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Taxing a Natural Resource with a Minimum Revenue Requirement

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Abstract:

The State may tax the extraction of a public natural resource in different ways. I consider the relative performances of a fixed fee, an *ad valorem* tax and a rent tax when the State must receive a minimal revenue for exploitation to take place. Taxing the resource may lower the probability that a firm will extract the resource. I show that the performance of each tax depends on the expected value of the resource: when it is high, the rent tax brings more revenue to the State; when it is low, the fixed fee is efficient while the rent tax does poorly. For an intermediate value, the *ad valorem* tax may bring the highest expected revenues among the three although it is always dominated by a hybrid tax that combines the latter two.

Keywords:

Rent, Royalties, Mining, Extraction Industry

Classification JEL: L71, H23, H25, Q34, Q38

1 Introduction

How the State taxes an extraction industry influences its development (Al Attar and Alo-mair 2005; Plourde 2010). The debate about which regime is the best goes back to Turgot (Crabbe 1985). The important distinction between the (Ricardian) rent and royalties was made a century ago (Fine 1990). Hotelling (1931) added the user cost consideration. Garnaut and Ross (1975) proposed the Resource Rent Tax (RRT) as a practical way for the State to extract the rent while preserving the firms' incentives to invest in valuable projects.

Current analyses of this issue (Polanyi 1966; Boadway and Flatters 1993; Otto 2006; Cawood 2010; Freebairn and Quiggin 2011) emphasize that a rent tax is more efficient and brings more revenues than an *ad valorem* tax. An *ad valorem* tax is advised only when the State's has a limited capacity to monitor the firms' costs and the output (transfer) prices, which are required to compute the economic rent¹.

The RRT is by no mean the only way for the State to (theoretically) achieve efficiency while capturing the rent. The two obvious alternatives are for the State to exploit the resource itself or to structure an auction and sell it wholesale to the highest bidder (Hendricks, Porter, and Tan 1993; Mead 1994). National exploitation requires an expertise that may not be economically obtained. The State may not be able to ensure the level of commitment required to transfer long-term extracting rights through an auction.² This is to be expected in a democratic country if the industry's activities annoy a significant portion of the electorate: a future administration might not considered itself linked by the decisions

¹Gaudet, Lasserre, and Van Long (1995) and Osmundsen (1998) explore the theoretical implications of asymmetric information within the context of exhaustible resource extraction industries.

²A common scheme is for the State to exploit the resource through a joint venture with a firm; but that involves nevertheless a partial transfer of rights.

of the past. The “political” risk of expropriation is also present when a royalty scheme is in place but it has a more adversely effect when a firm agrees to pay today an enormously amount of money for the exclusive right to exploit he resource in the future. As a matter of fact, the RRT is structured to minimize that risk by ensuring that the operating firm recoups its investment first.

The public’s concern for the soundness of mining policies has shifted over the years. Debates used to be all about jobs and benefits; they now increasingly focus on environmental externalities. Fine (1990) points out that most developed states appropriated the mineral rights in the XIXth century to *favour* the development of the extracting industry in a world where ownership mineral rights were too scattered to allow the construction of large mines. This is striking in the light of the recent expansion of the shale gas industry in the US: it expanded there, where the mineral rights are privately owned (and scattered), while it faced an effective barrage of public opposition in Quebec and in France where they are owned by the State.

There are many candidate explanations for these changes. People are more concerned about externalities than they were, either because of wealth effects, learning, or because the new projects are developed in more densely populated areas than in the past. The extracting industries are more capital intensive than they were a century ago, so relatively less economic surplus is likely to flow among the population through wages. The citizens are more empowered than they were and more likely to succeed if they want to block a project that they reject.

As the logic goes, a pure rent tax dominates an *ad valorem* tax because it does not adversely distort the incentive to invest in a marginally efficient project: since the tax is

levied on positive profits, it can't possibly turn a profitable mining project into an unprofitable one. By contrast, an *ad valorem* tax counts as an additional cost and its imposition may result in the shutdown of marginal but nevertheless economically profitable projects.

My point is that such marginal projects are precisely those that the public would not like to see undertaken. A project may pass a standard cost-benefit analysis test but nevertheless generates a lot of negative uncompensated externalities. These are the fuel that nourishes the grassroots bonfires. The public may welcome a project that brings a lot of revenues to the community but a blocking minority might form if the revenues are too small.³ In these conditions, an unsophisticated *ad valorem* scheme may yield a better outcome by discriminating against marginal projects.

I am not advocating here the adoption of *ad valorem* taxes. In fact, I shall show that the ranking between a fixed fee, an *ad valorem* tax and a pure rent tax depends on the value of the resource in comparison to that of other resources elsewhere to which the firms could devote their capital. The State will gather more revenues with a rent tax when the site promises high expected profits. By contrast, fiscal revenues will be higher with a fixed fee when the site is marginal. To the extent that an *ad valorem* tax lies in between, it might turn out a preferred compromise in some situations.⁴

The rent tax is *designed* to handle the sole economic concern of the industry, namely that exploitation should be undertaken whenever the market value of the resource is greater than its extraction cost. But the concerns of the public are no less important nor less “economic” in nature. It is no real surprise that the rent tax is not always the best tax to arbitrate

³This kind of “political” risk is different than that of a classic hold-up (the expropriation risk alluded above) that happens when the project turns out to be too profitable.

⁴Besides, (Rowse 1997) has computed that the welfare loss induced by an *ad valorem* tax is likely to be small.

these different concerns. My analysis thus rationalizes the fiscal regimes that have recently been proposed by the Government of Quebec in the mining industry (Gouvernement du Québec 2013) and by the Government of New-Brunswick in the oil and gas industry (New Brunswick Natural Gas Group 2012) which combine a rent tax plus an *ad valorem* component⁵.

Political considerations that shape the fiscal regime has been expressed before. Hotelling (1931) for one remarked that a conservationist would favour a distortionary *ad valorem* tax in order to slow down the exploitation of an exhaustible resource. More recently, Schantz (1994) has shown that environmentalists should favour higher royalties if that induces less extraction. But my point has more to do with incomplete contracts than with conservationism. The uncompensated externalities I consider sum to zero so they do not justify slowing down the industry. A natural extension of my analysis would be the development of more sophisticated compensating schemes likely to reduce the shadow cost of the political constraint induced by uncompensated externalities.

In the next section, I present a graphical explanation of how a fixed fee, an *ad valorem* tax and a rent tax discriminate among marginal projects. The illustration is done by assuming that all three taxes are set up to bring the same revenues at a given (profitable) price. Yet, the very problem of designing a fiscal regime lies in the fact that the State does not know *ex ante* what conditions will prevail *ex post*. To verify that the discriminating logic prevails under uncertainty, I develop in section 3 a simple model of the State's expected revenues. I compare the performance of each tax from the utility frontiers that they generate. The mathematical details of the model are relegated to the appendix.

⁵I end up with a combination a rent tax and a fixed fee but the political constraint I consider is rather simple.

2 The Three Taxes

The State has identified a natural resource extraction project and wishes to attract a specialized foreign firm to exploit it. The State cares only about the expected tax revenues it can gather from the resource. I focus on the effect of different tax schemes on the firm's decision to undertake the project. The capacity of the project is assumed to be fixed so that the firm cares only about its average profit by extracted unit. The output price, the tax and all others factors relevant to the project profitability are assumed fixed for the whole duration of the project.

I compare three possible tax regimes: i) a flat tax f per unit extracted; ii) an *ad valorem* tax vp that rises with the output price p and a pure rent tax $r(p - c)$ that rises with the rent $p - c$, where c denotes the (*ex post* observable) unit average extraction cost⁶. If the State knew what those factors were, the choice of a regime would be moot since it is always possible to adjust the rates f , v and r so that they all result in the same revenues f (as I do below). This level of revenues, although exogenous from the State's point of view, equalizes the demand and supply of projects on the international market. In Figure 1, the horizontal axis denotes the market price and the vertical axis denotes the State's revenue. If the State expects the price p_1 to prevail, then it may design the rates f , v and r so that the firm ends paying the competitive revenue f per unit extracted⁷.

These three tax schemes perform very differently though if the price departs from p_1 . With a flat tax, the firm pays f whatever the market price but it won't undertake the project if the price does not cover its unit extraction cost c plus the tax. As a consequence, the

⁶Constant returns to scale preclude any effect at the intensive margin. This assumption is not essential to the point I am making but relaxing it would complicate tremendously the analysis.

⁷In the appendix, I prove this and I show how to build the figure.

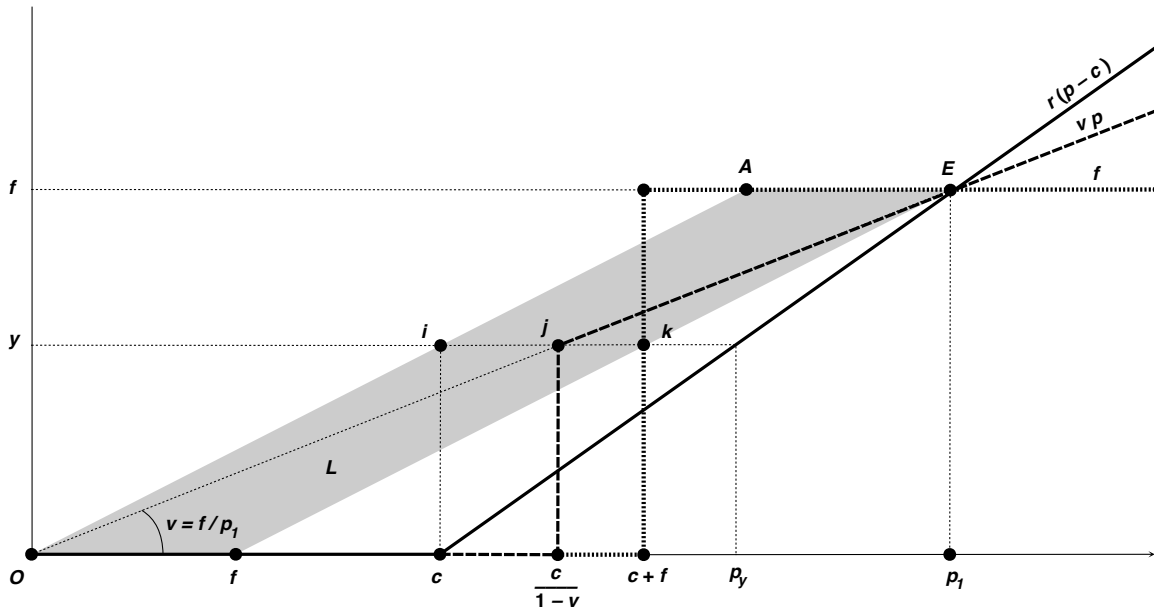


Figure 1: The Three Taxes.

revenues for the State (along the the dotted line) are zero for all prices below $c + f$ and they jump to f for all prices above that threshold. Likewise, the firm will agree to undertake the project and pay an *ad valorem* tax pv only if the price is high enough to leave it with a profit $(1 - v)p - c \geq 0$. This also results for the State in a discontinuous revenues function (the dashed line) that jumps at price $c/(1 - v)$ above which the firm makes a profit. Finally, the thick line represent the State's revenue under a pure rent tax which is zero if the price does not covers its cost c but rises continuously afterwards.

By design, all three taxes yield the same revenue in point E at price p_1 but the fixed fee performs better if the market price turns out to be lower. Likewise, should the price be above p_1 , the pure rent tax does better. The performance ordering of the regimes depends on the realized market price.

The fixed fee and the *ad valorem* tax bring higher revenues when the price and the tax revenues are low. The celebrated dominance of the pure rent tax comes from the fact that the firm will always undertake the project as soon as the project becomes economically efficient (when $p \geq c$). By comparison, an *ad valorem* tax discourages undertaking a marginally efficient project for prices within $[c, \frac{c}{1-v})$; the distortion is even higher with a fixed fee.

Now suppose that a blocking minority of people objects to the pursuit of a project if it does not bring at least y in revenues. To rationalize this assumption, assume that the unit extraction cost c includes all observed externalities except for a collection of *uncompensated externalities* diffused among the population: If the project is undertaken, each citizen suffers a personal random loss e . The average loss among the population is zero so that e counts as a benefit for some.

This concept of diffused uncompensated externalities yields a simple and sensible measure of political risk. Since c includes all other externalities and e has a zero average, the State is right from a costs-benefits perspective to only focus on revenues. Yet, since e is not compensated, some citizens shall disapprove of the project. A citizen who suffers e approves the project if it brings a revenue y to the State sufficient to justify his personal loss:

$$y - e \geq 0 \tag{1}$$

For simplicity, I assume that the distribution of personal losses is fixed although their allocation among the population may depend on the project. A project has the green light if it is approved by a *given proportion* of the population. For instance, if a 10% minority group of discontents is able to block the project, then the project should be approved by at least 90% of the population. With this political process, the options of undertaking and not undertaking the project are treated asymmetrically since that proportion needs not to be 50%. From (1), it follows that a project has the green light if it brings at least a given revenue y in the State's coffers⁸. Let t denotes the tax revenues from the project.

When such minimum revenue constraint must be met, a rent tax could induce a higher distortion than that of an *ad valorem* tax. Suppose the tax must bring at least y in revenue (see the figure), then the project would be undertaken as soon as $p \geq c/(1 - v)$ with an *ad valorem* tax but only when $p \geq p_y > c/(1 - v)$ with a rent tax. Such constraint would not affect the performance of a fixed fee (since $f > y$) but we have already evoked the heavy

⁸Denotes the distribution of e by F and assume that a project needs an approval rate of m ; then $\int_{y-e \geq 0} edF(e)de = F(y) = m$. So y is simply a non decreasing function of m . This story does not obey to the logic of compensation since y is not sufficient to cover all the losers who do not belong to the blocking minority. Yet, a more complicated story would yield the same result; namely that the higher the revenues brought by a project, the less likely it is to be blocked by discontents.

distortion induced by such fee. Hence, the *ad valorem* tax appears to balance the need to generate sufficient revenues when the project is undertaken (as the rent tax fails to do at low prices) without generating too much investment disincentives (as the flat tax would do).

3 A Model of Expected Revenues

The previous discussion about the advantage of an *ad valorem* tax needs a more formal examination: the proposition relies on the assumption that all the three taxes are set up to yield the same (competitive) revenue at price p_1 but the price is random. Such conjunction is unlikely if the price is random. For instance, under a pure rent tax, a firm will demand a lower tax rate if it expects that its investment opportunity will be blocked when the price is low. This implies that both f and v will have to be lower if the firm is to gather the same *expected* return under the three regimes. How the expected State's revenue behave under these circumstances remains to be shown.

In this section, I expand the model to account for uncertainty and I handle the firm's ex ante participation constraint more rigorously. A project has been identified but it won't be developed unless a firm takes interest and chooses to devote its resources to it. Its reservation profit π resumes this decision: the firm will develop the project only if it expects to make at least as much in profits doing so. I implicitly assume throughout that the expected returns of the project are no less than π . Hence, the *value* of a project is inversely related to π . Setting $\pi = 0$ means that exploiting the project would bring at least as much profit to the firm than it could get elsewhere. Setting π close to the expected returns of the project means that the project is marginal: there are other competitive projects elsewhere

to which the firm could devote its resources and expect the same profits.

Ex ante, the firm takes into account the risk that the project might not be profitable or get green-lighted. I assume that the minimal revenue y is known ex ante. Ex post exploitation thus depends on y and on the market and geological conditions (p and c) unknown at the time the regime is put in place. Exploitation takes place when the two following constraints are satisfied: i) the ex post profitability constraint (XP) —the firm’s profit should cover its tax liability—; ii) the minimal revenue requirement constraint (MR) —the tax brings at least y to the State.

$$p - c \geq t \quad (\text{XP})$$

$$t \geq y \quad (\text{MR})$$

Since the capacity of the project is fixed, the only economic problem here is that of the firm’s involvement in the project. Let $E(x)$ denotes the expected value⁹ of $\max\{x, 0\}$. The firm participates if

$$E(p - c - t) \geq \pi \quad (\text{PC})$$

and the State gathers $U = E(t)$. The State’s problem is to maximize U under the PC constraint. The classical analysis of this problem considers how the XP constraint affects that program or, I should say, how a particular tax regime handles that constraint. My contribution here is to add the MR political constraint.

Figures 2 and 3 resume my results. Both represent the utility frontiers that are achieved with each tax. A utility frontier graphs the set of points (π, U) that one obtains when con-

⁹This unusual notation yields here a less cluttered exposition. $E(x)$ is the expectation of x , conditional on $x \geq 0$, times the probability that $x \geq 0$.

straint PC is binding. Speaking loosely, the utility frontiers yields the maximum expected revenues U the State can get while ensuring that the firm gets π in expected profit. Three such frontiers are drawn in Figure 2, one for each tax: the grey diagonal is associated to the rent tax; the full line is associated to the *ad valorem* tax; and the dashed line is associated to the fixed fee.

I obtain Figure 2 by discarding the MR constraint. Depending on one's assumptions about the the distribution of (p, c) , the general form of the curves will vary but the following characteristics will remain:

The utility frontiers are downward slopping. This is true by construction since point (π, U) belongs to the frontier only if the constraint PC binds: If we relax that constraint (by reducing π), U must necessarily increase.

The three frontiers meet at the bottom right. Let $S^* = E(p - c)$ denotes the (first best) maximum total surplus that can be achieved. One way to achieve that level is to let the firm gathers all the surplus $p - c$, whenever $p - c \geq 0$, leaving none for the State. This is done by not taxing the firm (setting $f = v = r = 0$) in either regime. This point $(\pi, U) = (S^*, 0)$ clearly belong to all three frontiers.

The rent tax is not distortionary. That is, on every point (π, U) on the diagonal frontier, we have $\pi + U = S^*$. This is true because the rent tax has no effect on the XP constraint: the inequality $p - c \geq r(p - c)$ is always true for $r \in [0, 1]$. If (π, U) is on the frontier, then constraint PC binds so that $E(p - c - t) = (1 - r) E(p - c) = \pi$. Add $U = E(t) = r E(p - c)$ to get $\pi + U = S^*$.

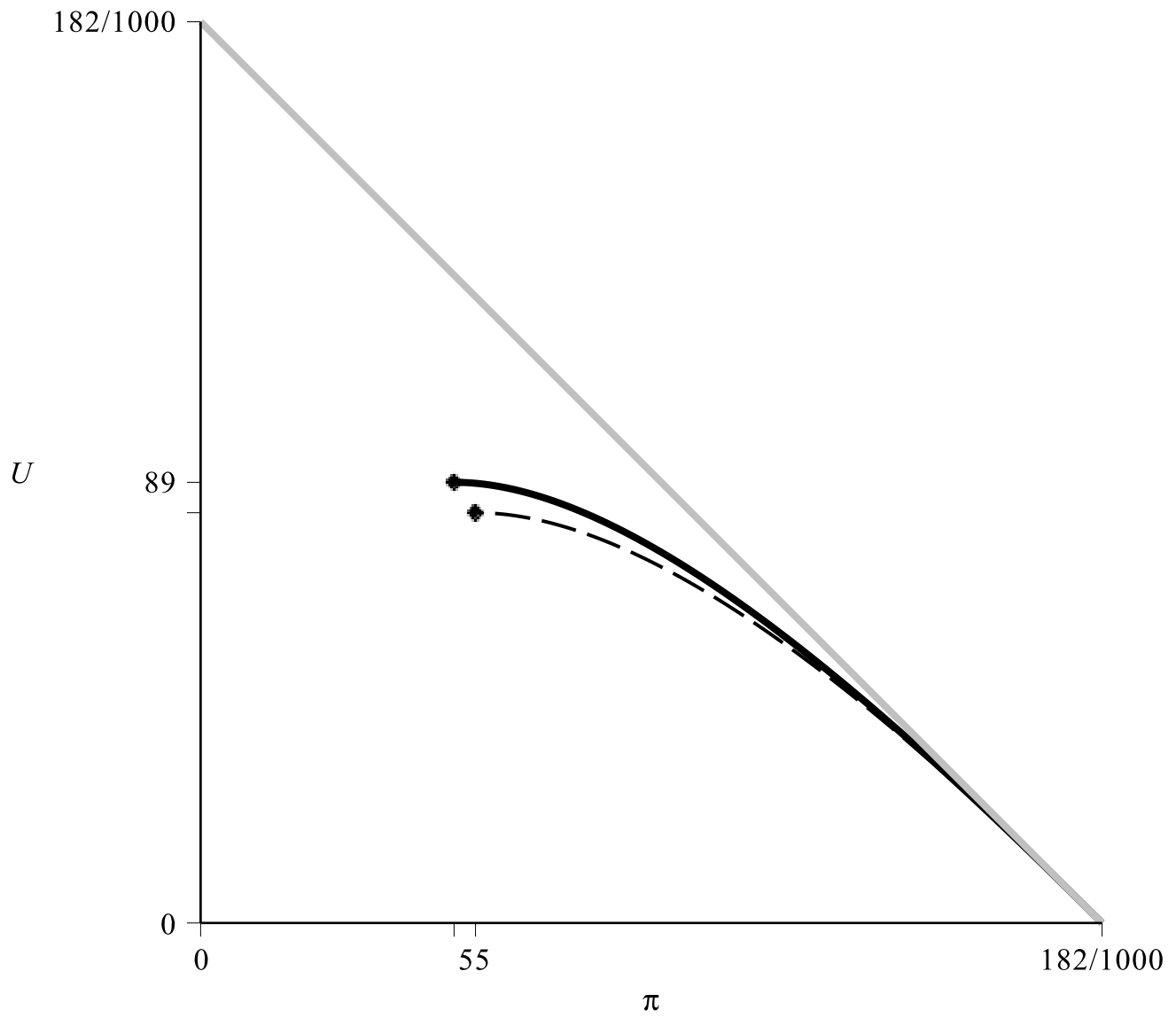


Figure 2: The Classical Utility Frontiers.

The *ad valorem* tax and the fixed fee induce a distortion. This implies that both associated frontiers are below that of the rent tax: given π , the State manages to gather less expected surplus with either an *ad valorem* tax or a fixed fee than with a rent tax. This is because increasing either tax reduce the probability that exploitation will take place. With an *ad valorem* tax, exploitation is undertaken whenever $p - c \geq vp \geq 0$; with a fixed fee, whenever $p - c \geq f > 0$. This implies that there is a positive probability to have a price p such that $vp > p - c > 0$ or $f > p - c > 0$ for which exploiting the resource would bring surplus yet the firm chooses not to do so because its low margin would not cover the tax¹⁰.

The fixed fee induces a more severe distortion than the *ad valorem* tax. After performing many numerical explorations, I could not find a single example where that was not the case: the result seems robust. Yet, I could not establish it formally (see the appendix).

Figure 2 thus tells the usual story: using a rent tax is the better option, followed by the *ad valorem* tax and by the fixed fee. When the project is marginal in comparison to other projects elsewhere, that is when $S^* = \pi$, this dominance relationship does not matter since the State won't be able to gather revenues anyway if it wants the project to remain competitive. But if the State has a promising project $S^* > \pi$, it will manage to secure higher expected revenues with a rent tax.

Consider now the incidence of the MR constraint on that pattern. Figure 3 illustrates the utility frontiers one obtains with the constraint MR added. The minimum revenue y is

¹⁰In the mathematical section, I show that these distortions are zero at the margin in $(S^*, 0)$ so that both frontiers are tangent to the diagonal at that point.

quite small in that example. In the figure, the most outward diagonal dotted line represent the efficient utility frontier obtain in Figure 2 with a rent tax. There, we have $\pi + U = S^*$. Such level is no longer achievable because the price p must now cover the firm's cost c and the minimal revenue requirement y . The second best expected level of surplus is then $S = E(p - c - y) < S^*$. The points on the second dotted diagonal reach that level. That both diagonals are very close one to the other indicates that y is very small in proportion of total surplus.

Yet, although y is small, it has a dramatic impact on the structure of utility frontiers. We observe the following characteristics.

The firm can't obtain the second best payoff. None of the three taxes manage to reach the bottom right point on the second best Pareto frontier. We can write the the second best payoff as

$$S = E((p - c)\chi(p - c \geq y))$$

where χ is the indicator function that returns 1 when the condition in parentheses is satisfied and 0 otherwise. In this example, we have $S = 180 < 182 = S^*$ (in thousands). The best expected profit the firm can expect with a minimal tax $t = y$ is $E(p - c - y)$; but since $E(p - c - y)$ is equivalent to $E((p - c - y)\chi(p - c \geq y))$,

$$E(p - c - y) < S$$

Yet, setting a fixed fee $f = y$ does yield a point on the second best frontier since

$$\pi + U = E([(p - c - y) + y]\chi(p - c \geq y)) = S$$

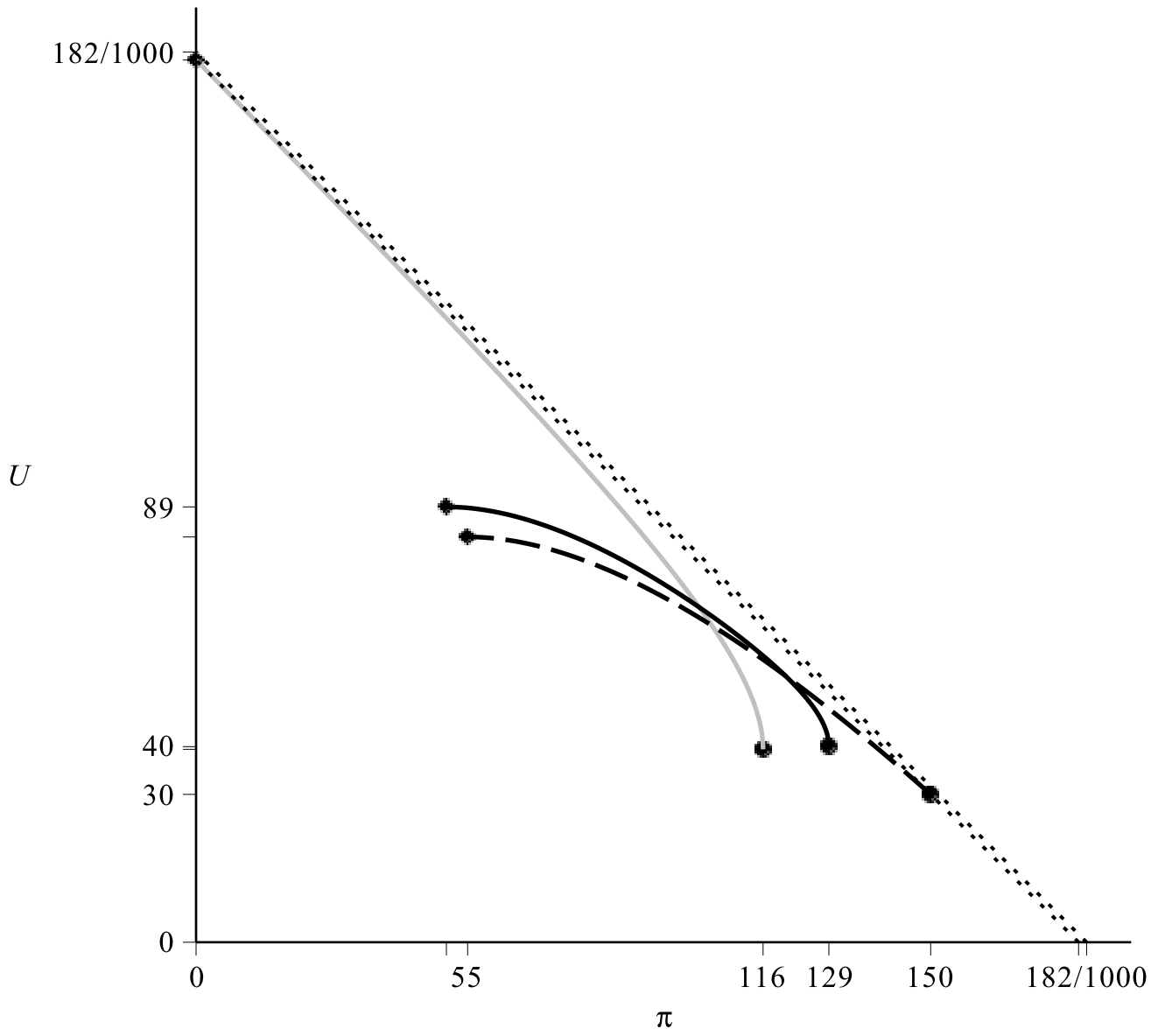


Figure 3: The Utility Frontiers With a Minimum Revenue Requirement.

This happens here at point (150, 30).

The State can obtain the second best payoff with a rent tax. This happens at point (0, 180) in the upper left corner of the Figure. If the firm has a zero reservation profit, the state will gather all the surplus by setting $r = 1$ but exploitation will take place only when the MR constraint is satisfied.

An *ad valorem* tax may be efficient. This happens here for the profit levels between 100 and 120 where the full black line associated to the *ad valorem* is above both the grey line (rent tax) and the dashed line (fixed fee).

With a minimal revenue requirement, *no tax scheme dominates the others*. Which one is the best depends on the PC constraint, that is on π . In particular, the previous dominance of the rent tax is absolutely not robust to the imposition of the MR constraint, even for a marginal small y .

Advocates of the rent tax will praise its ability to discriminate between marginal projects when ex post profitability is low. But this is exactly when the rent tax fails to satisfy the MR constraint. In a highly competitive environment, raising the tax rate r is a poor way to insure a minimal revenue for the State because it involves so much loss in profits for the firm when the profitability is high. Imposing a fixed fee is clearly then the better option.

Although the rent tax is a poor choice whenever the resource has a low value, the *ad valorem* tax is not very good either. Actually, in this very simple setup it is easy to find an hybrid family of tax schemes that may yield any point on the second best utility frontier. Combine both the rent tax and the minimal fixed fee into one: $t = \min\{r(p - c), y\}$ and adjust r to reach the desired point. Such tax will work because both constraints XP and

MR are always satisfied with such tax. Exploitation will take place whenever $p - c \geq y$ so that the second best surplus S will be realized.

4 Conclusion

I contend that the debate about the relative merits of different scheme of resources taxation is biased toward the appraisal of the rent tax. The usual argument establishes the efficiency of the rent tax in an ideal world and then declines the relative merits of the other taxes on the basis of transaction costs. The rent tax is better, one would say, but requires the observation of prices, quantities and costs. Since costs can be costly to monitor, an *ad valorem* tax may be advisable. And if both prices and costs are difficult to monitor, then a fixed fee could be the better option. According to this logic, when in addition extracted quantities are difficult to monitor, awarding the the resource wholesale through an auction is advisable. But it is well known that auctions are theoretically able to achieve efficiency when information is scarce so why not do so in the first place?.

By contrast, if we take political constraints seriously, the picture changes significantly. In this paper, I consider uncompensated externalities that translate into a minimum revenue requirement for the State. Which instrument is the best then depends on the relative competitiveness of the resource in comparison to others sources elsewhere. If the project is highly profitable, then the rent tax is advisable. If it is marginal, a fixed fee is a better option. In between, an *ad valorem* tax may bring higher expected revenues for the State although the real solution is to combine both the fixed fee and the rent tax in this particular simple case.

True, transaction costs matter but distributive issues matter even more. There are resources that are efficiently awarded wholesale when it is politically feasible to do so. But political constraints are pervasive. In Canada, awarding the public forest to private interests for long stretches of time is unfeasible. Near populated areas, granting private mining companies access to the underground proves to be more difficult than it was a century ago. International skirmishes in the fishery industry are common. So the resource remains public property and a tax scheme is put in place. But the political constraints are still the performance of a tax scheme will depend on its ability to handle them.

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A Drawing Figure 1

Follow these two steps:

1. Given p_1 and f , draw a parallelogram L with horizontal sides Of and AE (of length f).
2. Given c , draw a perpendicular up to i on the upper side of L and then an horizontal line that crosses points j on line OE and k on the lower side of L . Point j and k identify $c/(1 - v)$ and $c + f$ on the horizontal axis.

The proof of that last statement goes as follows. The slope of the sides of L is $f/(p_1 - f)$ so that points i, j and k are at $fc/(p_1 - f)$ on the vertical axis. Since j lies on vp , its horizontal coordinate x solves $vx = fc/(p_1 - f)$. Using $v = f/p_1$, this yields $x = c/(1 - v)$. Likewise, if x now denotes the horizontal coordinate of k , then

$$\frac{f}{p_1 - f} (x - f) = \frac{fc}{p_1 - f}$$

This yields $x = c + f$. Note that the inequality $c + f > c/(1 - v)$ is equivalent to the assumption $p_1 > c + f$:

$$p_1 - c - f > 0$$

$$p_1c + f(p_1 - c - f) = (p_1 - f)(c + f) > p_1c$$

$$c + f > \frac{p_1c}{p_1 - f} = \frac{c}{1 - f/p_1} = \frac{c}{1 - v}.$$

B The Mathematical Details

Let G and H denote respectively the marginal distributions of p and c . I assume that both variables are independent so that the pair (p, c) has $G(p)H(c)$ for distribution. Without much loss in generality, I restrict the support to the the unit square $[0, 1]^2$. The associated differentiable densities are denoted g and h . G and H where chosen within the remarkably flexible Kumaraswamy family¹¹

$$\{D_{a,b} : [0, 1] \rightarrow [0, 1] \text{ such that } D_{a,b}(x) = 1 - (1 - x^a)^b, \text{ with } a > 0, b > 0\}$$

The first best surplus is

$$S^* = \int_0^1 \int_0^p (p - c) dGdH$$

With a fixed fee f or an *ad valorem* tax v we get

$$S_F = \int_f^1 \int_0^{p-f} (p - c) dGdH$$
$$S_V = \int_0^1 \int_0^{(1-v)p} (p - c) dGdH$$

¹¹When $a = b = 1$, one obtains the uniform distribution.

Both taxes induce a distortion:

$$\frac{\partial S_F}{\partial f} = -f \int_f^1 h(p-f)dG < 0$$

$$\frac{\partial S_V}{\partial v} = -v \int_0^1 p^2 h((1-v)p)dG < 0$$

A rent tax always yields the maximum surplus S^* which is shared according to r between the State and the firm. With f and v , the expected revenues for the State are

$$U_F = f \int_f^1 \int_0^{p-f} dGdH = f \int_f^1 H(p-f)dG$$

$$U_V = v \int_0^1 \int_0^{(1-v)p} pdGdH = v \int_0^1 H((1-v)p)pdG$$

Both U_F and U_V start and end at zero on $[0, 1]$. Let U_F^* and U_V^* denote there maxima obtained in f^* and v^* .

Clearly, the expected profits are strictly decreasing in both f and v over $[0, 1]$. The utility frontiers are thus restricted to

$$P_F = \{(\pi, U) : \pi = \pi_F, U = U_F, 0 \leq f \leq f^*\}$$

$$P_V = \{(\pi, U) : \pi = \pi_V, U = U_V, 0 \leq v \leq v^*\}$$

where $\pi_F = S_F - U_F$ and $\pi_V = S_V - U_V$ denote the firm's expected profit.

To get the slope of these frontiers in $(S^*, 0)$, increase both tax at the margin while

ensuring that these increases result in the same reduction of profit $d\pi$ for the firm:

$$\frac{\partial \pi_F}{\partial f} df = \frac{\partial \pi_V}{\partial v} dv = d\pi$$

Then compute

$$\begin{aligned} dU_F &= \left. \frac{\partial U}{\partial \pi} \right|_{P_F} d\pi \\ &= \left(\frac{\partial S_F}{\partial f} - \frac{\partial \pi_F}{\partial f} \right) df \\ &= \frac{\partial S_F}{\partial f} df - d\pi \end{aligned} \qquad \begin{aligned} dU_V &= \left. \frac{\partial U}{\partial \pi} \right|_{P_V} d\pi \\ &= \left(\frac{\partial S_V}{\partial v} - \frac{\partial \pi_V}{\partial v} \right) dv \\ &= \frac{\partial S_V}{\partial v} dv - d\pi \end{aligned}$$

Since both $\frac{\partial S_F}{\partial f}$ and $\frac{\partial S_V}{\partial v}$ vanish in 0 (see above), both frontiers have the same slope

$$\frac{dU_F}{d\pi} = \frac{dU_V}{d\pi} = -1$$

than that of the first best.

Unfortunately, I could not prove that P_F decreases more rapidly than P_V , or that U_V reaches a higher maximum than U_F for that matter; but neither could I find a single example where that was not the case within the Kumaraswamy family. I thus report that P_V dominates P_F in Figure 2.

Constraint MR alters the payoffs in the following way. The second best surplus is $S = S_F(y)$. S_F is unchanged except that it is now defined only for $f \geq y$. Likewise, S_V

becomes

$$S'_V = \int_{y/v}^1 \int_0^{(1-v)p} (p-c)dGdH$$

and is only defined for $v \geq y$. The rent tax does not generate a surplus S_R independent of its use anymore:

$$S_R = \int_{y/r}^1 \int_0^{p-y/r} (p-c)dGdH$$

and is also only defined for $r \geq y$. U_V and U_R become

$$U'_V = v \int_{y/v}^1 H((1-v)p)pdG$$

$$U_R = r \int_{y/r}^1 \int_0^{p-y/r} (p-c)dGdH$$

Comparing S_F and S_R , we see that both taxes generate the same amount of surplus as f and v span $[y, 1]$ (but they produce different allocations of that surplus). U_F is unchanged but restricted to $f \geq y$.

Expected profit with an *ad valorem* tax or a rent tax changes a lot with the MR constraint: instead of decreasing with the tax level, both S'_V and U'_V , as well as S_R and U_R , are zero on the boundary of $[y, 1]$. It follows that the expected profit is zero as well in these points. To see why the expected profit would be zero in $v = y$, consider that the MR constraint would then be satisfied whenever $pv \geq y$, that is when $p \geq 1$; which is a null event. Likewise, if $r = y$, the MR constraint is satisfied when $r(p-c) \geq y$, that is when $p-c \geq 1$; which is also a null event.

With an *ad valorem* tax, we need to identify the interior point v_0 where the profit is maximized since, assuming that v_0 is a unique global maximum, the point $(S'_V(v_0) - U'_V(v_0), U'_V(v_0))$ is an end point of the utility frontier. To find the point at the other end, we look for v_1 where U'_V is maximized. The existence of these points is ensured but their unicity is not since it is not clear whether these functions are quasiconcave on $[y, 1]$. I draw the frontier by spanning $[\min\{v_0, v_1\}, \max\{v_0, v_1\}]$ (which could result in a single point).

Since U_r clearly increases with r , I had only to find the point r^* that maximizes profit and to span $[r^*, 1]$ to get the utility frontier associated with the rent tax.

The example reported in figures 2 and 3 is associated to the distribution functions $G_{3,3}$ and $H_{2,3}$. The minimum revenue y was set to 5% of the maximum profit ($y = \frac{1}{20}$). In the next page, I provide the graphs for the expected surplus and expected revenues, with and without the constraint MR, associated to this example.

Upper-Right: The price (full line) and cost (dotted line) densities (the vertical lines indicate the means).

Center-Left: The three taxes without MR. Dashed: fixed fee. Full: *ad valorem*. Grey: rent tax. The upper curves are the expected surplus S_F , S_V and S^* (the latter being constant). The lower curves are the expected revenues U_F , U_V and rS^* (the vertical dotted line indicate their maxima). The thick parts are used to draw the utility frontiers in Figure 2.

Center-Right: The fixed fee with MR. S_F and U_F are drawn on $[y, 1]$. The horizontal dotted line indicates S .

Bottom-Left: The *ad valorem* tax with MR. S'_V and U'_V are drawn on $[y, 1]$. The first vertical dotted line indicate the tax rate v_0 where the expected profit is maximized (where S'_V and U'_V have the same slope).

Bottom-Right: The rent tax with MR. S_R and U_R are drawn on $[y, 1]$.

