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International Emission Strategies under the Threat of a Sudden Jump in Damages

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Abstract:

We characterize the equilibrium level of emissions, the equilibrium stock of global pollution and the discounted net social welfare for both the cooperative and non-cooperative equilibria when the countries face the threat of a sudden irreversible jump in the global damages at an unknown date. The goal is to analyze the impact of this type of uncertainty on the equilibrium behavior of the countries. We find that it can have a significant effect on those equilibria. Countries reduce their emissions to mitigate their exposure to this threat. As the level of threat rises, countries adjust their emissions to lower the stock of pollutant. However, although initially this threat has the effect of lowering the discounted net welfare, it can in the long run have a net positive effect on welfare. The emissions trajectory is non-monotonic and discontinuous, but only under the threat.

Keywords: Global pollution, Environmental uncertainty, Regime shift, Stochastic differential games

Résumé:

Je considère un monde dans lequel les pays souffrent uniformément de la pollution globale. Ils font face à une menace continuelle de voir les dommages causés par cette pollution globale s'accroître subitement de façon irréversible. Je caractérise le niveau des émissions, le stock de pollution, et le niveau de bien-être actualisé en équilibre coopératif et non-coopératif. L'objectif visé est d'analyser l'impact de ce type d'incertitude sur les équilibres issus des comportements stratégiques des pays. Je trouve que cette incertitude peut avoir un effet significatif sur ces équilibres. Les pays réduisent leurs émissions pour mitiger leur exposition à cette menace. Plus la menace est grande, plus les pays ajustent leurs émissions afin de réduire davantage le stock de pollution globale. Cependant, en dépit du fait que cette incertitude diminue le bien-être net initial, elle peut à long terme avoir un effet net positif sur ce bien-être. La trajectoire des émissions est non-monotone et discontinue, mais seulement en présence de la menace.

Mots clés: Pollution globale, environnement incertain, changement de régime, jeux différentiels stochastiques

Classification JEL: C61, C7, D81, Q54

1 Introduction

There is scientific evidence that the accumulation of greenhouse gas could drive the world to an environmental catastrophic state. Such a catastrophic state would be irreversible and the level of CO2 which may provoke it is uncertain.[1] This type of global environmental catastrophic risk has become of special concern in recent years. Our limited subjective knowledge about such future environmental damages raises the necessity to contemplate strategies to mitigate the cost of such risks.

This issue has been largely neglected in the literature on the *international* control of pollution. Long (1992) and Van Der Ploeg and De Zeeuw (1992) have analyzed in a differential game the common pollution problem between countries, but in a deterministic setting. They find that, in the non-cooperative equilibrium steady state, the level of pollution is greater than in the cooperative equilibrium state state. Recently, Bramoullé and Treich (2009) have incorporated uncertain damage costs to investigate the optimal pollution control between polluters. They find that emissions are always lower under uncertainty than under certainty and that uncertainty may actually improve social welfare. A drawback of that paper is that it makes use of a static framework, which may not be appropriate to deal with stock pollution.

We try to remedy this. The basic model used in this paper is related to that of Dockner and Long (1993). They derive the Nash equilibrium in emissions of a differential pollution game between two neighboring polluters under perfect certainty. They find that the first-best steady state can be supported in the long run as a steady-state of the non-linear Nash equilibrium if the discount rate is sufficiently small. Rubio and Casino (2002) later show that this result holds only if the initial stock of pollutant lies above the steady-state level of the cooperative equilibrium.

In a world where there is a threat of disruption of future environmental damages, the question arises as to how best decision makers can adapt their strategies in response to such

[1] See for example IPCC (2007) and U.S. Climate Change Science Program (2009).

a threat. More precisely, can it be optimal for decision makers to reduce their emissions in order to ensure themselves again such threat? How will such a threat impact social welfare? Those are the type of issues addressed in this paper.

The issue of pollution control under a threat of regime shifts has also been analyzed by a few other authors. Clarke and Reed (1994) analyze the tradeoff between consumption and pollution decisions under the threat of a catastrophic event. Tsur and Zemel (1998) and de Zeeuw and Zemel (2011) examine the impacts of the threat of environmental disruptions on the efficient level of pollution. Our approach differs from that of the three papers cited above in some key respects. First, those papers deal with a local pollution problem (in a world constituted of one country with a single decision maker). In our analysis, pollution is global and the world is made of an arbitrary number of countries that undertake their emission decisions strategically. Second, in those papers the "country" considered in the open loop Nash equilibrium (OLNE) chooses and commits to a time path of emissions at the outset of the planning horizon. In the present paper, emission strategies of the countries are derived from a Markov perfect Nash equilibrium (MPNE). It is well known that the MPNE is subgame perfect whereas the OLNE is not.[2] Finally, it is also important to mention that those papers study the effects of uncertainty by comparing the steady state under the threat with that of the no-threat case, but they ignore the effects of the threat on welfare. We perform our analysis in terms of equilibrium trajectories and we also look at the effects of the threat on welfare.

We will assume that the state of damages can be either low or high. When it is currently low, there is a positive known probability that it will jump up to its high level at some unknown future date.[3] When it is currently high, it will stay into that state forever. We show that when countries act non-cooperatively the threat of a sudden jump in the damages impacts their behavior in the same way as it does in the cooperative equilibrium.[4] They

[2]See for instance Karp and Newbery (1993).

[3]There is an ongoing debate on whether or not human actions can trigger climate change. See for instance Evans and Steven (2007).

[4]Threats of disruption in resource economics have been analyzed and discussed by a few authors. See for

reduce their emissions to mitigate their exposure to threat, which in turn lowers the stock of pollutant. That threat initially lowers the discounted net welfare, but, in the long-run, it can increase it. The time path of emissions is non-monotone and discontinuous, but only in the presence of the threat. Overall the non-cooperative equilibrium creates a social distortion in terms of the environmental quality and social welfare. It results in a lower social welfare, higher emissions and a higher stock of pollutant as compared to the cooperative equilibrium.

The remainder of this paper proceeds as follows. Section 2 presents the model. In Section 3, we derive the equilibrium that results from the full coordination of emissions control. In addition, we investigate the effects of the threat of a sudden jump in the damages on that equilibrium. The Nash equilibrium of emissions is derived in Section 4. We then analyze the effects of the threat of a jump on the equilibrium emission levels, the equilibrium stock of pollutant and the equilibrium discounted net welfare. We also compare the outcome resulting from that analysis to those obtained from the first best. Section 5 concludes.

2 Set up of the model

Consider a world of N identical countries whose production activity has as by-product some pollution that damages a shared environmental resource. It will be assumed that one unit of production generates one unit of emission. Let q_i denote the emissions (production) of country i. The current aggregate emissions of the world is then $Q = \sum_{i=1}^{N} q_i$.

The current stock of pollutant is denoted $z(t)$. We assume that the quantity of pollutants emitted today by the world adds to the current stock of pollutant according to the following differential equation:

$$\dot{z}(t) = Q(t) - \rho z(t), \quad \rho \in (0, 1), \quad z(0) = z_0, \tag{1}$$

where ρ is the purification rate of the stock of pollutant.

As in Dockner and Long (1993), the typical country's instantaneous benefit function is

example Loury (1983), Bergström et al. (1985), Laurent-Lucchetti et al. (2011), Hillman and Long (1983), Gaudet and Lasserre (2011), Long (1975), Bahel (2011) and Polasky et al. (2011).

given by

$$U(q(t)) = \sigma q(t) - \frac{1}{2}q^2(t),$$

where σ is a positive parameter. The stock of pollutant at each date generates the same level of damages in each country (i.e. the pollution is global). Those damages are subject to uncertainty. The damage function can be written as the product of two functions: a deterministic part, which will be assumed quadratic and denoted $D(z(t)) = \frac{1}{2}z(t)^2$; a stochastic part, denoted $\theta(t)$, which captures the stochastic state of nature. Hence, at any time t, the global damages are given by:

$$\frac{\theta(t)}{2}z(t)^2.$$

There are two states of nature: $\theta > 0$ and $\theta + m$. The state θ corresponds to low damages, whereas the state $\theta + m$ corresponds to high damages. Initially the countries are not fully informed about future realizations of the states of damages. They know however that there are two states of nature and they know the probabilities associated to them. The transition between the two states is defined by the following stochastic process:

$$\theta(t+dt) - \theta(t) = \begin{cases} \left.\begin{matrix} m & \text{with probability } \beta dt \\ 0 & \text{with probability } 1 - \beta dt \end{matrix}\right\}, & \text{if } \theta(t) = \theta \\ 0 \quad \text{with probability } 1\ , & \text{if } \theta(t) = \theta + m \end{cases} \tag{2}$$

where $0 \leq \beta dt \leq 1$ and where β is a known non-negative parameter. Hence as long as the current state of nature is low damages ($\theta(t) = \theta$), with probability βdt it will jump up to the state of high damages $\theta + m$ over the interval $[t, t + dt]$. Once the state of high damages ($\theta(t) = \theta + m$) has occurred, it will never revert back to the low-damage state.[5] The level of severity of the jump in the damages is captured by the parameter non-negative real number m. We assume that initially the state of nature is that of low damages.

The flow of net benefits to the typical country is therefore stochastic and given by;

$$\pi(q(t), z(t), t) = \sigma q(t) - \frac{1}{2}q^2(t) - \frac{\theta(t)}{2}z^2(t). \tag{3}$$

[5]As shown in Appendix A, this implies that the lifetime of the state of low damage is finite. In other words, the countries know that the irreversible upward jump in the damages will happen but they do not know the date of occurrence of such an event.

For $m > 0$, the net benefit (3) will be discontinuous at the eventual date of the jump in the state of damages. The particular case of $m = 0$ (or $\beta = 0$) corresponds to the well known deterministic model of pollution control, the value of the jump being zero (or the probability of the jump occurring being zero).

To characterize the effect of the possibility of such a jump in the damages on strategic behavior of countries, we investigate, in order, the cooperative equilibrium and the non-cooperative equilibrium.

3 Cooperative equilibrium

In the cooperative setting, at any date t, countries decide jointly the emission levels that maximize the sum of their expected discounted net benefit. If $z(t)$ is the current stock of pollutant and $\theta(t)$ is the current state of nature, then the value function in current value at the date t is:[6]

$$W(z(t), \theta(t)) = \max_{q_1,\dots,q_N} \{\sum_{i=1}^{N} E_t \int_t^{\infty} e^{-r(s-t)}[\sigma q_i(s) - \frac{1}{2}q_i^2(s) - \frac{\theta(s)}{2}z^2(s)]ds\},$$

subject to (1)-(2).

The Hamilton-Jaccobi-Bellman equation associated to this stochastic optimization is

$$rW(z, \theta(t)) = \max_{q_1,\dots,q_N} \{\sum_{i=1}^{N}(\sigma q_i - \frac{1}{2}q_i^2) - N\theta(t)z^2/2 + (\sum_{k=1}^{N} q_k \quad -\rho z)W_z(z, \theta(t)) + \mathbb{E}\{\Delta W|\theta(t)\}\}, \tag{4}$$

where r is the discount rate and where:[7]

$$\mathbb{E}\{\Delta W|\theta(t)\} = \begin{cases} \beta[W(z, \theta + m) - W(z, \theta)], & \text{if } \theta(t) = \theta \\ 0, & \text{if } \theta(t) = \theta + m. \end{cases} \tag{5}$$

The first-order conditions for the maximization of the right-hand side of (4) are, for $i = 1, \dots, N$:

$$\sigma - q_i + W_z(z, \theta(t)) \leq 0; \quad q_i \geq 0; \quad (\sigma - q_i + W_z(z, \theta(t))q_i = 0. \tag{6}$$

[6]Since the problem is autonomous and has an infinite horizon, $W(z(t), \theta(t))$ depends only on the current state variables and not explicitly on the current date t (see Kamien and Schwartz, 1981, p. 164).

[7]We use the generalized Itô lemma for jump process in deriving this Bellman equation.

Thus, given the current state of nature, if the equilibrium emissions are positive at the date t the marginal benefit derived from polluting by a country must be equal to the marginal social cost of pollution, $-W_z(z, \theta(t))$.

For an interior solution we get, from (6), that:

$$q_i = \sigma + W_z(z, \theta(t)). \tag{7}$$

Substituting for this expression into (4) results in the following state dependent differential equation:

$$rW(z, \theta(t)) = N\sigma^2/2 - N\theta(t)z^2/2 + (N\sigma - \rho z)W_z + NW_z(z, \theta(t))^2/2 + \mathbb{E}\{\Delta W|\theta(t)\}. \tag{8}$$

Given the particular structure of (8), it is helpful to determine first its solution for the state of high damages and then use that solution to solve for the state of low damages.

3.1 The cooperative policy: state of high damages

Under this state, from (5), we have $\mathbb{E}\{\Delta W|\theta(t)\} = 0$. Hence (8) becomes:

$$rW(z, \theta + m) = N\sigma^2/2 - N(\theta + m)z^2/2 + (N\sigma - \rho z)W_z(z, \theta + m) + NW_z(z, \theta + m)^2/2. \tag{9}$$

It is shown in Appendix B that the following value function solves (9):[8]

$$W(z, \theta + m) = -\frac{A}{2}z^2 - Bz + C, \tag{10}$$

where the coefficients A, B and C are given by:

$$A = [-(2\rho + r) + \sqrt{(2\rho + r)^2 + 4N^2(m + \theta)}]/2N > 0,$$

$$B = \sigma AN/[NA + r + \rho] > 0,$$

$$C = [\sigma^2 N - 2\sigma BN + B^2 N]/2r > 0,$$

$$A' \equiv \frac{\partial A}{\partial m} > 0; \;\; B' \equiv \frac{\partial B}{\partial m} > 0; \;\; C' \equiv \frac{\partial C}{\partial m} < 0 \text{ and } \sigma > B.$$

[8] This functional form implicitly means that we restrict our attention only on linear strategies in this study. Since we have to do with a linear quadratic game, non-linear strategies may exist as well; see among others, Tsutsui and Mino (1990) or Dockner and Long (1993). However, it can be shown that linear and non-linear strategies yield the same steady-state.

Therefore expression (7) states that if the state of high damages prevails at a given date when the current stock of pollutant is z, then the equilibrium emission rate will be given by:

$$q_i(z, \theta + m) = \sigma - B - Az. \tag{11}$$

Equation (11) gives the typical country's decision rule in the cooperative equilibrium once the state of high damages has occurred.

3.2 The cooperative policy: state of low damages

Under this state, from (5), we have that $\mathbb{E}\{\Delta W|\theta(t)\} = \beta[W(z,\theta+m)-W(z,\theta)]$. Substituting into (8) and rearranging we find that $W(z,\theta)$ is the solution of the following differential equation:

$$(r+\beta)W(z,\theta) = N\sigma^2/2 - N\theta z^2/2 + (N\sigma - \rho z)W_z(z,\theta) + NW_z(z,\theta)^2/2 + \beta W(z,\theta+m), \tag{12}$$

where $W(z, \theta + m)$ is given by (10).

Again, it is shown in Appendix B that the following quadratic form provides a solution:

$$W(z,\theta) = -\frac{a_1}{2}z^2 - a_2 z + a_3. \tag{13}$$

where the coefficients a_1, a_2, a_3 are given by:

$$\begin{aligned}
a_1 &= [-(2\rho + r + \beta) + \sqrt{(2\rho + r + \beta)^2 + 4N(A\beta + N\theta)}]/2N,\\
a_2 &= \frac{N\sigma a_1 + \beta B}{Na_1 + \rho + r + \beta},\\
a_3 &= [\sigma^2 N + 2C\beta - 2\sigma N a_2 + N a_2^2]/2(r + \beta),\\
a_1' &\equiv \frac{\partial a_1}{\partial \beta} > 0;\ \ a_2' \equiv \frac{\partial a_2}{\partial \beta} > 0;\ \ a_3' \equiv \frac{\partial a_3}{\partial \beta} < 0 \ \text{ and } \ \sigma > a_2.
\end{aligned}$$

Making use of (7), we get that the typical country's decision rule at any date t when the state of damages is low is given:

$$q_i(z, \theta) = \sigma - a_2 - a_1 z.$$

From now on we will denote by $\nu > 0$ the uncertain date at which the jump in the damages occurs.

Using (1), the dynamic of the stock of pollutant on the interval of time $[0, \nu]$ can be rewritten as:

$$\dot{z}(t) \equiv Nq_i(z(t), \theta) - \rho z(t) = N[\sigma - a_2 - a_1 z(t)] - \rho z(t).$$

A particular solution of that differential equation is:

$$z^L = N(\sigma - a_2)/(\rho + Na_1),$$

where L stands for the state of low damages. The general solution of the homogenous equation associated to that differential equation is:

$$z(t) = \xi e^{-(\rho + Na_1)t},$$

where ξ is an arbitrary parameter. Hence the general solution of the above equation is:

$$z_L^c(t) = z^L + \xi e^{-(\rho + Na_1)t}.$$

Since $z_L^c(0) = z_0$, we have $\xi = z_0 - z^L$. Denote by $z_L^c(t)$ the equilibrium stock of pollutant, $q_L^c(t)$ the equilibrium emissions rate and $W_L^c(t)$ the discounted net welfare at date $t \in [0, \nu)$, where the superscript c stands for the cooperative equilibrium. Their respective expressions are:[9]

$$z_L^c(t) = N(\sigma - a_2)/(\rho + Na_1) + [z_0 - N(\sigma - a_2)/(\rho + Na_1)]e^{-(\rho + Na_1)t}, \tag{14a}$$

$$q_L^c(t) = \sigma - a_2 - a_1 z_L^c(t), \tag{14b}$$

$$W_L^c(t) = -a_1 z_L^c(t)^2/2 - a_2 z_L^c(t) + a_3, \tag{14c}$$

Let $z_\nu^c \equiv z_L^c(\nu)$.

Using (1) and (11), we get the dynamics of the stock of pollutant after the eventual jump in the damages:

$$\dot{z}(t) \equiv q_i(z(t), \theta + m) - \rho z(t) = N[\sigma - B - Az(t)] - \rho z(t).$$

[9]In this paper, we consider only small values of the initial stock of pollutant z_0, in order to get positive emissions.

We use a similar approach as for the state of low damages to solve this differential equation. The solution of that equation with the initial condition $z(\nu) = z_\nu^c$ will be the equilibrium stock of pollutant under the high damage state once has occurred. Its expression at any date $t \in [\nu, \infty)$ is:

$$z_H^c(t) = N(\sigma - B)/(\rho + NA) + [z_\nu^c - N(\sigma - B)/(\rho + NA)]e^{-(\rho+NA)(t-\nu)}, \tag{15a}$$

from which we derive the equilibrium emission levels $(q_H^c(t))$ and discounted net welfare $(W_H^c(t))$ at any date $t \in [\nu, \infty)$:

$$q_H^c(t) = \sigma - B - Az_H^c(t), \tag{15b}$$

$$W_H^c(t) = -Az_H^c(t)^2/2 - Bz_H^c(t) + C. \tag{15c}$$

The following proposition characterizes the cooperative equilibrium steady state.

Proposition 1 *In the fully cooperative equilibrium, we have:*
(i) The steady state of the stock of pollutant exists and its expression is given by:

$$z_c^{stea} = N(\sigma - B)/(\rho + NA).$$

The stock of pollutant converges to z_c^{stea} and the emissions rate approaches asymptotically its steady-state, $q_c^{stea} = \sigma - B - Az_c^{stea}$.
(ii) The steady state of emissions and that of the stock of pollutant are lower than they would be in the absence of the threat of a jump in the damages.

Proof. (i) Since the lifetime of the state of low damages is finite, the dynamics of the long run of the stock of pollutant and emissions rate are given respectively by (15a) and (15b). They clearly converge respectively to z_c^{stea} and q_c^{stea}.
(ii) Notice that the steady state of the stock of pollutant and the steady state of emissions in the no threat context are given respectively by $z_c^{stea}{}_{|m=0}$ and $q_c^{stea}{}_{|m=0}$. Making use of the values of A and B given in Section 3.1, it is easy to see that $\partial z_c^{stea}/\partial m < 0$. Thus $z_c^{stea} < z_c^{stea}{}_{|m=0}$. We also have $q_c^{stea} = \sigma - B - AN(\sigma - B)/(\rho + NA)$. Differentiating and

rearranging, we get: $\partial q_c^{stea}/\partial m = -B'\rho/(\rho + NA) - \rho NA'(\sigma - B)/(\rho + NA)^2 < 0$. Hence, $q_c^{stea} < q_c^{stea}{}_{|m=0}$.

3.3 Effects of the threat of a jump

To investigate the effect of the threat of a jump in the damages on the equilibrium emissions rate, the equilibrium stock of pollutant, and the equilibrium welfare, we first make the comparison with what would occur in the absence of such a threat.

Denote respectively by $\tilde{z}_c(t)$, $\tilde{q}_c(t)$, and $\tilde{W}(t)$ the equilibrium stock of pollutant, the emissions rate and the discounted net welfare at the date t for the case of no threat, which corresponds in this model to either $m = 0$ or $\beta = 0$. Then, (a) for all $t \in [0, \nu)$, $\tilde{z}_c(t) \equiv z_L^c(t)_{|\beta=0}$, $\tilde{q}_c(t) \equiv q_L^c(t)_{|\beta=0}$, and $\tilde{W}(t) \equiv W_L^c(t)_{|\beta=0}$; (b) for all $t \in [\nu, \infty)$, $\tilde{z}_c(t) \equiv z_H^c(t)_{|m=0}$ and $\tilde{q}_c(t) \equiv q_H^c(t)_{|m=0}$, and $\tilde{W}(t) \equiv W_H^c(t)_{|m=0}$.

Proposition 2 *In the cooperative equilibrium, we have:*
(i) $\tilde{z}_c(t) > z_L^c(t)$ *for all* $t \in (0, \nu]$,
(ii) $\tilde{q}_c(t) > q_L^c(t)$ *for all* $t \in [0, \nu)$,
(iii) $\tilde{W}(0) > W_L^c(0)$.

Proof. See Appendix B.1.

Proposition 2 states that for any feasible value of the initial stock of pollutant, the emission level and its resulting stock of pollutant are lower under the threat of a jump in the damages than its absence. This holds for the whole duration of the state of low damages. The following corollary shows that those results also hold in the state of high damages.

Corollary 3.1 *In the cooperative equilibrium, after the failure of the state of low damages, namely during the interval of time* $[\nu, \infty)$*, the following results hold.*
(i) $\tilde{z}_c(t) > z_H^c(t)$ *for all* $t \in [\nu, \infty)$.
(ii) $\tilde{q}_c(t) > q_H^c(t)$ *for all* $t \in [\nu, \infty)$.

Proof. See Appendix B.2.

Thus the countries anticipate the fact that, although the date of the jump in damages is uncertain, it will occur in finite time with certainty. This incites them to alleviate their exposure to high damages by adopting a lower emissions path, which in turn generates a lower stock of pollutant.

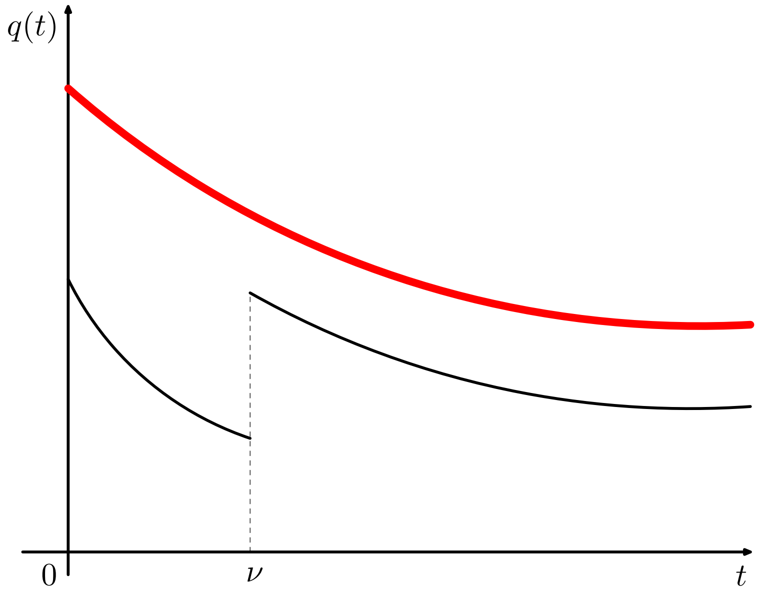

Figure 1: Effects of the threat on the optimal emissions.

Since $a_2' > 0$ and $a_1' > 0$ we have $a_2 > a_{2|_{\beta=0}} \equiv B$ and $a_1 > a_{1|_{\beta=0}} \equiv A$. Hence, $a_2 - B + (a_1 - A)z_\nu^c > 0$, which, rearranging, yields

$$q_i(z_\nu^c, \theta + m) \equiv \sigma - B - Az_\nu^c > \sigma - a_2 - a_1 z_\nu^c \equiv q_i(z_\nu^c, \theta).$$

This implies that the emission level jumps up at the date at which the state of low damages ends (i.e. at date $t = \nu$). Consequently, as illustrated by the thin line in Figure 1, the emissions trajectory under the threat is discontinuous and is non-monotone. This is in contrast to the literature on pollution control with no threat of a jump in the damages, which suggests that the transition of the emission levels from the initial date to the steady-state is continuous and monotone (as illustrated by thick line in Figure 1). The above findings show that those results do not hold when the threat of an upward jump in the damages prevails.

Let us now consider the effect of an increase in the threat of an upward jump in the damages in the cooperative equilibrium. Denote by X_β the random variable representing the duration of the state of low damages and notice that $pr(X_\beta > s) = pr(\theta(s) = \theta|\theta(0) = \theta) = \ell(s) = e^{-\beta s}$. Hence, if $\beta_1 > \beta_2 \geq 0$, then $pr(X_{\beta_2} > s) > pr(X_{\beta_1} > s)$ for all $s > 0$ and $pr(X_{\beta_2} > 0) = pr(X_{\beta_1} > 0) = 1$. Since $\beta_1 > \beta_2 \geq 0$ are arbitrary, we can conclude that at each date $t \in [0, \nu)$, the level of threat of a jump occurring in the next instant is increasing in β. We have the following results.

Proposition 3 *In the cooperative equilibrium,*
(i) At all positive dates, the higher the level of threat of a jump in the damages, the lower is the stock of pollutant.
(ii) At the initial date, an increase in the threat of an upward jump in the damages always decreases the discounted net welfare. In the long run, such an increase has no effect on the discounted net welfare.

Proof. (i) It was shown in Proposition 2 that $\partial z_L^c(t)/\partial\beta < 0$ for all $t \in (0, \nu]$. Since $z_L^c(\nu) = z_\nu^c$, we then have $\partial z_\nu^c/\partial\beta < 0$. From (15a), we can derive the following: $\partial z_H^c(t)/\partial\beta = (\partial z_\nu^c/\partial\beta)e^{-(\rho+NA)(t-\nu)} < 0$ for all $t \geq \nu$.
(ii) Since $z_0 \geq 0$, $a_1' > 0$, $a_2' > 0$ and $a_3' < 0$, it is an easy matter to derive from (14c) the following $\partial W_L^c(z_0)/\partial\beta = -a_1' z_0^2/2 - a_2' z_0 + a_3' < 0$. In the long run the discounted net welfare is equal to: $W_H^c(z_c^{stea}) = -A(z_c^{stea})^2/2 - Bz_c^{stea} + C$. It does not depend on β, the sole parameter that allows us to capture variations in threat.

At any date, the comparison between the discounted net welfare under the threat of a jump, $W(z(t))$, and that with no threat of a jump, $\tilde{W}(\tilde{z}(t))$, can be carried out as follows:

$$W(z(t)) - \tilde{W}(\tilde{z}(t)) = \{W(z(t)) - W(\tilde{z}(t))\} + \{W(\tilde{z}(t)) - \tilde{W}(\tilde{z}(t))\}.$$

The threat lowers the stock of pollutant, keeping the risk fixed (first term). The same current stock of pollutant generates more risk (second term). The first term on the right-hand side captures the strategic effect. Since $W''(z) < 0$ and $W'(z) < 0$, this effect is always positive except at the initial date.[10] The second term on the right-hand side captures the effect of the threat of a jump in the damages, which by Proposition 2 is always negative. Both effects work in opposite directions, so that the net effect can be either positive or negative.

In the long run, it is possible for the discounted net welfare under the threat to be greater than in the no-threat case. Indeed, there exist values of the parameters for which this is the case. For instance, with $\sigma = 100$; $\rho = 0.005$; $r = 0.025$; $\theta = 1$; $N = 2$; $\beta = 1$; $m = 100$, we get $W(z_c^{stea}) = -0.594 > -59.995 = \tilde{W}(\tilde{z}_c^{stea})$. Therefore the existence of the threat can improve social welfare. More generally, we show in Appendix B.3 that, in the long run, the strategic effect can dominate the threat effect, but only if $r > \rho$ and $N \geq (r+\rho)\sqrt{\rho}/\sqrt{\theta(r-\rho)}$.

It is interesting to note that some results differ from those obtained by Bramoullé and Treich (2009) in their static model. They analyze the effects of uncertainty on the optimal emissions and welfare for risk-averse polluters. One of their results is that with a constant-elasticity damage function, small risks would have a net positive effect on welfare. In this paper the damage function has a constant elasticity with respect to the stock of pollutant, but polluters are risk-neutral. We have shown that uncertainty lowers the discounted net welfare at the initial date, irrespective of the level of risk.

4 Non-cooperative equilibrium

This section derives the Nash equilibrium for the differential game in pollution control defined by (1), (2) and (3). In this setting, at any date t, each country decides unilaterally the emission strategy that maximizes its own discounted net benefit, considering as given the emission strategies of the other countries. The countries being identical, we restrict attention to symmetric equilibria. If $z(t)$ is the current stock of pollutant and $\theta(t)$ the current state of nature, the current value function of the typical country j, $j = 1, ..., N$, is:

$$V(z, \theta(t)) = \max_{q_j}\{E_t \int_t^{\infty} e^{-r(s-t)}[\sigma q_j(s) - \frac{1}{2}q_j^2(s) - \frac{\theta(s)}{2}z(s)^2]ds\},$$

[10] At the initial date, since $z(0) = \tilde{z}(0) = z_0$, the expression of the strategic effect is: $W_L^c(z_0) - W_L^c(z_0)$, which is equal to zero so that the uncertainty effect outweighs the strategic one.

subject to (1), (2) and (3).

The associated Hamilton-Jacobi-Bellman equation is:

$$rV(z,\theta(t)) = \max_{q_j}\{\sigma q_j - \frac{1}{2}q_j^2 - \theta(t)z^2/2 + (\sum_{k=1}^{N} q_k - \rho z)V_z(z,\theta(t)) + \mathbb{E}\{\Delta V|\theta(t)\}\}, \quad (16)$$

where:

$$\mathbb{E}\{\Delta V|\theta(t)\} = \begin{cases} \beta[V(z,\theta+m) - V(z,\theta)], & \text{if } \theta(t) = \theta \\ 0, & \text{if } \theta(t) = \theta + m. \end{cases} \quad (17)$$

The first-order conditions for the maximization of the right-hand side of (16) require, for $j = 1, ..., N$:

$$\sigma - q_j + V_z(z,\theta(t)) \leq 0; \quad q_j \geq 0; \quad (\sigma - q_j + V_z(z,\theta(t)))q_j = 0. \quad (18)$$

In the above expressions, $-V_z(z,\theta(t))$ represents the private marginal cost of pollution. Hence, if at date t the emissions rate of country j is positive, it must be the case that the marginal benefit derived from polluting is equal to its marginal private cost of the polluting. For such an interior solution, we have:

$$q_j = \sigma + V_z(z,\theta(t)). \quad (19)$$

Substituting the optimal emissions (19) into (16) yields:

$$rV(z,\theta(t)) = (N-1/2)V_z(z,\theta(t))^2 + (N\sigma - \rho z)V_z(z,\theta(t)) + \sigma^2/2 - \theta(t)z^2/2 + \mathbb{E}\{\Delta V|\theta(t)\}. \quad (20)$$

As for the cooperative case, we will first solve (20) for the state of high damages before solving it for the state of low damages.

4.1 The unilateral policy: state of high damages

In the state of high damages (17) yields $\mathbb{E}\{\Delta V|\theta(t)\} = 0$. Equation (20) can therefore be rewritten as:

$$rV(z,\theta+m) = (N-1/2)V_z(z,\theta+m)^2 + (N\sigma - \rho z)V_z(z,\theta+m) + \sigma^2/2 - (\theta+m)z^2/2. \quad (21)$$

As shown in Appendix C, a solution is:

$$V(z, \theta + m) = -\frac{\hat{A}}{2} z^2 - \hat{B} z + \hat{C}, \tag{22}$$

where the coefficients $\hat{A}$, $\hat{B}$ and $\hat{C}$ are given by:

$$\hat{A} = [-(2\rho + r) + \sqrt{(2\rho + r)^2 + 4(2N - 1)(\theta + m)}]/2(2N - 1),$$

$$\hat{B} = \sigma N \hat{A}/[r + \rho + (2N - 1)\hat{A}],$$

$$\hat{C} = [\sigma^2 - 2\sigma N \hat{B} + (2N - 1)\hat{B}^2]/2r.$$

Using (19), we can then derive the typical country's decision rule for emission, which is given by:

$$q_j^n(z, \theta + m) = \sigma - \hat{B} - \hat{A} z. \tag{23}$$

4.2 The unilateral policy: state of low damages

In the low-damages state, $\mathbb{E}\{\Delta V|\theta(t)\} = \beta[V(z, \theta + m) - V(z, \theta)]$. Substituting into (20) and rearranging yields the following differential equation:

$$(N-1/2)V_z(z,\theta)^2 + (N\sigma - \rho z)V_z(z,\theta) - (r+\beta)V(z,\theta) + \beta V(z,\theta+m) + \sigma^2/2 - \theta z^2/2 = 0, \tag{24}$$

where $V(z, \theta + m)$ is given by (22).

It is shown in Appendix C that the following is a solution:

$$V(z, \theta) = -\frac{1}{2} u_1 z^2 - u_2 z + u_3,$$

where:

$$u_1 = [-(2\rho + r + \beta) + \sqrt{(2\rho + r + \beta)^2 + 4(2N - 1)(\hat{A}\beta + \theta)}]/2(2N - 1),$$
$$u_2 = \frac{N\sigma u_1 + \beta \hat{B}}{(2N - 1)u_1 + \rho + r + \beta},$$
$$u_3 = [\sigma^2 + 2\beta \hat{C} - 2\sigma N u_2 + u_2^2(2N - 1)]/2(r + \beta),$$

Letting the superscript n stand for the non-cooperative equilibrium, the decision rule for emissions is

$$q_j^n(z, \theta) = \sigma - u_2 - u_1 z.$$

Knowledge of the positive parameters $u_1, u_2, u_3, \hat{A}, \hat{B}, \hat{C}$ allows us to summarize the characterization of the linear Markov perfect equilibrium as follows:

Proposition 4 *The N-tuple $(q_1^n, ..., q_N^n)$ given, for $j = 1, 2, ..., N,$ by:*

$$q_j^n(z, \theta(t)) = \begin{cases} \sigma - u_2 - u_1 z, & \text{if } \theta(t) = \theta \\ \sigma - \hat{B} - \hat{A}z, & \text{if } \theta(t) = \theta + m \end{cases}$$

constitutes the unique stationary linear Markov perfect equilibrium and the corresponding current discounted net welfare for each country is:

$$V(z, \theta(t)) = \begin{cases} -\frac{1}{2}u_1 z^2 - u_2 z + u_3, & \text{if } \theta(t) = \theta \\ -\frac{1}{2}\hat{A}z^2 - \hat{B}z + \hat{C}, & \text{if } \theta(t) = \theta + m \end{cases}$$

It is interesting to note that for the case where $\beta = 0, m = 0$ and $N = 2$, Proposition 4 yields exactly the same linear Markov perfect equilibrium and discounted net welfare as in Dockner and Long (1993). This proposition is a generalization of their result to an arbitrary number of countries and uncertainty about the date of a possible jump in the damages.

The following proposition characterizes the non-cooperative equilibrium steady state.

Proposition 5 *In the non-cooperative emissions game,*

(i) The stock of pollutant converges asymptotically to its steady-state, $z_n^{stea} = N(\sigma - \hat{B})/(\rho + N\hat{A})$, which is smaller than it would be in the absence of the threat of a jump in the damages and larger than it would be in the cooperative equilibrium;

(ii) The steady state emissions rate is $q_n^{stea} = \sigma - \hat{B} - \hat{A}z_n^{stea}$, which is larger than the individual emissions rate in the cooperative equilibrium, but smaller than it would be if their were no threat of a jump in the damages.

Proof. See Appendix C.2.

In the non-cooperative equilibrium, the dynamics of the stock of pollutant during the state of low damages is given by:

$$\dot{z}(t) \equiv Nq_j^n(z(t), \theta) - \rho z(t) = N[\sigma - u_2 - u_1 z(t)] - \rho z(t), \;\; z(0) = z_0.$$

By the same method as for the cooperative equilibrium, we derive the expressions for the stock of pollutant, the emissions rate and the discounted net welfare in the state of low damages. They are respectively given at any date $t \in [0, \nu)$ by

$$z_L^n(t) = N(\sigma - u_2)/(\rho + Nu_1) + [z_0 - N(\sigma - u_2)/(\rho + Nu_1)]e^{-(\rho+Nu_1)t},$$
$$q_L^n(t) = \sigma - u_2 - u_1 z_L^n(t),$$
$$V_L^n(t) = -u_1 z_L^n(t)^2/2 - u_2 z_L^n(t) + u_3.$$

Let us set $z_L^n(\nu) \equiv z_\nu^n$. Likewise, at any date $t \in [\nu, +\infty)$ the equilibrium stock of pollutant, the equilibrium emission levels and the discounted net welfare of the state of high damages are respectively given by

$$z_H^n(t) = N(\sigma - \hat{B})/(\rho + N\hat{A}) + [z_\nu^n - N(\sigma - \hat{B})/(\rho + N\hat{A})]e^{-(\rho+N\hat{A})(t-\nu)},$$
$$q_H^n(t) = \sigma - \hat{B} - \hat{A} z_H^n(t),$$
$$V_H^n(t) = -\hat{A} z_H^n(t)^2/2 - \hat{B} z_H^n(t) + \hat{C}.$$

It is shown in Appendix D that the paths of emissions and of the stock of pollutant in the cooperative equilibrium are lower than those in the non-cooperative equilibrium. The reason for this is of course that the social marginal cost of pollution is higher than the private marginal cost of pollution (i.e. $-W_z(z, \theta(t)) > -V_z(z, \theta(t))$). The disincentive to pollute is therefore lower in the non-cooperative equilibrium than it is in the cooperative equilibrium. As a consequence, the non-cooperative equilibrium generates a higher stock of pollutant and a lower discounted net welfare as compared to the cooperative equilibrium. It is also shown in Appendix D that the steady-state welfare is strictly lower than the steady-state welfare in the cooperative equilibrium. This is to be expected, since the non-cooperative decision rule could always have been adopted in the cooperative equilibrium, but it was not.

4.3 Effects of the threat of a jump

This section analyzes the effects of the threat of a jump in the damages on the equilibrium emission levels, the equilibrium stock of pollutant and the equilibrium welfare resulting from the Nash equilibrium pollution control.

Denote respectively by $\tilde{z}_n(t)$, $\tilde{q}_n(t)$ and $\tilde{V}_n(t)$ the equilibrium stock of pollutant, the emissions rate and the discounted net welfare at the date t in the case where there is no threat of a jump in the damages. Then, (a) for all $t \in [0, \nu)$, $\tilde{z}_c(t) \equiv z_L^n(t)_{|\beta=0}$, $\tilde{q}_n(t) \equiv q_L^n(t)_{|\beta=0}$, and $\tilde{V}_n(t) \equiv V_L^n(t)_{|\beta=0}$; (b) for all $t \in [\nu, \infty)$, $\tilde{z}_n(t) \equiv z_H^n(t)_{|m=0}$, $\tilde{q}_n(t) \equiv q_H^n(t)_{|m=0}$, and $\tilde{V}_n(t) \equiv V_H^n(t)_{|m=0}$.

The following proposition compares the dynamics of the equilibrium stock of pollutant, the equilibrium emission levels and the equilibrium discounted net welfare when there is a threat of a jump in the damages to that in the absence of such a threat.

Proposition 6 *In the non-cooperative emissions game, we have:*

(i) $\tilde{q}_n(t) > q_L^n(t)$ *for all* $t \in [0, \nu)$*, and* $\tilde{q}_n(t) > q_H^n(t)$ *for all* $t \geq \nu$*.*

(ii) $\tilde{z}_n(t) > z_L^n(t)$ *for all* $t \in (0, \nu]$*, and* $\tilde{z}_n(t) > z_H^n(t)$ *for all* $t \geq \nu$*.*

(iii) $\tilde{V}_n(0) > V_L^n(0)$*.*

Proof. See Appendix C.3.

Proposition 6 shows that implementation by the countries of the non-cooperative emission control under the threat of a sudden jump in the damages will result in a lower emission path and a lower stock of pollutant path than if there were no threat of such a jump. This result is the same as in the cooperative equilibrium. The reason for this similarity is that the damages harm the countries equally and the state of high damages will occur with certainty in finite time. Damages will be severe in the state of high damages if countries were to decide not to reduce their emissions when faced with the threat of the jump in damages. Thus the threat of a sudden jump in the damages increases the incentive to cut emissions as compared with the case of no threat, regardless of whether the countries act cooperatively or non-cooperatively in controlling their emissions.

Exactly as for the cooperative equilibrium, it can also be shown that a non-monotone and discontinuous emission strategy arises, but only when the threat prevails. An exogenous increase in the threat of an upward jump in the damages lowers the current stock of pollutant.

Moreover, at the initial date, an increase in the threat of a jump always decreases the discounted net welfare, whereas in the long run it has no effect on the discounted welfare. In the long run, using a similar reasoning as for the cooperative setting, it can be shown that this type of threat will again result in a higher welfare for each country than in its absence only if the number of country is sufficiently large *i.e.* $N \geq [2\rho + \rho^2(r + 2\rho)/\theta]/(r + \rho)$.

5 Conclusion

This paper has extended the model of pollution control by Dockner and Long (1993) in two respects. First, an arbitrary number of countries are involved in the pollution activity. Second, at each instant those polluters suffer from the risk of a sudden jump in their common damages. It turns out that the equilibrium outcome is affected in much the same way by the threat of a jump in damages whether the countries act cooperatively or non-cooperatively. The discounted welfare, the emissions path and the path of the stock of pollutant are lower than in the absence of the threat. Unlike in the no threat case, the emissions path is discontinuous and it converges to its steady-state non-monotonically. An increase in this threat decreases the discounted welfare and lowers the time path of the stock of pollutant. However, in the long run, it is possible for this type of uncertainty to have a net positive effect on welfare. This may be the case especially if the number of countries is large. But, as can be expected, the non-cooperative outcome always results in a lower environmental quality and a lower welfare than the cooperative outcome.

Our analysis may contribute to the literature of dynamic international environmental agreements (IEAs) with a stock of pollutant.[11] The general finding of that literature is that only a small number of countries can reach such agreements. We claim that the threat of an upward jump in the damages may increase participation in an IEA. This should be the case because the stock of pollution in the presence of the threat is lower than in the no threat case. Moreover, as suggested by that literature, when the stock of pollution is small

[11] See Rubio and Casino (2005), Rubio and Ulph (2007) and Nkuiya (2010).

at the outset, the number of participating decreases while the stock of pollution undergoes an increasing evolution.

Our analysis may have another important application as well. Bacterial resistance to antibiotics represents an intertemporal externality, which can build up as any stock of pollution can. In this context firms produce drugs, which allow to kill bacteria.[12] To be more relevant, that important literature needs to be generalized to the situation where there is a threat of occurrence of a dangerous resistance regime. This will involve the type of framework developed in this paper.

[12] See for instance Rudholm (2002), Herrmann and Gaudet (2009) and Herrmann (2010) for an overview of that literature.

Appendix

The following result known as the Gronwall's inequality, will be helpful in this appendix. Consider a function $y : [a, b] \to R$ which satisfies the following inequality: $\dot{y}(t) \geq uy(t) + v$ for all $t \in [a, b]$, with $y(a) = y_a$. We must have: $y(t) \geq e^{u(t-a)}[y_a + v \int_a^t e^{u(\tau-a)} d\tau]$, for all $t \in [a, b]$, where u, v and $b > a$ are arbitrary real numbers. For the proof, see for example Gronwall (1919).

A Proof that the lifetime of the state of low damages is finite

Let $\ell(s) \equiv pr(\theta(t+s) = \theta | \theta(t) = \theta)$, the probability that the low damage state will occur at $t+s$ if it occurs at t.[13] From the expression for $\theta(t)$ defined by (2), we know that $\theta(t+s+ds) = \theta + m$ with probability βds if $\theta(t+s) = \theta$; $\theta(t+s+ds) = \theta$ with probability $1 - \beta ds$ if $\theta(t+s) = \theta$ and $\theta(t+s+ds) = \theta$ with probability 0 if $\theta(t+s) = \theta + m$. Therefore, we have the following:

$$\ell(s+ds) = (1 - \beta ds)\ell(s) + 0 \times (1 - \ell(s)).$$

Hence:

$$\frac{d\ell}{ds}(s) = -\beta\ell(s) \text{ for all } s \geq 0. \tag{25}$$

The general solution of the above differential equation is

$$\ell(s) = ce^{-\beta s} \text{ for all } s \geq 0,$$

where c is an arbitrary constant. Since $\ell(0) = 1$, we must have $c = 1$, and hence:

$$\ell(s) = e^{-\beta s} \text{ for all } s \geq 0. \tag{26}$$

It follows that the probability that the jump in the damage function never occurs (i.e. $\nu = \infty$) is $\ell(\infty) = 0$. Therefore $pr(0 < \nu < \infty) = 1 - pr(\nu = \infty) = 1$; where ν is the random variable denoting the date at which the jump in the damages arises.

[13] The expression of $\ell(s)$ does not depend on t because the stochastic process (2) is time stationary.

B Details for the Cooperative equilibrium

Consider first the state $\theta(t) = \theta + m$. The equilibrium value function is then obtained as the solution to the differential equation:

$$rW(z, \theta + m) = N\sigma^2/2 - N(\theta + m)z^2/2 + (N\sigma - \rho z)W_z(z, \theta + m) + NW_z(z, \theta + m)^2/2. \quad (27)$$

The quadratic structure of terms in the above equation suggests the following guess:

$$W(z, \theta + m) = -\frac{A}{2}z^2 - Bz + C,$$

from which we derive $W_z(z, \theta + m) = -Az - B$. Plugging those two expressions into (27) and equating the coefficients of powers of z, we get:

$$A = [-(2\rho + r) \pm \sqrt{(2\rho + r)^2 + 4N^2(m + \theta)}]/2N,$$

$$B = \sigma AN/[NA + r + \rho],$$

$$C = [\sigma^2 N - 2\sigma BN + B^2 N]/2r.$$

In order to assure the stability of the steady state we retain for A only the positive root:

$$A = [-(2\rho + r) + \sqrt{(2\rho + r)^2 + 4N^2(m + \theta)}]/2N > 0.$$

It is an easy matter to verify that:

$$A' \equiv \frac{\partial A}{\partial m} > 0; B' \equiv \frac{\partial B}{\partial m} = \frac{\partial B}{\partial A} \times \frac{\partial A}{\partial m} > 0; C' \equiv \frac{\partial C}{\partial m} = \frac{\partial C}{\partial B} \times \frac{\partial B}{\partial m} < 0.$$

We also have: $\sigma - B = \sigma(r + \rho)/(NA + r + \rho) > 0$, which implies that $\sigma > B$.

Consider now the state $\theta(t) = \theta$. The equilibrium value function is then obtained as the solution of:

$$(r+\beta)W(z, \theta) = N\sigma^2/2 - N\theta z^2/2 + (N\sigma - \rho z)W_z(z, \theta) + NW_z(z, \theta)^2/2 + \beta W^c(z, \theta + m). \quad (28)$$

Again, a plausible guess is:

$$W(z, \theta) = -\frac{a_1}{2}z^2 - a_2 z + a_3, \quad (29)$$

which yields $W_z(z,\theta) = -a_1 z - a_2$. Substituting into (28) and equating coefficients of powers of z of the resulting polynomials, we get:

$$
\begin{aligned}
a_1 &= [-(2\rho + r + \beta) \pm \sqrt{(2\rho + r + \beta)^2 + 4N(A\beta + N\theta)}]/2N, \\
a_2 &= \frac{N\sigma a_1 + \beta B}{Na_1 + \rho + r + \beta}, \\
a_3 &= [\sigma^2 N + 2C\beta - 2\sigma N a_2 + N a_2^2]/2(r + \beta).
\end{aligned}
\tag{30}
$$

In order to assure the stability of the steady state, we retain only the positive root for a_1:

$$a_1 = [-(2\rho + r + \beta) + \sqrt{(2\rho + r + \beta)^2 + 4N(A\beta + N\theta)}]/2N > 0.$$

We have, $\sigma - a_2 = [\sigma(r+\rho) + \beta(\sigma - B)]/(Na_1 + \rho + r + \beta) > 0$ which yields $\sigma > a_2$. There remains to show that $a_1' \equiv \partial a_1/\partial\beta > 0$, $a_2' \equiv \partial a_2/\partial\beta > 0$, and $a_3' \equiv \partial a_3/\partial\beta < 0$.

From (30), differentiating with respect β, we get:

$$a_1' = [-1 + ((2\rho + r + \beta) + 2AN)/\sqrt{(2\rho + r + \beta)^2 + 4N(A\beta + N\theta)}]/2N.$$

Hence $a_1' > 0$ if and only if $(2\rho + r + \beta) + 2AN > \sqrt{(2\rho + r + \beta)^2 + 4N(A\beta + N\theta)}$. Squaring both sides of this inequality, rearranging and using the fact that A satisfies the polynomial $NA^2 + (r + 2\rho)A = N(\theta + m)$, we obtain that the inequality is equivalent to $m > 0$, which is true if the apprehended jump in the damages is to be positive. Hence we can conclude that $a_1' > 0$.

$$a_2' = \{Na_1'[\sigma(r+\rho) + \beta(\sigma - B)] - Na_1(\sigma - B) + B(r+\rho)\}/(Na_1 + \rho + r + \beta)^2$$

First note that a_1 satisfies the polynomial $Na_1^2 + (r + \beta + 2\rho)a_1 - (A\beta + N\theta) = 0$, which, when differentiated with respect to β yields $2[Na_1 + (r + \beta + 2\rho)/2]a_1' = A - a_1$. The left-hand side being positive, we therefore have the $A - a_1 > 0$. Now using the fact that $B(r+\rho) = (\sigma - B)AN$, a_2' can be rewritten as $a_2' = \{Na_1'[\sigma(r+\rho) + \beta(\sigma - B)] + N(\sigma - B)(A - a_1)\}/(Na_1 + \rho + r + \beta)^2$, which is positive since $\sigma > B$, as just shown above.

$$a_3' = [N(B - a_2)(B - \sigma + a_2 - \sigma) + 2Na_2'(r + \beta)(a_2 - \sigma)]/2(r + \beta)^2,$$

because $2rC = N(\sigma - B)^2$. Since $\sigma > B$, $\sigma > a_2$, and

$B - a_2 = (A - a_1)(\sigma - B)N/(Na_1 + r + \rho + \beta) > 0$, it follows that $a_3' < 0$.

B.1 Proof of Proposition 2

(i) From (14a), we derive the following:

$$\frac{\partial z_L^c(t)}{\partial \beta} = -N[a_2'(\rho + Na_1) + Na_1'(\sigma - a_2)]\frac{1 - e^{-(\rho+Na_1)t}}{(\rho + Na_1)^2} - tNa_1'(z_0 - \frac{N(\sigma - a_2)}{\rho + Na_1})e^{-(\rho+Na_1)t}$$

If $z_0 \geq [N(\sigma - a_2)]/[\rho + Na_1]$, then $\partial z_L^c(t)/\partial\beta < 0$ for all $t \in (0, \nu]$, since $a_2' > 0$, $a_1' > 0$ and $\sigma - a_2 > 0$. If $z_0 < [N(\sigma - a_2)]/[\rho + Na_1]$, then $\partial z_L^c(t)/\partial\beta < 0$ if and only if:

$$tNa_1'(-z_0 + \frac{N(\sigma - a_2)}{\rho + Na_1})e^{-(\rho+Na_1)t} < N[a_2'(\rho + Na_1) + Na_1'(\sigma - a_2)]\frac{1 - e^{-(\rho+Na_1)t}}{(\rho + Na_1)^2}$$

Rearranging, one gets:

$$a_1'(-z_0 + \frac{N(\sigma - a_2)}{\rho + Na_1}) < \frac{[a_2'(\rho + Na_1) + Na_1'(\sigma - a_2)]}{(\rho + Na_1)}\frac{e^{(\rho+Na_1)t} - 1}{(\rho + Na_1)t}. \qquad (31)$$

Set $\psi(s) = \frac{e^s - 1}{s}$ for all $s > 0$. We have $\psi'(s) > 0$ and $\psi(s) > 1$ for all $s > 0$. We also have $a_1'(-z_0 + \frac{N(\sigma-a_2)}{\rho+Na_1}) < \frac{a_2'(\rho+Na_1)+Na_1'(\sigma-a_2)}{(\rho+Na_1)}$ for non-negative values of z_0. Combining these two facts allows to see that inequality (31) always holds. Therefore $\partial z_L^c(t)/\partial\beta < 0$ in that case as well. We can therefore conclude that $z_L^c(t) < z_L^c(t)_{|\beta=0} = \tilde{z}_c(t)$ for all $t \in (0, \nu]$.

(ii) Set $A^* = A_{|m=0}$; $B^* = B_{|m=0}$; $A^* = a_{1|\beta=0}$ and $B^* = a_{2|\beta=0}$, with a_1, a_2, A and B as given in Section 3, and set $y(t) = \tilde{q}_c(t) - q_L^c(t) = q_L^c(t)_{|\beta=0} - q_L^c(t)$. We want to show that $y(t) > 0$ for all $t \in [0, \nu)$. From (14a) and (14b), one obtains:

$$y(t) = a_2 - B^* + a_1 z_L^c(t) - A^*\tilde{z}_c(t), \qquad (32)$$

and $y(0) = a_2 - B^* + z_0(a_1 - A^*)$. Since $a_2 > B^*$ and $a_1 > A^*$, it follows that $y(0) > 0$. Differentiating equation 32 with respect to time yields: $\dot{y}(t) = a_1\dot{z}_L^c(t) - A^*\dot{\tilde{z}}_c(t)$. Making use of (1), (14a) and (14b), this expression can be rewritten as $\dot{y}(t) = -(\rho + NA^*)y(t) +$

$N(a_1 - A^*)q_L^c(t) + \rho(a_2 - B^*)$. Since $q_L^c(t) \geq 0$, $a_1 > A^*$ and $a_2 > B^*$, it follows that $\dot{y}(t) \geq -(\rho + NA^*)y(t)$ for all $t \in [0, \nu)$. By Gronwall's inequality, we then have $y(t) \geq y(0)e^{-(\rho+NA^*)t} > 0$ for all $t \in [0, \nu)$.

(iii) Since $z_0 \geq 0$, $a_1' > 0$, $a_2' > 0$ and $a_3' < 0$, it follows that $W_L^c(0) = -a_1 z_0^2/2 - a_2 z_0 + a_3 < W_L^c(0)_{|\beta=0} \equiv \tilde{W}_c(0)$.

B.2 Proof of corollary 3.1

(i) Using result *(i)* from Proposition 2 at the instant of time $t = \nu$, one gets: $\tilde{z}_c(\nu) > z_\nu^c$. Set $g(t) = \tilde{z}_c(t) - z_H^c(t)$ for all $t \geq \nu$. Clearly we have $g(\nu) = \tilde{z}_c(\nu) - z_\nu^c > 0$. It is easy to show that $\dot{g}(t) = -(\rho + NA^*)g(t) + N(B - B^*) + N(A - A^*)z_H^c(t)$. Since $A > A^*$, $B > B^*$ and $z_H^c(t) \geq 0$ we have $\dot{g}(t) \geq -(\rho + NA^*)g(t)$. Making use of Gronwall's inequality, we obtain $g(t) \geq g(\nu)e^{-(\rho+NA^*)(t-\nu)} > 0$, for all $t \geq \nu$. Result *(i)* then follows.

(ii) Let us first prove that $q_H^c(\nu) = \sigma - B - Az_\nu^c < \tilde{q}_c(\nu) = \sigma - B^* - A^*\tilde{z}_c(\nu)$. Notice that $\lim_{\beta\to+\infty} a_1(\beta) = A$ and $\lim_{\beta\to+\infty} a_2(\beta) = B$; in addition, $a_1(\beta)$, $a_2(\beta)$ are increasing in β, and hence A and B are respectively their minimum upper bound. Since $q_L^c(t) = \sigma - a_2 - a_1 z_L^c(t) \leq \sigma - B^* - A^*\tilde{z}_c(t) = \tilde{q}_c(t)$ for all $t \in [0, \nu)$, by continuity of $\tilde{z}_c(t)$ and $z_L^c(t)$ at the point $t = \nu$ that inequality also works for $t = \nu$. Hence we have $\sigma + B + Az_\nu^c > \sigma + a_2(\beta) + a_1(\beta)z_\nu^c \geq \sigma + B^* + A^*\tilde{z}_c(\nu)$. Rearranging the first and the last term of these inequalities, one obtains $q_H^c(\nu) = \sigma - B - Az_\nu^c < \sigma - B^* - A^*\tilde{z}_c(\nu) = \tilde{q}_c(\nu)$.

Now, we are going to prove that $q_H^c(t) < \tilde{q}_c(t)$ for all $t \geq \nu$. Set $p(t) = \tilde{q}_c(t) - q_H^c(t)$ for all $t \geq \nu$. Since $q_H^c(\nu) < \tilde{q}_c(\nu)$, we have $p(\nu) > 0$. Using a similar method as above, we get that $\dot{p}(t) = -(\rho + NA^*)p(t) + \rho(B - B^*) + N(A - A^*)q_H^c(t)$, from which we derive: $\dot{p}(t) > -(\rho + NA^*)p(t)$ for all $t \geq \nu$. Applying Gronwall's inequality, we obtain $p(t) \geq p(\nu)e^{-(\rho+NA^*)(t-\nu)} > 0$ for all $t \geq \nu$. Hence $\tilde{q}_c(t) > q_H^c(t)$ for all $t \geq \nu$.

B.3 Proof that $W(z_c^{stea}) > \tilde{W}(\tilde{z}_c^{stea})$ if and only if $\rho(r+\rho)^2 \leq N^2\theta(r-\rho)$.

Recall that $\tilde{W}(\tilde{z}_c^{stea}) = W(z_c^{stea})_{|m=0}$ and $W(z_c^{stea}) \equiv W(z_c^{stea}, \theta + m) = -A(z_c^{stea})^2/2 - Bz_c^{stea} + C$. Replacing A, B, C and z_c^{stea} by their values given respectively in Section 3.1

and in Proposition 1, we get:

$$W(z_c^{stea}) = \sigma^2 N(r+\rho)[N^2(\rho - r)(\theta + m) + \rho^2(\rho + r)]/2r[\rho(r+\rho) + N^2(\theta+m)]^2,$$

from which we derive

$$\partial W(z_c^{stea})/\partial m = -\sigma^2 N^3(r+\rho)[N^2(\rho - r)(\theta + m) + \rho(\rho + r)^2]/2r[\rho(r+\rho) + N^2(\theta+m)]^3.$$

It is helpful to distinguish three cases.

Case 1: if $\rho \geq r$ then, $\partial W(z_c^{stea})/\partial m < 0$ for all $m > 0$. Hence $W(z_c^{stea})_{|m=0} > W(z_c^{stea})$.

Case 2: if $\rho(r+\rho)^2 > N^2\theta(r-\rho) > 0$, then $W(z_c^{stea})$ is convex in m; in addition, we have $\lim_{m\to\infty} W(z_c^{stea}) = 0 < W(z_c^{stea})_{|m=0}$. Therefore $W(z_c^{stea})_{|m=0} > W(z_c^{stea})$.

Case 3: if $N^2\theta(r-\rho) \geq \rho(r+\rho)^2$, $\partial W(z_c^{stea})/\partial m > 0$ for all $m > 0$. Hence $W(z_c^{stea}) > W(z_c^{stea})_{|m=0}$. The result then follows.

C Details for the non-cooperative equilibrium

Consider first the state $\theta(t) = \theta + m$. The equilibrium value function must then be a solution to the following differential equation:

$$rV(z, \theta+m) = (N - 1/2)V_z(z,\theta+m)^2 + (N\sigma - \rho z)V_z(z,\theta+m) + \sigma^2/2 - (\theta+m)z^2/2. \quad (33)$$

Given the quadratic nature of the instantaneous benefit function, a plausible guess is:

$$V(z, \theta+m) = -\frac{\hat{A}}{2}z^2 - \hat{B}z + \hat{C}. \quad (34)$$

Using a similar argument as for the cooperative equilibrium, we get that this will indeed be a solution if:

$$\begin{aligned} \hat{A} &= [-(2\rho + r) + \sqrt{(2\rho+r)^2 + 4(2N-1)(\theta+m)}]/2(2N-1) > 0, \\ \hat{B} &= \sigma N\hat{A}/[r + \rho + (2N-1)\hat{A}], \\ \hat{C} &= [\sigma^2 - 2\sigma N\hat{B} + (2N-1)\hat{B}^2]/2r. \end{aligned} \quad (35)$$

We have $\sigma - \hat{B} = \sigma[(r+\rho)+\hat{A}(N-1)]/[r+\rho+(2N-1)\hat{A}] > 0$, and hence $\sigma > \hat{B}$. Applying the chain rule for differentiation, we obtain the following results:

$$\hat{A}' \equiv \frac{\partial \hat{A}}{\partial m} > 0; \hat{B}' \equiv \frac{\partial \hat{B}}{\partial m} = \frac{\partial \hat{B}}{\partial \hat{A}} \times \frac{\partial \hat{A}}{\partial m} > 0; \hat{C}' \equiv \frac{\partial \hat{C}}{\partial m} = \frac{\partial \hat{C}}{\partial \hat{B}} \times \frac{\partial \hat{B}}{\partial m} < 0.$$

Consider next the state $\theta(t) = \theta$. The equilibrium value function must then solve:

$$(N-1/2)V_z(z,\theta)^2+(N\sigma-\rho z)V_z(z,\theta)-(r+\beta)V(z,\theta)+\beta V(z,\theta+m)+\sigma^2/2-\theta z^2/2 = 0. \quad (36)$$

Again a plausible guess is:

$$V(z,\theta) = -\frac{u_1}{2}z^2 - u_2 z + u_3. \quad (37)$$

Using a similar method as for the cooperative equilibrium, we find that it will indeed be a solution if:

$$u_1 = [-(2\rho + r + \beta) + \sqrt{(2\rho + r + \beta)^2 + 4(2N-1)(\hat{A}\beta + \theta)}]/2(2N-1),$$
$$u_2 = \frac{N\sigma u_1 + \beta\hat{B}}{(2N-1)u_1 + \rho + r + \beta},$$
$$u_3 = [\sigma^2 + 2\beta\hat{C} - 2\sigma N u_2 + u_2^2(2N-1)]/2(r+\beta).$$

There remains to determine the signs of $u_1' \equiv \partial u_1/\partial\beta$, $u_2' \equiv \partial u_2/\partial\beta$, and $u_3' \equiv \partial u_3/\partial\beta$.

$$u_1' = [-1 + \frac{(\beta + r + 2\rho) + 2(2N-1)\hat{A}}{\sqrt{(2\rho + r + \beta)^2 + 4(2N-1)(\hat{A}\beta + \theta)}}]/2(2N-1).$$

Using a similar argument as for the proof of $a_1' > 0$, while taking into account the fact that $\hat{A}$ satisfies $(2N-1)\hat{A}^2 + (r+2\rho)\hat{A} = m + \theta$, we verify that $u_1' > 0$. Notice that u_1 satisfies the polynomial $(2N-1)u_1^2 + (2\rho+\beta+r)u_1 - (\hat{A}\beta+\theta) = 0$, which, upon differentiation with respect to β, yields $u_1'[2(2N-1)u_1 + 2\rho + \beta + r] = \hat{A} - u_1$. Since the left-hand side of this equality is positive, so is its right-hand side. Therefore $\hat{A} > u_1$ as stated.

$$u_2' = \{N\sigma(\rho+r)u_1' + \beta u_1'(N\sigma-(2N-1)\hat{B}) + u_1[(2N-1)\hat{B}-N\sigma] + \hat{B}(r+\rho)\}/[(2N-1)u_1+r+\rho+\beta]^2.$$

Since the denominator of u_2' is positive, its sign is that of its numerator. Using (35), the numerator of u_2' can be rewritten as: $N\sigma(\rho+r)u_1' + \frac{N\sigma(r+\rho)\beta}{r+\rho+(2N-1)\hat{A}}u_1' + \frac{N\sigma(r+\rho)}{r+\rho+(2N-1)\hat{A}}(\hat{A} - u_1)$, which is positive since each of the terms are positive. Therefore $u_2' > 0$.

$u_3' = \{(\hat{B} - u_2)((2N-1)\hat{B} - \sigma N + (2N-1)u_2 - \sigma N) + 2(r+\beta)u_2'[(2N-1)u_2 - \sigma N]\}/2(r+\beta)^2$,

because $2r\hat{C} = \sigma^2 - 2\sigma N\hat{B} + (2N-1)\hat{B}^2$. Since we have:

$(2N-1)\hat{B} - \sigma N = -\sigma N(r+\rho)/[(2N-1)\hat{A} + r + \rho] < 0$,

$(2N-1)u_2 - \sigma N = [-\sigma N(r+\rho) + \beta((2N-1)\hat{B} - \sigma N)]/[(2N-1)u_1 + r + \rho + \beta] < 0$, and

$\hat{B} - u_2 = -\sigma N(r+\rho)(2N-1)(u_1 - \hat{A})/[(2N-1)u_1 + r + \rho + \beta][r + \rho + (2N-1)\hat{A}] > 0$,

it follows that $u_3' < 0$.

C.1 Proof that $A > \hat{A}$, $B > \hat{B}$, $a_1 > u_1$ and $a_2 > u_2$

Since $N^2 > 2N - 1$ for $N \geq 2$, we have:

$$\begin{aligned}
-(2\rho + r) + \sqrt{(2\rho + r)^2 + 4N^2(\theta + m)} &> -(2\rho + r) + \sqrt{(2\rho + r)^2 + 4(2N-1)(\theta + m)} \\
1/2N &\geq 1/2(2N-1)
\end{aligned}$$

Multiplying side by side both inequalities, we verify that $A > \hat{A}$.

We have $B - \hat{B} = \sigma N[(r+\rho)(A - \hat{A}) + (N-1)A\hat{A}]/(r + \rho + NA)(r + \rho + (2N-1)\hat{A}) > 0$ and hence $B > \hat{B}$.

Since $4N(A\beta + N\theta) > 4(2N-1)(\hat{A}\beta + \theta)$ for $N \geq 2$, it follows that:

$$\begin{aligned}
&-(2\rho + r + \beta) + \sqrt{(2\rho + r + \beta)^2 + 4N(A\beta + N\theta)} > \\
&\qquad -(2\rho + r + \beta) + \sqrt{(2\rho + r + \beta)^2 + 4(2N-1)(\hat{A}\beta + \theta)}
\end{aligned}$$

But recall that we also have:

$$1/2N > 1/2(2N-1).$$

By multiplying side by side those two inequalities we verify that $a_1 > u_1$.

Finally, since $B > \hat{B}$, using (30) it is easy to verify that:

$$a_2 > \frac{N\sigma a_1 + \beta\hat{B}}{(2N-1)a_1 + \rho + r + \beta} = f(a_1),$$

from which we derive:

$$f(a_1) - f(u_1) = \frac{(a_1 - u_1)[\sigma N(\rho + r) + \sigma N\beta - (2N-1)\beta\hat{B}]}{((2N-1)a_1 + \rho + \beta + r)((2N-1)u_1 + \rho + \beta + r)}.$$

Using (35), we get that $\sigma N\beta - (2N-1)\beta\hat{B} = N\sigma\beta(r+\rho)/(r+\rho+(2N-1)\hat{A}) > 0$. It then follows that $a_2 > f(a_1) > f(u_1) = u_2$.

C.2 Proof of Proposition 5

(i) From Appendix A we know that $pr(0 < \nu < \infty) = 1$. Thus, almost surely the state of high damages must occur at a finite date. This means that time path of the stock of pollutant is in the long run given by $z_H^n(t)$, which converges to z_n^{stea}. Since $A > \hat{A}$ and $B > \hat{B}$ we have:

$$\begin{aligned} N(\sigma - \hat{B}) &> N(\sigma - B) \\ 1/(\rho + N\hat{A}) &> 1/(\rho + NA). \end{aligned}$$

Multiplying those inequalities side by side, we get:

$$z_n^{stea} = N(\sigma - \hat{B})/(\rho + N\hat{A}) > N(\sigma - B)/(\rho + NA) = z_c^{stea}.$$

The proof that the presence of the threat of a jump in the damages results in a lower stock of pollutant than when that threat is not present is similar to that used to derive the same result for the cooperative equilibrium.

(ii) Notice that at the steady state, we have: $\dot{z} = Nq - \rho z = 0$. Hence, $q_n^{stea} = \rho z_n^{stea}/N$ and $q_c^{stea} = \rho z_c^{stea}/N$. Since we have just shown that $z_n^{stea} > z_c^{stea}$, it follows that $q_n^{stea} > q_c^{stea}$.

C.3 Proof of Proposition 6

(i) Using a similar method as for the proof of (ii) in Proposition 2, with u_2 replacing a_2 and u_1 replacing a_1 we get $\tilde{q}_n(t) \equiv q_L^n(t)_{|\beta=0} > q_L^n(t)$ for all $t \in (0, \nu]$. By a similar argument as for the proof of (ii) in Corollary 3.1, we obtain $\tilde{q}_n(t) \equiv q_H^n(t)_{|m=0} > q_H^n(t)$ for all $t \in (\nu, \infty]$.

(ii) The solution for $z_L^n(t)$, for $t \in (0, \nu]$, can be obtained from $z_L^c(t)$ by replacing a_1 by u_1

and a_2 by u_2. The proof of $\partial z_L^c(t)/\partial\beta < 0$ for all $t \in (0, \nu]$ in Proposition 2 rested only on the facts that $a_1, a_1', a_2, a_2' > 0$. Since $u_1, u_1', u_2, u_2' > 0$, we can conclude that $\partial z_L^n(t)/\partial\beta < 0$ for all $0 < t \leq \nu$ as well. Therefore $\tilde{z}_n(t) \equiv z_L^n(t)_{|\beta=0} > z_L^n(t)$ for all $t \in (0, \nu]$. We also have $\tilde{z}_n(t) \equiv z_H^n(t)_{|m=0} > z_H^n(t)$ for all $t \in (\nu, \infty]$. Indeed, its proof is similar to that of (i) in Corollary 3.1 in which $\hat{A}^* \equiv \hat{A}_{|m=0}$ plays the role of A^* whereas $\hat{B}^* \equiv \hat{B}_{|m=0}$ plays the role of B^*.

(iii) Since $z_0 \geq 0$, $u_1' > 0$, $u_2' > 0$ and $u_3' < 0$, we have $V_L^n(0) = -u_1 z_0^2/2 - u_2 z_0 + u_3 < V_L^n(0)_{|\beta=0} \equiv \tilde{V}_n(0)$.

D Comparison of the cooperative and non-cooperative equilibria

Set $\Delta(t) = q_L^n(t) - q_L^c(t)$ for all $t \in [0, \nu)$. Using the expressions for $q_L^c(t)$ and $q_L^n(t)$ given respectively in Section 3 and in Section 4, we get $\Delta(t) = a_1 z_L^c(t) - u_1 z_L^n(t) + a_2 - u_2$. Since $z_0 \geq 0, a_1 > u_1$ and $a_2 > u_2$, it follows that $\Delta(0) = (a_1 - u_1)z_0 + a_2 - u_2 > 0$. Differentiating $\Delta(t)$, we obtain $\dot{\Delta}(t) = N(a_1 - u_1)q_L^c(t) - (\rho + Nu_1)\Delta(t) + \rho(a_2 - u_2)$. Since $q_L^c(t) > 0$, $a_1 > u_1$ and $a_2 > u_2$, it follows that $\dot{\Delta}(t) \geq -(\rho + Nu_1)\Delta(t)$ for all $t \in [0, \nu)$. Using Gronwall's inequality, we get $\Delta(t) \geq \Delta(0)e^{-(\rho+Nu_1)t} > 0$. Thus $q_L^n(t) > q_L^c(t)$ for all $t \in [0, \nu)$. Applying again Gronwall's inequality, we verify that $q_H^n(t) > q_H^c(t)$ for all $t \geq \nu$.

Now set $h(t) = z_L^n(t) - z_L^c(t)$ for all $t \in [0, \nu]$. We have $h(0) = z_0 - z_0 = 0$ and $\dot{h}(t) = N(a_2 - u_2) - (\rho + Nu_1)h(t) + N(a_1 - u_1)z_L^c(t)$. Since $a_2 > u_2$, $a_1 > u_1$, and $z_L^c(t) \geq 0$, it follows that $\dot{h}(t) \geq -(\rho + Nu_1)h(t) + N(a_2 - u_2)$. Using Gronwall's inequality, we get $h(t) > h(0)e^{-(\rho+Nu_1)t} = 0$ for all $t \in (0, \nu]$. Hence $z_L^n(t) > z_L^c(t)$ for all $t \in (0, \nu]$. Using once more Gronwall's inequality, we verify similarly that $z_H^n(t) > z_H^c(t)$ for all $t \geq \nu$.

Finally, we may compare the steady-state levels of welfare. In the two cases, the steady state occurs after the state $\theta + m$ is reached. Set $\mu(z) = W(z, \theta + m)/N$ and $V(z) = V(z, \theta + m)$, the welfare of each individual country in respectively the cooperative and the non-cooperative equilibrium. By definition the cooperative solution maximizes the global welfare of the N identical countries and hence, for any given identical initial stock of pollution

z, $\mu(z) \geq V(z)$. In particular we have $\mu(z_c^{stea}) \geq V(z_c^{stea})$. But we know from Proposition 5 that $z_n^{stea} > z_c^{stea}$, therefore, since clearly $V_z(z) < 0$, we have $\mu(z_c^{stea}) > V(z_n^{stea})$.

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