an equilibrium is a price equilibrium satisfying (6) and (7), *and* a solution to the following three conditions:

$$\int_{\Omega_r} \big[cQ_r(i) + F\big]\mathrm{d}i = L_r, \tag{8}$$

$$\int_{\Omega_s} \big[cQ_s(j) + F\big]\mathrm{d}j = L_s, \tag{9}$$

$$L_s \int_{\Omega_r} p_r(i) q_{rs}(i)\mathrm{d}i = L_r \int_{\Omega_s} p_s(j) q_{sr}(j)\mathrm{d}j, \tag{10}$$

where all quantities are evaluated at a price equilibrium. It is readily verified that firms earn zero profits when conditions (8)–(10) hold. One may set either w_r or w_s as the numeraire. However, we need not choose a numeraire since the model is fully determined in real terms.[11]

3 Autarky

Assuming that the two countries can initially not trade with each other, we first characterize the equilibrium and the optimum in the closed economy, and show that there are too many firms operating at an inefficiently small scale in equilibrium. Without loss of generality, we consider country r in what follows.

3.1 Equilibrium

Inserting (2) and (4) into (6), and letting $q_{rs}(i) = \partial q_{rs}(i)/\partial p_r(i) = 0$, one can show that the price equilibrium is symmetric and unique, and given by (see Appendix A.2 for the derivation):

$$p_r^a = \left(1 + \frac{\alpha}{c n_r^a}\right) c w_r^a, \tag{11}$$

where an a-superscript henceforth denotes autarky values. At the symmetric price equilibrium, the profit of each firm is given by $\Pi_r^a = L_r q_{rr}^a \left(p_r^a - c w_r^a\right) - F w_r^a$. Using the consumer's budget constraint $w_r^a = n_r^a p_r^a q_{rr}^a$, the above expression can be rewritten as $\Pi_r^a = p_r^a q_{rr}^a [L_r \left(1 - c n_r^a q_{rr}^a\right) - F n_r^a]$. Zero profits then imply that the quantities must be such that

$$q_{rr}^a = \frac{1}{c}\left(\frac{1}{n_r^a} - \frac{F}{L_r}\right), \tag{12}$$

which are positive because $n_r^a F < L_r$ must hold from the resource constraint when n_r^a firms operate. Utility is then given by

$$U(n_r^a) = n_r^a \left[1 - \mathrm{e}^{-\frac{\alpha}{c}\left(\frac{1}{n_r^a} - \frac{F}{L_r}\right)}\right]. \tag{13}$$

[11]The choice of the numeraire is immaterial in our monopolistic competition framework. This is an important departure from general equilibrium oligopoly models, where the choice of the numeraire is usually not neutral (Gabszewicz and Vial, 1972).

Note that (12) and (13) hold whenever prices are symmetric and firms earn zero profit.

Inserting $q_{rr}^a = w_r^a/(n_r^a p_r^a)$ into the labor market clearing condition (8), we get:

$$n_r^a = \frac{L_r}{F}\left(1 - \frac{c w_r^a}{p_r^a}\right). \tag{14}$$

The equilibrium mass of firms can then be found by using (11) and (14), which yields:[12]

$$n_r^a = \frac{\sqrt{4\alpha c F L_r + (\alpha F)^2} - \alpha F}{2cF} \equiv \nu(\alpha, L_r) > 0. \tag{15}$$

The function ν will be useful to make notation compact when extending the model to a two-sector setting. The output per firm is given by $Q_r^a \equiv L_r q_{rr}^a = (L_r/n_r^a)(w_r^a/p_r^a) = L_r/(cn_r^a + \alpha)$, where we use (11). Plugging (15) into the last expression, we have

$$Q_r^a = \frac{2FL_r}{\sqrt{4\alpha c F L_r + (\alpha F)^2} + \alpha F} = \frac{F}{\alpha}\nu(\alpha, L_r). \tag{16}$$

Finally, inserting (15) into (13), the equilibrium utility in autarky is given by

$$U(L_r) = \frac{\sqrt{4\alpha c F L_r + (\alpha F)^2} - \alpha F}{2cF}\left[1 - \mathrm{e}^{-\frac{2\alpha F}{\sqrt{4\alpha c F L_r + (\alpha F)^2} + \alpha F}}\right] > 0, \tag{17}$$

which is a strictly increasing and strictly concave function of the population size L_r for all admissible parameter values, i.e., $\alpha > 0$, $c > 0$, $F > 0$, and $L_r > 0$. Alternatively, the equilibrium utility can be expressed in terms of ν as $U(L_r) = \nu(\alpha, L_r)[1 - \mathrm{e}^{-F\nu(\alpha, L_r)/L_r}]$.

3.2 Optimum

We now analyze the first-best problem. The planner maximizes the utility, as given by (1), subject to the technology and resource constraint (8). The first-order conditions of this problem with respect to $q_{rr}(i)$ show that the quantities must be symmetric. This, together with (8), implies that:

$$q_{rr} = \frac{Q_r}{L_r} = \frac{1}{c}\left(\frac{1}{n_r} - \frac{F}{L_r}\right). \tag{18}$$

Hence, the planner maximizes

$$U(n_r^o) = n_r^o\left[1 - \mathrm{e}^{-\frac{\alpha}{c}\left(\frac{1}{n_r^o} - \frac{F}{L_r}\right)}\right], \tag{19}$$

with respect to the mass of varieties n_r^o, where an o-superscript henceforth denotes the first-best values. Utility maximization requires the following first-order condition to hold:

$$\frac{cn_r^o}{\alpha + cn_r^o} = \mathrm{e}^{-\frac{\alpha}{c}\left(\frac{1}{n_r^o} - \frac{F}{L_r}\right)} \quad \Longrightarrow \quad -\left(1 + \frac{\alpha}{cn_r^o}\right)\mathrm{e}^{-\left(1 + \frac{\alpha}{cn_r^o}\right)} = -\mathrm{e}^{-1-\frac{\alpha F}{cL_r}}. \tag{20}$$

[12] The other root is negative and must, therefore, be ruled out.

Using the Lambert W function, which is defined as the inverse of the function $x \mapsto x\mathrm{e}^x$ (e.g., Corless *et al.*, 1996; Hayes, 2005), the latter can be rewritten as:

$$-\left(1+\frac{\alpha}{cn_r^o}\right) = W\left(-\mathrm{e}^{-1-\frac{\alpha F}{cL_r}}\right).$$

Solving this equation for n_r^o yields a unique optimal mass of firms

$$n_r^o = -\frac{\alpha}{c\left[1+W_{-1}\left(-\mathrm{e}^{-1-\frac{\alpha F}{cL_r}}\right)\right]} > 0, \tag{21}$$

where W_{-1} is the real branch of the Lambert W function satisfying $W(-\mathrm{e}^{-1-\alpha F/(cL_r)}) \leq -1$ (Corless *et al.*, 1996, pp.330-331; Hayes, 2005).[13] Note that W_{-1} is increasing in L_r, and that $-\infty < W_{-1} < -1$ for $0 < L_r < \infty$.

Furthermore, letting Q_r^o be the optimal output per firm given by $n_r^o(cQ_r^o + F) = L_r$, we can establish the following proposition.

Proposition 1 *There are too many firms operating at an inefficiently small scale in equilibrium, i.e.,* $n_r^a > n_r^o$ *and* $Q_r^a < Q_r^o$.

Proof. See Appendix B. ■

Note that excess entry arises because of pro-competitive effects ($\partial p_r^a/\partial n_r^a < 0$ by (11)). The negative externality each firm imposes on the other firms gives rise to the 'business-stealing effect' (Mankiw and Whinston, 1986, p.49), i.e., the equilibrium output per firm declines as the number of firms grows ($\partial Q_r^a/\partial n_r^a = \partial(L_r q_{rr}^a)/\partial n_r^a < 0$ by (12)).

Interestingly, this result contrasts starkly with the constant elasticity case, where the equilibrium mass of varieties is (second-best) optimal.[14] Stated differently, the basic CES model does not account for the tendency that too many firms produce at an inefficiently small scale in autarky (the so-called 'Eastman-Stykolt hypothesis'; Eastman and Stykolt, 1967), an argument often used to criticize import-substituting industrialization policies (Krugman *et al.*, 2012, pp.292-293) or tariff barriers (Horstmann and Markusen, 1986) on efficiency grounds.

Combining (19) and (20) yields $U^o(n_r^o) = \alpha n_r^o/(\alpha + cn_r^o)$. Inserting (21) into this expression, the optimal utility is given by

$$U^o(L_r) = -\frac{\alpha}{cW_{-1}\left(-\mathrm{e}^{-1-\frac{\alpha F}{cL_r}}\right)} > 0, \tag{22}$$

which is a strictly increasing function of the population size L_r for all admissible parameter values, i.e., $\alpha > 0$, $c > 0$, $F > 0$, and $L_r > 0$.

[13] As $-\mathrm{e}^{-1} < -\mathrm{e}^{-1-\alpha F/(cL_r)} < 0$, there is another possible real value of $W(-\mathrm{e}^{-1-\alpha F/(cL_r)})$ satisfying $-1 < W(-\mathrm{e}^{-1-\alpha F/(cL_r)}) < 0$. However, it leads to $n_r^o < 0$ and must, therefore, be ruled out.

[14] This can be seen from Dixit and Stiglitz (1977, p.301), when letting $s = 1$ and $\theta = 0$ in their equations (20) and (21), since there is no homogeneous good in our setting.

4 Free trade

We now analyze the impacts of trade on welfare and efficiency in a world with pro-competitive effects and a competitive limit. Section 4.1 analyzes the equilibrium. Section 4.2 then shows the existence of gains from trade and decomposes them into gains from product diversity and gains from pro-competitive effects. Section 4.3 finally illustrates that trade narrows the gap between the equilibrium utility and the optimal utility by driving prices closer to marginal costs.

4.1 Equilibrium

We have shown that the profit-maximizing price under autarky is given by (11), where n_r^a is evaluated at (15). Accordingly, markups in autarky are no longer the same across countries of different sizes. It is therefore not obvious that free trade leads to the equalization of price-cost margins and to product and factor price equalization when country sizes differ, labor markets are segmented, and products are differentiated. Assume that both countries can trade freely. The profits and the first-order conditions are still given by (5)–(7), respectively. Using these expressions, we establish the following result.

Proposition 2 *Free trade leads to product and factor price equalization, i.e.,* $p_r(i) = p_s(j) = p$ *for all* $i \in \Omega_r$ *and for all* $j \in \Omega_s$*, and* $w_r = w_s = w$*. The price equilibrium is then given by*

$$p = \left(1 + \frac{\alpha}{cN}\right) cw, \tag{23}$$

where markups are equalized across varieties and countries.

Proof. See Appendix C. ■

Note that (23) is an extension of the autarky case (11). Since prices and wages are equalized, $q_{rr} = q_{sr} = q_{ss} = q_{rs} = w/(Np)$ must hold by (2) and (3). Accordingly, all firms sell the same quantity $Q = (L_r + L_s)q$. Labor market clearing then implies that $n_r/n_s = L_r/L_s$, which, together with $q = w/(Np)$, yields

$$n_r = \frac{L_r}{F}\left(1 - \frac{cw}{p}\right). \tag{24}$$

Plugging (23) into (24) and the analogous expression for country s, we obtain two equations with two unknowns n_r and n_s. Solving for the equilibrium masses of firms, we get

$$\begin{aligned} n_r &= \frac{L_r}{L_r + L_s}\frac{\sqrt{4\alpha cF(L_r + L_s) + (\alpha F)^2} - \alpha F}{2cF} \\ n_s &= \frac{L_s}{L_r + L_s}\frac{\sqrt{4\alpha cF(L_r + L_s) + (\alpha F)^2} - \alpha F}{2cF} \end{aligned}$$

Thus, the equilibrium mass of firms in the global economy is given by

$$N = n_r + n_s = \frac{\sqrt{4\alpha cF(L_r + L_s) + (\alpha F)^2} - \alpha F}{2cF} = \nu(\alpha, L_r + L_s), \tag{25}$$

which is an extension of the autarky expression (15). The output per firm is then given by $Q = (L_r + L_s)q = [(L_r + L_s)/N](w/p) = (L_r + L_s)/(cN + \alpha)$, where we use (23). Plugging (25) into the last expression, we have

$$Q = \frac{2F(L_r + L_s)}{\sqrt{4\alpha cF(L_r + L_s) + (\alpha F)^2} + \alpha F} = \frac{F}{\alpha}\nu(\alpha, L_r + L_s). \tag{26}$$

The impacts of trade on product diversity, markups, and output per firm are the same as those in Krugman (1979) and Feenstra (2004), except that we obtain the closed form solution for each variable that is useful for many applications such as efficiency and multiple sectors. When compared with autarky, free trade leads in each country to: (i) greater product diversity, an increase in the mass of varieties consumed, $N > \max\{n_r^a, n_s^a\}$; (ii) pro-competitive effects, a decrease in markups, $p/(cw) < \min\{p_r^a/(cw_r^a), p_s^a/(cw_s^a)\}$; (iii) domestic exit effects, a decrease in the mass of varieties produced, $\max\{n_r, n_s\} < n_r^a$; and (iv) better exploitation of scale economies, an increase in output per firm, $Q > \max\{Q_r^a, Q_s^a\}$.[15]

These results capture the relationship among product diversity, markups, and output per firm. First, the mass of varieties consumed increases due to new import varieties by property (i). This intensifies competition and reduces markups by property (ii), thus driving some firms out of each domestic market by property (iii).[16] Labor market clearing then makes sure that output per firm expands by property (iv), as labor is reallocated from the fixed requirements of closing firms to the marginal requirements of surviving firms. This is an important departure from the standard CES model, in which only channel (i) operates. Recall that the equalization of markups in Proposition 2 holds regardless of country size. In autarky, a smaller country has a smaller mass of firms, which implies higher markups. Therefore, markups in a smaller country decrease more than those in a larger country under free trade. Similarly, a smaller country experiences a greater increase in product diversity and output per firm.

Our approach is closely related to that of Feenstra (2003) in that both the mass of varieties and markups are made endogenous without relying on a quasi-linear specification. However, there are several important differences. For instance, to solve for prices, Feenstra (2003) uses an approximation that applies to the case where markups are sufficiently small. In our framework, exact prices are obtained. Furthermore, in Feenstra (2003) there is no closed form solution for the direct utility function, although it is homothetic. By contrast, we use a class of

[15] It is readily verified that expressions (15) and (25) yield claim (i), which, together with (11) and (23), implies (ii). Claim (ii) and expressions (14) and (24) then yield (iii). Finally, from (16) and (26), we obtain claim (iv).

[16] In the CES model by Lawrence and Spiller (1983, Proposition 7), trade leads to a redistribution of existing firms between the two countries while the total mass of firms remains unchanged. This result is driven by changes in relative factor prices and, as pointed out by the authors, need not hold under variable markups.

non-homothetic preferences that admits a workable direct utility function. Our model is also tractable enough for obtaining several analytical results and for incorporating labor market clearing and zero profit conditions that must be satisfied in general equilibrium. Note that although there is a growing literature on firm heterogeneity in international trade (e.g., Melitz, 2003), the price-cost margin for each firm is usually assumed to be constant in those models because of the CES specification.[17]

One notable exception is Melitz and Ottaviano (2008) who illustrate pro-competitive effects in a quasi-linear framework with firm heterogeneity. Both their and our models predict that the market size is the crucial determinant of markups. However, the *extent* of the market in question differs, and so does the mechanism that drives the markup reduction. In Melitz and Ottaviano (2008), the *local* market size matters for the (average) prices and markups, so that the size of the trading partner has no impact on the domestic utility level, as well as the number of firms selling in the home country (see their expressions (23)–(25)). In contrast, in our model, it is the *global* market size ($L_r + L_s$) that affects the prices and markups, as well as the mass of varieties consumed, utility, and efficiency. This difference, which gives rise to quite different policy implications regarding the choice of trading partners, arises due to income effects. Indeed, we can show that if the numeraire good were added to our model, the prices and markups would be exogenously fixed by preferences (α) and technology (c), and thus independent of the global market size. As discussed in detail by Melitz and Ottaviano (2008, Sections 3.5 and 3.6), firm heterogeneity models, and more generally, monopolistic competition models, typically display *either* increased factor market competition as in Melitz (2003) *or* increased product market competition as in Melitz and Ottaviano (2008). We allow for *both* factor *and* product market competition by incorporating income effects as in Melitz (2003) and pro-competitive effects as in Melitz and Ottaviano (2008). Our framework is thus useful especially when analyzing how trading partners of different size affect domestic consumption diversity and markups, as well as welfare and efficiency.[18]

4.2 Welfare decomposition and gains from trade

Contrary to the CES case, in which all gains from trade are due to increased import varieties, our model allows for both gains from product diversity and gains from pro-competitive effects. In order to focus on each channel through which gains from trade materialize, we now decompose welfare as in Krugman (1981).[19] Since varieties are symmetric under both free trade and

[17]See Redding (2011) for a recent state-of-the-art survey on theories of heterogeneous firms and trade.

[18]See Behrens *et al.* (2009) for an example in the context of Canada-US interregional trade, where the sizes of trading partners matter in general equilibrium. Note that such an analysis is infeasible in the quasi-linear framework by Melitz and Ottaviano (2008), and as pointed out by Feenstra (2010, p.20), "its zero income elasticities suggest that in empirical application it is best suited for partial equilibrium analysis."

[19]Krugman (1981) illustrates a similar decomposition in a model where two types of sector-specific workers earn different real wages. However, because of the CES specification, there are no pro-competitive effects.

autarky, the utility difference is given by:

$$U_r - U_r^a = N\left(1 - \mathrm{e}^{-\frac{\alpha w}{Np}}\right) - n_r^a\left(1 - \mathrm{e}^{-\frac{\alpha w_r^a}{n_r^a p_r^a}}\right).$$

Adding and subtracting $n_r^a \mathrm{e}^{-\alpha w/(n_r^a p)}$, and rearranging the resulting terms, we obtain the following welfare decomposition:

$$U_r - U_r^a = \underbrace{N\left(1 - \mathrm{e}^{-\frac{\alpha w}{Np}}\right) - n_r^a\left(1 - \mathrm{e}^{-\frac{\alpha w}{n_r^a p}}\right)}_{\text{Product diversity}} + \underbrace{n_r^a\left(\mathrm{e}^{-\frac{\alpha w_r^a}{n_r^a p_r^a}} - \mathrm{e}^{-\frac{\alpha w}{n_r^a p}}\right)}_{\text{Pro-competitive effects}}. \tag{27}$$

We now examine the role and the sign of each component in expression (27) in more details, both from a theoretical and an empirical point of view.

Product diversity. The first term in (27) captures the beneficial effects of increased product diversity, given the wage-price ratio under free trade, w/p. As shown before, trade expands the mass of varieties consumed, despite the exit of some domestic producers. This raises utility, holding w/p constant, as we have

$$U_r = N\left(1 - \mathrm{e}^{-\frac{\alpha w}{Np}}\right), \qquad \frac{\partial U_r}{\partial N} = 1 - \mathrm{e}^{-\frac{\alpha w}{Np}}\left(1 + \frac{\alpha w}{Np}\right) > 0 \qquad \forall N.$$

To obtain the last inequality, let $z \equiv \alpha w/(Np)$ and $h(z) \equiv 1 - e^{-z}(1+z)$. Clearly, $h(0) = 0$ and $h'(z) > 0$ for all $z > 0$, which shows that for any given wage-price ratio w/p, utility increases with the mass of varieties consumed.

Despite its central role in new trade theory, little is known about the empirical importance of gains from product diversity (Feenstra, 1995). Yet, there is an emerging literature on measuring gains from varieties. Using extremely disaggregated data and the method developed by Feenstra (2004), Broda and Weinstein (2006) document that the number of varieties in US imports rose by 212% between 1972 and 2001, and according to their estimates this maps into US welfare gains of about 2.6% of GDP. A more recent study by Feenstra and Weinstein (2010), however, points out that the CES specification used in Broda and Weinstein (2006) ignores endogenous markups and thus may overstate gains from import varieties. Indeed, Feenstra and Weinstein (2010) compare the estimated gains from trade in a VES model based on Feenstra (2003), with those in Broda and Weinstein (2006). Interestingly, although the overall gains are roughly the same between the two specifications, the underlying mechanism is quite different: the CES model ascribes all gains to new import varieties, whereas in the VES model increased product diversity explains only two-thirds of the overall gains with the remaining one-third being driven by pro-competitive effects.

Pro-competitive effects. The second term in (27) captures the beneficial effects of intensified competition, given the mass of firms under autarky, n_r^a. As shown before, trade reduces markups, which ceteris paribus raises utility. It is worth emphasizing that the reduction in markups is a social gain: lower markups in a zero-profit equilibrium indicates a smaller wedge between firm's marginal and average costs, which implies larger output per firm and greater economies of scale. Note that $w_r^a/p_r^a = w/p$ would hold in the CES case, i.e., there would be no gains from trade due to pro-competitive effects.

It is well known from various industrial organization studies that prices in many imperfectly competitive industries are increasing functions of producer concentration (see Schmalensee, 1989, pp.987-988, for a survey). In our symmetric equilibrium, the Herfindahl-index of concentration, defined as the sum of squared market shares, reduces to $H = N(1/N)^2 = 1/N$. Since the mass of firms is increasing in market size in our model, markups are lower in larger markets (see Campbell and Hopenhayn, 2005, for empirical evidence). Similarly, by increasing the number of competitors in each market, import competition decreases concentration, which maps into lower consumer prices. Several case studies confirm this 'imports-as-market-discipline hypothesis' (e.g., Levinsohn, 1993; Harrison, 1994; Tybout, 2003). More recently, Badinger (2007) finds solid evidence that the Single Market Programme of the EU has reduced markups by 26% in aggregate manufacturing of 10 member states.

Let us summarize our results as follows:

Proposition 3 *Free trade raises welfare both by increasing the mass of varieties consumed and by reducing markups.*

Proof. To prove our claim, it is sufficient to examine the sign of the two components in (27). As shown above, they are both positive, which ensures gains from trade. ■

4.3 Entry, markups, and efficiency

We now compare the equilibrium and optimal allocations in the global economy. Since in our model free trade amounts to increasing the population size, the result on excess entry established in Section 3.2 continues to hold, even under free trade. Stated differently, there is a unique optimal mass of firms $N^o < N$ satisfying the first-order condition (20) under free trade. Hence, there are too many firms operating at an inefficiently small scale and the market outcome is not efficient. Furthermore, it can be verified that

$$\lim_{L \to 0} \frac{N}{N^o} = 1 \quad \text{and} \quad \lim_{L \to \infty} \frac{N}{N^o} = \sqrt{2}$$

hold regardless of parameter values.[20] By continuity, for a sufficiently small population size, excess entry tends to be small, whereas it gets larger when the population gets arbitrarily large.

[20]When taking the limit of expressions involving the Lambert W function, we use Mathematica, where $W_{-1}(\cdot)$ can be computed by `ProductLog[-1,·]`. Note that we will use `ProductLog[·]` for the principal branch of $W(\cdot)$ that we will encounter in the subsequent analysis.

Turning to pro-competitive effects, expressions (23) and (25) yield

$$\frac{p}{cw} = 1 + \frac{2\alpha F}{\sqrt{4\alpha cFL + (\alpha F)^2} - \alpha F} \quad \text{and} \quad \frac{\partial}{\partial L}\left(\frac{p}{cw}\right) < 0.$$

It is then readily verified that our model exhibits a competitive limit

$$\lim_{L\to 0} \frac{p}{cw} = \infty \quad \text{and} \quad \lim_{L\to\infty} \frac{p}{cw} = 1,$$

so that prices converge to marginal costs as the population gets arbitrarily large. Despite the fact that there remains excess entry even when the population gets arbitrarily large in the integrated economy, the associated gains from excessive varieties are shown to approach zero in the limit. As the gap between prices and marginal costs eventually vanishes, we obtain the overall efficiency result as follows.[21]

Proposition 4 *When the population gets arbitrarily large in the integrated economy, the equilibrium utility converges to the optimal utility, i.e.,*

$$\lim_{L\to\infty} U(L) = \lim_{L\to\infty} U^o(L) = \frac{\alpha}{c}.$$

Proof. Applying l'Hospital's rule to (17), it is readily verified that $\lim_{L\to\infty} U(L) = \alpha/c$. Furthermore, by definition of the Lambert W function, $\lim_{L\to\infty} W_{-1}(-\mathrm{e}^{-1-\alpha F/(cL)}) = -1$ holds. Hence, taking the limit of expression (22) yields $\lim_{L\to\infty} U^o(L) = \alpha/c$.[22] ■

Proposition 4 uses the optimum given technology and the resource constraint that has been analyzed in Section 3.2. Alternatively, we can confirm the efficiency result by implementing the first-best allocation via marginal cost pricing $p^o = cw^o$. This requires lump-sum transfers as each firm earns negative profits $-Fw^o$. When there is a mass N^o of operating firms, a lump-sum tax $(N^oFw^o)/L$ is levied on each consumer's income. Accordingly, the income net of this tax is given by $E^o = w^o(1 - N^oF/L)$. The consumer's budget constraint $q^o = E^o/(N^op^o)$, together with $p^o = cw^o$, then immediately yields $q^o = (1/c)(1/N^o - F/L)$, which is the same as (18). Hence, the planner faces the same utility (19) to maximize, thus achieving the optimal mass of varieties given by (21).

This alternative way of implementing the first-best allocation allows for a welfare decomposition in terms of product diversity and pro-competitive effects as follows.

$$\begin{aligned} U - U^o &= N\left(1 - \mathrm{e}^{-\frac{\alpha w}{Np}}\right) - N^o\left(1 - \mathrm{e}^{-\frac{\alpha w^o}{N^o p^o}}\right) \\ &= \underbrace{N\left(1 - \mathrm{e}^{-\frac{\alpha w}{Np}}\right) - N^o\left(1 - \mathrm{e}^{-\frac{\alpha w}{N^o p}}\right)}_{\text{excess entry}} + \underbrace{N^o\left(\mathrm{e}^{-\frac{\alpha w^o}{N^o p^o}} - \mathrm{e}^{-\frac{\alpha w}{N^o p}}\right)}_{\text{too high markups}}, \end{aligned}$$

[21] This result is reminiscent of Mankiw and Whinston (1986, Proposition 3) who establish conditions for the equilibrium utility in a partial equilibrium closed-economy model to converge to the optimal utility when excess entry gets large.

[22] See Behrens and Murata (2006, Appendix E) for an alternative proof that does not use the Lambert W function.

where we add and subtract $N^o[1 - \mathrm{e}^{-\alpha w/(N^o p)}]$, and rearrange the resulting terms. The first term is positive due to the excessive mass of varieties, $N > N^o$, given the equilibrium wage-price ratio. By definition of the optimal utility, the second term must be negative, reflecting too high equilibrium markups, $p/(cw) > p^o/(cw^o) = 1$, given the optimal mass of varieties. As $U < U^o$, the second term must always dominate the first term in any economy of finite size. Yet, by taking the limit of each term, we have

$$
\begin{aligned}
\lim_{L\to\infty} \left[N \left(1 - \mathrm{e}^{-\frac{\alpha w}{Np}}\right) - N^o \left(1 - \mathrm{e}^{-\frac{\alpha w}{N^o p}}\right)\right] &= 0 \\
\lim_{L\to\infty} N^o \left(\mathrm{e}^{-\frac{\alpha w^o}{N^o p^o}} - \mathrm{e}^{-\frac{\alpha w}{N^o p}}\right) &= 0,
\end{aligned}
$$

which shows that both equilibrium gains from excessive varieties and equilibrium losses from too high markups vanish in the limit, thereby yielding the overall efficiency result.

The limit result established in Proposition 4 may be extended to a finite economy by investigating whether

$$
\max\left\{\frac{U(L_r)}{U^o(L_r)}, \frac{U(L_s)}{U^o(L_s)}\right\} < \frac{U(L)}{U^o(L)} < 1,
$$

where the last inequality comes from the definition of the optimum. Whether U/U^o monotonically increases in L is not a trivial question since both the equilibrium and the optimal utility increase in L, as can be seen from (17) and (22). Figure 1 shows that N/N^o is increasing in L, meaning that excess entry gets larger as the population increases.[23] At the same time, Figure 2 illustrates that $[p/(cw)]/[p^o/(cw^o)] = p/(cw)$ is decreasing in L, implying that the gap between the equilibrium prices and marginal costs gets smaller as the population increases. The overall effect is depicted in Figure 3. As expected, $U/U^o < 1$. The market outcome thus remains inefficient for finite population sizes. However, since U/U^o increases monotonically with L, trade between larger countries yields higher efficiency than trade between smaller countries.

[**Insert Figures 1-3 around here**]

5 Two-sector model

We now extend our basic model to include two sectors, each of which produces a differentiated good. Doing so allows us to shed light on various issues arising from intersectoral and intrasectoral allocations. In particular, we establish the following three results. First, when both sectors are freely traded, countries having different sectoral compositions under autarky converge to the same industry structure under free trade. This is in sharp contrast to the prediction of Ricardian and Heckscher-Ohlin models that trade causes sectoral specialization.

[23] The parameter values for Figures 1-3 are as follows: $\alpha = \{0.1, 1, 5\}$; $c = 0.5$; and $F = 1$. Other admissible parameter values yield qualitatively similar figures, thus suggesting that the underlying property is robust.

Furthermore, unlike the new trade theory, where more similar countries engage in more intra-industry trade, we show that countries become more similar due to trade, thus suggesting circular causation between similarity and intra-industry trade. Second, despite intersectoral heterogeneity, we establish the efficiency result, namely that intersectoral distortions are eliminated as the population gets arbitrarily large in the integrated economy, while losses from intrasectoral distortions vanish in the limit as in the single-sector case. Finally, when either of the two sectors is nontraded, trade induces domestic exit, or a variety loss in the nontraded sector. Since the variety loss in the nontraded sector is not compensated by import varieties, this result suggests an important welfare implication: monopolistic competition models that abstract from nontraded varieties may overestimate gains from trade.

5.1 Autarky

Assume that there are two sectors, denoted by 1 and 2, each producing a horizontally differentiated consumption good with a continuum of varieties. Without loss of generality, we consider country r in this subsection. Let $\mathbf{q}_{1r}$ and $\mathbf{q}_{2r}$ denote the distribution of demands for the varieties of good 1 and 2, respectively. We assume that preferences are (weakly) separable across the two goods so that the utility maximization problem can be expressed as follows:

$$\max_{\mathbf{q}_{r1},\mathbf{q}_{2r}} \quad U_r \equiv U(U_{1r}(\mathbf{q}_{1r}), U_{2r}(\mathbf{q}_{2r})) \quad \text{s.t.} \quad \int_{\Omega_{1r}} p_{1r}(i)q_{1r}(i)\mathrm{d}i + \int_{\Omega_{2r}} p_{2r}(j)q_{2r}(j)\mathrm{d}j = E_r, \tag{28}$$

where U has standard properties. As in the single-sector case, U_{1r} and U_{2r} are given by

$$U_{1r} \equiv \int_{\Omega_{1r}} \left[1 - \mathrm{e}^{-\alpha_1 q_{1r}(i)}\right]\mathrm{d}i \quad \text{and} \quad U_{2r} \equiv \int_{\Omega_{2r}} \left[1 - \mathrm{e}^{-\alpha_2 q_{2r}(j)}\right]\mathrm{d}j.$$

Since U_{1r} and U_{2r} are not homothetic, two-stage budgeting is not applicable as pointed out by Dixit and Stiglitz (1977, p.302). However, as shown in Appendix A.3, the demand function for each variety in each sector can be expressed compactly as follows:

$$q_{1r}(i) = \frac{1}{\alpha_1}\ln\left[\frac{\widetilde{p}_{1r}}{p_{1r}(i)}\right] \qquad q_{2r}(j) = \frac{1}{\alpha_2}\ln\left[\frac{\widetilde{p}_{2r}}{p_{2r}(j)}\right], \tag{29}$$

where $\widetilde{p}_{1r}$ and $\widetilde{p}_{2r}$ are: (i) common to all firms within a sector; (ii) taken as given by each firm because of the continuum assumption; yet (iii) endogenously determined in equilibrium (see Appendix A.3 for their expressions). Note that $\widetilde{p}_{1r}$ and $\widetilde{p}_{2r}$ can be interpreted as reservation prices since $q_{1r}(i) > 0$ if and only if $p_{1r}(i) < \widetilde{p}_{1r}$ and $q_{2r}(j) > 0$ if and only if $p_{2r}(j) < \widetilde{p}_{2r}$. Given the demand functions, as well as $\widetilde{p}_{1r}$ and $\widetilde{p}_{2r}$, firms in each sector maximize profits

$$\Pi_{1r}(i) = [p_{1r}(i) - cw_r]\frac{L_r}{\alpha_1}\ln\left[\frac{\widetilde{p}_{1r}}{p_{1r}(i)}\right] - Fw_r \tag{30}$$

$$\Pi_{2r}(j) = [p_{2r}(j) - cw_r]\frac{L_r}{\alpha_2}\ln\left[\frac{\widetilde{p}_{2r}}{p_{2r}(j)}\right] - Fw_r. \tag{31}$$

5.1.1 Equilibrium

As in the single-sector case, we first analyze the price equilibrium. Let W denote the principal branch of the Lambert W function.[24] From the first-order conditions for profit-maximization, we obtain the prices, quantities, and operating profits under autarky as follows (see Appendix A.4 for the derivations and the properties of W):

$$p_{1r}^a = \frac{cw_r^a}{W_{1r}^a} \qquad q_{1r}^a = \frac{1}{\alpha_1}\left(1 - W_{1r}^a\right) \qquad \pi_{1r}^a = \frac{L_r c w_r^a}{\alpha_1}\left(\frac{1}{W_{1r}^a} + W_{1r}^a - 2\right) \tag{32}$$

$$p_{2r}^a = \frac{cw_r^a}{W_{2r}^a} \qquad q_{2r}^a = \frac{1}{\alpha_2}\left(1 - W_{2r}^a\right) \qquad \pi_{2r}^a = \frac{L_r c w_r^a}{\alpha_2}\left(\frac{1}{W_{2r}^a} + W_{2r}^a - 2\right), \tag{33}$$

where

$$W_{1r}^a \equiv W\left(\mathrm{e}\frac{cw_r^a}{\widetilde{p}_{1r}^a}\right) \quad \text{and} \quad W_{2r}^a \equiv W\left(\mathrm{e}\frac{cw_r^a}{\widetilde{p}_{2r}^a}\right). \tag{34}$$

Since $\widetilde{p}_{1r}^a$ and $\widetilde{p}_{2r}^a$ are common to all firms within the same sector, the price equilibrium and the associated quantity and operating profits in each sector are symmetric. We thus drop the firm indices i and j.

The equilibrium is characterized by the price equilibrium, zero profits in each sector, labor market clearing, and the relationship between $\widetilde{p}_{1r}^a$ and $\widetilde{p}_{2r}^a$ that results from the consumer's optimization problem. First, plugging the profit-maximizing prices and quantities into (30) and (31), the zero profit conditions are given by[25]

$$\begin{aligned}
\Pi_{1r} &= \frac{L_r c w_r^a}{\alpha_1}\left(\frac{1}{W_{1r}^a} + W_{1r}^a - 2\right) - F w_r^a = 0 \\
\Pi_{2r} &= \frac{L_r c w_r^a}{\alpha_2}\left(\frac{1}{W_{2r}^a} + W_{2r}^a - 2\right) - F w_r^a = 0,
\end{aligned}$$

which can be uniquely solved for W_{1r}^a and W_{2r}^a as follows.[26]

$$W_{1r}^a = 1 - \frac{\sqrt{4\alpha_1 c F L_r + (\alpha_1 F)^2} - \alpha_1 F}{2cL_r} \in (0,1) \tag{35}$$

$$W_{2r}^a = 1 - \frac{\sqrt{4\alpha_2 c F L_r + (\alpha_2 F)^2} - \alpha_2 F}{2cL_r} \in (0,1). \tag{36}$$

Expressions (35) and (36), together with (32) and (33), imply that the profit-maximizing prices, quantities, and operating profits in one sector do not depend on the characteristics of the other sector. This is due to the fact that preferences are (weakly) separable across goods: given consumers' budget allocation across sectors, firms care only about what happens in their own sector when maximizing profits.

Using (32), (33), (35), and (36), and noting that W_{1r}^a and W_{2r}^a are increasing in L_r, we know that, in autarky, equilibrium prices and quantities, p_{1r}^a, p_{2r}^a, q_{1r}^a, and q_{2r}^a, are smaller in

[24] Note that W is different from W_{-1} that we have introduced in Section 3.2.

[25] The same solution technique can be applied to the single-sector model (see Appendix A.5 for details).

[26] Note that the other root is greater than one, which is infeasible as it implies prices below marginal costs.

larger countries. Equilibrium outputs, Q^a_{1r} and Q^a_{2r}, defined as $Q^a_{1r} \equiv L_r q^a_{1r}$ and $Q^a_{2r} \equiv L_r q^a_{2r}$ are, however, larger in larger counties. We can also establish the following result.

Proposition 5 *Suppose that $\alpha_1 > \alpha_2$. Then, the firms in sector 1 charge higher markups and produce smaller output in each country, i.e., $p^a_{1r} > p^a_{2r}$ and $Q^a_{1r} < Q^a_{2r}$. Furthermore, larger countries have a smaller price ratio, p^a_{1r}/p^a_{2r}, and a greater output ratio, Q^a_{1r}/Q^a_{2r}.*

Proof. Noting that $W^a_{1r} < W^a_{2r}$ when $\alpha_1 > \alpha_2$, and that $L_r(1 - W^a_{1r})/\alpha_1$ and $L_r(1 - W^a_{2r})/\alpha_2$ are decreasing in α_1 and α_2, respectively, we obtain the first claim. Furthermore, noting that the price ratio is given by $p^a_{1r}/p^a_{2r} = W^a_{2r}/W^a_{1r}$, it is readily verified that

$$\frac{\partial \ln(p^a_{1r}/p^a_{2r})}{\partial \ln L_r} = \frac{\sqrt{\alpha_2 F}}{\sqrt{4cL_r + \alpha_2 F}} - \frac{\sqrt{\alpha_1 F}}{\sqrt{4cL_r + \alpha_1 F}}.$$

Since $\sqrt{\alpha_2 F}/\sqrt{4cL_r + \alpha_2 F}$ and $\sqrt{\alpha_1 F}/\sqrt{4cL_r + \alpha_1 F}$ are increasing in α_2 and α_1, respectively, we obtain $\partial \ln(p^a_{1r}/p^a_{2r})/\partial \ln L_r < 0$ when $\alpha_1 > \alpha_2$. Finally, differentiating the output ratio $Q^a_{1r}/Q^a_{2r} = (\alpha_2/\alpha_1)[(1 - W^a_{1r})/(1 - W^a_{2r})]$, we have

$$\frac{\partial \ln(Q^a_{1r}/Q^a_{2r})}{\partial \ln L_r} = \frac{1}{2}\left(\frac{\sqrt{\alpha_1 F}}{\sqrt{4cL_r + \alpha_1 F}} - \frac{\sqrt{\alpha_2 F}}{\sqrt{4cL_r + \alpha_2 F}}\right),$$

which, using the same argument as for the price ratio, yields the second claim. ■

We now turn to the equilibrium sectoral labor allocation L^a_{1r} and L^a_{2r} and the equilibrium masses of firms n^a_{1r} and n^a_{2r}. This requires making use of the remaining two equilibrium conditions – labor market clearing and the relationship between $\widetilde{p}^a_{1r}$ and $\widetilde{p}^a_{2r}$ – which are given as follows (see Appendix A.3 for the latter derivation):

$$L_r = L^a_{1r} + L^a_{2r}, \quad \text{where} \quad L^a_{\ell r} \equiv n^a_{\ell r}\left[\frac{L_r c}{\alpha_\ell}(1 - W^a_{\ell r}) + F\right] \tag{37}$$

$$\frac{\widetilde{p}^a_{1r}}{\widetilde{p}^a_{2r}} = \frac{\alpha_1}{\alpha_2}\frac{\partial U_r/\partial U_{1r}}{\partial U_r/\partial U_{2r}}. \tag{38}$$

The left-hand side of (38) is obtained by plugging W^a_{1r} and W^a_{2r} in (35) and (36) into the left-hand side of (34). Indeed, we can solve uniquely for $\widetilde{p}^a_{1r}$ and $\widetilde{p}^a_{2r}$ from (34) as follows:

$$\widetilde{p}^a_{1r} = \left[1 + \frac{\sqrt{4\alpha_1 cFL_r + (\alpha_1 F)^2} + \alpha_1 F}{2cL_r}\right] \mathrm{e}^{\frac{F}{L_r}\nu(\alpha_1, L_r)} cw^a_r > p^a_{1r}$$

$$\widetilde{p}^a_{2r} = \left[1 + \frac{\sqrt{4\alpha_2 cFL_r + (\alpha_2 F)^2} + \alpha_2 F}{2cL_r}\right] \mathrm{e}^{\frac{F}{L_r}\nu(\alpha_2, L_r)} cw^a_r > p^a_{2r},$$

so that the reservation prices are higher than the market prices in equilibrium.

The right-hand side of (38), in turn, can be obtained as follows. We have so far made no explicit assumption regarding U_r. We assume in what follows that $U_r \equiv \beta_1 \ln U_{1r} + \beta_2 \ln U_{2r}$, with $\beta_1, \beta_2 > 0$ and $\beta_1 + \beta_2 = 1$. In that case, we have

$$\frac{\partial U_r/\partial U_{1r}}{\partial U_r/\partial U_{2r}} = \frac{\beta_1 U_{2r}}{\beta_2 U_{1r}} = \frac{\beta_1 n^a_{2r}\left(1 - \frac{p^a_{2r}}{\widetilde{p}^a_{2r}}\right)}{\beta_2 n^a_{1r}\left(1 - \frac{p^a_{1r}}{\widetilde{p}^a_{1r}}\right)} = \frac{\beta_1 n^a_{2r}\left(1 - \frac{cw^a_r}{W^a_{2r}\widetilde{p}^a_{2r}}\right)}{\beta_2 n^a_{1r}\left(1 - \frac{cw^a_r}{W^a_{1r}\widetilde{p}^a_{1r}}\right)}. \tag{39}$$

Plugging (39) into (38) and using the expressions for W_{1r}^a, W_{2r}^a, $\widetilde{p}_{1r}^a$ and $\widetilde{p}_{2r}^a$ that we have obtained above, the two equilibrium conditions (37) and (38) depend on n_{1r}^a and n_{2r}^a only. Letting

$$\kappa_{1r}^a \equiv \mathrm{e}^{\frac{F}{L_r}\nu(\alpha_1, L_r)} - 1 \quad \text{and} \quad \kappa_{2r}^a \equiv \mathrm{e}^{\frac{F}{L_r}\nu(\alpha_2, L_r)} - 1, \tag{40}$$

the two equations can be solved for the equilibrium masses of firms:

$$n_{1r}^a = \frac{L_{1r}^a}{L_r}\nu(\alpha_1, L_r) \quad \text{and} \quad n_{2r}^a = \frac{L_{2r}^a}{L_r}\nu(\alpha_2, L_r), \tag{41}$$

where

$$\frac{L_{1r}^a}{L_r} = \frac{\frac{\beta_1}{\kappa_{1r}^a}\nu(\alpha_1, L_r)}{\frac{\beta_1}{\kappa_{1r}^a}\nu(\alpha_1, L_r) + \frac{\beta_2}{\kappa_{2r}^a}\nu(\alpha_2, L_r)} \quad \text{and} \quad \frac{L_{2r}^a}{L_r} = \frac{\frac{\beta_2}{\kappa_{2r}^a}\nu(\alpha_2, L_r)}{\frac{\beta_1}{\kappa_{1r}^a}\nu(\alpha_1, L_r) + \frac{\beta_2}{\kappa_{2r}^a}\nu(\alpha_2, L_r)}. \tag{42}$$

are sectoral labor shares. We can show that, in equilibrium, less labor is allocated to the sector with the lower elasticity of demand (larger α) or the smaller weight on utility (smaller β).

Proposition 6 *Suppose that $\alpha_1 > \alpha_2$ and $\beta_1 = \beta_2$, or that $\alpha_1 = \alpha_2$ and $\beta_1 < \beta_2$. Then, less labor is allocated to sector 1 than to sector 2 in equilibrium, i.e., $L_{1r}^a < L_{2r}^a$.*

Proof. Taking the difference between L_{1r}^a and L_{2r}^a, and using (40), we have

$$\operatorname{sign}\left\{L_{1r}^a - L_{2r}^a\right\} = \operatorname{sign}\left\{\beta_1 \frac{\nu(\alpha_1, L_r)}{\mathrm{e}^{\frac{F}{L_r}\nu(\alpha_1, L_r)} - 1} - \beta_2 \frac{\nu(\alpha_2, L_r)}{\mathrm{e}^{\frac{F}{L_r}\nu(\alpha_2, L_r)} - 1}\right\}.$$

Since $\nu(\cdot, L_r)/[\mathrm{e}^{F\nu(\cdot, L_r)/L_r} - 1]$ is decreasing in ν and ν is increasing in its first argument, $\alpha_1 > \alpha_2$ and $\beta_1 = \beta_2$ imply $L_{1r}^a < L_{2r}^a$. When $\alpha_1 = \alpha_2$, we have $\nu(\alpha_1, L_r)/[\mathrm{e}^{F\nu(\alpha_1, L_r)/L_r} - 1] = \nu(\alpha_2, L_r)/[\mathrm{e}^{F\nu(\alpha_2, L_r)/L_r} - 1]$. Then, the relationship reduces to $\operatorname{sign}\{L_{1r}^a - L_{2r}^a\} = \operatorname{sign}\{\beta_1 - \beta_2\}$, thus completing the proof. ■

Since the quantities within each sector are symmetric in equilibrium and given by $q_{1r}^a = F\nu(\alpha_1, L_r)/(\alpha_1 L_r)$ and $q_{2r}^a = F\nu(\alpha_2, L_r)/(\alpha_2 L_r)$, the equilibrium U_{1r}^a and U_{2r}^a are written as

$$U_{1r}^a = n_{1r}^a(1 - \mathrm{e}^{-\alpha_1 q_{1r}^a}) = \frac{L_{1r}^a}{L_r}\nu(\alpha_1, L_r)\left[1 - \mathrm{e}^{-\frac{F}{L_r}\nu(\alpha_1, L_r)}\right] \tag{43}$$

$$U_{2r}^a = n_{2r}^a(1 - \mathrm{e}^{-\alpha_2 q_{2r}^a}) = \frac{L_{2r}^a}{L_r}\nu(\alpha_2, L_r)\left[1 - \mathrm{e}^{-\frac{F}{L_r}\nu(\alpha_2, L_r)}\right], \tag{44}$$

which finally yields the equilibrium utility

$$U_r^a = \beta_1 \ln\left\{\frac{L_{1r}^a}{L_r}\nu(\alpha_1, L_r)\left[1 - \mathrm{e}^{-\frac{F}{L_r}\nu(\alpha_1, L_r)}\right]\right\} + \beta_2 \ln\left\{\frac{L_{2r}^a}{L_r}\nu(\alpha_2, L_r)\left[1 - \mathrm{e}^{-\frac{F}{L_r}\nu(\alpha_2, L_r)}\right]\right\}.$$

The foregoing expression is an extension of the equilibrium utility in the single-sector case.

5.1.2 Optimum

We now investigate the optimal allocation. The planner maximizes utility subject to technology and the resource constraint in the two-sector economy. As firms are symmetric within each sector, the first-best problem is given by

$$\max_{q_{1r},q_{2r},n_{1r},n_{2r}} \quad U_r = U(U_{1r}(q_{1r}, n_{1r}), U_{2r}(q_{2r}, n_{2r})) \quad \text{s.t.} \quad L_r = n_{1r}\left(L_r c q_{1r} + F\right) + n_{2r}\left(L_r c q_{2r} + F\right).$$

As in the foregoing, we assume that $U_r \equiv \beta_1 \ln U_{1r} + \beta_2 \ln U_{2r}$ with $\beta_1 + \beta_2 = 1$. Letting λ be the Lagrange multiplier, the first-order conditions are then given by

$$\frac{\beta_1}{n_{1r}} \frac{\alpha_1 \mathrm{e}^{-\alpha_1 q_{1r}}}{1 - \mathrm{e}^{-\alpha_1 q_{1r}}} = \lambda L_r c \tag{45}$$

$$\frac{\beta_2}{n_{2r}} \frac{\alpha_2 \mathrm{e}^{-\alpha_2 q_{2r}}}{1 - \mathrm{e}^{-\alpha_2 q_{2r}}} = \lambda L_r c \tag{46}$$

$$\frac{\beta_1}{n_{1r}} = \lambda\left(L_r c q_{1r} + F\right) \tag{47}$$

$$\frac{\beta_2}{n_{2r}} = \lambda\left(L_r c q_{2r} + F\right). \tag{48}$$

Expressions (47) and (48), together with the resource constraint, yield

$$L_r = n_{1r}\left(L_r c q_{1r} + F\right) + n_{2r}\left(L_r c q_{2r} + F\right) = \frac{\beta_1}{\lambda} + \frac{\beta_2}{\lambda} = \frac{1}{\lambda}.$$

This result, together with (47) and (48), yields the optimal sectoral labor allocation as follows

$$L^o_{1r} = n_{1r}\left(L_r c q_{1r} + F\right) = \beta_1 L_r \quad \text{and} \quad L^o_{2r} = n_{2r}\left(L_r c q_{2r} + F\right) = \beta_2 L_r, \tag{49}$$

which implies $L^o_{1r}/L^o_{2r} = \beta_1/\beta_2$. Using the equilibrium and optimal labor allocation, (42) and (49), we can establish the following proposition.

Proposition 7 *Suppose that $\alpha_1 > \alpha_2$. Then, the equilibrium labor allocation in sector 1 is insufficient, whereas that in sector 2 is excessive, i.e., $L^a_{1r} < L^o_{1r}$ and $L^a_{2r} > L^o_{2r}$.*

Proof. From expressions (42) and (49), the difference between the equilibrium and optimal labor allocation is given by

$$L^a_{1r} - L^o_{1r} = \frac{\beta_1 \beta_2 L_r}{\frac{\beta_1}{\kappa^a_{1r}}\nu(\alpha_1, L_r) + \frac{\beta_2}{\kappa^a_{2r}}\nu(\alpha_2, L_r)} \left[\frac{\nu(\alpha_1, L_r)}{\kappa^a_{1r}} - \frac{\nu(\alpha_2, L_r)}{\kappa^a_{2r}}\right].$$

Using (40), we thus have

$$\text{sign}\{L^a_{1r} - L^o_{1r}\} = \text{sign}\left\{\frac{\nu(\alpha_1, L_r)}{\mathrm{e}^{\frac{F}{L_r}\nu(\alpha_1, L_r)} - 1} - \frac{\nu(\alpha_2, L_r)}{\mathrm{e}^{\frac{F}{L_r}\nu(\alpha_2, L_r)} - 1}\right\}.$$

Since $\nu(\cdot, L_r)/[\mathrm{e}^{F\nu(\cdot, L_r)/L_r} - 1]$ is decreasing in ν and ν is increasing in its first argument, $\alpha_1 > \alpha_2$ implies $L^a_{1r} < L^o_{1r}$. As $L^a_{1r} + L^a_{2r} = L^o_{1r} + L^o_{2r} = L_r$, we then also have $L^a_{2r} > L^o_{2r}$. ■

Using (45)–(48), we show in Appendix D that the optimal mass of firms in each sector and the optimal utility are given by

$$n_{1r}^o = -\frac{\alpha_1\beta_1}{c\left[1+W_{-1}\left(-\mathrm{e}^{-1-\frac{\alpha_1 F}{cL_r}}\right)\right]} > 0, \qquad n_{2r}^o = -\frac{\alpha_2\beta_2}{c\left[1+W_{-1}\left(-\mathrm{e}^{-1-\frac{\alpha_2 F}{cL_r}}\right)\right]} > 0 \tag{50}$$

and

$$U_r^o = \beta_1 \ln\left[-\frac{\alpha_1\beta_1}{cW_{-1}\left(-\mathrm{e}^{-1-\frac{\alpha_1 F}{cL_r}}\right)}\right] + \beta_2 \ln\left[-\frac{\alpha_2\beta_2}{cW_{-1}\left(-\mathrm{e}^{-1-\frac{\alpha_2 F}{cL_r}}\right)}\right], \tag{51}$$

where $W_{-1}(\cdot)$ is the real branch of the Lambert W function satisfying $W(-\mathrm{e}^{-1-\alpha F/(cL_r)}) \leq -1$. Note that expressions (50) and (51) are straightforward extensions of the single sector case.

We now consider whether or not there is excess entry. Since the equilibrium mass of firms in each sector (e.g., n_{1r}^a) depends on the characteristics of both sectors (e.g., both α_1 and α_2), the proof of Proposition 1 is not applicable. However, noting that the equilibrium quantity in each sector (e.g., q_{1r}^a) does not depend on the characteristics of the other sector (e.g., α_2), we can show that the firms in each sector operate at an inefficiently small scale in equilibrium. It is then readily verified that at least one sector displays excess entry by using sectoral labor misallocations established in Proposition 7.

Proposition 8 *The firms in both sectors operate at an inefficiently small scale in equilibrium, i.e., $Q_{1r}^a < Q_{1r}^o$ and $Q_{2r}^a < Q_{2r}^o$. Furthermore, there is excess entry in at least one sector, i.e., when $\alpha_1 > \alpha_2$, we have $n_{2r}^a > n_{2r}^o$, whereas we have $n_{1r}^a > n_{1r}^o$ when $\alpha_1 < \alpha_2$.*

Proof. From (45) and (47), the optimal quantity in sector 1 must satisfy

$$\mathrm{LHS}(q_{1r}^o, \alpha_1) = \frac{\mathrm{e}^{-\alpha_1 q_{1r}^o}}{1-\mathrm{e}^{-\alpha_1 q_{1r}^o}} = \frac{L_r c}{\alpha_1(L_r c q_{1r}^o + F)} = \mathrm{RHS}(q_{1r}^o, \alpha_1),$$

where both the LHS and RHS are decreasing in q_{1r}^o. Furthermore, we have $\lim_{q_{1r}^o \to 0} \mathrm{LHS} = \infty$, $\lim_{q_{1r}^o \to 0} \mathrm{RHS} = L_r c/(\alpha_1 F)$, and $\lim_{q_{1r}^o \to \infty} \mathrm{LHS} = \lim_{q_{1r}^o \to \infty} \mathrm{RHS} = 0$. Since q_{1r}^o is uniquely determined by (49) and (50), the RHS cuts the LHS only once from below. We now evaluate the LHS and RHS at the equilibrium value, $q_{1r}^a = (1-W_{1r}^a)/\alpha_1$, and show that $\mathrm{LHS}(q_{1r}^a, \alpha_1) > \mathrm{RHS}(q_{1r}^a, \alpha_1)$. Using (35) and the definition of ν, we can show that

$$\begin{aligned}
\mathrm{LHS}(q_{1r}^a, \alpha_1) - \mathrm{RHS}(q_{1r}^a, \alpha_1) &= \frac{1}{\mathrm{e}^{\frac{F}{L_r}\nu(\alpha_1, L_r)} - 1} - \frac{c}{\alpha_1}\nu(\alpha_1, L_r) \\
&= \frac{c}{\alpha_1}\frac{1}{\mathrm{e}^{\frac{F}{L_r}\nu(\alpha_1, L_r)} - 1}\left\{\frac{\alpha_1}{c} - \nu(\alpha_1, L_r)\left[\mathrm{e}^{\frac{F}{L_r}\nu(\alpha_1, L_r)} - 1\right]\right\}.
\end{aligned}$$

We can derive exactly the same relationship in the single sector case, except that α_1 is replaced with α. Since we know by Proposition 1 that $q_r^a < q_r^o$ holds in the single sector case, regardless of parameter values, $\mathrm{LHS}(q_r^a, \alpha) > \mathrm{RHS}(q_r^a, \alpha)$ must hold for any α. We thus have $\mathrm{LHS}(q_{1r}^a, \alpha_1) > \mathrm{RHS}(q_{1r}^a, \alpha_1)$. This establishes that $q_{1r}^a < q_{1r}^o$ and $Q_{1r}^a < Q_{1r}^o$ even in the two-sector case. Similarly, we can prove that $Q_{2r}^a < Q_{2r}^o$. Finally, when $\alpha_1 > \alpha_2$, we know by Proposition 7 that $n_{2r}^a(cQ_{2r}^a + F) = L_{2r}^a > L_{2r}^o = n_{2r}^o(cQ_{2r}^o + F)$. Noting that $Q_{2r}^a < Q_{2r}^o$, we have $n_{2r}^a > n_{2r}^o$. Similarly, when $\alpha_1 < \alpha_2$, we have $n_{1r}^a > n_{1r}^o$. ■

Unlike in the single-sector case, Proposition 8 states that *at least one sector* displays excess entry. In fact, we can construct numerical examples of insufficient entry in one sector when the other sector displays excess entry. Such insufficient entry arises due to intersectoral labor misallocations between the two differentiated sectors. For instance, when $\alpha_1 > \alpha_2$ we know by Proposition 7 that $n_{1r}^a(cQ_{1r}^a + F) = L_{1r}^a < L_{1r}^o = n_{1r}^o(cQ_{1r}^o + F)$, i.e., there is insufficient labor allocated to sector 1 in equilibrium. It is then possible to have insufficient entry in sector 1, $n_{1r}^a < n_{1r}^o$, even though the firms in sector 1 operate at an inefficiently small scale in equilibrium, $Q_{1r}^a < Q_{1r}^o$.[27]

5.2 Trade

We turn to the open economy version of our two-sector model. We first analyze the properties of equilibrium when both goods are freely traded. We then investigate an intermediate case where one good is freely traded, while the other is nontraded.

5.2.1 Free trade

The model involves a mixture of Sections 4 and 5.1. We denote sectors by subscripts 1 and 2, and countries by subscripts r and s. We assume that trade is free, and that markets are integrated. Since the price of each variety is the same in the two countries under the integrated market assumption, we can show that PPE and FPE hold by using the same technique as in the single-sector open economy case (see Appendix C). To alleviate notation, when there is no confusion, we provide expressions for country r only, with mirror expressions holding for country s. Starting with profits, we have:

$$
\begin{aligned}
\Pi_{1r}(i) &= [p_{1r}(i) - cw_r][L_r q_{1r}(i) + L_s q_{1s}(i)] - Fw_r \\
\Pi_{2r}(i) &= [p_{2r}(i) - cw_r][L_r q_{2r}(i) + L_s q_{2s}(i)] - Fw_r,
\end{aligned}
$$

where the expressions for quantities are analogous to those in (29). Noting that PPE and FPE hold under the integrated market assumption, and that $\widetilde{p}_{1r} = \widetilde{p}_{1s} = \widetilde{p}_1$ and $\widetilde{p}_{2r} = \widetilde{p}_{2s} = \widetilde{p}_2$, we can aggregate demand across countries. Hence, the price equilibrium is also analogous to (32) and (33) in Section 5.1. We further know from (34) that W_1 and W_2, and thus all prices and quantities are no longer country specific. Accordingly, the zero profit conditions are given by

$$
\begin{aligned}
\Pi_1 &= \frac{(L_r + L_s)cw}{\alpha_1}(W_1^{-1} + W_1 - 2) - Fw = 0 \\
\Pi_2 &= \frac{(L_r + L_s)cw}{\alpha_2}(W_2^{-1} + W_2 - 2) - Fw = 0,
\end{aligned}
$$

[27] In the two-sector case, pro-competitive effects in each sector do not necessarily map into excess entry in both sectors. Hence, the general tendency toward excess entry in partial equilibrium models (Vives, 1999, Ch.6) does not carry over to general equilibrium models with multiple sectors.

which, as in the foregoing, can be solved for W_1 and W_2 to yield:

$$W_1 = 1 - \frac{\sqrt{4\alpha_1 cF(L_r+L_s)+(\alpha_1 F)^2}-\alpha_1 F}{2c(L_r+L_s)} \in (0,1)$$
$$W_2 = 1 - \frac{\sqrt{4\alpha_2 cF(L_r+L_s)+(\alpha_2 F)^2}-\alpha_2 F}{2c(L_r+L_s)} \in (0,1).$$

By comparing the free trade and autarky values, we see that $1/W_1 < \min\{1/W^a_{1r}, 1/W^a_{1s}\}$ and $1/W_2 < \min\{1/W^a_{2r}, 1/W^a_{2s}\}$. In words, trade reduces markups in both sectors in both countries due to pro-competitive effects.

The trade equilibrium is, in turn, characterized by the following four conditions. First, the labor market in each country must clear, which requires that

$$L_r = n_{1r}\left[\frac{(L_r+L_s)c}{\alpha_1}(1-W_1)+F\right]+n_{2r}\left[\frac{(L_r+L_s)c}{\alpha_2}(1-W_2)+F\right] \tag{52}$$
$$L_s = n_{1s}\left[\frac{(L_r+L_s)c}{\alpha_1}(1-W_1)+F\right]+n_{2s}\left[\frac{(L_r+L_s)c}{\alpha_2}(1-W_2)+F\right]. \tag{53}$$

Second, the relationship between $\widetilde{p}_1$ and $\widetilde{p}_2$ is given as before by (38). The construction of the left-hand side of (38) under free trade is analogous to that in Section 5.1. The right-hand side of (38) is obtained by assuming again that $U \equiv \beta_1 \ln U_1 + \beta_2 \ln U_2$. We then have $(\partial U/\partial U_1)/(\partial U/\partial U_2) = \beta_1 U_2/(\beta_2 U_1)$, where the ratio is given by

$$\frac{\beta_1 U_2}{\beta_2 U_1} = \frac{\beta_1(n_{2r}+n_{2s})\left(1-\frac{p_2}{\widetilde{p}_2}\right)}{\beta_2(n_{1r}+n_{1s})\left(1-\frac{p_1}{\widetilde{p}_1}\right)} = \frac{\beta_1(n_{2r}+n_{2s})\left(1-\frac{cw}{W_2\widetilde{p}_2}\right)}{\beta_2(n_{1r}+n_{1s})\left(1-\frac{cw}{W_1\widetilde{p}_1}\right)}.$$

Finally, in the open economy, trade must balance, which requires that:

$$L_r\left[n_{1s}\frac{W_1^{-1}-1}{\alpha_1}+n_{2s}\frac{W_2^{-1}-1}{\alpha_2}\right] = L_s\left[n_{1r}\frac{W_1^{-1}-1}{\alpha_1}+n_{2r}\frac{W_2^{-1}-1}{\alpha_2}\right]. \tag{54}$$

The four equilibrium conditions (38) and (52)–(54) yield $\{n_{1r}, n_{2r}, n_{1s}, n_{2s}\}$ as follows:

$$n_{1r} = \frac{L_r}{L_r+L_s}\frac{L_{1r}}{L_r}\nu(\alpha_1, L_r+L_s) \qquad n_{1s} = \frac{L_s}{L_r+L_s}\frac{L_{1s}}{L_s}\nu(\alpha_1, L_r+L_s) \tag{55}$$
$$n_{2r} = \frac{L_r}{L_r+L_s}\frac{L_{2r}}{L_r}\nu(\alpha_2, L_r+L_s) \qquad n_{2s} = \frac{L_s}{L_r+L_s}\frac{L_{2s}}{L_s}\nu(\alpha_2, L_r+L_s), \tag{56}$$

where

$$\frac{L_{1r}}{L_r} = \frac{L_{1s}}{L_s} = \frac{\frac{\beta_1}{\kappa_1}\nu(\alpha_1, L_r+L_s)}{\frac{\beta_1}{\kappa_1}\nu(\alpha_1, L_r+L_s)+\frac{\beta_2}{\kappa_2}\nu(\alpha_2, L_r+L_s)} \tag{57}$$
$$\frac{L_{2r}}{L_r} = \frac{L_{2s}}{L_s} = \frac{\frac{\beta_2}{\kappa_2}\nu(\alpha_2, L_r+L_s)}{\frac{\beta_1}{\kappa_1}\nu(\alpha_1, L_r+L_s)+\frac{\beta_2}{\kappa_2}\nu(\alpha_2, L_r+L_s)}, \tag{58}$$

and

$$\kappa_1 = e^{\frac{F}{L_r+L_s}\nu(\alpha_1, L_r+L_s)}-1 \quad \text{and} \quad \kappa_2 = e^{\frac{F}{L_r+L_s}\nu(\alpha_2, L_r+L_s)}-1. \tag{59}$$

Hence, the population distribution between the two countries (e.g., $L_r/(L_r + L_s)$) and the sectoral labor allocation (e.g., L_{1r}/L_r) are crucial for the mass of firms in each sector in each country. Holding the sectoral labor allocation constant, one can readily verify that there is domestic exit in each sector due to trade, as in the single-sector case. However, trade alters the sectoral labor allocation. This strengthens domestic exit in the shrinking sector, but weakens the tendency toward domestic exit in the other sector. We still find domestic exit and variety expansion in both sectors in both countries.

Having extended our single-sector model to the two-sector setting, we now address various intersectoral issues. As shown in Section 5.1, the sectoral variables under autarky such as the relative mass of firms, n_{1r}^a/n_{2r}^a, the relative mass of workers, L_{1r}^a/L_{2r}^a, the relative output, Q_{1r}^a/Q_{2r}^a, as well as the relative price, p_{1r}^a/p_{2r}^a, depend on the country size L_r. However, we can show that trade eliminates these cross-country differences as follows.

Proposition 9 *Trade leads to structural convergence regardless of population sizes, L_r and L_s. The relative price, the relative output, the relative mass of firms, and the relative mass of workers are equalized between the two countries, i.e., $p_{1r}/p_{2r} = p_{1s}/p_{2s}$, $Q_{1r}/Q_{2r} = Q_{1s}/Q_{2s}$, $n_{1r}/n_{2r} = n_{1s}/n_{2s}$, and $L_{1r}/L_{2r} = L_{1s}/L_{2s}$ hold.*

Proof. The equalization of the relative price and the relative output can be obtained by noting that W_1 and W_2 depend only on the world population. Using expressions (55) to (58), it is readily verified that

$$\frac{n_{1r}}{n_{2r}} = \frac{n_{1s}}{n_{2s}} = \frac{\beta_1/\kappa_1}{\beta_2/\kappa_2}\left[\frac{\nu(\alpha_1, L_r + L_s)}{\nu(\alpha_2, L_r + L_s)}\right]^2,$$

and that

$$\frac{L_{1r}}{L_{2r}} = \frac{L_{1s}}{L_{2s}} = \frac{\beta_1/\kappa_1}{\beta_2/\kappa_2}\frac{\nu(\alpha_1, L_r + L_s)}{\nu(\alpha_2, L_r + L_s)},$$

where κ_1 and κ_2 are given by (59). Both expressions depend only on the world population $L_r + L_s$, thus completing the proof. ■

Proposition 9 states that countries having different sectoral compositions under autarky converge to the same industry structure under free trade. This is in sharp contrast to the prediction of Ricardian and Heckscher-Ohlin models that trade causes sectoral specialization. Furthermore, unlike the new trade theory that emphasizes intra-industry trade between similar countries, with similarity giving rise to more trade, Proposition 9 shows that countries become more similar due to trade, thus suggesting circular causation between similarity and intra-industry trade. Finally, an immediate corollary to this proposition is that, under free trade, the smaller country is a small-scale replica of the larger country, e.g., $n_{1r}/n_{1s} = n_{2r}/n_{2s} = L_r/L_s$. It should be noted, however, that this almost never holds under autarky, unlike in the CES case. The only exception that we observe $n_{1r}/n_{1s} = n_{2r}/n_{2s} = L_r/L_s$ even under autarky is the case of symmetric countries $L_r = L_s$.

5.2.2 Efficiency

We now analyze whether trade enhances efficiency in the two-sector model. As in the single-sector case, free trade amounts to increasing the population size. Yet, unlike in that case, we can address how trade affects the difference between the equilibrium and optimum through both intersectoral and intrasectoral allocations.

Concerning intersectoral allocations, we can show that intersectoral distortions are eliminated in the limit. In particular, when the population gets arbitrarily large in the integrated economy, intersectoral output, variety, and labor distortions vanish, i.e.,

$$\begin{aligned}
\lim_{L\to\infty}\frac{Q_1}{Q_2} &= \lim_{L\to\infty}\frac{Q_1^o}{Q_2^o} = \sqrt{\frac{\alpha_2}{\alpha_1}} \\
\lim_{L\to\infty}\frac{n_1}{n_2} &= \lim_{L\to\infty}\frac{n_1^o}{n_2^o} = \sqrt{\frac{\alpha_1}{\alpha_2}}\frac{\beta_1}{\beta_2} \\
\lim_{L\to\infty}\frac{L_1}{L_2} &= \lim_{L\to\infty}\frac{L_1^o}{L_2^o} = \frac{\beta_1}{\beta_2}.
\end{aligned}$$

Turning to intrasectoral distortions, it is verified that $\lim_{L\to 0}(n_1/n_1^o) = \lim_{L\to 0}(n_2/n_2^o) = 1$, and that $\lim_{L\to\infty}(n_1/n_1^o) = \lim_{L\to\infty}(n_2/n_2^o) = \sqrt{2}$, as in the single-sector case. By continuity, for a sufficiently small population size, excess entry tends to be small, whereas it gets larger when the population gets arbitrarily large. Despite excess entry in the limit in both sectors, we can show the following overall efficiency result.

Proposition 10 *When the population gets arbitrarily large, the equilibrium utility converges to the optimal utility, i.e.,*

$$\lim_{L\to\infty} U = \lim_{L\to\infty} U^o = \beta_1 \ln\frac{\alpha_1\beta_1}{c} + \beta_2 \ln\frac{\alpha_2\beta_2}{c}.$$

Proof. In this context, U_1 and U_2 in equilibrium can be expressed as in (43) and (44). Since we already know by Proposition 4 that $\lim_{L\to\infty}\nu(\alpha_1, L)[1 - \mathrm{e}^{-F\nu(\alpha_1,L)/L}] = \alpha_1/c$ and $\lim_{L\to\infty}\nu(\alpha_2, L)[1 - \mathrm{e}^{-F\nu(\alpha_2,L)/L}] = \alpha_2/c$, we now establish the limit of L_1/L and L_2/L. They are given by

$$\lim_{L\to\infty}\frac{L_1}{L} = \beta_1 \quad \text{and} \quad \lim_{L\to\infty}\frac{L_2}{L} = \beta_2.$$

Hence, it is readily verified that $\lim_{L\to\infty} U_1 = \alpha_1\beta_1/c$ and $\lim_{L\to\infty} U_2 = \alpha_2\beta_2/c$, so that, using the chain rule for the limits of continuous functions, we have:

$$\lim_{L\to\infty} U = \beta_1 \ln\frac{\alpha_1\beta_1}{c} + \beta_2 \ln\frac{\alpha_2\beta_2}{c}.$$

Turning to the optimum, since $W_{-1}(-\mathrm{e}^{-1}) = -1$, it is readily verified that $\lim_{L\to\infty} U_1^o = \alpha_1\beta_1/c$ and $\lim_{L\to\infty} U_2^o = \alpha_2\beta_2/c$, so that

$$\lim_{L\to\infty} U^o = \beta_1 \ln\frac{\alpha_1\beta_1}{c} + \beta_2 \ln\frac{\alpha_2\beta_2}{c}.$$

Therefore, the equilibrium is efficient in the limit, which proves our claim. ■

Thus, our overall efficiency result in the single-sector case carries over to the two-sector case as intersectoral distortions vanish in the limit.

5.2.3 Nontraded good

Assume now that sector 1 produces a nontraded good, whereas sector 2 produces a freely traded good. Each good is differentiated as before. The objective of this subsection is to show that trade in sector 2 induces domestic exit in the nontraded sector 1. Given this objective, it is sufficient to focus on the simple case where the two countries are symmetric ($L_r = L_s = L$). Then, we can show again that PPE and FPE hold. Since the expressions for the autarky case are the same as those in Section 5.1, we only derive the expressions for the open economy. The price equilibrium is analogous to the previous cases, which yields the zero profit conditions:

$$\begin{aligned}\Pi_1 &= \frac{Lcw}{\alpha_1}(W_1^{-1} + W_1 - 2) - Fw = 0 \\ \Pi_2 &= \frac{2Lcw}{\alpha_2}(W_2^{-1} + W_2 - 2) - Fw = 0.\end{aligned}$$

From those conditions, we obtain

$$\begin{aligned}W_1 &= 1 - \frac{\sqrt{4\alpha_1 cFL + (\alpha_1 F)^2} - \alpha_1 F}{2cL} \in (0,1) \\ W_2 &= 1 - \frac{\sqrt{8\alpha_2 cFL + (\alpha_2 F)^2} - \alpha_2 F}{4cL} \in (0,1).\end{aligned}$$

Observe that the market size for sector 2 is doubled, whereas that for sector 1 is unchanged.

The trade equilibrium is characterized by the following conditions. First, the labor market must clear in each country. Letting $n_{1r} = n_{1s} = n_1$ and $n_{2r} = n_{2s} = n_2$ by symmetry, this requires that

$$L = n_1 \left[\frac{cL}{\alpha_1}(1 - W_1) + F\right] + n_2 \left[\frac{2cL}{\alpha_2}(1 - W_2) + F\right].$$

Second, the relationship between the two reservation prices is again given by (38). The construction of the left-hand side of (38) in the open economy is analogous to that in Section 5.1. The right-hand side of (38) is obtained by assuming as before that $U \equiv \beta_1 \ln U_1 + \beta_2 \ln U_2$. We then have $(\partial U/\partial U_1)/(\partial U/\partial U_2) = \beta_1 U_2/(\beta_2 U_1)$, where the ratio is given by

$$\frac{\beta_1 U_2}{\beta_2 U_1} = \frac{2\beta_1 n_2 \left(1 - \frac{p_2}{\tilde{p}_2}\right)}{\beta_2 n_1 \left(1 - \frac{p_1}{\tilde{p}_1}\right)} = \frac{2\beta_1 n_2 \left(1 - \frac{cw}{W_2 \tilde{p}_2}\right)}{\beta_2 n_1 \left(1 - \frac{cw}{W_1 \tilde{p}_1}\right)}.$$

Finally, trade is balanced because $L_r = L_s = L$, $n_{2s} = n_{2r} = n_2$, and FPE and PPE hold. From these conditions we obtain $\{n_1, n_2\}$ as follows:

$$\begin{aligned}n_1 &= \frac{L_1}{L}\nu(\alpha_1, L) = \frac{2\frac{\beta_1}{\kappa_1^a}\nu(\alpha_1, L)}{2\frac{\beta_1}{\kappa_1^a}\nu(\alpha_1, L) + \frac{\beta_2}{\kappa_2}\nu(\alpha_2, 2L)}\nu(\alpha_1, L) \\ n_2 &= \frac{1}{2}\frac{L_2}{L}\nu(\alpha_2, 2L) = \frac{\frac{\beta_2}{\kappa_2}\nu(\alpha_2, 2L)}{2\frac{\beta_1}{\kappa_1^a}\nu(\alpha_1, L) + \frac{\beta_2}{\kappa_2}\nu(\alpha_2, 2L)}\nu(\alpha_2, 2L).\end{aligned}$$

We can then establish the following results.

Proposition 11 *Trade induces domestic exit in the nontraded good sector, i.e.,* $n_1 < n_1^a$*. Put differently, there is a consumption variety loss in the nontraded good sector.*

Proof. The mass of firms in the nontraded good sector under autarky is given by

$$n_1^a = \frac{\frac{\beta_1}{\kappa_1^a}\nu(\alpha_1, L)}{\frac{\beta_1}{\kappa_1^a}\nu(\alpha_1, L) + \frac{\beta_2}{\kappa_2^a}\nu(\alpha_2, L)}\nu(\alpha_1, L) = \frac{\frac{\beta_1}{\kappa_1^a}\frac{\nu(\alpha_1,L)}{L}}{\frac{\beta_1}{\kappa_1^a}\frac{\nu(\alpha_1,L)}{L} + \frac{\beta_2}{\kappa_2^a}\frac{\nu(\alpha_2,L)}{L}}\nu(\alpha_1, L),$$

whereas in the open economy it becomes

$$n_1 = \frac{2\frac{\beta_1}{\kappa_1^a}\nu(\alpha_1, L)}{2\frac{\beta_1}{\kappa_1^a}\nu(\alpha_1, L) + \frac{\beta_2}{\kappa_2}\nu(\alpha_2, 2L)}\nu(\alpha_1, L) = \frac{\frac{\beta_1}{\kappa_1^a}\frac{\nu(\alpha_1,L)}{L}}{\frac{\beta_1}{\kappa_1^a}\frac{\nu(\alpha_1,L)}{L} + \frac{\beta_2}{\kappa_2}\frac{\nu(\alpha_2,2L)}{2L}}\nu(\alpha_1, L).$$

The only change between autarky and free trade in sector 2 appears in the second term of the denominator. Let $\omega \equiv \nu(\alpha_2, 2L)/(2L)$. Noting that

$$\frac{1}{\kappa_2}\frac{\nu(\alpha_2, 2L)}{2L} = \frac{1}{\mathrm{e}^{F\frac{\nu(\alpha_2,2L)}{2L}} - 1}\frac{\nu(\alpha_2, 2L)}{2L} = \frac{\omega}{\mathrm{e}^{F\omega} - 1}$$

decreases with ω, and that ω decreases with the market size for sector 2, we get $n_1 < n_1^a$. ■

Proposition 12 *The mass of varieties consumed in the traded good sector increases, i.e.,* $2n_2 > n_2^a$*. Furthermore, domestic entry occurs in the traded good sector, i.e., the mass of varieties produced in the traded sector in each country increases.*

Proof. The mass of varieties consumed in the traded good sector under autarky is given by

$$n_2^a = \frac{\frac{\beta_2}{\kappa_2^a}\nu(\alpha_2, L)}{\frac{\beta_1}{\kappa_1^a}\nu(\alpha_1, L) + \frac{\beta_2}{\kappa_2^a}\nu(\alpha_2, L)}\nu(\alpha_2, L) = \frac{\frac{\beta_2}{\kappa_2^a}\frac{\nu(\alpha_2,L)}{L}}{\frac{\beta_1}{\kappa_1^a}\frac{\nu(\alpha_1,L)}{L} + \frac{\beta_2}{\kappa_2^a}\frac{\nu(\alpha_2,L)}{L}}\nu(\alpha_2, L),$$

whereas in the open economy it becomes

$$2n_2 = 2\frac{\frac{\beta_2}{\kappa_2}\nu(\alpha_2, 2L)}{2\frac{\beta_1}{\kappa_1^a}\nu(\alpha_1, L) + \frac{\beta_2}{\kappa_2}\nu(\alpha_2, 2L)}\nu(\alpha_2, 2L) = \frac{\frac{\beta_2}{\kappa_2}\frac{\nu(\alpha_2,2L)}{2L}}{\frac{\beta_1}{\kappa_1^a}\frac{\nu(\alpha_1,L)}{L} + \frac{\beta_2}{\kappa_2}\frac{\nu(\alpha_2,2L)}{2L}}[2\nu(\alpha_2, 2L)].$$

Using the same notation as in the previous proposition, and noting that $2\nu(\alpha_2, 2L) > \nu(\alpha_2, L)$, we obtain the first claim. Turning to domestic entry, we have

$$n_2 = \frac{\frac{\beta_2}{\kappa_2}\nu(\alpha_2, 2L)}{2\frac{\beta_1}{\kappa_1^a}\nu(\alpha_1, L) + \frac{\beta_2}{\kappa_2}\nu(\alpha_2, 2L)}\nu(\alpha_2, 2L) = \frac{\frac{\beta_2}{\kappa_2}\frac{\nu(\alpha_2,2L)}{2L}}{\frac{\beta_1}{\kappa_1^a}\frac{\nu(\alpha_1,L)}{L} + \frac{\beta_2}{\kappa_2}\frac{\nu(\alpha_2,2L)}{2L}}\nu(\alpha_2, 2L).$$

Again, using the same notation as in the previous proposition, we can show that $n_2 > n_2^a$. ■

These two propositions have important implications for assessing gains from trade in the presence of nontraded goods. Although trade expands the range of varieties that consumers face in the traded good sector, it induces domestic exit, or a variety loss in the nontraded sector. Unlike the single sector case with a traded good, such a variety loss in the nontraded sector is not compensated by import varieties. Accordingly, monopolistic competition models that abstract from nontraded varieties may overestimate gains from trade.

Furthermore, our results suggest that, contrary to general belief, VES models *per se* may not explain domestic exit in the traded good sector when there are nontraded varieties. This reconfirms the importance of other factors such as firm heterogeneity in explaining domestic exit in the traded good sector. Extending the two-sector CES model with firm heterogeneity by Bernard *et al.* (2007) to a VES setting or extending the one-sector VES model with firm heterogeneity by Dhingra and Morrow (2011) to a multi-sector setting seems to be a promising step toward a better understanding of this issue.

6 Conclusions

We have developed a VES model of international trade displaying pro-competitive effects and a competitive limit, and have investigated the impacts of trade on welfare and efficiency. Unlike the standard CES model, our framework allows us to capture the impacts of trade on varieties, markups, and exploitation of scale economies without resorting to an additively separable numeraire good. The welfare decomposition helps us to understand the relative contribution of product diversity and pro-competitive effects to gains from trade, which is becoming increasingly more important given the recent empirical evidence such as Broda and Weinstein (2006) and Feenstra and Weinstein (2010). We have also explored whether or not trade is ultimately efficiency enhancing, and have shown that this is indeed the case.

The basic framework presented in this paper is flexible enough to allow for many extensions. In this paper, we have provided one, namely a multi-sector setting.[28] In our model, trade tends to reduce within-sector efficiency losses, which disappear at the competitive limit. However, as is well known, markup heterogeneity across sectors is a source of between-sector distortions (Bilbiie *et al.*, 2008; Epifani and Gancia, 2011), which may become more important with freer trade. Yet, we have established that such intersectoral distortions also vanish when the population gets arbitrarily large in the integrated economy. Our multi-sector analysis further illustrates some new aspects arising from the interaction between intersectoral and intrasectoral allocations, namely structural convergence, rather than sectoral specialization, and domestic exit in the nontraded good sector induced by trade in the other sector.

Finally, as we have obtained the closed form solutions for the equilibrium utility and the optimal utility, our model can also be extended to a multi-region setting in a spatial economy.

[28]Other extensions include, for instance, heterogeneous firms and trade costs (Behrens *et al.*, 2009) and heterogeneous consumers (Behrens and Murata, 2009).

Doing so sheds new light on whether or not larger cities are more efficient. In such a setting, a larger population not only allows for greater diversity but also exacerbates congestion in cities while achieving prices closer to marginal costs. Although we have established efficiency gains from trade in this paper, it is not obvious whether or not our efficiency result carries over to a spatial economy with urban congestion. Exploring this formally is left for future research.

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Appendix A. Computations for equilibrium

A.1. Demand functions

A representative consumer in country r solves problem (1). Letting λ stand for the Lagrange multiplier, the first-order conditions for an interior solution are given by:

$$\alpha \mathrm{e}^{-\alpha q_{rr}(i)} = \lambda p_r(i), \qquad \forall i \in \Omega_r \tag{A.1}$$

$$\alpha \mathrm{e}^{-\alpha q_{sr}(j)} = \lambda p_s(j), \qquad \forall j \in \Omega_s \tag{A.2}$$

and the budget constraint

$$\int_{\Omega_r} p_r(i) q_{rr}(i) \mathrm{d}i + \int_{\Omega_s} p_s(j) q_{sr}(j) \mathrm{d}j = E_r. \tag{A.3}$$

Taking the ratio of (A.1) with respect to i and j, we obtain

$$\mathrm{e}^{-\alpha[q_{rr}(i)-q_{rr}(j)]}=\frac{p_r(i)}{p_r(j)} \quad\Longrightarrow\quad q_{rr}(i)=q_{rr}(j)+\frac{1}{\alpha}\ln\left[\frac{p_r(j)}{p_r(i)}\right] \quad \forall i,j\in\Omega_r.$$

Multiplying the last expression by $p_r(j)$ and integrating with respect to $j\in\Omega_r$ we obtain:

$$q_{rr}(i)\int_{\Omega_r}p_r(j)\mathrm{d}j=\int_{\Omega_r}p_r(j)q_{rr}(j)\mathrm{d}j+\frac{1}{\alpha}\int_{\Omega_r}\ln\left[\frac{p_r(j)}{p_r(i)}\right]p_r(j)\mathrm{d}j. \tag{A.4}$$

Analogously, taking the ratio of (A.1) and (A.2) with respect to i and j, we get:

$$\mathrm{e}^{-\alpha[q_{rr}(i)-q_{sr}(j)]}=\frac{p_r(i)}{p_s(j)} \quad\Longrightarrow\quad q_{rr}(i)=q_{sr}(j)+\frac{1}{\alpha}\ln\left[\frac{p_s(j)}{p_r(i)}\right] \quad \forall i\in\Omega_r,\forall j\in\Omega_s.$$

Multiplying the last expression by $p_s(j)$ and integrating with respect to $j\in\Omega_s$ we obtain:

$$q_{rr}(i)\int_{\Omega_s}p_s(j)\mathrm{d}j=\int_{\Omega_s}p_s(j)q_{sr}(j)\mathrm{d}j+\frac{1}{\alpha}\int_{\Omega_s}\ln\left[\frac{p_s(j)}{p_r(i)}\right]p_s(j)\mathrm{d}j. \tag{A.5}$$

Summing (A.4) and (A.5), and using the budget constraint (A.3) yield

$$q_{rr}(i)=\frac{E_r-\dfrac{1}{\alpha}\displaystyle\int_{\Omega_r}\ln\left[\frac{p_r(i)}{p_r(j)}\right]p_r(j)\mathrm{d}j-\dfrac{1}{\alpha}\displaystyle\int_{\Omega_s}\ln\left[\frac{p_r(i)}{p_s(j)}\right]p_s(j)\mathrm{d}j}{\displaystyle\int_{\Omega_r}p_r(j)\mathrm{d}j+\int_{\Omega_s}p_s(j)\mathrm{d}j}.$$

Finally, noting the definitions of P and H given in the main text, we obtain the demands (2). The derivations of the demands (3) are analogous.

A.2. Price equilibrium

Behrens and Murata (2007, Proposition 2) show that the price equilibrium is symmetric and unique. Given that result, expression (11) is obtained as follows. Plugging $Q_r(i)\equiv L_rq_{rr}(i)+L_sq_{rs}(i)$ into (6), and setting $q_{rs}(i)=\partial q_{rs}(i)/\partial p_r(i)=0$, we have

$$\frac{\partial\Pi_r(i)}{\partial p_r(i)}=L_rq_{rr}(i)+[p_r(i)-cw_r]L_r\frac{\partial q_{rr}(i)}{\partial p_r(i)}=0. \tag{A.6}$$

Noting that $\partial q_{rr}(i)/\partial p_r(i)=-1/[\alpha p_r(i)]$ by (4), imposing symmetry on (A.6) implies

$$\frac{p_r-cw_r}{\alpha p_r}=q_{rr}=-\frac{1}{\alpha}\ln p_r+\frac{E_r}{P}+\frac{1}{\alpha}\frac{H}{P},$$

where we use (2) to get the last equality. Since $P=n_rp_r$ and $H=n_rp_r\ln p_r$ because of symmetry, it can be readily verified that

$$p_r=cw_r+\frac{\alpha E_r}{n_r}.$$

Noting that $E_r=w_r$ in equilibrium as profits are zero due to free entry, we obtain (11).

A.3. Two-sector demand functions and reservation prices

The first-order conditions for an interior solution of the utility maximization problem (28) are given by:

$$\begin{aligned}
\frac{\partial U_r}{\partial U_{1r}}\alpha_1 \mathrm{e}^{-\alpha_1 q_{1r}(i)} &= \lambda p_{1r}(i), \quad \forall i \in \Omega_{1r} \\
\frac{\partial U_r}{\partial U_{2r}}\alpha_2 \mathrm{e}^{-\alpha_2 q_{2r}(j)} &= \lambda p_{2r}(j), \quad \forall j \in \Omega_{2r},
\end{aligned}$$

where λ is the Lagrange multiplier. Applying the same technique as that we used for the single-sector case in Appendix A.1, we readily obtain the demands as follows:

$$q_{1r}(i) = -\frac{1}{\alpha_1}\ln p_{1r}(i) + \frac{E_r}{\mathbb{P}_r} + \frac{1}{\alpha_1}\frac{\mathbb{H}_r}{\mathbb{P}_r} - \frac{1}{\alpha_2\mathbb{P}_r}\ln\left(\frac{\alpha_2}{\alpha_1}\frac{\partial U_r/\partial U_{2r}}{\partial U_r/\partial U_{1r}}\right)\int_{\Omega_{2r}} p_{2r}(j)\mathrm{d}j \tag{A.7}$$

$$q_{2r}(j) = -\frac{1}{\alpha_2}\ln p_{2r}(j) + \frac{\alpha_1}{\alpha_2}\frac{E_r}{\mathbb{P}_r} + \frac{1}{\alpha_2}\frac{\mathbb{H}_r}{\mathbb{P}_r} + \frac{1}{\alpha_2\mathbb{P}_r}\ln\left(\frac{\alpha_2}{\alpha_1}\frac{\partial U_r/\partial U_{2r}}{\partial U_r/\partial U_{1r}}\right)\int_{\Omega_{1r}} p_{1r}(i)\mathrm{d}i, \tag{A.8}$$

where

$$\begin{aligned}
\mathbb{P}_r &\equiv \int_{\Omega_{1r}} p_{1r}(i)\mathrm{d}i + \frac{\alpha_1}{\alpha_2}\int_{\Omega_{2r}} p_{2r}(j)\mathrm{d}j \\
\mathbb{H}_r &\equiv \int_{\Omega_{1r}} p_{1r}(i)\ln p_{1r}(i)\mathrm{d}i + \frac{\alpha_1}{\alpha_2}\int_{\Omega_{2r}} p_{2r}(j)\ln p_{2r}(j)\mathrm{d}j
\end{aligned}$$

are the sum of prices and a measure of price dispersion in the two-sector economy. The demands (A.7) and (A.8) can then be rewritten as

$$\begin{aligned}
q_{1r}(i) &= \frac{1}{\alpha_1}\ln\left[\frac{\widetilde{p}_{1r}}{p_{1r}(i)}\right], \quad \text{where} \quad \widetilde{p}_{1r} \equiv \mathrm{e}^{\frac{\alpha_1 E_r}{\mathbb{P}_r} + \frac{\mathbb{H}_r}{\mathbb{P}_r} - \frac{\alpha_1}{\alpha_2\mathbb{P}_r}\ln\left(\frac{\alpha_2}{\alpha_1}\frac{\partial U_r/\partial U_{2r}}{\partial U_r/\partial U_{1r}}\right)\int_{\Omega_{2r}} p_{2r}(j)\mathrm{d}j} \\
q_{2r}(j) &= \frac{1}{\alpha_2}\ln\left[\frac{\widetilde{p}_{2r}}{p_{2r}(j)}\right], \quad \text{where} \quad \widetilde{p}_{2r} \equiv \mathrm{e}^{\frac{\alpha_1 E_r}{\mathbb{P}_r} + \frac{\mathbb{H}_r}{\mathbb{P}_r} + \frac{1}{\mathbb{P}_r}\ln\left(\frac{\alpha_2}{\alpha_1}\frac{\partial U_r/\partial U_{2r}}{\partial U_r/\partial U_{1r}}\right)\int_{\Omega_{1r}} p_{1r}(i)\mathrm{d}i}.
\end{aligned}$$

Observe that $\widetilde{p}_{1r}$ and $\widetilde{p}_{2r}$ are common to all firms within each sector, and taken as given by each firm because of the continuum assumption. Taking the ratio of $\widetilde{p}_{1r}$ and $\widetilde{p}_{2r}$, we obtain

$$\frac{\widetilde{p}_{1r}}{\widetilde{p}_{2r}} = \mathrm{e}^{-\frac{\alpha_1}{\alpha_2}\frac{1}{\mathbb{P}_r}\ln\left(\frac{\alpha_2}{\alpha_1}\frac{\partial U_r/\partial U_{2r}}{\partial U_r/\partial U_{1r}}\right)\int_{\Omega_{2r}} p_{2r}(j)\mathrm{d}j - \frac{1}{\mathbb{P}_r}\ln\left(\frac{\alpha_2}{\alpha_1}\frac{\partial U_r/\partial U_{2r}}{\partial U_r/\partial U_{1r}}\right)\int_{\Omega_{1r}} p_{1r}(i)\mathrm{d}i} = \frac{\alpha_1}{\alpha_2}\frac{\partial U_r/\partial U_{1r}}{\partial U_r/\partial U_{2r}}.$$

A.4. Profit-maximization and properties of W

The first-order conditions $\partial\Pi_{1r}/\partial p_{1r}(i) = 0$ are given by

$$\ln\left[\frac{\widetilde{p}_{1r}}{p_{1r}(i)}\right] = 1 - \frac{cw_r}{p_{1r}(i)}.$$

Taking the exponential of both sides and rearranging terms, we have

$$\mathrm{e}\frac{cw_r}{\widetilde{p}_{1r}} = \frac{cw_r}{p_{1r}(i)}\mathrm{e}^{\frac{cw_r}{p_{1r}(i)}}.$$

Noting that the Lambert W function is defined as $\varphi = W(\varphi)\mathrm{e}^{W(\varphi)}$ and setting $\varphi = \mathrm{e}\, cw_r/\widetilde{p}_{1r}$, we obtain $W(\mathrm{e}\, cw_r/\widetilde{p}_{1r}) = cw_r/p_{1r}(i)$, which implies the expression of p^a_{1r} as given in (32). Combining the first-order conditions and demand functions, quantities are given by $q_{1r}(i) = (1/\alpha_1)\,[1 - cw_r/p_{1r}(i)]$. Plugging $p_{1r}(i) = p^a_{1r}$, we have the expression for q^a_{1r}. The expression for operating profits is given by $\pi_{1r}(i) = [p_{1r}(i) - cw_r]\, L_r q_{1r}(i)$, which together with p^a_{1r} and q^a_{1r}, yields π^a_{1r}. Mirror expressions hold for sector 2.

Turning to the properties of the Lambert W function, $\varphi = W(\varphi)\mathrm{e}^{W(\varphi)}$ implies that $W(\varphi) \geq 0$ for all $\varphi \geq 0$. Taking the logarithm of both sides and differentiating yield

$$W'(\varphi) = \frac{W(\varphi)}{\varphi[W(\varphi) + 1]} > 0, \quad \forall \varphi > 0.$$

Finally, $0 = W(0)\mathrm{e}^{W(0)}$ and $\mathrm{e} = W(\mathrm{e})\mathrm{e}^{W(\mathrm{e})}$ imply $W(0) = 0$ and $W(\mathrm{e}) = 1$, respectively.

A.5. Application of Lambert W to the single-sector model

Take the benchmark case of a single sector (say sector 1), as developed in Sections 2 and 3. Then, solving the zero profit condition $\pi_{1r} = Fw_r$ for W_{1r} yields

$$W_{1r} = 1 - \frac{\sqrt{4\alpha_1 cFL_r + (\alpha_1 F)^2} - \alpha_1 F}{2cL_r}.$$

Substituting this expression into the labor market clearing condition

$$n_{1r}\,(L_r c q_{1r} + F) = L_r \quad \Rightarrow \quad n_{1r}\left[\frac{L_r c}{\alpha_1}(1 - W_{1r}) + F\right] = L_r$$

yields

$$n_{1r} = \frac{\sqrt{4\alpha_1 cFL_r + (\alpha_1 F)^2} - \alpha_1 F}{2cF},$$

which is exactly the same as (15) in the single sector case.

Appendix B: Proof of Proposition 1

We compare the equilibrium n^a_r with the optimum n^o_r defined as the solution of the equation:

$$f(n_r) = g(n_r), \quad \text{where} \quad f(n_r) \equiv \frac{cn_r}{\alpha + cn_r} \quad \text{and} \quad g(n_r) \equiv \mathrm{e}^{-\frac{\alpha}{c}\left(\frac{1}{n_r} - \frac{F}{L_r}\right)}. \tag{B.1}$$

Note first that f is strictly increasing in n_r, taking values from 0 to 1, and that g is also strictly increasing, taking values from 0 to $\mathrm{e}^{\alpha F/(cL_r)} > 1$. Some standard calculations show that there is a unique intersection since: (i) both functions are continuous; (ii) f is concave, whereas g is convex for n_r sufficiently small; (iii) the slope of f is strictly greater than that of g for n_r

sufficiently small;[29] and (iv) g admits a single value for which its second-order derivative is equal to zero.

We next show that $n_r^a > n_r^o$. To prove our claim, we use a convexity argument. The equilibrium mass of varieties is given by (15), whereas the optimal mass of varieties is the unique solution to (B.1). First, evaluate f at n_r^a, which yields

$$f(n_r^a) = \frac{cn_r^a}{\alpha + cn_r^a} = \frac{\sqrt{4\alpha cFL_r + (\alpha F)^2} - \alpha F}{\sqrt{4\alpha cFL_r + (\alpha F)^2} + \alpha F} = \frac{X_r - 2\alpha F}{X_r}, \tag{B.2}$$

where $X_r \equiv \sqrt{4\alpha cFL_r + (\alpha F)^2} + \alpha F$. Second, evaluate g at n_r^a to get

$$g(n_r^a) = \mathrm{e}^{-\frac{\alpha}{c}\left(\frac{1}{n_r^a} - \frac{F}{L_r}\right)} = \mathrm{e}^{-\frac{2\alpha F}{X_r}}. \tag{B.3}$$

Let $Y_r \equiv (2\alpha F)/X_r < 1$ and $g(Y_r) = \mathrm{e}^{-Y_r}$. Note that (B.2) can then be expressed as $f(Y_r) = 1 - Y_r$, which is tangent to (B.3) at $Y_r = 0$:

$$1 - Y_r = g(0) + g'(0)(Y_r - 0).$$

Since (B.3) is strictly convex, it lies strictly above its tangent. Put differently, $f(Y_r) = 1 - Y_r < \mathrm{e}^{-Y_r} = g(Y_r)$ holds for all $Y_r > 0$ (see Figure A1). Hence, the right-hand side of (B.1) exceeds the left-hand side of (B.1) at the equilibrium mass of firms n_r^a. By uniqueness of the optimal mass of firms, and since the right-hand side of (B.1) exceeds the left-hand side if and only if $n_r > n_r^o$, we may conclude that $n_r^a > n_r^o$ (see Figure A2). Expression (8) then implies that $n_r^a(cQ_r^a + F) = n_r^o(cQ_r^o + F) = L_r$, which yields $Q_r^a < Q_r^o$.

[**Insert Figures A1 and A2 around here**]

Appendix C: Proof of Proposition 2

Conditions (6) and (7) must hold for both country-r and country-s firms at every price equilibrium which, using (2)–(4), yields

$$\frac{\partial \Pi_r(i)}{\partial p_r(i)} - \frac{\partial \Pi_s(j)}{\partial p_s(j)} = 0 \quad \Longleftrightarrow \quad c\left[\frac{w_r}{p_r(i)} - \frac{w_s}{p_s(j)}\right] = \ln\left[\frac{p_r(i)}{p_s(j)}\right]. \tag{C.1}$$

[29] To check this, note that $\lim_{n_r \to 0} f'(n_r) = c/\alpha > \lim_{n_r \to 0} g'(n_r) = 0$. The last equality is obtained as follows. Noting that

$$\ln g'(n_r) = -\frac{2}{n_r}\left[\frac{\ln(n_r)}{1/n_r} + \frac{\alpha}{2c}\right] + \ln\left(\frac{\alpha}{c}\right) + \frac{\alpha F}{cL_r},$$

and that $\lim_{n_r \to 0} \ln n_r/(1/n_r) = 0$ by l'Hospital's rule, we have

$$\lim_{n_r \to 0} \ln g'(n_r) = -\lim_{n_r \to 0} \frac{2}{n_r} \times \lim_{n_r \to 0}\left[\frac{\ln(n_r)}{1/n_r} + \frac{\alpha}{2c}\right] + \ln\left(\frac{\alpha}{c}\right) + \frac{\alpha F}{cL_r} = -\infty,$$

which, by continuity of the logarithmic function, implies $\lim_{n_r \to 0} g'(n_r) = 0$.

It is also readily verified that

$$Q_r(i) \gtreqqless Q_s(j) \quad \Longleftrightarrow \quad -\frac{L_r + L_s}{\alpha} \ln\left[\frac{p_r(i)}{p_s(j)}\right] \gtreqqless 0. \tag{C.2}$$

Furthermore, an equilibrium is such that firms earn zero profit, i.e.,

$$\begin{aligned} \Pi_r(i) &= w_r \left\{ \left[\frac{p_r(i)}{w_r} - c\right] Q_r(i) - F \right\} = 0 \\ \Pi_s(j) &= w_s \left\{ \left[\frac{p_s(j)}{w_s} - c\right] Q_s(j) - F \right\} = 0. \end{aligned}$$

Assume that there exists $i \in \Omega_r$ and $j \in \Omega_s$ such that $p_r(i) > p_s(j)$. Then condition (C.1) implies that

$$\frac{w_r}{p_r(i)} > \frac{w_s}{p_s(j)} \qquad \Longrightarrow \qquad \frac{p_r(i)}{w_r} < \frac{p_s(j)}{w_s},$$

whereas condition (C.2) implies that $Q_r(i) < Q_s(j)$. Hence, $\Pi_r(i) < \Pi_s(j)$, which is incompatible with an equilibrium. We may hence conclude that $p_r(i) = p_s(j)$ must hold for all $i \in \Omega_r$ and $j \in \Omega_s$, which shows that product prices are equalized. Condition (C.1) then shows that $w_r = w_s$, i.e., factor prices are equalized whenever product prices are equalized. Finally, since $p_r(i) = p_s(j) = p$ and $w_r = w_s = w$, we can rewrite (2)–(4) as follows:

$$q_{rr} = q_{sr} = q_{ss} = q_{rs} = \frac{w}{Np} \tag{C.3}$$

and

$$\frac{\partial q_{rr}}{\partial p_r} = \frac{\partial q_{sr}}{\partial p_s} = \frac{\partial q_{ss}}{\partial p_s} = \frac{\partial q_{rs}}{\partial p_r} = -\frac{1}{\alpha p}. \tag{C.4}$$

Inserting (C.3) and (C.4) into the first-order condition (6), we obtain the price equilibrium, in which markups are equalized across varieties and countries.

Appendix D: The optimal masses of firms and the optimal utility in the two-sector case

As in the single sector case, we first derive the optimal mass of firms in each sector. Expression (45) becomes

$$\frac{\beta_1}{n_{1r}} \frac{\alpha_1 \mathrm{e}^{-\alpha_1 q_{1r}}}{1 - \mathrm{e}^{-\alpha_1 q_{1r}}} = c \quad \Rightarrow \quad \mathrm{e}^{-\alpha_1 q_{1r}} = \frac{c n_{1r}}{\alpha_1 \beta_1 + c n_{1r}}. \tag{D.1}$$

Expression (47), in turn, becomes

$$\frac{\beta_1}{n_{1r}} = \frac{L_r c q_{1r} + F}{L_r} \quad \Rightarrow \quad q_{1r} = \frac{1}{c}\left(\frac{\beta_1}{n_{1r}} - \frac{F}{L_r}\right). \tag{D.2}$$

Plugging (D.2) into (D.1), we have

$$\mathrm{e}^{-\frac{\alpha_1}{c}\left(\frac{\beta_1}{n_{1r}} - \frac{F}{L_r}\right)} = \frac{c n_{1r}}{\alpha_1 \beta_1 + c n_{1r}} \quad \Rightarrow \quad -\left(1 + \frac{\alpha_1 \beta_1}{c n_{1r}}\right) \mathrm{e}^{-\left(1 + \frac{\alpha_1 \beta_1}{c n_{1r}}\right)} = -\mathrm{e}^{-1 - \frac{\alpha_1 F}{c L_r}}.$$

We thus obtain

$$-\left(1+\frac{\alpha_1\beta_1}{cn_{1r}}\right)=W\left(-\mathrm{e}^{-1-\frac{\alpha_1 F}{cL_r}}\right),$$

which can be solved for n_{1r}^o as shown in (50). The derivation of n_{2r}^o is analogous. Finally, turning to the optimal utility, expressions (50) and (D.1) yield

$$U_{1r}^o=n_{1r}^o(1-\mathrm{e}^{-\alpha_1 q_{1r}^o})=n_{1r}^o\frac{\alpha_1\beta_1}{\alpha_1\beta_1+cn_{1r}^o}=-\frac{\alpha_1\beta_1}{cW_{-1}\left(-\mathrm{e}^{-1-\frac{\alpha_1 F}{cL_r}}\right)}.$$

Again, the derivation of U_{2r}^o is analogous. Hence, the optimal utility is given by (51).

Figure 1: N/N^o as a function of L.

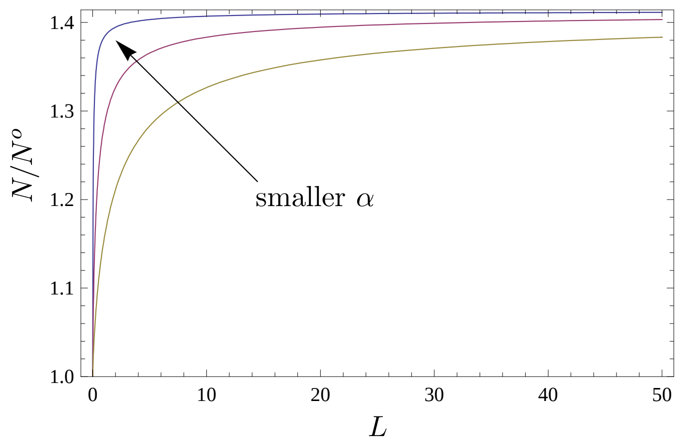

Figure 2: $[p/(cw)]/[p^o/(cw^o)]$ as a function of L.

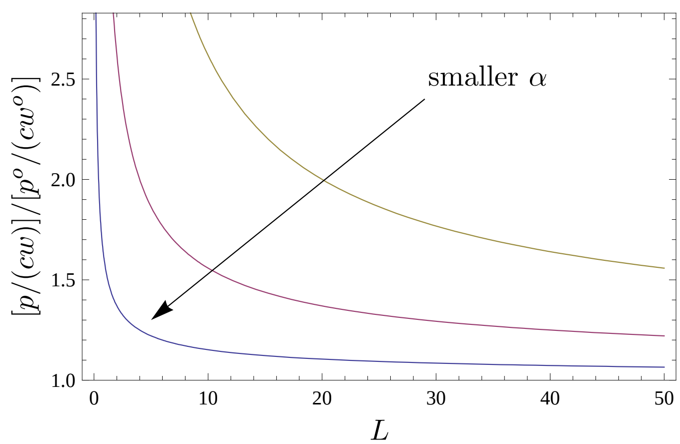

Figure 3: U/U^o as a function of L.

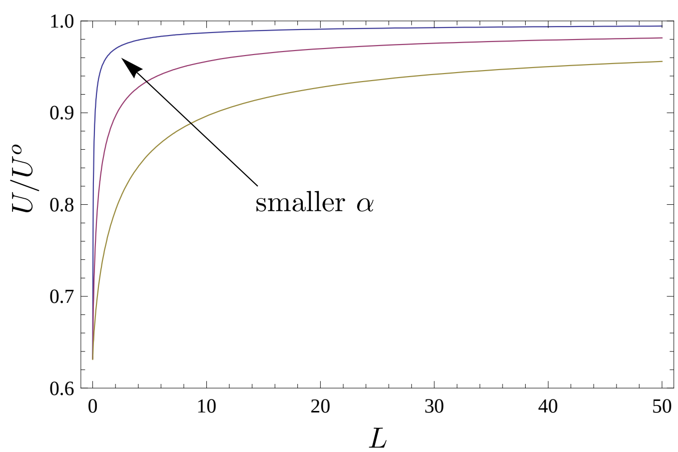

Figure A1: f and g as a function of Y_r.

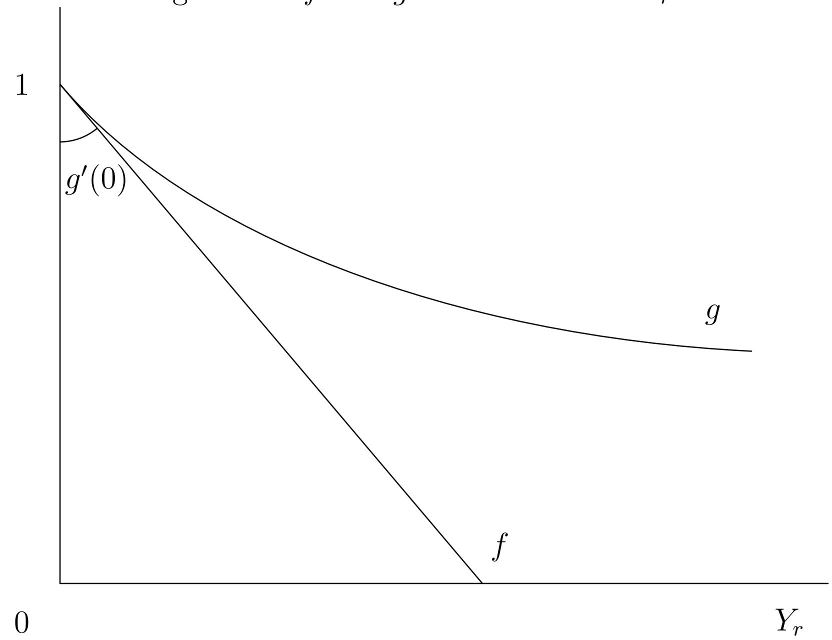

Figure A2: Excess entry.