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Government Spending, Monetary Policy, and the Real Exchange Rate

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Abstract:
A robust prediction across a wide range of open-economy macroeconomic models is that an unanticipated increase in public spending in a given country appreciates it currency in real terms. This result, however, contradicts the findings of a number of recent empirical studies, which instead document a significant and persistent depreciation of the real exchange rate following an expansionary government spending shock. In this paper, we rationalize the findings of the empirical literature by proposing a small-open-economy model that features three key ingredients: incomplete and imperfect international financial markets, sticky prices, and a not-too-aggressive monetary policy. The model predicts that in response to an unexpected increase in public expenditures, the effective long-term real interest rate falls, causing the real exchange rate to depreciate. We establish this result both analytically, within a special version of the model, and numerically for the more general case.

Keywords: Incomplete markets, monetary policy, public spending shocks, real exchange rate, small open economy, sticky prices

JEL Classification: F31, F41

1. Introduction

A robust prediction across a wide range of open-economy macroeconomic models is that an unanticipated increase in public spending in a given country appreciates it currency in real terms. This result holds both in the traditional Mundell-Fleming-Dornbusch framework and in standard dynamic general-equilibrium models that assume complete financial markets. In the former, the appreciation of the real exchange rate results from the rise in aggregate demand induced by the increase in public spending, which falls more heavily on domestically produced goods, raising their relative price with respect to foreign goods. In dynamic general-equilibrium models with complete markets, the real appreciation ensues from a risk-sharing condition that relates the real exchange rate to the ratio of marginal utilities of consumption across countries. Because the increase in public spending must ultimately be financed through taxes, it lowers households' permanent income, which in turn raises the shadow value of wealth in the domestic country and appreciates its currency in real terms.

This prediction, however, appears to be at odds with the data. Several empirical studies indeed find that the real exchange rate depreciates persistently in response to an unexpected exogenous increase in government expenditures (Corsetti and Müller [2006], Kim and Roubini [2008], Müller [2008], Monacelli and Perotti [2010], Enders, Müller and Scholl [2011], Ravn, Schmitt-Grohé and Uribe [2011], and Bouakez, Chihi and Normandin [2011]). This result holds across different countries, sample periods, and identification schemes, and is thus increasingly becoming a generally accepted stylized fact. To the extent that the response of the real exchange rate to public spending shocks is likely to play a critical role in determining the size of the spending multiplier in open economies, it is important to build macroeconomic models that are able to account for the empirical evidence.

In this paper, we rationalize the findings of the empirical literature by proposing a small-open-economy model that combines three key features: (*i*) incomplete and imperfect international financial markets, (*ii*) sticky prices, and (*iii*) a not-too-aggressive monetary policy. Under incomplete financial markets, the real exchange rate is no longer pinned down by the ratio of marginal utilities of consumption, but instead by (the negative of) the long-term real interest rate net of a risk premium. The latter is assumed to be an increasing function of the economy's foreign debt, as in Kollmann [2002], Schmitt-Grohé and Uribe [2003], and Senhadji [2003], and is meant to capture frictions in international financial markets stemming, for instance, from the possibility of default or from agency costs. By raising demand for both domestically produced and foreign goods, an increase in public spending raises domestic inflation and deteriorates the current account. When prices are sticky and monetary policy does not react too aggressively to (expected) inflation, the resulting increase in the long-term real interest rate is smaller than the increase in the risk premium, causing the real exchange rate to depreciate.

We illustrate this mechanism within a special version of our model in which we abstract from uncertainty and where we restrict the real interest rate to remain constant at its steady-state value at all times. This is achieved by assuming that the monetary authority changes the nominal interest rate one for one with expected inflation. The constancy of the real interest rate, which is only possible under sticky prices, implies that consumption also remains constant in response to transitory shocks. We can then solve the model analytically (up to a first-order approximation) and show that an increase in government purchases leads to a depreciation of the real exchange rate and to a fall in net foreign assets, consistent with the empirical findings. Intuitively, when the real interest rate is constant, output and the real exchange rate are determined independently of the supply side of the economy. Instead, their equilibrium paths are entirely pinned down by the equilibrium condition in the goods market (IS equation) and by the balance of payments (BOP) equation. By raising aggregate demand, an expansionary public spending shock shifts the IS curve outward in the output-real exchange rate space. But under a constant real interest rate, the shock causes an even larger outward shift of the BOP curve. To restore the equilibrium, the real exchange rate must depreciate, and this depreciation is larger the larger the degree of financial market imperfections.

We then proceed to the analysis of the general case via numerical simulations in order to quantify the relationship between the aggressiveness of monetary policy and the real exchange rate response to a public spending shock. We find that the values of the policy-rule parameter that generate a real depreciation fall in the range of empirically plausible numbers. Finally, we perform a sensitivity analysis by varying the values of key parameters within the range of available estimates and by assuming an alternative specification of consumer preferences. We find our main conclusion to be extremely robust to these perturbations.

Three closely related papers to ours are those by Kollmann [2010], Corsetti, Meier and Müller [2011], and Ravn et al. [2011]. Kollmann [2010] considers a two-country model with incomplete financial markets and flexible prices, and shows that an increase in public spending in one country can depreciate its real exchange rate, provided that labor supply is highly elastic. The values of the elasticity of labor supply needed to generate a real depreciation, however, seem to be hard to reconcile with available empirical estimates. In contrast, our explanation does not require such an extreme assumption about the elasticity of labor supply.

Corsetti et al. [2011] develop a two-country model with complete markets, sticky prices and wages, and spending reversals. The latter feature refers to the assumption that debt-financed increases in government spending will cause subsequent spending to fall below its steady state level for some time. In turn, this lowers long-term real interest rates and depreciates the currency in real terms. However, while empirical studies based on structural vector autoregressions (SVAR) invariably find that the path of government spending in the U.S. is "self-correcting", increasing for six to twelve quarters before eventually falling below trend, results for other countries do not display such a reversal, especially for long sample periods (e.g., Perotti [2005] and Bouakez et al. [2011]).

Finally, Ravn et al. [2011] also consider a two-country model with complete financial markets, but depart from time-separable preferences by assuming that consumers form deep habits, i.e., habits at the level of individual varieties of goods. The presence of deep habits induces firms to lower their markups in markets in which aggregate demand rises. Thus, in response to an increase in government spending in the domestic economy, markups on domestically sold goods will be lower than markups abroad, making those goods relatively cheaper in the domestic economy, and implying a depreciation of the real exchange rate. While our explanation involves a completely different mechanism than those discussed above, we regard these different mechanisms as complementary, rather than competing or mutually exclusive, explanations for the "puzzling" real depreciation caused by an expansionary public spending shock.

The paper is organized as follows. Section 2 presents the model. Section 3 analyzes the effects of a government spending shock first analytically within a simplified version of the model and then numerically for the general case. It also performs an extensive sensitivity analysis with respect to key parameters of the model. Section 4 concludes.

2. The Model Economy

The world consists of a continuum of identical small open economies that trade in final goods and financial assets. The size of each economy is small relative to the rest of the world. Each economy is populated by a continuum of identical, infinitely lived, households, and has a continuum of monopolistically competitive firms that produce a differentiated final good. The number of households and firms is normalized to one. Firms do not price discriminate between the domestic and export market, which means that the law of one price holds for all goods. Financial markets are assumed to be incomplete as households can only trade one-period non-state-contingent domestic and international bonds. Financial markets are also assumed to be imperfect in that the interest rate faced by domestic agents on their external borrowing is augmented by a premium that is increasing in the aggregate level of foreign debt.[1] This debt-elastic premium implicitly captures the idea that foreign lenders may perceive higher levels of foreign debt as an indication of a higher probability of default.[2] In what follows, we describe a representative small open economy, which we call "home country", and denote by an asterisk variables pertaining to the rest of the world. Throughout the paper, variables without time subscripts denote steady-state values.

Households The representative household in the home country maximizes its expected lifetime utility

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\ln c_s - \omega \frac{\ell_s^{1+1/\zeta}}{1+1/\zeta} \right), \tag{1}$$

subject to the budget constraint

$$\varepsilon_t B_t^* + B_t + P_t c_t = \varepsilon_t \xi_{t-1} R_{t-1}^* B_{t-1}^* + R_{t-1} B_{t-1} + P_t w_t \ell_t + D_t - \tau_t, \tag{2}$$

and to the appropriate no-ponzi-game conditions. In expression (1), the parameters $0 < \beta < 1$ and $\zeta > 0$ are, respectively, the subjective discount factor and the Frisch elasticity of labor supply, c_t is a consumption index, and ℓ_t is the quantity of labor competitively supplied to the firms. In equation (2), B_t^* and B_t are the household's nominal holdings of, respectively, international and domestic bonds at the end of period t; ε_t is the nominal exchange rate, defined as the price of a unit of the foreign currency in terms of the domestic currency; R_{t-1}^* and R_{t-1} are the nominal interest rates paid on, respectively, foreign and domestic bonds between periods $t-1$ and t; P_t is the consumption-based price index (CPI hereafter), w_t is the real wage rate, $D_t = \int_0^1 D_t(i)di$ denotes the household's claims on firms' (indexed by i) nominal profits, and τ_t denotes lump-sum taxes paid to the government. The term ξ_t is a risk premium that increases with the economy's aggregate level of debt as a percentage of steady-state output. More specifically, ξ_t has the following functional form.

$$\xi_t = \exp\left(-\gamma \frac{\varepsilon_t \tilde{B}_t^*}{Y} \right), \tag{3}$$

where $\tilde{B}_t^*$ is the aggregate level of foreign debt, which the household in the small open economy takes as given, and Y denotes nominal GDP in the steady state. Denoting by $q_t \equiv \varepsilon_t P_t^* / P_t$ the

[1] This assumption has been used by several authors, such as Kollmann [2002] or Schmitt-Grohé and Uribe [2003].

[2] This premium also induces stationarity in the log-linearized version of the model. Without it, the model's equilibrium dynamics exhibit a unit root (see Schmitt-Grohé and Uribe [2003]).

real exchange rate, the household's first order conditions imply

$$\omega c_t \ell_t^{1/\zeta} - w_t = 0, \tag{4}$$

$$\beta R_t E_t \left(\frac{P_t c_t}{P_{t+1} c_{t+1}} \right) - 1 = 0, \tag{5}$$

$$E_t \left[\frac{1}{P_{t+1} c_{t+1}} \left(R_t - \frac{q_{t+1}}{q_t} \frac{P_{t+1}}{P_t} \frac{P_t^*}{P_{t+1}^*} \xi_t R_t^* \right) \right] = 0, \tag{6}$$

in addition to the usual transversality conditions. Equation (4) is a standard labor supply equation, which equates the marginal rate of substitution between consumption and leisure to the real wage. Equation (5) is the Euler equation, relating the expected growth rate of consumption to the real interest rate. Equation (6) is a no-arbitrage condition that equates the expected rates of return on foreign and domestic bonds. A log-linear approximation of this equation yields a modified uncovered interest rate parity condition that includes a risk premium.

The consumption index, c_t, is an aggregate of consumption of goods produced in the home country ($c_{H,t}$) and goods produced in the rest of the world ($c_{F,t}$)

$$c_t = \left((1-\alpha)^{\frac{1}{\mu}} \left(c_{H,t} \right)^{\frac{\mu-1}{\mu}} + \alpha^{\frac{1}{\mu}} \left(c_{F,t} \right)^{\frac{\mu-1}{\mu}} \right)^{\frac{\mu}{\mu-1}}, \tag{7}$$

where the parameter $\alpha \in [0,1]$ measures the share of imported goods in total consumption, thus reflecting the degree of openness of the small open economy,[3] and the parameter μ is the elasticity of substitution between domestic and foreign goods. The associated CPI is

$$P_t = \left((1-\alpha) \left(P_{H,t} \right)^{1-\mu} + \alpha \left(P_{F,t} \right)^{1-\mu} \right)^{\frac{1}{1-\mu}}, \tag{8}$$

where $P_{H,t}$ and $P_{F,t}$ are the price sub-indexes associated with $c_{H,t}$ and $c_{F,t}$, respectively. The optimal allocation of consumption between home and foreign goods gives rise to the following demand functions:

$$c_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\mu} c_t, \quad c_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\mu} c_t. \tag{9}$$

The consumption sub-indexes $c_{H,t}$ and $c_{F,t}$ are bundles of differentiated varieties produced domestically and abroad, and are given by, respectively

$$c_{H,t} = \left(\int_0^1 c_{H,t}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \quad c_{F,t} = \left(\int_0^1 c_{F,t}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \tag{10}$$

where $c_{H,t}(j)$ (respectively $c_{F,t}(j)$) is consumption of a typical variety i produced in the home country (respectively in the rest of the world) and $\theta > 1$ is the elasticity of substitution between

[3] The fraction $(1-\alpha)$ therefore measures the degree of home bias in consumption.

varieties originating in the same country. The price sub-indexes associated with $c_{H,t}$ and $c_{F,t}$ are given by, respectively

$$P_{H,t} = \left(\int_0^1 P_{H,t}(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}, \qquad P_{F,t} = \left(\int_0^1 P_{F,t}(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}, \tag{11}$$

where $P_{H,t}(j)$ (respectively $P_{F,t}(j)$) is the domestic-currency price of a typical variety i produced in the home country (respectively in the rest of the world). Demand functions for domestically produced and foreign varieties are given by

$$c_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\theta} c_{H,t}, \quad c_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\theta} c_{F,t}. \tag{12}$$

Note that because the law of one price holds for all goods, we have $P_{F,t}(i) = \varepsilon_t P_{F,t}^*(i)\ \forall i$, and thus $P_{F,t} = \varepsilon_t P_{F,t}^*$.

Firms Each monopolistically competitive firm is the supplier of a single variety i, produced with the following linear technology:

$$y_t(i) = n_t(i), \tag{13}$$

where $n_t(i)$ is the quantity of labor hired by firm i. Since firms do not segment markets by country, the export price of a typical firm i, $P_{H,t}^*(i)$, will be equal to $P_{H,t}(i)/\varepsilon_t$, which also implies that $P_{H,t}^* = P_{H,t}/\varepsilon_t$. Prices are set à la Calvo [1983]. That is, each period, a fraction $(1-\eta)$ of firms are randomly selected to set new prices while the remaining fraction η of firms keep their prices unchanged. Let $\overline{P}_{H,t}$ denote the price set by a typical firm at period t.[4] Assuming that the government consumption indexes are analogous to those of private consumption, the total demand facing this firm in period s $(s \geq t)$ is

$$\begin{aligned} \overline{y}_s &= \left(\frac{\overline{P}_{H,t}}{P_{H,s}}\right)^{-\theta} \left(c_{H,s} + c_{H,s}^* + g_{H,s} + g_{H,s}^*\right) \\ &= \left(\frac{\overline{P}_{H,t}}{P_{H,s}}\right)^{-\theta} \left(\frac{P_{H,s}}{P_s}\right)^{-\mu} \left((1-\alpha)(c_s + g_s) + \alpha q_s^{\mu}(c_s^* + g_s^*)\right). \end{aligned} \tag{14}$$

The optimal price, $\overline{P}_{H,t}$, is chosen to maximize

$$\bar{D}_t = E_t \sum_{s=t}^{\infty} (\eta\beta)^{s-t} \lambda_{t,s} \left(\overline{P}_{H,t} - P_s w_s\right) \overline{y}_s, \tag{15}$$

where $\lambda_{t,s} = \frac{P_t c_t}{P_s c_s}$. The first-order condition for this problem yields

$$\overline{P}_{H,t} = \frac{\theta}{\theta-1} \frac{\sum_{s=t}^{\infty} (\eta\beta)^{s-t} E_t\left(w_s \overline{y}_s / c_s\right)}{\sum_{s=t}^{\infty} (\eta\beta)^{s-t} E_t\left(\overline{y}_s / (P_s c_s)\right)}. \tag{16}$$

[4] The optimal price does not depend on the firm index because all the firms that have the opportunity to change their prices at a given time choose the same price.

Aggregating across all firms, the price index for domestically produced goods is given by

$$P_{H,t} = \left((1-\eta)\,\overline{P}_{H,t}{}^{1-\theta} + \eta P_{H,t-1}^{1-\theta}\right)^{\frac{1}{1-\theta}}. \tag{17}$$

Government Since Ricardian equivalence holds in this model, we abstract from public debt and assume that the government finances its expenditures through lump-sum taxes. That is

$$P_t g_t = \tau_t. \tag{18}$$

Public spending is exogenous and follows the autoregressive process

$$\ln\left(\frac{g_t}{g}\right) = \rho_g \ln\left(\frac{g_{t-1}}{g}\right) + \epsilon_t^g, \tag{19}$$

where $0 \leq \rho_g \leq 1$ and ϵ_t^g is a serially uncorrelated disturbance with zero mean.

Monetary authority The monetary authority in the small open economy sets the nominal interest rate according to the following rule:

$$\ln\left(\frac{R_t}{R}\right) = \rho_r \ln\left(\frac{R_{t-1}}{R}\right) + (1-\rho_r)\phi_\pi E_t\left(\frac{\pi_{t+1}}{\pi}\right), \tag{20}$$

where $\pi_t = P_t/P_{t-1}$ is the CPI inflation rate between periods $t-1$ and t, $0 \leq \rho_r < 1$ captures the degree to which the monetary authority smooths out changes in the nominal interest rate, and $\phi_\pi \geq 1$ measures the aggressiveness with which it responds to expected inflation. The assumption that the monetary authority responds to expected, rather than current inflation, is only made for tractability as it allows us to obtain analytical results in Section 3.1, but it is not essential to the general point made in this paper.

2.1 Equilibrium, steady-state and log-linearized model

Equilibrium Given that we consider a small open economy, we assume that domestically produced goods receive a negligible weight in the world consumption bundle. This implies that $P_{F,t}^* = P_t^*$. For the purpose of deriving the market clearing conditions, let $y_t \equiv \left(\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$ denote aggregate output. The goods market clearing condition is then given by

$$y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\mu} \left((1-\alpha)\left(c_t + g_t\right) + \alpha q_t^\mu \left(c_t^* + g_t^*\right)\right). \tag{21}$$

Furthermore, aggregating the production functions of all domestic firms (equation 13) yields

$$\Delta_t y_t = \int_0^1 y_t\left(i\right) di = \int_0^1 n_t\left(i\right) di \equiv n_t, \tag{22}$$

where $\Delta_t \equiv \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\theta} di$ measures the dispersion of domestic producer prices.[5] Labor-market clearing implies that

$$n_t = \ell_t. \tag{23}$$

Since households in the domestic economy are identical, the domestic bonds market is in zero net supply, i.e., $B_t/P_t \equiv b_t = 0$, and $\tilde{B}_t^*/P_t^* = B_t^*/P_t^* \equiv b_t^*$. Imposing these conditions and consolidating the households' and government budget constraints yield the following balance of payments (BOP) equation:

$$q_t b_t^* = \xi_t R_{t-1}^* q_t b_{t-1}^* + \frac{P_{H,t}}{P_t} y_t - c_t - g_t, \tag{24}$$

where we have substituted the firms' current dividends, used the fact that $P_{H,t} y_t = \int_0^1 P_{H,t}(i)\, y_t(i) di$, and imposed the labor-market clearing condition.

Conditionally on both monetary policy and the exogenous variables, a symmetric equilibrium for this economy is a collection of 9 sequences (c_t, n_t, y_t, b_t^*, w_t, q_t, P_t, $P_{H,t}$, and $\bar{P}_{H,t})_{t=0}^{\infty}$ satisfying the households' first-order conditions (equations 4–6, with equation 23 substituted in for ℓ_t), the definition of the CPI (equation 8), the pricing condition (equation 16), the definition of the price index for domestically produced goods (equation 17), the aggregate production function (equation 22), the goods market clearing condition (equation 21), and the balance of payments (equation 24).

Steady state We define a symmetric flexible-price initial steady state in which exogenous variables take identical values in the small open economy and in the rest of the world, and net foreign assets are zero, i.e., $b^* = 0$.[6] Moreover, we assume without loss of generality that all nominal prices are equal to unity in steady state. This ensures that $q = 1$. Denoting by κ the steady-state ratio of government spending to output, the steady-state allocations are

$$y = n = \left(\frac{\theta - 1}{\omega(1-\kappa)\theta}\right)^{\frac{\zeta}{1+\zeta}}, \; c = (1-\kappa)\, y. \tag{25}$$

Log-linearization In order to solve the model, we log-linearize its equilibrium conditions around the steady state. Denoting by a circumflex the log-deviation of a variable from its steady-state

[5] The variable $\widetilde{y}_t \equiv \int_0^1 y_t(i) di = \Delta_t y_t$ is the appropriate measure of GDP in this model. However, as shown by Galí and Monacelli [2005], Δ_t is of second order, which implies that GDP and aggregate output are equivalent up to a first order approximation.

[6] This symmetric steady state is convenient to derive the analytical solution discussed in the next section, but our results still hold when we log-linearize the model around a steady state with non-zero net foreign assets.

value,[7] the log-linearized version of the model is given by (see the Appendix for the derivation)

$$\widehat{c}_t = E_t\widehat{c}_{t+1} - (\hat{R}_t - E_t\widehat{\pi}_{t+1}), \tag{26}$$

$$\widehat{q}_t = E_t\widehat{q}_{t+1} + (\widehat{R}^*_t - E_t\widehat{\pi}^*_{t+1}) - (\hat{R}_t - E_t\widehat{\pi}_{t+1}) - \gamma\widehat{b}^*_t, \tag{27}$$

$$\widehat{b}^*_t = \beta^{-1}\widehat{b}^*_{t-1} + \widehat{y}_t - \left((1-\kappa)\,\widehat{c}_t + \kappa\widehat{g}_t + \frac{\alpha}{1-\alpha}\widehat{q}_t\right), \tag{28}$$

$$\widehat{\pi}_{H,t} = \beta E_t\widehat{\pi}_{H,t+1} + \frac{(1-\eta)\,(1-\eta\beta)}{\eta}\left(\widehat{w}_t + \frac{\alpha}{1-\alpha}\widehat{q}_t\right), \tag{29}$$

$$\widehat{w}_t = \widehat{c}_t + \zeta^{-1}\widehat{n}_t, \tag{30}$$

$$\widehat{\pi}_t = \widehat{\pi}_{H,t} + \frac{\alpha}{1-\alpha}\left(\widehat{q}_t - \widehat{q}_{t-1}\right), \tag{31}$$

$$\widehat{y}_t = \widehat{n}_t, \tag{32}$$

$$\widehat{y}_t = (1-\alpha)\left((1-\kappa)\,\widehat{c}_t + \kappa\widehat{g}_t\right) + \mu\alpha\left(\frac{2-\alpha}{1-\alpha}\right)\widehat{q}_t + \alpha\left((1-\kappa)\,\widehat{c}^*_t + \kappa\widehat{g}^*_t\right), \tag{33}$$

$$\widehat{R}_t = \rho_r\widehat{R}_{t-1} + (1-\rho_r)\,\phi_\pi E_t\widehat{\pi}_{t+1}, \tag{34}$$

$$\widehat{g}_t = \rho_g\widehat{g}_{t-1} + \epsilon^g_t. \tag{35}$$

3. Public Spending Shocks and the Real Exchange Rate

Assume that all foreign variables remain at their steady-state values ($\widehat{c}^*_t = \widehat{g}^*_t = \widehat{\pi}^*_t = \widehat{R}^*_t = 0$), an assumption that we shall maintain in the rest of the paper. Combining the balance of payments (equation 28) and the goods market clearing condition (equation 33) yields:

$$\alpha\kappa\widehat{g}_t = -\widehat{b}^*_t + \beta^{-1}\widehat{b}^*_{t-1} - \alpha\,(1-\kappa)\,\widehat{c}_t + \frac{\alpha}{1-\alpha}\left(\mu\,(2-\alpha) - 1\right)\widehat{q}_t. \tag{36}$$

This equation states that for given paths of private consumption and net foreign assets, an increase in government spending implies a depreciation of the real exchange rate, provided that $\mu\,(2-\alpha) > 1$. How plausible is this parametric restriction? Available estimates of the elasticity of substitution between domestic and foreign goods indicate that μ is slightly above 1, although values as large as 1.5 or 2 are frequently used in calibrated open-economy models. Estimates of the degree of trade openness suggest that plausible values for α are in the range of $(0.1, 0.4)$. Clearly, even the most conservative values of μ and α imply that the condition $\mu\,(2-\alpha) > 1$ is met. Therefore, the rest of the analysis will assume that this condition holds.

Of course, whether the real exchange rate appreciates or depreciates in equilibrium will depend on the general-equilibrium adjustment of consumption and net foreign assets. In this paper, we show that a real depreciation occurs as long as consumption does not fall too much in response to the

[7] The only exception is $\widehat{b}^*_t$ which denotes $\frac{b^*_t}{y}$.

increase in government spending, which implies that the long term real interest rate, $E_t \sum_{i=0}^{\infty}(\widehat{R}_{t+i} - \widehat{\pi}_{t+1+i})$, does not increase too much.[8] Below, we illustrate this point by focusing on a special case of the model in which the real interest rate, and hence consumption, remain constant in equilibrium. This version allows us to solve analytically for the equilibrium path of the real exchange rate. We then turn to the analysis of the more general version of the model using numerical simulations.

3.1 A special case

Consider a simplified version of our model in which we abstract from uncertainty and where we set $\rho_r = 0$ and $\phi_\pi = 1$. In this case, the monetary authority changes the nominal interest rate one for one with expected inflation, thereby implying that the short and long-term real interest rates remain constant in every period, i.e., $\widehat{R}_t - \widehat{\pi}_{t+1} = \sum_{i=0}^{\infty}(\widehat{R}_{t+i} - \widehat{\pi}_{t+1+i}) = 0$ for all t. This in turn implies that consumption remains constant in equilibrium if the economy is affected by transitory shocks, i.e., $\widehat{c}_t = 0$ for all t. Note that this equilibrium exists only if prices are sticky, as the real interest rate is independent of monetary policy and must adjust in a flexible-price equilibrium.

In the absence of intertemporal substitution (stemming from the constancy of the real interest rate), it can be easily seen that the dynamics of net foreign assets and the real exchange rate can be solved for independently of the rest of the model. More specifically, these two variables are determined by the following two-equation dynamic system:

$$\widehat{b}_t^* - \beta^{-1}\widehat{b}_{t-1}^* = \lambda\widehat{q}_t - \alpha\kappa\widehat{g}_t, \tag{37}$$

$$\widehat{q}_{t+1} - \widehat{q}_t = \gamma\widehat{b}_t^*. \tag{38}$$

where $\lambda \equiv \frac{\alpha}{1-\alpha}\left(\mu\left(2-\alpha\right)-1\right) > 0$.

A graphical solution of this system is shown in Figure 1, which depicts the phase diagram in the $(\widehat{b}_{t-1}^*, \widehat{q}_t)$ space. The loci $\Delta\widehat{b}_t^* = 0$ and $\Delta\widehat{q}_{t+1} = 0$ are characterized by the following equations, respectively:

$$\widehat{q}_t = -\frac{\beta^{-1}-1}{\lambda}\widehat{b}_{t-1}^* + \frac{\alpha\kappa}{\lambda}\widehat{g}_t, \tag{39}$$

$$\widehat{q}_t = -\frac{\beta^{-1}}{\lambda}\widehat{b}_{t-1}^* + \frac{\alpha\kappa}{\lambda}\widehat{g}_t. \tag{40}$$

[8] Indeed, iterating equation (26) forward yields:

$$\widehat{c}_t = \lim_{T\to\infty} E_t\widehat{c}_{t+T} - E_t\sum_{i=0}^{\infty}(\widehat{R}_{t+i} - \widehat{\pi}_{t+1+i}),$$

which shows that the reponse of consumption to a transitory shock (so that $\lim_{T\to\infty} E_t\widehat{c}_{t+T} = 0$) is entirely determined by the (negative of) response of the long-term real interest rate.

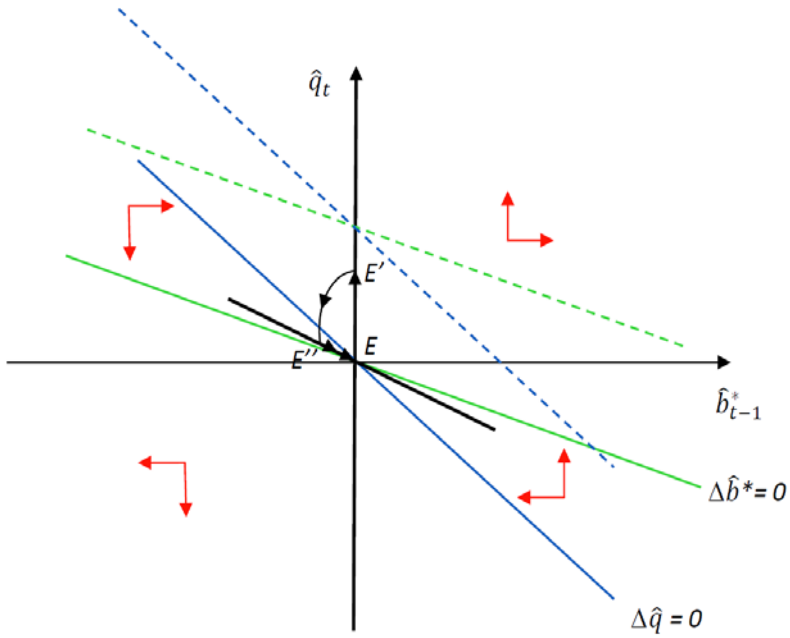

Figure 1: Illustrating the effects of an increase in government spending using the phase diagram.

The two loci are downward sloping, but the $\Delta\widehat{q}_{t+1} = 0$ locus has a steeper slope. Assume that the economy is initially at the equilibrium denoted by E, and that there is a temporary increase in government spending. The change in $\widehat{g}_t$ shifts both loci upward by exactly the same amount (see equations 39 and 40). At the time of the shock, the real exchange rate depreciates immediately, jumping to point E' in Figure 1. As the economy moves to point E'', the real exchange rate appreciates and net foreign assets fall for some time (inducing a current-account deficit) before starting to increase. Both variables then move down the saddle path until the economy returns to its initial equilibrium.

We now provide an analytical solution to system (37)-(38). To do so, we use the method of undetermined coefficients. That is, we conjecture that the solution takes the form

$$\widehat{q}_t = \psi_1 \widehat{b}^*_{t-1} + \psi_2 \widehat{g}_t, \tag{41}$$

and determine ψ_1 and ψ_2 as a function of the deep parameters. By straightforward algebra, it can be shown that the coefficient ψ_1 solves the following second-order equation:

$$\lambda\psi_1^2 - (1 + \gamma\lambda - \beta^{-1})\psi_1 - \gamma\beta^{-1} = 0. \tag{42}$$

The root that leads to a stable equilibrium is given by

$$\psi_1 = \frac{1+\gamma\lambda-\beta^{-1}-\sqrt{(1+\gamma\lambda-\beta^{-1})^2+4\gamma\lambda\beta^{-1}}}{2\lambda} < 0. \tag{43}$$

The parameter ψ_2 is thus given by

$$\psi_2 = \frac{-\alpha\kappa(\psi_1-\gamma)}{1-\rho_g-\lambda(\psi_1-\gamma)} > 0. \tag{44}$$

Note that ψ_2 is unambiguously positive, which means that the real exchange rate depreciates following a positive government spending shock. One can also easily check that the current account deteriorates following such a shock. Intuitively, given that the real interest rate is constant, the balance of payments equilibrium cannot be achieved with too large a decline in net foreign assets, as such a scenario would violate the transversality condition. The increase in government spending must therefore be accompanied by a real depreciation (see equation 36).

The effects of government spending shocks in this special case with a constant real interest rate can also be understood using an IS-BOP framework. The IS curve describes the goods-market equilibrium in the small open economy, and is characterized by the following equation:

$$\widehat{y}_t = \left(\lambda + \frac{\alpha}{1-\alpha}\right)\widehat{q}_t + \kappa\,(1-\alpha)\,\widehat{g}_t. \tag{45}$$

Note that this curve is increasing in $\widehat{q}_t$, i.e., the relative price of foreign goods. To obtain an equation for the BOP curve in the $(\widehat{y}_t, \widehat{q}_t)$ space, we start by using (41) to solve for $\widehat{q}_{t+1}$ in (38). This yields $\widehat{b}_t^* = \frac{1}{\psi_1-\gamma}\widehat{q}_t - \frac{\psi_2\rho_g}{\psi_1-\gamma}\widehat{g}_t$. Inserting this expression into equation (28), we obtain

$$\widehat{y}_t = -\beta^{-1}\widehat{b}_{t-1}^* + \left(\frac{1}{\psi_1-\gamma} + \frac{\alpha}{1-\alpha}\right)\widehat{q}_t + \left(\kappa - \frac{\psi_2\rho_g}{\psi_1-\gamma}\right)\widehat{g}_t. \tag{46}$$

The slope of this curve, $\left(\frac{1}{\psi_1-\gamma} + \frac{\alpha}{1-\alpha}\right)$, is negative for sufficiently low values of γ, but even when it is positive, it never exceeds that of the IS curve.[9] In other words, the BOP curve is either downward sloping or is steeper than the IS curve in the $(\widehat{y}_t, \widehat{q}_t)$ space.

Equations (45) and (46) show that an increase in $\widehat{g}_t$ shifts both the IS and BOP curves outward. This is shown in Figure 2, which illustrates the case of a downward sloping BOP curve. The shift in the IS curve is maximal given that it is not attenuated by a decline in private consumption (recall that $\widehat{c}_t = 0$). The shift in the BOP curve reflects the fact that the real exchange rate needs to depreciate, for a given level of output, to restore the balance-of-payments equilibrium. Because the BOP curve shifts more than does the IS curve (note that $\kappa - \frac{\psi_2\rho_g}{\psi_1-\gamma} > \kappa\,(1-\alpha)$), the rise in

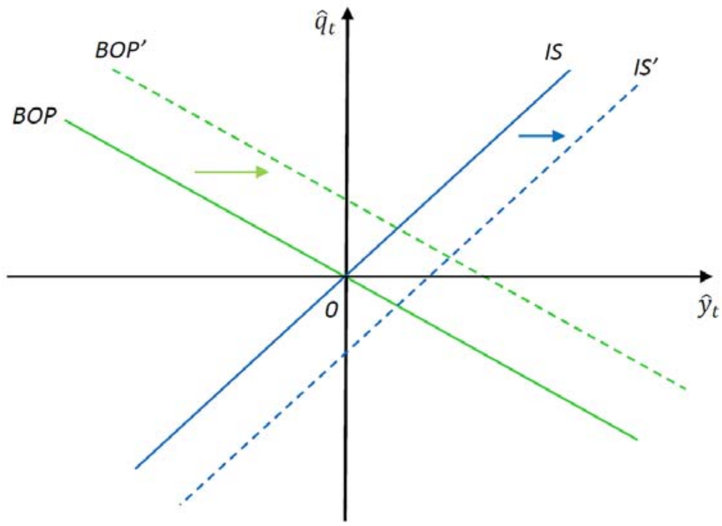

Figure 2: Illustrating the effects of a government spending shock using the IS-BOP framework.

government spending leads to an increase in output and to a depreciation of the real exchange rate. These effects are larger the steeper the BOP curve (i.e., the larger the value of γ).

Recall, however, that the depreciation of the real exchange rate is necessary in this special case because the equilibrium cannot be restored through a change in consumption, or alternatively, a change in the long-term real interest rate. In the general case where monetary policy changes the nominal interest rate more than one for one with expected inflation, the response of the real exchange rate will depend on the magnitude of the adjustment of the long-term real interest rate. We explore this matter in the next section.

3.2 The general case

In this section, we show that the result highlighted in the special case discussed above holds under less restrictive assumptions about monetary policy. In particular, we show that the real exchange rate continues to depreciate in response to a positive public spending shock if monetary policy adjusts the nominal interest rate more than one for one to expected inflation, without being overly aggressive. When ϕ_π takes on values that are strictly larger than 1, however, the linearized model no longer admits an analytical solution and we have to proceed with numerical simulations. To do so, we need to assign values to the structural parameters.

[9]Indeed, $\lim_{\gamma\to\infty}\left(\frac{1}{\psi_1-\gamma}+\frac{\alpha}{1-\alpha}\right)=\frac{\alpha}{1-\alpha}<\lambda+\frac{\alpha}{1-\alpha}$.

Parameter values We calibrate the model at a quarterly frequency. We start by discussing the parameters whose values are relatively conventional in the literature. The discount factor, β, is set to 0.99 so that the implied steady-state real interest rate is 4.1% annually. The preference parameter ω is a scaling parameter and is set to 1. We choose the elasticity of substitution across domestic varieties of final goods, θ, to equal 6, which yields a steady-state markup of 20%, as in Rotemberg and Woodford [1997]. The steady-state share of public spending in total output, κ, is set to 0.25. The parameter governing trade openness, α, is calibrated to 0.3 to match the average share of imports over GDP in industrialized economies. The smoothing parameter, ρ_r, is set to 0.8, consistently with the estimates reported in the literature (e.g., Smets and Wouters [2005]).

Next, we turn to the parameters whose values are relatively more controversial. For each of those parameters, our strategy is to choose a benchmark value that lies within the range of available estimates and then perform a sensitivity analysis. Starting with the elasticity of substitution between domestic and foreign goods, μ, available micro estimates tend to be much larger than macro estimates (see Ruhl [2004] for an extensive review of the literature). We choose to follow the macroeconomic literature and set $\mu = 1.5$ in the benchmark calibration (as in Backus and Kehoe [1994], for example). In the sensitivity analysis, we vary μ between 1 and 2. Merz [1995] reports that existing estimates of the Frisch elasticity of labor supply, ζ, vary between 0.03 and 3. In general, micro estimates converge towards low values of ζ, whereas macro estimates tend to be large. Following Galí, Gertler and López-Salido [2007], we consider a benchmark value of 0.5 and investigate the sensitivity of the results to values of ζ between 0.2 and 1. We set the Calvo probability of not changing prices, η, to 0.75, which implies an average duration of price spells of 4 quarters. In Section 3.3, we let this probability span the entire range of possible values, i.e., $(0, 1)$. In our model, the risk-premium parameter, γ, measures the (negative of the) elasticity of the interest rate differential with respect to net foreign assets (see equation 27). Lane and Milesi-Ferretti [2002] estimate the elasticity of the real interest rate differential (in %) to the ratio of foreign debt to exports using annual data. Their estimates range between 0.83 and 3.24. In our benchmark calibration, we choose to be conservative and use Lane and Milesi-Ferretti's lowest estimate. As we will show in the sensitivity analysis, higher values of γ actually strengthen our results. Once converted to a quarterly basis, Lane and Milesi-Ferretti's lowest estimate implies $\gamma = 0.0017$.[10] Finally, we calibrate the autocorrelation coefficient of the government spending

[10] Denote by $\varkappa$ the estimate reported by Lane and Milesi-Ferretti [2002]. In our model, b^*/y is the ratio of net foreign assets to *quarterly* GDP and α is the steady-state share of exports in GDP. Thus, the ratio of net foreign assets to *annual* exports is $b^*/(\alpha 4y)$. Since interest rates are quarterly in the model, Lane and Milesi-Ferretti's estimate implies

$$400\,(r - r^*) = -\varkappa b^*/(\alpha 4y),$$

process, ρ_g to 0.9, and study the sensitivity of the results when this parameter varies between 0.6 and 1. The benchmark values assigned to the parameters are summarized in Table 1.

Table 1: Parameter values

Discount factor	$\beta = 0.99$
Preference parameter	$\omega = 1$
Elasticity of substitution between varieties	$\theta = 6$
Steady state ratio of public spending to output	$\kappa = 0.25$
Trade openness	$\alpha = 0.3$
Nominal-interest-rate smoothing parameter	$\rho_r = 0.8$
Risk-premium parameter	$\gamma = 0.0017$
Elasticity of substitution between goods	$\mu = 1.5$
Frisch elasticity of labor supply	$\zeta = 0.5$
Calvo probability of not changing prices	$\eta = 0.75$
Autocorrelation of public spending shocks	$\rho_g = 0.9$

Impulse response functions We now discuss the impulse responses generated by the model under the benchmark calibration when the economy is hit by a 1 percent positive government spending shock. To shed light on the importance of monetary policy for the response of the real exchange rate, we study two scenarios: one in which the monetary authority fights inflation aggressively ($\phi_\pi = 1.5$) and one in which monetary policy is moderately aggressive ($\phi_\pi = 1.1$). The results are depicted in Figure 3.

The responses of output, consumption, net foreign assets, the nominal interest rate and the CPI inflation rate are qualitatively similar across the two cases. The increase in public spending increases aggregate demand, thus putting pressure on production capacities and input prices and inducing higher domestic-price inflation. The resulting increase in CPI inflation leads the monetary authority to raise the nominal interest rate, which, under the assumption that $\phi_\pi > 1$ raises the long-term real interest rate, $E_t \sum_{i=0}^{\infty}(\widehat{R}_{t+i} - \widehat{\pi}_{t+1+i})$. This in turn causes consumption to fall. As in the special case discussed above, net foreign assets decline. Increasing ϕ_π from 1.1 to 1.5 only changes the magnitude of these responses, but not their sign.

In contrast, the real exchange rate exhibits very different responses under the two scenarios, appreciating when $\phi_\pi = 1.5$ and depreciating when $\phi_\pi = 1.1$. In order to understand this result, it

where r and r^* are the (net) domestic and world real interest rates, respectively. The expression above yields the following mapping between the estimate $\varkappa$ and our parameter γ:

$$\gamma = \varkappa/(1600\alpha).$$

For instance, if $\alpha = 0.3$ and $\varkappa = 0.83$, we obtain $\gamma = 0.0017$, which is the value used in our benchmark calibration.

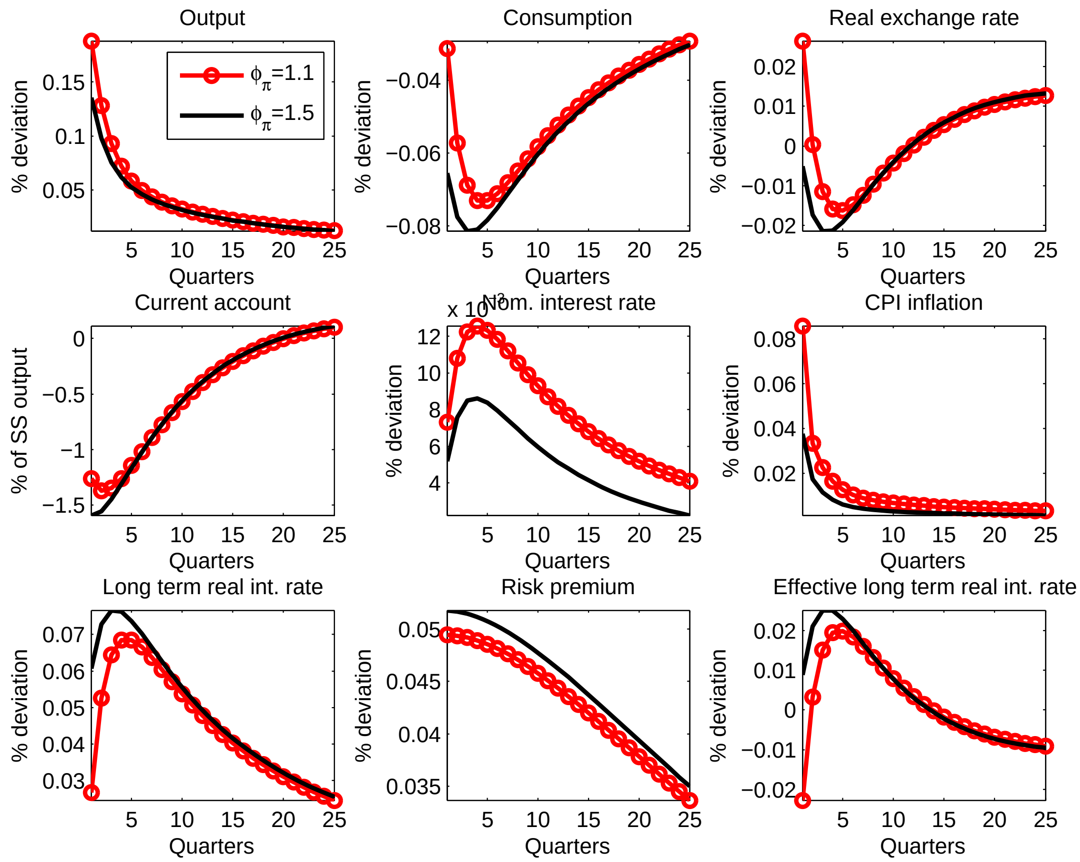

Figure 3: Impulse responses to a 1 percent increase in public spending under aggressive ($\phi_\pi = 1.5$) and moderately aggressive ($\phi_\pi = 1.1$) monetary policy.

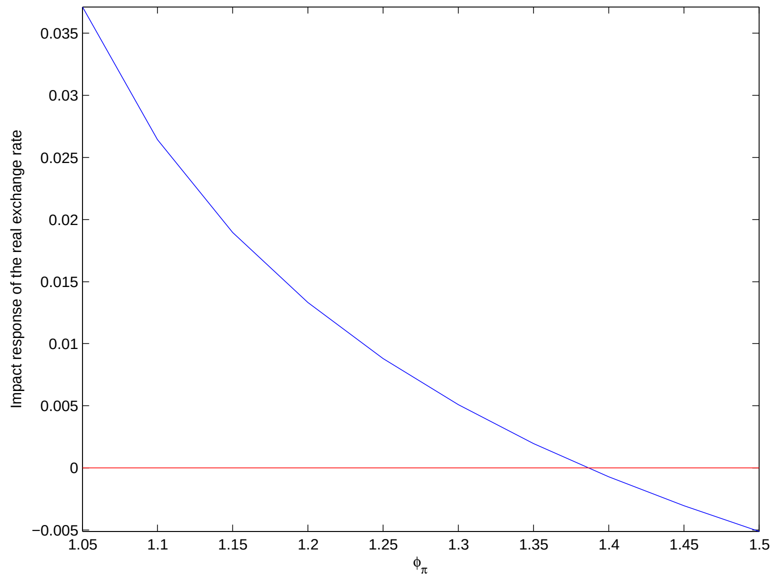

Figure 4: Initial response of the real exchange rate as a function of ϕ_π.

is useful to iterate the modified uncovered interest rate parity condition (equation 27) forward to express the real exchange rate as

$$\widehat{q}_t = \lim_{T\to\infty} E_t \widehat{q}_{t+T} - E_t \sum_{i=0}^{\infty} (\widehat{R}_{t+i} - \widehat{\pi}_{t+1+i}) - \gamma E_t \sum_{i=0}^{\infty} \widehat{b}^*_{t+i}. \tag{47}$$

If the shock is temporary, the term $\lim_{T\to\infty} E_t \widehat{q}_{t+T}$ must be equal to zero, and the response of the real exchange rate will depend on the difference between the response of the long-term real interest rate and that of the long-term risk premium, $-\gamma E_t \sum_{i=0}^{\infty} \widehat{b}^*_{t+i}$, or, equivalently, on the response of the *effective* long-term real interest rate, $E_t \sum_{i=0}^{\infty} (\widehat{R}_{t+i} - \widehat{\pi}_{t+1+i} + \gamma \widehat{b}^*_{t+i})$. Figure 3 shows that under aggressive monetary policy ($\phi_\pi = 1.5$), the rise in the long-term real interest rate is larger in magnitude than the increase in the long-term risk premium. Therefore the effective long-term real interest rate increases, causing the real exchange rate to appreciate. Under less aggressive monetary policy ($\phi_\pi = 1.1$), the risk premium dominates, the effective rate falls initially, and the real exchange rate depreciates.

Figure 4 depicts the relationship between the initial response of the real exchange rate to a positive government spending shock and the parameter ϕ_π, conditional on our benchmark calibration. The figure shows that this response is monotonically decreasing ϕ_π, and that a real depreciation is obtained with empirically plausible values of ϕ_π.

3.3 Sensitivity analysis

We now study the sensitivity of our results to alternative values of key parameters, namely, the elasticity of substitution between domestic and foreign goods, the Frisch elasticity of labor supply, the Calvo probability of not changing prices, the risk-premium parameter, and the autocorrelation of the government spending shock. In each case, we focus on the initial response of the real exchange when both the parameter of interest and the policy parameter, ϕ_π, vary. We also consider an alternative specification of households' preferences whereby government spending enters the utility function.

Elasticity of substitution between domestic and foreign goods (μ) As stated above, the necessary condition for the real exchange rate to depreciate in response to an expansionary government spending shock, i.e., $\mu(2-\alpha)-1>0$, holds for any sensible value of μ. But how sensitive is this depreciation across the range of plausible values of μ? Figure 5 depicts the initial response of the real exchange rate for values of μ ranging from 1 to 2, and values of ϕ_π ranging from 1 to 1.5. It shows that the magnitude of the exchange rate response is essentially insensitive to the value of μ (within the range considered), regardless of the degree of aggressiveness of monetary policy.

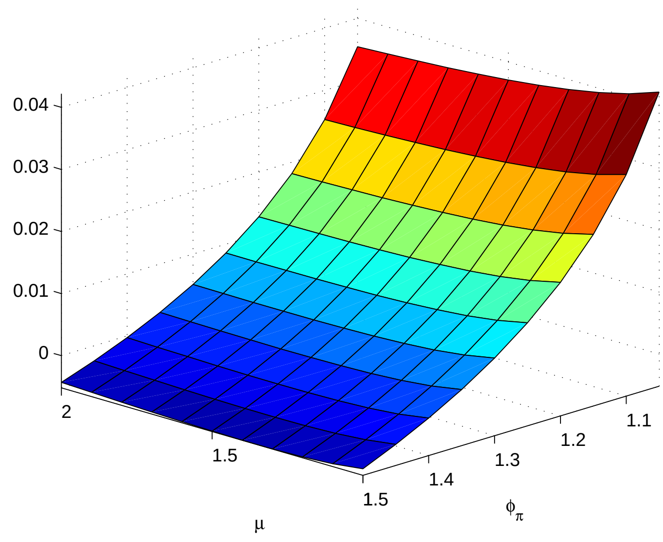

Figure 5: Initial response of the real exchange rate in the (μ, ϕ_π) space.

Frisch elasticity of labor supply (ζ) In the special case of no intertemporal substitution, discussed in Section 3.1, the absence of a consumption response implies that $\zeta^{-1}\widehat{n}_t = \widehat{w}_t$, which

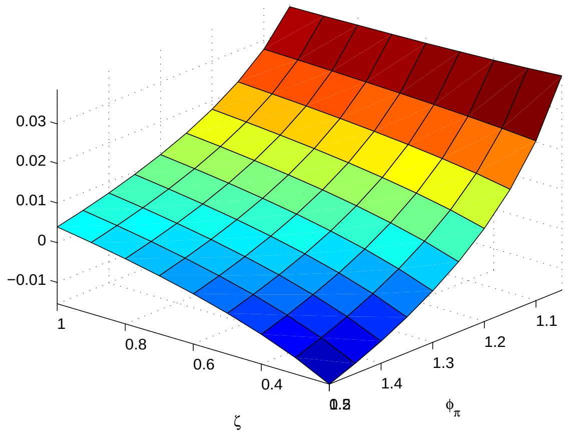

Figure 6: Initial response of the real exchange rate in the (ζ, ϕ_π) space.

means that the labor supply curve does not shift after an increase in public spending. In that version of the model, the parameter ζ plays no role in the determination of the real exchange rate. In the general case, however, changes in consumption shift the labor supply curve. An expansionary government spending shock will therefore shift the aggregate supply curve rightward through its effect on labor supply, and the resulting effect on the exchange rate will be larger the larger is ζ. As pointed out by Kollmann [2010], however, under flexible prices, generating a real depreciation requires very large values of ζ, which lie far outside the range of available estimates.[11] In contrast, as shown in Section 3.2, our model generates a real depreciation using a value of ζ that is in line with empirical estimates. Interestingly, Figure 6 below shows that, as ζ tends towards the lower bound of existing estimates, a real depreciation is still possible to obtain, but it requires less and less aggressive monetary policy.

Calvo probability of not changing prices (η) While the equilibrium path of the exchange rate is independent of the degree of price rigidity in the special case with constant real interest rate, this is no longer true in the more general version of the model. Figure 7 indeed shows that the magnitude of the exchange rate response to a government spending shock increases exponentially with η, and that the model can generate a real depreciation even with an aggressive monetary policy if prices are sufficiently rigid.

[11] Kollmann [2010] considers values of the Frisch elasticity equal to 2, 5 and infinity.

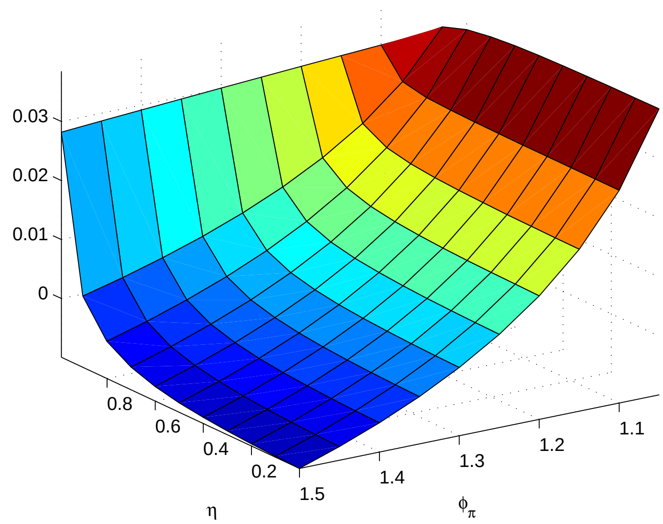

Figure 7: Initial response of the real exchange rate in the (η, ϕ_π) space.

Risk premium parameter (γ) In the special case with a constant real interest rate, higher values of γ make the BOP curve steeper and lead to a larger depreciation of the real exchange rate following an expansionary public spending shock.[12] To verify whether this result also holds in the more general case, we consider values of γ that range from 0.015 to 0.15. Figure 8 shows that this is indeed the case. For any given value of the policy parameter ϕ_π, the real exchange rate response is increasing in γ. Intuitively, higher values of γ imply a larger premium on the country's borrowing, for a given level of foreign debt. This translates into larger decline in the effective long term real interest rate and, therefore, a larger depreciation of the real exchange rate.

Autocorrelation of the government spending shock (ρ_g) Figure 9 shows that the way in which the persistence of the public spending shock affects the exchange rate response depends on the policy parameter, ϕ_π. For relatively small values of ϕ_π, this response increases monotonically with ρ_g,[13] but for larger values of ϕ_π, the relationship becomes non-monotonic, being negative for values of ρ_g that are less than 0.9 and positive for larger values. Again, this means that one can obtain a depreciation of the real exchange rate in response to a positive public spending shock even with an aggressive monetary policy, provided that the shock is sufficiently persistent.

[12] One can also check that $\frac{d\psi_2}{d\gamma} = \underbrace{\frac{\partial\psi_2}{\partial\gamma}}_{(+)} + \underbrace{\frac{\partial\psi_2}{\partial\psi_1}}_{(-)}\underbrace{\frac{\partial\psi_1}{\partial\gamma}}_{(-)} > 0$.

[13] In the special case of no intertemporal substitution ($\phi_\pi = 1$), this can be clearly seen from equation (41) and the expression of ψ_2.

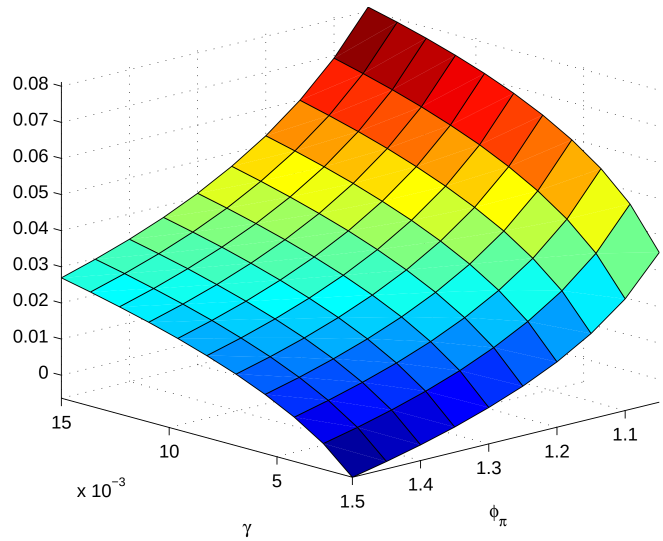

Figure 8: Initial response of the real exchange rate in the (γ, ϕ_π) space.

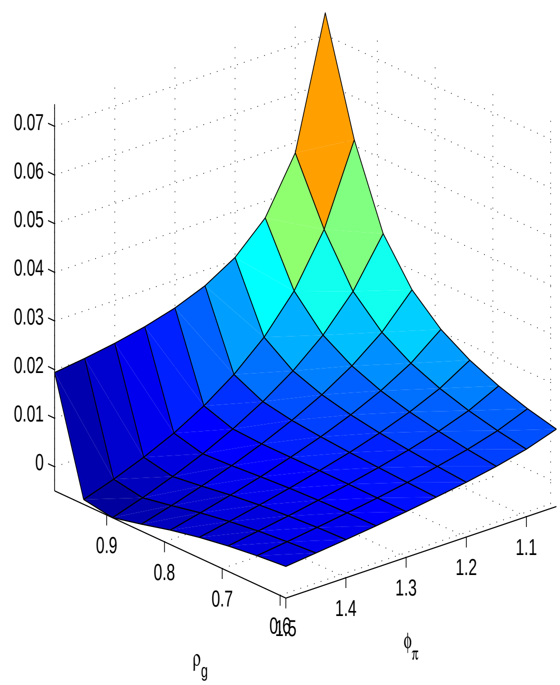

Figure 9: Initial response of the real exchange rate in the (ρ_g, ϕ_π) space.

Alternative specification of the utility function The model presented above predicts that consumption is crowded out following an increase in public spending, except in the special case where the real interest rate is held constant. Several empirical papers, however, find that consumption increases following an expansionary government spending shock. In what follows, we show that allowing for an alternative specification of consumer preferences whereby public spending increases the marginal utility of private consumption (i.e., public and private goods are Edgeworth complements) can generate a crowding-in of private consumption in response to a positive government spending shock,[14] without altering the mechanism that gives rise to a real depreciation.

To see this, assume that the representative consumer maximizes the following lifetime utility:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\ln \left(c_s - \nu g_s \right) - \omega \frac{\ell_s^{1+1/\zeta}}{1+1/\zeta} \right), \tag{48}$$

where ν is a parameter satisfying the condition $0 < \nu < \frac{1-\kappa}{\kappa}$. The fact that ν is positive ensures that the marginal utility of consumption is increasing in g (i.e., c and g are edgeworth complements). In this case, equations (26) and (30) are replaced by, respectively

$$\widehat{c}_t = E_t \widehat{c}_{t+1} - \frac{\nu\kappa}{1-\kappa} \left(\widehat{g}_t - E_t \widehat{g}_{t+1} \right) - \frac{1-\kappa\left(1+\nu\right)}{1-\kappa} \left(\hat{R}_t - E_t \widehat{\pi}_{t+1} \right), \tag{49}$$

$$\widehat{w}_t = \frac{1-\kappa}{1-\kappa\left(1+\nu\right)} \widehat{c}_t - \frac{\nu\kappa}{1-\kappa\left(1+\nu\right)} \widehat{g}_t + \zeta^{-1} \widehat{n}_t, \tag{50}$$

whereas equations (27)–(29) and (31)–(35) remain unchanged. In the special case where we abstract from uncertainty and where the monetary authority changes the nominal interest rate one for one with expected inflation, the resulting constancy of the real interest rate implies that the marginal utility of consumption remains constant, which by equation (49) implies that

$$\widehat{c}_t = \frac{\nu\kappa}{1-\kappa} \widehat{g}_t. \tag{51}$$

Since $\frac{\nu\kappa}{1-\kappa} > 0$, an increase in government spending raises private consumption by an amount that increases linearly with ν. Equations (27), (28), (33) and (51) yield again a two-equation system that allows us to solve for net foreign assets and the real exchange rate independently of the rest of the variables

$$\widehat{b}_t^* - \beta^{-1} \widehat{b}_{t-1}^* = \lambda \widehat{q}_t - \alpha\kappa \left(1+\nu\right) \widehat{g}_t, \tag{52}$$

$$\widehat{q}_{t+1} - \widehat{q}_t = \gamma \widehat{b}_t^*. \tag{53}$$

[14] The idea that Edgeworth complementarity between private and public spending can cause private consumption to rise in response to an expansionary government spending shock has been suggested by Bouakez and Rebei [2007].

The equilibrium path of the real exchange rate is again given by equation (41), where ψ_2 is now given by

$$\psi_2 = \frac{-\alpha\kappa(1+\nu)(\psi_1 - \gamma)}{1 - \rho_g - \lambda(\psi_1 - \gamma)} > 0.$$

That is, the real exchange rate depreciates following an expansionary government spending shock, and this depreciation is larger the larger is ν. Intuitively, as ν increases (that is, as edgeworth complementarity between private and public expenditures becomes stronger), the outward shift of the BOP curve exceeds that of the IS curve by a larger amount.[15] This leads to a larger increase in output and a stronger depreciation of the real exchange rate, which are both necessary to accommodate the increase in private consumption.

4. Conclusion

Empirical studies conclude that the real exchange rate depreciates sharply after an expansionary public spending shock. This paper has shown that a sticky-price small-open-economy model with incomplete and imperfect financial markets can account for this empirical regularity, at least qualitatively, provided that monetary policy is not too aggressive in fighting inflation. Admittedly, however, the magnitude of the real depreciation implied by the model is small in comparison with that documented in the empirical literature. Nonetheless, the mechanism highlighted in this paper can be viewed as a step forward towards a successful quantitative account of the effects of fiscal policy shocks on the real exchange rate.

[15] Note that the IS and BOP curves are now characterized by the following two equations, respectively:

$$\begin{aligned}
\widehat{y}_t &= \left(\lambda + \frac{\alpha}{1-\alpha}\right)\widehat{q}_t + \kappa\,(1+\nu)\,(1-\alpha)\,\widehat{g}_t, \\
\widehat{y}_t &= -\beta^{-1}\widehat{b}^*_{t-1} + \left(\frac{1}{\psi_1 - \gamma} + \frac{\alpha}{1-\alpha}\right)\widehat{q}_t + \left(\kappa\,(1+\nu) - \frac{\psi_2\rho_g}{\psi_1 - \gamma}\right)\widehat{g}_t.
\end{aligned}$$

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Appendix

Equations (4), (5), (6), (8), (16), (17), (22), (21), (24), as well as the monetary policy rule (20) and the government spending shock process (19) are log-linearized around the initial steady state defined in Section 2.2.

The log-linearized versions of equations (4)–(6) are, respectively

$$\widehat{c}_t + \zeta^{-1}\widehat{n}_t = \widehat{w}_t, \tag{A1}$$

$$E_t\widehat{c}_{t+1} - \widehat{c}_t = \widehat{R}_t - E_t\widehat{\pi}_{t+1}, \tag{A2}$$

$$\widehat{q}_t = E_t\widehat{q}_{t+1} + (\widehat{R}_t^* - E_t\widehat{\pi}_{t+1}^*) - (\widehat{R}_t - E_t\widehat{\pi}_{t+1}) - \gamma y\widehat{b}_t^*, \tag{A3}$$

which are, respectively, equations (30), (26), (27) in the main text.

Replacing $P_{F,t}$ by $\varepsilon_t P_t^*$ in equation (8) and dividing both sides by P_{t-1} yield

$$1 = (1-\alpha)\left(p_{H,t}\right)^{1-\mu} + \alpha\left(q_t\right)^{1-\mu}, \tag{A4}$$

where $p_{H,t} \equiv \frac{P_{H,t}}{P_t}$. Log-linearizing equation (A4) yields

$$\hat{p}_{H,t} = -\frac{\alpha}{1-\alpha}\hat{q}_t. \tag{A5}$$

Noting that $\widehat{\pi}_{H,t} = \hat{p}_{H,t} - \hat{p}_{H,t-1} + \widehat{\pi}_t$, and using equation (A5) to substitute for $\hat{p}_{H,t}$ and $\hat{p}_{H,t-1}$, we obtain

$$\widehat{\pi}_t = \widehat{\pi}_{H,t} + \frac{\alpha}{1-\alpha}(\widehat{q}_t - \widehat{q}_{t-1}), \tag{A6}$$

which is equation (31) in the main text.

The optimal pricing condition (16) can be rewritten as

$$\frac{\overline{P}_{H,t}}{P_{H,t}} = \frac{\theta}{\theta-1}\frac{k_t}{h_t}, \tag{A7}$$

$$k_t = \sum_{s=t}^{\infty}(\eta\beta)^{s-t}E_t\left(w_s\overline{y}_s/c_s\right),$$

$$h_t = \sum_{s=t}^{\infty}(\eta\beta)^{s-t}E_t\left(P_{H,t}\overline{y}_s/(P_s c_s)\right).$$

Expressing k_t and h_t recursively yields

$$k_t = \eta\beta E_t k_{t+1} + w_t\overline{y}_t/c_t, \tag{A8}$$

$$h_t = \eta\beta E_t(h_{t+1}/\pi_{H,t+1}) + (P_{H,t}\overline{y}_t)/(P_t c_t). \tag{A9}$$

Log-linearizing these equations yields

$$\widehat{k}_t = \eta\beta E_t \widehat{k}_{t+1} + (1-\eta\beta)\left(\widehat{\overline{y}}_t + \widehat{w}_t - \widehat{c}_t\right), \tag{A10}$$

$$\widehat{h}_t = \eta\beta E_t(\widehat{h}_{t+1} - \widehat{\pi}_{H,t+1}) + (1-\eta\beta)\left(\widehat{\overline{y}}_t - \widehat{c}_t - \frac{\alpha}{1-\alpha}\widehat{q}_t\right), \tag{A11}$$

where we have used the fact that $\pi_H = 1$. Dividing both sides of equation (17) by $P_{H,t}$, we obtain

$$(1-\eta)\left(\frac{\theta}{\theta-1}\frac{k_t}{h_t}\right)^{1-\theta} + \eta\pi_{H,t}^{\theta-1} = 1. \tag{A12}$$

Using the fact $k/h = w = (\theta-1)/\theta$, equation (A12) is log-linearized as

$$\widehat{\pi}_{H,t} = \frac{(1-\eta)}{\eta}(\widehat{k}_t - \widehat{h}_t), \tag{A13}$$

Combining equations (A10), (A11) and (A13), we obtain

$$\widehat{\pi}_{H,t} = \beta E_t \widehat{\pi}_{H,t+1} + \frac{(1-\eta)(1-\eta\beta)}{\eta}\left(\widehat{w}_t + \frac{\alpha}{1-\alpha}\widehat{q}_t\right), \tag{A14}$$

which is equation (29) in the main text.

The log-linearized versions of the remaining equations (22, 21, 24, 20, and 19) are, respectively

$$\widehat{b}_t^* = \beta^{-1}\widehat{b}_{t-1}^* + \widehat{y}_t - \left((1-\kappa)\widehat{c}_t + \kappa\widehat{g}_t + \frac{\alpha}{1-\alpha}\widehat{q}_t\right), \tag{A15}$$

$$\widehat{y}_t = \widehat{n}_t, \tag{A16}$$

$$\widehat{y}_t = (1-\alpha)\left((1-\kappa)\widehat{c}_t + \kappa\widehat{g}_t\right) + \mu\alpha\left(\frac{2-\alpha}{1-\alpha}\right)\widehat{q}_t + \alpha((1-\kappa)\widehat{c}_t^* + \kappa\widehat{g}_t^*), \tag{A17}$$

$$\widehat{R}_t = \rho_r \widehat{R}_{t-1} + (1-\rho_r)\phi_\pi E_t \widehat{\pi}_{t+1}, \tag{A18}$$

$$\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \epsilon_t^g, \tag{A19}$$

which are equations (28), (32), (33), (34), and (35) in the main text.