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Whither the Progressive Tax ?

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Abstract:
The progressive wage tax is the instrument commonly used by democracies to fund public expenditures. Yet it still divides opinions about its impact on skill formation. We develop a general equilibrium model to analyze this impact, in the context of uncertain return on higher education. We show that the quantitative impact on skill formation of switching from the flat to the progressive tax varies with the level of efficiency with which higher education imparts graduates with suitable skills. This impact is negative when the level of efficiency of higher education is low and positive when it is high.

Keywords: Tax progressivity, higher education, skilled formation, capital inflows, general equilibrium

JEL Classification: F21, F22, H20, J60

1. Introduction

There is a consensus that higher education graduates must bear the cost of their education since there are private returns to the beneficiaries.[4] There is also a consensus that society must share the cost of investing in higher education because enrolment yields social returns that transcend the individuals who directly benefit from it (Moretti 2004). This double consensus explains the pervasiveness, in advanced democracies, of cost-sharing models for funding higher education whereby the cost of education is shared between the beneficiaries and society at large (Barro and Sala-i-Martin 1995; Barro 1998; Garcia-Penalosa and Wälde 2000; Lochner and Moretti 2004; Bassanini and Scarpenta 2001; Greenaway and Haynes 2003; Barr, 2004; Riddell, 2005). However, for a number of these advanced democracies, including Canada and the United States, the general consensus ends there. It does not extend to the thorny issue of how the state should collect the revenue needed to finance its share of the cost of higher education. Instead, there are divided opinions over which, of the progressive wage tax system, or the flat wage tax system, is more likely to create an economy with abundance of skilled workers.

On the one hand, there is the presumption that unlike the flat tax, the progressive tax system cannot create the abundance of skills that underlies economic prosperity, because the higher tax rate levied on the skill-premium in wage acts as a disincentive to invest in skills (Heckman, Lochner and Taber 1998). On the other hand, there is the view that the alternative flat tax system would lead to a regressive income distribution whereby low-income families that cannot afford to send their children to college subsidize middle-income and higher-income families who can (Garcia-Penalosa and Wälde, 2000). These differences of opinions underscore the importance of the requirement that the choice of a tax system for funding public expenditures be based on a clear understanding of its impact. Yet missing in the existing literature on public financing of higher education has been a systematic assessment of how the tax system affects participation in higher education when (i) the skills graduates gain are imperfectly aligned with those employers need, and (ii) there is international mobility of (physical) capital. This article fills this gap, by contrasting the

[4] By higher education we mean tertiary education leading to an under-graduate or graduate degree.

impacts of various tax proposals indexed by their degree of tax progressivity.

Under the flat wage tax system (i.e., one with tax progressivity of degree zero), the burden of financing access to a higher education is equally shared by all wage earners, as this has to do with the tax rate. In contrast, under tax systems with a positive degree of progressivity, graduates who get a greater financial return from their higher education see the increment in their earning taxed at a higher rate depending upon the chosen degree of progressivity, thereby raising their share of the burden relative to the other wage earners. We develop a simple general equilibrium model of enrolment in higher education and labor supply, with uncertain return on higher education to contrast the performances of these wage tax proposals. In our model, prospective students are aware of the risk that the skills they gain through the higher education system are misaligned with those employers need. This is the only source of uncertainty for the return on higher education.[5]

We model an individual decision to pursue higher education as one that is determined jointly by education-related costs, family wealth, and the financial return on the skills gained. In the absence of tuition subsidies, enrolment in higher education tracks family wealth, thereby undermining the principle of equal opportunities. What is more, uncertainty about the return on higher education raises the opportunity cost of education for risk-averse individuals, and thus may increase inequality, as only individuals from rich family backgrounds can draw on family wealth to mitigate the effects of risk-aversion. Poor, risk-averse individuals would miss out, as they fear that participation in higher education would leave them saddled with debt.

Our general equilibrium model highlights two channels through which distortionary taxation impacts individuals' decision on whether or not to pursue higher education. On

[5]Our assumption that the misalignment between skills gained through a higher education system and those employers need creates a risk for prospective students is consistent with reality. The higher education system in Canada is a case in point. See Margaret Wente, "Access or quality – our universities can't have both", published in *The Globe and mail*, October 31st, 2012; Tavia Grant, "Canada's labour pain: 1.3 million jobless, but not enough skills", published in *The Globe and Mail* , April 1, 2013; and Gwyn Morgan, "Radical re-financing proposal would ease skills shortage", *The Globe and Mail*, Monday, April 15, 2013). Indeed, despite having one of the highest rates of university enrolment in the world, in 2011 Canada ranked second to last among OECD countries in producing graduates with suitable skills for the labour market, suggesting that the qualifications obtained do not always match the needs of the labour market (OECD 2011).

the one hand, a wage tax reduces the reward from being skilled, and thus tends to create a disincentive to pursue costly higher education as a skill-imparting mechanism. A progressive wage tax system reinforces this disincentive in two related ways. First, because the increase in earnings induced by these skills pushes individuals into higher tax brackets, it tends to reduce the return on skills gained through higher education, which creates a disincentive for participation. Second, it tends to raise the importance of family wealth for participation in higher education because the induced reduction in earnings tends to restrict gains from participation only to prospective students who require a minimal level of debt for participation. On the basis of this channel alone, a flat wage tax system would dominate a progressive wage tax system on efficiency grounds, as in Heckman, Lochner and Taber (1998). But on the other hand, when a fraction of the tax revenue collected through wage taxation is invested in enhancing access to higher education, for example, through tuition subsidies, a second channel for the impact of wage taxation on the decision to pursue higher education opens up, working through the level of tax revenue collected. To the extent that the progressive tax dominates the flat tax from the view point of revenue generation, it may be better at extending access to prospective students from poor family backgrounds. Yet tax revenue does not only depend on the tax rate; it also depends on the allocation of taxpayers across wage brackets, and its impacts (both direct and indirect) on wages. Because wage taxes are distortionary, they impact the allocation of workers across wage brackets through the decision on whether or not to pursue higher education. These complex feedback between taxation, the allocation of workers across wage brackets, wages, and tax revenue justify our use of a general equilibrium model.

We structure the interactions between the two channels of the impacts of wage taxation around three key features. First, we consider an aggregate production function such that abundance of capital raises the productivity of both skilled and unskilled workers. Second, capital is internationally mobile, and its productivity rises with the abundance of labor inputs. Third, the structure of this aggregate production function is selected to ensure that the earnings of skilled workers are always higher than those of unskilled workers. This has two effects. On the one hand, because skilled workers are more productive than their unskilled counterparts, their availability becomes a lever of capital inflows. On the other hand, because skilled workers earn more, they become key contributors to tax revenue. The more there are skilled workers, the higher the tax revenue. We show that a progressive wage tax reinforces this revenue effect. We supplement our analysis with a quantitative assessment of a tax reform underlaid by a switch from a flat to a progressive tax. To do so we use numerical methods to compute the degree of wage tax progressivity that maximizes the size of the skilled labor force and the level of capital inflows. We show that the quantitative impact on skill formation and capital inflows of switching from the flat to the progressive tax varies with the level of efficiency with which higher education imparts graduates with suitable skills. This impact is negative when the level of efficiency of the higher education system is low and positive when it is high. In other words, when the level of efficiency of the higher education system is low, the flat tax dominates the progressive tax as a mechanism for enhancing skill formation and capital inflows. The reverse is true, however, when this level of efficiency is sufficiently high.

Higher education financing is a highly debated issue in advanced democracies such as Canada and the United States, and both empirical and theoretical studies of this issue exist (e.g., Coelli 2009; Caponi and Plesca 2009; Burbidge, Collins, Davies, and Magee 2012). However, we view our work as complementary to existing theoretical works that rely on the discipline of a general equilibrium model to analyze the effect of public policy on enrolment in higher education and labor supply. Examples of such works include Heckman, Lochner, and Taber (1998), Garcia-Penalosa and Wälde (2000), Caucutt and Kumar (2003), and Hanushek et al. (2003). Heckman, Lochner, and Taber (1998) build a dynamic general equilibrium model to explore the impact of alternative tax systems on human capital formation. In particular, they contrast the performances of three different tax policy proposals namely, a progressive wage tax, a flat wage tax, and a flat consumption tax. In their model, a wage tax reduces marginal returns on schooling, but has no effects on the marginal costs of schooling. Unlike the two other tax systems considered, the progressive wage tax reinforces the negative effects on the marginal returns to education, because the increase in earnings induced by schooling pushes individuals into higher tax brackets.

In their model, this is the only channel through which tax policy affects human capital formation. We complement Heckman, Lochner, and Taber (1998), through the addition of a second channel for the effect of wage taxes on the decision to pursue higher education, and the level of efficiency with which higher education system imparts graduates with skills suitable for the labor market. In particular, we allow tax revenue to be invested in higher education system in the form of tuition subsidies, based upon documented evidence that governments in advanced democracies use tax revenue to fund access to education.[6]

Garcia-Penalosa and Wälde (2000) contrast the performances of a flat tax, loan schemes, and a graduate tax in a general equilibrium model of investment in higher education and labor supply. They show that a flat tax is dominated by the other funding mechanisms on the basis of equity-efficiency trade-offs. They also show that when the return on higher education is uncertain, and prospective students are risk-averse, a graduate tax out-performs the loan schemes, making it the best funding mechanism for public financing of higher education. It is not clear, however, how this graduate tax will be implemented in reality. In addition, Garcia-Penalosa and Wälde (2000) abstract away from international capital mobility, so that unskilled workers in their model do not benefit from the abundance of skilled workers, which explain why in their model, these workers are exempted from contributing to education financing. In our model, unskilled workers benefit from capital inflows because abundance of capital raises labor productivity. Yet, as we show in our model, it is abundance of workers with suitable skills which enhances capital inflows. This combines with the positive effect of capital inflows on unskilled labor wage to justify taxing the unskilled wage.

Caucutt and Kumar (2003) develop a dynamic general equilibrium model of college attendance and labor supply to contrast the performances of three alternative higher education policies, including (i) a tax and subsidy scheme reflecting equality of opportunities, (ii) a policy of maximizing the fraction of people with a college education, (iii) the provision of merit-based aid to the poor. As in Garcia-Penalosa and Wälde (2000), higher education in Caucutt and Kumar (2003) only benefits those who successfully pursue it and there is

[6]See, for example, Department of Finance Canada, 2012

no international capital mobility. They show that the tax-subsidy scheme underpinning the equal opportunities policy is dominated by the other two higher education policies because the former maximizes the equity-efficiency trade-off. In particular, they argue that increased subsidies end up attracting inframarginal students (those with lower abilities) to college, thereby causing an increase in the dropout rate— a waste of education resources. Our article, however, is not primarily about how to best use public funds; instead, it is more about how to best raise such funds to finance access to higher education when taxes are distortionary.

Hanushek *et al.* (2003) also use a general equilibrium model of college attendance and labor supply to compare three redistribution schemes, including (i) tuition subsidies for higher education, (ii) a negative wage tax, (iii) a wage subsidy. Like Garcia-Penalosa and Wälde (2000), and Caucutt and Kumar (2003), Hanushek *et al.*(2003) emphasize heterogeneous abilities and tie the level of uncertainty underlying returns to education to agents' innate abilities.[7] In this context, they show that tuition subsidies are dominated by other redistribution schemes on the basis of the equity-efficiency trade-off criterion. They abstract away from physical capital as an input in the aggregate production function, as well as from international capital movements. Therefore, as in Heckman, Lochner, and Taber (1998), Garcia-Penalosa and Wälde (2000) and Caucutt and Kumar (2003), there is no role for the abundance of workers with suitable skills to benefit unskilled workers.

The remainder of this paper is structured as follows. Section 2 presents the model, which is solved numerically in section 3. Finally, section 4 concludes the paper.

[7]By abstracting from heterogeneous abilities among prospective students, in our model, we do not at all pretend that abilities do not matter for college performance. Instead, our modelling strategy relies on two important facts. First, most colleges have admission criteria emphasizing abilities to pursue college studies. To restrict admission of low ability-students, the government can simply mandate a threshold test score below which an individual is not admissible to college, as is the case in France and England. Second, average college tuition differs from one country to another. Some countries like Canada and Germany have relatively low average tuitions when compared to the United States. However, there is no evidence that the average college student in Germany or Canada has lower ability than the average college student in the United States.

2. Preliminaries

The model emphasizes the role played by the tax system on individuals' decisions to participate in higher education (hereafter, HE for short). The basic structure of the model is a one-period general equilibrium with competitive markets, inspired by Hanushek *et al.* (2003). Workers, in total size normalized to unity, have all completed secondary education and all have the ability to pursue higher education. There is no learning-by-doing. Participation in HE is the only mechanism for acquiring productive skills. A worker may decide to invest in skills by pursuing HE or she may decide to look for a job as an unskilled worker, when the labor market opens. Training through HE is instantaneous in this one-period economy, and takes place prior to the opening of the labor market. We introduce a cost-sharing model whereby the beneficiaries of HE bear the costs of their education, albeit with the help of taxpayers' contributions. We draw from McPherson and Shapiro (1991), Garcia-Penalosa and Wälde (2000), Keane and Wolpin (2001), Caucutt and Kumar (2003), and Lochner, Belley, and Frenette (forthcoming) in recognizing that higher education financing has a strong intergenerational connection to it. To capture this feature of high education financing, we assume that each worker has access to a family fund of size b (for bequest), on which she can draw to help defray HE costs (including tuition, fees, school supplies, lodging, and other living expenses), but only if she decides to pursue HE.

Prior to entering the labor market, workers make an optimizing choice about the pursuit of HE for skill-acquisition based upon the tuition charged, their endowment of family fund, and expected wages. The decision on whether or not to pursue HE involves uncertainty because the skills gained through HE may or may not be aligned with those employers need, something which, in our model, is outside the control of prospective students. More formally, with an exogenous probability $q \in (0, 1)$, a worker who participates in HE prior to entering the labor market gains skills that are aligned with those employers need, and thus gets a skilled job. But with the converse probability $1 - q$, the education received is misaligned, in which case she joins the unskilled labor force. We assume that the market to insure against the misalignment risk is missing. We interpret the probability q as the level of efficiency with which the HE system imparts workers with skills suitable for the labor market.

2.1. HE Cost and Student Debt

For each worker, and in the absence of subsidies, the cost of education is $e \in \mathbb{R}_+$. We analyze workers' aggregate decisions on the pursuit of risky HE under various alternative tax systems indexed by their degree of progressivity $\theta \in \left[0, \bar{\theta}\right]$. By convention, we denote as tax system θ, the tax system whose degree of progressivity is θ. When $\theta = 0$, we have the flat wage tax system. For simplicity, we limit heterogeneity of labor income to two types only, skilled and unskilled.

Assume that the government subsidizes tuition at a rate $z \in [0, 1]$, as is the case in most advanced democracies such as Canadian Provinces. Therefore given the tax system θ chosen by the government, for an agent who decides to pursue HE, the subsidized cost of HE is given as follows:

$$E_\theta := (1 - z_\theta)\, e. \tag{2.1}$$

The subsidy rate z_θ is determined endogenously by the size of tax revenue under a balanced budget legislation, and given the degree of progressivity of the tax system, θ.

To cover her education costs, a typical student relies on two different sources. The first source is the family fund, $b \geq 0$, and, if not sufficient, a second source is a student loan, extended interest free (just for simplicity) by the government, and repayable after graduation. For each tax system θ implemented by the government, we define the level of student debt, $d_\theta \in \mathbb{R}$, as the difference between the (subsidized) level of education costs, E_θ, and the financial resources provided by the family, b:

$$d_\theta = E_\theta - b.$$

The level of this student debt is a determining factor of an individual's decision whether or not to pursue risky HE.

2.2. Individual Actions and Payoffs

A worker b is one who has an endowment of family fund, b. Initially, individuals differ only with respect to these endowments. They are distributed across levels of family fund according to a cumulative probability distribution function, Ψ, with strictly positive density, $\psi(b) := \Psi'(b)$, over a compact support $[0, e]$, where e is the per capita cost of education. That e is the upper bound of the compact support for family funds means that no worker in this environment has an endowment of family fund bigger than the pre-subsidy per capita cost of education, e. Workers have identical preferences over the quantity consumed of the numeraire, c. The common utility function representing these preferences is given by:

$$U(c) := \ln c. \tag{2.2}$$

At the opening of the labor market, after workers' skill-investment decisions are made, an individual is either a skilled worker ($i = s$) or an unskilled worker ($i = u$). A worker's after-tax income depends on her skill-investment decision, $a \in \{0, 1\}$, and the tax system, θ, implemented by the government to raise tax revenue.

Denote as $R(a, b, i, \theta)$, the realized after-tax income of worker b who, having made the skill-investment decision $a \in \{0, 1\}$ prior to entering the labor market, ends up with a skill-status $i \in \{s, u\}$, when the government's chosen degree of tax progressivity is $\theta \in \left[0, \bar{\theta}\right]$. A worker b who makes the decision $a = 0$ has skill-status $i = u$ with certainty, and thus earns an after-tax wage given by:

$$R(0, b, i, \theta) := (1 - \tau)\,\omega_\theta^u,$$

where ω_θ^u denotes the unskilled labor wage under the tax system θ. In contrast, a worker who makes the decision $a = 1$ has skill-status $i = u$, with probability $1 - q$, in which case her realized labor income is

$$R(1, b, u, \theta) := (1 - \tau)\,\omega_\theta^u;$$

but with the converse probability, q, she has skill-status $i = s$, in which case her after-tax wage is

$$R(1, b, s, \theta) := (1 - \tau)\,\omega_\theta^u + [1 - (1 + \theta)\,\tau]\,(\omega_\theta^s - \omega_\theta^u)\,, \tag{2.3}$$

where ω_θ^s denotes the skilled labor wage, and $\omega_\theta^s - \omega_\theta^u$, the incremental wage from having suitable skills under the tax system θ. Expression (2.3) states that, under the flat wage tax (i.e., $\theta = 0$) the after-tax wage of skilled workers is simply $R(1, b, s, 0) := (1 - \tau)\,\omega_0^s$. As long as $\theta > 0$, the incremental wage from having suitable skills is taxed at a higher rate, $(1 + \theta)\,\tau$, reflecting a marginal tax rate of θ.

A worker's after-tax income, which we specify fully further below, determines her budget constraint:

$$c \leq R(a, b, i, \theta) - a d_\theta, \tag{2.4}$$

where d_θ denotes the level of student debt under the tax system θ,

$$R(a, b, i, \theta) := \begin{cases} (1 - \tau)\,\omega_\theta^u & \text{if } a = 0 \\ \\ R(1, b, i, \theta) & \text{if } a = 1 \end{cases}\,. \tag{2.5}$$

A worker b thus has essentially one important decision to make namely, whether or not to pursue HE prior to entering the labor market. Combining (2.2), (2.3), (2.5), and (2.4) yields worker b's realized payoff from taking the decision a as follows*:*

$$\bar{U}(a, b, i, \theta) = \begin{cases} \ln(1 - \tau)\,\omega_\theta^u \equiv B_\theta & \text{if } a = 0 \\ \\ \ln[(1 - \tau)\,\omega_\theta^u - d_\theta] \equiv C_\theta & \text{if } a = 1 \text{ and } i = u \\ \\ \ln[(1 - \tau)\,\omega_\theta^u + [1 - (1 + \theta)\,\tau]\,\Delta_\theta - d_\theta] \equiv A_\theta & \text{if } a = 1 \text{ and } i = s \end{cases}\,, \tag{2.6}$$

where

$$\Delta_\theta := \omega_\theta^s - \omega_\theta^u \tag{2.7}$$

is interpreted as the skill premium in wage under the tax system θ.

A worker b with a realized payoff $\bar{U}(0, b, i, \theta) \equiv B_\theta$ is one who elected to pass on the opportunity to pursue HE prior to entering the labor market. There are $1-n_\theta$ such workers, where n_θ denotes the measure of HE graduates, under the tax system θ. A worker b with a realized payoff $\bar{U}(1, b, s, \theta) \equiv A_\theta$ is one who elected to pursue HE prior to entering the labor market and effectively became a skilled worker upon graduation. By the application of the law of large numbers, there are $\bar{S}_\theta := qn_\theta$ such workers. In contrast, a worker b with a realized payoff $\bar{U}(1, b, u, \theta) \equiv C_\theta$ is one who chose to pursue HE prior to entering the labor market, but was unfortunate not to acquire suitable skills upon graduation, and thus had to settle for an unskilled job. The total measure of such workers is $(1-q)\,n_\theta$, again by the application of the law of large numbers. Therefore, the size of the unskilled labor force under the tax system θ is $1 - n_\theta + (1-q)\,n_\theta = 1 - qn_\theta$. A worker b's decision process on the pursuit of HE is summarized in Figure 1 below:

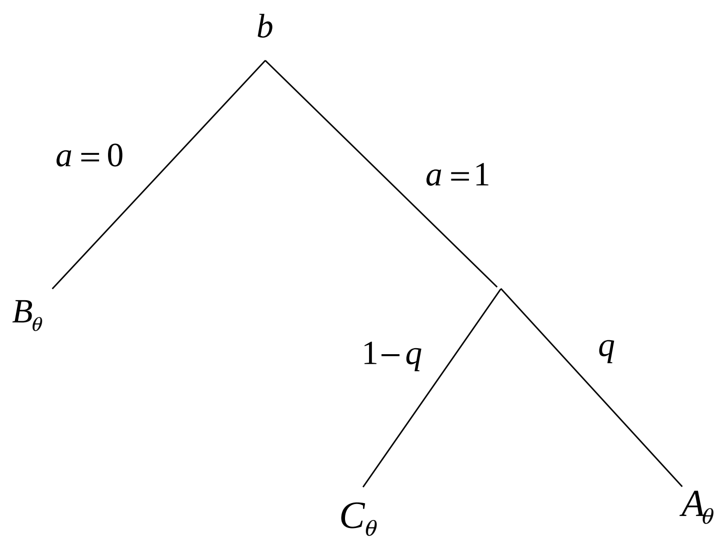

Figure 1. Worker b's decision tree

2.3. Tax Progressivity and Gains from Participation in HE

In this sub-section, we explore the determinants of a worker's gain from pursuing risky HE. Define a real-valued function $V : \{0,1\} \times \Theta \times [\underline{b}, \bar{b}]$ by $(a, \theta, b) \mapsto V(a, \theta, b)$, where

$$V(a,\theta,b) := a[qA_\theta + (1-q) C_\theta] + (1-a) B_\theta,$$

denotes the expected payoff of worker b from taking the decision $a \in \{0,1\}$, prior to entering the labor market, when the tax system implemented by the government is θ, and there are, in total, $n_\theta \in [0,1]$ workers who made the decision $a = 1$. Making use of (2.6), we obtain a reformulation of this expected payoff function as follows:

$$V(a,\theta,b) = \begin{cases} \ln[(1-\tau)\,\omega_\theta^u] & \text{if } a = 0 \\ \ln\left[\dfrac{(1-\tau)\,\omega_\theta^u + [1-(1+\theta)\tau]\,\Delta_\theta - d_\theta}{(1-\tau)\,\omega_\theta^u - d_\theta}\right]^q + \ln[(1-\tau)\,\omega_\theta^u - d_\theta] & \text{if } a = 1 \end{cases}. \tag{2.8}$$

The decision on whether or not to participate in HE is made by comparing the expected payoff from participating, $V(1,\theta,b)$, with the payoff from not participating, $V(0,\theta,b)$.

Let $\vartheta(\theta,b,q) := V(1,\theta,b) - V(0,\theta,b)$ denote the net expected payoff gain to a worker b from participating in HE prior to entering the labor market. Given θ, using (2.8), rearranging, yields this net expected payoff gain as follows:

$$\vartheta(\theta,b,q) = \ln\left[\frac{(1-\tau)\,\omega_\theta^u - d_\theta + [1-(1+\theta)\tau]\,\Delta_\theta}{(1-\tau)\,\omega_\theta^u - d_\theta}\right]^q - \ln\left[\frac{(1-\tau)\,\omega_\theta^u}{(1-\tau)\,\omega_\theta^u - d_\theta}\right]. \tag{2.9}$$

Therefore a worker b will choose to participate in HE if and only if her level of family fund satisfies $\vartheta(\theta,b,q) > 0$. She will elect to pass on it if and only if $\vartheta(\theta,b,q) < 0$. She is indifferent between the two options if and only $\vartheta(\theta,b,q) = 0$. A number of important remarks can be derived from (2.9) to give us preliminary insights about the impact of tax progressivity on skill formation.

Remark 1. *There is no incentive to leverage HE for one's social promotion if there is no* *skill premium in wage. In other words, given θ, the condition*

$$\Delta_\theta > 0, \tag{2.10}$$

is necessary (but not sufficient) for a worker to elect to participate in HE.

Remark 2. *The net expected payoff gain from participating in HE prior to entering the labor market is strictly increasing in a worker's level of endowment of family fund, b:*

$$\frac{\partial}{\partial b}\vartheta\left(b,\theta,q\right) > 0.$$

This remark suggests that workers who gain from participating in HE are those with a sufficiently high endowment of family fund, b. By subsidizing HE, the government therefore can expand access to it to include individuals from poorer backgrounds.

Remark 3. *A higher degree of wage tax progressivity tends to lower the net expected gain from participating in HE:*

$$\frac{\partial}{\partial \theta}\vartheta\left(\theta,b,q\right) < 0,$$

This remark highlights the main criticism of the progressive tax: it tends to create a disincentive to invest in skills. Furthermore:

Remark 4. *The function $\vartheta\left(.\right)$ is supermodular in (b,θ): given q,*

$$\frac{\partial^2}{\partial b\partial \theta}\vartheta\left(b,\theta,q\right) > 0,$$

implying that the incremental net expected payoff gain from having a higher endowment of family fund is larger, the higher the degree of progressivity of the tax system.

This fourth remark states that the progressive tax reinforces the role family income plays in influencing the decision to participate in HE: the higher the degree of progressivity of taxation, the richer the prospective student must be in order to gain from participating in HE. Overall, Remarks 2-4 suggest that tax progressivity may actually be regressive because it tends to bias the gain from participation in HE towards prospective students from richer family background. However, as can also be seen from (2.9), the above effects are only a partial equilibrium effects due to the fact that tax revenue collected partially feedback into higher education, thus opening up additional (indirect) channels for the impact of the progressive tax on the incentive to participate in HE. Indeed, tuition subsidies lower the cost of education, thus providing prospective students from poorer family background with the incentive to participate in HE. Workers' aggregate decisions on participation in HE yield the measure of skill-investors, $n_\theta \in [0, 1]$, of which qn_θ have skills that match those employers need. We are interested in the impact of tax progressivity on qn_θ, the aggregate supply of skilled workers.

2.4. Production

As is standard in the macroeconomics literature (e.g., Heckman, Lochner, and Taber 1998; Caucutt and Kumar, 2003), we assume that the institutional environment in this economy is characterized by perfectly competitive markets, implying that the prices of labor and capital services are determined as derivatives of an aggregate production function. Under the tax system θ, and at the aggregate level, the numeraire is produced using capital K, unskilled labor, U, and skilled labor, S, according to a constant return to scale technology:

$$Y := A\,(K)^\alpha \left[\phi\,(S + \gamma U)^\rho + (1 - \phi)\,(U + \varepsilon S)^\rho\right]^{\frac{1-\alpha}{\rho}}, \tag{2.11}$$

where $\alpha \in (0, 1)$ denotes the capital income share, $A > 0$ is a measure of total factor productivity, $\phi > 0$, a measure of the relative number of skill tasks created at the aggregate level, $\rho < 1$, a factor determining the level of the elasticity of substitution between the low-tech process and the high-tech process, $\gamma > 0$, the relative productivity of unskilled labor in the high-tech process and $\varepsilon > 0$, the relative productivity of skilled labor in the low-tech process. We make the following assumption, as in Caucutt and Kumar (2003):

A.1. $0 < \gamma < \varepsilon < 1$.

Assumption $A.1$ means the following. First, because $\varepsilon < 1$, it implies that skilled workers are not as good as unskilled workers at operating the low-tech production process. Second, because $\gamma < 1$, it also implies that unskilled workers are not as good as skilled workers at operating the high-tech process. Finally, because $\gamma < \varepsilon$, Assumption $A.1$ implies that it is relatively easier for skilled workers to operate the low-tech process, than it is for unskilled workers to operate the high-tech process. This assumption may be justified by the fact that the operation of a high-tech production process usually has high technical requirements that only suitably skilled workers can fulfill, while the operation of low-tech process requires more manual abilities for which unskilled individuals may be relatively more suited (Caucutt and Kumar 2003).

Factor use constraints are as follows:

$$\begin{aligned} S &\leq \bar{S}_\theta := qn_\theta \\ U &\leq 1 - \bar{S}_\theta := 1 - qn_\theta \\ K &\leq \bar{K} \end{aligned}$$

where $\bar{K}$ denotes the global stock of capital, as capital is perfectly internationally mobile in this environment.

We express market-clearing factor prices and input levels in terms of the chosen degree of progressivity of the wage tax θ, under perfect competition:

$$\omega_\theta^s = (1-\alpha) A (K_\theta)^\alpha (H_\theta)^{1-\alpha} \left[\frac{\phi \left(\lambda \bar{S}_\theta + \gamma\right)^{\rho-1} + \varepsilon (1-\phi) \left(1 - \delta \bar{S}_\theta\right)^{\rho-1}}{\phi \left(\lambda \bar{S}_\theta + \gamma\right)^{\rho} + (1-\phi) \left(1 - \delta \bar{S}_\theta\right)^{\rho}} \right] \tag{2.12}$$

$$\omega_\theta^u = (1-\alpha) A (K_\theta)^\alpha (H_\theta)^{1-\alpha} \left[\frac{\gamma\phi \left(\lambda \bar{S}_\theta + \gamma\right)^{\rho-1} + (1-\phi) \left(1 - \delta \bar{S}_\theta\right)^{\rho-1}}{\phi \left(\lambda \bar{S}_\theta + \gamma\right)^{\rho} + (1-\phi) \left(1 - \delta \bar{S}_\theta\right)^{\rho}} \right] \tag{2.13}$$

$$r_\theta = \alpha A \left(\frac{\left[\phi \left(\lambda \bar{S}_\theta + \gamma\right)^{\rho} + (1-\phi) \left(1 - \delta \bar{S}_\theta\right)^{\rho}\right]^{\frac{1}{\rho}}}{K_\theta} \right)^{1-\alpha}, \tag{2.14}$$

where

$$H_{\theta} \ : \ = \left[\phi\left(\lambda\bar{S}_{\theta}+\gamma\right)^{\rho}+(1-\phi)\left(1-\delta\bar{S}_{\theta}\right)^{\rho}\right]^{\frac{1}{\rho}} \tag{2.15}$$

$$\delta \ : \ = 1-\varepsilon$$

$$\lambda \ : \ = 1-\gamma,$$

and $\bar{S}_{\theta} := qn_{\theta}$. Expressions (2.12) and (2.13) *implies that a skill* premium exists (i.e., $\Delta_{\theta} > 0$) if and only if

$$\frac{\phi\lambda}{\delta(1-\phi)} > \left(\frac{\lambda\bar{S}_{\theta}+\gamma}{1-\delta\bar{S}_{\theta}}\right)^{1-\rho}. \tag{2.16}$$

Observe that the right-hand side of the inequality (2.16) is strictly increasing in $\bar{S}_{\theta}$. We once again draw from Caucutt and Kumar (2003) to make the following assumption which guarantees that a skilled worker always earns a higher wage than an unskilled worker in this environment:

A.2. The parameters γ, ε, ϕ, and ρ satisfy

$$\varepsilon > \left[\frac{\delta(1-\phi)}{\lambda\phi}\right]^{\frac{1}{1-\rho}}. \tag{2.17}$$

Assumption $A.2$ gives a sufficient condition for inequality (2.16) to hold. It guarantees that skilled workers are always paid a higher wage than their unskilled counterparts in this environment. In other words, a skill premium always exists.

2.5. Abundance of Skilled labor and Capital Inflows

Consider expression (2.14). Since capital is internationally mobile, this expression implies that capital will continue to flow in until the after-tax domestic rental rate of capital, $(1-\tau_k) r_{\theta}$, equals the international rate, r^*: $(1-\tau_k) r_{\theta} = r^*$, where $\tau_k \in (0,1)$ denotes the flat tax rate levied on capital gains. Solving this equation for K_{θ} yields the level of capital inflows under the tax system θ as follows:

$$K_\theta = \left[\frac{\alpha A (1-\tau_k)}{r^*}\right]^{\frac{1}{1-\alpha}} \left[\phi\left(\lambda \bar{S}_\theta + \gamma\right)^\rho + (1-\phi)\left(1-\delta \bar{S}_\theta\right)^\rho\right]^{\frac{1}{\rho}} \equiv \chi\left(\bar{S}_\theta\right), \tag{2.18}$$

where $\bar{S}_\theta$ denotes the endogenous supply of skilled workers. We then can obtain the partial derivative of (2.18) with respect to $\bar{S}_\theta$ as follows:

$$\frac{\partial K_\theta}{\partial \bar{S}_\theta} = \left[\frac{\lambda\phi\left(\lambda \bar{S}_\theta + \gamma\right)^{\rho-1} - \delta(1-\phi)\left(1-\delta \bar{S}_\theta\right)^{\rho-1}}{\phi\left(\lambda \bar{S}_\theta + \gamma\right)^\rho + (1-\phi)\left(1-\delta \bar{S}_\theta\right)^\rho}\right]\left[\frac{\alpha(1-\tau_k) A}{r^*}\right]^{\frac{1}{1-\alpha}} H_\theta,$$

where H_θ is as defined in (2.15). The above partial derivative is strictly positive if and only if

$$\frac{\phi\lambda}{\delta(1-\phi)} > \left(\frac{\lambda \bar{S}_\theta + \gamma}{1-\delta \bar{S}_\theta}\right)^{1-\rho},$$

which is guaranteed to hold under Assumption A.2. Hence the following result:

Proposition 1. *Under Assumptions A.1 and A.2, an exogenous increase in the supply of skilled workers, $\bar{S}_\theta$, causes the volume of capital inflows, K_θ, to increase.*

Assumption A.2 is sufficient for the skill premium in wage to exist (i.e., condition (2.16) holds) under the tax system θ. It also gives a necessary and sufficient condition for abundance of suitable skills to become a driver of capital inflows. Therefore, Proposition 1 can be interpreted as suggesting that as long as there is a productivity premium for skills, and to the extent that HE imparts suitable skills to prospective workers, expanding access to it can boost national economic performance, measured by the economy's ability to attract foreign capital.

The above notwithstanding, since from expressions (2.12) and (2.13), capital inflows have a positive effect on both skilled and unskilled labor wages (i.e., $\partial\omega_\theta^i/\partial K_\theta > 0$, $i = s, u$), Proposition 1 also implies that HE yields benefits that transcend its beneficiaries, which, in turn, may justify public intervention. This can be seen more clearly, by substituting

(2.18), in (2.12) and (2.13), and rearranging:

$$\omega_\theta^s = \bar{A}\left(\frac{\phi\left(\lambda\bar{S}_\theta+\gamma\right)^{\rho-1}+\varepsilon\left(1-\phi\right)\left(1-\delta\bar{S}_\theta\right)^{\rho-1}}{\left[\phi\left(\lambda\bar{S}_\theta+\gamma\right)^{\rho}+\left(1-\phi\right)\left(1-\delta\bar{S}_\theta\right)^{\rho}\right]^{\frac{\rho-1}{\rho}}}\right)\equiv W^s\left(\bar{S}_\theta\right), \tag{2.19}$$

$$\omega_\theta^u = \bar{A}\left(\frac{\gamma\phi\left(\lambda\bar{S}_\theta+\gamma\right)^{\rho-1}+\left(1-\phi\right)\left(1-\delta\bar{S}_\theta\right)^{\rho-1}}{\left[\phi\left(\lambda\bar{S}_\theta+\gamma\right)^{\rho}+\left(1-\phi\right)\left(1-\delta\bar{S}_\theta\right)^{\rho}\right]^{\frac{\rho-1}{\rho}}}\right)\equiv W^u\left(\bar{S}_\theta\right), \tag{2.20}$$

where $\bar{S}_\theta := qn_\theta$, and

$$\bar{A}=\left(1-\alpha\right)A\left[\frac{\alpha\left(1-\tau_k\right)A}{r^*}\right]^{\frac{\alpha}{1-\alpha}}. \tag{2.21}$$

We prove the following proposition in the Appendix section.

Proposition 2. *Under Assumptions A.1 and A.2, an exogenous increase in the supply of skilled workers, $\bar{S}_\theta$, causes an increase in the level of the unskilled labor wage.*

Proposition 2 states that unskilled workers gain from government's expansion of access to skill-imparting HE. It justifies why these workers too must contribute to public funding of access to HE. The effect of abundance of skilled workers on the skilled labor wage is rather ambiguous: it may or may not be negative.

2.6. Skill Abundance and Skill Premium

Next, define $\hat{\Delta}\left(\bar{S}_\theta\right) := W^s\left(\bar{S}_\theta\right) - W^u\left(\bar{S}_\theta\right)$ to be the skill premium in wage as a function of the aggregate supply of skilled labor, $\bar{S}_\theta$. Then, using (2.19) and (2.20), rearranging terms, we obtain a reformulation of this skill premium as follows:

$$\hat{\Delta}\left(\bar{S}_\theta\right)=\bar{A}\left[\frac{\lambda\phi\left(\lambda\bar{S}_\theta+\gamma\right)^{\rho-1}-\delta\left(1-\phi\right)\left(1-\delta\bar{S}_\theta\right)^{\rho-1}}{\left[\phi\left(\lambda\bar{S}_\theta+\gamma\right)^{\rho}+\left(1-\phi\right)\left(1-\delta\bar{S}_\theta\right)^{\rho}\right]^{\frac{\rho-1}{\rho}}}\right]. \tag{2.22}$$

The skill premium is positive if and only if

$$\lambda\phi\left(\lambda\bar{S}_\theta+\gamma\right)^{\rho-1}-\delta\left(1-\phi\right)\left(1-\delta\bar{S}_\theta\right)^{\rho-1}>0,$$

which is guaranteed by Assumption $A.2$. The existence of skill premium in wage formalizes the private benefits generated by higher education, which then justify the claim that beneficiaries of HE must bear the cost of their enrolment. We prove the following Proposition in the Appendix section.

Proposition 3. *Under Assumptions A.1 and A.2, an exogenous increase in the aggregate supply of skilled workers, $\bar{S}_\theta$, causes the skill premium in wage to decrease.*

Proposition 3 is a direct consequence of demand and supply factors combined with a fixed total population size, which implies that an increase in the aggregate supply of skilled workers trades off a decline in the aggregate supply of unskilled workers. Note, however, that even with a fixed total population size, Proposition 3 may still fail to hold, for example, if the relative contribution of the high-tech production process at the aggregate level, ϕ, was allowed to adjust to the supply of skilled workers. This may happen when the abundant supply of skilled workers induces more firms to operate the high-tech production process, thereby raising the demand for skilled workers, as in the case of skill-biased technological change. But we abstract from skill-induced technical change in this article.

2.7. Public Finance

In the previous sub-section, we explained why government funding of access to HE may be necessary. We also explained why all workers (both skilled and unskilled) must contribute to public funding of HE. The issue at stake in this article therefore concerns how best to allocate the tax burden between the two different categories of workers (skilled and unskilled). In this sub-section, we characterize tax revenue, as well as the tuition subsidy rate under alternative wage tax systems. Tax revenue has three sources: (i) unskilled workers, in total measure $1 - \bar{S}_\theta$, each with a pre-tax income, $W^u\left(\bar{S}_\theta\right)$; (ii) skilled workers, in total measure $\bar{S}_\theta$, each with a pre-tax income $W^s\left(\bar{S}_\theta\right) := W^u\left(\bar{S}_\theta\right) + \hat{\Delta}\left(\bar{S}_\theta\right)$, where $\hat{\Delta}\left(\bar{S}_\theta\right) := W^s\left(\bar{S}_\theta\right) - W^u\left(\bar{S}_\theta\right)$ denotes the skill premium under the tax system θ; (iii) capital gains given by $r^* K_\theta$.

Recall that under the tax system $\theta \in \{0, 1\}$, the first wage tax bracket is represented by the unskilled labor wage and taxed at a rate τ, while the second and higher labor income bracket is represented by the skilled labor wage $W^u\left(\bar{S}_\theta\right)+\hat{\Delta}\left(\bar{S}_\theta\right)$. Recall also that the first portion of the skilled labor wage (i.e., $W^u\left(\bar{S}_\theta\right)$) is taxed at a rate τ, while the second portion (i.e., $\hat{\Delta}\left(\bar{S}_\theta\right)$) is taxed at a rate $(1+\theta)\tau$, where $\theta \geq 0$ is the measure of the degree of progressivity of the tax system. Therefore, under the tax system θ, total tax revenue, T_θ, can be obtained as a function of the aggregate supply of skilled labor, $\bar{S}_\theta$.

$$T_\theta=\tau W^u\left(\bar{S}_\theta\right)+\tau(1+\theta)\hat{\Delta}\left(\bar{S}_\theta\right)\bar{S}_\theta+\tau_k r^*\chi\left(\bar{S}_\theta\right)\equiv\hat{T}\left(\theta,\bar{S}_\theta\right). \tag{2.23}$$

By inspection of expression (2.23), it is clear that tax progressivity tends to have a positive effect on tax revenue: $\partial\hat{T}/\partial\theta>0$. Since a share $\mu\in(0,1)$ of this tax revenue is allocated to subsidizing education costs, this opens up a channel through which tax progressivity can have a positive impact on the rate of participation in HE. We will return to this issue further below.

Recall that the unskilled labor wage, $W^u\left(\bar{S}_\theta\right)$, the level of capital inflow, $\chi\left(\bar{S}_\theta\right)$, are all increasing in the aggregate supply of skilled workers, $\bar{S}_\theta$. However, the skill premium, $\hat{\Delta}\left(\bar{S}_\theta\right)$, is decreasing in the aggregate supply of skilled labor, which implies that a change in the level of the latter has two opposite effects on the level of tax revenue, $\hat{T}\left(\theta,\bar{S}_\theta\right)$, one positive and the other, negative:

$$\frac{\partial}{\partial\bar{S}_\theta}\hat{T}\left(\theta,\bar{S}_\theta\right)=\tau W^{u\prime}\left(\bar{S}_\theta\right)+\tau_k r^*\chi'\left(\bar{S}_\theta\right)+\tau(1+\theta)\hat{\Delta}\left(\bar{S}_\theta\right)\left(1-\xi_{\Delta\bar{S}}\right)$$

where

$$\xi_{\Delta\bar{S}}=-\hat{\Delta}'\left(\bar{S}_\theta\right)\frac{\bar{S}_\theta}{\hat{\Delta}\left(\bar{S}_\theta\right)}$$

denotes the elasticity of the skill premium, $\hat{\Delta}\left(\bar{S}_\theta\right)$, with respect to a change in the size of the skilled labor force, $\bar{S}_\theta$.

Proposition 4. *Let Assumptions A.1 and A.2 hold. Then, an exogenous increase in the aggregate supply of skilled workers causes an increase in the level of tax revenue if*

$$\xi_{\Delta\bar{S}}<1. \tag{2.24}$$

Condition (2.24) is sufficient for the function $\hat{T}(\theta, .)$ to be strictly increasing. It states that the skill premium is only weakly elastic to a change in the size of the skilled labor force. Such a condition is likely to hold for an economy open to foreign capital inflows, as these tend to increase the productivity of skilled labor relative to that of the unskilled labor. When this condition holds, Proposition 4 states that any factor that raises the aggregate supply of skilled workers in this economy will cause the level of tax revenue to rise. Therefore, from the view point of the government, expanding access to HE generates a benefit to society in the form of increased tax revenue. This too may provide the government with a vested interest in expanding access to HE. Next, assume that a share μ of public funds collected through taxation of labor and capital income is allocated to financing tuition subsidies. Define total public expenditures on HE by

$$Z_\theta := z_\theta n_\theta e,$$

where n_θ denotes the level of enrolment in HE under the tax system θ. Therefore, assuming that the government operates under a balanced budget legislation, the subsidy rate, z_θ, is solution to the equation

$$z_\theta n_\theta e = \mu \hat{T}\left(\theta, \bar{S}_\theta\right), \tag{2.25}$$

all θ. In other words, using the fact that $\bar{S}_\theta := q n_\theta$, it follows that the subsidy rate, z_θ, solves:

$$z_\theta = \mu q \frac{\hat{T}\left(\theta, \bar{S}_\theta\right)}{e \bar{S}_\theta} \equiv \zeta\left(\theta, q, \bar{S}_\theta\right), \tag{2.26}$$

where $\hat{T}\left(\theta, \bar{S}_\theta\right)$ is as defined in (2.23).

We established in Proposition 4 above that $\hat{T}(\theta, .)$ is an increasing function of $\bar{S}_\theta$. Therefore, expression (2.26) suggests that an exogenous change in the level of the aggregate supply of skilled workers has to opposite effects on the tuition subsidy rate, z_θ, one positive and the other negative. Which one of the two dominates the other, unfortunately does not have a clear calculus answer.

Finally, we turn to the level of a student debt, $d_\theta = E_\theta - b$. From (2.1), substituting in (2.26), rearranging, leads to a reformulation of the level of student debt as a function of the aggregate supply of skilled workers:

$$D\left(\theta, q, \bar{S}_\theta\right) = (e-b) - \frac{\mu q \hat{T}\left(\theta, \bar{S}_\theta\right)}{\bar{S}_\theta}, \tag{2.27}$$

where $D\left(\theta, q, \bar{S}_\theta\right) \equiv d_\theta$. Expression (2.27) illustrates the role played by tuition subsidies in enhancing participation in HE. The first term of this expression, $(e-b)$, is the level of debt that worker b would have accumulated due to participation in HE in the absence of tuition subsidies. Tuition subsidies reduce this debt by $\mu q \hat{T}\left(\theta, q, \bar{S}_\theta\right)/\bar{S}_\theta$, thus providing worker b with the incentive to participate in HE. However, given, μ, the magnitude of this reduction depends on (i) the degree of tax progressivity, θ, (ii) the level of efficiency of the HE system as measured by q, (iii) the aggregate supply of skilled workers, $\bar{S}_\theta$. Just as in the case of the tuition subsidy rate, $\zeta\left(\theta, q, \bar{S}_\theta\right)$, an exogenous change in the level of the aggregate supply of skilled workers has two opposite effects on the level of student debt, $D\left(\theta, q, \bar{S}_\theta\right)$, one positive and the other negative. Hence the following result, which can be obtained by partially differentiating (2.27) with respect to $\bar{S}_\theta$.

Proposition 5. *Let*

$$\xi_{w\bar{S}} \quad : \quad = W^{u\prime}\left(\bar{S}_\theta\right) \frac{\bar{S}_\theta}{W^u\left(\bar{S}_\theta\right)}$$

$$\xi_{K\bar{S}} \quad : \quad = \chi'\left(\bar{S}_\theta\right) \frac{\bar{S}_\theta}{\chi\left(\bar{S}_\theta\right)}$$

*denote the elasticities of the unskilled labor wage and the level of capital inflows, respectively, with respect to a change in the level of the aggregate supply of skilled workers. Under Assumptions A.*1 *and A.*2*, if*

$$\xi_{w\bar{S}} < 1 \tag{2.28}$$

and

$$\xi_{K\bar{S}} < 1, \tag{2.29}$$

then an increase in the participation rate in HE (as proxied by an exogenous increase in the aggregate supply of skilled workers) causes the average level of student debt to rise.

The reverse is true if $\xi_{w\bar{S}} > 1$, $\xi_{K\bar{S}} > 1$, *and*

$$\xi_{\Delta\bar{S}} < \frac{\tau W^u\left(\bar{S}_\theta\right)\left(\xi_{w\bar{S}}-1\right)+\tau_k r^*\chi\left(\bar{S}_\theta\right)\left(\xi_{K\bar{S}}-1\right)}{\tau\left(1+\theta\right)\hat{\Delta}\left(\bar{S}_\theta\right)\bar{S}_\theta}.$$

Conditions (2.28) and (2.29) state that the skilled labor supply elasticities of the unskilled labor wage and capital inflows are moderate. Proposition 5 states that an increase in the aggregate supply of skilled workers has an ambiguous effect on the level of student debt. The sign of this effect depends on the levels of elasticities for the unskilled labor wage, the skill premium, and the level of capital inflows respectively. A highly elastic skill premium tends to cause this effect to be positive, while highly elastic capital inflows and unskilled labor wage tend to make it negative.

Furthermore, as $D\left(\theta, q, \bar{S}_\theta\right)$ is strictly decreasing in q, the level of efficiency with which the HE system imparts skills that match those required by the skilled labor market becomes a key factor not only in terms of raising the likelihood that a graduate will earn a financial benefit from the skills she acquired, but also in terms of reducing the debt she has to repay for participating in HE. Arguably, whether or not tax progressivity enhances skill formation depends on whether or not the HE system is sufficiently efficient at imparting skills that match those employers need. Our general equilibrium analysis makes this point below.

3. Equilibrium Analysis

In this section, we set the stage for the computation of the equilibrium of this small economy. This will provide the framework for contrasting the performances of the tax systems. Essentially, given the tax system θ implemented by the government, the existence of an equilibrium of this environment hinges upon the existence of a cut-off level of family fund, $b_\theta \in [0, e]$ such that a worker with a level of private fund equal to this threshold has a net expected payoff gain from pursuing HE equal to zero. In what follows, we first discuss the existence of an equilibrium in this environment.

3.1. Existence of Equilibrium

We focus on an interior equilibrium, i.e., one where $b_\theta \in (0, e)$. Consider the net payoff gain in (2.9). To complete the specification of this net payoff, we need to (i) compute and substitute in the level of student debt under each alternative tax system, (ii) substitute in wages as well.

Therefore, from (2.9), substituting in (2.19), (2.20), and (2.27), rearranging, yields a reformulation of the net expected gain from pursuing HE as a follows:

$$\bar{\vartheta}\left(\theta, b, q, \bar{S}_\theta\right) = q \ln \Gamma\left(\theta, b, \bar{S}_\theta\right) - \ln \Upsilon\left(\theta, b, \bar{S}_\theta\right) \tag{3.1}$$

for all $\theta \in \left[0, \bar{\theta}\right]$, where

$$\begin{aligned}
\Gamma\left(\theta, b, q, \bar{S}_\theta\right) &: = \frac{(1-\tau) W^u\left(\bar{S}_\theta\right) - (e-b) + \mu q \hat{T}\left(\bar{S}_\theta\right)\left(\bar{S}_\theta\right)^{-1} + \left[1 - (1+\theta)\tau\right]\hat{\Delta}\left(\bar{S}_\theta\right)}{(1-\tau) W^u\left(\bar{S}_\theta\right) - (e-b) + \mu q \hat{T}\left(\bar{S}_\theta\right)\left(\bar{S}_\theta\right)^{-1}} \\
\Upsilon\left(\theta, b, q, \bar{S}_\theta\right) &: = \frac{(1-\tau) \bar{W}^u\left(\bar{S}_\theta\right)}{(1-\tau) \bar{W}^u\left(\bar{S}_\theta\right) - (e-b) + \mu q \hat{T}\left(\bar{S}_\theta\right)\left(\bar{S}_\theta\right)^{-1}},
\end{aligned}$$

where $\bar{\vartheta}\left(\theta, b, q, \bar{S}_\theta\right) \equiv \vartheta(\theta, b, q)$. Note that since $\vartheta(\theta, b, q)$ is differentiable and strictly increasing in b, therefore, by construction, $\bar{\vartheta}\left(\theta, b, q, \bar{S}_\theta\right)$ is also differentiable and strictly increasing in b. Therefore an interior equilibrium exists if and only if

$$0 \in (\bar{\vartheta}\left(\theta, 0, q, \bar{S}_\theta\right), \bar{\vartheta}\left(\theta, e, q, \bar{S}_\theta\right)).$$

In other words, the following conditions must be simultaneously satisfied:

$$\begin{aligned}
(i) \qquad & \bar{\vartheta}\left(\theta, 0, q, \bar{S}_\theta\right) < 0 \\
(ii) \qquad & \bar{\vartheta}\left(\theta, e, q, \bar{S}_\theta\right) > 0
\end{aligned}$$

Since

$$\lim_{b \to e} \Upsilon\left(\theta, b, q, \bar{S}_\theta\right) < 1 \qquad \text{and} \qquad \lim_{b \to e} \Gamma\left(\theta, b, q, \bar{S}_\theta\right) > 1,$$

it follows from (3.1) that

$$\lim_{b \to e} \bar{\vartheta}\left(\theta, b, q, \bar{S}_\theta\right) = \bar{\vartheta}\left(\theta, e, q, \bar{S}_\theta\right) > 0.$$

This implies that condition (ii) above is always satisfied.

In contrast, an inspection of expression (3.1) suggests that the sign of

$$\lim_{b \to 0} \bar{\vartheta}\left(\theta, b, q, \bar{S}_\theta\right) = \bar{\vartheta}\left(\theta, 0, q, \bar{S}_\theta\right)$$

is ambiguous. To the extent that (e, q, μ) can be chosen so as to satisfy $\bar{\vartheta}\left(\theta, 0, q, \bar{S}_\theta\right) < 0$, we can claim that there exists a cut-off level of family fund, $b_\theta \in (0, e)$, such that:

$$\begin{aligned} \forall b &< b_\theta, \quad \bar{\vartheta}\left(\theta, b, q, \bar{S}_\theta\right) < 0 \\ \forall b &> b_\theta, \quad \bar{\vartheta}\left(\theta, b, q, \bar{S}_\theta\right) > 0, \end{aligned}$$

and

$$\bar{\vartheta}\left(\theta, b_\theta, q, \bar{S}_\theta\right) \equiv 0. \tag{3.2}$$

where

$$b_\theta := B\left(\theta, q, \bar{S}_\theta\right) \tag{3.3}$$

all θ. With the threshold b_θ, thus characterized, we can next obtain the measure of workers who choose not to pursue HE as $\Psi(b_\theta)$, using the cdf for the distribution of workers across endowments of family funds. Since the total measure of workers is 1, the measure of those who elect to participate in HE is

$$n_\theta = 1 - \Psi(b_\theta).$$

Note then that since $\bar{S}_\theta = qn_\theta$, the above equation can be reformulated as follows, using (3.3):

$$\bar{S}_\theta = q\left(1 - \Psi\left[B\left(\theta, q, \bar{S}_\theta\right)\right]\right), \tag{3.4}$$

which therefore is a fixed point problem, since $\bar{S}_\theta \in [0,1]$ and $q\left(1-\Psi\left[B\left(\theta,q,\bar{S}_\theta\right)\right]\right) \in [0,1]$.

We can then define an equilibrium of this small economy as (i) a cut-off level for the endowment of family fund, b_θ^*, (ii) the level of supply of workers, $\bar{S}_\theta^*$, such that given (θ, q):
(i) given $\bar{S}_\theta^*$, b_θ^* solves (3.2);
(ii) $\bar{S}_\theta^*$ is solution to (3.4).

The existence of $\bar{S}_\theta^*$ can be established using *Brouwer's fixed point theorem*. But we also want this fixed point to be unique. Unfortunately, expression (3.4) suggests that monotonicity cannot be directly verified by partially differentiating $\Psi\left[B\left(\theta,q,\bar{S}_\theta\right)\right]$ with respect to $\bar{S}_\theta$. To fully characterize the equilibrium, and therefore complete our analysis, we simulate the model using numerical methods.

3.2. Model Simulation

We use numerical algorithms (available upon request) to fully characterize the model. The main technical issue that concerns us in this numerical simulation is whether the model indeed admits a unique equilibrium. For the purpose of the simulation exercise, we define an equilibrium as a system of two equations in two unknown b_θ and S_θ. The two equations are (3.2) and (3.4). We follow four steps in simulating the model.

Step 1. Given $\left(\theta, q, \bar{S}_\theta\right)$, compute the threshold family fund, b_θ, such that any worker with an endowment of family fund above that threshold gains from pursuing HE.

Step 2. Given a cut-off $b_\theta := B\left(\theta,q,\bar{S}_\theta^*\right)$, and by postulating a uniform distribution, we obtain the equilibrium supply of skilled workers as follows:

$$\bar{S}_\theta^* = q\left[\frac{e-B\left(\theta,q,\bar{S}_\theta^*\right)}{e}\right]. \tag{3.5}$$

Step 3. We use (3.5), in combination with (2.18), and (2.27) to compute (i) the equilibrium level of capital inflows, K_θ, (ii) the average level of student debt defined as

$$\bar{d}_\theta = \frac{1}{n_\theta} \int_{b_\theta^*}^{e} \left((e-b) - \frac{\mu q \hat{T}\left(\bar{S}_\theta^*\right)}{\bar{S}_\theta^*} \right) \psi(b)\, db. \tag{3.6}$$

Step 4. We then present our results in a series of graphs representing each of the relevant endogenous variables as a function of the degree of wage tax progressivity θ, and the degree of alignment of the skills gained through HE with the requirements of the labor market, q.

3.2.1. Parameter Values

An important exercise in the simulation process is the computation of the equilibrium cut-off level of family fund, b_θ. A number of parameters are involved in the numerical computation of this cut-off. They include the relative contribution of the high-tech process, ϕ, the determinant of the elasticity of substitution between skilled and unskilled labor, ρ, the capital income share, α, the total productivity factor, A, the relative productivity of unskilled labor in the high-tech process, γ, the relative productivity of skilled labor in the low-tech process, ε, the global stock of capital, $\bar{k}$, the international rental rate of capital, r^*, the share of public funds allocated to tertiary education expenditures by the government, μ, the tax rates τ and τ_k, the degree of tax progressivity θ, the cost of education, e, the level of efficiency with which the HE system imparts graduates with skills that match those employers need, q. Of all these parameters, only $\bar{k}, q$, and θ are specific to our model.

We set the capital income share to $\alpha = 0.4$, consistent with empirical evidence revealing that this share falls in the range $0.2 - 0.4$ (Gollin, 2002). Also consistent with the macroeconomics literature (e.g., Mehra and Prescott, 1985; Andolfatto *et al.* 2004; Dib *et al.* 2008), we set $r^* = 1.04$. We also set the level of total factor productivity, A, equal to 1. Van Beveren (2012) finds that the level of total factor productivity is in the range $1.06 - 2.47$. We follow Caucutt and Kumar (2003) in choosing the values of ε and e. In particular, like them we set $\varepsilon = 0.1$, $e = 0.06$. However, since unlike theirs, our model includes physical capital, we set $\gamma = 0.04$ and $\phi = 0.85$, so that Assumptions $A.1$ and $A.2$

simultaneously hold. Corresponding figures for Caucutt and Kumar (2003) are $\gamma = 0.02$ and $\phi = 0.5$. Furthermore, we set $\rho = 0.32$. This corresponds to an elasticity of substitution between skilled workers and unskilld workers of 1.47, well within the range $1.4 - 1.5$ identified by Auter *et al.* (1998). We draw from an OECD report on education to set the average share of public funds allocated to tertiary education expenditures among OECD countries, μ, at 0.05.[8] We follow Heckman, Lockner and Taber (1998) in setting $\tau_k = 0.15$. We set $\tau = 0.085$, which is closer to the base rate set by many countries including Canada and the United States. Finally, we normalize the global stock of capital, $\bar{k}$, to unity.

As for θ and q, we specify a range for each. We set the feasible range for the degree of wage tax progressivity, θ, to be the unit interval $[0, 1]$. Just to recall, $\theta = 0$, corresponds to the flat wage tax system. A value of θ equal to unity corresponds to a marginal tax rate of 100% on the incremental wage from being a skilled worker. Any marginal tax rate beyond this level would seem anything but reasonable. We set the range of q such that the cut-off endowment of family fund, b_θ, lies in the interior of its range, $[0, e]$.

The following table gives a summary of the parameter values:

[8] See OECD, 2009. *Education at a Glance (2009):* OECD Indicators

Capital share	α	0.4
International rental rate	r^*	1.04
Measure of total factor productivity	A	1.
Elasticity of substitution $= \dfrac{1}{1-\rho} = 1.47$	ρ	0.32
Measure of the relative contribution of the high-tech process	ϕ	0.85
Relative productivity of unskilled labor in the high-tech process	γ	0.04
Relative productivity of skilled labor in the low-tech process.	ε	0.1
The range for the family fund b	$[0, e]$	$[0 \;\; 0.06]$
Income tax rate	τ	0.085
Degree of tax progressivity	θ	$[0, 1]$
Capital tax rate	τ_k	0.15
The share of Tax Revenue allocated to HE	μ	0.05
The cost of education	e	0.06
Range for the quality of higher education	q	$0.45; 0.55; 0.65; 0.75$

Using the above parameter values, we compute all equilibrium variables. We particularly single out, the size of the skilled labor force, $\bar{S}^*_\theta$, the level of capital inflows, K^*_θ, the level of average student debt, $\bar{d}^*_\theta$, and the level of tax revenue, T^*_θ

3.2.2. Wage Tax Progressivity and The Size of The Skilled Labor Force

Our model economy is closed to migration. Therefore any change in the size of the skilled labor force has its source in the changes in the allocation of workers by skill status. To explore the effects of wage tax progressivity on skill formation, we compute the equilibrium supply of the skilled workers, $\bar{S}^*_\theta$, for each degree of wage tax progressivity, $\theta \in [0, 1]$, and for different level of efficiency of the HE system.

Figure 1 below contains 4 graphs. In each of them, the horizontal axis represents the degree of progressivity of the wage tax system, θ. An increase in θ along the horizontal axis corresponds to an increase in the degree of progressivity of the wage tax system. The vertical axis represents the equilibrium supply of skilled workers, $\bar{S}_{\theta}^{*}$. Each of the four graphs corresponds to a different level of efficiency with which the HE system imparts workers with skills suitable for the labor market, q.

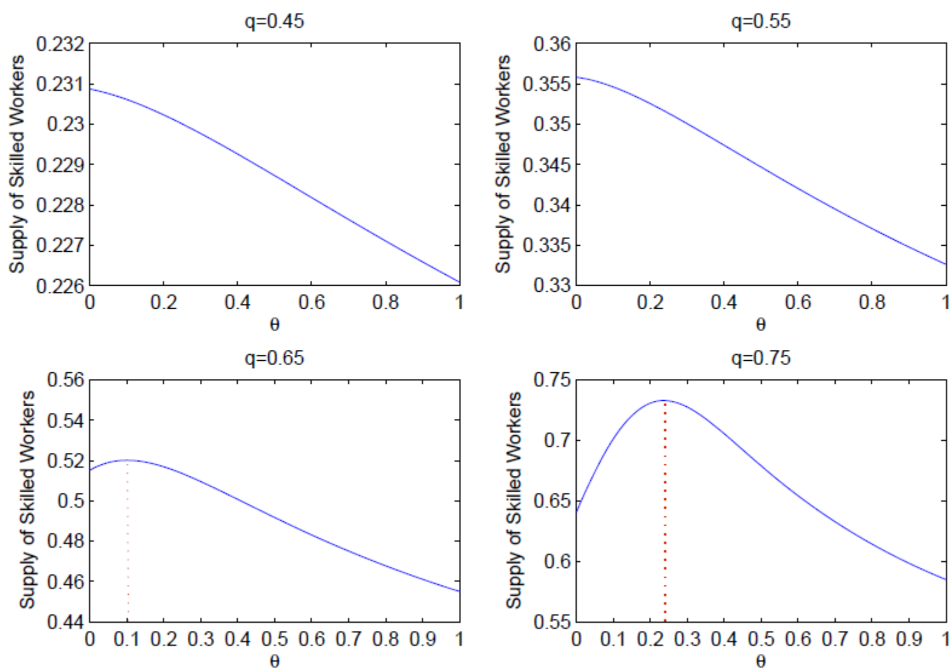

Figure 1

The higher q, the higher the level of efficiency with which the HE system imparts skills that employers need.

On the upper-left graph in Figure 1, the equilibrium supply of skilled workers is plotted in the context where only 45% of HE graduates gain skills that are suitable for the skilled labor market (i.e., $q = 0.45$). On the upper-right graph, the supply of skilled workers is plotted in the context where 55% of HE graduates gain skills that are suitable for the labor market (i.e., $q = 0.55$). On the bottom-left and the bottom-right graphs, the corresponding figures are 65% and 75%, respectively.

Figure 1 shows that the shape of the curve representing the equilibrium supply of skilled workers varies with the level of efficiency of the HE system, q, becoming increasingly Hump-shaped as efficiency rises. This is because the degree of tax progressivity that maximizes the supply of skilled workers varies with the level of efficiency with which the HE system imparts skills that are suitable for the market, q. At a level of efficiency of the HE system $q = 0.45$, the equilibrium supply of skilled workers ($\bar{S}_\theta$) reaches its maximum at a degree of wage tax progressivity $\theta = 0$, corresponding to a flat tax. At this level of efficiency, therefore, the flat tax dominates the progressive tax as a mechanism for enhancing skill formation. Raising this level of efficiency from $q = 0.45$ to $q = 0.55$ (an increase of about 22%) does not change the ranking of the two tax system because the degree of progressivity that maximizes the equilibrium supply of skilled workers is unchanged at $\theta = 0$, as illustrated on the graph in the upper-right corner of Figure 1. However, this 22% increase in the level of efficiency of the HE system raises the equilibrium supply of skilled workers by about 54%.

In contrast, starting from a level of efficiency $q = 0.55$, if we counterfactually increase this level of efficiency by 18% to move it up to $q = 0.65$, causes the degree of progressivity that maximizes the supply of skilled workers to become positive at $\theta = 10\%$ (see lower-left graph of Figure 1), leading to a 46% increase in the equilibrium supply of skilled workers (from 0.3555 on the upper-right graph of Figure 1 to 0.52 on the lower-left graph). This counterfactual increase in the level of efficiency of the HE system reverses the ranking of the two tax systems, as the progressive tax now dominates the flat tax as a mechanism for enhancing skill formation.

Moving the level of efficiency of HE from $q = 0.65$ to $q = 0.75$ not only reinforces the superiority of the progressive tax, but, in addition, it raises the degree of tax progressivity that maximizes the equilibrium supply of skilled workers by 140% (from $\theta = 10\%$ to $\theta = 24\%$). Figure 1 therefore shows that the degree of tax progressivity that maximizes the supply of skilled workers rises with level of efficiency with which the HE system imparts skills that match those employers need, q. In other words, the quality of HE, as measured by the level of efficiency with which the HE system imparts suitable skills, is an important mediator of the impact of wage tax progressivity on skill formation. Countries that use a progressive wage tax system as the instrument for financing public expenditures must therefore ensure that whenever higher education is relied upon as a skill accumulation mechanism, the skills it imparts are closely aligned with the skills employers need, otherwise, this tax system would undermine the development of a skilled labor force. In that sense, Figure 1 can also be interpreted as suggesting that a misaligned higher education system may be the source of the criticism directed at the progressive tax system.

3.2.3. Tax Progressivity and The Equilibrium Level of Capital Inflows

We argued in Proposition 1 above that the availability of workers with suitable skills is essential for attracting foreign capital. Furthermore, because labor productivity rises with abundance of capital inflows, in equilibrium, this positive association impacts the supply of workers with suitable skills through the aggregate of individuals' decisions to pursue HE. Since taxation is distortionary in this environment, it impacts the interactions between the aggregate supply of skilled workers, the inflow of foreign capital, and wages for skilled workers and unskilled workers. In this sub-section, we are interested in how the chain of interactions generated by a change in the degree of progressivity of the wage tax system impacts the equilibrium level of capital inflows, under alternative specifications of the level of efficiency of the HE system. To simulate the equilibrium level of capital inflows, we first compute b_θ, for each θ. Then we plug the result in (3.5) to obtain the equilibrium aggregate supply of skilled workers, for each θ, which, in turn, is plugged back into (2.18) to obtain the equilibrium level of capital inflows, K_θ.

Figure 2 below plots the equilibrium level of K_θ against the degree of progressivity of the wage tax system, θ, for four different levels of q.

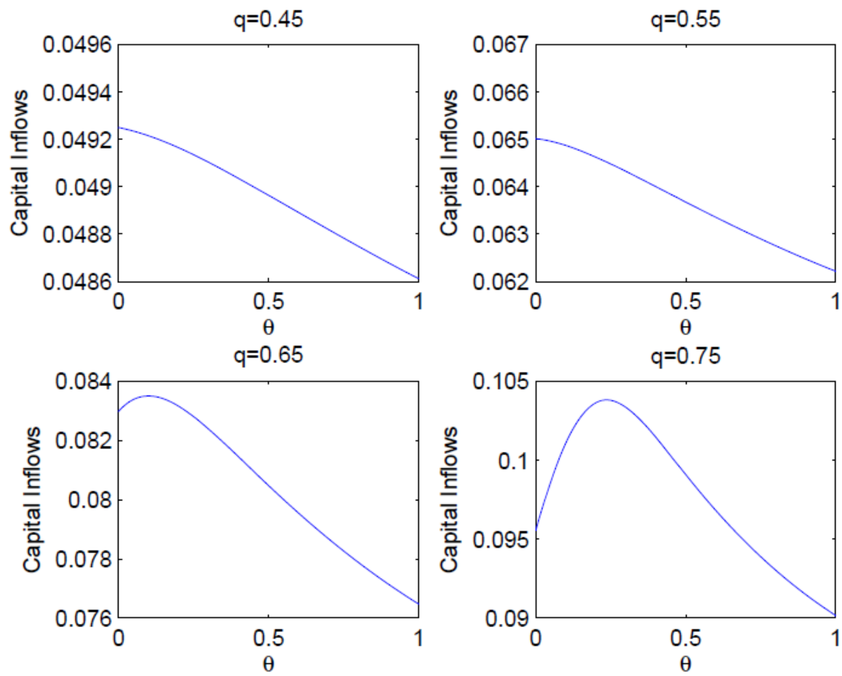

Figure 2

Compared to Figure 1, Figure 2 confirms the co-movement between the aggregate supply of skilled workers, $\bar{S}_\theta$, and the level of capital inflows, K_θ. The wage tax system that maximizes $\bar{S}_\theta$ also maximizes K_θ. When q is set at 0.45 or 0.55, capital inflows are maximized at a degree of progressivity $\theta = 0$, implying the superiority of the flat tax over the progressive tax, as mechanism for attracting capital inflows. The reverse is true, however, when the level of efficiency is raised at 0.65 or 0.75. This figure suggests that wage tax progressivity is bad for capital inflows only when the higher education system that is invested with the task of imparting suitable skills is misaligned with the labor market. Redressing this misalignment is necessary and sufficient for the progressive tax to dominate the flat tax as a mechanism for attracting capital inflows.

3.2.4. Tax Progressivity and Skill Premium

In our above discussion of the skill premium, Proposition 3 establishes that abundant supply of skilled workers reduces the skill premium. Therefore, any exogenous factor that causes the supply of skilled workers to increase will have a negative effect on the skill premium. Figure 3 below plots the equilibrium skill premium against the degree of tax progressivity, θ, for four different levels of efficiency of the HE system, q.

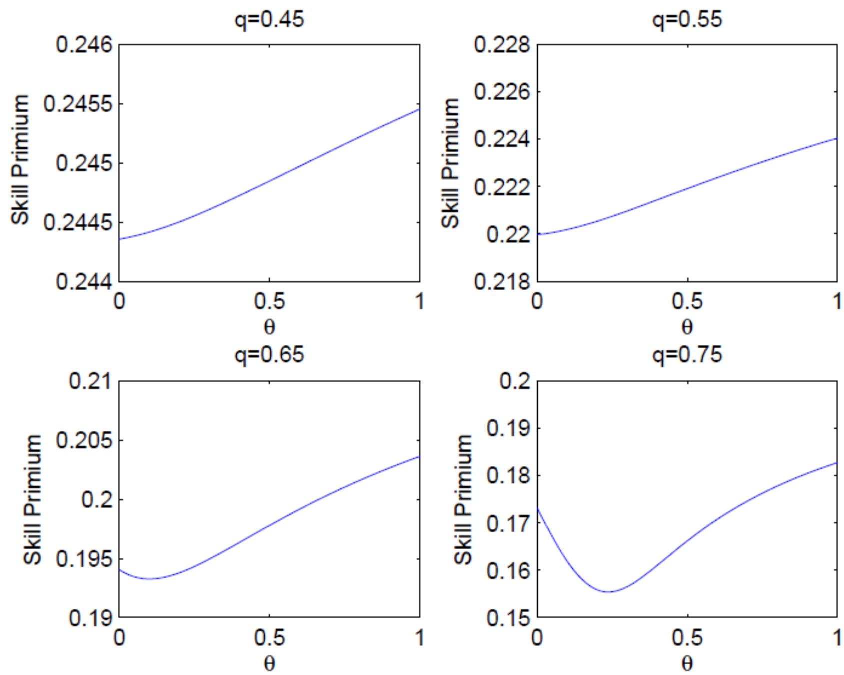

Figure 3

The shape of the curve representing the skill premium varies with the level of efficiency of the HE system, becoming increasingly U-shaped as efficiency rises. In particular, when the HE system is relatively inefficient at imparting skills that match those employers need, as in the case where $q = 0.55$ or lower, the skill premium is minimized under a flat tax system (i.e., $\theta = 0$), implying that a flat tax is relatively more effective than the progressive tax at achieving a more equal distribution, on the basis of pre-tax wages. However, when the level of efficiency of the HE system is counterfactually raised to $q = 0.65$, the skill premium achieves its minimum at the point where the marginal rate of taxation is strictly positive at $\theta = 10\%$. The superiority of the progressive tax on equity ground is further enhanced when the level of efficiency of the HE system is raised to $q = 0.75$. At that level, the skill premium is minimized by a progressive tax system that exhibits a 24% marginal tax rate, as can be seen in the bottom-right graph of Figure 3. This implies that there exists a threshold level of efficiency of the HE system above which the progressive tax is more effective than the flat tax at enhancing distributional equity on the basis of pre-tax wages, and below which the reverse is true.

3.2.5. Wage Tax Progressivity and Average Student Debt

According to a Statistics Canada Report published in 2010, the average debt of students at graduation, was $18,800 in Canada.[9] Research in the area of student loans has been focused on loan repayment difficulties (e.g., Lochner, Stinebrickner, and Suleymanoglu 2013) often relating debt burdens to a decrease in public funding of access to HE. Missing in this literature is an exploration of the role played by an increase in the rate of participation in HE and average student debt. The analysis we provide below explores this link.

Figure 4 below plots the average student debt, d_θ, against the degree of wage tax progressivity, θ, for four different values of the degree of alignment of the skills gained through HE with those employers need, q.

[9] See May Long, 2010. The Financial Impacts of Student Loans. Statistics Canada, catalogue No 75-001-X.

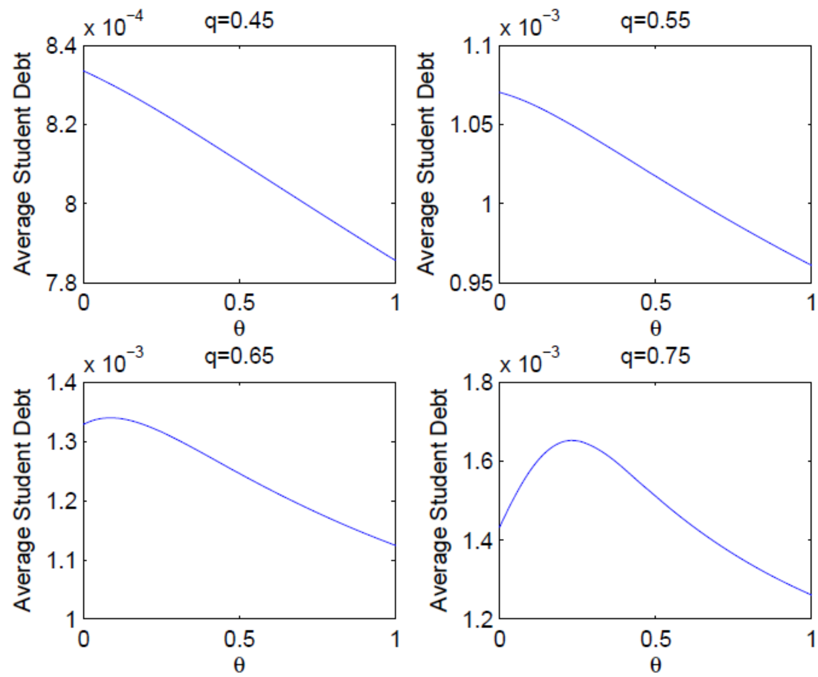

Figure 4

In all four graphs of Figure 4, the average student debt is maximized at the degree of wage tax progressivity for which the aggregate supply of skilled workers is the highest. This corresponds to a flat wage tax system for levels of HE efficiency of respectively $q = 0.45$ and $q = 0.55$, and to a progressive wage tax system for $q = 0.65$ and $q = 0.75$. When combined with Figure 1, Figure 4 suggests that an increase in the rate of participation in HE causes the average student debt to rise. The intuition behind this result is quite straightforward. An increase in public funding to HE raises the prospects for higher tuition subsidy rates. This provides students from poorer family backgrounds (those who require higher levels of student loans in order to participate) with the incentive to pursue higher education. In equilibrium, proportionately more such students enroll in HE, thereby causing the average student debt at graduation to rise.

3.2.6. Tax Progressivity and Tax revenue

Tax revenue depends on the tax system θ, on factor prices, and on the allocation of workers across skill-status. We argued above that when a fraction of tax revenue is used to subsidize tuition, this opens up an indirect channel through which progressive taxation can impact skill formation. However this depends on whether tax revenue is higher or lower under the progressive tax than under the flat tax. Figure 5 below plots the equilibrium tax revenue, T_θ^* against the degree of progressivity of the wage tax system θ, for four different levels of efficiency of the HE system as measured by q.

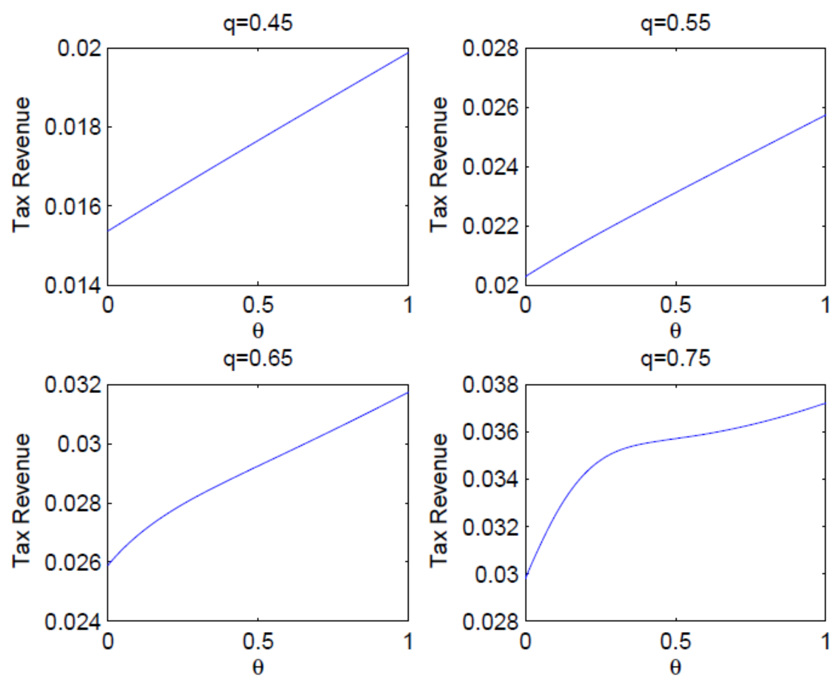

Figure 5

In all four graphs of Figure 5, an increase in the degree of tax progressivity causes the level of tax revenue to increase. This implies that a progressive tax is better at raising tax revenue than a flat tax. If we only look at the degree of tax progressivity that maximizes the aggregate supply of skilled workers, Figure 5 combines with Figure 1 to show that an increase in the aggregate supply of skilled workers raises the level of tax revenue, as demonstrated in Proposition 4.

4. Concluding Remarks

This article develops a general equilibrium model of enrolment in higher education and labor supply in an environment where the return to higher education are uncertain and capital is international mobile. The model was numerically simulated to provide us with a sensible basis for comparing alternative tax systems for funding access to higher education. In particular, we use this model to contrast the performances of the progressive wage tax system and the flat wage tax system. We show that the quantitative impact on skill formation and capital inflows of switching from the flat to the progressive tax varies with the level of efficiency with which the higher education system imparts graduates with suitable skills. This quantitative impact is negative when the level of efficiency of the higher education system is low and positive when it is high. In other words, when the level of efficiency of the higher education system is low, the flat tax dominates the progressive tax as a mechanism for enhancing skill formation and capital inflows. The reverse is true, however, when this level of efficiency is sufficiently high.

The key mechanism underlying the quantitative impacts of the progressive tax has three main elements namely, (i) the negative impact wage taxation has on the reward from being a skilled worker, (ii) the positive effect progressive taxation has on tax revenue, part of which is used to reduce the cost of participating in higher education, (iii) the mediating role played by the level of efficiency with which the higher education system imparts graduates with skills suitable for the market. We structured the interactions between the two opposite effects (i) and (ii) as taking place in an environment where labor productivity is enhanced by the level of capital inflows, skilled and unskilled workers are imperfectly substitutable at the aggregate level, and uncertainty about the return on higher education stems from the imperfect alignment of the skills it imparts to graduates and those employers need.

When interpreting the predictions of our model, one should keep in mind that our focus on labor income inequality between skilled and unskilled workers led us to abstract from a number of other features of the real world. For instance, our model assumes that there is no international mobility of labor, skilled or unskilled. Taking emigration/immigration into account, however, may not necessarily alter our ranking of the two tax systems. For example, while a progressive tax may act as a *push-factor* for skilled emigration, if it drives the return to skills below its international level, it may act as a pool factor for unskilled immigration, if it raises the domestic return to unskilled labor above its international level. It is not clear, in a general equilibrium how individuals decision on whether or not to pursue higher education will be impacted. Likewise, while a flat tax may act as a *pool-factor* for skilled immigration, it may also act as a *push-factor* for unskilled emigration. Again, it is not clear, in a general equilibrium, how this will impact participation rates in higher education. Furthermore, emigration/immigration entails costs to migrants due to frictions in the search for overseas employment opportunities. These costs may or may not differ by skill status, further blurring the picture of the ranking of these two alternative tax systems.

5. Appendix

In this section, we provide the proofs of some of the results.

5.1. Proof of Proposition 2

In this sub-section, we provide the proof of Proposition 2. Our main claim is as follows: given θ,

$$\frac{d\bar{W}^u}{d\bar{S}_\theta} > 0.$$

To prove this claim, differentiate expression (2.20) with respect to $\bar{S}_\theta$, rearranging terms, to get:

$$\begin{aligned}\frac{d\bar{W}^u}{d\bar{S}_\theta} &= \bar{A}\,(1-\rho)\,\frac{\left[\lambda\phi\left(\lambda\bar{S}_\theta+\gamma\right)^{\rho-1}-\delta\,(1-\phi)\left(1-\delta\bar{S}_\theta\right)^{\rho-1}\right]N_u P}{D^2}\\ &\quad -\bar{A}\,(1-\rho)\,\frac{\left[\lambda\gamma\phi\left(\lambda\bar{S}_\theta+\gamma\right)^{\rho-2}-\delta\,(1-\phi)\left(1-\delta\bar{S}_\theta\right)^{\rho-2}\right]}{D}\end{aligned} \tag{5.1}$$

where D and N_u denote, respectively, the denominator and the numerator of expression (2.20), and

$$P := \left[\phi\left(\lambda\bar{S}_\theta+\gamma\right)^{\rho}+(1-\phi)\left(1-\delta\bar{S}_\theta\right)^{\rho}\right]^{\frac{-1}{\rho}}. \tag{5.2}$$

D, N_u, and P are all strictly positive. The derivative in (5.1) is an algebraic sum of two terms. Observe that the difference $\lambda\gamma\phi\left(\lambda\bar{S}_\theta+\gamma\right)^{\rho-2}-\delta\left(1-\phi\right)\left(1-\delta\bar{S}_\theta\right)^{\rho-2}$ is non-positive if and only if

$$\frac{\gamma\phi\lambda}{\delta\left(1-\phi\right)}\leq\left(\frac{\lambda\bar{S}_\theta+\gamma}{1-\delta\bar{S}_\theta}\right)^{2-\rho}. \tag{5.3}$$

Therefore, since by Assumption $A.1$ unskilled workers are less productive than skilled workers in the operation of the high-tech production process, choosing γ — the relative productivity of unskilled workers in the operation of the high-tech process— sufficiently small ensures that inequality (5.3) holds, which, in turns guarantees that the second terms of (5.1) is non-negative.

Next, consider the difference $\lambda\phi\left(\lambda\bar{S}_\theta+\gamma\right)^{\rho-1}-\delta\left(1-\phi\right)\left(1-\delta\bar{S}_\theta\right)^{\rho-1}$ in the first term of (5.1). It can be shown that this difference is strictly positive if and only if

$$\frac{\lambda\phi}{\delta\left(1-\phi\right)}>\left(\frac{\lambda\bar{S}_\theta+\gamma}{1-\delta\bar{S}_\theta}\right)^{1-\rho},$$

which is guaranteed under Assumption $A.2$. Therefore, the first term of (5.1) is strictly positive since $\rho<1$. Hence the result. This completes the proof.

5.2. Proof of Proposition 3

To prove this proposition, we differentiate expression (2.22) with respect to $\bar{S}_\theta$ and rearrange terms to get:

$$\frac{d}{d\bar{S}_\theta}\hat{\Delta}\left(\bar{S}_\theta\right)=-\frac{\left(1-\rho\right)\hat{\Delta}\left(\bar{S}_\theta\right)}{\lambda\bar{S}_\theta+\gamma}\left[\frac{\lambda^2\phi+\delta^2\left(1-\phi\right)\left[R\left(\bar{S}_\theta\right)\right]^{\rho-2}}{\lambda\phi-\delta\left(1-\phi\right)\left[R\left(\bar{S}_\theta\right)\right]^{\rho-1}}-\frac{\left[\lambda\phi-\delta\left(1-\phi\right)\left[R\left(\bar{S}_\theta\right)\right]^{\rho-1}\right]}{\phi+\left(1-\phi\right)\left[R\left(\bar{S}_\theta\right)\right]^{\rho}}\right],$$

where

$$R\left(\bar{S}_\theta\right):=\frac{1-\delta\bar{S}_\theta}{\lambda\bar{S}_\theta+\gamma}.$$

Since $\rho<1$, this derivative is negative if and only if

$$\frac{\lambda^2\phi+\delta^2\left(1-\phi\right)\left[R\left(\bar{S}_\theta\right)\right]^{\rho-2}}{\lambda\phi-\delta\left(1-\phi\right)\left[R\left(\bar{S}_\theta\right)\right]^{\rho-1}}>\frac{\left[\lambda\phi-\delta\left(1-\phi\right)\left[R\left(\bar{S}_\theta\right)\right]^{\rho-1}\right]}{\phi+\left(1-\phi\right)\left[R\left(\bar{S}_\theta\right)\right]^{\rho}}.$$

Since $\phi < 1$, this inequality can be shown to reduce to

$$\lambda^2 \left[R\left(\bar{S}_\theta\right)\right]^\rho + \delta^2 \left[R\left(\bar{S}_\theta\right)\right]^{\rho-2} + 2\delta\lambda \left[R\left(\bar{S}_\theta\right)\right]^{\rho-1} > 0,$$

which is unambiguously positive. Hence the result. This completes the proof.

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