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Optimal Redistributive Pensions and the Cost of Self-Control

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Abstract:
We examine how the introduction of self-control preferences influence the trade-off between two fundamental components of a public pension system: the contribution rate and its degree of redistribution. The pension regime affects individuals' welfare by altering how yielding to temptation (i.e. not saving, or saving less) is attractive. We show that proportional taxation increases the cost of self-control, and that this adverse effect is more acute when public pensions become more redistributive.

Keywords: Taxation, Redistribution, Pensions, Self-control

JEL Classification: H55, H21, D03

1 Introduction

We analyze an optimal public pensions scheme where some individuals have self-control preferences (Gul & Pesendorfer, 2001, 2004).[1] With such utility representation, they tentatively accommodate two competing desires. On the one hand, they value commitment and would appreciate sticking to a smooth consumption pattern over time. On the other hand, saving necessitates cognitive self-control that imposes an immediate cost.

The mental cost of self-control arises because an individual who saves remains aware of the immediate gratification he *could have* had by consuming all his available liquidity. For example, deciding to save $100 today entails immediately depriving oneself from a nice dinner in a restaurant, which is mentally suffering. Absent any restaurant, the mental cost would vanish. Additionally, partly succumbing to immediate temptation leaves him with less cash on hand in the future, a source of liquidity which would have become an eventual source of temptation. Hence, savings are delayed as the outcome of an inter-temporal compromise between the benefits of commitment and the mental costs of resisting temptation.

In this paper, we examine how the introduction of self-control preferences influence the trade-off between two fundamental components of a public pension system: the contribution rate and its degree of redistribution. The pension regime affects individuals' welfare by altering how yielding to temptation (i.e. not saving, or saving less) is attractive. We show that proportional taxation increases the cost of self-control, and that this adverse effect is more acute when public pensions become more redistributive.

We find that forced savings, that take the form of mandatory pension contributions and contributory-based pension benefits, tend to increase the mental cost of voluntarily savings. It thus partly offsets the beneficial mandatory-savings benefits of the system. The mental

[1]These are increasingly used, in particular to formalize problems of preference reversals (Noor (2007)), but also because introspective, empirical and experimental evidence suggest the existence of costly self-control (Frederick et al., 2002; Bucciol, 2012; Huang et al., 2007; Krussel et al., 2010).

cost of self-control depends on the difference between what one would have consumed without saving at all, and what is actually consumed. Distortive pension contributions reduce the value of both options, but the negative marginal effect is stronger via the latter. It is therefore optimal for the government — at the margin — to focus less on forcing people to save, and more on the redistributive aspect of the scheme. Although simple, this underlying intuition has not yet been pointed out, possibly because no traditional optimal pensions exercise have yet been done with self-control preferences.[2]

Despite their appealing features, the self-control preferences have largely been ignored in the normative taxation literature. Thus, our paper is the first to characterize the tradeoff between the redistributive and forced-saving roles of public pensions in such context. Obtaining tax formulas allows us to present a clear characterization of the effects of taxation and redistribution on the cost of self-control, and to provide an analytical expression for the marginal effect of taxation on it.

Also, even though the topic of social security with self-control preferences have been treated in the macroeconomic literature, our method and results depart from it in several ways. Although our purpose is to present a normative exercise, and not a calibration, it is noteworthy that our results lie on joint assumptions that have not yet been tested (income heterogeneity, endogenous labor supply and endogenous redistribution). A related paper by Kumru & Thanopoulos (2008) studied numerically the welfare effect of social security as compared to a benchmark economy populated by time-inconsistent agents. Individuals are identical (but face idiosyncratic risks). They find that self-control preferences can mitigate the adverse welfare costs of social security, focusing on the case of a convex temptation function. Also, (Kumru & Thanopoulos, 2011) shows that the elimination of social security may not be optimal when the intensity of the self-control problem is high. Bucciol (2011) allowed households to allocate their time between labor and leisure, but with a pension system that has no redistributive objective. Solely focusing on a convex temptation ranking,

[2] Krussel et al. (2010) studied a Ramsey tax problem with linear taxes in a single-agent model, and advocated late consumption and savings subsidies.

he concludes that social security can be welfare-improving in the presence of self-control preferences, also obtaining the special case that payroll taxation reduces the mental cost of resisting temptation.

2 The model

The economy consists of a large number of individuals who live for three periods. They differ with respect to their productivity and to the intensity of their self-control problems. Heterogeneity in productivity is captured by the existence of $N > 1$ exogenously given wage rates, which are denoted by w^i with $w^1 < ... < w^N$. The intensity of one's self-control problem is captured by a parameter $\lambda^j \in \{0, \lambda\},\ \lambda > 0$. As will become clear shortly, an individual with $\lambda^j = \lambda$ has a self-control problem whereas one with $\lambda^j = 0$ has not. There is a fixed proportion π^{ij} of type-ij individuals. We normalize the total population to one. In the first two periods denoted by $t = 0, 1$ they supply labor (L_t^{ij}) and save (s_t^{ij}). Individuals are liquidity constrained, so $s_t^{ij} \geq 0$. This ensures that public pension claims cannot be used as a collateral to obtain consumption credit (Lindbeck & Persson, 2003). Labor income is taxed at proportional rate τ. The proceeds of the tax are capitalized at an exogenous market interest rate r, and are used to finance the pension benefits b^{ij} that are paid to individuals in their old age, at $t = 2$.

Let us call by c_t^{ij} the consumption level of an individual at time t. Without loss of generality, they are expressed net of disutility of labor, which is captured by the strictly convex cost function $\varphi(L)$.[3] Consumption levels are given by

$$c_t^{ij} = (1 - \tau) w^i L_t^{ij} - \varphi(L_t^{ij}) + s_{t-1}^{ij}(1 + r) - s_t^{ij}, \quad t = 0, 1. \tag{1}$$

[3] It satisfies $\varphi(0) = 0,\ \varphi'(0) = 0,\ \varphi'(L) > 0\ \forall L > 0$, and $\varphi''(L) > 0$.

and

$$c_2 = s_1^{ij}(1+r) + b^{ij}, \quad t = 2. \tag{2}$$

Individuals exhibit self-control preferences (Gul & Pesendorfer, 2001, 2004):

$$U^{ij} \equiv \sum_{t=0}^{1} \beta^t \left[u(c_t^{ij}) + \lambda^j v(c_t^{ij}) - \lambda^j v(\hat{c}^{ij}) \right] + \beta^2 u(c_2^{ij}). \tag{3}$$

where c_t^{ij} is the consumption level one would get if no self-control was exerted at all. This most tempting option satisfies $\hat{c}_t^{ij} \equiv \max_{L,s} \lambda v(w^i(1-\tau) - \varphi(L^{ij}) - s)$. Labor supply under the most tempting option is implicitly given by $\varphi'(L^{ij}) = (1-\tau)w^i$ and savings are null.

Redistributive pensions

Let us first introduce the simplified notation Y^{ij} for the value of one's lifetime labor earnings, from the point of view of $t = 2 : Y^{ij} = \sum_{t=0}^{1} w_t^i L_t^{ij}(2-t)$. Thus, when one reaches retirement time, his lifetime capitalized contributions to to the public pensions fund are τY^{ij}. Let us also denote by $E[Y] = \sum_{ij} \pi^{ij} Y^{ij}$ the average (capitalized) lifetime earnings in the economy.

We introduce the Bismarckian parameter α that captures the tradeoff between the forced-savings and redistributive roles of the pension system (Cremer et al., 2008). The pension benefit of an individual ij is a linear combination of his own contributions and of the average's:

$$b^{ij} = \alpha\tau Y^{ij} + (1-\alpha)\tau E[Y] \tag{4}$$

The contributory/Bismarckian benefit $\alpha\tau Y^{ij}$ is a forced saving device. The lump-sum/Beveridgian one is redistributive, since everyone receives it independently of their past income or contributions. Increasing α forces people to save more, a policy objective that conflicts with its redistributive counterpart.[4] A purely Bismarckian system has $\alpha = 1$ whereas a Beveridgian system has $\alpha = 0$. A pension system featuring $\alpha < 0$ is targeted since individuals are implicitly taxed for their contributions. Such system is very redistributive, but also highly very distortionary.

Is the mental cost of self-control afflicting the rich, or the poor?

Let us recall the main insights provided by the self-control preferences, as expressed in (3). The function $u(\cdot)$ is just a typical utility function, or a "commitment ranking" which is strictly concave and meets Inada's requirements. The function $\lambda^j v(\cdot)$ is a temptation ranking that captures the welfare effect of temptation. Someone who has no issue with self-control ($\lambda^j = 0$) will want to smooth consumption over his life cycle, provided that $\beta(1+r) = 1$ (an assumption that we make for simplicity). We call such individual a life-cycler.

For individual with $\lambda^j = \lambda$, consuming all current cash-on-hand is tempting. Let us momentarily ignore time and type indexes, and denote by $\hat{c}$ one's consumption when labor supplies and savings are chosen by a perfectly myopic individual. This is the most tempting option for someone who experiences problems of self-control. Such an individual derives immediate utility $u(c) + \lambda v(c) - \lambda v(\hat{c})$, where $\hat{c}$ is the most tempting option and c is the consumption level that is actually chosen, where $\hat{c} > c$. It is typical to call $\lambda(v(\hat{c} - v(c)) > 0$ the cost of self-control one imposes to oneself by saving money in the face of temptation.

Before designing an optimal pension plan, it is instructive to refer to the laisser-faire solution to assess the economic significance of the shape of the temptation ranking $v(\cdot)$. Over one's lifetime, the total *cost of self-control* that will be experienced is

$$\gamma^{ij} = \lambda^j \sum_{t=0}^{1} \beta^t [v(\hat{c}^{ij}) - v(c_t^{ij})] > 0. \tag{5}$$

[4] Note that the pension plan's budget is always balanced by definition, as is typical in linear-progressive tax models.

Let us call by $V^{ij}(w^i, \lambda^j)$ the indirect utility function of an individual who maximizes his utility, and where no public pension plan is offered. Proposition 1 shows that the cost of self-control increases in w^i if the temptation-ranking is convex, and that it decreases with w^i otherwise.

Proposition 1. *In a laissez-faire equilibrium, lifetime commitment utility is strictly increasing with respect to w^i. However, the cost of self-control is strictly increasing in w^i if $v''(\cdot) > 0$, and strictly decreasing in w^i if $v''(\cdot) < 0$.*

Proof: Using the Envelope theorem, differentiating one's indirect utility function with respect to w^i yields

$$\frac{\partial V^{ij}(w^i, \lambda^j)}{\partial w^i} = \underbrace{\sum_{t=0}^{1} \beta^t L_t^{ij} u'(c_t^{ij})}_{\Delta \text{ comm. utility}} - \underbrace{\sum_{t=0}^{1} \beta^t L_t^{ij} \lambda^j (v'(\hat{c}_t^{ij}) - v'(c_t^{ij}))}_{\Delta \text{cost self-control}}. \tag{6}$$

Since u is strictly increasing in its argument, the first term within brackets implies that commitment utility is strictly increasing in w^i as well. For individuals with problems of self-control, since marginal tax rates are null, $L_t^{ij} = \hat{L}_t^{ij}$ for all i. By the definition of the maximization problem, $\hat{c}_t^{ij} \geq c_t^{ij}$ holds with strict equality if savings are positive in at least one period. Because u satisfies the Inada conditions $s_1^{ij} > 0$ and $\gamma^{ij} > 0$. Thus, the net effect of w^i on the cost of self-control relies solely on the sign of $v'(\hat{c}_t^{ij}) - v'(c_t^{ij})$. $\square$

Proposition 1 has important economic features, and has significant consequences for potential policy involvements. Absent any public intervention, the effect of w^i on the cognitive cost of self-control depends on the difference between the marginal temptation-utility of actual consumption and that of the most tempting consumption level. The difference between both can take either sign, depending on the shape of temptation, which is itself characterized by the sign of $v''(\cdot)$. It is noteworthy that the axioms underlying the self-control allow the function $v(\cdot)$ to be either concave or convex.[5]

[5] The only requirement is that the problem must be globally concave, or that $u''(c_t^{ij}) + \lambda^j v''(c_t^{ij}) < 0$, $\forall ij$.

For the sake of optimal taxation, the curvature of the temptation ranking $v(\cdot)$ turns out to be highly relevant. If $v(\cdot)$ is strictly concave, the cost of temptation is more significant for poorer individuals. Depriving themselves from consuming more than they could is very costly when they are poor, and the magnitude of the problem declines with income. This is consistent with Mullainathan & Banerjee (2010) who show that the poor may be more likely to exhibit a hands-to-mouth type of behavior when fulfilling basic needs is involved. Spears (2011), Bernheim et al. (2012) and Shah et al. (2012) also reached similar conclusions. However, assuming that the function is strictly convex may also be sensible. Richer households facing a higher mental cost of self-control could be consistent with the inability for a rich person to resist engaging in conspicuous consumption, possibly accounting for the presence of positional externalities.

2.1 Redistributive pensions

Let us denote by $V^{ij}(w^i, \lambda^j; \alpha, \tau)$ the indirect utility function of an individual ij, which solves:

$$V^{ij}(w^i, \lambda^j; \alpha, \tau) \equiv \max_{\{s_t^{ij}, L_t^{ij}\}_{t=0}^{1}} \sum_{t=0}^{1} \beta^t \left[u(c_t^{ij}) + \lambda^j v(c_t^{ij}) - \max_{L_t^{ij}, s_t^{ij}} \lambda^j v(\hat{c}_t^{ij}) \right] + \beta^2 u(c_2^{ij}), \qquad (7)$$

subject to (1), (2) and to the liquidity constraint $s_t^{ij} > 0, \quad t = 0, 1$. As is typical with linear-progressive taxation models, individuals do not internalize the effect of their own labor supply decisions on the lump-sum (Beveridgian) part of the pension plan. Inserting the first three constraints directly in (7), individual decisions are given by the following first-order conditions:

$$[(w^i(1-\tau)-\varphi'(L_0^{ij})][u'(c_0^{ij})+\lambda^j v'(c_0^{ij})]+\alpha\tau w^i u'(c_2^{ij})=0, \qquad (L_0^{ij})$$

$$[(w^i(1-\tau)-\varphi'(L_1^{ij})][u'(c_1^{ij})+\lambda^j v'(c_1^{ij})]+\alpha\tau w^i u'(c_2^{ij})=0, \qquad (L_1^{ij})$$

$$-[u'(c_0^{ij})+\lambda^j v'(c_0^{ij})]+[u'(c_1^{ij})+\lambda^j v'(c_1^{ij})]-\lambda^j v'(\hat{c}_1^{ij})\leq 0, \qquad (s_0^{ij})$$

$$-[u'(c_1^{ij})+\lambda^j v'(c_1^{ij})]+u'(c_2^{ij})\leq 0, \qquad (s_1^{ij})$$

where labor supply always find interior solutions, but where the two last conditions strictly equal zero only when the liquidity constraints are not binding. As in the laisser-faire equilibrium, from (s_0^{ij}) and (s_1^{ij}) above one can readily see that all individuals with self-control issues delay savings.

The assumptions on the disutility of labor $\varphi(\cdot)$ ensure interior solutions for individual labor supply. We get

$$\varphi'(L_t^{ij})=(1-\tau)w^i+\alpha\tau w MRS_{t,2}^{ij} \qquad (8)$$

where $MRS_{t,2}^{ij}$ is one's marginal rate of substitution between one unit of consumption at $t=0,1$ and one unit after retirement. Equation (8) exhibits the good incentive properties of a system that is Bismarckian. When $\alpha>0$ individuals reduce their labor supply when taxed more, but such distortion is partly alleviated because they are aware that a share of their contribution will be paid back to them at $t=2$.

Welfare effect of policies

We can derive expressions for the effect of an exogenous change in the policy parameters, α and τ, on one's welfare. We do so by using both the envelope theorem and the fact that individuals take the Beveridgian component of public pensions as given when making decisions.

A marginal increase in α has the following effect on one's welfare:

$$\frac{\partial V^{ij}(w^i, \lambda^j; \alpha, \tau)}{\partial \alpha} = u'(c_2^{ij})\tau \left[Y^{ij} - E(Y) + (1-\alpha)\frac{\partial E(Y)}{\partial \alpha} \right]. \tag{9}$$

Making the system more Bismarckian increases the welfare of those whose lifetime income is larger than average, by making them benefitting more from their own contributions. By the same token, it penalizes retirees whose lifetime income was lower than the average's. The second effect, this time via the tax-base, is beneficial to all since $E(Y)$ is increasing in α. Making the system more contributory has a positive effect on labor supply, thereby reducing the distortions entailed by income taxes.[6]

The effect of a marginal increase in τ is somewhat more complex. Taking the derivative of one's indirect utility function with respect to the tax rate and reorganizing terms yields

$$\begin{aligned} \frac{\partial V^{ij}(w^i, \lambda^j; \alpha, \tau)}{\partial \tau} = & \overbrace{\sum_{t=0}^{1} \beta^t [u'(c_2^{ij}) - u'(c_t^{ij})] w^i L_t^{ij}}^{\text{Consumption smoothing}} + \overbrace{w^i \lambda^j \sum_{t=0}^{1} [\hat{L}^{ij} v'(\hat{c}_t^{ij}) - L_t^{ij} v'(c_t^{ij})]}^{\text{Cost of self-control}} \\ & + \underbrace{(1-\alpha) u'(c_2^{ij})(E(Y) - Y^{ij})}_{\text{Equity}} + \underbrace{(1-\alpha)\tau u'(c_2^{ij}) \frac{\partial E(Y)}{\partial \tau}}_{\text{Efficiency}}. \end{aligned} \tag{10}$$

The right-hand side of (10) clarifies the four different effects of taxation on individuals' welfare. The first term, $\sum_{t=0}^{1} \beta^t [u'(c_2^{ij}) - u'(c_t^{ij})] w^i L_t^{ij}$, is the consumption-smoothing benefit of taxation. By displacing consumption from early periods to retirement, it increases the value of one's commitment ranking unless it induces $c_t^{ij} < c_2^{ij}$ for some liquidity-constrained agents. The second term, which is of high interest to us, it the effect of taxation on the cost of self-control γ^{ij} for $j = 1$. Its sign, which is not fully characterizable analytically without imposing functional forms, is analyzed in proposition 2:

[6] That $\partial E(Y)/\partial \alpha > 0$ can be observed from the first-order conditions with respect to L_t^{ij}, although the comparative statics is highly intractable in our three-period model.

Proposition 2. ***Effect of taxation on the cost of self-control.*** *The net effect of payroll taxation is to* ***increase*** *the cost of self-control for all individuals with* $j = 1$ *if* $v'' < 0$ *and* $\alpha \geq 0$. *It* ***reduces*** *it when* $v'' > 0$ *and* $\alpha \leq 0$. *The effect is ambiguous otherwise.*

Proof: Appendix.

2.2 Problem of the government and tax formulas

The government's problem is to choose the optimal values for τ and α that maximize its social objective. We continue assuming that the social welfare function is weighted utilitarian and that it assigns a weight ω^{ij} on each type-ij individual. The optimal tax problem is rather straightforward, given the non-paternalistic nature of the policy. Individuals' perception of their own welfare is equivalent to their indirect utility functions $V^{ij}(w^i, \lambda^j; \alpha, \tau)$ for all ij, which is aggregated as such by the social welfare function.[7] The government solves

$$(\alpha^*, \tau^*) \equiv \arg\max_{\alpha,\tau} \sum_{t=0}^{2} \sum_{ij} \pi^{ij} \omega^{ij} V^{ij}(w^i, \lambda^j; \tau, \alpha) \tag{11}$$

where α^* and τ^* denote the solution to the maximization problem, and where the pension plans' balanced budget constraint is implicitly included in the indirect utility functions of individuals. Given that the problem is globally concave, interior solutions for the policy parameters are characterized by the first-order conditions

$$\sum_{t=0}^{2} \sum_{ij} \pi^{ij} \omega^{ij} \frac{\partial V^{ij}(w^i, \lambda^j; \alpha, \tau)}{\partial \alpha} = 0 \tag{12}$$

$$\sum_{t=0}^{2} \sum_{ij} \pi^{ij} \omega^{ij} \frac{\partial V^{ij}(w^i, \lambda^j; \alpha, \tau)}{\partial \tau} = 0 \tag{13}$$

[7] This contrasts sharply with paternalistic objectives found with models of myopia or quasi-hyperbolic discounting (discussed below in this paper) in which the government considers that individuals make time-inconsistent mistakes in decision-making.

where the partial derivatives of the indirect utility functions with respect to α and τ are respectively expressed in (9) and (10). Reorganizing the first-order condition allows us to obtain implicit tax formulas. We denote by $\xi^{ij} \equiv \omega^{ij} u'(c_2^{ij})$ the marginal social value of an increase in old-age revenue of a type-ij individual. Given decreasing marginal utility, ξ^{ij} is decreasing with retirement income and increasing with the welfare weights. The implicit policy formula for α^* is characterized by

$$\mathrm{cov}(\xi, Y) + (1-\alpha)E(\xi)\frac{\partial E(Y)}{\partial \alpha^*} = 0. \tag{α^*}$$

Equation (α^*) reflects the typical equity-efficiency tradeoff in optimal taxation. A negative covariance term between $E(Y)$ and his marginal utilities of retirement consumption strengthen the case for more redistribution (lower α). However, the desirable effect of making the system more contributory-based counterbalances equity concerns.

The behavioral role of the pension system must figure in the implicit tax formula, which is:

$$\tau^* = \frac{\overbrace{\mathrm{cov}(\xi, Y)}^{\text{Equity}}}{E(\xi)\dfrac{\partial E(Y)}{\partial \tau^*}} + \frac{\overbrace{\sum_{t=0}^{1} \beta^t E[\omega w_t L_t(u'(c_2) - u'(c_t))]}^{\text{consumption smoothing effect}} + \overbrace{\sum_{t=0}^{1} \beta^t E[\omega\lambda(w\hat{L}_t v'(\hat{c}_t) - wL_t v'(c_t))]}^{\text{self-control effect} \lessgtr 0}}{\underbrace{-(1-\alpha)E(\xi)\dfrac{\partial E(Y)}{\partial \tau^*}}_{\text{Distortions} > 0}}. \tag{τ^*}$$

To clarify how the cost of self-control affects the optimal tax rate, we have divided the right-hand side of (τ^*) into two parts. The first one captures the traditional equity (numerator) and efficiency (denominator) tradeoff that we find in linear-progressive optimum tax models.

The rightmost term in, which has the labor-market distortion in the denominator, contains two components in the numerator. These are the terms in importance here, because they capture the two roles of public pensions that we want to emphasize, namely consumption-smoothing (commitment effect) and its effect on the mental cost of exerting self-control. For clarity, both of them are textually identified in (τ^*).

The social consumption-smoothing benefits are due to forced savings, which helps satisfying individuals' commitment rankings. It is generally positive in the presence of individuals with problems of self-control, unless the society consists of an large number of liquidity-constraints who end up consuming more during their retirement years than when younger. Accordingly, the consumption-smoothing benefit of taxation seems to justify higher tax rates.

However, that commitment benefit may conflict with the effect of an increase in taxes on individuals' costs of self-control. If taxation reduces someone's cost of exerting self-control, that individual will be induced to save more by himself. In such case, the consumption-smoothing and self-control effects of taxation go in the same direction in the implicit tax formulas. We find that for a significant family of cases, increasing taxes also increases individuals' costs of self-control, thereby offsetting the consumption-smoothing benefits of taxation.

2.3 Taxation and redistribution increase the cost of self-control: numerical examples

We provide a numerical illustration of how the forced-savings (commitment) role of the pension system may conflict with its effect on the aggregate mental costs of self-control in the economy. The results that are reported are a representative and nuanced subset of the several simulated experiments that we ran with the model. Given the normative nature of our optimal tax exercise, one should note that we do not try to calibrate a real economy, but we rather seek to get a sense out of tax formulas that have no close-form.

The government maximizes a utilitarian utility function, so $\omega^{ij} = 1$ for all ij. Wages are distributed according to a beta(2,4) distribution, discretized on the domain [1,4], which induces income inequality. The commitment ranking is logarithmic with $u(x) = \log(x)$ and the temptation ranking, which is allowed to be both convex or concave, take the CRRA form $v(x) = x^{1-\rho}/(1-\rho)$. The interest rate and discount factors satisfy $\beta = 1/(1+r)$ where $r = 0$, and none of our results hinge on this number. We conduct simulations with a strictly concave temptation ranking where $\rho = 0.5$ and with a strictly convex one, where $\rho = -0.5$.

Tables A and B report the optimal policies when $v''(\cdot) < 0$ and when the intensity of the self-control problem is $\lambda = 0.1$. We denote by $\pi \in [0,1]$ the proportion of individuals who have a problem of self-control.[8] Thus, the optimal policy in an economy riddled with self-control problems is reported on the first line of the table, whereas that in an economy populated with life-cyclers only figures on the very last line. For several values of π, we show the optimal policy (α^*, τ^*). Additionally, we report what we call the marginal behavioral welfare effects of pension taxation, evaluated at the optimal policy. These are the consumption-smoothing benefits due to forced savings:

$$\sum_{t=0}^{1} \beta^t E[w_t L_t (u'(c_2) - u'(c_t))] \tag{14}$$

and the marginal social welfare effect of taxation due to it affecting costs of self-control

$$\sum_{t=0}^{1} \beta^t E[\lambda(w \hat{L}_t v'(\hat{c}_t) - w L_t v'(c_t))]. \tag{15}$$

Recall that both of these terms are identified in the tax formula (τ^*).

As a general finding, we find that the tax rates and the extent to which the pension plan is contributory increase with both the proportion of individuals who have self-control problems, and with the intensity of self-control problems. This should seem intuitive, since when the

[8] This proportion is independent of wages. As discussed earlier in the paper, the relationship between wages and the intensity of the self-control problems is already captured by the concavity/convexity of the temptation ranking.

intensity of self-control problems λ increases, individuals with such problems displace more consumption towards early periods. Thus, when a high proportion of individuals have self-control issues, the forced-saving role of pensions is important and a large portion (around one third) of pension benefits are Bismarckian.

However, as our theoretical results show, optimal tax rate and Bismarckian factors increase with λ and π because it provides self-control individuals with a forced saving device, but not because the pension system reduces their marginal cost of exerting self-control. To clearly observe this, refer to the rightmost columns in tables A and B, which give the marginal social welfare of taxation do to it affecting the cost of self-control. In both cases, one can see that this effect is always negative: taxation *increases* the costs of self-control, and partly offsets the forced savings role of the pension system. As a logical consequence, we see that when λ goes from 0.1 to 0.25 (i.e. when passing from table A to table B), the marginal increase in the cost of self-control induced by taxation roughly doubles. Thus, if α and τ increase with the intensity of self-control, it is simply because the forced-savings benefits of the pension system increase accordingly. In this regard, the roles of social security with self-control preferences is comparable to that under a time-inconsistent, paternalistic policy.

It is no surprise, however, that the negative social welfare effect of taxation generally does not fully offset the positive consumption-smoothing effect of the pension system. If it was the case, the only role of public pensions would be to redistribute income. In such situation, we would have a purely Beveridgian or a targeted system, even when a large proportion of individuals has self-control problem.

Finally, in table C we consider the case where all individuals have self-control problems ($\pi = 1$), and we provide the optimal policy for some very large values of λ. One can then observe that for reasonably low intensities of self-control problems a larger λ is associated to more forced savings. However, when λ becomes outstandingly large, the negative self-control effect of taxation tends to drive α down and the optimal Bismarkian term α^* starts

Table A: Policy and marginal behavioral effects: $v''(x) < 0$ and $\lambda = 0.1$

	Optimal policy		Marginal behavioral effects	
π	α^*	τ^*	Cons. Smooth.	Self-Control
1.0	0.3245	0.1592	0.1640	-0.0483
0.9	0.3090	0.1573	0.1363	-0.0427
0.8	0.2849	0.1545	0.1201	-0.0370
0.7	0.2607	0.1517	0.0968	-0.0315
0.6	0.2121	0.1461	0.0623	-0.0262
0.5	0.1565	0.1399	0.0462	-0.0211
0.4	0.0875	0.1325	0.0316	-0.0162
0.3	0.0500	0.1271	-0.0103	-0.0123
0.2	0.0500	0.1250	-0.0631	-0.0085
0.1	-0.3267	0.0978	0.0017	-0.0036
0.0	—	0.0281	—	—

decreasing with λ.

Let us now consider the case where $v''(\cdot) > 0$. As before, the optimal τ and α both increase when the intensity of self-control problem increases, and when a larger share of the population is subject to it. In all the simulations that we have ran we have found that, for high values of λ, a similar tradeoff as with the concave temptation ranking operates. However, the effect on the cost of self-control is welfare-enhancing when a small share of the population has self-control problems, which induces an optimal policy where α becomes small (redistribution becomes dominant), and taxes as well. This result should nonetheless be nuanced: in the optimum, taxation increasingly reduces the marginal cost of self-control as government gradually "gives up" on forced savings an only focuses on its normative objective (redistribution). So, one can hardly think of pension taxation as a useful device to reduce mental costs of self-control that are wither very severe, or highly prevalent in the economy.

Table B: Policy and marginal behavioral effects: $v''(x) < 0$ and $\lambda = 0.25$

	Optimal policy		Marginal behavioral effects	
π	α^*	τ^*	Cons. Smooth.	Self-Control
1.0	0.3763	0.1671	0.4957	-0.1020
0.9	0.3794	0.1666	0.4136	-0.0925
0.8	0.3620	0.1646	0.3538	-0.0794
0.7	0.3475	0.1625	0.2849	-0.0676
0.6	0.3250	0.1597	0.2612	-0.0549
0.5	0.2863	0.1552	0.1790	-0.0430
0.4	0.2350	0.1493	0.1714	-0.0304
0.3	0.1850	0.1426	0.0951	-0.0210
0.2	0.0950	0.1338	0.0172	-0.0118
0.1	0.0500	0.1251	-0.0495	-0.0056
0.0	—	0.0281	—	—

3 Concluding comments

This paper analyzed an optimal public pension scheme when individuals' wellbeing is characterized by self-control preferences. We focus on the effect of taxation on the costs of exerting the required self-control to voluntarily save. We study cases in which that cost decreases with productivity levels and that where it increases with productivity. We find that the commitment benefits of pension taxation can be offset by it increasing the cost of self-control. Thus, in a non-negligible and realistic set of situations, the joint presence of temptation and self-control weakens the rationale for forced savings.

Deriving an optimal-linear pension scheme allowed us to find simple tax formulas and to characterize the possibly competing commitment and self-control effects of taxation. One possible criticism of it is its partial equilibrium nature, and the fact that the only source of distortions comes from the non-contributory part of the pension benefit formula (as is typically the case with static linear-progressive taxation models). The important element for our results is that self-control preferences induce a wedge between marginal temptation-utility

Table C: Policy and marginal behavioral effects: $v''(x) < 0$ and $\pi = 1$

	Optimal policy		Marginal behavioral effects	
λ	α^*	τ^*	Cons. Smooth.	Self-Control
5	0.1158	0.1661	1.0481	-0.1172
3	0.1610	0.1668	0.9523	-0.1482
1	0.2839	0.1671	0.7413	-0.1807
0.35	0.3601	0.1671	0.5576	-0.1247
0.25	0.3763	0.1671	0.4957	-0.1020
0.10	0.3245	0.1592	0.1640	-0.0483

of actual consumption, and that of the most tempting option. One should note that this effect would be robust to a more complex environment, including an overlapping-generation model with endogenous capital accumulation. One next step in this line of research is to study non-linear pension schemes, with which this wedge may be relaxed.

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Table D: Policy and marginal behavioral effects: $v''(x) > 0$ and $\lambda = 0.1$

	Optimal policy		Marginal behavioral effects	
π	α^*	τ^*	Cons. Smoothing	Self-Control
1.0	0.3600	0.1676	0.4925	-0.0744
0.9	0.3596	0.1662	0.4446	-0.0670
0.8	0.3582	0.1650	0.3856	-0.0595
0.7	0.3500	0.1601	0.4149	-0.0515
0.6	0.3300	0.1580	0.3997	-0.0423
0.5	0.2500	0.1522	0.1750	-0.0144
0.4	0.0500	0.1368	0.1730	0.0134
0.3	0.0496	0.1328	0.0989	0.0110
0.2	0.0000	0.1247	0.0377	0.0109
0.1	-0.1010	0.1150	-0.0403	0.0085
0.0	—	0.0281	—	—

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Table E: Policy and marginal behavioral effects: $v''(x) > 0$ and $\lambda = 0.25$

	Optimal policy		Marginal behavioral effects	
π	α^*	τ^*	Cons. Smooth.	Self-Control
1.0	0.3460	0.1680	0.5726	-0.1351
0.9	0.3411	0.1666	0.5161	-0.1202
0.8	0.3343	0.1649	0.4643	-0.1051
0.7	0.3266	0.1632	0.4027	-0.0903
0.6	0.3156	0.1610	0.3467	-0.0753
0.5	0.3012	0.1585	0.2900	-0.0604
0.4	0.2812	0.1552	0.2328	-0.0457
0.3	0.2530	0.1516	0.1618	-0.0314
0.2	0.2000	0.1439	0.1150	-0.0171
0.1	0.1100	0.1338	0.0193	-0.0050
0.0	—	0.0281	—	—

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