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Risk Aversion and Dynamic Games Between Hydroelectric Operators under Uncertainty

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Abstract: This article analyses management of hydropower dams within monopolistic and oligopolistic competition and when hydroelectricity producers are risk averse and face demand uncertainty. In each type of market structure we analytically determine the water release path in closed-loop equilibrium. We show how a monopoly can manage its hydropower dams by additional pumping or storage depending on the relative abundance of water between different regions to smooth the effect of uncertainty on electricity prices. In the oligopolistic case with symmetric risk aversion coefficient, we determine the conditions under which the relative scarcity (abundance) of water in the dam of a hydroelectric operator can favor additional strategic pumping (storage) in its competitor's dams. When there is asymmetry of the risk aversion coefficient, the firm's hydroelectricity production increases as its competitor's risk aversion increases, if and only if the average recharge speed of the competitor's dam exceeds a certain threshold, which is an increasing function of its average water inflows.

Keywords: Closed-loop Cournot competition, electricity wholesale market, hydropower dams, demand uncertainty, asymmetric risk aversion

JEL Classification: L94, Q25, C61, C73

Résumé: Cet article traite de la gestion des barrages réservoirs sous deux structures industrielles différentes : monopolistique et oligopolistique. Les producteurs hydroélectriques sont averses au risque et font face à de l'incertitude sur la demande. Nous déterminons analytiquement le sentier de prélèvement de l'eau à l'équilibre en boucle fermée. Nous montrons comment le monopole gère son parc hydroélectrique par pompage ou stockage supplémentaire en fonction de l'abondance relative de l'eau entre les différentes régions afin de lisser l'effet de l'incertitude sur le prix de l'électricité. Dans le cas oligopolistique à taux d'aversion au risque symétrique, nous déterminons les conditions sous lesquelles la rareté (abondance) relative de l'eau dans le barrage d'un opérateur hydroélectrique favorise un pompage (stockage) stratégique supplémentaire sur les barrages de ses concurrents. Lorsqu'il y a asymétrie du coefficient d'aversion au risque, la production hydroélectrique d'une firme augmente au fur et à mesure que l'aversion au risque de son concurrent augmente si et seulement si la moyenne de la vitesse de remplissage du barrage de son concurrent dépasse un certain seuil qui est une fonction croissante de la moyenne de ses flux de recharge.

Mots clés : Jeu à la Cournot en boucle fermée, marché de gros de l'électricité, les barrages réservoirs, incertitude de la demande, asymétrie d'aversion au risque

1 Introduction

In 2012, hydroelectricity supplied 16.3% of world electricity; growth has occurred almost all over the world (IPCC 2014).[1] Countries where hydroelectricity is the main source of electricity are Norway, at 98%, Brazil, at 97%, the Province of Quebec, Canada, at 90%, and New Zealand, at 80% (Cramps and Moreaux, 2001). Because of climate change, the contribution of hydroelectricity to world electricity is expected to grow. Indeed, decarbonizing (i.e. reducing the carbon intensity) electricity generation is a key component of cost-effective mitigation strategies intended to stabilize temperatures. In most integrated modelling scenarios, decarbonizing happens more rapidly in electricity generation than in the industrial, buildings, and transport sectors (IPCC, 2014). This will lead to the growth of renewable energy including hydropower.[2] Nonetheless, the economic literature has mainly analyzed the strategic behavior of electricity operators on the wholesale electricity market in purely thermal systems or mixed hydrothermal systems.[3] Purely hydroelectric industries with large water storage capacities have attracted little attention.

Ambec and Doucet (2002) examined the problem of managing run of river hydropower dams under a monopolistic and oligopolistic structure when water inflows are deterministic. They used a two-period model to show that the absence of a water market during hydroelectric production can engender two sources of loss of social welfare: suboptimal management of water resources and the exercise of market power. Van Ackere and Ochoa (2010) used a stylized deterministic simulation model to evaluate the impact of the liberalization of the hydroelectric industry on the

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[1] The share of renewables in global electricity generation approached 21% in 2012 (Enerdata, 2013), making renewables the third largest contributor to global electricity production, just behind coal and gas. IPCC (2014) expects that renewable energy will become the second-largest contributor before 2020. Renewable energy includes bioenergy, direct solar energy, geothermal energy, hydropower, ocean energy, and wind energy (IPCC, 2014).

[2] Nonetheless, as Fischedick et al. (2011) contends, the long-term percentage contribution of some individual renewable energy sources (e.g., hydropower, bioenergy, and ocean energy) to climate change mitigation may be limited by the available technical potential in countries where deep reductions in GHG emissions are sought.

[3] Examples are Scott and Read (1996), Von der Fehr and Sandsbraten (1997), Buschnell (1998), Crampes and Moreaux (2001), Garcia et al. (2001), Chaton and Doucet (2003), Dakhlaoui and Moreaux (2004), Gen and Thille (2011).

quantity and price of electricity produced. They show that total electricity production is clearly lower in a non-liberalized market. Haddad (2011) developed a two-period model that characterizes the effects of deterministic seasonal water inflows on storage capacity optimal water management.

Nonetheless, the works above have not integrated the risk dimension, which is quite salient for suppliers of electricity from hydraulic structures. Water reserves are renewed randomly by precipitation. Thus, in the extreme case that precipitation is zero. The water stock is a temporarily finite resource because using a unit of water stored in the dam would constitute one unit less for the following period. Given the climate change phenomenon and the associated series of extreme events (IPCC, 2014), the challenge of optimal management of this resource over several periods of time has become more pressing.

On the demand side, operators of hydroelectric plants are also facing several sources of uncertainty closely linked to different categories of electricity consumers. Residential demand strongly depends on climate conditions, which determine the intensity of use of home appliances, along with electricity prices (Reiss and White, 2005; Dergiades and Tsoulfidis, 2008). In contrast, commercial and industrial demand is strongly associated with economic conditions, and some macroeconomic policy changes (Dilaver and Hunt, 2011). Electricity demand (residential, industrial and commercial) therefore fluctuates over the short term, and operators may find it difficult to smooth prices (Genc and Thille, 2012). Climate change may also exacerbate water and energy tensions across sectors and regions, potentially impacting hydropower (either positively or negatively, depending on whether the potential climate-adaptation benefits of hydropower facilities are realized) and on other technologies that require water (Arent et al., 2011; Cisneros and Oki, 2014). Overall, fluctuations in water reserves between different hydroelectric sites coupled with electricity demand uncertainty may favor strategic behavior by hydroelectric operators in the harnessing of this potential energy.

Philpott et al. (2013) consider this dimension of risk in their analysis and show that risk hedging instruments can reduce losses of welfare associated with the presence of electricity supply uncertainty. However, these authors focus on equilibrium on a competitive market.[4] Garcia et al. (2001), Dakhlaoui and Moreaux (2004) and Genc and Thille (2011) explored the implications of imperfect competition on electricity markets. To the best of our knowledge, analytical works that examined imperfect competition when uncertainty is present did not consider hydropower producers' risk aversion. Yet as the IPCC (2014) argues, good knowledge of how various market structures operate is needed for enlightened decision making on arbitrage between different sources of energy.

In this paper, we analyze the behavior of hydropower producers that are risk averse and face electricity demand uncertainty. To do so, we develop a dynamic model in which the hydroelectric park comprise multiple mountain reservoir-type dams, and posit two different industrial structures: monopolistic and oligopolistic.[5] We analytically determine the water release path at closed-loop equilibrium. We show how a monopoly manages hydropower reservoirs by additional pumping or storing depending on the relative abundance of water between different regions to smooth the effect of demand uncertainty. In addition, risk aversion reduces variance of water pumping when the net flow of precipitation is either positive or negative. In the duopolistic case with symmetric risk aversion, we determine the conditions under which the relative scarcity (abundance) of water in an operator's dam can favor strategic additional pumping (storage) at the competitor's dams. In the case of a duopoly with asymmetric aversion rates, we show that a firm's hydroelectric production increases in parallel with its competitor's risk aversion, if and only if the average recharge speed of the competitor's dam exceeds a certain threshold that increasingly depends on the average water inflows.

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[4] Philpott and Guan (2013) empirically analyze social welfare following the opening of a wholesale electricity market in New Zealand when the social planner is risk averse about uncertainty of water inflows. Aslo see Genc and Sen (2008) for an empirical analysis.

[5] Deregulation processes have engendered several market structures. The industry is ranked according to four models based on the degree of competition: vertically integrated monopoly, single buyer, competition on the wholesale electricity market and competition on the retail market (Hunt, 2002).

The rest of the paper is organized as follows. Section 2 presents the dynamic and stochastic model of management of hydroelectric park with multiple dams. Section 3 analyzes the situation of a monopoly. Section 4 covers the case where dams are managed by oligopolies with asymmetric attitude toward risk, whereas Section 5 examines the case of risk asymmetry. The last section concludes the paper.

2 Model

Electricity demand

Demand in period t is represented by an inverse demand function of linear form (Genc and Thille, 2012):

$$p_t = a_t - bQ_t \qquad (1)$$

where p_t represents the price of electricity, Q_t the quantity demanded, b is a positive constant and the parameter a_t is normally distributed with an expectation of a and a variance of σ^2.

Electricity production

Let us define by q_{it} the electricity production of dam i $(i = 1, \dots, n)$ at time t and such that total electricity and production of different dams ($Q_t = \sum_{i=1}^{n} q_{it}$) is totally consumed.[6] Each facility i uses water stored in a hydropower dam of region i, denoted by s_{it}. Without lost of generality, we assume that each unit released from the dam allows free generation of one unit of electricity. The dam of region i is regularly recharged by random flows of precipitation f_{it}, which follow a

[6] To simplify the example, we assume that we never encounter the power transmission capacity constraint. Head loss during transmission is also ignored.

random walk where $E\left(f_{it}\right)=F_i$ with $F_i > 0$.[7] The evolution of the water available in the dam in region i is governed by the following recurrent equation on the dynamics of the stock:

$$\begin{aligned} q_{it} - f_{it} &= s_{it} - s_{it+1} \\ q_{it+1} - f_{it+1} &= s_{it+1} - s_{it+2} \end{aligned} \qquad (2)$$

Hydropower producer's objective function

At the start of each period t, the hydropower operator observes s_{it}, the water available in region i, with certainty. He then decides on production q_{it} which maximizes the discounted sum of expected utilities of the profit from operation of all hydropower facilities while meeting the n dynamic stochastic constraints on the evolution of water in various dams.

We assume that the cost of storage is a quadratic[8] function of the difference between the current stock, s_{it}, and future storage at the end of period $t+1$, s_{it+1}. In other words, the electricity producer must have a stable stock in each dam to avoid negative externalities on other activities around the dam (agriculture, drinking water supply, recreational activities, etc.) or cause flooding during periods of high water levels. In the case where the future stock fluctuates above or below the current stock, the hydropower operator must pay a penalty equal to $\frac{\gamma}{2}\left(s_{it} - s_{it+1}\right)^2$ for each hydropower dam, with $\gamma > 0$. The operator's instantaneous profit at period t is written as the difference between total revenues $\left(RT_t\right)$ and the cost of storage in n reservoirs:

$$\pi_t = RT_t - \sum_{i=1}^{n} \frac{\gamma}{2}\left(s_{i,t} - s_{i,t+1}\right)^2 \qquad (3)$$

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[7] Garcia et al. (2001) assume that natural filling of a dam follows a binomial distribution, whereas Genc and Thille (2011) assume a normal distribution. In general, hydrologists posit a Markovian process (Karamous and Vasiliadis, 1992; Faber and Stedinger, 2001).

[8] Dakhlaoui and Moreaux (2004) presumed that the storage cost is a quadratic function of the difference between the current stock, s_{it}, and an exogenous target storage s^*.

At each period t, the hydroelectric operator maximizes the expected utility of its mean-variance type profit, characterized by constant absolute risk aversion:

$$W(\pi_t) = E(\pi_t) - (A/2)Var(\pi_t) \quad (4)$$

where $A = -\frac{U''}{U'}$ is the Arrow-Pratt coefficient of constant absolute risk aversion.

3 Dam management under a monopolistic structure

We assume an electricity industry with a monopolistic structure. Using the demand function defined by (1) and the profit function given by (3), the operator's objective function defined by (4) is:

$$W(\pi_t) = E\left(a_t Q_t - bQ_t^2 - \sum_{i=1}^{n} \frac{\gamma}{2}\left(s_{i,t} - s_{i,t+1}\right)^2\right) - (A/2)Var\left(a_t Q_t - bQ_t^2 - \sum_{i=1}^{n} \frac{\gamma}{2}\left(s_{i,t} - s_{i,t+1}\right)^2\right) \quad (5)$$

By assuming that the precipitation inflows to the different hydropower dams are independent, we can therefore write equation (5) as:

$$W(\pi_t) = aQ_t - bQ_t^2 - \frac{\gamma}{2}\sum_{i=1}^{n} E_t\left(s_{i,t} - s_{i,t+1}\right)^2 - (A/2)Q_t^2\sigma^2 \quad (6)$$

The monopoly chooses the electricity production path $\{q_{it}\}_{\substack{i=1,\dots,n \\ t=1,\dots,\infty}}$ by n hydropower dams as a solution to the dynamic and stochastic optimization problem with an infinite horizon. The solution to the optimization problem of hydroelectric operator q_{it} is (see Technical Appendix A1):

$$q_{it} = \frac{a}{\gamma + n\left(2b + A\sigma^2\right)} + F_i - \frac{2b + A\sigma^2}{\gamma + n\left(2b + A\sigma^2\right)}\sum_{i=1}^{n} F_i \quad (7)$$

Based on (7) and the dynamic equation of stocks given by the expression in (2), the solution to the variation in stock in the hydropower dam in region i is:

$$\Delta s_{i,t+1} = -q_{it} + f_{it} = -\frac{a}{\gamma + n\left(2b + A\sigma^2\right)} + \frac{2b + A\sigma^2}{\gamma + n\left(2b + A\sigma^2\right)} \sum_{i=1}^{n} F_i \qquad (8)$$

and total production of the monopoly is:

$$Q_t^M = \frac{na}{\gamma + n\left(2b + A\sigma^2\right)} + \frac{\gamma}{\gamma + n\left(2b + A\sigma^2\right)} \sum_{i=1}^{n} F_i \qquad (9)$$

3.1 Case of abundance of water in region *i* and shortage in region *j*

In this case, there is an increase in the average precipitation inflows in region *i* ($\Delta F_i > 0$), and an average decrease in flows in region *j* ($\Delta F_j < 0$). The average flow is presumed constant in regions other than *i* and *j*.

We assume that the gap between the variation of the rate of refilling of dam *i* and that of dam *j* is written as: $\Delta F_j = -\Delta F_i + \phi$ with $\Delta F_j < \phi < \Delta F_i$. The total effect of fluctuations in water inflows in the two regions on production by the power plant of region *i* $\left(\Delta q_{it}\right)$ is:

$$\Delta q_{it} = \Delta F_i - \frac{\left(2b + A\sigma^2\right)\phi}{\gamma + n\left(2b + A\sigma^2\right)} > 0 \qquad (10)$$

The operator must therefore perform additional pumping from the dam of the region with abundant water and additional storage in the dam in the region with scarce water. The total impact of fluctuations in water inflows in the two regions *i* and *j* on production by the dam of region *j* is:

$$\Delta q_{jt} = \Delta F_j - \frac{\left(2b + A\sigma^2\right)\phi}{\gamma + n\left(2b + A\sigma^2\right)} < 0 \qquad (11)$$

Further, variation in total production of the two dams $\left(\Delta Q_t^M = \Delta q_{it} + \Delta q_{jt}\right)$ is:

$$\Delta Q_t^M = \frac{\gamma + (n-2)(2b + A\sigma^2)}{\gamma + n(2b + A\sigma^2)}\phi \qquad (12)$$

The variation in total production of the two dams depends on the sign of ϕ. If $\Delta F_j < \phi < 0$, that is $\Delta F_i < |\Delta F_j|$, then $\Delta Q_t^M < 0$, and if $0 < \phi < \Delta F_i$ that is $\Delta F_i > |\Delta F_j|$ then $\Delta Q_t^M > 0$. Given expressions (10) and (11), the impact of fluctuations of recharge flows on the amount of water available in the hydropower dam in region *j* is identical to that observed in region *i*:

$$d\Delta s_{j,t+1} = d\Delta s_{i,t+1} = \frac{(2b + A\sigma^2)\phi}{\gamma + n(2b + A\sigma^2)}$$

These results show that under its optimal solution, the monopoly have to keep the same change in stock in all dams by increasing its electricity production in the region with abundant water and decreasing production in the region with scarce water.

The variation in stock in the dams depends on the net variation in flows in the two regions. If $\Delta F_j < \phi < 0$, that is $\Delta F_i < |\Delta F_j|$, then $d\Delta s_{j,t+1} < 0$ $(d\Delta s_{i,t+1} < 0)$ and if $0 < \phi < \Delta F_i$ that is $\Delta F_i > |\Delta F_j|$, then $d\Delta s_{j,t+1} > 0$ $(d\Delta s_{i,t+1} > 0)$.[9]

To analyze the effect of risk aversion on total production, we compare the variation in total production with and without risk aversion. We have:

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[9] In the case where the average variation in flows of the two dams is not identical, $(\Delta F_i \neq |\Delta F_j|)$, the monopoly assumes additional storage costs in the two dams. The more ϕ increases, the higher these costs. In the particular case where $\phi = 0$ $(\Delta F_i = -\Delta F_j)$, the total effect on hydropower production in regions *i* and *j* is $\Delta q_i = \Delta F_i$ and $\Delta q_j = -\Delta F_i$. In this case as well, the operator must satisfy electricity demand through additional pumping from the reservoir in the region with abundant water and reduce its production in the region with scarce water. To smooth the effects of this fluctuation of flows on electricity prices, it must ensure that this additional storage in the region with scarce water equalizes the additional discharge in the region with abundant water: $\Delta q_{it} = -\Delta q_{jt}$.

$$\Delta Q_t^M\Big|_{A\neq 0} - \Delta Q_t^M\Big|_{A=0} = \frac{-2A\gamma\sigma^2\phi}{\left[\gamma + n\left(2b + A\sigma^2\right)\right]\left(\gamma + 2nb\right)}.$$

This implies that $\forall \gamma > 0$, $\Delta Q_t^M\Big|_{A\neq 0} \leq \Delta Q_t^M\Big|_{A=0}$ if $\Delta F_i \geq \left|\Delta F_j\right|$, and $\Delta Q_t^M\Big|_{A\neq 0} > \Delta Q_t^M\Big|_{A=0}$ if $\Delta F_i < \left|\Delta F_j\right|$. This result shows that when the operator is risk neutral and the net flow of precipitation is positive (negative), the increase (decrease) in total production of the two regions is greater than when the operator is risk averse.

3.2 Case of abundant water in regions *i* and *j*

We denote the gap between the variation in the refilling speed of dam *j* and that of region *i* by ω: $\Delta F_j = \Delta F_i + \omega$ with $-\Delta F_i < \omega < \Delta F_j$. Then,

$$\Delta q_{it} = \frac{\gamma + (n-2)\left(2b + A\sigma^2\right)}{\gamma + n\left(2b + A\sigma^2\right)}\Delta F_i - \frac{\left(2b + A\sigma^2\right)\omega}{\gamma + n\left(2b + A\sigma^2\right)} \qquad (13)$$

$$\Delta q_{jt} = \frac{\gamma + (n-2)\left(2b + A\sigma^2\right)}{\gamma + n\left(2b + A\sigma^2\right)}\Delta F_j + \frac{\left(2b + A\sigma^2\right)\omega}{\gamma + n\left(2b + A\sigma^2\right)} \qquad (14)$$

Proposition 1

In the case where the hydropower producer observes an increase in the average flow of precipitation in the two regions i and j ($\Delta F_i > 0$ *and* $\Delta F_j > 0$*), he decides to perform:*

- *additional pumping (storage) at the dam in region i (* $\Delta q_{it} > (<) 0$ *) if* $\omega < (>) \tau\Delta F_j$
- *additional pumping (storage) at the dam in region j* $\left(\Delta q_{jt} > (<) 0\right)$ *if* $\omega > (<) -\tau\Delta F_i$
- *additional pumping at the dams in regions i and j* $\left(\Delta q_{it} > 0 \textit{ et } \Delta q_{jt} > 0\right)$ *if* $-\tau\Delta F_i < \omega < \tau\Delta F_j$

With $0 < \tau \equiv \frac{\gamma + (n-2)\left(2b + A\sigma^2\right)}{\gamma + (n-1)\left(2b + A\sigma^2\right)} < 1$.■

The proof of proposition 1 follows from equations (13) and (14).

The additional discharge in the two regions due to abundant water is given as:

$$\Delta discharge = \underset{+}{\Delta_{F_i} q_{it}} + \underset{+}{\Delta_{F_j} q_{jt}} = \frac{\gamma + (n-2)(2b + A\sigma^2)}{\gamma + n(2b + A\sigma^2)}(\Delta F_i + \Delta F_j) > 0 \cdot$$

Whereas the additional storage of water is evaluated at:

$$\Delta storage = \underset{+}{\Delta_{F_j} q_{it}} + \underset{+}{\Delta_{F_i} q_{jt}} = -\frac{(2b + A\sigma^2)}{\gamma + n(2b + A\sigma^2)}(\Delta F_i + \Delta F_j) < 0 \cdot$$

In this case we have: $|\Delta storage| < \Delta discharge$. In other words, in the case of abundant water in both regions, the additional inter-annual transfer of water from period *t* to period *t+1* is less than its additional use at period *t*. Further, the analysis of the effects of the risk aversion coefficient show that $\frac{\partial(\Delta storage)}{\partial A} < 0$ and $\frac{\partial(\Delta discharge)}{\partial A} > 0$. These results are summarized by proposition 2.[10]

Proposition 2

In the case of abundant water in regions i and j $(\Delta F_i > 0 \text{ and } \Delta F_j > 0)$, *the hydroelectricity producer uses two hydropower reservoirs to satisfy current demand for electricity. However, it should not fully use additional recharge to satisfy current demand for electricity* $\left(\frac{\gamma + (n-2)(2b + A\sigma^2)}{\gamma + n(2b + A\sigma^2)}(\Delta F_i + \Delta F_j)\right)$, *but instead should store a quantity* $\frac{2(2b + A\sigma^2)}{\gamma + n(2b + A\sigma^2)}(\Delta F_i + \Delta F_j)$ *of potential energy in the form of water to satisfy future demand. Further, the greater the hydroelectric operator's risk aversion, the larger the quantity of water stored for future demand.* ■

[10] In the case of a water shortage in regions *i* and *j*, we obtain the opposite results to those found in this section.

4 Dam management under an oligopolistic structure with symmetric attitude toward risk

In this section, we consider an electricity industry with an oligopolistic structure, with *n* firms that compete *a la Cournot*. The profit of hydroelectric firm i in period t is written as:

$$\pi_{i,t} = P\left(\sum_{i=1}^{n} q_{i,t}\right) q_{i,t} - \frac{\gamma}{2}\left(s_{i,t} - s_{i,t+1}\right)^2 \qquad (15)$$

At each period *t*, hydroelectric operator i maximizes the expected utility of its profit with a utility function characterized by constant absolute risk aversion identical for all hydroelectric operators $\left(A_i = A_j = A\right)$. The objective function of firm *i* is therefore:

$$W\left(\pi_i\right) = \left(a - b\left(q_{it} + \sum_{j \neq i}^{n} q_{jt}\right)\right) q_{it} - \frac{\gamma}{2} E_t\left(s_{it} - s_{it+1}\right)^2 - \left(A/2\right) q_{it}^2 \sigma^2 \quad (16)$$

The solution to the problem of maximization gives us equilibrium production of the dynamic Cournot closed-loop game of firm *i* (see Technical Appendix A2):

$$q_{it} = \frac{a\left(A\sigma^2 + b + \gamma\right) + \gamma\left(A\sigma^2 + nb + \gamma\right) F_i - b\gamma \sum_{j \neq i} F_j}{\left(A\sigma^2 + b + \gamma\right)\left(A\sigma^2 + b\left(1+n\right) + \gamma\right)} \qquad (17)$$

In the case where the hydroelectric dams are independent and are in the same region $\left(F_i = F_j = F\right)$, hydroelectric production of firm *i* in period *t* is reduced to $q_{it} = \frac{a + \gamma F}{A\sigma^2 + b\left(n+1\right) + \gamma}$.

Therefore, $\frac{\partial q_{it}}{\partial \gamma} = \frac{-a + F\left(b\left(n+1\right) + A\sigma^2\right)}{\left(\gamma + b\left(n+1\right) + A\sigma^2\right)^2}$. This shows that the increase in storage costs motivates producer *i* to deviate from its equilibrium strategy by additional pumping of its current stock if and only if the average recharge speed of dam *i* is markedly higher than the water release rhythm from dam i in period t (case of strong hydraulicity), $\Delta s > 0$. Conversely, the operator must perform additional storage in the case of low water levels $\Delta s \leq 0$ [11]. Further, from (17):

$$\frac{\partial q_{it}}{\partial \sigma^2} = -\frac{(a+\gamma F)A}{\left(A\sigma^2 + b(n+1)+\gamma\right)^2} < 0$$

Any increase in risk level reduces production of firm *i* in period *t* when all hydroelectric operators have the same average flow of precipitation. Lastly, all increases in the risk aversion rate of firm *i* reduce the immediate use of water, that is it favors an inter-temporal transfer of water from *t* to *t+1*.

$$\frac{\partial q_{it}}{\partial A} = -\frac{(a+\gamma F)\sigma^2}{\left(A\sigma^2 + b(n+1)+\gamma\right)^2} < 0$$

Hydroelectricity producer *i's* production at period *t* depends positively on its inflow rates and negatively on the sum its competitor's flow of precipitation. Total equilibrium hydroelectric production at period *t* is written as:

$$Q_t^C = \frac{\left(A\sigma^2 + b + \gamma\right)\left(na + \gamma \sum_{i=1}^{n} F_i\right)}{\left(A\sigma^2 + b + \gamma\right)\left(A\sigma^2 + b(1+n)+\gamma\right)} \qquad (18)$$

4.1 Case of water abundance in region *i* and scarcity in region *j*

In this case, the hydroelectric producer in region *i* observes an increase in its average flow of precipitation $(\Delta F_i > 0)$, whereas the hydroelectric producer in region *j* observes an average

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[11] We have $\frac{\partial q_{it}}{\partial \gamma} > (\leq) 0$ if $a < (\geq) F\left[b(n+1) + A\sigma^2\right]$ and $a < (\geq) F\left[b(n+1) + A\sigma^2\right] \Rightarrow F > (\leq) q$. Also, $F > (\leq) q \Rightarrow \Delta s > (\leq) 0$. Therefore, $\frac{\partial q_{it}}{\partial \gamma} > (\leq) 0$ if $\Delta s > (\leq) 0$.

decrease in its flows $(\Delta F_j < 0)$. The average flow is presumed constant in regions other than *i* and *j*. Let κ be the gap between variation in the recharge rate of dam *i* and that in dam *j*: $\Delta F_j = -\Delta F_i + \kappa$ with $\Delta F_j < \kappa < \Delta F_i$. The total effect of the variation of average recharge inflow in the two regions on production of hydroelectric dams *i* and *j* is:

$$\Delta q_{it} = \frac{\gamma}{\left(\gamma + b + A\sigma^2\right)} \Delta F_i - \frac{b\gamma\kappa}{\left(\gamma + b + A\sigma^2\right)\left(\gamma + b\left(1+n\right) + A\sigma^2\right)} > 0 \qquad (19)$$

$$\Delta q_{jt} = \frac{\gamma}{\left(\gamma + b + A\sigma^2\right)} \Delta F_j - \frac{b\gamma\kappa}{\left(\gamma + b + A\sigma^2\right)\left(\gamma + b\left(1+n\right) + A\sigma^2\right)} < 0 \qquad (20)$$

We therefore deduce that the hydroelectricity producer in the region with abundant water must do additional pumping, and inversely the hydroelectricity producer in the region with scarce water must do additional storage. In the case where $\Delta F_j = -\Delta F_i$ $(\kappa = 0)$, additional pumping in the region with abundant water corresponds to additional storage in the region with a water shortage $(\Delta q_{jt} = -\Delta q_{it})$. Based on (19) and (20), the impact of variation of the average water inflows on the variation of stock in the dams of regions *i* and *j* is:

$$d\Delta s_{it+1} = \frac{b + A\sigma^2}{\left(\gamma + b + A\sigma^2\right)} \Delta F_i + \frac{b\gamma\kappa}{\left(\gamma + b + A\sigma^2\right)\left(\gamma + b\left(1+n\right) + A\sigma^2\right)} \qquad (21)$$

$$d\Delta s_{jt+1} = \frac{b + A\sigma^2}{\left(\gamma + b + A\sigma^2\right)} \Delta F_j + \frac{b\gamma\kappa}{\left(\gamma + b + A\sigma^2\right)\left(\gamma + b\left(1+n\right) + A\sigma^2\right)} \qquad (22)$$

Proposition 3

In the case where the hydroelectric operators in regions i and j observe an increase in the average flow of precipitation in region i and a decrease in average flow in region j¨ $(\Delta F_i > 0$ *and* $\Delta F_j < 0)$, *we have:*

(i) *A decrease in the stock of water in the dams in regions i and j* $(d\Delta s_{it+1} \leq 0$ *and* $d\Delta s_{jt+1} < 0)$ *if* $\Delta F_j < \kappa \leq \eta \Delta F_j$.

(ii) *A decrease in the stock of water in the dams in region j and an increase in the stock of water in the dam of region i* $(d\Delta s_{it+1} > 0$ *and* $d\Delta s_{jt+1} \leq 0)$ *if* $\eta \Delta F_j < \kappa \leq \eta \Delta F_i$.

(iii) *A decrease in the stock of water in the dams of regions i and j* $(d\Delta s_{it+1} > 0$ *and* $d\Delta s_{jt+1} \geq 0)$ *if* $\eta \Delta F_i \leq \kappa < \Delta F_i$.

with $0 < \eta \equiv \dfrac{\left(b + A\sigma^2\right)\left(\gamma + b(1+n) + A\sigma^2\right)}{\left(b + A\sigma^2\right)\left(\gamma + b(1+n) + A\sigma^2\right) + b\gamma} < 1$ ■

The proof of proposition 3 follows from equations (21) and (22).

Proposition 3 specifies the conditions under which, in a structure of imperfect competition, the possibility of storing electricity in the form of water may motivate hydroelectric operators to manage their water resources strategically according to the relative scarcity of water inflows in the dams. Case (i) is that of two dams situated in regions where the average abundance of water resources in the dam of hydroelectricity producer *i* does not compensate for the average scarcity in the dam of hydroelectricity producer *j*. In this case, the two hydroelectric operators benefit from deviating from their equilibrium strategy by performing additional pumping in the two regions. Despite the shortage of water in region *j*, hydroelectricity producer *j* knows that its competitor does not have sufficient additional water resources to play strategically against it on the electricity market. Hydroelectricity producer *j* consequently performs strategic pumping in its own dam despite its water shortage.

Condition (ii) is that of a dam of hydroelectricity producer *j*, which suffers from a shortage whereas the dam of hydroelectricity producer *i* experiences an increase in average water inflows. In this case, the abundance of inflows prompts the hydroelectricity producer to play strategically on the electricity market against hydroelectricity producer *j*, which suffers from a severe water shortage, by doing additional storage of this positive variation of its water inflows: this corresponds to strategic water storage. By additional pumping despite the scarcity of its water resources, producer *j* exacerbates its situation.

Under condition (iii), the hydroelectric operators increase additional storage in both dams when the gap between the variations in the average water inflows in region *i* and that in region *j* is less than the increase in average water inflows in the region with abundant water. In other words, in the case where the two dams are in two regions where the average abundance of water resources in the dam of hydroelectricity producer *i* can relatively compensate for the average scarcity in the dam of hydroelectricity producer *j*, the two players deviate from their equilibrium strategy by additionally reducing electricity production in both dams, i.e. the one with scarce water and the one with abundant water. The hydroelectricity producer with a water shortage, namely that in region *j*, knows that its competitor has not a sufficient increase in water inflows to play strategically against it. Because the water scarcity of hydroelectricity producer *j* is not severe relative to the water abundance of hydroelectricity producer *i*, then hydroelectricity producer *j* can respond to the strategic storage of hydroelectricity producer *i* by doing its own additional storage.

The solutions of Δq_{it} and Δq_{jt} (equations (19) and (20)) let us deduce the impact of the variation of precipitation inflows on total production $\left(\Delta Q_t^C\right)$:

$$\Delta Q_t^C = \frac{\gamma\left(\gamma + A\sigma^2 + b\left(n-1\right)\right)\kappa}{\left(\gamma + A\sigma^2 + b\right)\left(\gamma + A\sigma^2 + b\left(n+1\right)\right)} \quad (23)$$

The variation in total production of the two dams depends on the sign of κ. If $\Delta F_j < \kappa < 0$, that is $\Delta F_i < |\Delta F_j|$, then $\Delta Q_t^C < 0$ and if $0 < \kappa < \Delta F_i$; that is $\Delta F_i > |\Delta F_j|$, then $\Delta Q_t^C > 0$. In the case where the increase in average water inflows in region *i* is less than the average scarcity in dam *j*, then total equilibrium production decreases. In other words, the relative scarcity of water resources in the whole stock favors additional storage of water. Conversely, relative abundance of water favors additional pumping at all hydropower reservoirs. In both cases, regardless of the hydroelectric operators' strategic behavior, the reservoirs will be operated with respect to merit order principle.

The effect of the risk aversion coefficient on ΔQ_t^C (the impact of the variation of precipitation inflows on quantity) is not monotone. The sign of $\frac{\partial\left(\Delta Q_t^C\right)}{\partial A}$ is determined by the sign of

$$\left(\gamma + A\sigma^2 + b\right)\left(\gamma + A\sigma^2 + b\left(n+1\right)\right) - 2\gamma\left(\gamma + A\sigma^2 + b\left(n/2+1\right)\right)\left(\gamma + A\sigma^2 + b\left(n-1\right)\right).$$

4.2 Case of abundant water in regions *i* and *j*

In this case, the hydroelectric operators in regions *i* and *j* observe an increase in the flow of precipitation $\left(\Delta F_j > 0 \text{ and } \Delta F_i > 0\right)$. Let us denote by χ the gap between the variation in the recharge speed of dam *j* and that of region *i*: $\Delta F_j = \Delta F_i + \chi$ with $-\Delta F_i < \chi < \Delta F_j$. We then have:

$$\Delta q_{it} = \frac{\gamma\left(A\sigma^2 + b\left(n-1\right) + \gamma\right)}{\left(A\sigma^2 + b + \gamma\right)\left(A\sigma^2 + b\left(1+n\right) + \gamma\right)}\Delta F_i - \frac{b\gamma\chi}{\left(A\sigma^2 + b + \gamma\right)\left(A\sigma^2 + b\left(1+n\right) + \gamma\right)} \qquad (24)$$

$$\Delta q_{jt} = \frac{\gamma\left(A\sigma^2 + b\left(n-1\right) + \gamma\right)}{\left(A\sigma^2 + b + \gamma\right)\left(A\sigma^2 + b\left(1+n\right) + \gamma\right)}\Delta F_j + \frac{b\gamma\chi}{\left(A\sigma^2 + b + \gamma\right)\left(A\sigma^2 + b\left(1+n\right) + \gamma\right)} \qquad (25)$$

Proposition 4

In the case where the hydroelectric operators in regions i and j observe an increase in the average flow of precipitation $(\Delta F_i > 0 \text{ and } \Delta F_j > 0)$*, we have:*

(i) *Additional pumping from the dam of region i and additional storage in the dam of region j* $(\Delta q_{it} > 0 \text{ and } \Delta q_{jt} \leq 0)$ *if* $-\Delta F_i < \chi \leq -\delta \Delta F_i$.

(ii) *Additional pumping in the dams of regions i and j* $(\Delta q_{it} \geq 0 \text{ and } \Delta q_{jt} > 0)$ *if* $-\delta \Delta F_i < \chi \leq \delta \Delta F_j$.

(iii) *Additional pumping from the dam of region j and additional storage in the dam of region i* $(\Delta q_{it} < 0 \text{ and } \Delta q_{jt} > 0)$ *if* $\delta \Delta F_j < \chi < \Delta F_j$.

with $0 < \delta \equiv \frac{A\sigma^2 + b(n-1) + \gamma}{A\sigma^2 + b(n-1) + \gamma + b} < 1$.■

Proposition 4 states the conditions under which the possibility of storing electricity in the form of water, coupled with imperfect competition, can lead hydroelectric operators to manage their water resources strategically according to the relative abundance of water inflows in the dams. Under condition (i), hydropower operator *i* has a larger increase in average water inflows than that of hydroelectricity producer *j*. Because hydroelectricity producer *i* knows that the average increase in its water inflows can compensate for the gap in the variation of inflows in the two regions, he decides to do additional pumping on its dam. Consequently, hydroelectricity producer *j* can respond only by additional storage. In case (ii), hydropower operator *i* experiences an increase in its average water inflows that is slightly greater than that which occurs at the dam of hydroelectricity producer *j*. In this case, both hydroelectric operators do additional pumping. In case (iii), hydroelectricity producer *j* has a larger increase in its average inflows than that of hydroelectricity producer *i*. The relative abundance of water resources in the dam of hydroelectricity producer *j* compared with that of hydroelectricity producer *i* prompts operator *i* to increase its production, whereas hydroelectricity producer *i* will store its additional inflows.

5 Dam management under a duopolistic structure with asymmetric attitude toward risk

In this section, we assume that the hydroelectricity producers do not have the same attitude toward risk ($A_i \neq A_j$). Equilibrium electricity production under the Cournot Nash closed-loop strategy is (see Technical Appendix A3):

$$q_{it} = \frac{\left(\gamma + b + A_j\sigma^2\right)a + \gamma\left(\gamma + 2b + A_j\sigma^2\right)F_i - \gamma b F_j}{\left(\gamma + 2b + A_i\sigma^2\right)\left(\gamma + 2b + A_j\sigma^2\right) - b^2}$$

The equilibrium strategy of hydroelectricity producer *i* at period *t* depends on several parameters including the risk aversion rates A_i, A_j and uncertainty of electricity demand σ^2. The sensitivity of a firm's hydroelectric production relative to its competitor's risk aversion coefficient is:

$$\frac{\partial q_{it}}{\partial A_j} = \frac{-b^2\left(a + \gamma F_i\right) + b\left(\gamma + 2b + A_i\sigma^2\right)\left(a + \gamma F_j\right)}{\left[\left(\gamma + 2b + A_i\sigma^2\right)\left(\gamma + 2b + A_j\sigma^2\right) - b^2\right]^2} \cdot \sigma^2 \qquad (26)$$

We have $\frac{\partial q_{it}}{\partial A_j} \leq 0$ if $F_j \leq \varphi^*_{A_j}\left(F_i\right)$ with $\varphi^*_{A_j}\left(F_i\right) = \frac{-a\left(A_i\sigma^2 + b + \gamma\right) + b\gamma F_i}{\gamma\left(2b + \gamma + A_i\sigma^2\right)}$. Therefore, $\varphi^*_{A_j}\left(F_i\right)$ represents the threshold at which the variation of the risk aversion coefficient of firm *j* has no effect on the water pumping strategy of hydroelectricity producer *i* at period *t*.

In the case where the average inflows of the competing firm is below the threshold $\varphi^*_{A_j}\left(F_i\right)$, all increases in the risk aversion coefficient of operator *j* leads producer *i* to deviate from its equilibrium strategy by performing additional storage of its current stock. However, this condition is possible only if the average inflow of dam *j* is slightly lower than that of dam *i*. If the relative weakness of the average water inflows in dam *j* is associated with an increase in the risk aversion coefficient of hydroelectricity producer *j*, hydroelectricity producer *i* will compete strategically by performing additional storage of its potential energy. The increase in A_j triggers raises the variability of the profit of producer *j*, which it must minimize through additional decreases in equilibrium water pumping at dam *j*. Therefore, the additional gain from minimization of the variance of the profit of operator *j* does not offset the additional loss in that operator's expected profit, and is compounded by its lower average water inflows. Consequently, operator *j* decides to store additional water. This additional storage increases the average electricity price and consequently augments the marginal gain on immediate use of a unit of water in the dam of competitor *i*, which has more abundant average inflow. To further increase its current gain, operator *i* decides to play strategically against its competitor by further lowering its production because it knows that hydroelectricity producer *j* cannot compete strategically against it on electricity wholesale market. In the opposite case, hydroelectricity producer *i* does additional strategic pumping of potential energy.[12]

The sensitivity study of a firm's production relative to the variation of its own risk aversion coefficient is:

$$\frac{\partial q_{it}}{\partial A_i} = \frac{-\left(\gamma + 2b + A_j\sigma^2\right)\sigma^2\left[\left(\gamma + b + A_j\sigma^2\right)a + \gamma\left(\gamma + 2b + A_j\sigma^2\right)F_i - b\gamma F_j\right]}{\left[\left(\gamma + 2b + A_i\sigma^2\right)\left(\gamma + 2b + A_j\sigma^2\right) - b^2\right]^2}$$

Therefore, if producer *i* becomes more risk averse then he decides to reduce its equilibrium production at period *t* if and only if the average water inflows in its competitor's dam is below the threshold of $\tilde{\varphi}^*_{A_i}\left(F_i\right)$:

$$\frac{\partial q_{it}}{\partial A_i} \leq 0 \text{ if } F_j \leq \tilde{\varphi}^*_{A_i}\left(F_i\right) \text{ with } \tilde{\varphi}^*_{A_i}\left(F_i\right) = \frac{a\left(A_j\sigma^2 + b + \gamma\right) + \gamma\left(2b + \gamma + A_j\sigma^2\right)F_i}{b\gamma}$$

[12] However, in the case where average precipitation inflows is the same for both producers ($F_i = F_j = F$), any increase in the competitor's risk aversion coefficient always increases equilibrium hydroelectric production of firm *i* at period *t*: $\frac{\partial q_{it}}{\partial A_j} = \frac{\left(\gamma + b + A_i\sigma^2\right)\left(a + \gamma F\right)b\sigma^2}{\left[\left(\gamma + 2b + A_i\sigma^2\right)\left(\gamma + 2b + A_j\sigma^2\right) - b^2\right]^2} > 0$.

In other words, when the increase in the risk aversion coefficient of hydroelectricity producer *i* coincides with a relative weakness in average water inflow in its competitor's dam *j*, this favors additional storage by firm *i*. If hydroelectricity producer *i* is more risk averse, then additional reduction in water pumping from its dam at period *t* lets it minimize the variance of its profit. It can thus raise the expected utility of its profit following an increase in the average price of electricity at the current period. However, the marginal gain from the rise in price does not compensate for the marginal loss due to the increase in its risk aversion coefficient. Consequently, according to the first first-order condition (see the Technical appendix A3), the marginal gain from pumping an additional unit from the dam of hydroelectricity producer *i* at period *t* always remains below the marginal value of the same unit of water in stock; this favors additional storage in dam *i*. In the case where the increase in risk aversion coefficient of producer *i* coincides with abundance in average water inflows in the dam of competitor *j* relative to dam *i* ($F_j > \varphi_{A_i}^*\left(F_i\right) > F_i$), this favors additional water release by firm *i* at period *t*. Conversely, if the average precipitation inflows is the same for both producers ($F_i = F_j = F$), then:

$$\frac{\partial q_{it}}{\partial A_i} = \frac{-\left(\gamma + 2b + A_j\sigma^2\right)\sigma^2\left[\left(\gamma + b + A_j\sigma^2\right)\left(a + \gamma F\right)\right]}{\left[\left(\gamma + 2b + A_i\sigma^2\right)\left(\gamma + 2b + A_j\sigma^2\right) - b^2\right]^2} < 0$$

In other words, if hydroelectric operators have the same average water inflows, an increase in risk aversion coefficient for any operator reduces the equilibrium quantity of water pumped from its dam. Conversely, an increase in electricity demand uncertainty decreases water pumping in the dams:

$$\frac{\partial q_{it}}{\partial \sigma^2} = \frac{\left(a + \gamma F\right)\left[A_j b\left(b + \gamma\right) - A_i\left(2b^2 + \left(\gamma + A_j\sigma^2\right)^2 + b\left(3\gamma + 2A_j\sigma^2\right)\right)\right]}{\left[3b^2 + 4b\gamma + 2\left(A_i + A_j\right)b\sigma^2 + \left(\gamma + A_i\sigma^2\right)\left(\gamma + A_j\sigma^2\right)\right]^2} \leq 0$$

An increase in the variance of electricity demand at a given period will not affect the expected instantaneous profit of hydroelectricity producer *i*. It has a positive effect uniquely on the variance of producer *i's* profit and consequently favors a drop in expected utility of the total profit of *i*. Given that hydroelectricity producer *i* must minimize the variance of its instantaneous profit, its only option is to reduce current water pumping. This additional storage of water, with all things being equal, lowers the average electricity price. However, the marginal gain from the drop in variance in instantaneous profit does not offset the marginal loss from the decline in expected profit. According to the condition of optimality, this creates a negative gap between the marginal income from immediate pumping of a unit of water from dam *i* and its in situ price, which justifies additional storage of water for future use.

6 Conclusion

In 2012, hydroelectricity supplied 16.3% of world electricity and in several countries, hydropower is the main source of electricity (e.g. Brazil, Norway, province of Quebec in Canada, New Zealand). And, because of its contribution to climate change mitigation, the share of hydroelectricity in total world electricity is expected to grow in the future decades (IPCC, 2014). Nonetheless, the economic literature has mainly analyzed the strategic behavior of electricity operators in purely thermal systems or mixed hydrothermal systems. Purely hydroelectric industries with large water storage capacities have attracted sparsely attention. In addition, little works have integrated risk dimension, which is quite salient for suppliers of hydropower. Water reserves are renewed randomly by precipitation and given the climate change phenomenon and the associated series of extreme events (IPCC, 2014), the challenge of optimal management of this resource over several periods of time has become more pressing. On the demand side, operators of hydroelectric plants are also facing several sources of uncertainty closely linked to different categories of electricity demands (residential, commercial and industrial).

In this paper, we analyzed the problem of water resource management under two industrial structures, monopolistic and oligopolistic, when hydroelectricity producers are risk averse and face uncertainty on demands. Analytic resolution of the problem of dynamic stochastic optimization show how a monopoly can manage its hydropower reservoirs through additional pumping or storage depending on the relative abundance of water between regions to smooth the effect of uncertainty on electricity prices. In addition, risk aversion reduces the variation of water pumping when the net flow of precipitation is positive or negative.

Under oligopolistic competition with symmetric risk aversion, we have specified the conditions under which relative scarcity (abundance) of water in an operator's dam can favor additional strategic pumping (storage) in its competitor's dams. When the average abundance of water resources in the dam of operator does not compensate for the average scarcity in the dam of the other one, the two operators benefit from deviating from their equilibrium strategy by performing additional pumping in the two regions. Conversely, when one producer suffers from a shortage whereas the other one experiences an increase in average water inflows, the second one plays strategically by doing additional storage.

Under asymmetric risk aversion, we show that the hydroelectricity production of a firm increases in parallel with the risk aversion coefficient of its competitor if and only if the recharge speed of the competitor's dam exceeds a certain level that increasingly depends on its average water inflows. If the two hydroelectric operators have the same average water inflows, an increase in risk aversion coefficient for any operator reduces the equilibrium quantity of water pumped from its dam. Conversely, an increase in electricity demand uncertainty decreases water pumping in the dams.

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Technical Appendix

A.1. Derivation of Monopoly solution

The objective function of hydroelectric operator is:

$$W(\pi_t) = E\left(a_t Q_t - bQ_t^2 - \sum_{i=1}^{n}\frac{\gamma}{2}\left(s_{i,t} - s_{i,t+1}\right)^2\right) - (A/2)Var\left(a_t Q_t - bQ_t^2 - \sum_{i=1}^{n}\frac{\gamma}{2}\left(s_{i,t} - s_{i,t+1}\right)^2\right)$$

With $Q_t = \sum_i q_{it}$, the total production of various hydroelectric dams. Assuming that precipitation flows of various dams are independent, we have:

$$W(\pi_t) = aQ_t - bQ_t^2 - \frac{\gamma}{2}\sum_{i=1}^{n} E_t\left(s_{i,t} - s_{i,t+1}\right)^2 - (A/2)Q_t^2\sigma^2 \qquad (27)$$

The monopoly chooses the electricity production path $\{q_{it}\}_{\substack{i=1,\dots,n \\ t=1,\dots,\infty}}$ by the n dams' solution of dynamic and stochastic optimization problem with infinite horizon in following:

$$\begin{cases} \underset{\{q_{it}\}_{\substack{i=1,\dots,n \\ t=1,\dots,\infty}}}{Max}\ E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[RT(Q_t) - \frac{\gamma}{2}\sum_{i=1}^{n}\left(s_{it} - s_{it+1}\right)^2 - (A/2)Q_t^2\sigma^2\right]\right\} \\ sc \\ s_{it+1} = s_{it} - q_{it} + f_{it}, \qquad \forall i, \forall t \\ s_{i0} \text{ given}, \qquad \forall i \end{cases}$$

where $E_0(.)$ is the conditional expectation to information available on the stock of water in various dams at initial period. The initial stock of each dam (s_{i0}) is known with certainty. The Lagrangian of this optimization problem is:

$$J = E_0\sum_{t=0}^{\infty}\beta^t\left[RT(Q_t) - \frac{\gamma}{2}\sum_{i=1}^{n}\left(s_{it} - s_{it+1}\right)^2 - (A/2)Q_t^2\sigma^2 + \sum_{i=1}^{n}\lambda_{it}\left(s_{it} - q_{it} + f_{it} - s_{it+1}\right)\right]$$

where $\{\lambda_{it}\}$ represents the state co-variables associated to each stochastic dynamic constraint on the storage of water in the dam *i*. This problem admits one solution of finite value of the objective function (Sargent, 2001). The First order conditions relative to the production of dam *i* at period *t*, q_{it} , is :

$$q_{it} : a - \left(2b + A\sigma^2\right)\sum_{i=1}^{n} q_{it} - \lambda_{it} = 0 \qquad (28)$$

The first order conditions determining the level of water storage in the dam *i* at period *t+1*, s_{it+1} is:

$$s_{it+1} : \ \beta E_t\left[\lambda_{it+1}\right] - \lambda_{it} + \gamma E_t\left(s_{it} - s_{it+1}\right) - \beta\gamma E_t\left[s_{it+1} - s_{it+2}\right] = 0 \qquad (29)$$

Using the delay operator L , Euler's equation is written:

$$\left(1 - \beta L^{-1}\right)\lambda_{it} = Cms_{it} - \beta E_t\left[Cms_{it+1}\right]$$

Where $Cms_{it} = \gamma E_t\left(s_{it} - s_{it+1}\right)$ is the marginal cost of storage in the dam *i* at period *t*. After simplification, the solution of λ_{it} is:

$$\lambda_{it} = \sum_{l=0}^{\infty} \beta^l \left[Cms_{it+l} - \beta E_t\left[Cms_{it+1+l}\right]\right] \qquad (30)$$

From (30), the in situ price of a unit of potential energy stored in the dam *i* at period *t* (λ_{it}) is equal to the discounted sum of the differences between the marginal cost at period *t* and the updated marginal cost at period *t+1*. Thus, in equilibrium, the producer must equalize the marginal value of water in stock at period *t* to the net marginal cost of inter-temporal transfer of electricity from period *t* to the following periods for later use. Thus, in each period *t*, hydroelectric operator does an inter-temporal trade-off between conservation and exploitation of a water's unit in the dam of region *i*. If the marginal cost of storage is constant between periods

$(Cms_{it} = Cms_i)$, the in situ price of a unit of potential energy stored in the dam i at period t also becomes constant $(\lambda_{it} = \lambda_i)$ and would be equal to the marginal cost of storage $(\lambda_i = Cms_i)$.

Taking into account the dynamic equation of storage, Euler's equation gives:

$$\beta E_t[\lambda_{it+1}] - \lambda_{it} = \beta\gamma E_t[q_{it+1} - f_{it+1}] - \gamma E_t[q_{it} - f_{it}]$$

Replacing λ_{it} and λ_{it+1} by their expression, we obtain:

$$\gamma q_{it} - \beta\gamma E_t[q_{it+1}] - \gamma E_t[f_{it}] + \beta\gamma E_t[f_{it+1}] = a(1-\beta) - (2b + A\sigma^2)(Q_t - \beta E_t[Q_{t+1}]) \tag{31}$$

for $i = 1, ..., n$. Making the sum for i, we obtain:

$$Q_t - \beta E_t[Q_{t+1}] = \frac{na(1-\beta)}{\theta} + \frac{\gamma}{\theta}\sum_{i=1}^{n} E_t[f_{it}] - \frac{\beta\gamma}{\theta}\sum_{i=1}^{n} E_t[f_{it+1}] \tag{32}$$

with $\theta = \gamma + n(2b + A\sigma^2)$. Using the properties on the delay operator L and by replacing (32) in (31), we obtain:

$$\gamma(1-\beta L^{-1})q_{it} = a(1-\beta) + \gamma E_t[f_{it}] - \beta\gamma E_t[f_{it+1}] - (2b + A\sigma^2)\left[\frac{na(1-\beta)}{\theta} + \frac{\gamma}{\theta}\sum_{i=1}^{n} E_t[f_{it}] - \frac{\beta\gamma}{\theta}\sum_{i=1}^{n} E_t[f_{it+1}]\right]$$

Using assumption of the random walk of the water inflows, the solution of q_{it} is:

$$q_{it} = \frac{a}{\gamma + n(2b + A\sigma^2)} + F_i - \frac{2b + A\sigma^2}{\gamma + n(2b + A\sigma^2)}\sum_{i=1}^{n} F_i \tag{33}$$

A.2. Derivation of the solution of the oligopoly model with an identical coefficients of risk aversion

The Lagrangian of duopoly problem is:

$$J_i = E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(a - b\left(q_{it} + \sum_{j \neq i}^{n} q_{jt} \right) \right) q_{it} - \frac{\gamma}{2} E_t \left(s_{it} - s_{it+1} \right)^2 - \left(A/2 \right) q_{it}^2 \sigma^2 + \lambda_{it} \left(s_{it} - q_{it} + f_{it} - s_{it+1} \right) \right]$$

where $\{\lambda_{it}\}$ represents the state co-variable associated with the stochastic dynamic constraint on the storage of water in the dam of the hydro producer *i*. First order conditions are:

$$q_{it} : a - \left(2b + A\sigma^2 \right) q_{it} - b \sum_{j \neq i}^{n} q_{jt} - \lambda_{it} = 0 \qquad (34)$$

$$s_{it+1} : \gamma E_t \left[s_{i,t} - s_{i,t+1} \right] - \lambda_{i,t} - \beta\gamma E_t \left[s_{i,t+1} - s_{i,t+2} \right] + \beta E_t \left[\lambda_{i,t+1} \right] = 0 \qquad (35)$$

Using recurrent equation on the dynamic of the stock, we have:

$$\gamma q_{it} - \beta\gamma E_t \left[q_{it+1} \right] = \gamma E_t \left[f_{it} \right] - \beta\gamma E_t \left[f_{it+1} \right] + \lambda_{i,t} - \beta E_t \left[\lambda_{i,t+1} \right] \qquad (36)$$

Replacing (34) in (36), we obtain :

$$q_{it} - \beta E_t \left[q_{it+1} \right] = \frac{(1-\beta) a}{\theta} + \frac{\gamma}{\theta} \left(E_t \left[f_{it} \right] - \beta E_t \left[f_{it+1} \right] \right) - \frac{b}{\theta} \sum_{j \neq i} \left(q_{jt} - \beta E_t \left[q_{jt+1} \right] \right) \qquad (37)$$

with $\theta = \gamma + 2b + A\sigma^2$. We denote by:

$$Z_{it+1} = \frac{(1-\beta) a}{\theta} + \frac{\gamma}{\theta} \left(E_t \left[f_{it} \right] - \beta E_t \left[f_{it+1} \right] \right) \qquad (38)$$

$$\begin{aligned} dq_{it+1} &= q_{it} - \beta E_t \left[q_{it+1} \right] \\ dq_{jt+1} &= q_{jt} - \beta E_t \left[q_{jt+1} \right] \end{aligned} \qquad (39)$$

Thereby, we obtain a system of reaction functions:

$$dq_{it+1} = Z_{it+1} - \frac{b}{\theta} E_t\left(\sum_{j \neq i} dq_{jt+1}\right), \forall i, \forall j, i \neq j \qquad (40)$$

Solving this system gives the following solution:

$$dq_{it+1} = \frac{\theta}{\psi}\left[\left(\theta + b\left(n-2\right)\right) Z_{it+1} - b \sum_{j \neq i} Z_{jt+1}\right] \forall i, \forall j, i \neq j \qquad (41)$$

With $\psi = \left(\theta - b\right)\left[\theta + b\left(n-1\right)\right]$

Substituting dq_{it+1}, Z_{it+1} et Z_{jt+1} by theirs expressions in (15) and using the assumption of the random walk of the water inflows, we have,

$$q_{it} - \beta E_t\left[q_{it+1}\right] = \frac{\theta}{\psi}\left[\theta + b\left(n-2\right)\right]\left[\frac{a\left(1-\beta\right)}{\theta} + \frac{\gamma\left(1-\beta\right)}{\theta} F_i\right] - b\frac{\theta}{\psi}\sum_{j \neq i}\left[\frac{a\left(1-\beta\right)}{\theta} + \frac{\gamma\left(1-\beta\right)}{\theta} F_j\right]$$

This implies:

$$q_{it} - \beta E_t\left[q_{it+1}\right] = \frac{\left(H - (n-1)b\right)\left(1-\beta\right)a}{\psi} + \frac{H\gamma\left(1-\beta\right)}{\psi} F_i - \frac{b\gamma\left(1-\beta\right)}{\psi}\sum_{j \neq i} F_j$$

With $H = \theta + b\left(n-2\right)$

Using the delay operator L, we obtain:

$$\left(1 - \beta L^{-1}\right) q_{it} = \frac{\left(H - (n-1)b\right)\left(1-\beta\right)a}{\psi} + \frac{H\gamma\left(1-\beta\right)}{\psi} F_i - \frac{b\gamma\left(1-\beta\right)}{\psi}\sum_{j \neq i} F_j$$

After simplification, we obtain:

$$q_{it} = \frac{\left(H-(n-1)b\right)a}{\psi} + \frac{H\gamma}{\psi}F_i - \frac{b\gamma}{\psi}\sum_{j\neq i}F_j$$

We replace H,ψ,θ by theirs expressions, we get the production of the equilibrium closed-loop Cournot dynamic game of the hydro producer *i*:

$$q_{it} = \frac{a\left(A\sigma^2+b+\gamma\right)+\gamma\left(A\sigma^2+nb+\gamma\right)F_i - b\gamma\sum_{j\neq i}F_j}{\left(A\sigma^2+b+\gamma\right)\left(A\sigma^2+b\left(1+n\right)+\gamma\right)}$$

A.3. Derivation of the solution of the duopoly model with different coefficients of risk aversion

The Lagrangian of the maximization problem of firm is :

$$J_i = E_0\sum_{t=0}^{\infty}\beta^t\left[\left(a-b\left(q_{it}+\sum_{j\neq i}^{n}q_{jt}\right)\right)q_{it} - \frac{\gamma}{2}E_t\left(s_{it}-s_{it+1}\right)^2 - \left(A_i/2\right)q_{it}^2\sigma^2 + E_t\lambda_{it}\left(s_{it}-q_{it}+f_{it}-s_{it+1}\right)\right]$$

The first order conditions are:

$$q_{it}: a-\left(2b+A_i\sigma^2\right)q_{it} - b\sum_{j\neq i}^{n}q_{jt} - \lambda_{it} = 0 \qquad (42)$$

$$s_{it+1}: \gamma E_t\left[s_{i,t}-s_{i,t+1}\right] - \lambda_{i,t} - \beta\gamma E_t\left[s_{i,t+1}-s_{i,t+2}\right] + \beta E_t\left[\lambda_{i,t+1}\right] = 0 \qquad (43)$$

From recurrent equation on the dynamic of the stock and (43) we have:

$$\gamma q_{it} - \beta\gamma E_t\left[q_{it+1}\right] = \gamma E_t\left[f_{it}\right] - \beta\gamma E_t\left[f_{it+1}\right] + \lambda_{i,t} - \beta E_t\left[\lambda_{i,t+1}\right] \qquad (44)$$

Substituting (42) in (44) , we obtain:

$$q_{it} - \beta E_t\left[q_{it+1}\right] = \frac{(1-\beta)a}{\theta_i} + \frac{\gamma}{\theta_i}\left(E_t\left[f_{it}\right] - \beta E_t\left[f_{it+1}\right]\right) - \frac{b}{\theta_i}\sum_{j \neq i}\left(q_{jt} - \beta E_t\left[q_{jt+1}\right]\right) \qquad (45)$$

With $\theta_i = \gamma + 2b + A_i\sigma^2$

We denote by:

$$Z_{it+1} = \frac{(1-\beta)a}{\theta_i} + \frac{\gamma}{\theta_i}\left(E_t\left[f_{it}\right] - \beta E_t\left[f_{it+1}\right]\right) \qquad (46)$$

As the flow of precipitation follow a random walk, we have:

$$Z_{it+1} = Z_i = \frac{(1-\beta)}{\theta_i}\left[a + \gamma F_i\right]$$

$$\begin{aligned} dq_{it+1} &= q_{it} - \beta E_t\left[q_{it+1}\right] \\ dq_{jt+1} &= q_{jt} - \beta E_t\left[q_{jt+1}\right] \end{aligned} \qquad (47)$$

Thus, we obtain a system of reaction functions:

$$dq_{it+1} = Z_i - \frac{b}{\theta_i}E_t\left(\sum_{j \neq i} dq_{jt+1}\right), \forall i, \forall j, i \neq j \qquad (48)$$

To simplify the analytical solution of this system of equations, we limit our analysis to the case of a hydroelectric duopoly ($n = 2$).Solving this system gives us as solution:

$$\begin{cases} dq_{it+1} = \psi_j\left[\theta_i Z_{it+1} - bZ_{jt+1}\right] \\ dq_{jt+1} = \psi_i\left[\theta_j Z_{jt+1} - bZ_{it+1}\right] \end{cases} \qquad (49)$$

With $\psi_i = \dfrac{\theta_i}{\theta_i\theta_j - b^2}$ and $\psi_j = \dfrac{\theta_j}{\theta_i\theta_j - b^2}$.

Replacing Z_{it+1} et Z_{jt+1} by theirs expressions in (49) and using the delay operator L , we obtain:

$$\left(1-\beta L^{-1}\right)q_{it}=\psi_j\left(1-\beta\right)\left[\left[a+\gamma F_i\right]-\frac{b}{\theta_j}\left[a+\gamma F_j\right]\right]$$

Whence,

$$q_{it}=\psi_j\left[\left[a+\gamma F_i\right]-\frac{b}{\theta_j}\left[a+\gamma F_j\right]\right]$$

Finally, replacing ψ_j and θ_j by theirs expressions, the electricity production at equilibrium of the Nash Cournot closed-loop game is:

$$q_{it}=\frac{\left(\gamma+b+A_j\sigma^2\right)a+\gamma\left(\gamma+2b+A_j\sigma^2\right)F_i-\gamma bF_j}{\left(\gamma+2b+A_i\sigma^2\right)\left(\gamma+2b+A_j\sigma^2\right)-b^2}$$