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An Extension of the Consumption-based CAPM Model

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Abstract:
We extend the Consumption-based CAPM (C-CAPM) model to representative agents with different risk attitudes. We first use the concept of expectation dependence and show that for a risk averse representative agent, it is the first-degree expectation dependence (FED) rather than the covariance that determines C-CAPM's riskiness. We extend the assumption of risk aversion to prudence and propose the measure of second-degree expectation dependence (SED) to obtain the values of asset price and equity premium. These theoretical results are linked to the equity premium puzzle. Using the same dataset as in Campbell (2003), the estimated measures of relative risk aversion from FED and SED approximations are much lower than those obtained in the original study and correspond to the theoretical values often discussed in the literature. The theoretical model is then generalized to higher-degree risk changes and higher-order risk averse representative agents.

Keywords: Consumption-based CAPM, Risk premium, Equity premium puzzle, Expectation dependence, Ross risk aversion

JEL Classification: D51, D80, G12

1 Introduction

The consumption-based capital asset pricing model (C-CAPM), developed in Rubinstein (1976), Lucas (1978), Breeden (1979) and Grossman and Shiller (1981), relates the risk premium on each asset to the covariance between the asset's return and a decision maker's intertemporal marginal rate of substitution. The most important comparative static results for C-CAPM is how an asset's price or equity premium changes as the quantity and price of risk change. The results of comparative statics analysis thus form the basis for much of our understanding of the sources of changes in consumption (macroeconomic) risk and risk aversion that drive asset prices and equity premia.

The three objectives of this study are to propose a new theoretical framework for C-CAPM, to extend its comparative statics, and to verify empirically how our framework can be useful to solve the equity premium puzzle. We use general utility functions and probability distributions to investigate C-CAPM. Our model provides insight into the basic concepts that determine asset prices and equity premia and generate reasonable empirical measures of relative risk aversion.

The C-CAPM pricing rule is sometimes interpreted as implying that the price of an asset with a random payoff falls short of its expected payoff if and only if the random payoff positively correlates with consumption. Liu and Wang (2007) show that this interpretation of C-CAPM is generally inadequate by presenting a counterexample. We use more powerful statistical tools to obtain the appropriate dependence between asset payoff and consumption. We first discuss the concept of expectation dependence developed by Wright (1987) and Li (2011). We show that, with general distributions and utility functions, for a risk averse representative agent, it is the first-degree expectation dependence (FED) between the asset's payoff and consumption rather than the covariance that determines C-CAPM's riskiness. Our result also reinterprets the covariance between an asset's payoff and the marginal utility of consumption in terms of the expectation dependence between the asset's payoff and consumption itself. We extend the assumption of risk aversion to prudence and propose the measure of second-degree expectation dependence (SED) to obtain the values of asset price and equity premium. We interpret C-CAPM in a general setting: for the i^{th}-degree risk averse representative agent,[1] with $i = 2, .., N+1$, it is the N^{th}-order expectation dependence that determines C-CAPM's riskiness. We also provide bivariate log-normal and truncated standardized bivariate normal distribution examples to measure first-degree expectation dependence and second-degree expectation dependence empirically, and to construct shifts in distributions that satisfy the comparative statics. Examples of nonelliptical distributions are also provided. Our empirical results are linked to the equity premium puzzle. Using the same dataset as in Campbell (2003), the estimated measures of relative risk aversion, using FED and SED approximations, are much lower than those obtained in the original study.

Our contribution is also linked to the recent literature on higher-order risk preferences and higher-order moments and comoments in finance developed by Harvey and Siddique (2000), Dittmar (2002), Mitton and Vorkink (2007), Chabi-Yo et al. (2007), and Martellini and Ziermann (2010). We provide a theoretical foundation for the pricing kernel model based on higher comoments than the covariance by suggesting a more general definition of dependence between consumption and asset payoff, and propose a general pricing formula not restricted to specific utility functions.

Our study also relates to Gollier and Schlesinger (2002), who examine asset prices in a representative-agent model of general equilibrium, with two differences. First, we study asset price and equity premium driven by macroeconomic risk as in the traditional C-CAPM model, while Gollier and Schlesinger's model considers the relationship between the riskiness of the market portfolio and its expected return. Second, Gollier and Schlesinger's model is a one-period model whereas our results rest on a two-period framework.

Finally, our study extends the literature that examines the effects of higher-degree risk changes on the economy. Eeckhoudt and Schlesinger (2006) investigate necessary and sufficient conditions on preferences for a higher-degree change in risk to increase saving. Our study provides necessary and sufficient conditions on preferences for a higher-degree change in risk to set asset prices, and sufficient conditions on preferences for a higher-degree change in risk to set equity premia.

[1] Risk aversion in the traditional sense of a concave utility function is indicated by $i = 2$, whereas $i = 3$ corresponds to downside risk aversion in the sense of Menezes, Geiss and Tressler (1980). i^{th}-degree risk aversion is equivalent to preferences satisfying risk apportionment of order i. See Ekern (1980) and Eeckhoudt and Schlesinger (2006) for more discussions.

The paper proceeds as follows. Section 2 introduces several concepts of dependence. Section 3 provides a reinterpretation of C-CAPM for risk averse representative agents. Section 4 extends the results of Section 3 to prudent and higher-order risk averse agents respectively. Section 5 discusses the results in relation to local indexes of risk aversion and higher-order moments and comoments. Section 6 shows empirically how our model can help to mitigate the equity premium puzzle. Section 7 concludes the paper.

2 Concepts of dependence

The concept of correlation coined by Galton (1886) had served as the only measure of dependence for the first 70 years of the 20th century. However correlation is too weak to obtain meaningful conclusions in many economic and financial applications. For example, covariance is a poor tool for describing dependence for non-normal distributions. Since Lehmann's introduction of the concept of quadrant dependence in 1966, stronger definitions of dependence have received much attention in the statistical literature[2].

Suppose $\tilde{x} \times \tilde{y} \in R \times R$ are two continuous random variables. Let $F(x, y)$ denote the joint and $F_x(x)$ and $F_y(y)$ the marginal distributions of $\tilde{x}$ and $\tilde{y}$. Lehmann (1966) introduces the following concept to investigate positive dependence.

Definition 2.1 *(Lehmann, 1966) $(\tilde{x}, \tilde{y})$ is positively quadrant dependent, written as $PQD(\tilde{x}, \tilde{y})$, if*

$$F(x, y) \geq F_x(x)F_y(y) \qquad for\ all\ (x, y) \in R \times R. \tag{1}$$

The above inequality can be rewritten as

$$F_x(x|\tilde{y} \leq y) \geq F_x(x) \tag{2}$$

and Lehmann provides the following interpretation of definition (2.1): "knowledge of $\tilde{y}$ being small increases the probability of $\tilde{x}$ being small." PQD is useful to model dependent risks because it can take into account the simultaneous downside (upside) evolution of risks. The marginal and the conditional CDFs can be changed simultaneously.[3] We notice that there are many bivariate

[2] For surveys of the literature, we refer to Joe (1997), Mari and Kotz (2001) and Embrechts (2009).

[3] Portfolio selection problems with positive quadrant dependency have been explored by Pellerey and Semeraro (2005) and Dachraoui and Dionne (2007), among others. Pellerey and Semeraro (2005) assert that a large subset of the elliptical distributions class is PQD.

random variables other than elliptical or Gaussian distributions being PQD. For examples of such distributions, see Joe (1997) or Balakrishnan and Lai (2009).

Wright (1987) introduced the concept of expectation dependence in the economics literature. The following definition uses a weaker definition of dependence than PQD.

Definition 2.2 *If*

$$FED(\tilde{x}|y) = [E\tilde{x} - E(\tilde{x}|\tilde{y} \leq y)] \geq 0 \; for \; all \; y \in R, \tag{3}$$
$$then \; \tilde{x} \; is \; positive \; first-degree \; expectation \; dependent \; on \; \tilde{y}.$$

The family of all distributions F satisfying (3) will be denoted by $\mathcal{F}_1$. Similarly, $\tilde{x}$ is negative first-degree expectation dependent on $\tilde{y}$ if (3) holds with the inequality sign reversed. The totality of negative first-degree expectation dependent distributions will be denoted by $\mathcal{G}_1$.

Wright (1987, page 113) interprets negative first-degree expectation dependence as follows: "when we discover $\tilde{y}$ is small, in the precise sense that we are given the truncation $\tilde{y} \leq y$, our expectation of $\tilde{x}$ is revised upward." First-degree expectation dependence is a stronger definition of dependence than correlation, but a weaker definition than quadrant dependence. Therefore, bivariate random variables being positive (negative) quadrant dependent are also first-degree expectation dependent. However, as the next example shows, positive (negative) correlated random variables are not necessary positive (negative) first-degree expectation dependent.

Example 2.3 *Let $\tilde{x}$ be normally distributed with $E\tilde{x} = \mu > 0$ and $var(\tilde{x}) = \sigma^2$. Let $\tilde{y} = \tilde{x}^2$. Since $E\tilde{x}^2 = \mu^2 + \sigma^2$ and $E\tilde{x}^3 = \mu^3 + 3\mu\sigma^2$, then*

$$\begin{aligned} cov(\tilde{x}, \tilde{y}) &= E\tilde{x}\tilde{y} - E\tilde{x}E\tilde{y} \\ &= E\tilde{x}^3 - E\tilde{x}E\tilde{x}^2 \\ &= \mu^3 + 3\mu\sigma^2 - \mu(\mu^2 + \sigma^2) = 2\mu\sigma^2 > 0. \end{aligned} \tag{4}$$

By definition,

$$\begin{aligned} FED(\tilde{y}| - \sqrt{\mu^2 + \sigma^2}) &= E\tilde{x}^2 - E(\tilde{x}^2|\tilde{x} \leq -\sqrt{\mu^2 + \sigma^2}) \\ &= \mu^2 + \sigma^2 - E(\tilde{x}^2|\tilde{x} \leq -\sqrt{\mu^2 + \sigma^2}) < 0, \end{aligned} \tag{5}$$

and we obtain $(\tilde{y}, \tilde{x}) \notin \mathcal{F}_1$.

First-degree expectation dependence can be applied to log-normal random variables.

Example 2.4 *Consider bivariate log-normal random variables $(\tilde{x}, \tilde{y})$ with joint probability distribution $F(x, y)$ such that*

$$\begin{pmatrix} \log(\tilde{x}) \\ \log(\tilde{y}) \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \right) \tag{6}$$

where indexes 1 and 2 are used for $\log(\tilde{x})$ and $\log(\tilde{y})$ respectively. We know that (see Lien 1985, p244-245)

$$E(\tilde{x}|\tilde{y} \leq y) = \exp(\mu_1 + \frac{\sigma_1^2}{2})\frac{\Phi(\frac{\log(y)-\mu_2-\sigma_{12}}{\sigma_2})}{\Phi(\frac{\log(y)-\mu_2}{\sigma_2})}, \tag{7}$$

where $\Phi(x)$ is the cumulative density function of a standardized normal random variable evaluated at x,

$$cov(\tilde{x}, \tilde{y}) = E\tilde{x}\tilde{y} - E\tilde{x}E\tilde{y} = \exp(\mu_1 + \mu_2 + \frac{\sigma_1^2 + \sigma_2^2}{2})[\exp(\sigma_{12}) - 1] \tag{8}$$

and

$$FED(\tilde{x}|y) = E\tilde{x} - E(\tilde{x}|\tilde{y} \leq y) = \exp(\mu_1 + \frac{\sigma_1^2}{2})(1 - \frac{\Phi(\frac{\log(y)-\mu_2-\sigma_{12}}{\sigma_2})}{\Phi(\frac{\log(y)-\mu_2}{\sigma_2})}), \tag{9}$$

So $\sigma_{12} \geq 0 \Leftrightarrow cov(\tilde{x}, \tilde{y}) \geq 0 \Leftrightarrow FED(\tilde{x}|y) \geq 0$.

We can also relate $FED(\tilde{x}|y)$ to the correlation coefficient ρ .

Example 2.5 *Consider a truncated standardized bivariate normal distribution with $h < \tilde{x} < \infty$, $-\infty < \tilde{y} < \infty$ and correlation coefficient ρ. From Balakrishnan and Lai (2009, p532-533), we know that the marginal density of $\tilde{y}$ is $\frac{\Phi(y)}{\Psi(-h)}\Psi(\frac{-h+\rho y}{\sqrt{1-\rho^2}})$, where Ψ the cumulative distribution function of the standardized univariate normal distribution. Let E_T denote the mean after truncation. Then (see Balakrishnan and Lai 2009, p532-533)*

$$E_T(\tilde{x}) = \frac{\Phi(h)}{\Psi(-h)} \tag{10}$$

and

$$E_T(\tilde{x}|\tilde{y} = y) = \rho y + \sqrt{1-\rho^2}\frac{\Phi(\frac{h-\rho y}{\sqrt{1-\rho^2}})}{\Psi(\frac{-h+\rho y}{\sqrt{1-\rho^2}})}. \tag{11}$$

Hence

$$\begin{aligned}
& E_T(\tilde{x}|\tilde{y} \leq y) \qquad (12)\\
= & \int_{-\infty}^{y} E_T(\tilde{x}|\tilde{y}=t)dF_y(t)\\
= & \rho\int_{-\infty}^{y} t\frac{\Phi(t)}{\Psi(-h)}\Psi(\frac{-h+\rho t}{\sqrt{1-\rho^2}})dt + \sqrt{1-\rho^2}\int_{-\infty}^{y}\frac{\Phi(\frac{h-\rho t}{\sqrt{1-\rho^2}})}{\Psi(\frac{-h+\rho t}{\sqrt{1-\rho^2}})}\frac{\Phi(t)}{\Psi(-h)}\Psi(\frac{-h+\rho t}{\sqrt{1-\rho^2}})dt\\
= & \frac{\rho}{\Psi(-h)}\int_{-\infty}^{y} t\Phi(t)\Psi(\frac{-h+\rho t}{\sqrt{1-\rho^2}})dt + \frac{\sqrt{1-\rho^2}}{\Psi(-h)}\int_{-\infty}^{y}\Phi(t)\Phi(\frac{h-\rho t}{\sqrt{1-\rho^2}})dt
\end{aligned}$$

and

$$\begin{aligned}
& FED(\tilde{x}|y) = E_T(\tilde{x}) - E_T(\tilde{x}|\tilde{y} \leq y) \qquad (13)\\
= & \frac{1}{\Psi(-h)}[\Phi(h) - \rho\int_{-\infty}^{y} t\Phi(t)\Psi(\frac{-h+\rho t}{\sqrt{1-\rho^2}})dt - \sqrt{1-\rho^2}\int_{-\infty}^{y}\Phi(t)\Phi(\frac{h-\rho t}{\sqrt{1-\rho^2}})dt].
\end{aligned}$$

Therefore, we can see that $\rho > 0$ cannot guarantee $FED(\tilde{x}|y) > 0$.

Li (2011) proposes the following weaker definition of dependence:

Definition 2.6 *If*

$$\begin{aligned}
SED(\tilde{x}|y) &= \int_{-\infty}^{y} [E\tilde{x} - E(\tilde{x}|\tilde{y} \leq t)]F_y(t)dt \qquad (14)\\
&= \int_{-\infty}^{y} FED(\tilde{x}|t)F_y(t)dt \geq 0 \ for\ all\ y,
\end{aligned}$$

then $\tilde{x}$ is positive second-degree expectation dependent on $\tilde{y}$.

The family of all distributions F satisfying (14) will be denoted by $\mathcal{F}_2$. Similarly, $\tilde{x}$ is negative second-degree expectation dependent on $\tilde{y}$ if (14) holds with the inequality sign reversed, and the totality of negative second-degree expectation dependent distributions will be denoted by $\mathcal{G}_2$.

It is obvious that $\mathcal{F}_1 \subseteq \mathcal{F}_2$ and $\mathcal{G}_1 \subseteq \mathcal{G}_2$ but the converse is not true. Because $\tilde{x}$ and $\tilde{y}$ are positively correlated when (see Lehmann 1966, lemma 2)

$$cov(\tilde{x},\tilde{y}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}[F(x,y) - F_x(x)F_y(y)]dxdy = \int_{-\infty}^{+\infty} FED(\tilde{x}|t)F_y(t)dt \geq 0, \qquad (15)$$

again we see that $cov(\tilde{x},\tilde{y}) \geq 0$ is only a necessary condition for $(\tilde{x},\tilde{y}) \in \mathcal{F}_2$ but the converse is not true. Therefore, we notice there are many bivariate random variables other than elliptical or Gaussian distributions being FED or SED, because all the PQD distributions are also FED and SED. Comparing (14) and (15), we know that $cov(\tilde{x}, \tilde{y})$ is the 2nd central cross moment of $\tilde{x}$ and $\tilde{y}$, while $SED(\tilde{x}|y)$ is related to the 2nd central cross lower partial moment of $\tilde{x}$ and $\tilde{y}$ which can be explained as a measure of downside risk computed as the average of the squared deviations below a target.

The following two examles relate $SED(\tilde{x}|y)$ to the covariance and the correlation coefficient.

Example 2.7 *For bivariate log-normal random variables $(\tilde{x}, \tilde{y})$ defined in (6),*

$$\begin{aligned} SED(\tilde{x}|y) &= \int_{-\infty}^{y} \exp(\mu_1 + \frac{\sigma_1^2}{2})[1 - \frac{\Phi(\frac{\log(t)-\mu_2-\sigma_{12}}{\sigma_2})}{\Phi(\frac{\log(t)-\mu_2}{\sigma_2})}]\Phi(\frac{\log(t)-\mu_2}{\sigma_2})dt \qquad (16) \\ &= \exp(\mu_1 + \frac{\sigma_1^2}{2}) \int_{-\infty}^{y} [\Phi(\frac{\log(t)-\mu_2}{\sigma_2}) - \Phi(\frac{\log(t)-\mu_2-\sigma_{12}}{\sigma_2})]dt, \end{aligned}$$

So $\sigma_{12} \geq 0 \Leftrightarrow cov(\tilde{x}, \tilde{y}) \geq 0 \Leftrightarrow SED(\tilde{x}|y) \geq 0$.

Example 2.8 *Considering the truncated standardized bivariate normal distribution defined in Example 2.5, we can verify that $\rho > 0$ does not guarantee $SED(\tilde{x}|y) > 0$.*

For our purpose of extending the C-CAPM model, comparative expectation dependence has to be defined. We propose the following definition to quantify comparative expectation dependence.

Definition 2.9 *Distribution $F(x, y)$ is more first-degree expectation dependent than $H(x, y)$ if and only if $FED_F(\tilde{x}|y)F_y(y) \geq FED_H(\tilde{x}|y)H_y(y)$ for all y. Distribution $F(x, y)$ is more second order expectation dependent than $H(x, y)$, if $FED_F(\tilde{x}) \geq FED_H(\tilde{x})$, and*

$$SED_F(\tilde{x}|y) \geq SED_H(\tilde{x}|y) \; for \; all \; y. \qquad (17)$$

Example 2.10 *Consider bivariate log-normal random variables $(\tilde{x}, \tilde{y})$ with probability distribution $F(x, y)$ such that*

$$\begin{pmatrix} \log(\tilde{x}) \\ \log(\tilde{y}) \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \right) \qquad (18)$$

and random variables $(\tilde{x}', \tilde{y}')$ *and with probability distribution* $H(x', y')$ *such that*

$$\begin{pmatrix} \log(\tilde{x}') \\ \log(\tilde{y}') \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_1' \\ \mu_2' \end{pmatrix}, \begin{pmatrix} \sigma_1'^2 & \sigma_{12}' \\ \sigma_{12}' & \sigma_2'^2 \end{pmatrix} \right). \tag{19}$$

Then

$$\begin{aligned} & FED_F(\tilde{x}|y)F_y(y) \geq FED_H(\tilde{x}|y)H_y(y) \\ \Leftrightarrow \quad & \exp(\mu_1 + \frac{\sigma_1^2}{2})[\Phi(\frac{\log(y)-\mu_2}{\sigma_2}) - \Phi(\frac{\log(y)-\mu_2-\sigma_{12}}{\sigma_2})] \\ \geq \quad & \exp(\mu_1' + \frac{\sigma_1'^2}{2})[\Phi(\frac{\log(y)-\mu_2'}{\sigma_2'}) - \Phi(\frac{\log(y)-\mu_2'-\sigma_{12}'}{\sigma_2'})] \end{aligned} \tag{20}$$

and

$$\begin{aligned} & SED_F(\tilde{x}|y) \geq SED_H(\tilde{x}|y) \\ \Leftrightarrow \quad & \exp(\mu_1 + \frac{\sigma_1^2}{2}) \int_{-\infty}^{y} [\Phi(\frac{\log(t)-\mu_2}{\sigma_2}) - \Phi(\frac{\log(t)-\mu_2-\sigma_{12}}{\sigma_2})]dt \\ \geq \quad & \exp(\mu_1' + \frac{\sigma_1'^2}{2}) \int_{-\infty}^{y} [\Phi(\frac{\log(t)-\mu_2'}{\sigma_2'}) - \Phi(\frac{\log(t)-\mu_2'-\sigma_{12}'}{\sigma_2'})]dt. \end{aligned} \tag{21}$$

3 C-CAPM for a risk averse representative agent

3.1 Consumption-based asset pricing model

Suppose that an investor can freely buy or sell an asset with random payoff $\tilde{x}_{t+1}$ at a price p_t. The investor's preference can be represented by a utility function $u(.)$. We assume that all derivatives for $u(.)$ exist. Denote ξ as the amount of the asset the investor chooses to buy. Then, his decision problem is to

$$\begin{aligned} & \max_{\xi} u(c_t) + \beta E_t[u(\tilde{c}_{t+1})] \\ s.t. \quad & c_t = e_t - p_t \xi \\ & \tilde{c}_{t+1} = e_{t+1} + \tilde{x}_{t+1}\xi, \end{aligned} \tag{22}$$

where e_t and e_{t+1} are the original consumption levels, β is the subjective discount factor, c_t is the consumption in period t, and $\tilde{c}_{t+1}$ is the consumption in period $t+1$.

From the first order condition of this problem, we can obtain the well-known consumption-based asset pricing model which can be expressed by the following two equations (see e.g.

Cochrane 2005, page 13-14).[4]

$$p_t = \frac{E_t \tilde{x}_{t+1}}{R^f} + \beta \frac{cov_t[u'(\tilde{c}_{t+1}), \tilde{x}_{t+1}]}{u'(c_t)}, \tag{23}$$

and

$$E_t \tilde{R}_{t+1} - R^f = -\frac{cov_t[u'(\tilde{c}_{t+1}), \tilde{R}_{t+1}]}{E_t u'(\tilde{c}_{t+1})} \tag{24}$$

where $1 + \tilde{R}_{t+1} = \frac{\tilde{x}_{t+1}}{P_t}$ is defined as the asset's gross return in period $t+1$, $1 + R^f$ is defined as the gross return of the risk-free asset, $u'(\cdot)$ is the marginal utility function, $E\tilde{R}_{t+1} - R^f$ is the asset's risk premium.

The first term on the right-hand side of (23) is the standard risk-free present value formula. This is the asset's price for a risk-neutral representative agent or for a representative agent when asset payoff and consumption are independent. The second term is a risk aversion adjustment. (23) states that an asset with random future payoff $\tilde{x}_{t+1}$ is worth less than its expected payoff discounted at the risk-free rate if and only if $cov[u'(\tilde{c}_{t+1}), \tilde{x}_{t+1}] \leq 0$. (24) shows that an asset has an expected return equal to the risk-free rate plus a risk adjustment under risk aversion.

When the representative agent's utility function is the power function, $u(c_t) = \frac{c_t^{1-\gamma}-1}{1-\gamma}$ where γ is the coefficient of relative risk aversion and $\tilde{c}_{t+1}$ and $\tilde{x}_{t+1}$ are conditional lognormally distributed, (24) becomes (Campbell 2003, page 819)

$$E_t \tilde{r}_{t+1} - r^f + \frac{var_t(\tilde{r}_{t+1})}{2} = \gamma cov_t(\log \tilde{c}_{t+1}, \tilde{r}_{t+1}), \tag{25}$$

where $\tilde{r}_{t+1} = \log(1 + \tilde{R}_{t+1})$ and $r^f = \log(1 + R^f)$.

(25) states that the log risk premium is equal to the product of the coefficient of relative risk aversion and the covariance of the log asset return with consumption growth. We now provide a generalization of these results.

From Theorem 1 in Cuadras (2002), we know that covariance can always be written as

$$cov_t[u'(\tilde{c}_{t+1}), \tilde{x}_{t+1}] = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} [F(c_{t+1}, x_{t+1}) - F_{c_{t+1}}(c_{t+1}) F_{x_{t+1}}(x_{t+1})] u''(c_{t+1}) dx_{t+1} dc_{t+1}. \tag{26}$$

Because we can write

$$\int_{-\infty}^{+\infty} [F_{x_{t+1}}(x_{t+1} | \tilde{c}_{t+1} \leq c_{t+1}) - F_{x_{t+1}}(x_{t+1})] dx_{t+1} = E\tilde{x}_{t+1} - E(\tilde{x}_{t+1} | \tilde{c}_{t+1} \leq c_{t+1}),$$

[4] Equations (23) and (24) can also be obtained in a muti-period dynamic framework from Euler equations. For more details, see Constantinides and Duffie (1996).

(see, e.g., Tesfatsion (1976), Lemma 1), hence, we have

$$\begin{aligned}
& cov_t[u'(\tilde{c}_{t+1}), \tilde{x}_{t+1}] \\
= & \int_{-\infty}^{+\infty} [E\tilde{x}_{t+1} - E(\tilde{x}_{t+1}|\tilde{c}_{t+1} \leq c_{t+1})] F_{c_{t+1}}(c_{t+1}) u''(c_{t+1}) dc_{t+1} \\
= & \int_{-\infty}^{+\infty} FED(\tilde{x}_{t+1}|c_{t+1}) u''(c_{t+1}) F_{c_{t+1}}(c_{t+1}) dc_{t+1}.
\end{aligned} \tag{27}$$

(27) allows us to break the covariance out in terms of the FED and agents' preferences[5].

Using (27), (23) can be rewritten as

$$\begin{aligned}
p_t &= \underbrace{\frac{E_t\tilde{x}_{t+1}}{R^f}}_{risk-free\ present\ value\ effect} \underbrace{- \beta \int_{-\infty}^{+\infty} FED(\tilde{x}_{t+1}|c_{t+1}) F_{c_{t+1}}(c_{t+1}) [-\frac{u''(c_{t+1})}{u'(c_t)}] dc_{t+1}}_{first-degree\ expectation\ dependence\ effect} \qquad (28) \\
&= \frac{E_t\tilde{x}_{t+1}}{R^f} - \beta \int_{-\infty}^{+\infty} FED(\tilde{x}_{t+1}|c_{t+1}) F_{c_{t+1}}(c_{t+1}) AR(c_{t+1}) MRS_{c_{t+1},c_t} dc_{t+1},
\end{aligned}$$

where $AR(x) = -\frac{u''(x)}{u'(x)}$ is the Arrow-Pratt absolute risk aversion coefficient, and $MRS_{x,y} = \frac{u'(x)}{u'(y)}$ is the marginal rate of substitution between x and y.[6]

We can also rewrite (24) as

$$E_t\tilde{R}_{t+1} - R^f = \int_{-\infty}^{+\infty} \underbrace{FED(\tilde{R}_{t+1}|c_{t+1}) F_{c_{t+1}}(c_{t+1})}_{consumption\ risk\ effect} \underbrace{[-\frac{u''(c_{t+1})}{E_t u'(\tilde{c}_{t+1})}]}_{price\ of\ risk\ effect} dc_{t+1} \tag{29}$$

Because $R^f = \frac{1}{\beta}\frac{u'(c_t)}{E_t u'(\tilde{c}_{t+1})}$ (see e.g. Cochrane 2005, page 11), we also have

$$E_t\tilde{R}_{t+1} - R^f = \beta R^f \int_{-\infty}^{+\infty} FED(\tilde{R}_{t+1}|c_{t+1}) F_{c_{t+1}}(c_{t+1}) AR(c_{t+1}) MRS_{c_{t+1},c_t} dc_{t+1} \tag{30}$$

(28) shows that an asset's price involves two terms. The effect, measured by the first term on the right-hand side of (28), is the "risk-free present value effect." This effect depends on the expected return of the asset and the risk-free interest rate. The sign of the risk-free present value effect is the same as the sign of the expected return. This term captures the "direct" effect of the risk-free present expected return, which characterizes the asset's price for a risk-neutral representative agent.

The second term on the right-hand side of (28) is called "first-degree expectation dependence effect." This term involves the subjective discount factor, the expectation dependence

[5] When $(\tilde{c}_{t+1}, \tilde{x}_{t+1})$ is joint normal distribution, Stein's lemma can be applied to compute $cov_t[u'(\tilde{c}_{t+1}), \tilde{x}_{t+1}] = cov_t(\tilde{c}_{t+1}, \tilde{x}_{t+1}) E_t(U''(\tilde{c}_{t+1}))$ (see e.g. Cochrane 2005, page 163).

[6] Andersen et al. (2011) propose a multi-attribute risk aversion model that allows one to separate the intertemporal risk aversion coefficient into the risk aversion coefficient and the MRS

between the random payoff and consumption, the Arrow-Pratt risk aversion coefficient and the intertemporal marginal rate of substitution. The sign of the first-degree expectation dependence indicates whether the movements on consumption tend to reinforce (positive first-degree expectation dependence) or to counteract (negative first-degree expectation dependence) the movements on an asset's payoff.

(29) states that the expected excess return on any risky asset over the risk-free interest rate can be explained as the sum of the quantity of consumption risk times the price of this risk. The quantity of consumption risk is measured by the first-degree expectation dependence of the excess stock return with consumption, while the price of risk is the Arrow-Pratt risk aversion coefficient times the intertemporal marginal rate of substitution.

We obtain the following proposition from (28) and (29).

Proposition 3.1 *Suppose $F(x_{t+1}, c_{t+1})$ and $F(R_{t+1}, c_{t+1})$ are continuous, then the following statements hold:*

(i) $p_t \leq \frac{E_t\tilde{x}_{t+1}}{R^f}$ for any risk averse representative agent ($u'' \leq 0$) if and only if $(\tilde{x}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{F}_1$;

(ii) $p_t \geq \frac{E_t\tilde{x}_{t+1}}{R^f}$ for any risk averse representative agent ($u'' \leq 0$) if and only if $(\tilde{x}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{G}_1$;

(iii) $E_t\tilde{R}_{t+1} \geq R^f$ for any risk averse representative agent ($u'' \leq 0$) if and only if $(\tilde{R}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{F}_1$;

(iv) $E_t\tilde{R}_{t+1} \leq R^f$ for any risk averse representative agent ($u'' \leq 0$) if and only if $(\tilde{R}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{G}_1$.

Proof See Appendix A.

Proposition 3.1 states that, for a risk averse representative agent, an asset's price is lowered (or equity premium is positive) if and only if its payoff is positively first-degree expectation dependent with consumption. Conversely, an asset's price is raised (or equity premium is negative) if and only if its payoff is negatively first-degree expectation dependent with consumption. Therefore, for a risk averse representative agent, it is the first-degree expectation dependence rather than the covariance that determines its riskiness. Because $(\tilde{x}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{F}_1(\mathcal{G}_1) \Rightarrow cov_t(\tilde{x}_{t+1}, \tilde{c}_{t+1}) \geq 0 (\leq 0)$ and the converse is not true, we conclude that a positive (negative) covariance is a necessary but not sufficient condition for a risk averse agent paying a lower (higher) asset price (or having a positive (negative) equity premium).

Example 3.2 *For bivariate log-normal* $(\tilde{x}_{t+1}, \tilde{c}_{t+1}) = (\tilde{x}, \tilde{y})$ *defined in (6),* $\sigma_{12} \geq 0$*, if and only if* $p_t \leq \frac{E_t \tilde{x}_{t+1}}{R^f}$ *for any risk averse representative agent.*

3.2 Comparative risk aversion

The assumption of risk aversion has long been a cornerstone of modern economics and finance. Ross (1981) provides the following strong measure for comparative risk reversion:

Definition 3.3 *(Ross 1981) u is more Ross risk averse than v if and only if there exists* $\lambda > 0$ *such that for all* x, y

$$\frac{u''(x)}{v''(x)} \geq \lambda \geq \frac{u'(y)}{v'(y)}. \tag{31}$$

More risk averse in the sense of Ross guarantees that the more risk averse decision-maker is willing to pay more to benefit from a mean preserving contraction.

Our important comparative statics question is: Under which condition does a change in the representative agent's risk preferences reduce the asset price? To answer this question let us consider a change in the utility function from u to v. From (28), for agent v, we have

$$p_t = \frac{E_t \tilde{x}_{t+1}}{R^f} - \beta \int_{-\infty}^{+\infty} FED(\tilde{x}_{t+1}|c_{t+1}) F_{C_{t+1}}(c_{t+1})[-\frac{v''(c_{t+1})}{v'(c_t)}]dc_{t+1}. \tag{32}$$

Intuition suggests that if asset return and consumption are positive dependent and agent v is more risk averse than agent u, then agent v should have a larger risk premium than agent u. This intuition can be reinforced by Ross risk aversion and first-degree expectation dependence, as stated in the following proposition.

Proposition 3.4 *Let* p_t^u *and* p_t^v *denote the asset's prices corresponding to* u *and* v *respectively. Suppose* u', v', u'' *and* v'' *are continuous, then the following statements hold:*

(i) $p_t^u \geq p_t^v$ *for all* $(\tilde{x}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{F}_1$ *if and only if* v *is more Ross risk averse than* u*;*

(ii) $p_t^u \geq p_t^v$ *for all* $(\tilde{x}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{G}_1$ *if and only if* u *is more Ross risk averse than* v*;*

Proof See Appendix A.

Proposition 3.4 indicates that when an asset's payoff is first-degree positive (negative) expectation dependent on consumption, an increase in risk aversion in the sense of Ross decreases (increases) the asset price.

Example 3.5 *For bivariate log-normal* $(\tilde{x}_{t+1}, \tilde{c}_{t+1}) = (\tilde{x}, \tilde{y})$ *defined in (6), if* $\sigma_{12} \geq 0$*, then the order of risk aversion in the sense of Ross is equivalent to the order of asset price.*

3.3 Changes in joint distributions

The question dual to the change in risk aversion examined above is: Under which condition does a change in the joint distribution of random payoff and consumption increase the asset's price? We may also ask the same question for the risk premium by using the joint distribution of an asset's gross return and consumption. To address these questions, let us denote $E_t^H$and FED_H as the expectation and first-order expectation dependency under distribution $H(x, y)$. Let p_t^F and p_t^H denote the corresponding prices under distributions $F(x, y)$ and $H(x, y)$ respectively. From (28), we have

$$p_t^H = \frac{E_t^H \tilde{x}_{t+1}}{R^f} - \beta \int_{-\infty}^{+\infty} FED_H(\tilde{x}_{t+1}|c_{t+1}) H_{C_{t+1}}(c_{t+1})[-\frac{u''(c_{t+1})}{u'(c_t)}]dc_{t+1}. \tag{33}$$

From (28) and (33), we obtain the following result.

Proposition 3.6 *(i) Suppose $F(x_{t+1}, c_{t+1})$ is continuous and $E_t^F \tilde{x}_{t+1} = E_t^H \tilde{x}_{t+1}$, then $p_t^F \leq p_t^H$ for all risk averse representative agents if and only if $F(x_{t+1}, c_{t+1})$ is more first-degree expectation dependent than $H(x_{t+1}, c_{t+1})$;*

(ii)Suppose $F(R_{t+1}, c_{t+1})$ is continuous, then for all risk averse representative agents, $F(R_{t+1}, c_{t+1})$ is more first-degree expectation dependent than $H(R_{t+1}, c_{t+1})$ if and only if the risk premium under $F(R_{t+1}, c_{t+1})$ is greater than under $H(R_{t+1}, c_{t+1})$.

Proof See Appendix A.

Proposition 3.6 shows that a pure increase in first-degree expectation dependence represents an increase in asset riskiness for all risk averse investors. The next corollary considers a simultaneous variation in expected return and dependence.

Corollary 3.7 *For all risk averse representative agents, $E_t^F \tilde{x}_{t+1} \leq E_t^H \tilde{x}_{t+1}$ and $F(x_{t+1}, c_{t+1})$ is more first-degree expectation dependent than $H(x_{t+1}, c_{t+1})$ imply $p_t^F \leq p_t^H$.*

Proof The sufficient conditions are directly obtained from (28) and (33).

Corollary 3.7 states that, for all risk averse representative agents, a decrease in the expected return and an increase in the first-degree expectation dependence between return and consumption will decrease the asset's price. Again, the key available concept for prediction is comparative first-degree expectation dependence.

Example 3.8 *For bivariate log-normal $(\tilde{R}_{t+1}, \tilde{c}_{t+1}) = (\tilde{x}, \tilde{y})$ and $(\tilde{R}'_{t+1}, \tilde{c}'_{t+1}) = (\tilde{x}', \tilde{y}')$ defined in (18) and (19), if (20) holds, then, for all risk averse representative agents, $p_t^F \leq p_t^H$.*

4 C-CAPM for a higher-order risk averse representative agent

4.1 C-CAPM for a risk averse and prudent representative agent

The concept of prudence and its relationship to precautionary savings was introduced by Kimball (1990). Since then, prudence has become a common and accepted assumption in the economics literature (Gollier 2001). All prudent agents dislike any increase in downside risk in the sense of Menezes et al. (1980) (see also Chiu, 2005.). Deck and Schlesinger (2010) conduct a laboratory experiment to determine whether preferences are prudent, and show behavioral evidence for prudence. In this section, we will demonstrate that we can get dependence conditions for asset price and equity premium in addition to first-degree expectation dependence, when the representative agent is risk averse and prudent.

We can integrate the right-hand term of (27) by parts and obtain:

$$\begin{aligned}
cov_t[u'(\tilde{c}_{t+1}), \tilde{x}_{t+1}] &= \int_{-\infty}^{+\infty} FED(\tilde{x}_{t+1}|c_{t+1})u''(c_{t+1})F_{c_{t+1}}(c_{t+1})dc_{t+1} \\
&= \int_{-\infty}^{+\infty} u''(c_{t+1})d(\int_{-\infty}^{c_{t+1}}[E\tilde{x}_{t+1} - E(\tilde{x}_{t+1}|\tilde{c}_{t+1} \leq s)]F_{c_{t+1}}(s)ds) \\
&= u''(c_{t+1})\int_{-\infty}^{c_{t+1}}[E\tilde{x}_{t+1} - E(\tilde{x}_{t+1}|\tilde{c}_{t+1} \leq s)]F_{c_{t+1}}(s)ds|_{-\infty}^{+\infty} \\
&\quad - \int_{-\infty}^{+\infty}\int_{-\infty}^{c_{t+1}}[E\tilde{x}_{t+1} - E(\tilde{x}_{t+1}|\tilde{c}_{t+1} \leq s)]F_{c_{t+1}}(s)dsu'''(c_{t+1})dc_{t+1} \\
&= u''(+\infty)\int_{-\infty}^{+\infty}[E\tilde{x}_{t+1} - E(\tilde{x}_{t+1}|\tilde{c}_{t+1} \leq s)]F_{c_{t+1}}(s)ds \\
&\quad - \int_{-\infty}^{+\infty}\int_{-\infty}^{c_{t+1}}[E\tilde{x}_{t+1} - E(\tilde{x}_{t+1}|\tilde{c}_{t+1} \leq s)]F_{c_{t+1}}(s)dsu'''(c_{t+1})dc_{t+1} \\
&= u''(+\infty)cov_t(\tilde{x}_{t+1}, \tilde{c}_{t+1}) - \int_{-\infty}^{+\infty} SED(\tilde{x}_{t+1}|c_{t+1})u'''(c_{t+1})dc_{t+1}.
\end{aligned} \tag{34}$$

From equation (15), we know that a positive SED implies a positive $cov(\tilde{x}_{t+1}, \tilde{c}_{t+1})$ but the converse is not true. Hence, we have from (34) that $cov_t(\tilde{x}_{t+1}, \tilde{c}_{t+1}) \geq 0$ is a necessary but not sufficient condition for $cov_t[u'(\tilde{c}_{t+1}), \tilde{x}_{t+1}] \leq 0$ for all $u'' \leq 0$ and $u''' \geq 0$. With a positive SED function, prudence is also necessary.

(23) and (24) can be rewritten as:

$$\begin{aligned}
p_t = &\underbrace{\frac{E_t\tilde{x}_{t+1}}{R^f}}_{risk-free\ present\ value\ effect} \underbrace{-\beta cov_t(\tilde{x}_{t+1}, \tilde{c}_{t+1})[-\frac{u''(+\infty)}{u'(c_t)}]}_{covariance\ effect} \\
&\underbrace{-\beta\int_{-\infty}^{+\infty} SED(\tilde{x}_{t+1}|c_{t+1})[\frac{u'''(c_{t+1})}{u'(c_t)}]dc_{t+1}}_{second-degree\ expectation\ dependence\ effect}
\end{aligned} \tag{35}$$

or

$$p_t = \frac{E_t\tilde{x}_{t+1}}{R^f} - \beta cov_t(\tilde{x}_{t+1}, \tilde{c}_{t+1}) AR(+\infty) MRS_{+\infty,c_t} - \beta \int_{-\infty}^{+\infty} SED(\tilde{x}_{t+1}|c_{t+1}) AP(c_{t+1}) MRS_{c_{t+1},c_t} dc_{t+1}, \tag{36}$$

where $AP(x) = \frac{u'''(x)}{u'(x)}$ is the index of absolute prudence[7], and

$$\begin{aligned} & E_t\tilde{R}_{t+1} - R^f \\ = \; & \underbrace{cov_t(\tilde{R}_{t+1}, \tilde{c}_{t+1})[-\frac{u''(+\infty)}{E_t u'(\tilde{c}_{t+1})}]}_{consumption\ covariance\ effect} + \underbrace{\int_{-\infty}^{+\infty} SED(\tilde{R}_{t+1}|c_{t+1}) \frac{u'''(c_{t+1})}{E_t u'(\tilde{c}_{t+1})} dc_{t+1}}_{consumption\ second-degree\ expectation\ dependence\ effect} \end{aligned} \tag{37}$$

or

$$\begin{aligned} & E_t\tilde{R}_{t+1} - R^f \\ = \; & \beta R^f cov_t(\tilde{R}_{t+1}, \tilde{c}_{t+1}) AR(+\infty) MRS_{+\infty,c_t} \\ & + \beta R^f \int_{-\infty}^{+\infty} SED(\tilde{R}_{t+1}|c_{t+1}) AP(c_{t+1}) MRS_{c_{t+1},c_t} dc_{t+1}. \end{aligned} \tag{38}$$

Condition (35) includes three terms. The first one is the same as in condition (28). The second term on the right-hand side of (35) is called the "covariance effect." This term involves β, the covariance of asset return and consumption, the Arrow-Pratt risk aversion coefficient and the marginal rates of substitution. The third term on the right-hand side of (35) is called "second-degree expectation dependence effect," which reflects the way in which second-degree expectation dependence of risk affects asset's price through the intensity of downside risk aversion. Again (35) affirms that positive correlation is only a necessary condition for all risk averse and prudent agents to pay a lower price. Equation (37) shows that a positive SED reinforces the positive covariance effect to obtain a positive risk premium.

We state the following propositions without proof. The proofs of these propositions are similar to the proofs of Propositions in Section 3, and are therefore skipped. They are available from the authors.

Proposition 4.1 *Suppose $F(x_{t+1}, c_{t+1})$ and $F(R_{t+1}, c_{t+1})$ are continuous, then the following statements hold:*

(i) $p_t \leq \frac{E_t\tilde{x}_{t+1}}{R^f}$ for any risk averse and prudent representative agent ($u'' \leq 0$ and $u''' \geq 0$) if and only if $(\tilde{x}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{F}_2$;

[7]Modica and Scarsini (2005), Crainich and Eeckhoudt (2008) and Denuit and Eeckhoudt (2010) propose $\frac{u'''(x)}{u'(x)}$ instead of $-\frac{u'''(x)}{u''(x)}$ (Kimball, 1990) as an alternative candidate to evaluate the intensity of prudence.

(ii) $p_t \geq \frac{E_t \tilde{x}_{t+1}}{R^f}$ *for any risk averse and prudent representative agent* ($u'' \leq 0$ *and* $u''' \geq 0$) *if and only if* $(\tilde{x}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{G}_2$;

(iii) $E_t \tilde{R}_{t+1} \geq R^f$ *for any risk averse and prudent representative agents* ($u'' \leq 0$ *and* $u''' \geq 0$) *if and only if* $(\tilde{R}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{F}_2$;

(iv) $E_t \tilde{R}_{t+1} \leq R^f$ *for any risk averse and prudent representative agents* ($u'' \leq 0$ *and* $u''' \geq 0$) *if and only if* $(\tilde{R}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{G}_2$.

Modica and Scarsini (2005) provide a comparative statics criterion for downside risk in the spirit of Ross (1981).

Definition 4.2 *(Modica and Scarsini 2005)* u *is more downside risk averse than* v *if and only if there exists* $\lambda > 0$ *such that for all* x, y

$$\frac{u'''(x)}{v'''(x)} \geq \lambda \geq \frac{u'(y)}{v'(y)}. \tag{39}$$

More downside risk aversion can guarantee that the decision-maker with a utility function that has more downside risk aversion is willing to pay more to avoid the downside risk increase as defined by Menezes et al. (1980). We can therefore extend Proposition 3.4 as follows:

Proposition 4.3 *Suppose* u', v', u''' *and* v''' *are continuous, then the following statements hold:*

(i) $p_t^u \geq p_t^v$ *for all* $(\tilde{x}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{F}_2$ *if and only if* v *is more Ross and downside risk averse than* u;

(ii) $p_t^u \geq p_t^v$ *for all* $(\tilde{x}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{G}_2$ *if and only if* u *is more Ross and downside risk averse than* v;

We also obtain the following results for changes in joint distributions.

Proposition 4.4 *(i) Suppose* $F(x_{t+1}, c_{t+1})$ *is continuous and* $E_t^F \tilde{x}_{t+1} = E_t^H \tilde{x}_{t+1}$, *then* $p_t^F \leq p_t^H$ *for all risk averse and prudent representative agents if and only if* $F(x_{t+1}, c_{t+1})$ *is more second-degree expectation dependent than* $H(x_{t+1}, c_{t+1})$;

(ii) Suppose $F(R_{t+1}, c_{t+1})$ *is continuous, then for all risk averse and prudent representative agents,* $F(R_{t+1}, c_{t+1})$ *is more second-degree expectation dependent than* $H(R_{t+1}, c_{t+1})$ *if and only if the risk premium under* $F(R_{t+1}, c_{t+1})$ *is greater than under* $H(R_{t+1}, c_{t+1})$.

Corollary 4.5 *For all risk averse and prudent representative agents,* $E_t^F \tilde{x}_{t+1} \leq E_t^H \tilde{x}_{t+1}$ *and* $F(x_{t+1}, c_{t+1})$ *is more second-degree expectation dependent than* $H(x_{t+1}, c_{t+1})$ *implies* $p_t^F \leq p_t^H$;

4.2 C-CAPM for a higher-order representative agent

Ekern (1980) provides the following definition to sign the higher-order risk attitude.

Definition 4.6 *(Ekern 1980) An agent u is N^{th} degree risk averse, if and only if*

$$(-1)^N u^{(N)}(x) \leq 0 \quad for \quad all \quad x, \tag{40}$$

where $u^{(N)}(\cdot)$ denotes the N^{th} derivative of $u(x)$.

Ekern (1980) shows that all agents having utility function with N^{th} degree risk aversion dislike a probability change if and only if it produces an increase in N^{th} degree risk. Risk aversion in the traditional sense of a concave utility function is indicated by $N = 2$. When $N = 3$, we obtain $u''' \geq 0$, which means that marginal utility is convex, or implies prudence. Eeckhoudt and Schlesinger (2006) derive a class of lottery pairs to show that lottery preferences are compatible with Ekern's N^{th} degree risk aversion.

Jindapon and Neilson (2007) generalize Ross' risk aversion to higher-order risk aversion.

Definition 4.7 *(Jindapon and Neilson 2007) u is more N^{th}-degree Ross risk averse than v if and only if there exists $\lambda > 0$ such that for all x, y*

$$\frac{u^{(N)}(x)}{v^{(N)}(x)} \geq \lambda \geq \frac{u'(y)}{v'(y)}. \tag{41}$$

Li (2009) and Denuit and Eeckhoudt (2010) provide context-free explanations for higher-order Ross risk aversion. In Appendix B, we generalize the results of Section 3 and 4 to higher-degree risks and higher order representative agents.

5 Pricing with two local absolute indexes of risk attitude

If we assume that $\tilde{c}_t$ and $\tilde{c}_{t+1}$ are close enough, then we can use the local coefficient of risk aversion and local downside risk aversion (see Modica and Scarsini, 2005) to obtain the following approximation formulas for (28) and (35):

$$\begin{aligned} p_t &\approx \frac{E_t \tilde{x}_{t+1}}{R^f} + \beta \frac{u''(c_t)}{u'(c_t)} \int_{-\infty}^{+\infty} FED(\tilde{x}_{t+1}|c_{t+1}) F_{c_{t+1}}(c_{t+1}) dc_{t+1} \\ &= \frac{E_t \tilde{x}_{t+1}}{R^f} - \beta AR(c_t) cov_t(\tilde{x}_{t+1}, \tilde{c}_{t+1}) \end{aligned} \tag{42}$$

and

$$
\begin{aligned}
p_t &\approx \frac{E_t\tilde{x}_{t+1}}{R^f} + \beta\frac{u''(c_t)}{u'(c_t)}cov_t(\tilde{x}_{t+1},\tilde{c}_{t+1}) - \beta\frac{u'''(c_t)}{u'(c_t)}\int_{-\infty}^{+\infty} SED(\tilde{x}_{t+1}|c_{t+1})dc_{t+1} \qquad (43)\\
&= \frac{E_t\tilde{x}_{t+1}}{R^f} - \beta AR(c_t)cov_t(\tilde{x}_{t+1},\tilde{c}_{t+1}) - \beta AP(c_t)\int_{-\infty}^{+\infty} SED(\tilde{x}_{t+1}|c_{t+1})dc_{t+1}.
\end{aligned}
$$

When the variation of consumption is small, (42) implies that absolute risk aversion and covariance determine asset prices while (43) implies that absolute risk aversion, absolute prudence, covariance and SED determine asset prices. We mentioned before that $SED(\tilde{x}|y)$ is related to the 2nd central cross lower partial moment of $\tilde{x}$ and $\tilde{y}$, hence (43) provides a theoretical explanation of the importance of higher-order risk preferences, higher-order moments and co-moments in finance. Note that we obtain only approximations of asset prices when we use the Arrow-Pratt measure of risk aversion and the extended measure of prudence as in Modica and Scarsini (2005).

To analyze the equity premium puzzle, it is helpful to compute similar approximation formulas for (29) and (37):

$$
\begin{aligned}
E_t\tilde{R}_{t+1} - R^f &\approx -\frac{u''(c_t)}{u'(c_t)}\int_{-\infty}^{+\infty} FED(\tilde{R}_{t+1}|c_{t+1})F_{c_{t+1}}(c_{t+1})dc_{t+1} \qquad (44)\\
&= AR(c_t)cov_t(\tilde{R}_{t+1},\tilde{c}_{t+1})
\end{aligned}
$$

and

$$
\begin{aligned}
E_t\tilde{R}_{t+1} - R^f &\approx -\frac{u''(c_t)}{u'(c_t)}cov_t(\tilde{R}_{t+1},\tilde{c}_{t+1}) + \frac{u'''(c_t)}{u'(c_t)}\int_{-\infty}^{+\infty} SED(\tilde{R}_{t+1}|c_{t+1})dc_{t+1} \qquad (45)\\
&= AR(c_t)cov_t(\tilde{R}_{t+1},\tilde{c}_{t+1}) + AP(c_t)\int_{-\infty}^{+\infty} SED(\tilde{R}_{t+1}|c_{t+1})dc_{t+1}.
\end{aligned}
$$

For a given preference function and data on stock return and aggregate consumption, the above approximations yield risk aversion estimates. These estimates allow one to gauge whether the extended C-CAPM we propose improves the understanding of the equity premium puzzle.

6 Equity premium puzzle for a higher-order representative agent

6.1 Implications of our results on the equity premium puzzle

The major discrepancy between the C-CAPM model predictions and previous empirical reality is identified as the equity premium puzzle in the literature. As mentioned in Section 3, the key empirical observations of the equity premium puzzle based on (25) can be summarized as follows:

When the representative agent's utility function is the power function, and $\tilde{c}_{t+1}$ and $\tilde{x}_{t+1}$ are conditional lognormally distributed, the observed equity premium can be explained only by assuming a very high coefficient of relative risk aversion. It is also difficult to explain observed high risk premia with the covariance because of the smoothness of consumption over time. However, the equity premium puzzle conclusion is built on specific utility functions and return distributions. Our theoretical results show that, for general utility functions and distributions, covariance is not the key element of equity premium prediction. It is very easy to find counterintuitive results. For example, given positively correlated gross return and consumption distributions, a lower Arrow-Pratt coefficient of relative risk aversion may result in a higher equity premium. Alternatively, given a representative agent's preference, a lower covariance between gross return and consumption may result in a higher equity premium. Therefore, (25) is not a robust theoretical prediction of equity premia.

Our results prove that asset pricing's comparative statics imply the following robust predictions:

(a) expectation dependence between payoff and consumption determines asset riskiness rather than covariance;

(b) when a representative agent's risk preference satisfies higher-order risk aversion, more expectation dependence between payoff and consumption is equivalent to a lower price.

(c) when a representative agent's risk preference satisfies higher-order risk aversion, more expectation dependence between gross return and consumption is equivalent to a higher equity premium.

(d) when payoff and consumption are positive expectation dependent, higher risk aversion in the sense of Ross is equivalent to a lower equity price.

We now test how our model can be useful to study the equity premium puzzle.

6.2 Consumption-based asset pricing with exponential utility

Does accounting for higher-order risk attitude and higher-order risk measure help mitigate the equity premium puzzle? To answer this question, we assume that there is a representative agent endowed with an exponential utility function defined over aggregate consumption c_t. This is a significant departure from other classic papers on the equity premium puzzle, which consider either the power utility preference (Mehra and Prescott, 1985; Hansen and Singleton, 1983; Campbell, 2003), or the Epstein and Zin (1989, 1991) and Weil (1989) recursive utility function (Kandel and Stambaugh, 1991; Campbell, 2003). To be coherent with (44) and (45), we choose the exponential utility function $u(c_t) = -e^{-\lambda c_t}$, which entails a constant absolute risk aversion coefficient λ in the sense of Arrow-Pratt, and a constant absolute prudence index λ^2 in the sense of Modica and Scarsini (2005).

Because the consumption level c_t is known at time t, it follows from (44) that

$$E_t\tilde{R}_{t+1} - R^f \approx \lambda cov_t(\tilde{R}_{t+1}, \Delta\tilde{c}_{t+1}) \tag{46}$$

where $\tilde{R}_{t+1}$ is the asset's net return, R^f is the net return of the risk-free asset[8], and $\Delta\tilde{c}_{t+1} = \tilde{c}_{t+1} - c_t$ is the differenced consumption level.

Similarly, (45) is rewritten as

$$E_t\tilde{R}_{t+1} - R^f \approx \lambda cov_t(\tilde{R}_{t+1}, \Delta\tilde{c}_{t+1}) + \lambda^2 \underbrace{\int_{-\infty}^{+\infty} SED(\tilde{R}_{t+1}|c_{t+1})dc_{t+1}}_{integrated\ consumption\ SED}. \tag{47}$$

We compare the risk aversion estimates between (46) and (47) to assess the equity premium puzzle improvement resulting from the inclusion of higher-order risk measures. To this end, we compute both the covariance and the integrated second-degree expectation dependence between the return and the differenced consumption series. Then, we solve for the absolute risk aversion coefficient $AR = \lambda$. At a given time period t, the relative risk aversion is equal to the absolute risk aversion multiplied by the current consumption. Thus, we compute an unconditional estimate of the relative risk aversion RR as the absolute risk aversion AR times the average[9] aggregate consumption level $\bar{c}$.

To calculate the integrated consumption second-degree expectation dependence, we proceed as follows. Assume that the consumption level $\tilde{c} \in [\underline{\underline{c}}, \bar{\bar{c}}]$ takes n increasingly ordered values over its support, where $\underline{\underline{c}} = c^{(1)} \leq \cdots \leq c^{(i)} \leq \cdots \leq c^{(n)} = \bar{\bar{c}}$. Then, the integrated consumption second-degree expectation dependence can be approximated as a sum of the products between

[8] Using gross returns $\left(1+\tilde{R}_{t+1}\right)$ and $\left(1+R^f\right)$ rather than net returns amounts to a location shift in the returns distribution, which changes neither the risk quantity nor the risk premium.

[9] Using the median rather than the average aggregate consumption gives similar relative risk aversion estimates.

lower partial covariances and changes in increasingly ordered consumption levels:

$$
\begin{aligned}
\int_{\underline{c}}^{\bar{\bar{c}}} SED\left(\tilde{R}_{t+1}|c_{t+1}\right) dc_{t+1} &\approx \sum_{i=2}^{n} SED\left(\tilde{R}_{t+1}|\tilde{c}_{t+1}=c^{(i)}\right) \times \left[c^{(i)}-c^{(i-1)}\right] \\
&= \sum_{i=2}^{n} cov_t\left(\tilde{R}_{t+1},\tilde{c}_{t+1}|\tilde{c}_{t+1}\leq c^{(i)}\right) \times \left[c^{(i)}-c^{(i-1)}\right] \\
&= \sum_{i=2}^{n} \underbrace{cov_t\left(\tilde{R}_{t+1},\Delta\tilde{c}_{t+1}|\tilde{c}_{t+1}\leq c^{(i)}\right)}_{lower\ partial\ covariance} \times \underbrace{\left[c^{(i)}-c^{(i-1)}\right]}_{consumption\ change} .
\end{aligned} \tag{48}
$$

For convenience, we summarize the estimation procedure of the absolute and relative risk aversion coefficients with second-degree expectation dependence in 5 steps:

(step 1) sort the consumption level series in ascending order $\left\{c^{(i)}\right\}_{i=1}^{n}$, then find the corresponding net returns $\tilde{R}$ and differenced consumption levels $\Delta\tilde{c}$ observations;

(step 2) calculate $n-1$ consecutive lower partial covariances between the sorted series of net returns $\tilde{R}$ and differenced consumption levels $\Delta\tilde{c}$, starting with the observations corresponding to the two lowest levels of consumption, and adding one new observation for each subsequent covariance;

$$
\underbrace{\underbrace{\underbrace{c^{(1)} \leq c^{(2)}}_{cov\left(\tilde{R},\Delta\tilde{c}|\tilde{c}\leq c^{(2)}\right)} \leq c^{(3)}}_{cov\left(\tilde{R},\Delta\tilde{c}|\tilde{c}\leq c^{(3)}\right)} \leq \cdots \leq c^{(n)}}_{\vdots \atop cov\left(\tilde{R},\Delta\tilde{c}|\tilde{c}\leq c^{(n)}\right)};
$$

(step 3) evaluate the integrated consumption SED in (48) as the sum of the products between the $n-1$ lower partial covariances and the changes in sorted consumption levels;

(step 4) solve the second-order equation (47) in λ to get an estimate of the absolute risk aversion coefficient AR;

(step 5) compute an implied relative risk aversion proxy $RR = AR \times \bar{c}$.

The integrated consumption SED involves lower partial comovements between market portfolio returns and consumption, and can be interpreted as a measure of downside consumption risk (Hogan and Warren, 1974; Bawa and Lindenberg, 1977; Price, Price and Nantell, 1982). For instance, a positive integrated consumption SED is obtained when there are “more” positive lower partial covariances between the returns and the consumption. That is, when stock market portfolio returns are “more” positively correlated with consumption in the left part of the consumption distribution. In that case, the stock market portfolio does not offer a hedge against the downside consumption risk. Thus, the representative investor requires a premium as a compensation for bearing this risk.

The next subsection presents international evidence for the risk premium puzzle. Our results advocate for higher-order expectation dependence measures in the consumption-based capital asset pricing model.

6.3 Empirical results

For our empirical analysis, we use the same dataset as in Campbell (2003). The data can be downloaded from Campbell's website. This international developed-country dataset combines Morgan Stanley Capital International (MSCI) stock market data with macroeconomic data on consumption, interest rates, and the price index from the International Financial Statistics (IFS) of the International Monetary Fund. The data allow us to construct quarterly series of stock market return, risk-free rate and per capita consumption spanning the early 1970s through the late 1990s for 11 countries: Australia, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, UK and USA. Longer annual series are also available for Sweden (1920-1998), the UK (1919-1998) and the U.S. (1891-1998). We refer the reader to Campbell's (2003) chapter, for a full data description.

We begin by replicating the risk premium puzzle results in Campbell (2003). We use these relative risk aversion estimates as benchmark values for the C-CAPM extension we propose in this paper. Table 1 presents some descriptive statistics for international stock market returns and consumption. Specifically, Table 1 shows the mean, standard deviation, and first-order serial correlation for the real returns on the stock market index, the risk-free asset and the per capita real consumption growth. While the top panel reports the results for quarterly data from all 11 countries, the bottom panel presents longer annual sample data results for Sweden, UK and USA only. At least six major empirical regularities emerge from Table 1:

(i) stock market real returns have been historically high, averaging above 4.5% in most countries except Australia and Italy;

(ii) by contrast, real risk-free rates have been low, generally under 3% except for Germany and the Netherlands;

(ii) stock market annualized volatility is found between 15% and 27%, while the returns are weakly autocorrelated;

(iv) risk-free rates have shown low annualized volatility, never exceeding 3% for quarterly data and 9% for annual data;

(vi) consumption growth has been smooth for all countries, with annualized standard deviation barely reaching 3%;

(vii) the correlation and, thus, the covariance between stock market real returns and consumption growth has been weak for most countries, even negative for France, Italy and Switzerland.

Table 1: International stock, Tbill log returns and per capita consumption growth

Country	Sample period	$\overline{r_e}$	$\sigma(r_e)$	$\rho(r_e)$	$\overline{r_f}$	$\sigma(r_f)$	$\rho(r_f)$	$\overline{\Delta \log c}$	$\sigma(\Delta \log c)$	$\rho(\Delta \log c)$	$\rho(r_e, \Delta \log c)$
USA	1947.2-1998.4	8.085	15.645	0.083	0.891	1.746	0.507	1.964	1.073	0.216	0.231
AUL	1970.1-1999.1	3.540	22.700	0.005	2.054	2.528	0.645	2.099	2.056	-0.324	0.158
CAN	1970.1-1999.2	5.431	17.279	0.072	2.713	1.856	0.667	2.082	1.971	0.105	0.183
FR	1973.2-1998.4	9.023	23.425	0.048	2.715	1.837	0.710	1.233	2.909	0.029	-0.099
GER	1978.4-1997.4	9.838	20.097	0.090	3.219	1.152	0.348	1.681	2.431	-0.327	0.027
ITA	1971.2-1998.2	3.168	27.039	0.079	2.371	2.847	0.691	2.200	1.700	0.283	-0.028
JAP	1970.2-1999.1	4.715	21.909	0.021	1.388	2.298	0.480	3.205	2.554	-0.275	0.112
NTH	1977.4-1998.4	14.407	17.384	-0.037	3.523	1.535	-0.173	1.763	2.488	-0.215	0.030
SWD	1970.1-1999.3	10.648	23.840	0.022	1.995	2.835	0.260	0.962	1.856	-0.266	0.016
SWT	1982.2-1999.1	13.745	21.828	-0.128	1.393	1.498	0.243	0.524	2.112	-0.399	-0.119
UK	1970.1-1999.2	8.156	21.190	0.084	1.301	2.957	0.478	2.203	2.507	-0.006	0.123
USA	1970.1-1998.4	6.929	17.556	0.051	1.485	1.685	0.571	1.812	0.907	0.374	0.289
SWD	1920-1998	6.561	18.262	0.069	2.167	6.189	0.667	1.727	2.831	0.169	0.213
UK	1919-1998	7.398	22.098	-0.024	1.227	5.800	0.588	1.466	2.827	0.284	0.413
USA	1891-1998	6.670	18.372	0.024	1.970	8.779	0.402	1.769	3.211	-0.117	0.449

Clearly, high stock market returns with low risk-free rates and weak covariance between stock returns and consumption growth, yield high relative risk aversion coefficients for an aggregate investor who maximizes a time-additive power utility function. This finding provides insight into the equity premium puzzle presented in Table 2. For each country, Table 2 reports the quantities required to compute the relative risk aversion in equation (25). The first column shows the annualized percentage average excess log return inflated by one-half of the variance of the excess log return as an adjustment for Jensen's Inequality. Because we use quarterly series to compute the top panel results, we multiply the values by 400. To express the bottom panel results in annualized percentage points, we multiply the values by 100 given that they come from annual series. The second and third columns report the standard deviation of the market excess log return and the consumption growth. To annualize the standard deviations, we multiply the quarterly numbers by 200. Correlations and covariances between the excess log return and the consumption growth are presented in the fourth and fifth columns. The two last columns give relative risk aversion estimates based on sample correlations ($RR\,(1)$) and assuming a maximum correlation of one ($RR\,(2)$). The latter counterfactual assumption induces lower levels of relative risk aversion.

Table 2: The equity premium puzzle with covariance as per equation (25)

Country	Sample period	$\overline{aer_e}$	$\sigma\,(er_e)$	$\sigma\,(\Delta \log c)$	$\rho\,(er_e, \Delta \log c)$	$cov\,(er_e, \Delta \log c)$	Campbell 2003	
							$RR\,(1)$	$RR\,(2)$
USA	1947.2-1998.3	8.074	15.272	1.072	0.205	3.358	240.442	49.293
AUL	1970.1-1998.4	3.885	22.403	2.060	0.144	6.629	58.609	8.435
CAN	1970.1-1999.1	3.968	17.266	1.920	0.202	6.871	57.743	11.659
FR	1973.2-1998.3	8.308	23.175	2.922	-0.093	-6.288	< 0	12.322
GER	1978.4-1997.3	8.669	20.196	2.447	0.029	1.437	603.375	17.657
ITA	1971.2-1998.1	4.687	27.068	1.665	-0.006	-0.258	< 0	10.186
JAP	1970.2-1998.4	5.098	21.498	2.561	0.112	6.154	82.842	9.285
NTH	1977.4-1998.3	11.628	17.082	2.486	0.039	1.643	707.834	27.357
SWD	1970.1-1999.2	11.540	23.518	1.851	0.015	0.675	1708.667	26.431
SWT	1982.2-1998.4	14.898	21.878	2.123	-0.112	-5.154	< 0	32.243
UK	1970.1-1999.1	9.169	21.198	2.512	0.093	4.921	186.329	17.255
USA	1970.1-1998.3	6.359	16.979	0.909	0.274	4.224	150.531	41.308
SWD	1920-1997	5.879	18.192	2.826	0.165	8.523	68.973	11.413
UK	1919-1997	8.301	20.644	2.752	0.338	19.747	42.035	14.223
USA	1891-1997	6.329	17.968	3.211	0.490	28.283	22.377	10.971

$\overline{aer_e} = \overline{r_e} - \overline{r_f} + var\,(er_e)\,/2$ and $cov\,(er_e, \Delta \log c)$ is in $\%^2$.

Consistent with Campbell (2003), our estimations yield implausible relative risk aversion coefficients $RR\,(1)$. These coefficients are negative for France, Italy and Switzerland while ranging from 57.7 to 1708.7 for the other countries. Even though the risk aversion numbers from annual series seem lower than the positive ones from quarterly data, they all exceed 22. This stylized fact illustrates the equity premium puzzle in the basic C-CAPM.

We now present the empirical evidence on the equity premium puzzle with expectation dependence as per equations (46) and (47). In our extended C-CAPM framework, the representative investor is equipped with an exponential utility function instead of a power utility preference as in the basic C-CAPM studied in Campbell (2003). Moreover, we employ "net" real excess returns and "differenced" real per capita consumption rather than "log" excess returns and real per capita consumption "growth". Table 3 presents the means, standard deviations and first-order autocorrelations for the stock market index net return, the net return on the risk-free asset and the differenced consumption. The average net return values in Table 3 are slightly higher than the average log returns in Table 1 due to Jensen's Inequality. Note that the stylized facts (i)-(iv) broadly hold for net returns series as well.

Table 3: International stock, Tbill net returns and differenced per capita consumption

Country	Sample period	$\overline{R_e}$	$\sigma(R_e)$	$\rho(R_e)$	$\overline{R_f}$	$\sigma(R_f)$	$\rho(R_f)$	$\overline{\Delta c}$	$\sigma(\Delta c)$	$\rho(\Delta c)$
USA	1947.2-1998.4	9.381	15.476	0.092	0.907	1.744	0.508	201.976	106.067	0.327
AUL	1970.1-1999.1	6.005	21.401	0.004	2.091	2.528	0.647	262.104	237.865	-0.233
CAN	1970.1-1999.2	6.931	16.940	0.070	2.739	1.863	0.667	264.206	234.242	0.092
FR	1973.2-1998.4	11.836	23.263	0.054	2.741	1.847	0.710	710.758	1856.763	0.025
GER	1978.4-1997.4	11.899	19.195	0.124	3.238	1.160	0.347	338.385	477.527	-0.310
ITA	1971.2-1998.2	6.893	28.210	0.071	2.418	2.843	0.693	261.735	185.779	0.347
JAP	1970.2-1999.1	7.105	21.668	0.031	1.417	2.287	0.478	48.238×10^3	41.908×10^3	-0.380
NTH	1977.4-1998.4	16.163	17.183	-0.023	3.551	1.547	-0.173	352.418	479.193	-0.194
SWD	1970.1-1999.3	13.642	24.150	0.060	2.040	2.841	0.260	610.491	1068.867	-0.227
SWT	1982.2-1999.1	16.293	21.135	-0.133	1.406	1.499	0.243	145.099	552.346	-0.398
UK	1970.1-1999.2	10.534	22.259	0.036	1.346	2.932	0.481	121.591	124.522	0.061
USA	1970.1-1998.4	8.506	17.278	0.061	1.502	1.694	0.570	230.745	114.609	0.353
SWD	1920-1998	8.519	19.399	0.057	2.391	6.665	0.667	786.308	1184.255	0.136
UK	1919-1998	10.196	23.620	-0.074	1.405	6.011	0.585	56.743	100.789	0.437
USA	1891-1998	8.622	18.843	0.003	2.382	9.110	0.400	99.139	132.508	0.222

R_e and R_f are net returns in %. Δc is the differenced consumption also in %.

Furthermore, the differenced consumption series Δc exhibits patterns that are similar to the consumption growth series $\Delta \log c$:

(vii) first-order autocorrelations are weak, while as shown in Figure 1, the coefficients of variation[10] for these two series are almost identical and do not exceed 2 except for France and Switzerland.

[10]The coefficient of variation is the ratio of the standard deviation to the mean.

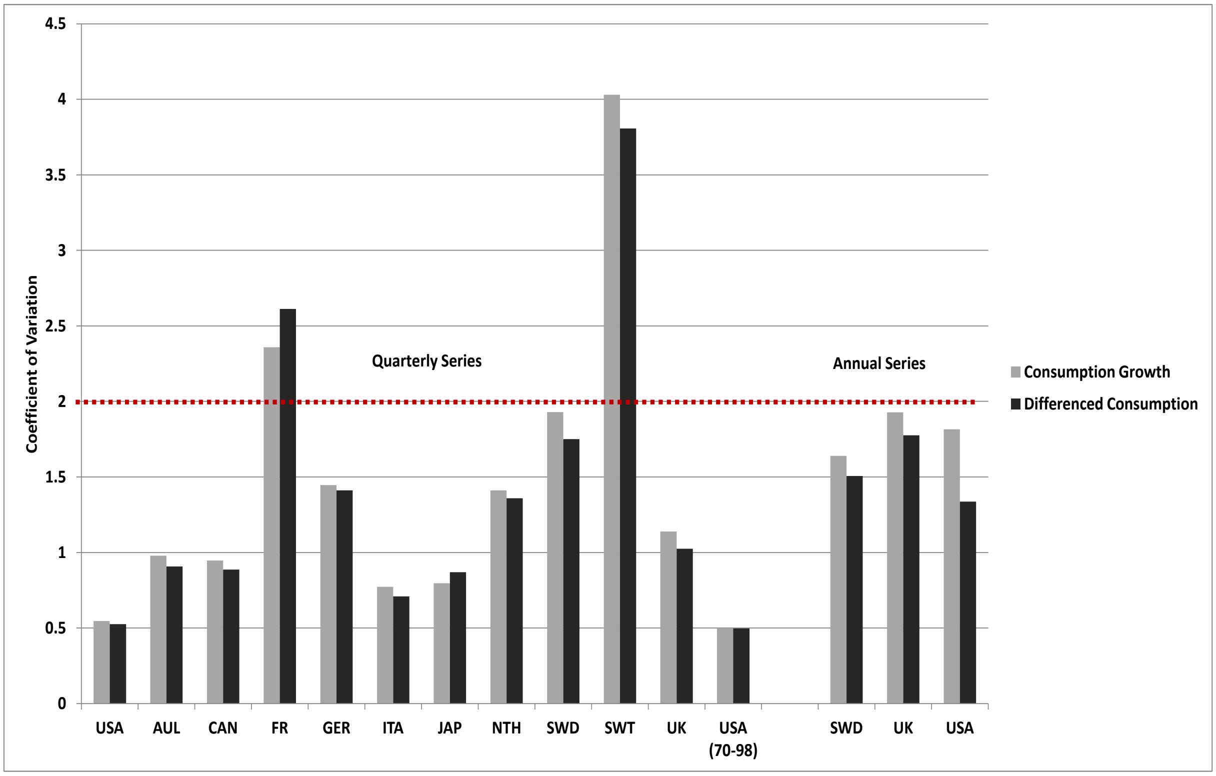

Figure 1: Coefficients of variation for consumption growth and differenced consumption series

It is noteworthy to mention that the extended C-CAPM we propose can easily accommodate higher-order risk attitudes without any particular restriction on the choice of the utility function or the joint distribution of the return and consumption. Table 4 turns to equations (46) and (47) to estimate the coefficient of relative risk aversion in the C-CAPM under expectation dependence. The two first columns in Table 4 report the annualized average excess returns and per capita consumption levels expressed in percentage points. The third column gives the covariance between the net return and the differenced consumption. The fourth column gives the integrated second-degree expectation dependence between the net return and the differenced consumption, which is computed according to (48). For nearly all countries, the covariance and the integrated SED have the same sign. As argued in Campbell (2003), the negative covariance and integrated SED for France and Switzerland may arise from short-term measurement errors in consumption. Interestingly, for Italy, the covariance between the net return and the differenced consumption is negative, whereas the integrated SED appears positive. By focusing only on the covariance, one might fail to capture the downside risk and thus, miss an important part of the equity premium. We now discuss the empirical evidence supporting this argument.

The right part of Table 4 presents two groups of four columns each. The first group headed FED uses (46) to compute the implied absolute risk aversion, dividing the average excess return by the covariance between the net return and the differenced consumption. In this group, the column headed $AR\,(1)$ reports the absolute risk aversion. The column headed $AR\,(2)$ sets the correlation to a maximum value of one before computing the absolute risk aversion. The next two columns headed $RR\,(1)$ and $RR\,(2)$ report relative risk aversion estimates, resulting from the product between corresponding absolute risk aversion coefficients and average consumption levels. The implied relative risk aversion coefficients estimated from the first-degree expectation dependence C-CAPM (46) are similar to the ones from the basic C-CAPM presented in Table 2. The equity premium puzzle seems robust across all 11 countries with huge relative risk aversion numbers. Even when we consider a maximum correlation of one between return and consumption, estimated relative risk aversion coefficients are still big for the U.S., Germany, the Netherlands, Sweden, and Switzerland. The results are similar to those presented in Table 2, which supports our methodology.

The second column group headed "FED with SED" gives the absolute risk aversion coefficients by solving the second-order equation (47). The absolute risk aversion coefficient is the only unknown in (47), because the estimated average excess return, covariance and integrated SED are readily available. The second group of four columns has the same structure as the previous one. The columns headed $AR\,(1)$ and $AR\,(2)$ show implied absolute risk aversion coefficients based on empirical correlation or assuming a maximum correlation of one. These implied absolute risk aversion values are then multiplied by average consumption levels to calculate the corresponding relative risk aversion coefficients $RR\,(1)$ and $RR\,(2)$. In contrast with the results from the FED C-CAPM (46), the relative risk aversion coefficients estimated from the FED-SED C-CAPM (47) appear much closer to plausible numbers. The relative risk aversion coefficients are still negative for France and Switzerland, but now range from 7.9 to 95.1 for the other countries. Clearly, we notice a sharp reduction in the relative risk aversion estimates when the C-CAPM includes the consumption second-degree expectation dependence effect. The FED-SED implied relative risk aversion coefficients are 63, 32, 24, 14, 12, 11 times smaller than their FED implied counterparts for Sweden, Germany, the Netherlands, USA, UK, and Japan. Further, the relative risk aversion estimate shifts from a negative value to become positive (18.5) for Italy. The $RR\,(2)$ values vary from 2.6 to 15.3 in the last column, which correspond to the theoretical values often proposed in the literature.

In a nutshell, accounting for the integrated SED improves the C-CAPM estimation dramatically and delivers reasonable risk aversion coefficients. Our empirical findings illustrate the need to include higher-order risk attitudes associated with higher-degree expectation dependence measures in the C-CAPM analysis. Beyond its theoretical appeal, the concept of expectation dependence helps bridge the gap between real-world data and the consumption-based capital asset pricing model.

Table 4: The equity premium puzzle with expectation dependence as per equations (46) and (47)

						FED				FED with SED			
Country	Sample period	$\overline{eR_e}$	$\overline{c}$	$cov\,(R_e, \Delta c)$	$\int SED\,(R_e\|c)\,dc$	$AR\,(1)$	$AR\,(2)$	$RR\,(1)$	$RR\,(2)$	$AR\,(1)$	$AR\,(2)$	$RR\,(1)$	$RR\,(2)$
USA	1947.2-1998.4	8.474	104.921	390.461	0.7998	2.170	0.516	227.707	54.166	0.157	0.071	16.448	7.144
AUL	1970.1-1998.4	3.913	120.667	861.785	1.9491	0.454	0.077	54.795	9.276	0.066	0.025	7.908	3.042
CAN	1970.1-1999.1	4.192	126.384	815.409	0.7682	0.514	0.106	64.974	13.352	0.104	0.030	13.180	3.830
FR	1973.2-1998.3	9.095	621.921	-4120.523	-11.7776	< 0	0.021	< 0	13.095	< 0	0.008	< 0	5.269
GER	1978.4-1997.3	8.661	202.189	124.037	0.4336	6.982	0.094	1411.776	19.105	0.220	0.031	44.463	6.307
ITA	1971.2-1998.1	4.475	122.925	-83.617	0.5064	< 0	0.085	< 0	10.497	0.151	0.029	18.526	3.530
JAP	1970.2-1998.4	5.688	$1.637{\times}10^4$	$79.268{\times}10^3$	$31.9498{\times}10^3$	0.007	0.0006	117.462	10.254	0.0006	0.0002	10.425	3.219
NTH	1977.4-1998.3	12.613	197.647	110.484	0.1306	11.416	0.153	2256.332	30.276	0.481	0.040	95.053	7.839
SWD	1970.1-1999.2	11.602	602.641	381.114	12.0994	3.044	0.045	1834.571	27.087	0.049	0.014	29.270	8.251
SWT	1982.2-1998.4	14.887	261.848	-1484.719	-0.4072	< 0	0.128	< 0	33.392	< 0	0.059	< 0	15.322
UK	1970.1-1999.1	9.188	52.531	315.500	0.3280	2.912	0.332	152.987	17.414	0.253	0.082	13.284	4.312
USA	1970.1-1998.4	7.004	129.569	586.176	1.1109	1.195	0.354	154.821	45.830	0.119	0.064	15.435	8.306
SWD	1920-1997	6.129	508.739	4391.133	389.5519	0.140	0.027	71.005	13.572	0.012	0.006	6.101	3.223
UK	1919-1997	8.791	36.629	769.054	3.6239	1.143	0.369	41.871	13.526	0.146	0.084	5.330	3.080
USA	1891-1997	6.241	57.400	1088.124	12.5198	0.574	0.250	32.920	14.346	0.066	0.045	3.811	2.579

$\overline{eR_e} = \overline{R_e} - \overline{R_f}$ in %, $cov\,(R_e, \Delta c)$ in $\%^2$ and $\int SED\,(R_e|c)\,dc$ in non annualized unit. The implied relative risk aversion is computed as $RR = AR \times \overline{c}$

7 Concluding remarks

We have proposed a new theoretical framework for solving the equity premium puzzle. Our contribution emphasizes the importance of measuring the dependence between agregate consumption and asset returns adequately. We use the concept of expectation dependence and show that taking into account higher degrees of risk dependence and orders of risk behavior than covariance and risk aversion is the key to understanding the variations in asset returns and the corresponding equity premia. Our empirical results confirm that using more general measures of risk dependence and risk behavior reduce the implicit measures of relative risk aversion and partly solve the equity premium puzzle.

Because the comparative Ross risk aversion is fairly restrictive upon preference, some readers may regard the comparative risk aversion results as a negative result, because no standard utility functions satisfy such condition on the whole domain. However, some utility functions satisfy comparative Ross risk aversion in some domain. For example, Crainich and Eeckhoudt (2008) and Denuit and Eeckhoudt (2010) assert that $(-1)^{N+1}\frac{u^{(N)}}{u'}$ is an appropriate local index of N^{th} order risk attitude. Nonetheless, some readers may think that because no standard utility functions satisfy these conditions, experimental methods to identify these conditions may need to be developed. Ross (1981), Modica and Scarsini (2005), Li (2009) and Denuit and Eeckhoudt (2010) provide context-free experiments for comparative Ross risk aversion. Denuit et al (2011) found a relationship between Ross risk aversion and one-switch utility function. More recently, Dionne and Li (2012) verified that decreasing cross risk aversion gives rise to the utility function family belonging to the class of n-switch utility functions. More research is needed in both directions to develop the theoretical foundations for C-CAPM. This paper takes a first step in that direction. We have proposed a new unified interpretation to C-CAPM, which we have related to the equity premium puzzle problem. Our results are important because C-CAPM shares the positive versus normative tensions that prevail in finance and economics to explain asset prices and equity premia.

8 Appendix A: Proofs of propositions

8.1 Proof of Proposition 3.1

(i): The sufficient conditions are directly obtained from (28) and (29). We prove the necessity using a contradiction. Suppose that $FED(\tilde{x}_{t+1}|c_{t+1}) < 0$ for c^0_{t+1}. Because of the continuity of $FED(\tilde{x}|y)$, we have $FED(\tilde{x}_{t+1}|c^0_{t+1}) < 0$ in interval [a,b]. Choose the following utility function:

$$\bar{u}(x) = \begin{cases} \alpha x - e^{-a} & x < a \\ \alpha x - e^{-x} & a \le x \le b \\ \alpha x - e^{-b} & x > b, \end{cases} \tag{49}$$

where $\alpha > 0$. Then

$$\bar{u}'(x) = \begin{cases} \alpha & x < a \\ \alpha + e^{-x} & a \le x \le b \\ \alpha & x > b \end{cases} \tag{50}$$

and

$$\bar{u}''(x) = \begin{cases} 0 & x < a \\ -e^{-x} & a \le x \le b \\ 0 & x > b. \end{cases} \tag{51}$$

Therefore,

$$p_t = \frac{E_t \tilde{x}_{t+1}}{R^f} - \beta \frac{1}{u'(c_t)} \int_a^b FED(\tilde{x}_{t+1}|c_{t+1}) F_{C_{t+1}}(c_{t+1}) e^{-c_{t+1}} dc_{t+1} > \frac{E_t \tilde{x}_{t+1}}{R^f}. \tag{52}$$

This is a contradiction.

(ii) (iii) and (iv): We can prove them using the same approach used in (i).

8.2 Proof of Proposition 3.4

(i): The sufficient conditions are directly obtained from (28), and (32). We prove the necessity by a contradiction. Suppose that there exists some c_{t+1} and c_t such that $\frac{u''(c_{t+1})}{v''(c_{t+1})} > \frac{u'(c_t)}{v'(c_t)}$. Because u', v', u'' and v'' are continuous, there exists a neighborhood $[\gamma_1, \gamma_2]$, such that

$$\frac{u''(c_{t+1})}{v''(c_{t+1})} > \frac{u'(c_t)}{v'(c_t)} \quad for \quad all \quad (c_{t+1}, c_t) \in [\gamma_1, \gamma_2], \tag{53}$$

hence

$$\frac{-u''(c_{t+1})}{-v''(c_{t+1})} > \frac{u'(c_t)}{v'(c_t)} \quad for \quad all \quad (c_{t+1}, c_t) \in [\gamma_1, \gamma_2], \tag{54}$$

and

$$-\frac{u''(c_{t+1})}{u'(c_t)} > -\frac{v''(c_{t+1})}{v'(c_t)} \quad for \quad all \quad (c_{t+1}, c_t) \in [\gamma_1, \gamma_2]. \tag{55}$$

If $F(x,y)$ is a distribution function such that $FED(\tilde{x}_{t+1}|c_{t+1})F_Y(y)$ is strictly positive on interval $[\gamma_1, \gamma_2]$ and is equal to zero on other intervals, then we have

$$p_t^u - p_t^v = \beta \int_{\gamma_1}^{\gamma_2} FED(\tilde{x}_{t+1}|c_{t+1})F_Y(y)[\frac{u''(c_{t+1})}{u'(c_t)} - \frac{v''(c_{t+1})}{v'(c_t)}] < 0. \tag{56}$$

This is a contradiction.

(ii): We can prove them using the same approach used in (i).

8.3 Proof of Proposition 3.6

(i) The sufficient conditions are directly obtained from (28), and (33). We prove the necessity using a contradiction. Suppose $FED_F(\tilde{x}_{t+1}|c_{t+1})F_{C_{t+1}}(c_{t+1}) < FED_H(\tilde{x}_{t+1}|c_{t+1})H_{C_{t+1}}(c_{t+1})$ for c_{t+1}^0. Owing to the continuity of $FED_F(\tilde{x}_{t+1}|c_{t+1})F_{C_{t+1}}(c_{t+1}) - FED_H(\tilde{x}_{t+1}|c_{t+1})H_{C_{t+1}}(c_{t+1})$, we have $FED_F(\tilde{x}_{t+1}|c_{t+1}^0)F_{C_{t+1}}(c_{t+1}^0) < FED_H(\tilde{x}_{t+1}|c_{t+1}^0)H_{C_{t+1}}(c_{t+1}^0)$ in interval [a,b]. Choose the following utility function:

$$\bar{u}(x) = \begin{cases} \alpha x - e^{-a} & x < a \\ \alpha x - e^{-x} & a \leq x \leq b \\ \alpha x - e^{-b} & x > b, \end{cases} \tag{57}$$

where $\alpha > 0$. Then

$$\bar{u}'(x) = \begin{cases} \alpha & x < a \\ \alpha + e^{-x} & a \leq x \leq b \\ \alpha & x > b \end{cases} \tag{58}$$

and

$$\bar{u}''(x) = \begin{cases} 0 & x < a \\ -e^{-x} & a \leq x \leq b \\ 0 & x > b. \end{cases} \tag{59}$$

Therefore,

$$
\begin{aligned}
& p_t^F - p_t^H && (60)\\
= \quad & \beta \frac{1}{u'(c_t)} \int_a^b [FED_H(\tilde{x}_{t+1}|c_{t+1}) F_{C_{t+1}}(c_{t+1}) - FED_F(\tilde{x}_{t+1}|y) F_{C_{t+1}}(c_{t+1})] e^{-c_{t+1}} dc_{t+1} > 0.
\end{aligned}
$$

(ii): We can prove them using the same approach used in (i).

9 Appendix B: Higher-order risks and higher order representative agents

In this section, we suppose $\tilde{x} \times \tilde{y} \in [a, b] \times [d, e]$, where a, b, d and e are finite. Rewriting $1^{th}ED(\tilde{x}|y) = FED(\tilde{x}|y)$, $2^{th}ED(\tilde{x}|y) = SED(\tilde{x}|y) = \int_d^y FED(\tilde{x}|t) F_Y(t) dt$, repeated integrals yield:

$$N^{th}ED(\tilde{x}|y) = \int_d^y (N-1)^{th} ED(\tilde{x}|t) dt, \ for \ N \geq 3. \tag{61}$$

Definition 9.1 *(Li 2011) If $k^{th}ED(\tilde{x}|e) \geq 0$, for $k = 2, ..., N-1$ and*

$$N^{th}ED(\tilde{x}|y) \geq 0 \ for \ all \ y \in [d, e], \tag{62}$$

then $\tilde{x}$ is positive N^{th}-order expectation dependent on $\tilde{y}$ ($N^{th}ED(\tilde{x}|y)$).

The family of all distributions F satisfying (62) will be denoted by $\mathcal{F}_N$. Similarly, $\tilde{x}$ is negative N^{th}-order expectation dependent on $\tilde{y}$ if (62) holds with the inequality sign reversed, and the totality of negative N^{th}-order expectation dependent distributions will be denoted by $\mathcal{G}_N$. From this definition, we know that $\mathcal{F}_{N-1} \subseteq \mathcal{F}_N$ and $\mathcal{G}_{N-1} \subseteq \mathcal{G}_N$ but the converse is not true. N^{th}-order expectation dependence is related to N^{th}-order central cross lower partial moment of $\tilde{x}$ and $\tilde{y}$ (See, Li (2011) for more details). Several recent studies in finance have focused on estimators of higher-order moments and comoments of the return distribution (i.e. coskewness and cokurtosis) and showed that these estimates generate a better explanation of investors' portfolios (see, Martellini and Ziemann (2010) for more details).

Definition 9.2 *Define $i^{th}ED_F$ and $i^{th}ED_H$, for $i = 1, .., N$, as the i^{th} expectation dependence under distribution $F(x, y)$ and $H(x, y)$ respectively. Distribution $F(x, y)$ is more first-degree* *expectation dependent than $H(x,y)$, if and only if $FED_F(\tilde{x}|y)F_Y(y) \geq FED_H(\tilde{x}|y)H_Y(y)$ for all $y \in [d,e]$. Distribution $F(x,y)$ is more N^{th}-order expectation dependent than $H(x,y)$ for $N \geq 2$, if $k^{th}ED_F(\tilde{x}|e) \geq k^{th}ED_H(\tilde{x}|e)$, for $k = 2, ..., N-1$ and*

$$N^{th}ED_F(\tilde{x}|y) \geq N^{th}ED_H(\tilde{x}|y) \ for \ all \ y \in [d,e]. \tag{63}$$

Suppose $(\tilde{x}_{t+1}, \tilde{R}_{t+1}, \tilde{c}_{t+1}) \in [\underline{\underline{x}}, \overline{\overline{x}}] \times [\underline{\underline{R}}, \overline{\overline{R}}] \times [\underline{\underline{c}}, \overline{\overline{c}}]$. We integrate the right-hand term of (27) by parts repeatedly until we obtain:

$$\begin{aligned} cov[u'(\tilde{c}_{t+1}), \tilde{x}_{t+1}] &= \sum_{k=2}^{N} (-1)^k u^{(k)}(\overline{\overline{c}}) k^{th} ED(\tilde{x}_{t+1}|\overline{\overline{c}}) \\ &\quad + \int_{\underline{\underline{c}}}^{\overline{\overline{c}}} (-1)^{N+1} u^{(N+1)}(c_{t+1}) N^{th} ED(\tilde{x}_{t+1}|c_{t+1}) dc_{t+1}, \ for \ n \geq 2. \end{aligned} \tag{64}$$

Then (23) and (24) can be rewritten as:

$$\begin{aligned} p_t &= \underbrace{\frac{E_t \tilde{x}_{t+1}}{R^f}}_{discounted\ present\ value\ effect} \underbrace{- \beta \sum_{k=2}^{N} k^{th} ED(\tilde{x}_{t+1}|\overline{\overline{c}}) [(-1)^{k+1} \frac{u^{(k)}(\overline{\overline{c}})}{u'(c_t)}]}_{higher-order\ cross\ moments\ effect} \\ &\quad \underbrace{- \beta \int_{\underline{\underline{c}}}^{\overline{\overline{c}}} N^{th} ED(\tilde{x}_{t+1}|c_{t+1}) [(-1)^{N+2} \frac{u^{(N+1)}(c_{t+1})}{u'(c_t)}] dc_{t+1}}_{N^{th}-order\ expectation\ dependence\ effect} \\ &= \frac{E_t \tilde{x}_{t+1}}{R^f} - \beta \sum_{k=2}^{N} k^{th} ED(\tilde{x}_{t+1}|\overline{\overline{c}}) AR^{(k)}(\overline{\overline{c}}) MRS_{\overline{\overline{c}}, c_t} \\ &\quad - \beta \int_{\underline{\underline{c}}}^{\overline{\overline{c}}} N^{th} ED(\tilde{x}_{t+1}|c_{t+1}) AR^{(k+1)}(c_{t+1}) MRS_{c_{t+1}, c_t} dc_{t+1} \end{aligned} \tag{65}$$

where $AR^{(k)}(x) = (-1)^{k+1} \frac{u^{(k)}(x)}{u'(x)}$ is the absolute index of k^{th} order risk aversion, and

$$\begin{aligned} & E_t \tilde{R}_{t+1} - R^f \\ &= \sum_{k=2}^{N} \underbrace{k^{th} ED(\tilde{R}_{t+1}|\overline{\overline{c}}) [(-1)^{k+1} \frac{u^{(k)}(\overline{\overline{c}})}{E_t u'(\tilde{c}_{t+1})}]}_{consumption\ cross\ moments\ effect} \\ &\quad + \int_{\underline{\underline{c}}}^{\overline{\overline{c}}} \underbrace{N^{th} ED(\tilde{R}_{t+1}|c_{t+1}) [(-1)^{N+2} \frac{u^{(N+1)}(c_{t+1})}{E_t u'(\tilde{c}_{t+1})}]}_{N^{th}\ degree\ expectation\ dependence\ effect} dc_{t+1} \\ &= \beta R^f \sum_{k=2}^{N} k^{th} ED(\tilde{R}_{t+1}|\overline{\overline{c}}) AR^{(k)}(\overline{\overline{c}}) MRS_{\overline{\overline{c}}, c_t} \\ &\quad + \beta R^f \int_{\underline{\underline{c}}}^{\overline{\overline{c}}} N^{th} ED(\tilde{R}_{t+1}|c_{t+1}) AR^{(k+1)}(c_{t+1}) MRS_{c_{t+1}, c_t} dc_{t+1}. \end{aligned} \tag{66}$$

Condition (65) includes three terms. The first one is the same as in condition (28). The second term on the right-hand side of (65) is called the "higher-order cross moments effect." This term involves β, the intensity of higher-order risk aversion, the marginal rates of substitution and the higher-order cross moments of asset return and consumption. The third term on the right-hand side of (65) is called "$N^{th} - degree$ expectation dependence effect," which reflects the way in which N^{th}-degree expectation dependence of risks affects asset price through the intensity of absolute N^{th} risk aversion and the marginal rates of substitution.

We state the following propositions without proof (The proofs of these propositions are similar to the proofs of propositions in Section 3, and are therefore skipped. They are however available from the authors.).

Proposition 9.3 *Suppose $F(x_{t+1}, c_{t+1})$ and $F(R_{t+1}, c_{t+1})$ are continuous, then the following statements hold:*

(i) $p_t \leq \frac{E_t \tilde{x}_{t+1}}{R^f}$ for any i^{th} risk averse representative agent with $i = 2, ..., N+1$ if and only if $(\tilde{x}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{F}_N$;

(ii) $p_t \geq \frac{E_t \tilde{x}_{t+1}}{R^f}$ for any i^{th} risk averse representative agent with $i = 2, ..., N+1$ if and only if $(\tilde{x}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{G}_N$;

(iii) $E_t \tilde{R}_{t+1} \geq R^f$ for any i^{th} risk averse representative agents with $i = 2, ..., N+1$ if and only if $(\tilde{R}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{F}_N$;

(iv) $E_t \tilde{R}_{t+1} \leq R^f$ for any i^{th} risk averse representative agents with $i = 2, ..., N+1$ if and only if $(\tilde{R}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{G}_N$.

Proposition 9.3 suggests that, for an i^{th}-degree risk averse representative agent with $i = 1, .., n = 1$, an asset's price is lowered if and only if its payoff N^{th}-order is positively expectation dependent on consumption. Conversely, an asset's price is raised if and only if it N^{th}-order is negatively expectation dependent on consumption. Therefore, for i^{th}-degree representative agents with $i = 1, .., N+1$, it is the N^{th}-order expectation dependence that determines its riskiness. The next two propositions and Corollary 9.6 have a similar general intuition when compared with those in Section 3.

Proposition 9.4 *Suppose u', v', u^{N+1} and v^{N+1} are continuous, then the following statements hold:*

(i) $p^u_t \geq p^v_t$ for all $(\tilde{x}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{F}_N$ if and only if v is more i^{th} risk averse than u for $i = 2, ..., N+1$;

(ii) $p_t^u \geq p_t^v$ *for all* $(\tilde{x}_{t+1}, \tilde{c}_{t+1}) \in \mathcal{G}_N$ *if and only if* u *is more* i^{th} *risk averse than* v *for* $i = 2, ..., N+1$*;*

Proposition 9.5 *(i) Suppose* $F(x_{t+1}, c_{t+1})$ *is continuous and* $E_t^F \tilde{x}_{t+1} = E_t^H \tilde{x}_{t+1}$*, then* $p_t^F \leq p_t^H$ *for all* i^{th} *risk averse representative agents with* $i = 2, ..., N+1$ *if and only if* $F(x_{t+1}, c_{t+1})$ *is* N^{th} *more expectation dependent than* $H(x_{t+1}, c_{t+1})$*;*

(ii) Suppose $F(R_{t+1}, c_{t+1})$ *is continuous, then for all* i^{th} *risk averse representative agents with* $i = 2, ..., N+1,$*,* $F(R_{t+1}, c_{t+1})$ *is more* i^{th}*-degree expectation dependent than* $H(R_{t+1}, c_{t+1})$ *if and only if the risk premium under* $F(R_{t+1}, c_{t+1})$ *is greater than* $H(R_{t+1}, c_{t+1})$*.*

Corollary 9.6 *For all* i^{th} *risk averse representative agents with* $i = 2, ..., N+1$*,* $E_t^F \tilde{x}_{t+1} \leq E_t^H \tilde{x}_{t+1}$ *and* $F(x_{t+1}, c_{t+1})$ *is more* N^{th}*-degree expectation dependent than* $H(x_{t+1}, c_{t+1})$ *implies* $p_t^F \leq p_t^H$*;*

10 References

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